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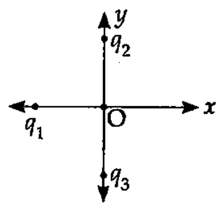
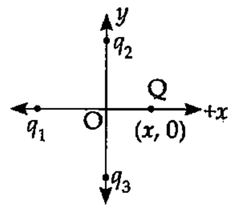
1 Electric Charges and Fields

MULTIPLE CHOICE QUESTIONS—I

Q1.1. In given figure, two positive charges q_2 and q_3 fixed along y-axis, exert a net electric force in the positive x direction on a charge q_1 fixed along the x-axis.



(a)



(b)

If a positive charge Q is added at $(x, 0)$, the force on q_1

(a) shall increase along the positive x-axis.

(b) shall decrease along the positive x-axis.

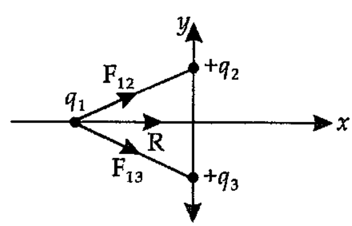
(c) shall point along the negative x-axis.

(d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3.

Main concepts used: Like forces repel and unlike forces attract and vector addition of forces.

Ans. (*a*): (*i*)As the resultant force on q_1 due to q_2 and q_3 is along positive x direction. [Given]

(*ii*) The vector sum of forces F_{12} and F_{13} is R, along positive x direction. So, F_{12} and F_{13} are attractive forces and $|F_{12}| = |F_{13}|$ also.



(*iii*) So q_1 is negative charge (as q_2, q_3 are positive charges.)

(*iv*) Charge Q $(x, 0)$ is positive, so the force due to Q will be along positive x direction.

(*v*) As the direction of forces R (due to F_{12} and F_{21} and due to Q) are along positive x-axis, so the net force on q_1 shall increase along positive x-axis.

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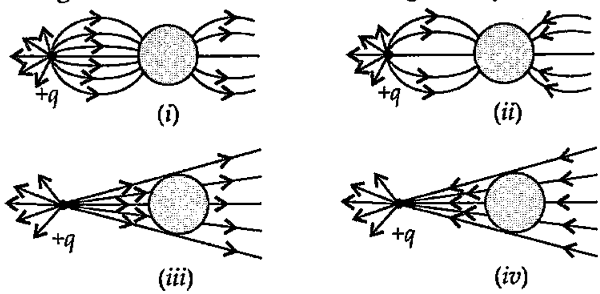
Q1.2. A point positive charge is brought near an isolated conducting sphere in given figure. The electric field is best given by



(*a*) Fig. (*i*) (*b*) Fig. (*ii*)
(*c*) Fig. (*iii*) (*d*) Fig. (*iv*)

Main concepts used: Properties of lines of forces, induction.

Ans. (*a*): As given charge is $+q$ and lines of forces in positive charge must be outward from positive charge q.

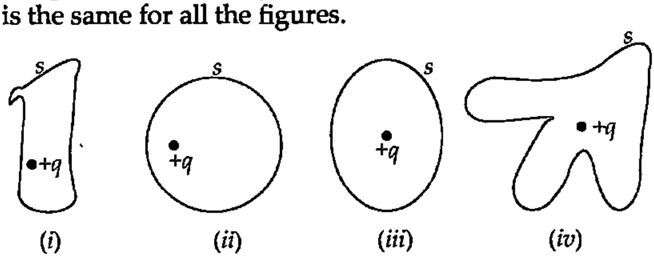
Now, as the positive charge is kept near an isolated conducting sphere, due to induction left part of sphere gets accumulated negative charge and right part positive, and lines of force from right part of sphere must emerge outward normally.

So verifies the answer (*a*).

As lines of forces are not perpendicular to the surface of sphere, so (*iii*) and (*iv*) is also rejected again.

Q1.3. The electric flux through the surface

(*a*) in fig. (*iv*) is the largest.
(*b*) in fig. (*iii*) is the least.
(*c*) in fig. (*ii*) is same as fig. (*iii*) but is smaller than fig. (*iv*).
(*d*) is the same for all the figures.



Main concept used: Gauss's law of Electrostatics.

Ans. (*d*): Electric flux (ϕ) through the closed surface (or space) depends only on the charge enclosed inside the surface.

Here, charge inside all figures are same. So electric flux will remain same. $\left[\phi = \dfrac{q}{\varepsilon_0}\right]$

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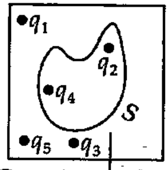
Q1.4. Five charges q_1, q_2, q_3, q_4 and q_5 are fixed at their positions as in the figure. S is a Gaussian surface. The Gauss's law is given by

$$\oint_s \mathbf{E}.d\mathbf{s} = \frac{q}{\varepsilon_0}.$$



Gaussian surface

Which of the following statements is correct?

(*a*) E on the L.H.S. of the above equation will have a contribution from q_1, q_5 and q_3, while q on the R.H.S. will have a contribution from q_2 and q_4 only.

(*b*) E on the L.H.S. of the above equation will have a contribution from all charges while q on the R.H.S. will have a contribution from q_2 and q_4 only.

(*c*) E on the L.H.S. of the above equation will have a contribution from all charges while q on the R.H.S. will have a contribution from q_1, q_3 and q_5 only.

(*d*) Both E on the L.H.S. and q on the R.H.S. will have contributions from q_2 and q_4 only.

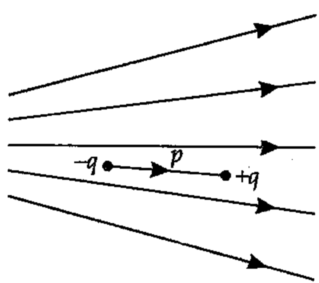
Main concepts used: Gauss's theorem, superposition of electric field.

Ans. (*b*): As all charges are positive (or of same signs) so electric field lines on R.H.S. of Gaussian surface will be due to q_2, q_3 and q_4 only.

On L.H.S. of Gaussian surface, the electric field lines on 'E' will be due to q_1q_2, q_3q_4 and q_5. So **answer** (*b*) is verified.

Q1.5. Figure here shows electric field lines in which an electric dipole $\vec{p}$ is placed as shown.

Which of the following statements is correct?



(*a*) The dipole will not experience any force.

(*b*) The dipole will experience a force towards right.

(*c*) The dipole will experience a force towards left.

(*d*) The dipole will experience a force upwards.

Main concepts used: (*i*) Magnitude of force on a charge in electric field is $\vec{F} = q\vec{E}$

(*ii*) Electric field is directly proportional to the concentration of field lines.

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Ans. (*c*): Electric field lines density decreases from left to right, so electric field intensity on $+q$ will be smaller than $-q$ charge of dipole. As $\vec{F} = q\vec{E}$, so the force on ($+q$) will be smaller than ($-q$).

The direction of force on $+q$ is along the direction of electric field, so force on $-q$ will be in left direction. So net force on dipole will be towards left **verifies the answer '*c*'.**

Q1.6. A point charge $+q$ is placed at a distance '*d*' from an isolated conducting plane. The field at a point P on the other side of the plane is

(*a*) directed perpendicular to the plane and away from the plane.
(*b*) directed perpendicular to the plane but towards the plane.
(*c*) directed radially away from the point charge.
(*d*) directed radially towards the point charge.

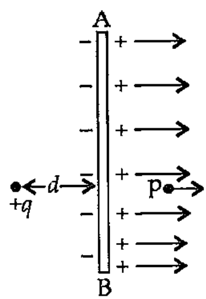
Main concepts used: (*i*) Induction; (*ii*) Electric field lines are perpendicular to the surface; (*iii*) lines of forces emerge outwards from positive charge.

Ans. (*a*): Let charge $+q$ is placed to the left of isolated conducting plane AB vertical to plane of paper.

Due to induction by $+q$ charge, R.H.S. plane will acquire positive charge.

So lines of forces will emerge perpendicularly, outward and parallel to each other.

It **verifies the answer (*a*).**



Conducting Plane

Q1.7. A hemisphere is uniformly charged positively. The electric field at a point to the diameter away from the centre is directed

(*a*) perpendicular to the diameter.
(*b*) parallel to the diameter.
(*c*) at an angle tilted towards the diameter.
(*d*) at an angle tilted away from the diameter.

Main concept used: Lines of electric field are perpendicular to the surface.

Ans. (*a*): As the side or diameter of hemisphere is plane surface, and whole hemisphere is charged with positive charge so, the electric field lines of forces emerging outward will be perpendicular to the plane surface or diameter.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q1.8. If $\oint_s E.ds = 0$ over a surface, then

(*a*) the electric field inside the surface and on it is zero.
(*b*) the electric field inside the surface is necessarily uniform.

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(c) the number of flux lines entering the surface must be equal to the number of flux lines leaving it.

(d) all charges must necessarily be outside the surface.

Main concept used: Gauss's theorem of electric flux.

Ans.(c) and (d): Flux in Gaussian surface is zero. So the net charge inside the closed surface either is zero or charges are outside the surface. If charge or charges are outside the Gaussian surface, then entering leaving lines of electric field will be equal so net flux (lines of electric field) is zero **verifies answers (c) and (d).**

Q1.9. The electric field at a point is

(a) always continuous.

(b) continuous if there is no charge at that point.

(c) discontinuous only if there is a negative charge at that point.

(d) discontinuous if there is a charge at that point.

Main concept used: Properties of lines of forces.

Ans. (b) and (d): Either positive or negative charges will interact the lines of electric field so makes the electric field discontinuous.

If there is no any charge inside the electric field then the lines will not be affected. So electric field becomes continuous. So, **answers (b) and (d) are verified.**

Q1.10. If there were only one type of charge in the universe, then

(a) $\oint_s E.ds \neq 0$ on any surface.

(b) $\oint_s E.ds = 0$ if the charge is outside the surface.

(c) $\oint_s E.ds$ could not be defined.

(d) $\oint_s E.ds = \frac{q}{\varepsilon_0}$ if charges of magnitude q were inside the surface.

Main concepts used: Gauss's theorem in Electrostatics. $\oint_s E.ds = \frac{q}{\varepsilon_0}$

Ans. (b) and (d): If a charge q is enclosed inside Gaussian surface then (d) is true.

If Gaussian surface (or space) is outside the charge $\oint_s E.ds = 0$ and (c) and (a) **are not true.** So, **answers (b) and (d) are verified.**

Q1.11. Consider a region inside which there are various types of charges, but the total charge is zero. At points outside the region,

(a) the electric field is necessarily zero.

(b) the electric field is due to the dipole moment of the charge distribution only.

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(*c*) the dominant electric field is $\propto \frac{1}{r^3}$, for large '*r*', where '*r*' is the distance of point (outside) from an origin in this region.

(*d*) the work done to move a charged particle along a closed path, away from the region, will be zero.

Main concepts used: (*i*) The Electric field due to dipole is always proportional to $\frac{1}{r^3}$.

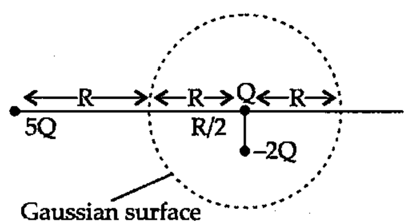
(*ii*) A matter have so many dipoles and net charge zero.

(*iii*) Electrostatic work is conservative.

Ans. (*c*) and (*d*):

Although net charge in a dipole is zero but its electric field is proportional to $\frac{1}{r^3}$. Work done against electric field is conservative, so net work done in a closed loop is always zero. So **answers (*c*) and (*d*) are verified.**

Q1.12. Refer to the arrangement of charges in given figure and a Gaussian surface of radius R with charge Q at the centre of surface. Then



(*a*) total flux through the surface of the sphere is $\frac{-Q}{\varepsilon_0}$.

(*b*) field on the surface of the sphere is $\frac{-Q}{4\pi\varepsilon_0 R^2}$.

(*c*) flux through the surface of sphere due to 5Q is zero.

(*d*) field on the surface of sphere due to –2Q is same everywhere.

Main concepts used: (*i*) Gauss's law of electrostatic, (*ii*) Electric field on a metallic sphere.

Ans. (*a*) and (*c*): (*a*) **is true** by Gauss's law. Here net charge = –2Q + Q.

(*b*) is the electric field on a conducting sphere. There is no any conducting sphere but only surface or spherical space is there.

5Q charge outside the Gaussian surface will not contribute to electric flux in Gaussian surface (or space).

For (*d*), the distance of Gaussian surface from –2Q is different, so field will not be same on the surface.

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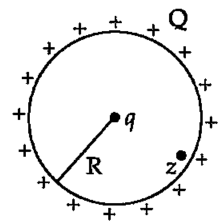
Q1.13. A positive charge +Q is uniformly distributed along a circular ring of radius R. A small test charge q is placed at the centre of ring, (Fig.). Then:

(*a*) If $q > 0$ and is displaced away from the centre in the plane of ring, it will be pushed back towards the centre.

(*b*) If $q < 0$ and is displaced away from the centre in the plane of ring, it will never return to the centre and will continue moving till it hits the ring.



(*c*) If $q < 0$, it will perform S.H.M. for small displacement along the axis.

(*d*) q at the centre of the ring is in an unstable equilibrium within the plane of the ring for $q > 0$.

Main concepts used: (*i*) Interaction between charges. (*ii*) stable and unstable equilibrium.

Ans. (*a*), (*b*), (*c*) and (*d*):

For *d*, charge is uniformly distributed along the ring. It is not sphere in which charge is only outside. So positive charge of ring will interact equally a charge placed at centre of ring but will be in unstable equilibrium.

For *c*, if q is displaced slightly (or small), it will perform S.H.M. and stops at centre.

(*a*) and **(*b*)** are verified in similar way.

VERY SHORT ANSWER TYPE QUESTIONS

Q1.14. An arbitrary surface encloses a dipole. What is the electric flux through this surface?

Main concept used: Gauss's law of electrostatics.

Ans. From Gauss's theorem, Electric flux,

$$\phi = \oint_s E.ds = \frac{q}{\varepsilon_0}$$

Net charge inside the Gaussian surface due to a dipole

$$= +q - q = 0$$

$$\therefore \quad \phi = \frac{0}{\varepsilon_0} = 0$$

Q1.15. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on (*i*) the inner surface, (*ii*) the outer surface?

Main concepts used: (*i*) Spreading the charge on a conductor, (*ii*) Induction, (*iii*) Electric field.

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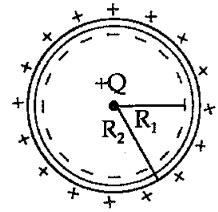
Ans. Due to induction by charge +Q at centre, the inner surface acquires –Q charge and outer surface of shell will acquire +Q charge. So, Surface charge density,

$$\sigma = \frac{Q}{A}$$

$$\sigma \text{ on outside surface} = \frac{+Q}{4\pi R_2^2}$$

$$\sigma \text{ on inside surface} = \frac{-Q}{4\pi R_1^2}.$$



Q1.16. The dimensions of an atom are of the order of an Angstrom. Thus, there must be large electric fields between protons and electrons. Why, then is the electrostatic field inside a conductor zero?

Main concepts used: (*i*) Electric field due to dipole, (*ii*) Dipole moment, $p = 2aq$ (*iii*) Coulomb's law.

Ans. As we know, net charge in atom is zero and size of atom is of the order of Angstrom. So force between electron and proton is very large by Coulomb's law.

Electric field outside the atom (or substance) will be $\propto \frac{2aq}{r^3}$.

$2a$ = Average distance between positive protons and electrons in atom.

r = Very large distance between point and atomic dipole.

$\therefore r >>> a$. So. E.F. $\to 0$.

The electric field inside surface of isolated conductor is zero.

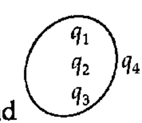
Q1.17. If the total charge enclosed by a surface is zero, does it imply that the electric field everywhere on the surface is zero?

Conversely, if the electric field everywhere on the surface is zero, does it imply that net charge inside is zero?

Main concept used: Gauss's law.

Ans. By Gauss's law of electrostatics,

$$\oint_s E.ds = \frac{q}{\varepsilon_0}.$$



Consider a Gaussian surface enclosing charges q_1, q_2, q_3 and $q_1 + q_2 + q_3 = 0$.

But electric field on R.H.S. due to q_4 will not be zero. But, on L.H.S., it will be zero. So first statement is **not true.**

If electric field on Gaussian surface is zero, then, in above case, it is possible only when $q_4 = 0$, *i.e.*, if everywhere, on Gaussian surface, electric field is zero then net charge will be zero.

Q1.18. Sketch the electric field lines for a uniformly charged hollow cylinder shown in the figure.

Main concept used: Electric lines of forces are perpendicular to the surface.

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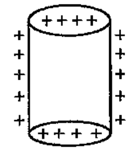
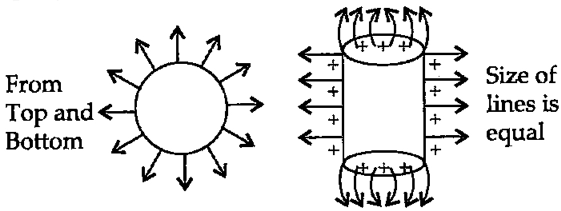
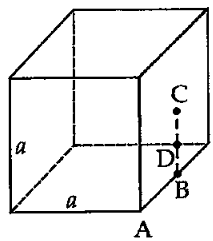
Ans. As there is no any charge inside the hollow cylinder, so not any negative charge due to induction only positive charge is spreading uniformly and lines of force emerge away perpendicularly from the surface to infinity.





Q1.19. What will be the total flux through the faces of the cube (Fig.) with side of length '*a*' if the charge *q* is placed at

(*a*) A; a corner of the cube.
(*b*) B; mid-point of an edge of the cube.
(*c*) C; centre of a face of cube.
(*d*) D; the mid-point of B and C.



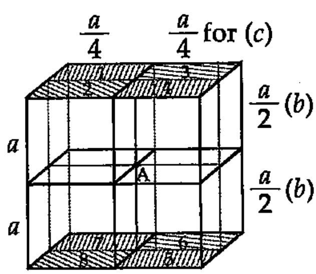
$DC = DB = \frac{a}{4}$

Main concepts used: (*i*) Gauss's law.(*ii*) Imagine the number of **cubes** so that position of charge (*q*) becomes symmetric to whole figure. (*iii*) Get the charge by dividing the given charge by number of cubes (or given figure).

Ans. (*a*) To make charge *q* at A symmetric to identical cube, there will be 8 identical cubes as shown in figure. So charge distribution for 1 cube is:

$$Q_1 = \frac{q}{8 \times 1} = \frac{q}{8}$$

$\therefore$ Total flux by Gauss's law $= \frac{q}{8\,\varepsilon_0}$



(*b*) To make charge *q* at B symmetric, we can consider the 8 identical cubes of side $\frac{a}{2}$ as shown in figure given below. So charge distribution for one cube is:

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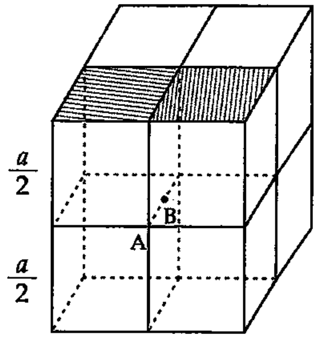
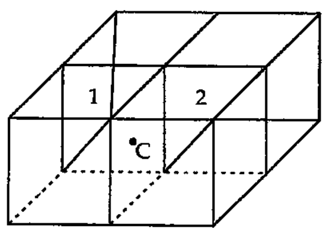
$$Q_2 = \frac{q}{8 \times \frac{1}{2}} = \frac{q}{4}$$



Total flux by Gauss's law = $\frac{q}{4\,\varepsilon_0}$

(*c*) Point 'C' is symmetric to two cubes as shown in figure, so charge distribution for one cube $Q_3 = \frac{q}{8 \times \frac{1}{4}} = \frac{q}{2}$

Total flux across cube by Gauss's law = $\frac{q}{2\,\varepsilon_0}$



(*d*) As the charge q is placed at D, the mid-point of BC, the charge is shared by 2 identical cubes, so the charge for each cube = $\frac{q}{2}$

Total electric flux by Gauss's law = $\frac{q}{2\,\varepsilon_0}$.

SHORT ANSWER TYPE QUESTIONS

Q1.20. A paisa coin is made up of Al-Mg alloy and weighs 0.75 g. It has a square shape and its diagonal measures 17 mm. It is electrically neutral and contains equal amounts of positive and negative charges.

Treating the paisa coins made up of only Al, find the magnitude of equal number of positive and negative charges. What conclusion do you draw from this magnitude?

Main concepts used: (*i*) Avogadro's number, (*ii*) Mole concept, (*iii*) Atomic structure.

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Ans. A 0.75 g paisa coin now is made with Al only. So,

the mass of a paisa coin = 0.75 g

Atomic mass of Al ≅ 27 amu

So, number of moles in 0.75 g = $\frac{0.75}{27}$ mole

Number of Al atoms in coin = N

$$= \frac{0.75}{27} \times 6.022 \times 10^{23} \text{ atoms}$$

Atomic number of Al = 13

∴ Number of electron (negative charge) and proton (positive charge) = 13.

So, number of either proton or electron in a coin

$$= \frac{13 \times 0.75}{27} \times 6.022 \times 10^{23}$$

Magnitude of charge on a proton or electron = 1.6×10^{-19} C

So, charge either positive or negative

$$= \frac{13 \times \overset{25}{\cancel{75}} \times 6.022 \times 10^{23} \times 1.6 \times 10^{-19} \text{C}}{\underset{9}{\cancel{27}} \times 100}$$

$$= \frac{13 \times 25 \times 6.022 \times 1.6 \times 10^{23-19-2}}{9}$$

$$= \frac{3131.44 \times 10^{21-19}}{9} = 347.9 \times 10^{2} \text{ C}$$

$$= 3.48 \times 10^{4} \text{ C}$$

Either positive or negative charge on a coin = 34.8 kC

It concludes that even a 0.75 g Al contains anormous amount of positive and negative charges and equal in magnitude.

Q1.21. Consider a coin of question 1.20. It is electrically neutral and contains equal amounts of positive and negative charge of magnitude 34.8 kC. Suppose that these equal charges were concentrated in two point charges separated by:

(*i*) 1 cm (~ 1/2 × diagonal of one paisa coin), (*ii*) 100 m (~ length of a long building), (*iii*) 10^6 m (radius of earth). Find the force on each such point charge in each case. What do you conclude from these results?

Main concept used: Coulombian force.

Ans. Here charges are equal and opposite. So, by Coulomb's law, force of attraction between charges,

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

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$$q_1 = q_2 = 34.8 \text{ kC}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N-m}^2/\text{C}^2$$

$$F = \frac{34.8 \times 10^3 \times 34.8 \times 10^3 \times 9 \times 10^9}{r^2}$$

$$= \frac{34.8 \times 34.8 \times 9 \times 10^{9+3+3}}{r^2}$$

$$= \frac{10899.36 \times 10^{15}}{r^2} \text{ N} \cong \frac{1.1 \times 10^{19}}{r^2} \text{ N}$$

$$\boxed{F = \frac{1.1 \times 10^{19}}{r^2} \text{ N}}$$

(*i*) $r_1 = 1 \text{ cm} = 0.01 \text{ m}$

$\therefore$ $F_1 = \dfrac{1.1 \times 10^{19}}{0.01 \times 0.01} = 1.1 \times 10^{19+4}$

$= \mathbf{1.1 \times 10^{23}}$ **N** towards the charges.

(*ii*) $r_2 = 100 \text{ m}$

$\therefore$ $F_2 = \dfrac{1.1 \times 10^{19}}{100 \times 100} = 1.1 \times 10^{19-4}$

$= \mathbf{1.1 \times 10^{15}}$ **N** towards the charges.

(*iii*) $r_3 = 10^6 \text{ m}$

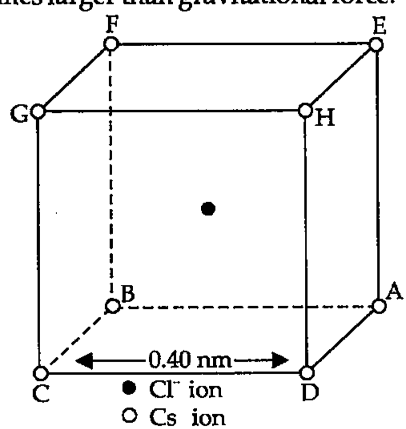
$\therefore$ $F_3 = \dfrac{1.1 \times 10^{19}}{10^6 \times 10^6} = 1.1 \times 10^{19-6-6}$

$= \mathbf{1.1 \times 10^{7}}$ **N** towards the charges.

This electrostatic force varies from order 10^7 N to 10^{23} N. The minimum force 10^7 is equivalent to the force of attraction between earth and 1 million kg body which is too much high.

So electrostatic force is so many times larger than gravitational force.

Q1.22. Figure shows a crystal unit of cesium chloride, CsCl. The cesium atoms represented by open circles are situated at the corners of a cube of side 0.40 nm, whereas a chlorine atom is situated at the centre of the cube. The cesium atoms are deficient by one electron while the Cl atom carries an excess electron.



(*i*) What is the net electric field (force) on Cl atom due to eight Cs atoms?

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(*ii*) Suppose that the Cs atom at the corner A is missing. What is the net force on the Cl atom due to seven remaining Cs atoms?

Main concept used: Electric force is vector quantity.

Ans. The charge on Cs atom = $+e$

The charge on Cl atom = $-e$

The distance of Cl^- ion from any Cs^+ ion

$$= \frac{1}{2} \text{ diagonal of cube of side } l$$

$$r = \frac{1}{2}\sqrt{3l^2}$$

$$= \frac{1}{2}\sqrt{3 \times 0.4 \times 10^{-9} \times 0.4 \times 10^{-9}} \text{ m}$$

$$= \frac{1}{2} \times 0.4 \times 10^{-9} \times \sqrt{3}$$

$$\boxed{r = 0.2\sqrt{3} \times 10^{-9} \text{ m}}$$

(*i*) As the distance of Cl^- ion from Cs^+ is equal and charge on each Cs^+ ion is same, so electrostatic force due to Cs^+ ion at A and D will be equal in magnitude and opposite in direction. So by $F = \frac{kq_1q_2}{r^2}$ net force on Cl^- due to Cs^+ ion will be zero, as atoms of Cs attracts the Cl^- equally in opp. direction with pairs diagonally *i.e.,* (B, H), (C, E), (D, F).

(*ii*) As the Cs^+ ion at A is missing so net force on Cl^- ion will be only due to its' opposite Cs^+ ion and other forces will be cancelled out.

Net force on Cl^- ion when a Cs^+ ion from A is removed,

$$= \frac{kq_1q_2}{r^2} \qquad \left(k = \frac{1}{4\pi\varepsilon_0}\right)$$

$$r = 0.2\sqrt{3} \times 10^{-9} \text{ m and } |q_1| = |q_2| = |e|$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\therefore \qquad F = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{0.2\sqrt{3} \times 10^{-9} \times 0.2\sqrt{3} \times 10^{-9}}$$

$$= \frac{9 \times 16 \times 16 \times 10^{-38+9}}{2 \times 2 \times 3 \times 10^{-18}}$$

$$= 3 \times 16 \times 4 \times 10^{-29+18} = 192 \times 10^{-11}$$

Force on Cl^- ion F = 1.92×10^{-9} N towards Cl^- ion

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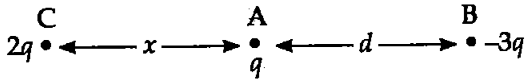
Q1.23. Two charges q and $-3q$ are placed fixed on x-axis separated by distance 'd'. Where should a third charge '$2q$' be placed such that it will not experience any force?

Main concepts used: (*i*) Coulomb's law (*ii*) Vector properties of addition.

Ans.



(*i*) If we place the third charge $2q$ between A and B the direction of the force on $2q$ due to A and B on C will be same.

So the net force cannot be zero, so the charge q cannot be placed between A and B.

(*ii*) If $2q$ is placed right side of A then $r_{AC} > r_{BC}$ as $q_A < q_B$. So $F_{CA} < F_{CB}$ always as the direction of F_{CA} is towards right and F_{CB} is left so $F_{CA} + F_{CB} \neq 0$ we cannot obtain required condition.

(*iii*) Now consider $2q$ at C to the left of q at distance x from q.

Force on $2q$ at C (left of q) is in opposite direction so net force will be zero if magnitude is equal so,

$$F_{CA} + F_{CB} = 0 \text{ or } F_{CA} = -F_{CB}$$

$$\frac{Kq_C q_A}{r^2_{CA}} = \frac{-Kq_C q_B}{r^2_{CB}}$$

$$\Rightarrow \quad \frac{2q.q}{x^2} = \frac{-2q(-3q)}{(x+d)^2}$$

$$\Rightarrow \quad \frac{2q^2}{x^2} = \frac{6q^2}{(x+d)^2} \Rightarrow \frac{1}{x^2} = \frac{3}{(x+d)^2}$$

$$\Rightarrow \quad 3x^2 = x^2 + d^2 + 2xd$$

$$3x^2 - x^2 - 2xd - d^2 = 0$$

$$2x^2 - 2xd - d^2 = 0$$

$$x = \frac{+2d \pm \sqrt{(-2d)^2 - 4.2.(-d^2)}}{2.2}$$

$$\Rightarrow \quad x = \frac{+2d \pm \sqrt{4d^2 + 8d^2}}{4} = \frac{2d \pm 2d\sqrt{3}}{4}$$

$$\Rightarrow \quad x = \frac{2\left(d \pm d\left(\sqrt{3}\right)\right)}{4\,2} = \frac{d\left(1 \pm \sqrt{3}\right)}{2}$$

$$x = \frac{d}{2}\left(1+\sqrt{3}\right) \text{ m to the left of } q.$$

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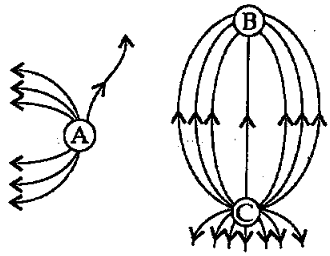
Q1.24. Figure shows the electric field lines around three point charges A, B and C.

(*a*) Which charges are positive?

(*b*) Which charge has the largest magnitude? Why?

(*c*) In which region or regions of the picture could the electric field be zero? Justify your answer. (*i*) near A (*ii*) near B (*iii*) near C (*iv*) nowhere.



Main concepts used: (*i*) Properties of electric lines of force (*ii*) neutral point.

Ans. (*a*) Electric lines of forces emerge out from positive charge. So, A and C are positive charges.

(*b*) As the density of electric lines of forces from a charge increases, the intensity or of electric field or magnitude of charge increases. So the magnitude of charge C is maximum.

(*c*) The neutral point lies between two like charges. At neutral point force acting is zero, or no electric lines of force. As the lines of force repel each other sideways and the lines of force emerging from A and C are of same kind so repel each other and there will be no any electric lines of force at a point between A and C.

As the magnitude of charge C is larger than A so lines of force of C will be stronger than of A.

So, the Neutral point lies near A

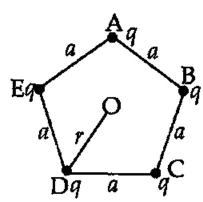
So, option (*i*) is the correct answer.

Q1.25. Five charges, q each are placed at the corners of a regular pentagon of side 'a' as shown in figure.

(*a*) (*i*) What will be the electric field at O, the centre of the regular pentagon?

(*ii*) What will be the electric field at O, if the charge from one of the corners (say A) is removed?

(*iii*) What will be the electric field at O if the charge q at A is replaced by $-q$?



(*b*) How would your answer to (*a*) be affected if pentagon is replaced by n– sided regular polygon with charge q at each of its corners?

Main concepts used: (*i*) Properties of electric field (*ii*) concept of Geometry of regular polygon.

Ans. (*a*) (*i*) Point O is symmetric to all the five equal charges of $+q$ placed at vertices of regular polygon. So net electric field will be zero at O.

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(*ii*) When a charge q from A is removed from symmetric charge distribution about 'O', the net electric field due to rest of the charges will be equal in magnitude as there is $-q$ charge at 'A'. So Electric field at 'O' in this case

$$E = \frac{-1q}{4\pi\varepsilon_0 r^2}$$

The direction of E is from O to A.

(*iii*) When charge q from A is removed and $-q$ charge is placed then electric field at O

$$E_2 = \frac{-q}{4\pi\varepsilon_0 r^2}.$$

When a new charge $-q$ is placed the E.F. will increase by $\frac{-q}{4\pi\varepsilon_0 r^2}$.

Now resultant becomes $= E_3 = E_2 + \frac{-q}{4\pi\varepsilon_0 r^2} = \frac{-q}{4\pi\varepsilon_0 r^2} + \frac{-q}{4\pi\varepsilon_0 r^2}$

$$\boxed{E_3 = \frac{-2q}{4\pi\varepsilon_0 r^2}}$$

(*b*) Now when pentagon is replaced by n – sided polygon

(*i*) electric field at 'O' will be again zero as all charge distribution about O is symmetric as in case of electric field at the centre of conducting ring or shell.

(*ii*) electric field at O will remain same as in a(*ii*) if a charge q is removed. The resultant will be equal to the electric field due to charge $-q$ at A.

So electric field at 'O' at the centre of regular polygon of n side if a charge from one vertex is removed is equal to $E_2 = \frac{-q}{4\pi\varepsilon_0 r^2}$. The direction from O is opposite to OA.

(*iii*) If charge $-q$ is placed at one vertex after removing $+q$ from there then resultant electric field at 'O' will be due to charge $(-2q)$ at A.

So the net electric field at O after removing q from A by placing $-q$ at A.

$$E = \frac{-q}{4\pi\varepsilon_0 r^2} - \frac{q}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{-2q}{4\pi\varepsilon_0 r^2} \text{ from O to A.}$$

LONG ANSWER TYPE QUESTIONS

Q1.26. In 1959 Lyttleton and Bondi suggested that the expansion of the Universe could be explained if matter carried a net charge. Suppose that the Universe is made up of hydrogen atoms with a

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number density N which is maintained a constant. Let the charge on the proton be: $e_p = -(1+y)\,e$, where e is the electronic charge.

(*a*) Find the critical value of y such that expansion may start.

(*b*) Show that the velocity of expansion is proportional to the distance from the centre.

Main concepts used: (*i*) Expansion will start if gravitational force (F_G) between H atoms is smaller than Coulombian repulsive force (F_C), (*ii*) Atom has equal no. of proton and electron, (*iii*) for critical value of $y \rightarrow F_C = F_G$.

Ans. (*a*) Consider that the Universe is spherical with radius R made up of H atoms.

$$\text{Charge on proton} = -(1+y)\,e$$

$$\text{So the total charge on a H atom} = e_p + e$$

$$= -(1+y)\,e + e$$

$$= [-1 - y + 1]\,e$$

$$\text{Charge on 1 H atom} = -ye$$

$$\text{Number of H atoms in spherical Universe} = \text{N.V.}$$

$$= \text{N}.\frac{4}{3}\pi\text{R}^3$$

$$\therefore \text{The net charge in Universe} = \frac{4}{3}\pi\text{NR}^3.ye$$

Consider the boundaries of Universe as Gaussian surface then by Gauss's law of electrostatics,

$$\oint_s \text{E}.ds = \frac{q}{\varepsilon_0}$$

$$\text{E}.\ 4\pi\text{R}^2 = \frac{-4\pi\text{NR}^3 ye}{3\varepsilon_0}$$

$$\Rightarrow \qquad \text{E} = \frac{-4\pi\text{NR}^3 ye}{3\varepsilon_0.4\pi\text{R}^2} = \frac{-\text{NR}ye}{3\varepsilon_0}$$

Electrostatic force acting on one H atom $F_C = q\text{E}$

$$F_C = \frac{(-ye)(-\text{NR}ye)}{3\varepsilon_0} = \frac{+y^2e^2\text{N.R.}}{3\varepsilon_0}$$

Positive sign of F_C shows repulsive force.

$$\text{Gravitational potential at boundary of Universe} = \frac{\text{GM}}{\text{R}^2}$$

M = mass of Universe (or all H atoms)

So, the gravitational force acting on a H atom

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at boundary of Universe = Gravitational potential × m

[m = mass of H-atom]

$$F_G = \frac{GM}{R^2} m_H$$

Mass of 1 H atom = mass of a proton

$= m_p$

$\therefore$ Mass of Universe = No. of H atoms in Universe × m_p

$$= N.\frac{4}{3}\pi R^3 . m_p$$

$$\therefore \qquad F_G = \frac{G\left(N\frac{4}{3}\pi R^3\right)m_p^2}{R^2}$$

$$F_G = \frac{4\pi GNR\, m_p^2}{3}$$

If $F_C > F_G$, then universe will start to expand. So for critical value of expansion

$$F_C = F_G$$

$$\frac{y^2e^2NR}{3\varepsilon_0} = \frac{4\pi\, GNR\, m_p^2}{3}$$

$$\Rightarrow \qquad y^2 = \frac{3\varepsilon_0 4\pi\, GNR\, m_p^2}{3e^2\, NR} = 4\pi\varepsilon_0 G\left(\frac{m_p}{e}\right)^2$$

$$\Rightarrow \qquad y^2 = \frac{6.67\times10^{-11}}{9\times10^9}\left(\frac{1.66\times10^{-27}}{1.6\times10^{-19}}\right)^2$$

$$\Rightarrow \qquad y^2 \cong \frac{6.67\times10^{-20}}{9}[10^{-27+19}]^2$$

$$= 0.741 \times 10^{-20} \times (10^{-8})^2$$

$$\Rightarrow \qquad y^2 = 74.1 \times 10^{-22} \times 10^{-16} \Rightarrow y = \sqrt{74.1\times10^{-38}}$$

$$y = 8.6 \times 10^{-19} \cong 10^{-18}$$

So critical value of y is of the order of 10^{-18} so that Universe start to expand.

(*b*) For expansion repulsive force F_C must be greater than, attractive gravitational force

So net force on H atom to expand

$= F_C - F_G$

$$F_H = \frac{y^2e^2NR}{3\varepsilon_0} - \frac{4\pi}{3} GNR\, m_p^2$$

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This force F_H will produce acceleration in H atom

$$\therefore \qquad F_H = m_p \frac{d^2R}{dt^2}$$

Here R (size of Universe) changes with time as Universe expands with velocity

$$\therefore \qquad m_p \frac{d^2R}{dt^2} = \left[\frac{Ny^2e^2}{3\varepsilon_0} - \frac{4\pi GNm_p^2}{3}\right]R$$

$$\frac{d^2R}{dt^2} = \frac{1}{m_p}\left[\frac{Ny^2e^2}{3\varepsilon_0} - \frac{4\pi GNm_p^2}{3}\right]R$$

As N, y, e, ε_0 π, G, m_p are constants so taking a new constant α^2 such that

$$\alpha^2 = \frac{1}{m_p}\left[\frac{Ny^2e^2}{3\varepsilon_0} - \frac{4\pi GNm_p^2}{3}\right]$$

$$\therefore \qquad \frac{d^2R}{dt^2} = \alpha^2 R$$

It is differential equation of order 2. Its solution is

$$R = Ae^{\alpha t} + Be^{-\alpha t}$$

For expansion of Universe B = 0

$$\therefore \qquad R = Ae^{\alpha t} \qquad ...(I)$$

Receding velocity of Universe $v = \frac{dR}{dt}$

$$v = A\alpha e^{\alpha t} \quad or \quad v = R\alpha$$

as α is constant. So the receding (expanding) velocity of Universe is directly proportional to the distance of matter (H atom) from centre of Universe.

Q1.27. Consider a sphere of radius R with charge density distributed as

$$\rho(r) = kr \text{ (for } r \leq R)$$
$$\rho(r) = 0 \text{ (for } r > R)$$

(*a*) Find the electric field at all points r.

(*b*) Suppose the total charge on the sphere is $2e$ where e is electron charge. Where can two protons be embedded such that the force on each of them is zero. Assume that the introduction of the proton does not alter the negative charge distribution.

Main concept used: Gauss's law of electrostatics.

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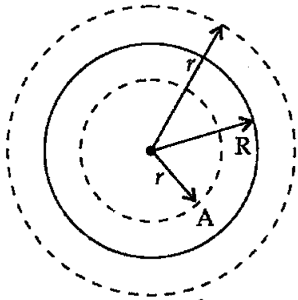
Ans. (*a*) Consider a sphere (solid) of radius R with charge density $\rho(r) = kr$ (for $r \leq R$) and $\rho(r) = 0$ (for $r > R$)



Case-I: Consider the Gaussian surface at radius $r < R$. Applying Gaussian law at A

$$v = \text{Volume of Gaussian surface}$$

$$\therefore \quad q = \rho(r) \cdot \frac{4}{3}\pi r^3$$

$$\oint_s E.ds = \frac{1}{\varepsilon_0}\rho(r).dv$$

$$v = \frac{4}{3}\pi r^3$$

$$dv = \frac{4}{3}\pi\, 3r^2\, dr = 4\pi r^2\, dr$$

$$\rho(r) = kr \text{ for } r < R$$

$$\oint_s E.ds = \int_0^r \frac{kr.4\pi r^2 dr}{\varepsilon_0}$$

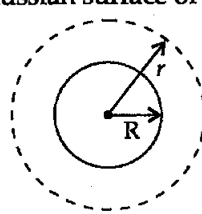
$$E4\pi r^2 = \frac{4\pi k}{\varepsilon_0}\int_0^r r^3 dr$$

$$E = \frac{4\pi k}{4\pi r^2 \varepsilon_0} \cdot \frac{r^4}{4}$$

$$\boxed{E = \frac{kr^2}{4\varepsilon_0}}$$

As field is positive so direction of E is radially outward.

Case-II: Consider a Gaussian surface of radius $r > R$



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As charge inside the Gaussian surface is upto $r = 0$ to $r = R$ remains same as earlier. So net charge in this Gaussian surface

$$q = \rho(r).dv$$

$$q = \int_0^R kr\ 4\pi r^2 dr$$ as charge reside upto radius R only

By Gaussian law

$$\oint_{s=4\pi r^2} E.ds = \int_0^R \frac{4\pi k r^3 dr}{\varepsilon_0}$$

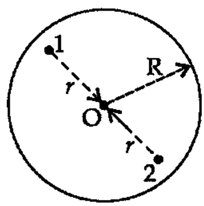
$$E.4\pi r^2 = \frac{4\pi k}{\varepsilon_0}\left[\frac{r^4}{4}\right]_0^R$$

$$E = \frac{4\pi k}{4\varepsilon_0}\frac{(R^4 - 0)}{(4\pi r^2)}$$

$$\boxed{E(r) = \frac{k}{4\varepsilon_0}\cdot\frac{R^4}{r^2}}$$

The direction of E.F. is outward radially.

(*b*) As the total negative charge on sphere is $2e$ (*e* is charge on electron) is distributed in sphere of radius R symmetrically. So two protons must be symmetrical in sphere. *i.e.*, two protons must be on the opposite sides equidistant from the centre as shown in fig.



Charge on sphere $q = \rho(r)dv$

From last part (*a*) $dv = 4\pi r^2.dr$

$$\therefore \quad q = \int_0^R (kr)\ 4\pi r^2 dr = 4\pi k\cdot\frac{R^4}{4}$$

$$2e = \pi k R^4$$

$$\Rightarrow \quad k = \frac{2e}{\pi R^4} \quad ...(I)$$

Protons 1 and 2 shown in fig. are embedded at distance r from centre O, of sphere, thus force of attraction between a proton and negative charge distribution in sphere as E is due to (–) charge. E.F due to charge distribution inside the charge sphere at $r < R$ from part (*a*) is

$$E = \frac{kr^2}{4\varepsilon_0}$$

$$F_1 = -eE = \frac{-e\cdot kr^2}{4\varepsilon_0}$$

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Repulsive force on proton 1 due to proton 2 = F_2 by Coulomb's law

$$F_2 = \frac{e^2}{4\pi\varepsilon_0(2r)^2}$$

Net force on proton 1, $F = F_1 + F_2 = 0$

$$\Rightarrow \quad F = \frac{-ekr^2}{4\varepsilon_0} + \frac{e^2}{4\pi\varepsilon_0 4r^2} = 0$$

$$\Rightarrow \quad \frac{e^2}{4\pi\varepsilon_0 4r^2} = \frac{ekr^2}{4\varepsilon_0}$$

$$\Rightarrow \quad r^4 = \frac{4\varepsilon_0 e^2}{4\pi\varepsilon_0 4.ek}$$

$$\Rightarrow \quad r^4 = \frac{e}{4\pi k}$$

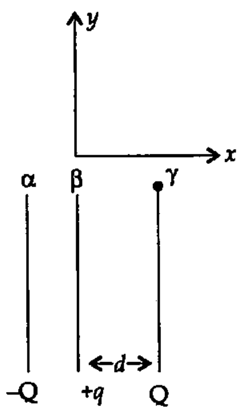
$$\Rightarrow \quad r^4 = \frac{e\,\pi R^4}{4\pi \cdot 2e} \qquad \left(\because k = \frac{2e}{\pi R^4} - \text{from I}\right)$$

$$\Rightarrow \quad r^4 = \frac{R^4}{8}$$

$$\Rightarrow \quad r = \frac{R}{(8)^{1/4}}$$

So protons must be embedded at a distance of $R/(8)^{1/4}$ from the centre of sphere of radius R.

Q1.28. Two fixed identical conducting plates (α & β), each of surface area S are charged to –Q and q, respectively, where $Q > q > 0$. A third identical plate (γ), free to move is located on the other side of the plate with charge Q at a distance d as shown in fig. The third plate is released and collides with the plate β. Assume the collision is elastic and the time of collision is sufficient to redistribute charge amongst β and γ.



(*a*) Find the electric field acting on the plate γ before collision.

(*b*) Find the charges on β and γ plates after collision.

(*c*) Find the velocity of plate γ after collision and at a distance d from the plate β.

Main concepts used: Electric field of infinite plate of charge density $= \dfrac{\sigma}{2\varepsilon_0}$

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(*i*) WD = F.d. (*ii*) KE = $\frac{1}{2}mv^2$

Ans. (*a*) As the plate γ has Q charge. So electric field on plate γ due to plate α = $\frac{\sigma}{2\varepsilon_0} = \frac{-Q}{S2\varepsilon_0}$

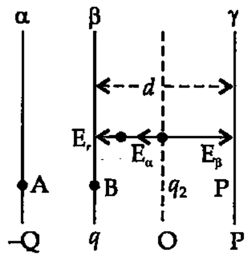
$$E_\alpha = \frac{-Q}{2S\varepsilon_0} \text{ towards left}$$

Similarly, electric field on γ due to plate β = $E_\beta = \frac{+q}{2S\varepsilon_0}$ towards right.
So this net electric field on plate γ = E = $F_\alpha + E_\beta$

$$E = \frac{-Q}{2S\varepsilon_0} + \frac{q}{2S\varepsilon_0}$$

$$E = \frac{1}{2S\varepsilon_0}[q - Q] \text{ towards left as } (Q > q)$$

So, the electric field on plate γ before collision is towards left as $(|Q| > |q|) = \frac{1}{2S\varepsilon_0}[q - Q]$.



(*b*) On collision between plate β and γ their potential becomes same. Suppose during collision at any point P between plates β and γ the charges on β and γ plates are q_1 and q_2 respectively. Consider a point O to the right of plate B and left to plate γ such that net electric field at O is zero.

Electric field due to plate α at O

$$= \frac{-Q}{2S\varepsilon_0} \text{ (towards left)}$$

Electric field at O due to plate β = $\frac{+q_1}{2S\varepsilon_0}$ (towards right)

Electric field at O due to plate γ = $\frac{-q_2}{2S\varepsilon_0}$ (towards left)

$\because |-Q| > |q|$

Net electric field at O must be zero so,

$$\frac{-Q}{2S\varepsilon_0} + \frac{-q_2}{2S\varepsilon_0} + \frac{q_1}{2S\varepsilon_0} = 0$$

or $$Q = q_1 - q_2 \quad \text{... I}$$

As there is no loss of charge on collision by law of conservation of charges so,

$$+Q + q = q_1 + q_2 \quad \text{... II}$$

$$Q = q_1 - q_2 \quad \text{[From I]}$$

$$q = 2q_2 \quad \text{[Subtract I from II]}$$

or $$q_2 = \frac{q}{2}$$

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So, charge on plate γ after collision $= \frac{q}{2}$ unit.

Charge on plate $\beta = Q + q - \frac{q}{2}$

Charge on plate $\beta = \left(Q - \frac{q}{2}\right)$

(*c*) After collision of plate γ with α, after charge distribution between plate γ and β, the plates will repel each other and plate γ will move towards its initial position as plate γ is free to move but plate α, β are fixed.

Let the velocity of plate γ after collision at distance d is v and mass of plate is m then gain in KE round the trip from P to B and B to P must be equal to the work done by electric field.

After collision the electric field on plate γ at O, due to plate at α and β

$$= \frac{-Q}{2S\varepsilon_0} + \frac{Q + q/2}{2\varepsilon_0 S}$$

$$E_2 = \frac{q/2}{2\varepsilon_0 S} \text{ towards right}$$

Electric field on plate γ just before collision due to plate α, and β

$$E_1 = \frac{-Q}{2\varepsilon_0 S} + \frac{q}{2\varepsilon_0 S}$$

$$E_1 = \frac{-Q + q}{2\varepsilon_0 S}$$

Force on plate γ just before collision $= -E_1 Q$

$$\Rightarrow \quad F_1 = \frac{(-Q + q)(-Q)}{2\varepsilon_0 S} = \frac{(Q - q)Q}{2\varepsilon_0 S}$$

$$\Rightarrow \quad F_2 = E_2 \cdot \frac{q}{2} = \frac{\frac{q}{2} \cdot \frac{q}{2}}{2S\varepsilon_0} = \frac{\left(\frac{q}{2}\right)^2}{2\varepsilon_0 S}$$

$$W = (F_1 + F_2)d$$

$$= \left[\frac{(Q - q)Q}{2\varepsilon_0 S} + \frac{\left(\frac{q}{2}\right)^2}{2\varepsilon_0 S}\right] d = \frac{\left[Q^2 - qQ + \frac{q^2}{4}\right] d}{2\varepsilon_0 S}$$

$$= \frac{\left[Q^2 + \left(\frac{q}{2}\right)^2 - 2 \cdot Q \cdot \frac{q}{2}\right] d}{2\varepsilon_0 S}$$

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$$W = \frac{\left(Q - \frac{q}{2}\right)^2 d}{2\varepsilon_0 S} = KE$$

$$\therefore \quad \frac{1}{2}mv^2 = \frac{\left(Q - \frac{q}{2}\right)^2 .d}{2\varepsilon_0 S} \Rightarrow v^2 = \frac{2d\left(Q - \frac{q}{2}\right)^2}{2\varepsilon_0 Sm}$$

$$v = \left(Q - \frac{q}{2}\right)\sqrt{\left(\frac{d}{\varepsilon_0 S.m}\right)}$$ is the velocity of plate γ at a distance d after collision.

Q1.29. There is another useful system of units, besides the SI/mks. A system, called the cgs (centimeter-gram-second) system. In this system Coulomb's law is given by

$$F = \frac{Qq}{r^2}\hat{r}$$

where the distance r is measured in cm ($= 10^{-2}$m), F in dynes ($= 10^{-5}$ N) and the charges in electrostatic units (*es* units), where 1es unit of charge $= \frac{1}{[3]} \times 10^{-9}$ C

The number [3] actually arises from the speed of light in vaccum which is now taken to be exactly given by $c = 2.99792458 \times 10^8$ m/s. An approximate value of c then is $c = [3] \times 10^8$ m/s.

(*i*) Show that the Coulomb law in cgs units yields
1 esu of charge = 1(dyne)½ cm.
Obtain the dimensions of units of charge in terms of mass M, length L and time T. Show that it is given in terms of fractional powers of M and L.

(*ii*) Write 1 esu of charge = χ Coulomb, where χ is a dimensionless number. Show that this gives:

$$\frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{\chi^2}\frac{Nm^2}{C^2}$$

With $$\chi = \frac{1}{[3]} \times 10^{-9},$$

We have

$$\frac{1}{4\pi\varepsilon_0} = [3]^2 \times 10^9 \frac{Nm^2}{C^2}$$

or $$\frac{1}{4\pi\varepsilon_0} = (2.99792458)^2 \times 10^9 \frac{Nm^2}{C^2} \text{ (Exactly)}$$

Main concept used: Homogeneity of a formula in Dimensions.

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Ans. (*i*) $F = \dfrac{Qq}{r^2}$

$$1 \text{ dyne} = \frac{[1 \text{ esu}]^2}{(1 \text{ cm})^2}$$

$$1 \text{ esu} = \text{cm}\sqrt{\text{dyne}}$$

$$1 \text{ esu} = L^1 \; F^{1/2} = L^1 \quad [MLT^{-2}]^{1/2}$$

$$1 \text{ esu} = M^{1/2} \; L^{3/2} \, T^{-1}$$

So esu of charge is represented in terms of fractional powers of $\frac{1}{2}$ of M, $\frac{3}{2}$ of L and (–1) of T

(*ii*) 1 esu = χC [Given]

Where, χ is dimensionless number. Coulombian force between two charges each of magnitude 1 esu separated by 1 cm is one dyne = 10^{-5}N. This situation is equivalent to two charges of magnitude χC separated by 10^{-2} m. By Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \text{ or } \frac{1}{4\pi\varepsilon_0} = \frac{F \cdot r^2}{q_1 q_2} \qquad \text{... I}$$

$$q_1 = q_2 = \chi C$$

$$r = 1 \text{ cm} = 10^{-2} \text{ m}$$

If $F = 1$ dyne,

From I, $$\frac{1}{4\pi\varepsilon_0} = \frac{1 \text{ dyne } (1 \text{ cm})^2}{\chi \; \chi}$$

$$= \frac{10^{-5}\text{N } (10^{-2}\text{m})^2}{\chi^2 \; C^2}$$

$$\frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{\chi^2}\,\frac{\text{N-m}^2}{C^2}$$

$$\chi = \frac{1}{[3] \times 10^9}$$

$\therefore$ $$\frac{1}{4\pi\varepsilon_0} = 10^{-9} \times [3]^2 \times (10^9)^2 \, \frac{\text{N-m}^2}{C^2}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \, \frac{\text{N-m}^2}{C^2}$$

If [3] → 2.99792458 we get

$$\frac{1}{4\pi\varepsilon_0} = 8.98755 \times 10^9 \text{ N-m}^2 \, C^{-2}$$

Q1.30. Two charges $-q$ each are fixed separated by a distance $2d$. A third charge q of mass m placed at the mid-point is displaced slightly

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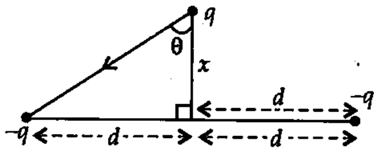
by x (x <<< d) perpendicular to the line joining the two fixed charges as shown in figure. Show that q will perform simple harmonic oscillation of time period



$$T = \left[\frac{8\pi^3\varepsilon_0 m d^3}{q^2}\right]^{\frac{1}{2}}$$

Main concepts used: (*i*) Properties of S.H.M. (*ii*) Coulombian force.

Ans. Force acting on charge q due to $-q$ at A will be F_{PA} along P to A

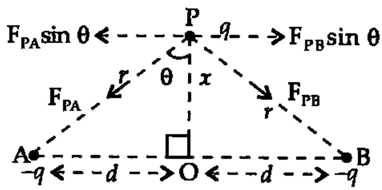
So, $$F_{PA} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q(-q)}{r^2}$$

$$F_{PA} = \frac{-q^2}{4\pi\varepsilon_0 r^2}$$

Similarly, $$F_{PB} = \frac{-q^2}{4\pi\varepsilon_0 r^2}$$ $[\because |F_{PA}| = |F_{PB}| = F]$

So, the horizontal components of F_{PA} and F_{PB} are equal and opposite so neutralise each other.

The vertical components of F_{PA} and F_{PB} are downward and so add up.



Net force on q at P = $F_{PB}\cos\theta + F_{PA}\cos\theta = 2F\cos\theta$

$\because |F_{PA}| = |F_{PB}| = |F|$

$$= \frac{-2\cdot q^2}{4\pi\varepsilon_0 r^2}\cos\theta$$

Force on q downwards $= \dfrac{-2q^2}{4\pi\varepsilon_0 r^2}\,\dfrac{x}{r}$

$r^2 = x^2 + d^2$ [By Pythagoras theorem]

$\therefore$ Force on $q = \dfrac{-2q^2}{4\pi\varepsilon_0}\,\dfrac{x}{(x^2+d^2)^{3/2}}$

$\because x <<< d \quad \therefore x^2 <<<< d^2$

As negative sign shows force of attraction.

So net force on q at P downwards $= \left(\dfrac{-2q^2}{4\pi\varepsilon_0 d^3}\right)x$

So force on q is directly proportional to the displacement from mean position O (mid point of segment joining $-q$ and $-q$ charges). So motion of q about O will be S.H.M.

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$$F = -kx \qquad \text{where, } k = \frac{2q^2}{4\pi\varepsilon_0 d^3}$$

$$\therefore \quad \omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{m}{\frac{2q^2}{4\pi\varepsilon_0 d^3}}} = 2\pi\sqrt{\frac{\not{4}\pi m\varepsilon_0 d^3}{\not{2}q^2}}$$

$$T = \left[\frac{8\pi^3\varepsilon_0 m d^3}{q^2}\right]^{\frac{1}{2}}$$

Hence Proved.

Q1.31. Total charge –Q is uniformly spread along the length of a ring of radius R. A small test charge $+q$ of mass m is kept at the centre of the ring and is given a gentle push along the axis of the ring

(*a*) Show that particle executes a simple harmonic oscillation.

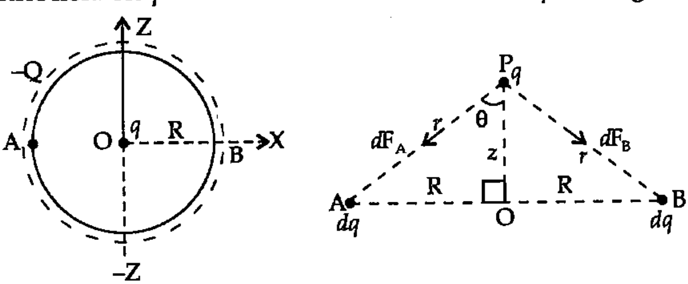
(*b*) Obtain its time period.

Main concepts used: (*i*) Properties of SHM, (*ii*) Centre of ring is symmetric to all charge distribution so electric field at centre is zero.

Here ring is along x - y plane and its axis is along z-axis.

Ans.(*a*) As –Q charge is equally distributed along a conducting ring, so, at the point O, the centre of ring is symmetric to charge distribution. So the electric field at O will be zero, or force on charge q placed at O will be equal to qE *i.e.*, $q \times 0 = 0$.

But when the charge q is displaced gently from 'O', or mean position the electric field on q will not be zero so force acts on q as in fig. below.



Force on q at P when z is small

$$F = F_A + F_B \qquad \because |F_A| = |F_B| = |F_A|$$

$$F_Y = F_A \cos\theta + F_A \cos\theta$$

$$F_Y = 2F_A \cos\theta \text{ (downward)}$$

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The horizontal component of F_A and F_B will be equal and opposite, so cancelled out.

So net force acting on q due to small element dl of ring = $dF \cos\theta$

$$dF_y = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q.dq}{r^2} \cdot \frac{z}{r} \qquad \left[\because \cos\theta = \frac{z}{r}\right]$$

Charge on small element dl of ring

$$dq = \lambda dl$$

$$dq = \frac{-Q}{2\pi r} \cdot dl$$

$$\therefore \quad dF_y = \frac{qz}{4\pi\varepsilon_0 r^3} \cdot \frac{(-Q)dl}{2\pi R}$$

$$dF_y = \frac{-Qqzdl}{4\pi\varepsilon_0 \cdot 2\pi R r^3}$$

Integrating both sides we get,

$$\int_0^F dF_y \downarrow = \int_0^{2\pi R} \frac{-qQzdl}{4\pi\varepsilon_0 2\pi R r^3}$$

$$\therefore \quad F = \frac{-qQz}{4\pi\varepsilon_0 2\pi R\,(R^2+z^2)^{3/2}} \int_0^{2\pi R} dl \;\left(\because r = \sqrt{R^2+z^2}\right)$$

[if $z <<< R$, $z^2 <<<< R^2$ or z^2 can be neglected]

$$\therefore \quad F = \frac{-qQz.2\pi r}{4\pi\varepsilon_0 2\pi r R^3}$$

$$F = \frac{-Qq}{4\pi\varepsilon_0 R^3} z \qquad \text{... [I]}$$

$$F \propto z$$

So motion of q is SHM

$$F = -kz \qquad \text{... [II]}$$

(b) $\therefore$
$$k = \frac{Qq}{4\pi\varepsilon_0 R^3} \qquad \text{[Comparing I and II]}$$

$$\therefore \quad T = 2\neq\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 m R^3}{Qq}}$$

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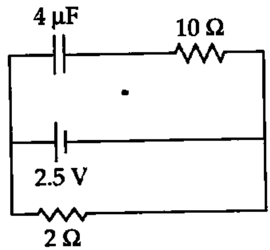
2 Electrostatic Potential and Capacitance

MULTIPLE CHOICE QUESTIONS—I



Q2.1. A capacitor of 4 μF is connected as shown in circuit here. The internal resistance of the battery is 0.5 Ω. The amount of charge on the capacitor plates will be

(*a*) 0 (*b*) 4 μC
(*c*) 16 μC (*d*) 8 μC

Main concepts used: (*i*) DC current does not flow across capacitor. (*ii*) P.D. across parallel branches are equal.

Ans. (*d*): As capacitor offer infinite resistance so current from cell will not flow across capacitor branch.

So current will flow across 2 Ω branch.

$$\boxed{I = \frac{V}{R + r}} = \frac{2.5}{2 + 0.5} = \frac{2.5}{2.5} = 1 \text{ Amp.}$$

So P.D. across 2 Ω resistance V = RI = 2 × 1 = 2 Volt.

As battery, capacitor and 2 Ω branches are in parallel. So P.D. will remain same across all three branches.

As current does not flow through capacitor branch so no potential drop will be across 10 Ω resistance.

So P.D. across 4 μF capacitor = 2 Volt

$$\therefore \quad \boxed{q = CV} = 4\,\mu\text{F} \times 2 = 8\,\mu\text{C}.$$

Q2.2. A positively charged particle is released from rest in a uniform electric field. The electric potential energy of the charge

(*a*) remains a constant because the electric field is uniform.
(*b*) increases because the charge moves along the electric field.
(*c*) decreases because the charge moves along the electric field.
(*d*) decreases because the charge moves opposite to the electric field.

Main concepts used: (*i*) Uniform electric field. (*ii*) As K.E. increases, P.E. decreases. (*iii*) The direction of electric field is from higher to lower potential.

Ans. (*c*): Equipotential surface is always perpendicular to the direction of electric field.

Positive charge experiences the force in the direction of electric field.

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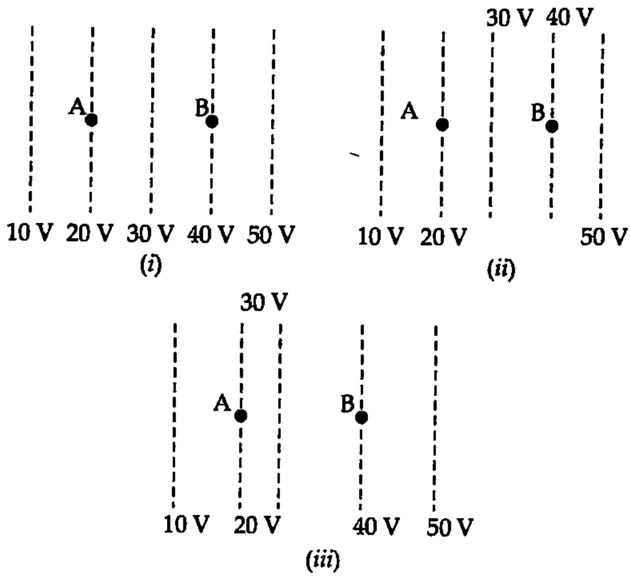
When a positive charge is released from rest in uniform electric field, its velocity increases in the direction of electric field. So K.E. increases, and the P.E. decreases due to law of conservation of energy.

So P.E. of positively charged particle decreases because speed of charged particle moves in the direction of field due to force $q\vec{E}$.

Q2.3. Figures below show some equipotential lines distributed in space. A charged object is moved from point A to point B.



(*a*) The work done in figure (*i*) is the greatest.
(*b*) The work done in figure (*ii*) is least.
(*c*) The work done is the same in all figures (*i*), (*ii*) and (*iii*).
(*d*) The work done in figure (*iii*) is greater than figure (*ii*) but equal to that in figure (*i*).

Main concepts used: Work done in electric field by charge q.

$$\boxed{W = (V_2 - V_1)q}$$

Ans. (*c*): As the potential difference between A and B in all three figures are equal (20 V) so work done ($\Delta V.q$) by any charge in moving from A to B surface will be equal.

Q2.4. The electrostatic potential on the surface of a charged conducting sphere is 100 V. Two statements are made in this regard:

S_1: At any point inside the sphere, electric intensity is zero.

S_2: At any point inside the sphere, the electrostatic potential is 100 V.

Which of the following is a correct statement?

(*a*) S_1 is true but S_2 is false.

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(*b*) Both S_1 and S_2 are false.
(*c*) S_1 is true, S_2 is also true and S_1 is the cause of S_2.
(*d*) S_1 is true, S_2 is also true, but the statements are independent.

Main concepts used: (*i*) Potential gradient, (*ii*) Electric field, (*iii*) Shielding (effect).

Ans. (*c*): The relation between electric field intensity E and potential (V) is $\boxed{E = -\frac{dV}{dr}}$.

Here, E = 0 inside the sphere then $\frac{dV}{dr} = 0$

i.e., V = constant.

E = 0 inside charged sphere, the potential is constant or V = 100 everywhere inside the sphere and it verifies the shielding effect also. Hence verifies the option (*c*).

Q2.5. Equipotentials at a great distance from a collection of charges, whose total sum is not zero, are approximately

(*a*) spheres (*b*) planes (*c*) paraboloids (*d*) ellipsoids

Main concepts used: (*i*) Equipotential surface, (*ii*) Properties of field lines, (*iii*) Properties of charges (*iv*) Potential, (*v*) Point charge.

Ans. (*a*): Here we have to find out the shape of equipotential surface. These surfaces are perpendicular to the field lines. So there must be electric field which cannot be without charge.

So the algebraic sum of all charges must not be zero. Equipotential surface at a great distance means that space of charge is negligible as compared to distance.

So the collection of charges is considered as a point charge.

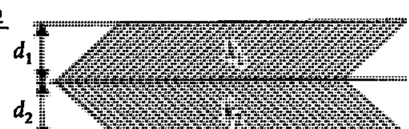
The lines of field from point charges are radial. So the equipotential surface (perpendicular to the field lines) form a sphere.

It verifies that (*a*) is the correct answer.

Q2.6. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant k_1 and the other has thickness d_2 and dielectric constant k_2 as shown in the figure. This arrangement can be thought as a dielectric slab of thickness $d(d_1 + d_2)$ and effective dielectric constant k. The k is

(*a*) $\frac{k_1 d_1 + k_2 d_2}{d_1 + d_2}$ (*b*) $\frac{k_1 d_1 + k_2 d_2}{k_1 + k_2}$

(*c*) $\frac{k_1 k_2 (d_1 + d_2)}{(k_1 d_1 + k_2 d_2)}$ (*d*) $\frac{2 k_1 k_2}{k_1 + k_2}$



Main concepts used: (*i*) Capacitance of a capacitor, (*ii*) Combination of capacitor.

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Ans. (*c*): Capacitance of a parallel plate capacitor filled with dielectric of constant k_1 and thickness d_1 is $\boxed{C_1 = \frac{k_1\varepsilon_0 A}{d_1}}$.

Similarly for other, $\boxed{C_2 = \frac{k_2\varepsilon_0 A}{d_2}}$.

Both capacitors are in series so equivalent capacitance C is related as:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{k_1\varepsilon_0 A} + \frac{d_2}{k_2\varepsilon_0 A} = \frac{1}{\varepsilon_0 A}\left[\frac{k_2 d_1 + k_1 d_2}{k_1 k_2}\right]$$

So $$C = \frac{k_1 k_2 \varepsilon_0 A}{(k_1 d_2 + k_2 d_1)} \quad \text{...I}$$

$$\boxed{C' = \frac{k\varepsilon_0 A}{d}} \quad \text{...II}$$

where $d = (d_1 + d_2)$

So, multiply the numerator and denominator of eqn. I with $(d_1 + d_2)$,

$$C = \frac{k_1 k_2 \varepsilon_0 A}{(k_1 d_2 + k_2 d_1)} \cdot \frac{(d_1 + d_2)}{(d_1 + d_2)} = \frac{k_1 k_2 (d_1 + d_2)}{(k_1 d_2 + k_2 d_1)} \cdot \frac{\varepsilon_0 A}{(d_1 + d_2)} \quad \text{...III}$$

Comparing eqns. II and III, the dielectric constant of new capacitor is:

$$k = \frac{k_1 k_2 (d_1 + d_2)}{(k_1 d_2 + k_2 d_1)}$$

It verifies that the correct answer is (*c*).

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q2.7. Consider a uniform electric field in the $\hat{z}$ direction. The potential is a constant

(*a*) in all space.

(*b*) for any *x* for a given z.

(*c*) for any *y* for a given z.

(*d*) on the *x*-*y* plane for a given z.

Main concepts used: (*i*) Equipotential surface, (*ii*) Electric field lines.

Ans. (*b*), (*c*) and (*d*): As we know that equipotential surfaces are perpendicular to the direction of electric field lines. Here electric field is in $+\hat{z}$ direction.

So, equipotential surfaces will be the plane perpendicular to z axis, *i.e.*, along *x*-*y*, plane, which includes any *x* or *y* axes. So answers (*b*), (*c*) and (*d*) are verified respectively.

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Q2.8. Equipotential surfaces:

(*a*) are closer in regions of large electric fields compared to regions of lower electric fields.

(*b*) will be more crowded near sharp edges of a conductor.

(*c*) will be more crowded near the regions of large charge densities.

(*d*) will always be equally spaced.

Main concepts used: (*i*) Relation between electric field E and potential gradient, (*ii*) E.F. $\propto \sigma$, (*iii*) $\sigma = \frac{q}{A}$.

Ans. (*a*), (*b*) and (*c*): We know that on any two points of equipotential surface, potential difference is zero or of equal potential.

$$\because \qquad E = \frac{-dV}{dr}$$

So the electric field intensity is inversely proportional to the separation between equipotential surfaces.

So equipotential surfaces are closer in regions of large electric fields. Thus, it verifies answer (*a*).

The electric field is larger near the sharp edge, due to larger charge density as A is very small.

$$\because \qquad \sigma = \frac{q}{A}$$

So equipotential surfaces are closer or crowded. It verifies answer (*b*).

As the electric field $E = \frac{kq}{r^2}$ and potential or field decreases as size of body increases or vice-versa (case of earth), so the equipotential surfaces will be more crowded if the charge density $\sigma = \frac{q}{A}$ increases. It verifies the answer (*c*).

As the equipotential surface depends on distance *r* by $E = \frac{-dV}{r}$ and $V = \frac{kq}{r}$. Equipotential surface depends on charge density at that place which is different at different place, so equipotential surfaces are not equispaced all over.

Q2.9. The work done to move a charge along an equipotential surface from A to B

(*a*) cannot be defined as $-\int_A^B E.dl$ (*b*) must be defined as $-\int_A^B E.dl$

(*c*) is zero. (*d*) can have a non-zero value.

Main concepts used: (*i*) $W_{12} = (V_2 - V_1)q$, (*ii*) Equipotential surface.

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Ans. (*c*): As the potential on equipotential surface does not change

so $$(V_2 - V_1) = 0$$

and $$W = (V_2 - V_1)q.$$

So, work done on moving a charge is zero, verifies answer (*c*).

We know the work done by charge *q* in moving in electric field

$$dW = F.dl$$

$$\int \quad = \int qE.dl \qquad [\because F = qE]$$

$$W = q.\int E.dl$$

So, $W \neq \int E.dl$ or answer (*b*) is wrong.

Answer (*a*) and (*b*) can be true only when $q = +1C$ which is not given in question.

Q2.10. In a region of constant potential

(*a*) the electric field is uniform.

(*b*) the electric field is zero.

(*c*) there can be no charge inside the region.

(*d*) the electric field shall necessarily change if a charge is placed outside the region.

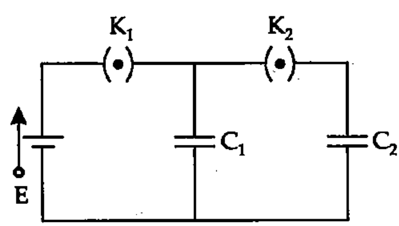
Main concept used: Relation between E and V, *i.e.*, $E = \frac{-dV}{dr}$.

Ans. (*b*) (*c*): Constant potential $\Rightarrow dV = 0$ so by relation $E = \frac{-dV}{dr}$, $E = 0$ *i.e.*, the E.F. is not uniform discards answer (*a*) and agree with answer (*b*).

As potential may be outside the charge also so there can be no charge inside the region of constant potential. It verifies answer (*c*).

If a charge is placed in outside region, potential difference in region will not be changed or electric field will not be changed. It makes answer (*d*) false.

Q2.11. In the circuit shown in figure, initially key K_1 is closed and key K_2 is open. Then K_1 is opened and K_2 is closed (order is important). [Take Q_1' and Q_2' as charges on C_1 and C_2 and V_1 and V_2 as voltage respectively].



Then

(*a*) charge on C_1 gets redistributed such that $V_1 = V_2$

(*b*) charge on C_1 gets redistributed such that $Q_1' = Q_2'$

(*c*) charge on C_1 gets redistributed such that $C_1V_1 + C_2V_2 = C_1E$

(*d*) charge on C_1 gets redistributed such that $Q_1' + Q_2' = Q$

Main concepts used: (*i*) Law of conservation of charges, (*ii*) Potential in parallel combination is equal.

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Ans. (*a*) (*d*): When K_1 is closed keeping K_2 open, the capacitor C_1 gets charged by battery of emf E. Now when K_1 opens, C_1 remains charged. When K_2 closes keeping K_1 open, C_2 gets charged by redistribution of charge of C_1 between C_1 and C_2.

Let charge on C_1, which is charged by battery, was Q then after redistribution of charge $Q = Q_1' + Q_2'$ by law of conservation of charge. So answer (*d*) is verified.

As C_1 and C_2 both are in parallel combination, so their potential will be equal, *i.e.*, $V_1 = V_2$. It verifies the answer (*a*).

Q2.12. If a conductor has a potential $V \neq 0$ and there are no charges anywhere else outside, then

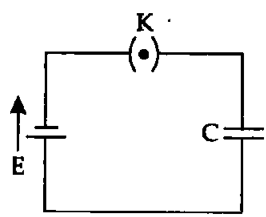
(*a*) there must be charges on the surface or inside itself.
(*b*) there cannot be any charges in the body of conductor.
(*c*) there must be charges only on the surface.
(*d*) there must be charges inside the surface.

Main concepts used: (*i*) The charge reside only on the surface of a conductor. (*ii*) Net charge inside the conductor is zero. (*iii*) Charged spherical shell.

Ans. (*a*) (*b*): As the excess charge can reside only on the surface of conductor and inside net positive and negative charge is zero. Any charge can reside inside the hollow shell or body. So verifies answer (*a*) and discards answer (*c*).

Inside the solid material of conducting body there is no charge, it comes to outer surface. So verifies answer (*b*) and discards answer (*d*).

Q2.13. A parallel plate capacitor is connected to a battery as shown in figure. Consider two situations:



A: Key *K* is kept closed and plates of capacitors are moved apart using insulating handle.

B: Key *K* is opened and plates of capacitors are moved apart using insulating handle.

Choose the correct option(s):

(*a*) In A: Q remains same but C changes.
(*b*) In B: V remains same but C changes.
(*c*) In A: V remains same and hence Q changes.
(*d*) In B: Q remains same and hence V changes.

Main concepts used: (*i*) $C = \dfrac{k\varepsilon_0 A}{d}$, (*ii*) $Q = CV$

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Ans. (*c*) (*d*): **(A) In situation A:** When the space between the plates of capacitor increases, the *capacitance decreases* by relation $C = \frac{k\varepsilon_0 A}{d}$, but battery remains same, *i.e.*, potential difference across plate remains 'V' same. So by Q = CV relation, Q also decreases verifies answer (*c*) and discards answer (*a*).

(B) Now for situation B: K is open and capacitance decreases by moving apart plates of capacitor, so by relation Q = CV, here K is open so charge Q remains same in turn V will increase on decreasing C hence answer (*d*) is verified.

VERY SHORT ANSWER TYPE QUESTIONS

Q2.14. Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one.

Main concepts used: (*i*) $\sigma = \frac{q}{A}$, (*ii*) $V = \frac{kq}{r}$.

Ans. We know that $V_1 = \frac{kq_1}{R_1}$ and $V_2 = \frac{kq_2}{R_2}$.

As $V_1 = V_2$, so:

$$\frac{kq_1}{R_1} = \frac{kq_2}{R_2} \qquad \left(\text{Multiply by } \frac{1}{4\pi} \text{ on both sides}\right)$$

$$\frac{R_1}{R_1} \times \frac{kq_1}{4\pi R_1} = \frac{kq_2}{4\pi R_2} \times \frac{R_2}{R_2}$$

$$\frac{q_1 R_1}{4\pi R_1^2} = \frac{q_2 R_2}{4\pi R_2^2}$$

$$\frac{q_1}{A_1} R_1 = \frac{q_2}{A_2} R_2$$

$$\sigma_1 R_1 = \sigma_2 R_2$$

$$\sigma_1 < \sigma_2 \qquad (\because R_1 > R_2)$$

So charge density of smaller sphere (R_2) will be larger than larger sphere (R_1).

Q2.15. Do free electrons travel to a region of higher potential or lower potential?

Main concepts used: Current (or positive charge) flows from higher to lower potential.

Ans. As free electrons has negative charge so the direction of flow will be opposite to positive charge, *i.e.*, free electrons will move from lower potential to higher potential.

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Q2.16. Can there be a potential difference between two adjacent conductors carrying the same charge?

Main concepts used: (*i*) V = IR, (*ii*) $R = \rho \frac{l}{A}$.

Ans. If in two conductors flowing current is same then both may be considered in series. So Ohm's law becomes $V \propto R$. *i.e.*, if the resistances (which depends on ρ, *l* and A) are different then potential difference will be different.

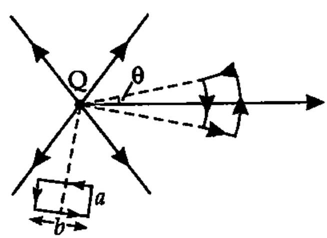
So there can be potential difference between two adjacent conductors carrying the same charge or current if either their length or area of cross-section (A) and resistivity are different.

Q2.17. Can the potential function have a maximum or minimum in free space?

Main concepts used: Electric field, potential difference.

Ans. In the absence of free space or atmosphere, the phenomenon of electric field or potential leakage cannot be prevented. Hence, the potential function do not have maximum or minimum in free space.

Q2.18. A test charge *q* is made to move in the electric field of a point charge Q along two different closed paths as in given figure. First path has sections along and perpendicular to lines of electric field. Second path is a rectangular loop of the same area as the first loop. How does the work done compare in the two cases?



Main concepts used: Work done in electrostatic force is conservative.

Ans. We know that electrostatic work done is conservative. So work done in closed loop is always zero, it does not depend on the nature of closed path.

SHORT ANSWER TYPE QUESTIONS

Q2.19. Prove that a closed equipotential surface with no charge within itself must enclose an equipotential volume.

Main concepts used: (*i*) Electric field lines are perpendicular to equipotential surface, (*ii*) $E = \frac{-dV}{dr}$.

Ans. Let us consider that inside the enclosed equipotential surface, potential is not same. Let the potential just inside the equipotential surface is different to that on the equipotential surface, causing in a

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potential gradient $\frac{dV}{dr} = E$. So electric field will exist inside surface which is equal to $E = -\frac{dV}{dr}$.

The field lines pointing inward or outward from the surface are perpendicular to equipotential surfaces or the field lines cannot be on the equipotential surface. The field lines can be on the equipotential surface if field lines can originate from the charge inside, which contradicts the original assumption. Hence, the entire volume inside equipotential surface has no charge.

Q2.20. A capacitor has some dielectric between its plates, and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed. State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant.

Main concepts used: (*i*) $C = \frac{k\varepsilon_0 A}{d}$, (*ii*) Q = CV, (*iii*) $E = \frac{1}{2}\frac{q^2}{C}$, (*iv*) $E = \frac{V}{d}$.

Ans. ∵ $C = \frac{k\varepsilon_0 A}{d}$

As k is positive and more than one, so by removing dielectric slab, and keeping A and d constant, capacitance of *capacitor* will decrease.

When battery and dielectric slab from capacitor is removed, the charge remains same as it was when battery connected earlier.

As the energy stored in capacitor is $\frac{q^2}{2C}$. When capacitance C is decreased by removing dielectric slab but q remains same, so the **energy stored in capacitor will increase.**

We know that $V = \frac{q}{C}$, where q is same and C is decreased so **potential will increase.**

As $E = \frac{V}{d}$, distance between plates of capacitor is same and potential is increased as discussed above, so ***electric field* between the plates of capacitor will increase.**

Q2.21. Prove that, if an insulated uncharged conductor is placed near a charged conductor and no other conductors are present, the uncharged body must be intermediate in potential between that of the charged body and that of infinity.

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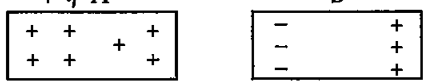
Main concept used: $V = \frac{kq}{r}$.

Ans. Consider a charged body (A) (say with positive charge) and an insulated uncharged conductor (B) is placed near the charged conductor (A) as shown in the figure:



As $V = \frac{kq}{r}$ where k and q are constants, so

$$V \propto \frac{1}{r} \text{ or at infinity, } V \to 0$$

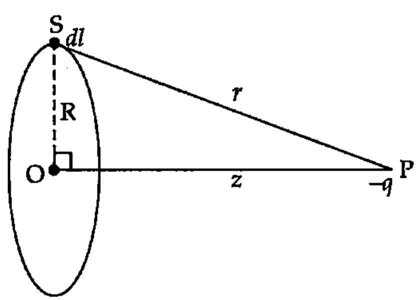
Uncharged conductor is between charged conductor and infinity, so potential decreases from body A to infinity.

So the potential of uncharged body varies between potential of A and infinity.

Q2.22. Calculate the potential energy of a point charge $-q$ placed along the axis due to a charge +Q uniformly distributed along a ring of radius R. Sketch P.E. as a function of axial distance Z from the centre of the ring. Looking at graph, can you see what would happen if $-q$ is displaced slightly from the centre of the ring (along the axis)?

Main concepts used: (*i*) $V = \frac{kq}{r}$, (*ii*) $PE = V \cdot q$.

Ans. Let us consider a ring of radius R having charge +Q distributed uniformly over the ring. Also a point P at distance z on its axis passing through centre O and perpendicular to plane of ring.



Again consider an element of ring at S of length dl having charge dq and SP is equal to r. Then potential energy due to element dl at P. If dq is charge on element dl of ring

$dV = \frac{-kdq}{r}$, where $k = \frac{1}{4\pi\varepsilon_0}$ and as Q is positive charge so potential due to dq charge will be negative.

Charge on $2\pi R$ length of ring $= Q$

Charge on dl length of ring $dq = \frac{Q}{2\pi R} dl$

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So potential due to element dl at P

$$dV = \frac{-k.Q.dl}{2\pi Rr}$$

$$\because dW = dV.q \quad \text{and} \quad r = \sqrt{R^2 + z^2}$$

So $$dW = \frac{-kQqdl}{2\pi R\sqrt{R^2 + z^2}}$$

Integrating both sides, over a ring, we have

$$\int_0^W dW = -\int_0^{2\pi R} \frac{kqQdl}{2\pi R\sqrt{R^2 + z^2}}$$

$$W = -\frac{kqQ\,2\pi R}{2\pi R\, R\sqrt{1 + \frac{z^2}{R^2}}}$$

This work done converts into P.E. at P, so

$$\text{P.E., } V = \frac{-Qq}{4\pi\varepsilon_0 R\sqrt{1 + \frac{z^2}{R^2}}}$$

Let $\boxed{\frac{Qq}{4\pi\varepsilon_0 R} = S}$ (a new constant)

$$V = \frac{-S}{\left[1 + \frac{z^2}{R^2}\right]^{1/2}}$$

at $z = -\infty$

$$V_z = \frac{-S}{\left(1 + \frac{z^2}{R^2}\right)^{1/2}}$$

$\because$ $z >>> R$

$\therefore$ $z^2 >>> R^2$

$$\left(1 + \frac{z^2}{R^2}\right)^{1/2} = \infty$$

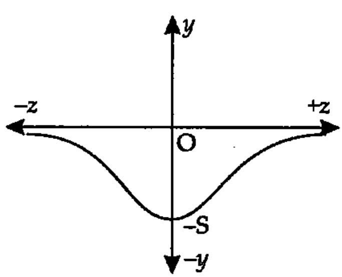
$\therefore$ $V_{-z} = \frac{-S}{\infty} \to 0$

$V_{-z} \to 0$

$V_{+z} \to 0$

at $z = 0$

$V = -S$



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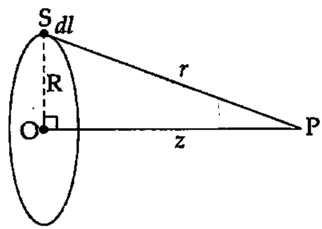
Q2.23. Calculate the potential on the axis of a ring due to charge Q uniformly distributed along the ring of radius R.

Main concept used: $V = \frac{kq}{r}$.



Ans. Let us consider a ring of radius R having charge +Q distributed uniformly. Also a point P at distance z on its axis passing through centre O and perpendicular to plane of ring.

Again consider an element of ring at S of length dl having charge dq and SP is equal to r. Then potential energy due to element dl at P, $dV = \frac{-kdq}{r}$ where $k = \frac{1}{4\pi\varepsilon_0}$

Charge on $2\pi R$ length of ring = Q

Charge on dl length of ring $= \frac{Q}{2\pi R} dl$

So potential due to element dl at P,

$$dV = \frac{-k.Q.dl}{2\pi Rr}$$

Integrating over a ring the potential at P, V_P

$$\int_0^V dV_p = \int_0^{2\pi R} \frac{kQdl}{2\pi Rr} \qquad \text{where } r = \sqrt{R^2 + z^2}$$

$$V_P = \frac{kQ2\pi R}{2\pi R\sqrt{R^2 + z^2}} = \frac{Q}{4\pi\varepsilon_0\sqrt{R^2 + z^2}}.$$

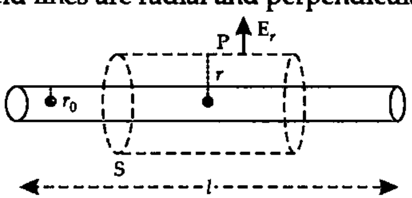
LONG ANSWER TYPE QUESTIONS

Q2.24. Find the equation of the equipotentials for an infinite cylinder of radius r_0, carrying charge of linear density λ.

Main concepts used: (*i*) Gauss's law of electrostatics, (*ii*) Properties of electric field lines, (*iii*) Drawing of proper Gaussian surface, (*iv*) $E = \frac{-dV}{dr}$.

Ans. Consider a Gaussian cylindrical dotted surface, S at a distance r from the centre of the cylinder of radius r_0 of infinite length.

The electric field lines are radial and perpendicular to the surface.



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Let electric field intensity on Gaussian surface at P is E_r and total charge q on cylinder will be $q = \lambda l$.

So, by Gauss's law,

$$\oint_S E_r ds = \frac{\lambda l}{\varepsilon_0} \Rightarrow \left[E_r s \cos\theta\right]_0^{2\pi l} = \frac{\lambda l}{\varepsilon_0}$$

$$E_r 2\pi r l \cos 90° = \frac{\lambda l}{\varepsilon_0}$$ [$\angle\theta$ is between E_r and curved surface of dotted cylinder is 90°]

$$E_r = \frac{\lambda}{2\pi r \varepsilon_0}$$

We know that electric field E_r at distance r from centre of cylinder $E_r = \frac{-dV}{dr}$.

So potential difference d at distance r_0 and r from the centre of cylinder,

$$dV = -E_r . dr \qquad \left[\because E = \frac{-dV}{dr}\right]$$

$$V(r) - V(r_0) = -\int E_r . dr$$

$$= -\int_{r_0}^{r} \frac{\lambda}{2\pi\varepsilon_0 r} \cdot dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_{r_0}^{r}\frac{dr}{r} = \frac{-\lambda}{2\pi\varepsilon_0}[\log_e r]_{r_0}^{r}$$

$$= \frac{-\lambda}{2\pi\varepsilon_0}[\log_e r - \log_e r_0] = \frac{-\lambda}{2\pi\varepsilon_0}\log_e \frac{r}{r_0}$$

$$\log_e \frac{r}{r_0} = \frac{-2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]$$

$$\frac{r}{r_0} = e^{\frac{-2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]}$$

$$r = r_0\, e^{\frac{-2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]}$$

So equipotential surfaces are the coaxial curved surfaces of cylinders with given cylinder of radius r related as above.

Q2.25. Two point charges of magnitudes $+q$ and $-q$ are placed at $\left(-\frac{d}{2}, 0, 0\right)$ and $\left(\frac{d}{2}, 0, 0\right)$ respectively. Find the equation of the equipotential surface where the potential is zero.

Main concepts used: Net potential at a point is equal to the vector sum of all potentials due to different charges in the system and $V = \frac{kq}{r}$.

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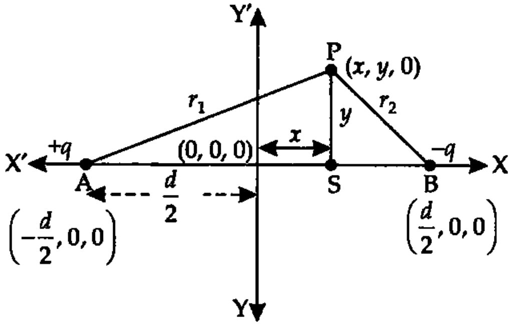
Ans. The potential due to charges $+q$ and $-q$ will be zero in between the line joining the two charges $+q$ and $-q$. Let zero potential is at S.



Then equipotential surface will pass through S and perpendicular to line joining two chargers or AB.

So
$$r_1^2 = AS^2 + SP^2$$
$$= \left(x + \frac{d}{2}\right)^2 + y^2$$
$$r_1 = \sqrt{\left(x + \frac{d}{2}\right)^2 + y^2}$$

Similarly,
$$r_2 = \sqrt{\left(x - \frac{d}{2}\right)^2 + y^2}$$

So net potential at P = 0

$$\frac{kq}{r_1} + \frac{k(-q)}{r_2} = 0 \qquad \text{where, } k = \frac{1}{4\pi\varepsilon_0}$$

$$\Rightarrow \quad kq\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 0 \qquad [\because kq \neq 0]$$

$$\Rightarrow \quad \frac{1}{r_1} - \frac{1}{r_2} = 0$$

$$\Rightarrow \quad \frac{1}{r_1} = \frac{1}{r_2} \quad \Rightarrow \quad r_1 = r_2$$

$$\Rightarrow \quad \left(x + \frac{d}{2}\right)^2 + y^2 = \left(x - \frac{d}{2}\right)^2 + y^2$$

$$\Rightarrow \quad \left(x + \frac{d}{2}\right)^2 = \left(x + \frac{d}{2}\right)^2$$

$$\Rightarrow \quad x^2 + \frac{d^2}{4} + dx = x^2 + \frac{d^2}{4} - dx$$

$$2dx = 0$$

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$$2d \neq 0$$

$\therefore$ $$x = 0$$

So equipotential surface will be perpendicular to X-axis passing through $x = 0$ *i.e.*, origin in Y-Z plane.

Q2.26. A parallel plate capacitor is filled by a dielectric whose relative permittivity varies with the applied voltage (U) as $\varepsilon = \alpha U$ where $\alpha = 2\ V^{-1}$. A similar capacitor with no dielectric is charged to $U_0 = 78$ V. It is then connected to the uncharged capacitor with the dielectric. Find the final voltage on the capacitors.9650852605

Main concepts used: (*i*) $Q = CV$, (*ii*) $C = \varepsilon C_0$.

Ans. Let C be the capacitance of capacitor C_1 without dielectric then charge $q_1 = CU$ where U is the final potential of C_1 when connected to C_2 the capacitor filled with dielectric ε_0

$$C_2 = \varepsilon C$$
$$q_2 = \varepsilon CU$$
$$= \alpha UCU$$
$$= \alpha CU^2$$

Initial charge q_0 of C_1 when charged at potential of $U_0 = 78$ V is,

$$q_0 = CU_0 = 78\ C$$

By the law of conservation of charge

$$q_0 = q_1 + q_2$$
$$78\ C = CU + \alpha.CU^2$$
$$78 = U + \alpha U^2 \qquad [\alpha = 2 \text{ per volt}]$$

$\therefore$ $$= U + 2U^2$$

or $$2U^2 + U - 78 = 0$$

$$U = \frac{-1 \pm \sqrt{1 - 4.2(-78)}}{2.2} = \frac{-1 \pm \sqrt{1 + 624}}{4}$$
$$= \frac{-1 \pm \sqrt{625}}{4} = \frac{-1 \pm 25}{4}$$
$$= \frac{-1 + 25}{4} \quad \text{as U is positive}$$
$$= \frac{24}{4} = \mathbf{6\ Volts.}$$

Final potential on both the capacitors becomes 6 Volts.

Q2.27. A capacitor is made of two circular plates of radius R each separated by a distance $d \ll R$. The capacitor is connected to a constant voltage. A thin conducting disc of radius $r \ll R$ and thickness 't' $\ll r$ is placed at the centre of the bottom plate. Find the minimum voltage required to lift the disc if the mass of the disc is m.

Main concepts used: (*i*) $E = \frac{-dV}{dr}$,

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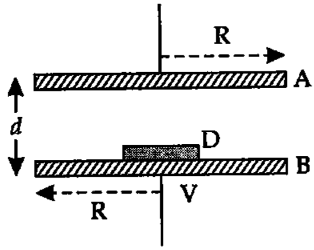
(*ii*) Electrostatic force is balanced by weight, (*iii*) Electrostatic force $F = qE$.



Ans. Let A and B are circular plates of radius R separated by distance $d \ll R$ kept horizontally.

A thin conducting disc D of radius $r \ll R$ of thickness 't' is placed concentrically on lower plate B as shown in figure.

Let plates A and B charged with potential V.

The magnitude of electric field E between plates of capacitor

$$E = \frac{V}{d} \qquad \left[\because E = \frac{-dV}{dr}\right]$$

Consider Gaussian surface along circular disc D.

By Gauss's law, $\oint E.ds = \frac{q'}{\varepsilon_0}$

$$\frac{V}{d} \cdot \pi r^2 = \frac{q'}{\varepsilon_0}$$

q' is the charge conducted by plate B to disc D during charging. Nature of charge on plate B and disc will be same so repulsive force acts between B and D.

So, the charge on disc D $= q' = \frac{V}{d}\pi r^2 \varepsilon_0$

Electrostatic repulsive force acting on disc in upward direction

$$F = q' E = \frac{V}{d} \cdot \pi r^2 \varepsilon_0 \cdot \frac{V}{d} = \frac{V^2}{d^2} \pi r^2 \varepsilon_0$$

This repulsive force will be balanced by weight mg of disc D.

$$mg = \frac{V^2}{d^2} \pi r^2 \varepsilon_0$$

$$V^2 = \frac{mg\, d^2}{\pi r^2 \varepsilon_0}$$

So minimum voltage V to lift the disc

$$V = \sqrt{\frac{mg\, d^2}{\pi r^2 \varepsilon_0}}$$

Q2.28. (*a*) In a quark model of elementary particles, a neutron is made of **one up quark** $\left[\text{charge}\left(\frac{2}{3}\right)e\right]$ and **two down quarks** $\left[\text{charges}\left(-\frac{1}{3}\right)e\right]$. Assume that they have a triangle configuration with side length of the order of 10^{-15} m. Calculate the electrostatic potential energy of a neutron and compare it with its mass 939 MeV.

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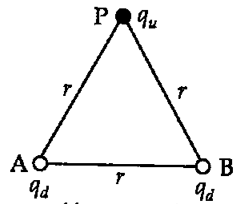
(*b*) Repeat the above exercise for a proton which is made of two up and one down quark.

Main concepts used: The potential energy is equal to the sum of potential energy or energies required to form the configuration, if charge particles are carried with zero acceleration, from infinity to that point.

Ans. (*a*) $q_d = -\frac{1}{3}e$ [charge on down quark]

$q_u = +\frac{2}{3}e$ [charge on up quark]



(A neutron)

Potential energy $U = \frac{kq_1q_2}{r}$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$U = \frac{kq_1q_2}{r} + \frac{kq_1q_3}{r} + \frac{kq_2q_3}{r}$$

$$\therefore \quad U_n = \frac{1}{4\pi\varepsilon_0}\frac{(-q_d)(-q_d)}{r} + \frac{(-q_d)q_u}{4\pi\varepsilon_0 r} + \frac{q_u(-q_d)}{4\pi\varepsilon_0 r}$$

$$= \frac{q_d}{4\pi\varepsilon_0 r}[+q_d - q_u - q_u] \quad \text{(Taking sign of charge)}$$

$$= \frac{q_d}{4\pi\varepsilon_0 r}[q_d - 2q_u] = \frac{9\times10^9 \times \frac{1}{3}e}{10^{-15}}\left[\frac{1}{3}e - 2\cdot\frac{2}{3}e\right]$$

[nature sign of charges taken already]

$$= \frac{9\times10^9 \times e}{3\times10^{-15}}\cdot\frac{e}{3}[1-4] \text{ Joule}$$

$$= \frac{-3\times9\times10^9\times1.6\times10^{-19}}{9\times10^{-15}}e \text{ Joule}$$

$$= -4.8\times10^{9-19+15}\, eV = -4.8\times10^5\, eV = -0.48\times10^6\, eV$$

$$U = -0.48 \text{ MeV}$$

So charges inside neutron [$1q_u$ and $2q_d$] are attracted by energy of 0.48 MeV.

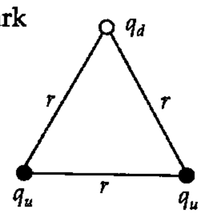
Energy released by a neutron when converted into energy is 939 MeV.

$$\therefore \text{ Required ratio} = \frac{1 - 0.481 \text{ MeV}}{939 \text{ MeV}} = 0.0005111 = \mathbf{5.11\times10^{-4}}$$

(*b*) P.E. of proton consists of 2 up and 1 down quark

$r = 10^{-15}$ m

$$q_d = -\frac{1}{3}e, \quad q_u = \frac{2}{3}e$$



$$U_p = \frac{1}{4\pi\varepsilon_0}\frac{q_u\times q_u}{r} + \frac{q_u(-q_d)}{4\pi\varepsilon_0 r} + \frac{q_u(-q_d)}{4\pi\varepsilon_0 r}$$

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$$= \frac{q_u}{4\pi\varepsilon_0 r}[q_u - q_d - q_d]$$

$$= \frac{q_u}{4\pi\varepsilon_0 r}[q_u - 2q_d] = \frac{9\times10^9}{10^{-15}}\frac{2}{3}e\left[\frac{2}{3}e - 2\cdot\frac{1}{3}e\right] = 0.$$

Q2.29. Two metal spheres, one of radius R and the other of radius 2R, both have same surface charge density '*s*'. They are brought in contact and separated. What will be new surface charge densities of them?

Main concepts used: (*i*) $Q = \sigma . A$, (*ii*) $V = \frac{kq}{R}$.

Ans. Let surface charge density of both the spheres are σ and their charges are q_1 and q_2.

$\therefore$ $$q_1 = \sigma . A_1 = \sigma . 4\pi R^2$$

$$q_2 = \sigma A_2 = \sigma . 4\pi(2R)^2 = \sigma . 4\pi R^2 . 4 = 4q_1$$

Both charged spheres are kept in contact, so charge flows between them and their potential becomes equal, let the charges on them now become q_1' and q_2'.

So, $$V_1 = V_2 \qquad \left(\because V = \frac{kq}{r}\right)$$

So $$\frac{kq_1'}{R} = \frac{kq_2'}{(2R)} \qquad \left[\because k = \frac{1}{4\pi\varepsilon_0}\right]$$

Where q_1' and q_2' are the charges on spheres after redistribution of charges

$$\frac{q_1'}{R} = \frac{q_2'}{2R}$$

$\therefore$ $$q_2' = 2q_1' \qquad \text{...I}$$

By law of conservation of charges

$$q_1 + q_2 = q_1' + q_2'$$

$$q_1 + 4q_1 = q_1' + 2q_1' \qquad \text{(from I)}$$

$$5q_1 = 3q_1'$$

$$3q_1' = 5.4.\sigma.\pi R^2$$

$$q_1' = \frac{20}{3}\pi R^2 \sigma$$

$$\sigma_1 = \frac{q_1'}{A_1} = \frac{\frac{20}{3}\pi R^2\sigma}{4\pi R^2} = \frac{5}{3}\sigma$$

$$\sigma_2 = \frac{q_2'}{A_2} = \frac{2.q_1'}{4.\pi(2R)^2} = \frac{2.\frac{20}{3}\pi R^2\sigma}{4\pi.4R^2} = \frac{5}{6}\sigma$$

Hence, $\sigma_1 = \frac{5}{3}\sigma$ and $\sigma_2 = \frac{5}{6}\sigma$

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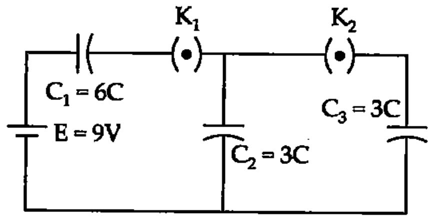
Q2.30. In the circuit in given figure initially K_1 is closed and K_2 is open. What are the charges on each capacitor?

Then K_1 was opened and K_2 was closed (order is important). What will be the charge on each capacitor now?



[C = 1 μF]

Main concepts used: (*i*) $V = \frac{q}{C}$, (*ii*) Law of conservation of charge, (*iii*) In series combination charges on capacitors are equal.

Ans. When K_2 is open and K_1 is closed the capacitors C_1 and C_2 will charge and potential develops across them *i.e.,* V_1 and V_2 respectively which will be equal to the potential of battery 9 V.

$$\therefore \quad V_1 + V_2 = 9 \quad \text{...I}$$

$$\because \quad V = \frac{q}{C} = \quad \text{or} \quad V \propto \frac{1}{C}$$

or
$$\frac{V_1}{V_2} = \frac{C_2}{C_1}$$

$$\frac{V_1}{V_2} = \frac{3C}{6C}$$

$$3V_2 = 6V_1$$

$$V_2 = 2V_1 \quad \text{...II}$$

From Eqns. I and II,

$$V_1 + 2\,V_1 = 9$$

$$3\,V_1 = 9$$

$$V_1 = 3 \text{ Volt}$$

$$V_2 = 2 \times 3 \text{ Volt} = 6 \text{ Volt} \quad \begin{bmatrix}\text{(from II)} \\ C = 1\,\mu F\end{bmatrix}$$

$$\therefore \quad q_1 = C_1V_1 = 6C \times 3 = 18C$$

$$= 18 \times 1\,\mu F = 18\,\mu C$$

$$q_2 = C_2V_2 = 3C \times 6$$

$$= 3 \times 1\,\mu F \times 6 = 18\,\mu C$$

So, charges on each capacitor i.e., $q_1 = q_2 = 18\,\mu C$

When K_1 is open and K_2 is closed then charge q_2 will be distributed among C_2 and C_3. Let it be q_2' and q_3.

$$\therefore \quad q_2 = q_2' + q_3$$

As C_2 and C_3 are now in parallel combination so their potentials remain same (V)

$$\therefore \quad q_2 = C_2V + C_3V$$

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$$18\ \mu C = 3 \times 1\ \mu F \times V + 3 \times 1\ \mu F \times V$$
$$18 = 6\ V$$
$$V = 3\ \text{Volt.}$$

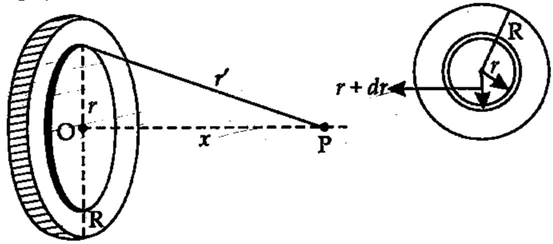
So potential on C_2 and C_3 capacitors are 3 Volt each

$$\left.\begin{aligned} q_2' &= C_2V = 3 \times 1\ \mu F \times 3\ \text{Volt} = 9\ \mu C \\ q_3 &= C_3V = 3 \times 1\ \mu F \times 3\ \text{Volt} = 9\ \mu C \end{aligned}\right] \textbf{Ans.}$$

Q2.31. Calculate the potential on the axis of a disc of radius R due to a charge Q, uniformly distributed on its surface.

Main concepts used: A disc can be considered as the combination of rings of different radii and V for ring $= \frac{kq}{r}$ where, r is the distance of axial point from ring.

Ans. Consider a point P on the axis perpendicular to the plane of disc and at distance x from the centre O of disc as shown in the figure.



Now consider a ring of radius r of thickness dr on disc of radius R, as shown in figure. Again let the charge on the ring is dq then potential dV due to ring at P, will be,

$$dV = \frac{k\,dq}{r'} \qquad [\because r' = \sqrt{r^2 + x^2}]$$

dq is the charge on the ring $= \sigma \cdot$area of ring

$$= \sigma \cdot [\pi(r + dr)^2 - \pi r^2]$$
$$dq = \sigma \cdot \pi[r^2 + dr^2 + 2rdr - r^2]$$

Because dr is small therefore, dr^2 is negligible.

$$\therefore \qquad dq = \sigma\pi(2rdr) = 2\pi r\sigma \cdot dr$$

$$\therefore \qquad dV = \frac{k.2\pi r\sigma dr}{\sqrt{(r^2 + x^2)}}$$

So the potential due to charged disc

$$\int_0^V dV = \int_0^R \frac{k2\pi r\sigma dr}{\sqrt{r^2 + x^2}}$$

$$V = k.2\pi\sigma \cdot \int_0^R \frac{rdr}{(r^2 + x^2)^{1/2}} = 2\pi k\sigma \int_0^R r \cdot (r^2 + x^2)^{1/2} dr$$

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$$= 2\pi k\sigma[(R^2 + x^2)^{1/2} - x] = \frac{2\pi\sigma}{4\pi\varepsilon_0}[(R^2 + x^2)^{1/2} - x]$$

[$\because$ $\pi R^2\sigma = Q$ (charge on disc)

$\sigma = \frac{Q}{\pi R^2}$]

$$= \frac{2\pi R^2\sigma}{4\pi\varepsilon_0 R^2}\left[\sqrt{R^2 + x^2} - x\right]$$

$$V = \frac{2Q}{4\pi\varepsilon_0 R^2}\left[\sqrt{R^2 + x^2} - x\right]$$

Q2.32. Two charges q_1 and q_2 are placed at $(0, 0, d)$ and $(0, 0, -d)$ respectively. Find the locus of points where the potential is zero.

Main concepts used: Where the net potential due to different charges are zero. $V = \frac{kq}{r}$.

Ans. Let the potential at any point $P(x, y, z)$ is zero then—

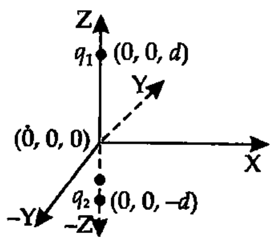
$$V_1 + V_2 = 0$$

$$\frac{kq_1}{\sqrt{x^2 + y^2 + (z - d)^2}} + \frac{kq_2}{\sqrt{x^2 + y^2 + (z + d)^2}} = 0$$

$$\frac{q_1}{\sqrt{x^2 + y^2 + (z - d)^2}} + \frac{q_2}{\sqrt{x^2 + y^2 + (z + d)^2}} = 0$$

$$\frac{q_1}{\sqrt{x^2 + y^2 + (z - d)^2}} = \frac{-q_2}{\sqrt{x^2 + y^2 + (z + d)^2}}$$

$$\frac{q_1}{q_2} = \frac{-\sqrt{x^2 + y^2 + (z - d)^2}}{\sqrt{x^2 + y^2 + (z + d)^2}}$$



$$\frac{q_1^2}{q_2^2} = \frac{x^2 + y^2 + z^2 + d^2 - \ \ zd}{x^2 + y^2 + z^2 + d^2 + \ \ zd}$$

Componendo and dividendo

of $\frac{x}{a} = \frac{y}{b}$ is $\frac{x + a}{x - a} = \frac{y + b}{y - b}$

Then componendo and dividendo of

$$\frac{\left(\frac{q_1}{q_2}\right)^2}{1} = \frac{x^2 + y^2 + z^2 + d^2 - 2zd}{x^2 + y^2 + z^2 + d^2 + 2dz}$$

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$$\frac{\left(\frac{q_1}{q_2}\right)^2+1}{\left(\frac{q_1}{q_2}\right)^2-1}=\frac{x^2+y^2+z^2+d^2-2dz+(x^2+y^2+z^2+d^2+2dz)}{x^2+y^2+z^2+d^2-2dz-(x^2+y^2+z^2+d^2+2dz)}$$

$$\left[\frac{\left(\frac{q_1}{q_2}\right)^2+1}{\left(\frac{q_1}{q_2}\right)^2-1}\right]=\frac{2(x^2+y^2+z^2+d^2)}{-4dz}$$

$$x^2+y^2+z^2+d^2=-2dz\left[\frac{\left(\frac{q_1}{q_2}\right)^2+1}{\left(\frac{q_1}{q_2}\right)^2-1}\right]$$

$$x^2+y^2+z^2+2d\left[\frac{\left(\frac{q_1}{q_2}\right)^2+1}{\left(\frac{q_1}{q_2}\right)^2-1}\right]z+d^2=0$$

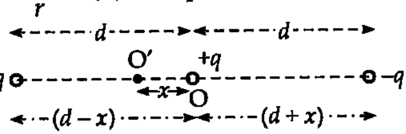
$$x^2+y^2+z^2+2d\left[\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right]z+d^2=0$$

This is the equation of sphere with centre (a, b, c) as required point is on z axis so $a=0$, $b=0$ and $z=2d\left[\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right]$

$$\left(0, 0, -2d\left[\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right]\right).$$

Q2.33. Two charges $-q$ each are separated by distance $2d$. A third charge $+q$ is kept at mid-point 'O'. Find the potential energy of $+q$ as a function of small distance x from 'O' due to $-q$ charges. Sketch P.E. v/s x and convince yourself that the charge at O is in an unstable equilibrium.

Main concepts used: (*i*) P.E. $U=\frac{kq_1q_2}{r}$, (*ii*) At equilibrium, $F=0$ or $\frac{dF}{dx}\cdot dx=0 \Rightarrow \frac{dU}{dx}=0$



Ans. $V=\frac{kq}{r}$

Let equilibrium of $+q$ is at P at a distance x from mid-point of line joining two charges.

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Force F_A on $+q$ is towards left side and force F_B is towards right side, so for equilibrium of $+q$ at P,

$$F_A = F_B$$

$$\frac{-kq^2}{(d-x)^2} = \frac{-kq^2}{(d+x)^2}$$

$$\therefore \quad (d-x)^2 = (d+x)^2$$

$$d - x = d + x \qquad \text{(Taking square root)}$$

$$-2x = 0$$

$$x = 0$$

So, equilibrium position of charge $+q$ between two $-q$ charges is at mid-point (O) of line joining the two charges $(-q)$ and $(-q)$.

Now we have to find out potential energy of $+q$ as a function of small distance x from balance condition (O) towards any of $(-q)$ charge.

Let new position of charge $(+q)$ from a small distance x from (O)

$$U = \frac{k.(q)(-q)}{(d-x)} + \frac{k(q)(-q)}{(d+x)} \qquad \left(\because U = \frac{kq_1q_2}{r_1}\right)$$

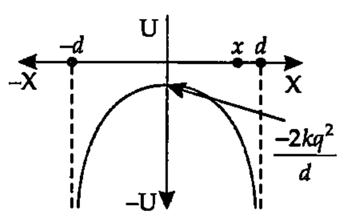
$$= -kq^2\left[\frac{1}{(d-x)} + \frac{1}{(d+x)}\right]$$

$$= -kq^2\left[\frac{d+x+d-x}{(d-x)(d+x)}\right] = -kq^2\left[\frac{2d}{d^2-x^2}\right]$$

$$\boxed{U = \frac{-q^2}{4\pi\varepsilon_0}\cdot\frac{2d}{(d^2-x^2)}}$$

So, U is the P.E. as a function of x.

x	U
0	$\frac{-2k}{d}q^2$
$\frac{d}{2}$	$\frac{4}{3}\left(\frac{-2kq^2}{d}\right)$
$-\frac{d}{2}$	$\frac{4}{3}\left(\frac{-2kq^2}{d}\right)$
$+d$	$-\alpha$
d	$-\alpha$



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3 Current Electricity

MULTIPLE CHOICE QUESTIONS—I

Q3.1. Consider a current carrying wire (current I), in the shape of a circle. Note that as the current progresses along the wire, the direction of J (current density) changes in an exact manner, while the current I remains unaffected. The agent that is essentially responsible for it is:

(*a*) source of emf.
(*b*) electric field produced by the charges accumulated on the surfaces of wire.
(*c*) the charges just behind a given segment of the wire which push them just the right way by repulsion.
(*d*) the charges ahead.

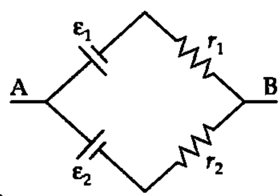
Main concepts used: $\vec{J} = \sigma\vec{E}$, $J = \frac{I}{A}$, $\sigma = \frac{1}{\rho} = \frac{l}{RA}$.

Ans. (*b*): Current density (J) depends on conductivity $\sigma = \frac{1}{\rho} = \frac{l}{R.A}$,

Electric field (J = σE), current and length and area of cross-section.

In our options only E *i.e.*, electric field can be varied by the charges accumulated on the surface of wire.

Q3.2. Two batteries of emf ε_1 and ε_2 ($\varepsilon_2 > \varepsilon_1$) and internal resistances r_1 and r_2 respectively are connected in parallel as shown in figure.



(*a*) The equivalent emf ε_{eq} of the two cells is between ε_1 and ε_2, *i.e.*, $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$.
(*b*) The equivalent emf ε_{eq} is smaller than ε_1.
(*c*) The equivalent emf is given by

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 \text{ always.}$$

(*d*) ε_{eq} is independent of internal resistances r_1 and r_2.

Main concept used: Combination of cells in parallel

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

Ans. (*a*): We know that equivalent emf ε_{eq} in parallel combination of cells is:

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

Clearly, part '*c*' and '*d*' are discarded by formula. This formula suggests that $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$. So verifies answer (*a*).

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Q3.3. The resistance R is to be measured using a meter bridge. Student chooses the standard resistance S to be 100 Ω. He finds the null point at $l_1 = 2.9$ cm. He is told to improve the accuracy. Which of the following is a useful way?

(*a*) He should measure l_1 more accurately.
(*b*) He should change S to 1000 Ω and repeat the experiment.
(*c*) He should change S to 3 Ω and repeat the experiment.
(*d*) He should give up hope of a more accurate measurement with a meter bridge.

Main concept used: $\frac{R}{S} = \frac{l_1}{100 - l_1}$

Ans. (*c*): R is the unknown resistance

$$\Rightarrow \qquad R = S\left(\frac{l_1}{100 - l_1}\right) = 100\left[\frac{2.9}{97.1}\right] = 2.98\ \Omega.$$

So to get balance point near to 50 cm (middle) we should take S = 3 Ω, as here R : S = 2.9 : 97.1 implies that S is nearly 33 times to R. In order to make ratio R and S = 1 : 1, we must take the resistance S = 3 Ω, verifies answer (*c*).

Q3.4. Two cells of emf approximately 5 V and 10 V are to be accurately compared using a potentiometer of length 400 cm.

(*a*) The battery that runs the potentiometer should have voltage of 8 V.
(*b*) The battery of potentiometer can have a voltage of 15 V and R adjusted so that the potential drop across the wire slightly exceeds 10 V.
(*c*) The first portion of 50 cm of wire itself should have a potential drop of 10 V.
(*d*) Potentiometer is usually used for comparing resistances and not voltage.

Main concept used: Potential drop across the potentiometer wire should be slightly more than the emf of primary cell which is to be measured.

Ans. (*b*): Here, emf of primary cells are 5 V and 10 V. So the potential drop across potentiometer wire must be slightly more than that larger emf 10 V. So the battery should be of 15 V and about 4 V potential is dropped by using rheostat or resistances. So verifies answer (*b*).

Q3.5. A metal rod of length 10 cm and a rectangular cross-section of $1\text{ cm} \times \frac{1}{2}$ cm is connected to a battery across opposite faces.

The resistance will be

(*a*) maximum when the battery is connected across $1\text{ cm} \times \frac{1}{2}$ cm faces.
(*b*) maximum when the battery is connected across 10 cm × 1 cm faces.

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(*c*) maximum when the battery is connected across 10 cm × $\frac{1}{2}$ cm faces.

(*d*) same irrespective of the three faces.

Main concept used: $R = \rho \frac{l}{A}$.

Ans. (*a*): As we know $R = \rho . \frac{l}{A}$. The maximum resistance will be when the value of $\frac{l}{A}$ is maximum, i.e., 'A' must be minimum, it is minimum when area of cross section is 1 cm × $\frac{1}{2}$ cm. Verifies option (*a*).

Q3.6. Which of the following characteristics of electrons determines the current in a conductor?

(*a*) Drift velocity alone.

(*b*) Thermal velocity alone.

(*c*) Both the drift and thermal velocity.

(*d*) Neither drift nor thermal velocity.

Main concept used: $I = Anev_d$

Ans. (*a*): As $I = Anev_d$, so current $I \propto v_d$ *i.e.*, verifies answer (*a*).

Although I also depends on *n*, the number of free electrons which increases on increasing temperature which makes more collision between electrons increase resistance or decrease current. So Ans. '*a*' verified.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q3.7. Kirchhoff's junction rule is a reflection of

(*a*) conservation of current density vector.

(*b*) conservation of charge.

(*c*) the fact that the momentum with which a charged particle approaches a junction is uncharged (as vector), as the charged particle leaves the junction.

(*d*) the fact that there is no accumulation of charges at a junction.

Main concept used: Kirchhoff's junction rule.

Ans. (*b*) (*d*): According to junction rule, algebraic sum of current or charge flowing per unit time towards a junction in an electric network is zero, *i.e.*, law of conservation of charges verifies answer (*b*) and no any charges accumulate at junction as the sum of entering and out going charge are equal, at any time interval. It verifies answer (*d*).

Q3.8. Consider a simple circuit shown in figure.  stands for a variable resistance R′. R′ can vary from R_0 to infinity, *r* is internal resistance of the battery ($r << R << R_0$).

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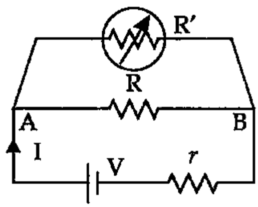
(*a*) Potential drop across AB is nearly constant as R′ is varied.
(*b*) Current through R′ is nearly a constant as R′ is varied.
(*c*) Current I depends sensitively on R′.
(*d*) $I \geq \frac{V}{r+R}$ always.



Main concept used: (*i*) Property of resistances in series and parallel, (*ii*) V = IR.

Ans. (*a*) (*d*): As $r \ll R \ll R_0 < R'$ from question, R′ > R and R′ and R are in parallel combination, so the equivalent resistance will be always less than R. So $I \geq \frac{V}{r+R}$ and potential across AB will remain nearly constant. It verifies answers (*a*) and (*d*).

Q3.9. The temperature dependence of resistivity ρ(T) of semiconductors, insulators and metals is significantly based on the following factors:

(*a*) Number of charge carriers can change with temperature T.
(*b*) Time-interval between two successive collisions can depend on T.
(*c*) Length of material can be a function of T.
(*d*) Mass of carriers is a function of T.

Main concept used: Resistivity $(\rho) = \frac{m}{ne^2\tau}$

Ans. (*a*) and (*b*): We know that resistivity (ρ) depends on mass of charge-carrier (*m*), relaxation time (τ). Length and mass cannot be function of T as the mass of a body is constant everywhere. So discards answer (*d*) and length of body changes negligibly with temperature discards answer (*c*).

As τ decreases on increasing T due to rise in speed of charge-carriers and *n* increases on increasing temperature. So will affect the ρ or ρ is function of T verifies answers (*a*) and (*b*).

Q3.10. The measurement of an unknown resistance R is to be carried out using Wheatstone Bridge. Two students perform an experiment in two ways. The first student takes $R_2 = 10\ \Omega$ and $R_1 = 5\ \Omega$. The other student takes $R_2 = 1000\ \Omega$ and $R_1 = 500\ \Omega$. In the standard arm, both students take $R_3 = 5\ \Omega$. Both find $R = \frac{R_2}{R_1} R_3 = 10\ \Omega$ within errors.

(*a*) The errors of measurement of the two students are the same.
(*b*) Errors of the measurement do depend on the accuracy with which R_2 and R_1 can be measured.
(*c*) If the student uses large values of R_2 and R_1, the current through the arms will be feeble. This will make determination of null point accurately more difficult.

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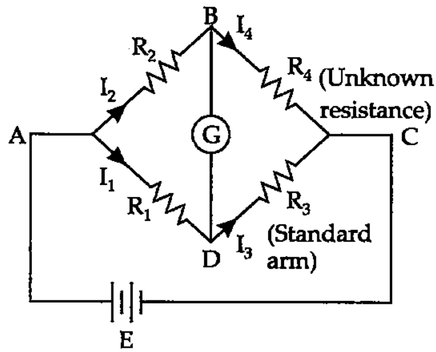
(*d*) Wheatstone Bridge is a very accurate instrument and has no errors of measurement.

Main concept used:

$$\frac{R_2}{R_3} = \frac{R_1}{R_4}$$

$$R = \frac{R_2}{R_1} \times R_3$$

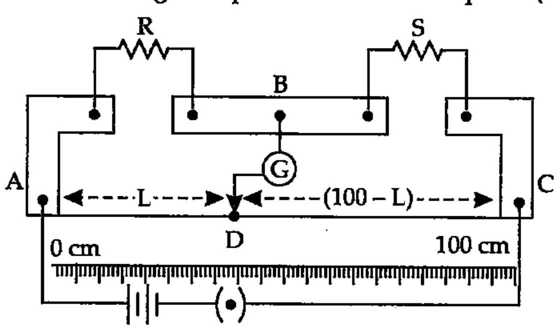


Ans. (*b*) and (*c*): As the ratio of $\frac{R_2}{R_1}$ and standard resistance are same, so value of unknown resistance for both students are 10 Ω. So we can say the Wheatstone Bridge is most sensitive.

The results of both students depend on the accuracy of resistances used. **So answer (*b*) is verified.**

When R_1 and R_2 is larger, the current through galvanometer becomes weak. It will make difficult to find out null point more accurately. So answer (*c*) is verified.

Q3.11. In the meter bridge the point D is a neutral point (Fig.).



(*a*) The meter bridge can have no other neutral point for this set of resistances.

(*b*) When the jockey contacts a point on meter wire left of D, current flows to B from the wire.

(*c*) When the jockey contacts a point on the meter wire to the right of D, current flows from B to the wire through galvanometer.

(*d*) When R is increased, the neutral point shifts to left.

Main concepts used: (*i*) Principal of meter bridge $\frac{R}{S} = \frac{L}{100 - L}$,

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(*ii*) When potential difference between two points is zero the current does not flow, (*iii*) Potential decreases from A to B.

Ans. (*a*) and (*c*): When jockey is at D, the current does not flow through galvanometer. So the potentials at B and D are equal. Potentials at different points on the wire are different.

The point D is unique to get null point verifies the answer (*a*). When jockey is shifted to right of D on wire the potential in wire towards right side becomes smaller or $V_B > V_D$ becomes smaller so current flows from B to D in wire verify the answer (*c*) and discards answer (*b*).

When R is increased potential drop across R increases. So potential at B increases to get null point. Jockey must move to right.

VERY SHORT ANSWER TYPE QUESTIONS

Q3.12. Is the motion of a charge across junction momentum conserving? Why or why not?

Main concept used: $p_C = mv_d$, $v_d = \dfrac{eE\tau}{m}$.

Ans. As we know that drift velocity depends on e, E, τ and m as for a junction point, if the temperature is constant e, τ, m is constant so drift velocity (v_d) of electron depends on electric field only i.e., ($\theta \propto$ E).

When a free electron approaches a junction in addition of the uniform electric field (E) facing it normally as E is constant by cell or battery so v_d is constant.

There is a accumulation of charges on the junction which will affect the drift velocity or the momentum, so the momentum is not conserved at junction.

Q3.13. The relaxation time τ is nearly independent of applied electric field E whereas it changes significantly with temperature T. First fact is (in part) responsible for Ohm's law whereas the second fact leads to variation of ρ with temperature. Elaborate (why)?

Main concept used: (*i*) Higher v_d makes higher collision, v_d increases on increasing temperature, (*ii*) Increase in v_d decreases relaxation time.

Ans. As the drift velocity increases, the relaxation time (τ) (average time between successive collision) decreases which increases the ρ by formula: $\rho = \dfrac{m}{ne^2\tau}$.

The drift velocity (v_d) changes of the order of one mm on increasing electric field, whereas the drift velocity increases of the order of 10^2 m/s when the number of free electrons (n) increases on increasing temperature (T). So, due to increase in v_d the relaxation time (τ) considerably decreases in metal or conductor.

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Q3.14. What are the advantages of the null point method in Wheatstone Bridge? What additional measurements would be required to calculate R (unknown) by any other method?

Main concept used: At null point in Wheatstone Bridge experiment no current flows in the arm of galvanometer.

Ans. The main advantage in Wheatstone Bridge is that at null point current does not flow in arm of galvanometer, so no effect of resistance of galvanometer or no consumption of electric energy or potential across galvanometer. It is convenient and easy method.

We can calculate the unknown resistance by Ohm's law in which we need to calculate the least counts and readings of ammeter and voltmeter.

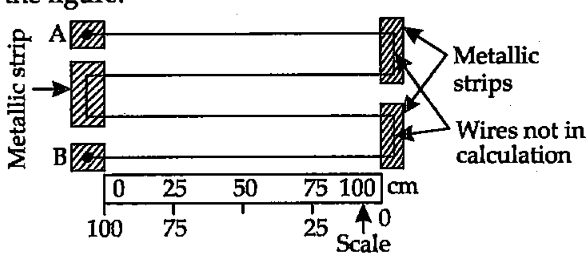
Unknown resistance can also be calculated by applying Kirchhoff's laws to the circuit in which unknown resistance is connected. Here we have to measure the currents and potential differences across all components of circuit which makes the method difficult.

Q3.15. What is the advantage of using thick metallic strips to join wires in a potentiometer?

Main concept used: $R = \rho \frac{l}{A}$.

Ans. As the area of cross-section of metallic strips in potentiometer (and meter bridge) is larger than a single wire. So the resistance of strip is much smaller by formula: $R = \rho \frac{l}{A}$.

A long resistance wire is used in potentiometer by winding it as shown in the figure.



The resistance below the metallic strips are out of circuit and calculation which makes easy to take readings and calculations.

Q3.16. For wiring in the home one uses Cu wires or Al wires. What considerations are involved in this?

Main concept used: Resistance and resistivity of metal, cost availability.

Ans. As the metals have low resistivities so metals have low resistance. The cost of metals used in electric circuits decreases from Ag, Cu, Al, Fe (steel). But Ag is costly so Cu or Al wires are used in wiring.

Q3.17. Why are alloys used for making standard resistance coils?

Main concept used: Dependence of resistance on temperature, atmospheric condition must be minimum and cost also.

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Ans. Dependence of resistance on change of temperature, humidity, pressure, etc. must be negligible. Alloys has small value of temperature coefficient and are not affected by moisture, etc.

Alloys has higher resistivity in turn the higher resistance so need smaller length to make coils which decrease the effect of inductance. Due to these reasons alloys are used to make standard resistance coils.

Q3.18. Power P is to be delivered to a device via transmission cables having resistance R_c. If V is the voltage across R and I the current flowing through it, find the power wasted, and how can it be reduced.

Main concept used: $P = VI$, $P = I^2R$.

Ans. As we know that P = VI so, to transmit a constant power P through transmission cable there are two ways:

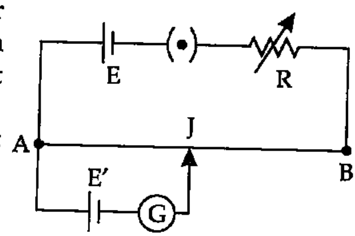
(*i*) If a constant power P is transmitted at low voltage (V) and high current (I). In this method high current will produce higher heat by $H = I^2R$ and power loss through cable is higher.

(*ii*) If a constant or same power be transmitted at high voltage (V) and low current. It gives lower loss of power as heat. But need thicker insulation during transmission.

So to transmit high power at long distance, we use low current and high (132 kV) voltage to minimize heat losses through towers and thicker (long) insulator.

To transmit power supply at short distance, we can transmit power at low 440 V, 220 V, 11 kV with higher current.

Q3.19. In given figure AB is potentiometer wire. If the value of R is increased, in which direction will the balance point J shift?



Main concept used: At null point current in galvanometer circuit is zero so potentials at A and J are equal and $K = \frac{V}{AB}$.

Ans. If R is increased current in main circuit will decrease (by V = IR) as the potential (E) is constant. So in turn potential difference across AB will decrease (by V = IR). As R of AB is constant so potential gradient $K = \frac{V}{AB}$ will decrease. So to balance potential across AB equal to potential of secondary circuit (E′) the length AJ′ must be larger than earlier AJ. So the point J shifts towards B.

Q3.20. While doing an experiment with potentiometer as in the figure, it was found that the deflection is one-sided and (*i*) the deflection decreased while moving from one-end (A) of the wire to the other end (B), (*ii*) the deflection increased while the jockey was moved towards end B.

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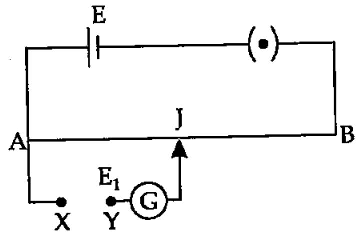
(*i*) Which terminal positive or negative of cell E_1 is connected at X in case (*i*) and how is E_1 related to E?

(*ii*) Which terminal of cell E_1 is connected at X in case (*ii*)?

Main concept used: (*i*) V = IR, (*ii*) $K = \frac{V}{l}$.



Ans. (*i*) One-sided deflection in galvanometer decreases when jockey moves towards B. So the potential in galvanometer circuit decreases as compared to potential across AJ earlier or potential between AJ′ increases. It is possible when positive terminal of E_1 is at X and negative at Y. So $E_1 > E$.

(*ii*) One-sided deflection in galvanometer increases when jockey moves from end A to B. So the potential in galvanometer circuit increases as compared to potential across AJ earlier or potential between AJ′ decrease. It is possible when positive terminal of E is at Y and negative is at X. So $E_1 < E$.

Q3.21. A cell of emf E and internal resistance *r* is connected across an external resistance R. Plot a graph showing the variation of P.D. across R, verses R.

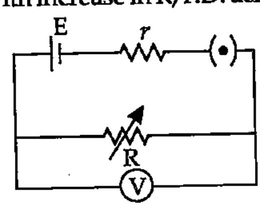
Main concept used: $V = \frac{ER}{R+r}$.

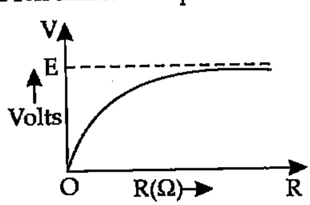
Ans. We know that $I = \frac{E}{R+r}$ and V = IR.

So
$$V = \frac{ER}{R+r} \qquad ...(I)$$
$$V = \frac{E}{1+\frac{r}{R}} \qquad ...(II)$$

Here E, *r* are constants. So
$$V \propto \frac{1}{1+\frac{r}{R}} \qquad \text{(from II)}$$
and
$$V \propto R \qquad \text{(from I)}$$

With increase in R, P.D. across R is increased upto maximum value E.





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SHORT ANSWER TYPE QUESTIONS

Q3.22. First set of n equal resistors of R each are connected in series to battery of emf E and internal resistance R. A current (I) is observed to flow. Then the n resistors are connected in parallel to the same battery. It is now observed that the current is increased 10 times. What is n?

Main concept used: (*i*) Series and parallel combination of resistances, (*ii*) $I = \frac{E}{R + r}$ for a battery.

Ans. When n resistances of each RΩ are connected in series and parallel then

$$R_S = R + R + R + \cdots n \text{ times} \quad \Rightarrow \quad R_S = nR$$

$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \cdots n \text{ times} \quad \Rightarrow \quad \frac{1}{R_p} = \frac{n}{R}$$

$$\Rightarrow \qquad \boxed{R_p = \frac{R}{n}}$$

When n resistors are connected in series connected with battery of emf E then current (I) flows. So

$$\frac{E}{R + nR} = I \qquad \text{...(I)}$$

and now n resistances are connected in parallel combination then current in circuit increased to 10 times of I

$$\therefore \qquad \frac{E}{R + \frac{R}{n}} = 10\,I$$

$$\frac{E}{R + \frac{R}{n}} = \frac{10\,E}{R + nR}$$

$$\frac{1}{R\left(1 + \frac{1}{n}\right)} = \frac{10}{R(1 + n)}$$

$$10\left(1 + \frac{1}{n}\right) = 1 + n$$

$$10 + \frac{10}{n} - 1 - n = 0$$

$$-n + \frac{10}{n} + 9 = 0 \qquad \text{[Multiply by } -n \text{ to both sides]}$$

$$n^2 - 10 - 9n = 0$$

$$n^2 - 9n - 10 = 0$$

$$n^2 - 10n + 1n - 10 = 0$$

$$n(n - 10) + 1(n - 10) = 0$$

So $\qquad (n + 1)\,(n - 10) = 0$

or $\quad n = -1$ is not possible $\quad$ or $\quad n = 10$.

So, there are 10 resistors in combination.

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Q3.23. Let there be n resistors $R_1 \dots R_n$ with $R_{max} = \max(R_1 \dots R_n)$ and $R_{min} = \min(R_1 \dots R_n)$. Show that when they are connected in parallel, the resultant resistance $R_p < R_{min}$ and when they are connected in series the resultant resistance $R_s > R_{max}$. Interpret the result physically.

Main concept used: (*i*) $R_s = R_1 + R_2 + \cdots,$

(*ii*) $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots.$

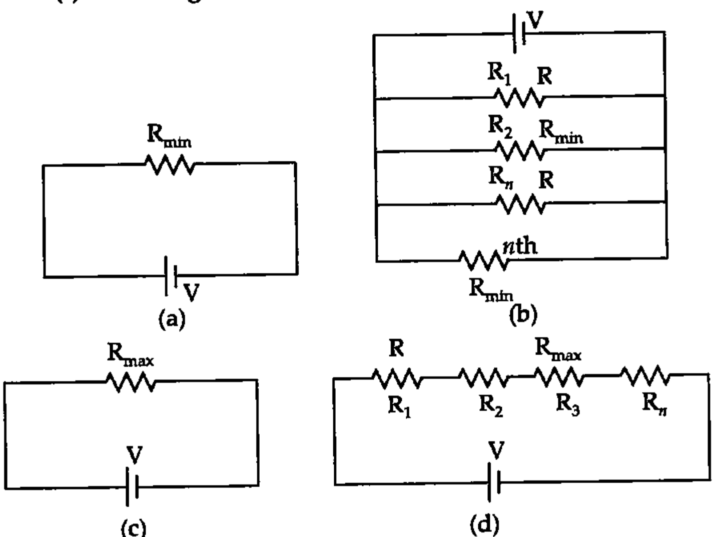
Ans. Let R_{min} and R_{max} are the minimum and maximum resistances among all resistances $R_1, R_2, \dots R_n$.

When resistors are connected in parallel then equivalent resistance R_p is $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_n}.$

Multiplying both sides by R_{min} we get,

$$\frac{R_{min}}{R_p} = \frac{R_{min}}{R_1} + \frac{R_{min}}{R_2} + \cdots + \frac{R_{min}}{R_n} \qquad \dots I$$

Among $R_1, R_2, \dots, R_n$, there must be a resistor which is minimum. So there must be a term $\dfrac{R_{min}}{R_{min}}$ in R.H.S. of equation (I). So R.H.S. in equation (I) must be greater than 1 as all other terms are also positive.



or $$\frac{R_{min}}{R_p} = \frac{R_{min}}{R_1} + \frac{R_{min}}{R_2} + \cdots + \frac{R_{min}}{R_{min}} + \cdots + \frac{R_{min}}{R_n} > 1$$

or $\dfrac{R_{min}}{R_p} > 1$ or $R_p < R_{min}$.

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So in parallel combination, the equivalent resistance R_p is always less than any smallest resistance in the combination.

When n resistors are connected in series then equivalent resistance—

$$R_s = R_1 + R_2 + R_3 + \cdots + R_n. \quad \text{...II}$$

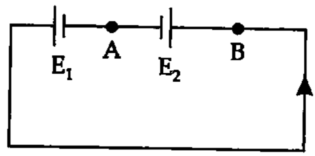
Here, in R.H.S. there must be a term R_{max} which has maximum value among R_1, R_1, ... R_n. As all terms in R.H.S. of equation II are positive so

$$R_s = R_1 + R_2 + R_3 + \cdots + R_{max} + \cdots + R_n > R_{max}$$

or $$R_s > R_{max}$$

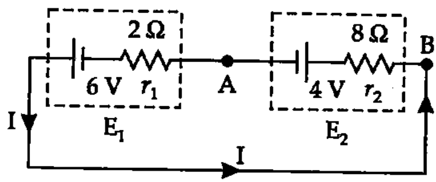
So in series combination equivalent resistance is always greater than the maximum resistance (R_{max}) among R_1, R_2, ..., R_n.

Q3.24. Here the circuit shows two cells connected in opposition to each other. Cell E_1 is of emf 6 V and internal resistance 2 Ω, the cell E_2 is of emf 4 V and internal resistance 8 Ω. Find the potential difference between points A and B.



Main concept used: Current in circuit from higher to lower potential, $I = \frac{E}{r}$.

Ans. The above figure can be redrawn as given here. The direction of current in circuit will be as shown in the figure.



So point B is at higher potential than A. So $V_B > V_A$.

Current (I) in circuit, $I = \frac{E_1 + E_2}{r_1 + r_2} = \frac{(6-4)\,V}{(2+8)\,\Omega} = 0.2$ amp

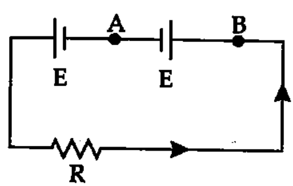
For positive potential A is near to positive terminal of E_2 so has +4 V. So potential across E_1 and E_2:

$$E_1 = V - Ir_1 = 6 - 0.2 \times 2 = 6 - 0.4 = 5.6 \text{ V}$$
$$E_2 = V + Ir_2 = 4 + 0.2 \times 8 = 4 + 1.6 = 5.6 \text{ V}$$

So potential between A and B = E_2 = 5.6 Volt.

As current is flowing from B to A. So potential at B is larger than A.

Q3.25. Two cells of same emf E but internal resistances r_1, r_2 are connected in series to an external resistor R (Figure). What should be the value of R so that potential difference across terminals of the first cell become zero?



Main concepts used: $I = \frac{E}{R + r}$

$V_0 = E - I_r$ and Kirchhoff's law, Ohms' law.

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Ans. Current I flowing in the circuit $I = \dfrac{E + E}{R + r_1 + r_2}$.

$$V_1 = E - Ir_1 = E - \frac{2E \cdot r_1}{R + r_1 + r_2}$$

The net potential difference across Ist cell $V_1 = 0$ (Given)

$$\therefore \quad E - \frac{2Er_1}{R + r_1 + r_2} = 0$$

$$\text{or} \quad 1 - \frac{2r_1}{R + r_1 + r_2} = 0$$

$$\frac{2r_1}{r_1 + r_2 + R} = \frac{1}{1}$$

$$\text{or} \quad 2r_1 = r_1 + r_2 + R$$

$$\boxed{r_1 - r_2 = R}$$

It is the required condition for the potential difference across Ist cell to be zero.

Q3.26. Two conductors are made of the same material and have the same length. Conductor A is solid wire of diameter 1 mm. Conductor B is a hollow tube of outer diameter 2 mm and inner diameter 1 mm. Find the ratio of resistances R_A to R_B.

Main concept used: $R_0 = \dfrac{\rho l}{A}$

Ans. Conductor A (solid wire R_A) **Conductor B (hollow tube R_B)**

$l_1 = l$	$l_2 = l$	(Given)
$A_1 = \pi r_1^2$	$A_2 = \pi r_2^2 - \pi r_1^2$	
$r_1 = \frac{1}{2}$ mm $= 0.5 \times 10^{-3}$ m	$r_2 = \frac{2}{2}$ mm $= 1 \times 10^{-3}$ m	
$\rho_1 = \rho$	$\rho_2 = \rho$	

$$\frac{R_A}{R_B} = \frac{\dfrac{\rho_1 l_1}{A_1}}{\dfrac{\rho_2 l_2}{A_2}} = \frac{\rho_1 l_1}{A_1} \times \frac{A_2}{\rho_2 l_2} = \frac{\rho l}{A_1} \times \frac{A_2}{\rho l} = \frac{A_2}{A_1}$$

$$\therefore \quad \frac{R_A}{R_B} = \frac{A_2}{A_1} = \frac{\pi r_2^2 - \pi r_1^2}{\pi r_1^2} = \frac{\pi(r_2^2 - r_1^2)}{\pi r_1^2} = \left(\frac{r_2}{r_1}\right)^2 - 1$$

$$= \left(\frac{1 \times 10^{-3}}{0.5 \times 10^{-3}}\right)^2 - 1 = (2)^2 - 1 = 4 - 1 = \frac{3}{1}$$

$$\therefore \quad R_A : R_B = 3 : 1.$$

Q3.27. Suppose there is a circuit consisting of only resistors and batteries. Suppose one is to double (or increase it to *n*-times) all voltage and resistances. Show that currents are unaltered.

(See NCERT Textbook Example 3.7)

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Main concept used: $I = \frac{E}{R + r}$.

Ans. Case I: Consider a circuit having external $R_1, R_2, ...,$ connected with some batteries $E_1, E_2, E_3 ...$ having their internal resistances $r_1, r_2, r_3, ...$

Let the equivalent resistance, emf and internal resistance of above combination is R_{eq}, E_{eq}, and r_{eq} respectively. So the current passing in the circuit, $I_1 = \frac{E_{eq}}{R_{eq} + r_{eq}}$.

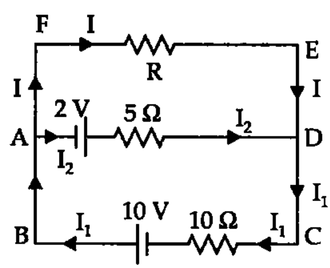
Now the resistances and cells are again connected in a manner that their equivalent resistance, emf and internal resistances are nR_{eq}, nE_{eq} and nr_{eq} respectively. So again current in new circuit $I_2 = \frac{nE_{eq}}{nR_{eq} + nr_{eq}}$.

$$I_2 = \frac{nE_{eq}}{n[R_{eq} + r_{eq}]} = \frac{E_{eq}}{R_{eq} + r_{eq}} = I_1$$

So the current remains same if the R, E and r of a circuit is increased by n times, *i.e.* nR, nE, nr.

LONG ANSWER TYPE QUESTIONS

Q3.28. Two cells of voltage 10 V and 2 V and internal resistances 10 Ω and 5 Ω respectively are connected in parallel with the positive end of 10 V battery connected to negative pole of 2 V battery as in figure. Find the effective voltage and effective resistance of combination.



Main concepts used: V = IR, Kirchhoff's law.

Ans. Applying junction rule at A

$$\boxed{I_1 = I + I_2} \quad ...I$$

Apply Kirchhoff's loop rule on loop B CEF and loop ADEF

$$10 = IR + 10\,I_1 \quad ...II$$
$$2 = -IR + 5\,I_2 \quad ...III$$
$$2 = -IR + 5(I_1 - I) \quad \text{[from I]}$$
$$2 = -IR + 5\,I_1 - 5\,I \quad ...IV$$
$$4 = -2IR + 10\,I_1 - 10\,I \quad \text{[on multiplying IV by 2] ...V}$$
$$10 = IR + 10\,I_1$$
$$- \quad - \quad -$$
$$-6 = -3\,IR - 10\,I \quad \text{[on subtracting (II) from (V)]}$$

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or $3IR + 10I = 6$

$$I(3R + 10) = 2 \times 3$$

$$\frac{I(3R+10)}{3} = 2$$

$$2 = I\left(R + \frac{10}{3}\right) \quad \text{...VI}$$

Let the effective potential difference due to both batteries is V_{eq}. It will be across resistance R. So

$$V_{eq} = I(R + R_{eq}) \quad \text{...VII}$$

Where R_{eq} is the resistance of circuit except R

Comparing (VI) and (VII),

$$V_{eq} = \mathbf{2\ Volts\ and}\ R_{eq} = \frac{10}{3}\ \Omega$$

Q3.29. A room has AC (air-conditioner) that runs 5 hours a day at voltage 220 V. The wiring of the room consists of Cu wire of 1 mm radius and length of 10 m. Power consumption per day is 10 commercial units. What fraction of it goes in the Joule heating in wires? What would happen if the wiring is made of the aluminium of the same dimensions?

[$\rho_{Cu} = 1.7 \times 10^{-8}\ \Omega\text{-m}$, $\rho_{Al} = 2.7 \times 10^{-8}\ \Omega\text{-m}$]

Main concepts used: $P = I^2R$, $R = \frac{\rho l}{A}$.

Ans. Total energy consumed in 5 hrs a day by AC and wiring

$$= 10 \text{ kWh}$$

∴ Energy consumed in 1 hr by AC and wiring

$$= 2 \text{ kWh}$$

So total power of AC and wire = 2000 W

$$P = VI$$

$$I = \frac{P}{V} = \frac{2000}{220} \cong 9.0 \text{ A}$$

Let P_0 is power of wiring then,

$$P_0 = I^2R_w \quad [R_w = \text{resistance of wiring}]$$

$$= 9 \times 9 \cdot \frac{\rho \cdot l}{A}$$

$$= \frac{9 \times 9 \times 1.7 \times 10^{-8} \times 10}{3.14 \times 1 \times 10^{-3} \times 1 \times 10^{-3}} = \frac{81 \times 17 \times 10^{-8+6}}{3.14}$$

$$= \frac{1377}{3.14} \times 10^{-2} = 4.38 = 4.4 \text{ Watt}$$

So loss of energy in wiring ≅ 4.4 J/sec

The fractional loss due to heating of wires $= \frac{4.4}{2000} \times 100\% = 0.22\%$

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$$\frac{P_{Al}(\text{wiring})}{P_{Cu}(\text{wiring})} = \frac{I^2R_{Al}}{I^2R_{Cu}} = \frac{\rho_{Al}\dfrac{l_{Al}}{A_{Al}}}{\rho_{Cu}\dfrac{l_{Cu}}{A_{Cu}}} \quad \text{as } l_{Al} = l_{Cu} \text{ and } A_{Al} = A_{Cu}$$

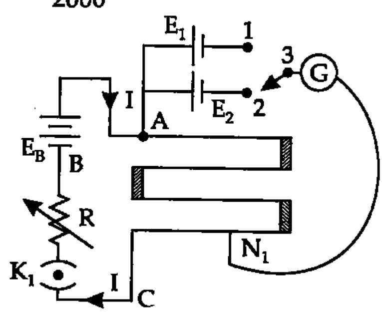
$$\frac{P_{Al}}{P_{Cu}} = \frac{\rho_{Al}}{\rho_{Cu}} \quad \text{or} \quad P_{Al} = \frac{2.7 \times 10^{-8}}{1.7 \times 10^{-8}} \times 4.4 \text{ Watt}$$

$$P_{Al} = 7 \text{ Watt}$$

So power loss in Al wiring = 7 Watt

The fractional loss due to Al wiring $= \dfrac{7}{2000} \times 100\% = 0.35\%$.

Q3.30. In an experiment with potentiometer, V_B = 10 V. R is adjusted to 50 Ω in the figure. A student wants to measure voltage E_1 of battery (approx. 8 V), finds no null point possible. He then diminishes R to 10 Ω and is able to locate the null point on the last (4th) segment of the potentiometer. Find the resistance of the potentiometer wire and potential drop per unit length across the wire in the second case.



Main concepts used: (*i*) $I = \dfrac{V}{R}$, (*ii*) Potentials at A and jockey point N_1 are equal at balance condition, (*iii*) Null point is obtained if E_1 and $E_2 < V_{AB}$ (wire of potentiometer).

Ans. Let R′ be the resistance of potentiometer wire.

∵ Variable resistance, R = 50 Ω

I is the current in primary circuit which is at E_B = 10 V.

∴ $$I = \frac{V_B}{R + R'} \Rightarrow \frac{10}{50 + R'} = I \text{ (in primary circuit)} \quad \text{...I}$$

Potential difference across the wire of potentiometer

$$V' = IR'$$

From I $$V' = \frac{10\,R'}{50 + R'} \quad \text{...II}$$

As with R = 50 Ω resistance, null point cannot be obtained by 8 Volt.

So $$V' < 8 \text{ Volt.}$$

$$\frac{10\,R'}{50 + R'} < 8 \quad \text{(No balance point)}$$

As 50 + R′ is positive so we can multiply above equation by positive number and we get

$$10\,R' < 400 + 8\,R'$$
$$2\,R' < 400$$

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$$R' < 200 \quad \text{...III}$$

Similarly, null point is obtained by R = 10 Ω. then V″ > 8 at balance point

So it is possible when $\frac{10\,R'}{10 + R'} > 8$ (as from I, R = 10)

Similarly, multiply above equation by positive number 10 + R′ to both sides

$$10\,R' > 80 + 8\,R'$$
$$2\,R' > 80$$
$$R' > 40 \quad \text{...IV}$$

As the null point is obtained on 4th segment or at $\frac{3}{4}$ of total length so at $\frac{3}{4}R'$ (No balance point)

or $$\frac{10 \times \frac{3}{4}R'}{10 + R} < 8 \quad \text{(At balance point)}$$

So $$7.5\,R' < 80 + 8\,R$$
$$-0.5\,R' < 80$$
$$-R' < 160$$
$$R' > -160$$

R′ can never be negative so, –160 Ω is considered 160 Ω

So $$\boxed{160 < R' < 200} \quad \text{...V}$$

Any R′ between 160 Ω and 200 Ω will achieve null point. Since the null point is on last 4th segment of potentiometer wire, so the potential drop across 400 cm wire > 8 Volt.

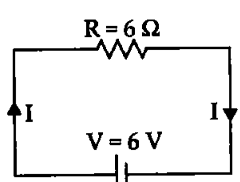
So $$K.400 \text{ cm} > 8 \text{ V} \quad \text{(At balance point)}$$
$$K > \frac{8}{400} \text{ Volt/cm}$$
$$K > \frac{8}{4} \text{ Volt/m}$$
$$K > 2 \text{ Volt/m}$$

As balance point is at 4th wire, so no balance point at 3 m,

i.e., $$K.3 < 8 \quad \text{(No balance point)}$$
$$K < \frac{8}{3} \text{ Volt/m}$$
$$K < 2\frac{2}{3} \text{ Volt/m}$$

So $$\boxed{2\frac{2}{3} \text{ V/m} > K > 2 \text{ Volt/m}}\,.$$

Q3.31. (*a*) Consider circuit in figure. How much energy is absorbed by electrons from the initial state of no current (ignore thermal motion) to the state of drift velocity?



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(*b*) Electrons give up energy at the rate of RI^2 per second to the thermal energy. What time scale would one associate with energy in problem (*a*)?

n = Number of free electrons/volume = $10^{29}/m^3$

Length of circuit = 10 cm

Cross-sectional area = A = 1 mm^2.

Main concepts used: $I = Anev_d$, $KE = \frac{1}{2} m v_d^2$ per electron, ohmic loss of energy per sec = RI^2, V = RI.

Ans. $A = 1\ mm^2 = 10^{-6}\ m^2$ $\quad$ $R = 6\ \Omega$

$n = 10^{29}/m^3$ $\quad$ $V = 6\ V$

$e = 1.6 \times 10^{-19}$ C $\quad$ $I = \frac{V}{R} = \frac{6}{6} = 1$ amp

$m_e = 9.1 \times 10^{-31}$ kg $\quad$ $l = 10\ cm = 10^{-1}$ m

(*a*) $\because \quad I = Anev_d$

$$\therefore \quad v_d = \frac{I}{Ane} = \frac{1}{10^{-6} \times 10^{29} \times 1.6 \times 10^{-19}} \text{ m/s}$$

$$= \frac{1}{1.6} \times 10^{+6+19-29} = \frac{1}{1.6} \times 10^{-4} \text{ m/s}$$

$$KE = \frac{1}{2} m v_d^2 \text{ per electron}$$

Number of electrons (free) in wire = n (volume of wire)

$= n \times Al$

$$\therefore \text{ KE of all electrons} = \frac{1}{2} m v_d^2\ Anl$$

$$KE = \frac{1}{2} \times 9.1 \times 10^{-31} \times \frac{10^{-4} \times 10^{-4}}{1.6 \times 1.6} \times 10^{-6} \times 10^{29} \times 10^{-1}$$

$$= \frac{9.1}{2 \times 2.56} \times 10^{-31-8-7+29} = 1.78 \times 10^{-46+29}$$

$$= 1.78 \times 10^{-17} J.$$

So, to start flow of current I, the electrons will take energy from cell

= KE of all electrons = 1.78×10^{-17} J

(*b*) Loss of energy during current flowing = I^2R.

$P = 1 \times 1 \times 6 = 6$ Joule per second

$\because \quad$ Energy = P.t

or $\quad t = \frac{E}{P} = \frac{1.78 \times 10^{-17}}{6}$

$\cong 0.29 \times 10^{-17}$ sec $\cong 0.3 \times 10^{-17}$

$= \mathbf{3 \times 10^{-18}}$ **second.**

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4 Moving Charges and Magnetism

MULTIPLE CHOICE QUESTIONS—I

Q4.1. Two charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field $B = B_0\hat{k}$.

(*a*) They have equal z-components of momenta.

(*b*) They must have equal charges.

(*c*) They necessarily represent a particle-antiparticle pair.

(*d*) The charge to mass ratio satisfy $\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0.$

Main concepts used: (*i*) Pitch, $P = \frac{2\pi mv\cos\theta}{Bq}$,

(*ii*) Law of conservation of momenta.

Ans. (*d*): For a given pitch, $P = \frac{2\pi mv\cos\theta}{Bq}$

$$\frac{q}{m} = \frac{2\pi v\cos\theta}{BP}$$ [θ is angle of velocity of charge particle with X-axis]

If motion is not helical, $\theta = 0$.

As path of both the particles is identical and helical but of opposite direction in same magnetic field so by law of conservation of momenta

$$\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0.$$

So, verifies answer (*d*).

Q4.2. Biot-Savart law indicates that the moving electrons (with velocity *v*) produce a magnetic field B such that

(*a*) B is perpendicular to velocity *v*.

(*b*) B is parallel to *v*.

(*c*) it obeys inverse cube law.

(*d*) it is along the line joining the electron and point of observation.

Main concept used: Biot-Savart law.

Ans. (*a*): By Biot-Savart law, $dB = \frac{I\,dl\sin\theta}{r^2}$

or $$dB = \frac{I \times dl}{r}$$

I can be considered flow of charge.

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So the magnetic field is perpendicular to the direction of flow of charge verifies answer '*a*'.

Q4.3. A current-carrying circular loop of radius R is placed in *x*-*y* plane with centre at origin. Half of the loop with $x > 0$ is now bent so that it now lies in the *y*-*z* plane.

(*a*) The magnitude of magnetic moment now diminishes.
(*b*) The magnetic moment does not change.
(*c*) The magnitude of B at (0, 0, *z*), $z >> R$ increases.
(*d*) The magnitude of B at (0, 0, *z*), $z >> R$ is unchanged.

Main concept used: Direction of magnetic field due to circular loop (Right-Hand Thumb-Rule).

Ans. (*a*): As the direction of magnetic field due to current-carrying circular loop is perpendicular and it is perpendicular to plane of loop and unidirectional.

In first case, direction of magnetic field is only in positive *x*-*z* direction but when it is bented then half of B is along *z*-*x* axis (due to unfolded loop) and half of B is along +*x* direction so vector sum of B will decrease. Verifies answer (*a*).

Q4.4. An electron is projected with uniform velocity along the axis of a current-carrying long solenoid. Which of the following is true?

(*a*) Electron will be accelerated along the axis.
(*b*) The electron path will be circular about the axis.
(*c*) The electron will experience a force at 45° to the axis and hence execute a helical path.
(*d*) The electron will continue to move with uniform velocity along the axis of the solenoid.

Main concept used: Lorentz force.

Ans. (*d*): The Lorentz force acts on a charged particle in a magnetic and electric field is $F = q[\vec{E} + v \times B]$. As there is no E, force due to E.F. is zero and force due to B is perpendicular to the direction of *v* and B which will be perpendicular to the direction of motion (*v*), so will not affect the velocity of moving charge particle. So verifies answer (*d*).

Q4.5. In a cyclotron, a charged particle

(*a*) undergoes acceleration all the time.
(*b*) speeds up between the dees because of the magnetic field.
(*c*) speeds up in a dee.
(*d*) slows down within a dee and speeds up between dees.

Main concept used: Working of cyclotron and motion of charged particle in magnetic field, electric field or both.

Ans. (*a*): There is crossed electric and magnetic field between dees so the charged particle accelerates by electric field between Dee's towards other Dee.

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Inside dees, there is no electric field due to shielding effect of charge or field. So only magnetic force keeps the circular motion of charged particle inside (any circular motion is also accelerated).

So the charged particle accelerates inside and between dees always verifies answer (*a*).

Q4.6. A circular current loop of magnetic moment M is in an arbitrary orientation in an external magnetic field B. The work done to rotate the loop by 30° about an axis perpendicular to its plane is

(*a*) MB (*b*) $\frac{\sqrt{3}}{2}$MB (*c*) $\frac{MB}{2}$ (*d*) zero.

Main concept used: Work done by loop in orientation

$$= MB(\cos\theta_2 - \cos\theta_1). \quad \text{Where } M = N/A$$

Ans. (*b*) and (*d*): When the axis of rotation of loop is along B then angle between $\vec{B}$ and $\vec{A}$ is 90° always. So WD by loop to rotate i.e., WD = MB cos 90°. So WD is zero. Verifies option (*d*).

But when the axis of rotation of loop is not along the direction of B, then direction of vector B and A will change with time.

Work done by loop during orientation in uniform magnetic field

$$= MB(\cos\theta_2 - \cos\theta_1) = MB\cos\theta$$

$$= MB\cos 30° = MB\frac{\sqrt{3}}{2}$$

So answer (*b*) is verified.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q4.7. The gyro-magnetic ratio of an electron in an H-atom, according to Bohr's model is

(*a*) independent of which orbit it is in.
(*b*) negative.
(*c*) positive.
(*d*) increases with the quantum number *n*.

Main concept used: Gyro-magnetic ratio,

$$\mu_e = \frac{\text{Magnetic moment of } e}{\text{Angular momentum of } e}.$$

Ans. (*a*) and (*b*): Magnetic moment of

$$e = I \cdot A = \frac{-e}{T} \cdot \pi r^2$$

$$M_e = \frac{-e\pi r^2}{\frac{2\pi r}{v}} = \frac{-e\pi r^2 v}{2\pi r} = \frac{-evr}{2}$$

The angular momentum of $e = L = m\vec{v}r$

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$\therefore$ Gyro-magnetic ratio, $\mu_e = \dfrac{\text{Magnetic moment of } e}{\text{Angular momentum of } e} = \dfrac{-evr}{2.mvr} = \dfrac{-e}{2m}$.

So it is independent of velocity or orbit of *e* depends only on charge and is with negative sign, *i.e.* μ_e of *e* is opposite of any positive charge. So verified answers (*a*) and (*b*).

Q4.8. Consider a wire carrying a steady current I placed in a uniform magnetic field B perpendicular to its length. Consider the charges inside the wire. It is known that magnetic forces do not work. This implies that,

(*a*) motion of charges inside the conductor is unaffected by B since they do not absorb energy.

(*b*) some charges inside the wire move to surface as a result of B.

(*c*) if the wire moves under the influence of B, no work is done by the force.

(*d*) if the wire moves under the influence of B, no work is done by the magnetic force on the ions, assumed fixed within the wire.

Main concept used: Force on current-carrying conductor placed in magnetic field B is equal to $BIl \sin\theta$, its direction is perpendicular to B and I (or *l*).

Ans. (*b, d*): Force (F) on current-carrying conductor by magnetic field B is perpendicular to $\vec{B}$. So by formula—

$$F = BIL \sin\theta = I \times B \times L$$

F is perpendicular to B and L both by Fleming's Left-hand Rule.

So work done by magnetic field is $W = F.l \cos\theta$.

$W = F.l \cos 90°$ will be equal to zero. Verifies the answer (*d*).

Due to magnetic induction some charges can move on the surface of conductor verifies answer (*b*).

Q4.9. Two identical current-carrying coaxial loops carry current I in an opposite sense. A simple amperian loop passes through both of them once. Calling the loop as C,

(*a*) $\oint_C B.dl = \mp 2\mu_0 I$.

(*b*) the value of $\oint_C B.dl$ is independent of sense of C.

(*c*) there may be a point on C where $\vec{B}$ and $\vec{dl}$ are perpendicular.

(*d*) B vanishes everywhere on C.

Main concept used: Ampere's circuital law.

Ans. (*b*) and (*c*): Loops are identical placed coaxially and carrying same current in opposite sense. So inside amperian loop of any type direction of current will be opposite by Ampere's circuital law

$$\oint_C B.dl = \mu_0(I - I) = \mu_0(0) = 0$$

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As the magnetic field inside (over everywhere) the loop is perpendicular to the direction of plane of loop, so

$$\oint_C \mathbf{B}.dl = |\mathbf{B}.\vec{dl}| \cos 90° = 0$$

So, answers (*b*) and (*c*) are verified.

Q4.10. A cubical region of space is filled with some uniform electric and magnetic fields. An electron enters the cube across one of its faces with velocity **v**, and a positron enters via opposite face with velocity –**v**. At this instant,

(*a*) the electric forces on both the particles cause identical accelerations.

(*b*) the magnetic forces on both the particles cause equal accelerations.

(*c*) both particles gain or loose the energy at same rate.

(*d*) the motion of centre of mass (CM) is determined by 'B' alone.

Main concept used: Lorentz force, $F = q(E + v \times B)$.

Ans. (*b*), (*c*) and (*d*): As $F = qE$ here E is same but q is in opposite nature force. Electric force or acceleration is not identical. It discards answer (*a*).

As the $F = qv \times B$, *i.e.*, F is perpendicular to velocity and magnetic field, so particle revolves perpendicular to both $\vec{B}$ and $\vec{v}$ with uniform speed. But magnitude of acceleration by magnetic field is equal. It verifies answer (*b*).

As magnitudes of charge $\vec{v}$, $\vec{E}$ and B are constant, so gain or loose the energy at the same rate verifies answer (*c*).

As there is no change in centre of mass of particles therefore the motion of centre of mass is determined by B alone. It verifies answer (*d*).

Q4.11. A charged particle would continue to move with a constant velocity in a region wherein,

(*a*) $E = 0, B \neq 0,$ (*b*) $E \neq 0, B \neq 0$

(*c*) $E \neq 0, B = 0,$ (*d*) $E = 0, B = 0$

Main concepts used: (*i*) Lorentz force, (*ii*) How the net force on charged particle may be zero.

Ans. (*a*), (*b*) (*d*):

We know that $F_L = F_e + F_m = qE + qv \times B$

The velocity $\vec{v}$ of a charge (q) particle in magnetic field ($\vec{B}$) and electric field ($\vec{E}$) will be constant. If Lorentz force (F_L) on q is zero. As

$$\vec{F}_m = q\vec{E} + q\vec{v} \times \vec{B}$$

i.e., for constant v $\quad F_m = 0$

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(*i*) When $E = 0$, but $B \neq 0$, then for $F_m = 0$

$$q(0) + qv \times B = 0 \quad \text{or} \quad q(v \times B) = 0 \qquad ...I$$

$q \times v \times B$ will be zero as the force F_m is $\perp$ to the direction of $\vec{B}$ and $\vec{v}$ both (By Fleming's left hand rule) $F_m = 0$. So, verifies option (*a*).

(*ii*) If $B \neq 0$, $E \neq 0$, Consider $F_m = 0$ may or may not be as under

$$q\vec{E} + q(\vec{v} \times \vec{B}) = 0$$

$$q(E + v \times B) = 0$$

$$q \neq 0 \text{ so } E + (v \times B) = 0$$

or $$qE = -q(v \times B)$$

Above two forces F_m and F_e may be equal and opposite when the direction of E is opposite to direction of $(v \times B)$ and the magnitude of E and B must be in such a way that

$$(qE) = q(v \times B)$$

i.e., $$|E| = v \times B$$

$\Rightarrow$ E.F. must be $|v|$ times of B and perpendicular to B.

(*iii*) $B = 0$, $E \neq 0$

$$qE \neq 0$$

It will change the velocity or direction of v and v cannot be constant discards option (*c*).

(*iv*) When $E = 0$ and $B = 0$. Then $qE + qv \times B = 0$

$$0 + 0 = 0$$

So no force acts on charge particle. Hence it will move with uniform velocity, verifies option (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q4.12. Verify that the cyclotron frequency $\omega = \frac{eB}{m}$ has the correct dimensions of $[T]^{-1}$.

Main concepts used: (*i*) For a circular motion, there must be centripetal force perpendicular to velocity. (*ii*) Magnetic force is perpendicular to motion of particle.

Ans. In cyclotron, charged particles revolve in circular orbit due to magnetic force which acts perpendicular to the velocity of particle.

So it provides the centripetal force for revolution. So, $\frac{mv^2}{R} = qv \times B$.

Here, $\theta = 90°$ as θ is angle between $\vec{v}$ and $\vec{B}$

$$\therefore \quad \frac{mv^2}{R} = qvB \quad \Rightarrow \quad \frac{v^2}{Rv} = \frac{qB}{m} \quad \text{or} \quad \frac{qB}{m} = \frac{v}{R}$$

$\because$ $\omega = \frac{qB}{m}$ so dimensions of below must be equal.

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So $$[\omega] = \left[\frac{qB}{m}\right] = \left[\frac{v}{R}\right]$$

$$\left[\frac{2\pi}{T}\right] = \left[\frac{LT^{-1}}{L}\right] = [T^{-1}]$$

So dimensions of ω is $[T^{-1}]$.

Q4.13. Show that a force that does no work must be a velocity dependent force.

Main concept used: W.D. = F.dl

Ans. As work done by force is zero, so

$$dW = F.dl = 0$$

$$\Rightarrow \quad F.\frac{dl}{dt} \times dt = 0$$

$$dW = F.v\,dt = 0$$

$$dt \neq 0 \qquad [\therefore F.v = 0]$$

So F must be velocity-dependent, *i.e.*, angle between F and v must be 90° always, then

For $$F.v = 0$$

$$Fv \cos\theta = \cos 90°$$

$$\theta = 90°$$

If v changes direction then to make θ = 90°, F must change angle according to v. So F is dependent on v to make work done zero.

Q4.14. The magnetic force depends on $\vec{v}$ which depends on the inertial frame of reference. Does then magnetic force differ from inertial frame to frame? Is it reasonable that the net acceleration has a different value in different frames of reference?

Main concept used: Propagation of electromagnetic waves.

Ans. The magnetic force changes from inertial frame to frame, *i.e.* the magnetic force depends on frame of reference. So the net acceleration which comes into existence out of this is, however, frame independent (non-relativistic physics) for inertial frame.

Q4.15. Describe the motion of a charged particle in a cyclotron if the frequency of radio frequency (rf) field were doubled.

Main concept used: In cyclotron, frequency of charged particle is equal to the frequency of radio, $\left(T = \frac{2\pi}{\omega}\right)$.

Ans. When the frequency ω of electric field (oscillator) is doubled, the time-period $\left(T = \frac{2\pi}{\omega}\right)$ becomes half. So the charged particle will take half time to reach between dees. Hence, a charged particle accelerates as it moves in circular path between the dees during motion in Dee's the radius of moving charged particle remain same.

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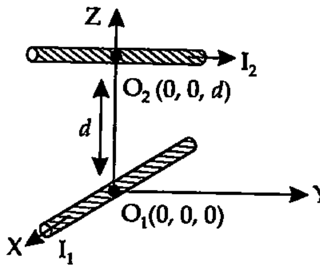
Q4.16. Two long wires carrying current I_1 and I_2 are arranged as shown in figure. The one carrying current I_1 is along the X-axis. The other carrying current I_2 is along a line parallel to Y-axis given by X = 0, Z = *d*. Find the force exerted at O_2 because of the wire along X-axis.



Main concept used: RHGR to find the direction of magnetic field and $F = BIl \sin\theta$.

Ans. We know that force on current (I) carrying conductor placed in magnetic field B is

$$F = B \times I.dl = BIdl \sin\theta.$$

The direction of magnetic field at O_2 due to the current I_1 is parallel to Y-axis and in –Y direction.

As wire of current I_2 is parallel to Y-axis, current in I_2 is also along Y-axis. So I_2 and B_1 (magnetic field due to current I_1) are also along Y-axis *i.e.*, angle between I_2 and B_1 is zero. So magnetic force F_2 on wire of current I_2 is $F_2 = B_1 I_2\, dl \sin 0° = 0$.

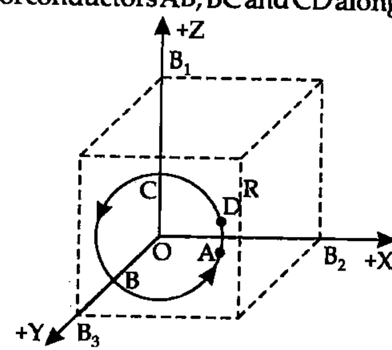
Hence, force on O_2 due to wire of current I_1 is zero.

SHORT ANSWER TYPE QUESTIONS

Q4.17. A current-carrying loop consists of 3 identical quarter circles of radius R, lying in the positive quadrants of X-Y, Y-Z and Z-X planes with their centres at the origin, joined together. Find the direction and magnitude of B at the origin.

Main concepts used: (*i*) Direction of magnetic field in a current-carrying loop, (*ii*) Magnitude of magnetic field B due to an arc.

Ans. Consider in figure, 3 quadrants of conductors AB, BC and CD along positive X-Y, Y-Z and Z-X planes respectively. A and D are connected to a battery which is responsible to flow current I through the three quadrants of radius R coordinate of A or D (R, 0, 0), B(0, R, 0) and of C(0, 0, R). Now the direction of magnetic field by right-hand thumb rule due to quadrants AB, BC and CD are $+B_1$, B_2 and B_3 along +Z, +X and +Y directions respectively. So, at the centre of quadrant



$$\because \quad \vec{B} = \frac{\mu_0 I}{2\pi R} \cdot \frac{\frac{\pi}{2}}{2\pi}$$

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$$B = \frac{\mu_0 I}{8\pi R}$$

So M.F. due to quadrants AB, BC and CD at their centre O are B_1, B_2 and B_3 respectively.

$$B_1 = \frac{\mu_0 I}{8\pi R}\hat{k},\quad B_2 = \frac{\mu_0 I}{8\pi R}\hat{i},\ \text{and}\ B_3 = \frac{\mu_0 I}{8\pi R}\hat{j}.$$

So net magnetic field at origin due to three current-carrying loops $B = B_1 + B_2 + B_3$.

$$B = \frac{\mu_0 I}{8R}[\hat{i} + \hat{j} + \hat{k}]$$

The resultant of B_1,B_2 and B_3 will be diagonal OR of cube of side B_1, B_2, B_3 as the $|B_1| = |B_2| = |B_3|$

Q4.18. A charged particle of charge e and mass m is moving in an electric field $\vec{E}$ and magnetic field $\vec{B}$. Construct dimensionless quantities and quantities of dimension $[T]^{-1}$.

Main concept used: How a charged particle moves in a magnetic and electric field.

Ans. When a charged particle is placed in an electric and magnetic field, its motion will be circular, and centripetal force is applied by magnetic force $F_m = qvB \sin 90° = qvB$.

$\therefore$ Centripetal force $= \frac{mv^2}{R}$ is balanced by $F_m = qvB$

or $$\frac{mv^2}{R} = qvB$$

$$\frac{v}{R} = \frac{qB}{m}$$

$\because$ $v = \omega R$ and $q = e$

$\therefore$ $$\omega = \frac{v}{R} = \frac{eB}{m}$$

Dimensional formula for angular velocity ω

$$\omega = \left[\frac{eB}{m}\right] = \left[\frac{v}{R}\right] = [T^{-1}]$$

Q4.19. An electron enters with a velocity $v = v_0\hat{i}$ into a cubical region (faces parallel to coordinate planes), in which there are uniform electric and magnetic fields. The orbit of electron is found to spiral down inside the cube in the plane parallel to X-Y plane. Suggest a configuration of fields $\vec{E}$ and $\vec{B}$ that can lead to it.

Main concept used: Motion of charged particle in magnetic and electric field is helical.

Ans. The velocity of electron is $v = v_0\hat{i}$, *i.e.*, along X-axis so magnetic field is in Y direction.

$$B = B_0\hat{k}$$

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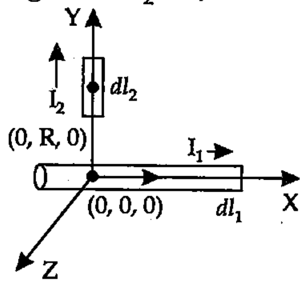
The moving electron enters into cubical region. The force on electron due to Lorentz force

$$F_m = -e[v_0\hat{i} \times B\hat{k}] = -ev_0B\hat{j}$$

which revolve the electron in X-Y plane.

The force due to electric field $F_e = e\vec{E}\hat{k}$ accelerates electron along Z-axis which in turn increases the radius of circular path. So the motion becomes **helical path.**

Q4.20. Do magnetic forces obey Newton's third law? Verify for two current elements $dl_1 = dl\hat{i}$ located at the origin and $dl_2 = dl\hat{j}$ located at (0, R, 0). Both carry current I.



Main concept used: The direction of magnetic field due to current-carrying conductor. And direction of force on current-carrying conductor placed in magnetic field.

Ans. The direction of magnetic field on dl_2 due to magnetic field B_1 by dl_1 will be along +Z direction by Right-Hand Grip Rule, *i.e.*, directions of I_2 and B_1 are perpendicular.

Force on dl_2 due to $dl_1 = B_1I_2dl_2 \sin 90° = B_1I_2l_2$

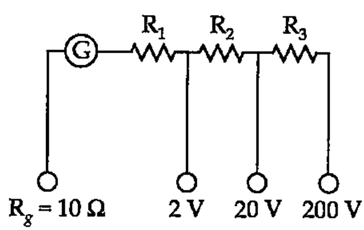
Similarly, angle between magnetic field B_2 and current I_1 is 0°. So the force acting on dl_1 due to dl_2

$$= B_2I_1\, dl_1 \sin 0° = 0.$$

$$\therefore \qquad |dl_1| = |dl_2| = |dl|$$

So magnetic force existing on dl_1 is zero but on dl_2 due to dl_1 is not zero so magnetic forces do not obey Newton's third law.

Q4.21. A multirange voltmeter can be constructed by using a galvanometer circuit, as shown in figure. We want to construct a voltmeter that can measure 2 V, 20 V, 200 V using galvanometer of resistance 10 Ω and that produces maximum deflection for current of 1 mA. Find R_1, R_2 and R_3 that have to be used.



Main concept used: Resistance of galvanometer R_g, R_1, R_2 and R_3 are in series and G can tolerate $I_g = 1$ mA.

Ans. For 2 V:

$$I_g(R_g + R_1) = 2\text{ V}$$

$$1 \times 10^{-3}[10 + R_1] = 2$$

$$R_1 = 2000 - 10 = \mathbf{1990\ \Omega}.$$

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For 20 V: $\boxed{I(R_g + R) = V}$ $\quad \because R = R_1 + R_2$

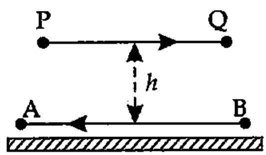
$$I_g[R_g + R_1 + R_2] = 20\text{ V}$$
$$10^{-3}[10 + 1990 + R_2] = 20$$
$$R_2 = 20000 - 2000$$
$$R_2 = 18000\ \Omega = \mathbf{18\ k\Omega}.$$

For 200 V: $\quad I_g[R_g + R_1 + R_2 + R_3] = 200\text{ V} \quad \because R = (R_1 + R_2) + R_3$

$$10^{-3}[10 + 1990 + 18000 + R_3] = 200$$
$$20000 + R_3 = 200000$$
$$R_3 = 200000 - 20000$$
$$= 180000\ \Omega = \mathbf{180\ k\Omega}$$

Hence, $R_1 = 1990\ \Omega$; $R_2 = 18\ k\Omega$; $R_3 = 180\ k\Omega$

Q4.22. A long straight wire carrying current of 25 A rests on a table as shown in figure. Another wire PQ of length 1 m and mass 2.5 g carries the same current but in opposite direction. The wire PQ is free to slide up and down. To what height will PQ rise?



Main concept used: Direction and magnitude of magnetic field due to current-carrying conductor and force on a current-carrying conductor placed in magnetic field.

Ans. Wire PQ must experience a repulsive force due to magnetic field by wire AB.

Let the wire is balanced at height h thus, magnetic force due to wire AB on PQ = m g

$$F_m = m\,g$$

Let magnetic field due to AB at height h is B_1 and length of PQ is l_2, balanced at height h. The angle between B_1 and I in PQ is 90°.

$\therefore$
$$B_1 I_2 l_2 \sin\theta = m\,g$$
$$\frac{\mu_0 I_1}{2\pi h} \cdot I_2 l_2 \sin 90° = m\,g \qquad [I_1 = I_2 = I = 25\text{ A}]$$
$$\frac{\mu_0 I^2 l_2}{2\pi h} = m\,g$$
$$h = \frac{\mu_0 I^2 l_2}{2\pi\, mg} = \frac{4\pi \times 10^{-7} \times 25 \times 25 \times 1}{2\pi \times 2.5 \times 10^{-3} \times 9.8}$$
$$= \frac{2 \times 625}{2.5 \times 9.8} \times 10^{-7+3} = \frac{20 \times 25}{9.8} \times 10^{-4}\text{ m} = 51 \times 10^{-4}\text{ m}$$
$$h = \mathbf{0.51\ cm}$$

LONG ANSWER TYPE QUESTIONS

Q4.23. A 100 turn rectangular coil ABCD (in X-Y plane) is hung from one arm of a balance as in figure.

A mass 500 g is added to the other arm to balance the weight of coil. A current of 4.9 A passes through the coil and a constant magnetic field of

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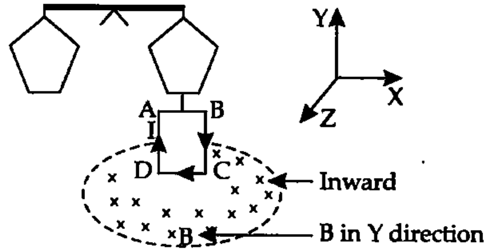
0.2 T acting inward (in X-Z plane) is switched on such that only arm CD of length 1 cm lies in the field. How much additional mass m must be added to regain the balance?



Main concept used: (i) Force experienced by current carrying conductors due to M.F. (ii) Weight of coil measured by beam balance is 500 gm.

Ans. The magnetic field is perpendicular to arms BC and AD, so torque will act on CD and AB arms due to it, coil rotate.

When current of 4.9 A does not pass through the coil the balance measures mass of coil 500 g.

On arm AD and BC of rectangular coil magnetic force due to M.F. will be equal and opposite so it will rotate the coil horizontally not vertically up or down. So does not affect the balance.

When current of 4.9 A passes through the coil, downward force acts on arm CD due to magnetic field. Length of arm CD is 1 cm (10^{-2} m).

$\therefore$ Force acting on arm CD $= F_m = B \times I.l = BIl \sin\theta$

[θ is angle between B and I in CD]

$= 0.2 \times 4.9 \times \sin 90° \times 10^{-2}$

or $F_m = 0.98 \times 10^{-2}$ N

Now let the weight mg is added on other side of beam balance to balance the coil

$$mg = F_m$$
$$m \times 9.8 = 0.98 \times 10^{-2}$$
$$m = \frac{0.98}{9.8} \times 10^{-2} = 10^{-3} \text{ kg} = \mathbf{1\ g.}$$

Q4.24. A rectangular conducting loop consists of two wires on two opposite sides of length 'l' joined together by rods of length 'd'. The wires are each of the same material but with cross-sections differing by a factor of 2. The thicker wire has a resistance R and the rods are of low resistance, which in turn are connected to a constant voltage source V_0. The loop is placed in a uniform magnetic field B at 45° to its plane. Find the torque (τ) exerted by the magnetic field on the loop about an axis through the centres of rods.

Main concepts used: $R = \frac{\rho l}{A}$, $F_m = BIL \sin\theta$, $V = IR$.

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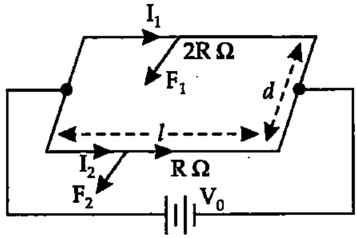
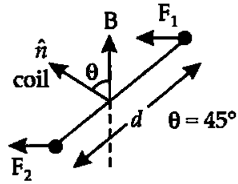
Ans.





As $R = \frac{\rho l}{A}$, the thicker wire has resistance R Ω then resistance of thinner wire 2R Ω as both the wires are of the same material with same length but differing in area of cross-section by factor 2 (as given).

$$V_0 = I_1 R_1 \Rightarrow I_1 = \frac{V_0}{R_1} = \frac{V_0}{R} \text{ and } I_2 = \frac{V_0}{2R}$$

So
$$F_1 = BI_1 l \sin\theta = \frac{BV_0 l}{R} \sin 45° = \frac{BV_0 l}{\sqrt{2}R}$$

$$F_2 = BI_2 l \sin\theta = \frac{BV_0 l \sin 45°}{2R} = \frac{BV_0 l}{2\sqrt{2}R}$$

$$\tau_1 = F_1 d = \frac{BV_0 l}{\sqrt{2}R} \cdot d \quad \text{and} \quad \tau_2 = \frac{BV_0 ld}{2\sqrt{2}R}$$

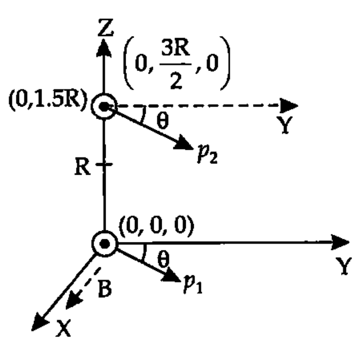
∴
$$\text{Net torque} = \tau = \tau_1 - \tau_2 = \frac{BV_0 ld}{\sqrt{2}R} - \frac{BV_0 ld}{2\sqrt{2}R}$$

$$\tau = \frac{BV_0 ld}{\sqrt{2}R}\left[1 - \frac{1}{2}\right] = \frac{BV_0 ld}{2\sqrt{2}R}.$$

Q4.25. An electron and a positron are released from (0, 0, 0) and (0, 0, 1.5 R) respectively in a uniform magnetic field $B = B_0\hat{i}$, each with an equal momentum of magnitude $\vec{p} = e\vec{B}R$. Under what conditions on the direction of momentum will the orbits be non-intersecting circles?

Main concept used: The boundaries of two circles of radius r_1 and r_2 will not overlap if the distance (d) between their centres is greater than $(r_1 + r_2)$.

Ans. As $B = B_0\hat{i}$ so magnetic field is along +X-axis. The circular motion of the momenta of both an electron and a positron are in Y-Z plane. Let p_1 and p_2 are the momentum of the electron and positron respectively, the magnitude of charge and momentum of both are equal so they revolve in Y-Z plane due to MFB = *Boi.* but in opposite sense with same radius R as the direction and p_1 and p_2 are opposite.



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Let p_1 and p_2 make an angle $+\theta$ ($-\theta$) with Y-axis, as shown in the above figure.

The centres of the respective circles must be perpendicular to the momentum and at a distance R. Let the centre of revolving electron and positron are C_e and C_p respectively so co-ordinates of

C_e will be $(0, + R \sin \theta, R \cos \theta)$ and that of C_p will be

$$\left(0, - R \sin \theta, \frac{3}{2}R - R \cos \theta\right).$$

The planes of circular paths are in Y-Z plane.

The condition that the circular path of the electron and positron are non-intersecting circles is that the distance (d) between their centres must be greater than 2R

$$id > d > 2R$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

$$= (0 - 0)^2 + (+R \sin \theta + R \sin \theta)^2 + \left(R \cos \theta - \frac{3}{2}R + R \cos \theta\right)^2$$

$$= (2R \sin \theta)^2 + \left(2R \cos \theta - \frac{3}{2}R\right)^2$$

$$= 4R^2 \sin^2 \theta + 4R^2 \cos^2 \theta + \frac{9}{4}R^2 - 6R^2 \cos\theta$$

$$= 4R^2 + \frac{9}{4}R^2 - 6R^2 \cos \theta \qquad \text{...(I)}$$

$$d > 2R$$

$$d^2 > 4R^2$$

$$4R^2 + \frac{9}{4}R^2 - 6R^2 \cos \theta > 4R^2$$

$$\frac{9}{4}R^2 - 6R^2 \cos \theta > 0$$

$$3R^2 \left(\frac{3}{4} - 2 \cos \theta\right) > 0$$

$$3R^2 > 0 \text{ rejected}$$

$$-2 \cos \theta > \frac{3}{4}$$

$\cos \theta < \frac{3}{8}$ is the condition that two circular paths do not intersect; θ is angle of momentum of electron or positron with Y-axis.

Q4.26. A uniform conducting wire of length 12 a and resistance R is wound up as current-carrying coil in the shape of (i) an equilateral triangle of side a (ii) a square of side a, (iii) a regular hexagon of side a. The coil is connected to a voltage source V_0. Find the magnetic moment of the coils in each case.

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Main concept used: Magnetic moment of a coil of n turns, $M = nIA$,

$$\text{Number of turns in coil} = \frac{\text{Length of wire}}{\text{Perimeter of coil}}.$$

Ans. (*i*) The coil of shape equilateral Δ is made up with side a. So number of turns in coil $= \frac{12a}{3a} = 4$ turns.

$\therefore$ Magnetic moment $= nIA = 4 \times I \times \frac{\sqrt{3}}{4}a^2$

Magnetic moment of triangular coil $= \sqrt{3}\,\mathbf{I}\boldsymbol{a}^2$.

(*ii*) Number of turns in coil of square-shape of side $a = \frac{12a}{4a} = 3$ turns

So magnetic moment due to square shaped coil

$$= nIA = 3I \times a^2 = \mathbf{3Ia^2}$$

(*iii*) For a regular hexagon-shaped coil of side a, number of turns in coil

$$= \frac{12a}{6a} = 2$$

$$\therefore \quad \text{Magnetic moment} = nIA = 2I.\left(\frac{\sqrt{3}}{4}a^2\right)\cdot 6 = \mathbf{3\sqrt{3}Ia^2}.$$

Q4.27. Consider a circular current-carrying loop of radius R in the X-Y plane with centre at origin. Consider the line integral

$$\Im(L) = \left|\int_{-L}^{L} B.dl\right| \text{ taken along Z-axis.}$$

(*a*) Show that $\Im(L)$ monotonically increases with L.

(*b*) Use an appropriate Amperian loop to show that $\Im(\infty) = \mu_0 I$, where I is the current in the wire.

(*c*) Verify directly the above result.

(*d*) Suppose we replace the circular coil by a square coil of side R carrying the same current I. What can you say about $\Im(L)$ and $\Im(\infty)$?

Main concept used: Ampere's circuital law.

Ans. (*a*) B(z) point in the same direction of Z-axis and hence $\Im(L)$ is monotonical function of L as B and dl are along the same direction. So

$$B.dl = Bdl \cos\theta = Bdl \cos 0° = Bdl.$$

(*b*) $\Im(L)$ + Contribution from large distance on contour $C = \mu_0 I$.

$\therefore$ As $\quad L \to \infty$

Contribution from large distance $\to 0$. $\quad \left(\text{As } B \propto \frac{1}{r^3}\right)$

$$\therefore \quad \Im(\infty) - \mu_0 I.$$

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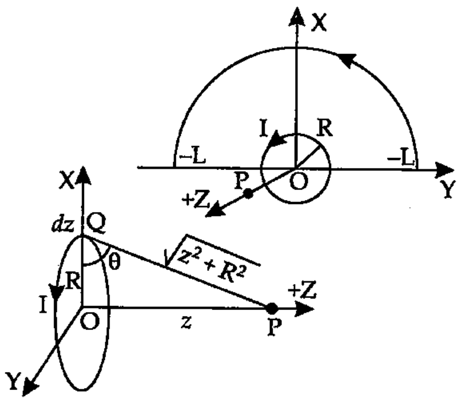
(c) The magnetic field due to circular current-carrying loop of radius R in X-Y plane with centre at origin at any point lying at distance *a* from origin.

Consider a loop of current-carrying conductor placed in X-Y plane. A point P is in +Z direction at distance *z*, *i.e.* OP = z.



Again consider an element *dz* on loop of conductor as shown in figure below.

Let angle between R and QP = θ then magnetic field at P due to loop is

$$B\hat{z} = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}$$

[Integrating both sides w.r.t.z and z can vary from –*d* to +*d*]

$$\int_{-\infty}^{\infty} B\hat{z}dz = \int_{-\infty}^{\infty} \frac{\mu_0 I R^2 dz}{2(z^2+R^2)^{3/2}}$$

$$\tan\theta = \frac{z}{R}$$

$$z = R\tan\theta$$

Differentiating both sides:

$$dz = R.\sec^2\theta.d\theta$$

$$\cos\theta = \frac{R}{\sqrt{z^2+R^2}}$$

$$\cos^2\theta = \frac{R}{\sqrt{z^2+R^2}}$$

$$\int_{-\infty}^{\infty} Bzdz = \frac{\mu_0 I}{2}\int_{-\infty}^{+\infty}\frac{R^2}{(z^2+R^2)}\frac{dz}{\sqrt{(z^2+R^2)}}$$

$$= \frac{\mu_0 I}{2}\int_{-\pi/2}^{+\pi/2}\cos^2\theta\cdot\frac{R\sec^2\theta.d\theta}{\sqrt{z^2+R^2}} \quad \left[\because \cos\theta = \frac{R}{\sqrt{z^2+R^2}}\right]$$

$$= \frac{\mu_0 I}{2}\int_{-\pi/2}^{+\pi/2}\frac{R\,d\theta}{\sqrt{z^2+R^2}} = \frac{\mu_0 I}{2}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta$$

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$$= \frac{\mu_0 I}{2}[\sin\theta]_{-\pi/2}^{\pi/2} = \frac{\mu_0 I}{2}\left[\sin\frac{\pi}{2} - \sin\frac{-\pi}{2}\right]$$

$$= \frac{\mu_0 I}{2}[1+1] = \mu_0 I = \boxed{\int_{-\infty}^{+\infty} B\hat{z}.dz = \mu_0 I}$$

(*d*) Because the area of square loop is smaller than the area of circular loop, for the same length of conducting wires, hence loop $B(z)_{\text{square loop}}$ < loop $B(z)_{\text{circular loop}}$

$\Im_S(L)_{sq.} = \Im_S(L)_{circular}$ [∵ length of conducting wire are equal]

By using arguments as in (*b*) part, Bz does not depend on length of wire

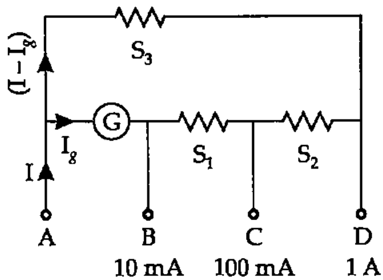
$\therefore\ \Im(\infty)_{\text{sq. loop}} = \Im(\infty)_{\text{circular loop}}$

Magnetic field due to circular or square loops remain same, *i.e.*

$$(Bz)_{\text{circular loop}} = (Bz)_{\text{square loop}} = \mu_0 I$$

So, $\Im(\infty)_{\text{sq. loop}} = \Im(\infty)_{\text{circular loop}} = \mu_0 I$ Hence proved.

Q4.28. A multi-range current meter can be constructed by using a galvanometer circuit as shown in figure. We want a current meter that can measure 10 mA, 100 mA and 1 A using a galvanometer of resistance 10 Ω and that produces maximum deflection for current of 1 mA. Find S_1, S_2 and S_3 that have to be used.



Main concept used: Galvanometer can be converted into ammeter by lowering its net resistance by using shunt of proper value and using Kirchhoff's law.

Ans. We can measure the currents of magnitude 10 mA, 100 mA and 1 A by connecting ammeter A with B, C and D respectively. So,

for 10 mA → $I_g G = (I - I_g)(S_1 + S_2 + S_3)$ ...(I)

for 100 mA → $I_g(G + S_1) = (I - I_g)(S_2 + S_3)$ ...(II)

for 1 A → $I_g(G + S_1 + S_2) = (I - I_g) S_3$ ...(III)

$I_g = 1\text{ mA} = 10^{-3}$ A and $G = 10\ \Omega$

$$10^{-3} \times 10 = (10^{-2} - 10^{-3})(S_1 + S_2 + S_3) \quad \text{[from (I)]}$$

$$\Rightarrow \quad 10 = (10 - 1)(S_1 + S_2 + S_3)$$

$$\Rightarrow \quad 10 = 9(S_1 + S_2 + S_3) \quad \text{...(IV)}$$

$$10^{-3}(10 + S_1) = (10^{-1} - 10^{-3})(S_2 + S_3) \quad \text{[from (II)]}$$

$$\Rightarrow \quad 10 + S_1 = (100 - 1)(S_2 + S_3)$$

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$$\Rightarrow \quad 10 + S_1 = 99(S_2 + S_3) \qquad \text{...(V)}$$

$$10^{-3}(10 + S_1 + S_2) = (1 - 10^{-3})(S_3) \qquad \text{[from (III)]}$$

$$\Rightarrow \quad 10 + S_1 + S_2 = (1000 - 1)S_3$$

$$\Rightarrow \quad 10 + S_1 + S_2 = 999\, S_3 \qquad \text{...(VI)}$$

$$10 + S_1 = 99(S_3 + S_2) \qquad \text{[from (V)]}$$

$$10 = 9(S_3 + S_2 + S_1) \qquad \text{[from (IV)]}$$

$$\frac{10}{9} = S_1 + S_2 + S_3 \qquad \text{[from (IV)]}$$

$$\underset{-}{\frac{10}{99}} + \underset{-}{\frac{S_1}{99}} = \underset{-}{S_2} + \underset{-}{S_3} \qquad \text{[from (V)]}$$

$$\frac{10}{9} - \frac{10}{99} - \frac{S_1}{99} = S_1$$

$$\frac{110 - 10}{99} = S_1 + \frac{S_1}{99}$$

$$\frac{100}{99} = \frac{99\, S_1 + S_1}{99} \Rightarrow \frac{100}{99} = \frac{100}{99} S_1$$

$$S_1 = 1\ \Omega$$

So $\quad \dfrac{10}{9} = 1 + S_2 + S_3 \qquad$ [from (IV)]

or $\quad \dfrac{1}{9} = S_2 + S_3$

$$S_3 = \frac{10}{999} + \frac{S_1}{999} + \frac{S_2}{999} \qquad \text{[from (VI)]}$$

$$S_3 = \frac{10}{999} + \frac{1}{999} + \frac{S_2}{999} \qquad [\because S_1 = 1\ \Omega]$$

$$\Rightarrow \quad S_3 = \frac{11}{999} + \frac{S_2}{999}$$

$$S_2 = \frac{1}{9} - S_3 \qquad \left[\because \frac{1}{9} = S_2 + S_3 \text{ from above}\right]$$

$$= \frac{1}{9} - \frac{11}{999} - \frac{S_2}{999} \qquad \text{(from III)}$$

$$S_2 + \frac{S_2}{999} = \frac{111 - 11}{999}$$

$$\frac{1000}{999} S_2 = \frac{100}{999}$$

$$S_2 = 0.1\ \Omega$$

$$S_1 + S_2 + S_3 = \frac{10}{9} \qquad \text{(from IV)}$$

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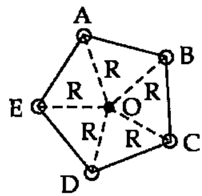
$$\Rightarrow \qquad 1 + 0.1 + S_3 = \frac{10}{9}$$

$$\Rightarrow \qquad S_3 = \frac{10}{9} - \frac{11}{10} = \frac{100-99}{90} = \frac{1}{90}$$

Hence, $S_1 = 1\ \Omega$; $S_2 = 0.1\ \Omega$; $S_3 = 0.010\ \Omega$

Q4.29. Five long wires A, B, C, D and E each carrying current I are arranged to form edges of a pentagonal prism as shown in figure. Each carries current out of the plane of paper.



Front side

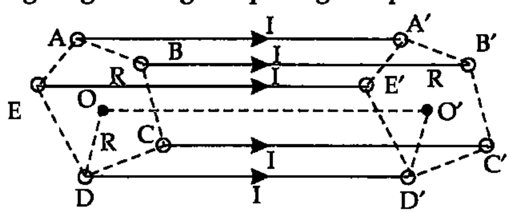
(*a*) What will be magnetic induction at a point on the axis O? Axis is at a distance R from each wire.

(*b*) What will be the field if current in one of the wires (say A) is switched off?

(*c*) What if current in one of the wire (say) A is reversed?

Main concept used: Magnetic field due to current-carrying conductor at distance R is $B = \frac{\mu_0 I}{2\pi R}$.

Ans. (*a*) Figure shows that five conductors AA′, BB′, CC′, DD′ and EE′ are along height of regular pentagonal prism ABCDE.



It is given that five identical conducting wires are along the heights of regular pentagon, represented in figure above by AA′, BB′, CC′, DD′, and EE′. Axis of regular pentagon is OO′ will be equidistant (R) from all five conductors, the current is passing through all five conductors are equal let (I).

As the current in all 5 conductors are equal to I and the distance of O from conductors is also equal to R then magnitude of magnetic field due to each conductor will be equal, *i.e.*

$$|B_1| = |B_2| = |B_3| = |B_4| = |B_5| = B = \frac{\mu_0 I}{2\pi R}$$

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The direction of magnetic field induced can be find out by right hand grip rule, then direction of induced magnetic field at 'O' due to AA' will be perpendicular to both AA' and AO. Angles between any two consecutive magnetic field is $\frac{360°}{5} = 72°$

As shown in figures given here

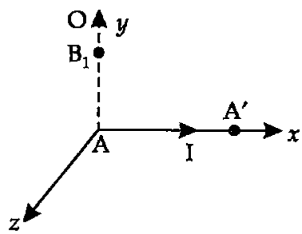

Fig. shows the direction of magnetic field B_1 due to AA' conducting wire

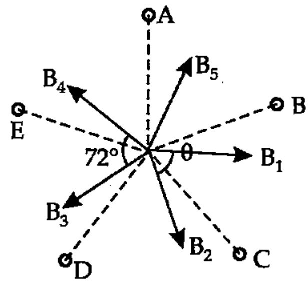

Fig. shows the direction of B_1, B_2, B_3, B_4 and B_5 due to conductors AA', BB', CC', DD', and EE' respectively

As $B = B_1 = B_2 = B_3 = B_4 = B_5$ and angle between consecutive magnetic fields is 72° or symmetric in 360° so their resultant at O will be zero, *i.e.*

$$\vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 = 0$$

Hence, the induced magnetic induction at O due to five conductors as shown in figure is zero.

(*b*) When current in AA' is switched off, then $B_1 = 0$ and resultant becomes $R = B_2 + B_3 + B_4 + B_5$

But from (*a*) part $B_1 + B_2 + B_3 + B_4 + B_5 = 0$

or
$$\vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 = -\vec{B}_1$$
$$R = -B_1$$
$$R = -\frac{\mu_0 I}{2\pi R}$$

i.e. direction of resultant is opposite to $\vec{B}_1$.

(*c*) Here, on reversing the current in AA', direction of magnetic field due to AA' becomes $-\vec{B}_1$,

$$R = B_2 + B_3 + B_4 + B_5$$

$\because$ $|-B_1| = |B_2| = |B_3| = |B_4| = |B_5| = B$

$\therefore$ net induced magnetic field at O becomes

$$-B + B + B + B + B = 3B$$
$$R = \frac{3\mu_0 I}{2\pi R}$$

❑❑❑

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5 Magnetism and Matter

MULTIPLE CHOICE QUESTIONS—I

Q5.1. A Toroid of n turns, mean radius R and cross-sectional area *a* carries a current I. It is placed on a horizontal table taken as *x-y* plane. It's magnetic moment *m*—

(*a*) is non-zero and points in z-direction by symmetry.

(*b*) points along the axis of toroid ($\vec{M} = m\hat{\phi}$).

(*c*) is zero otherwise there would be a field falling as $\frac{1}{r^3}$ at large distances outside the toroid.

(*d*) is pointing radially outwards.

Main concept used: No magnetic field outside the toroid. Magnetic field only inside the toroid and $\vec{M} = I\vec{A}$.

Ans. (*c*): We know that there is no magnetic field outside the toroid. So outside, the M.F. falls very rapidly as it is inversely proportional to third power of distance from centre of toroid.

By Amperian circuital law, there is no any current so no magnetic moment outside. Verifies answer (*c*).

Q5.2. The magnetic field of Earth can be modelled by that of a point dipole placed at the centre of Earth. The dipole axis makes an angle of 11.3° with the axis of Earth. At Mumbai, declination is nearly zero, then

(*a*) the declination varies between 11.3° W to 11.3° E.

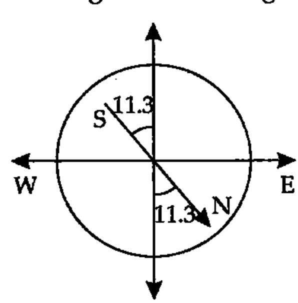
(*b*) the least declination is zero.

(*c*) the plane defined by dipole axis and the earth axis passes through Greenwich.

(*d*) declination averaged over the earth must be always negative.

Main concept used: Magnetic declination is an angle between angle of magnetic meridian and the geographic meridian.

Ans. (*a*): As the earth's magnetic field can be considered similar to that of a hypothetical magnetic dipole located at the centre of earth. The axis of dipole is tilted 11.3° with axis of geographic of earth by 11.3° both side east and west (South pole of earth towards West and North pole towards east).



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So the declination varies from East to West both side 11.3° on whole earth surface verify answer (*a*). Also see figure with answer Q.24.

Q5.3. In a permanent magnet at room temperature

(*a*) magnetic moment of each molecule is zero.

(*b*) the individual molecules have non-zero magnetic moment which are all perfectly aligned.

(*c*) domains are partially aligned.

(*d*) domains are all perfectly aligned.

Main concept used: In permanent magnet more domains are permanently aligned (not all).

Ans. (*c*): At room temperature permanent magnet behaves like a ferromagnetic substance.

When there is no magnetic field in ferromagnetic substance domains are randomly spread. So resultant magnetic moment is about zero.

But when this substance is placed in strong magnetic field some domains aligned in external field permanently even in absence of external magnetic field.

So domains partially aligned permanently verifies the option (*c*).

Q5.4. Consider two idealised systems (*i*) a parallel plate capacitor with large plates and small separation, (*ii*) a long solenoid of length L >> R, radius of cross-section. In (*i*) E is ideally treated as a constant between the plates and zero outside. In (*ii*) magnetic field is constant inside the solenoid and zero outside. These idealised assumptions, however, contradict fundamental law as below

(*a*) case (*i*) contradicts Gauss's law of electrostatic fields.

(*b*) case (*ii*) contradicts Gauss's law of magnetic fields.

(c) case (*i*) agrees with $\oint_S E \cdot dl = 0$

(*c*) case (*ii*) contradicts $\oint H.dl = I_{en}$

Main concept used: Electric field lines do not form continuous path while the magnetic field lines forms closed paths.

Ans. (*b*): According to Gauss's law of electrostatic field $\oint_s E \cdot ds = \frac{q}{\varepsilon_0}$ so it does not contradicts for electrostatic field as the electric field lines do not form continuous path.

According to Gauss's law of magnetic field $\oint_s B \cdot ds = 0$. It contradicts for magnetic field, because there is a magnetic field inside the solenoid, and no field outside the solenoid carrying current, but the magnetic field lines form the closed paths.

Q5.5. Paramagnetic sample shows a net magnetisation of 8 Am^{-1} when placed in external magnetic field of 0.6 T at a temperature of

4 K. When the same sample is placed in an external magnetic field of 0.2 T at a temperature of 16 K, the magnetisation will be—

(*a*) $\frac{32}{3}$ Am^{-1}, (*b*) $\frac{2}{3}$ Am^{-1}, (*c*) 6 Am^{-1}, (*d*) 2.4 Am^{-1}

Main concept used: Curie's law,

$$\text{Magnetisation} \propto \frac{\text{B(Magnetic field induction)}}{\text{T(Absolute temperature)}}.$$

Ans. (*b*) According to the Curie's law of magnetisation, (I), for a substance is directly proportional to magnetic field induction (B) and inversely proportional to the absolute temperature T.

$$I \propto \frac{B}{T} \quad \text{or} \quad \frac{I_2}{I_1} = \frac{B_2 T_1}{B_1 T_2}$$

$I_1 = 8$ Am^{-1} $\quad B_1 = 0.6$T $\quad T_1 = 4$ K

$I_2 = ?$ $\quad B_2 = 0.2$T $\quad T_2 = 16$ K

$$I_2 = \frac{B_2 T_1}{B_1 T_2} I_1 = \frac{0.2 \times 8 \times 4}{0.6 \times 16} = \frac{2}{3} \text{ Amp}^{-1}$$

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q5.6. S is the surface of a lump of magnetic material.

(*a*) Lines of B are necessarily continuous across S.
(*b*) Some lines of B must be discontinuous across S.
(*c*) Lines of H are necessarily continuous across S.
(*d*) Lines of H cannot all be continuous across S.

Main concepts used: (*i*) Magnetic field lines are always continuous. (*ii*) Field lines of electric field of a dipole begins from positive charge and ends on negative charge or escape to infinity. (*iii*) Also magnetic intensity (H) outside the magnet is $H = \frac{B}{\mu_0}$ and for

$$\text{inside the magnet } H = \frac{B}{\mu_0 \mu_r}.$$

Ans. (*a*) (*d*): Magnetic field lines for magnetic induction (B) form continuous lines so lines of B are necessarily continuous across S. Also magnetic intensity (H) to magnetise varies for inside and outside the lump. So lines of H cannot all be continuous across S.

Q5.7. The primary origin of magnetism lies in

(*a*) atomic currents (*b*) Paulie's exclusion principle
(*c*) polar nature of molecules (*d*) intrinsic spin of electron.

Main concept used: Motion of charge produces magnetic field, due to magnetic effect of current.

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Ans. (*a*) (*d*): Motion of charge particle produces magnetism and nature of magnetism depends on the nation of motion of charge particle.

In atom, electrons revolve and spin around the nucleus which in turn produces current and due to magnetic effect of current, magnetism produces in the material.

Q5.8. A long solenoid has 1000 turns per metre and carries a current of 1 A. It has a soft iron core of $\mu_r = 1000$. The core is heated beyond the Curie temperature T_C.

(*a*) The H field in the solenoid is (nearly) unchanged but the B field decreases drastically.
(*b*) The H and B fields in the solenoid are nearly unchanged.
(*c*) The magnetisation in the core reverses the direction.
(*d*) The magnetisation in the core diminishes by a factor of about 10^8.

Main concept used: (*i*) Behaviour of magnetic material beyond Curie temperature, (*ii*) Magnetic intensity $H = nI$, (*iii*) The magnetic induction $B = \mu_r \mu_o nI$

Ans. (*a*) (*d*): $n = 1000$ turns per metre, $\mu_r = 1000$

$H = nI = 1000 \times 1 = 1000$ Amp. So H is constant verifies the answer (*a*).

$B = \mu_o \mu_r nI = (\mu_o nI)\mu_r = K\,\mu_r$ (K = constant)

So, $B \propto \mu_r$.

By Curie's law, when ferromagnetic substance is heated beyond Curie temperature it behaves like a paramagnetic substance. Where,

Susceptibility of $(\chi_m)_{ferro} = 10^3$

Susceptibility of $(\chi_m)_{para} = 10^{-5}$

$$\therefore \quad \frac{B_2}{B_1} = \frac{\chi_2}{\chi_1} = \frac{10^{-5}}{10^3} = 10^{-8} \quad \text{or} \quad B_2 = 10^{-8} B_1$$

So, magnetisation becomes 10^{-8} times of earlier or diminished by 10^8 times. Verified answer (*d*).

Q5.9. Essential difference between electrostatic shielding by conducting shell and magnetic shielding is due to

(*a*) electrostatic field lines can end on charges and conductors have free charges.
(*b*) lines of B can also end but the conductors cannot end them.
(*c*) lines of B cannot end on any material and perfect shielding is not possible.
(*d*) shells of high permeability material can be used to divert the lines of B from the interior region.

Main concepts used: (*i*) Electric field lines start from positive charge and ends on negative charge and there is existence of positive and negative charge separately (*ii*) Non-existence of monopole, (*iii*) Magnetic field lines start from North pole and ends to South

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pole, (*iv*) The path of magnetic field lines can be affected (attracted or repelled) by magnetic materials.

Ans. (*a*) (*c*) (*d*): Conductors have free charge particles so the lines of force can be stopped by conductors gives shielding effect.

As non-existence of mono pole magnetic field lines cannot be stopped or shield.

Magnetic field lines are affected by magnetic materials so can be repelled by using a high permeability magnetic material to get the region of no magnetic field.

Q5.10. Let the magnetic field on the earth is modelled by that of a point magnetic dipole at the centre of the earth. The angle of dip at a point on the geographical equator

(*a*) is always zero
(*b*) can be zero at specific points
(*c*) can be positive or negative
(*d*) is bounded

Main concept used: Angle of dip specifies the direction of resultant magnetic field of earth.

Ans. (*b*) (*c*) (*d*): As the dipole is placed at an angle of 11.3° W to 11.3° E from geographical N-S axis. South and North pole of it are just like an imagined magnet of earth. South of dipole is in North direction towards 11.3° towards west and North pole in South direction 11.3° towards east.

The resultant magnetic field due to dipole will be zero at its equatorial plane of dipole but not on geographical equator verifies answer (*b*).

The angle of dip. will change on changing the equatorial plane of dipole, *i.e.*, where angle of dip. is zero. It is not zero at all points of geographic equator. Angle of dip on geographical equator will be zero where it meets with magnetic equator. So verifies answers (*c*) and (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q5.11. A proton has spin and magnetic moment just like an electron. Why then its effect is neglected in magnetism of materials?

Main concept used: (*i*) Magnetic dipole moment of a charge particle $\boxed{M = \frac{eh}{4\pi m}}$, (*ii*) Mass of proton is 1836 times larger than electron.

Ans. As we know that magnetic moment of a charged particle of charge e and mass m is

$$M_e = \frac{eh}{4\pi m_e} \quad \text{and} \quad M_p = \frac{eh}{4\pi m_p}$$

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As charge on proton and electron are equal in magnitude so $M \propto \frac{1}{m}$ or $\frac{M_e}{M_p} = \frac{m_p}{m_e}$.

As the mass of proton is 1836 times as of electron, so magnetic moment of proton $M_p = \frac{M_e}{m_p} m_e = \frac{M_e \cdot m_e}{1836\, m_e}$, $M_p = \frac{1}{1836}$ of M_e.

So magnetic moment of proton is $\frac{1}{1836}$ times of electron, so can be neglected.

Q5.12. A permanent magnet in the shape of a thin cylinder of length 10 cm has $M = 10^{+6}$ A/m. Calculate the magnetisation current I_M.

Main concept used: Magnetic moment $= \frac{I}{l}$.

Ans. $M = 10^{+6}$ ampre/metre, $l = 10$ cm $= 0.10$ m

I_M = Magnetisation current

$$M = \frac{I_M}{l}$$

So, $I_M = M \times l = 10^6 \times 0.1 = 10^5$ ampere.

Q5.13. Explain quantitatively the order of magnitude difference between the diamagnetic susceptibility of N_2(~5×10^{-9}) (at S.T.P) and of Cu(~10^{-5}).

Main concept used: Magnetic susceptibility is the property of substance which show how a material behaves in external magnetic field.

Ans.

$$\rho_N = \frac{28\text{ g}}{22.4\text{ lit}} = \frac{28\text{ g}}{22400\text{ ml}} = \frac{28}{22400}\text{ g per cm}^3$$

$$\rho_{cu} = 8\text{ g/cm}^3$$

$\therefore$
$$\frac{\rho_N}{\rho_{Cu}} = \frac{28}{22400 \times 8} = 1.6 \times 10^{-4}$$

Also,
$$\frac{\chi_N}{\chi_{Cu}} = \frac{5 \times 10^{-9}}{10^{-5}} = 5 \times 10^{-4}$$

$$\chi = \frac{\text{Intensity of magnetisation (M)}}{\text{Magnetising field intensity (H)}}$$

$$\chi = \frac{\dfrac{\text{Magnetic moment } (m)}{\text{Volume (V)}}}{\text{H}}$$

$$\chi = \frac{m}{HV}$$

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If m', V, ρ are the mass, volume and density of magnetic material, then

$$\frac{m'}{\rho} = V$$

$$\chi = \frac{m \times \rho}{H.m'}$$

$\chi \propto \rho$ as mass, m' and H are constant and magnetic material are same.

So $$\frac{\chi_N}{\chi_{Cu}} = \frac{\rho_N}{\rho_{Cu}} = 1.6 \times 10^{-4}$$

The major difference between diamagnetic susceptibility of N_2 gas and solid Cu is due to their difference between densities.

Q5.14. From molecular point of view, discuss the temperature dependence of susceptibility for diamagnetism, paramagnetism and ferromagnetism.

Main concept used: Susceptibility χ is the ratio of intensity of magnetisation (M) of magnetic material to the intensity of magnetising field (H).

$$\chi = \frac{M}{H}.$$

Magnetic moment of diamagnetism due to orbital motion is opposite to applied field.

Ans. The direction of external magnetic field H and magnetism M due to orbital motion of electrons of diamagnetic substances are opposite so net magnetism becomes zero. Hence, the susceptibility (χ) of *diamagnetism is not much affected by temperature.*

The direction of magnetism due to orbital motion of electrons in paramagnetism and ferromagnetism material and external applied field are in same direction so net magnetism increased and much affected by temperature. *As temperature raised the alignment of atomic magnetism is disturbed resulting in decrease in susceptibility.*

Q5.15. A ball of superconducting material is dipped in liquid nitrogen and placed near a bar magnet.

(*i*) In which direction will it move

(*ii*) What will be the direction of its magnetic moment?

Main concept used: Superconducting material and liquid nitrogen are diamagnetic materials.

Ans. Liquid nitrogen and a superconducting material are diamagnetic materials and remain diamagnetic when superconducting material is dipped in liquid nitrogen.

So when external magnetic field is applied on superconducting material dipped in liquid nitrogen it will *be repelled by external* magnetic field and direction of motion will be opposite to direction of magnet or magnetic field.

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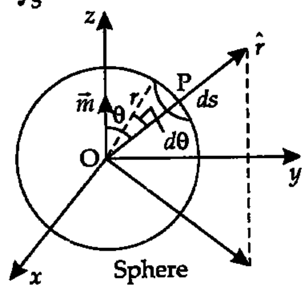
SHORT ANSWER TYPE QUESTIONS

Q5.16. Verify the Gauss's law for magnetic field of a point dipole (magnetic) of dipole moment $\vec{m}$ at the origin for surface which is a sphere of radius R.

Main concept used: Gauss's law in magnetism.

Ans. We know by Gauss's law in magnetism $\oint_S \vec{B}.\vec{ds} = 0$.



Magnetic moment (m) of dipole at origin O is $\vec{m} = m\hat{k}$.

Let P be a point at a distance r from O and OP makes an angle θ with z axis. Component of $\vec{m}$ along OP is equal to $\vec{m}$ cos θ.

Now the magnetic field of induction at P due to dipole of moment $\vec{m}$ cos θ is

$$B = \frac{\mu_0}{4\pi}\frac{2\vec{m}\cos\theta}{r^3}\hat{r}$$

Here r is the radius of sphere with centre 'O' lying in X-Y-Z plane. Centre at origin.

Take an elementary area ds at P then

$$\vec{ds} = r(r\sin\theta)\,d\theta\,\hat{r} = r^2\sin\theta\,d\theta\,\hat{r} \quad \left[\because d\theta = \frac{ds}{r^2} \text{ or } ds = r^2 d\theta\right]$$

$$\oint_S \vec{B}.\vec{ds} = \oint \frac{\mu_0}{4\pi}\frac{2\vec{m}\cos\theta}{r^3}\hat{r}\cdot(r^2\sin\theta\cdot d\theta\,\hat{r})$$

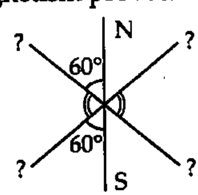
$$= \frac{\mu_0\vec{m}}{4\pi r}\int_0^{2\pi} 2\sin\theta\cos\theta\cdot d\theta = \frac{\mu_0\vec{m}}{4\pi r}\int_0^{2\pi}\sin 2\theta\cdot d\theta$$

$$= \frac{\mu_0\vec{m}}{4\pi r}\left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi} = \frac{\mu_0}{4\pi r.2}[\cos 4\pi - (-\cos 0)]$$

$$= \frac{\mu_0}{4\pi r\times 2}[-\cos 0 + 1] = \frac{\mu_0}{8\pi r}[-1+1]$$

$\oint B.ds = 0$ Hence the Gauss's law in magnetism proved.

Q5.17. Three identical bar magnets are riveted together at centre in the same plane as shows in figure. This system is placed at rest in a slowly varying magnetic field. It is found that the system of magnets does not show any motion. The north-south poles of one magnet is shown in figure. Determine the poles of the remaining two.



Main concept used: The resultant magnetic force on magnets must be zero for no motion.

Ans. The poles must be symmetric to each other or a magnet. It is possible only when the poles of the remaining two magnets are as in given figure.

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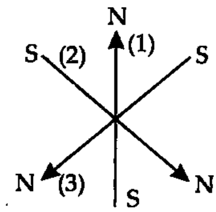
The north pole of magnet (1) is equally attracted by south poles of (2) and (3) magnets placed at equal distance.

Similarly one pole of any one magnet is attracted by opposite poles of other two magnets so resultant force or moment on each magnet is zero and will not be in motion placed on table.



Q5.18. Suppose we want to verify the analogy between electrostatic and magnetostatic by an explicit experiment. Consider the motion of (*i*) electric dipole $\vec{p}$ in an electrostatic field $\vec{E}$ and (*ii*) magnetic dipole $\vec{m}$ in a magnetic field $\vec{B}$. Write down a set of conditions on $\vec{E}$, $\vec{B}$, $\vec{p}$, and $\vec{m}$ so that the two motions are verified to be identical. (Assume identical initial conditions).

Main concept used: Force on dipole (electric) in electric field and torque on magnetic dipole in magnetic field.

Ans. Let θ is the angle between $\vec{m}$ and $\vec{B}$.

$\therefore$ Torque on magnetic dipole in a magnetic field B is

$$\tau = \vec{m}\vec{B} \sin \theta \quad \text{...I}$$

Similarly if θ is the angle between electric dipole moment $\vec{p}$ and electric field E then torque on electric dipole in E is

$$\tau' = \vec{p}\vec{E} \sin \theta \quad \text{...II}$$

For if motion in I and II of electric and magnetic dipole are identical then $\tau' = \tau$

$$\vec{p}\vec{E} \sin \theta = \vec{m}\vec{B} \sin \theta$$

or

$$\vec{p}\vec{E} = \vec{m}\vec{B} \quad \text{...III}$$

We know that $\vec{E} = c\vec{B}$ (relation between E and B) ...IV

c is velocity of light.

Put the value of E from IV in III

$$\vec{p}cB = \vec{m}\vec{B}$$

$$\boxed{\vec{p} = \frac{\vec{m}}{c}}$$

It is the required relation.

Q5.19. A bar magnet of magnetic moment $\vec{m}$ and moment of inertia I (about axis passing through centre and perpendicular to length) is cut into two equal pieces perpendicular to length. Let T be the period of oscillations of the original magnet about an axis through the mid point, perpendicular to length, in magnetic field B. What would be the similar period T′ for each piece?

Main concept used: $T = 2\pi\sqrt{\frac{I}{mB}}$ and $\boxed{nm' = m}$

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Ans. If a magnet of magnetic moment m is cut into n equal parts then magnetic moment m' of all equal parts is $nm' = m$. So magnetic moment of each 2 parts of magnet $= m' = \frac{m}{2}$.

$$I = \frac{ml^2}{12}$$

as the length of new magnet $= l' = \frac{l}{2}$

So original time period $T = 2\pi\sqrt{\frac{I}{mB}}$.

If M is the mass of original magnet then the mass of each two magnets m' will be $\frac{M}{2}$.

So, $$I = \frac{Ml^2}{12} \quad \text{and} \quad I' = \frac{\frac{M}{2}\cdot\left(\frac{l}{2}\right)^2}{12} = \frac{Ml^2}{8\times 12}$$

$$\frac{T}{T'} = \frac{2\pi\sqrt{\frac{I}{mB}}}{2\pi\sqrt{\frac{I'}{m'B}}} = \sqrt{\frac{I}{m}\cdot\frac{m'}{I'}} \quad \text{or} \quad \frac{T'}{T} = \sqrt{\frac{m}{m'}\cdot\frac{I'}{I}}$$

$$\frac{I'}{I} = \frac{\frac{m'l'^2}{12}}{\frac{ml^2}{12}} = \frac{\frac{m}{2}\cdot\left(\frac{l}{2}\right)^2}{ml^2}$$

$$\frac{I'}{I} = \frac{\frac{m}{2}\cdot\frac{l^2}{4}}{ml^2} = \frac{1}{8}$$

$$\frac{m}{m'} = \frac{m}{\frac{m}{2}} = \frac{2}{1}$$

$\therefore$ $$\frac{T}{T'} = \sqrt{\frac{2}{1}\times\frac{1}{8}} = \sqrt{\frac{1}{4}}$$

$$\frac{T}{T'} = \frac{1}{2} \quad \text{or} \quad T' = \frac{T}{2}\ \text{sec}$$

Q5.20. Use (*i*) Ampere's law for $\vec{H}$ and (*ii*) continuity of lines of $\vec{B}$ to conclude that inside a bar magnet, (*a*) lines of $\vec{H}$ run from the N-pole to S-pole while (*b*) lines of $\vec{B}$ must run from S-pole to N-pole.

Main concept used: Ampere's circuital law, angle between $\vec{B}$ and $\vec{dl}$ inside or outside the magnet.

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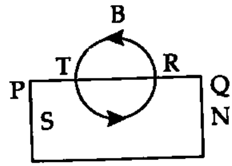


Ans. Consider an Amperian loop C inside and outside the magnet NS on side PQ of magnet then

$$\int_P^Q \vec{H}.\vec{dl} = \int_Q^P \frac{\vec{B}}{\mu_0}\cdot\vec{dl}$$

where B is magnetic field and m_0 is dipole moment. As angle between B and dl varies from 90°, 0, 90° from R to T in figure, so cos θ is greater than 1. So $\int_P^Q \vec{H}.\vec{dl} = \int_Q^P \frac{\vec{B}}{\mu_0}\cdot\vec{dl} > 0$ *i.e.* positive.

Hence, the value of B must be varied from south pole to north pole inside the magnet. According to Ampere's law $\oint_{PQP} \vec{H}.\vec{dl} = 0$

$$\oint_{PQP} \vec{H}.\vec{dl} = \int_P^Q \vec{H}.\vec{dl} + \int_Q^P \vec{H}.\vec{dl} = 0$$

As $\int_P^Q H.dl > 0$ (outside the magnet) and $\int_Q^P H.dl < 0$ (inside the magnet). It is due to the angle between H and dl is more than 90° inside the magnet so cos θ is negative. It means the lines of H must run from north pole to south pole.

LONG ANSWER TYPE QUESTIONS

Q5.21. Verify the Ampere's law for magnetic field of a point dipole of dipole moment $\vec{m} = m\hat{k}$. Take C as the closed curve running clockwise along

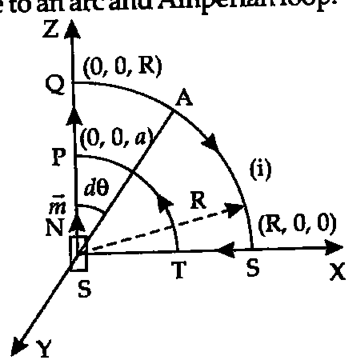
(*i*) the Z-axis from $Z = a > 0$ to $Z = R$,

(*ii*) along the quarter circle of radius R and centre at the origin in first quadrant of X-Z plane,

(*iii*) Along the X-axis from $X = R$ to $X = a$ and

(*iv*) along the quarter circle of radius a and centre at the origin in the first quadrant of X-Z plane.

Main concept used: Magnetic field due to an arc and Amperian loop.



Ans. In figure from P to Q every point on Z-axis lies on dipole is in $\hat{k}$ direction so all points on Z-axis lies on axial dipole NS placed at origin.

So magnetic field induction (B) at a point (0, 0, z) from magnetic dipole (centre at origin) and having magnetic moment $\vec{m}\hat{k}$ of magnitude ($\vec{m}$).

$$|B| = \frac{\mu_0}{4\pi}\frac{2(\vec{m})}{z^3} = \frac{\mu_0\vec{m}}{2\pi z^3}$$

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(*i*) Ampere's law along Z axis from $Z = a$ to $Z = R$ *i.e.* from P to Q

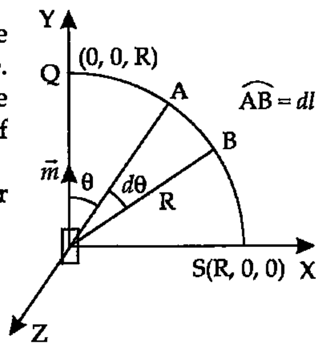
$$\int_P^Q \vec{B}.\vec{dl} = \int_P^Q B dl \cos 0° = \int_P^Q B\, dz$$

$$= \int_P^Q \frac{\mu_0 \vec{m}}{2\pi z^3} dz = \frac{\mu_0 \vec{m}}{2\pi} \int_a^R z^{-3} dz$$ [∵ distance of P and Q from origin are a and R respectively]

$$= \frac{\mu_0 \vec{m}}{2\pi}\left[\frac{z^{-2}}{-2}\right]_a^R = \frac{\mu_0 \vec{m}}{2\pi(-2)}\left[\frac{1}{R^2} - \frac{1}{a^2}\right]$$

$$\int_P^Q B.dl = \frac{\mu_0 \vec{m}}{4\pi}\left[\frac{1}{a^2} - \frac{1}{R^2}\right]$$

(*ii*) Ampere's law along the quarter circle QS of radius R as given in figure here. Point A can be considered on the equitorial line of magnetic dipole of moment $\vec{m} \sin\theta$.



Magnetic field at point A on the circular arc is

$$B = \frac{\mu_0}{4\pi} \frac{\vec{m}\sin\theta}{R^3}$$

$$d\theta = \frac{dl}{R} \Rightarrow dl = R.d\theta$$

∴ By Ampere's law $\int B.dl = \int B.dl \cos\theta$

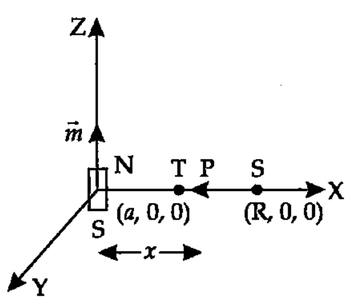
$$\int \vec{B}.\vec{dl} = \int_0^{\frac{\pi}{2}} \frac{\mu_0}{4\pi} \frac{\vec{m}\sin\theta}{R^3} \cdot R\, d\theta$$

$$\int \vec{B}.\vec{dl} = \frac{\mu_0 \vec{m}}{4\pi R^2}[-\cos\theta]_0^{\pi/2} = \frac{\mu_0 \vec{m}}{4\pi R^2}[-\cos 90° + \cos 0°]$$

$$\int B.dl = \frac{\mu_0 \vec{m}}{4\pi R^2}$$

(*iii*) Ampere's law along the X axis from $x = R$ to $x = a$ as in given figure here.

As all points from S to T lies on equitorial line of magnetic dipole N-S so magnetic field induction at a point P at a distance x from the dipole is



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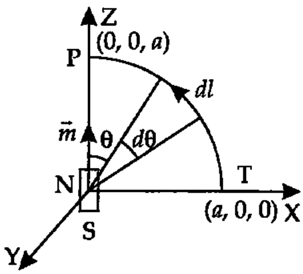
$$B = \frac{\mu_0}{4\pi}\frac{\vec{m}}{x^3} = \frac{\mu_0}{4\pi}\frac{\vec{m}\hat{k}}{x^3}$$

$$\int \vec{B}.\vec{dl} = \int_R^a \frac{-\mu_0 \vec{m}\hat{k}}{4\pi x^3}.\vec{dl} \quad \because \text{ angle between } dl \text{ and } m \text{ is } 90°$$

So $$\int \vec{B}.\vec{dl} = \int_R^a \frac{-\mu_0 |\vec{m}|\, dl \cos 90°}{4\pi x^3} = 0$$

(*iv*) Ampere's law along the quarter circle of radius '*a*' and centre at the origin in the quadrant X-Z plane as in figure here as in part (*ii*).



$$\int B.dl = \int_{\pi/2}^{0} \frac{\mu_0}{4\pi}\frac{\vec{m}\sin\theta}{a^3}.a d\theta$$

$$\int B.dl = \frac{\mu_0 \vec{m}}{4\pi a^2}[-\cos\theta]_{\pi/2}^{0}$$

θ = angle from axial line of dipole

$$\int B.dl = \frac{\mu_0 \vec{m}}{4\pi a^2}\left[-\cos 0 + \cos\frac{\pi}{2}\right] = \frac{\mu_0 \vec{m}}{4\pi a^2}(-1+0) = \frac{-\mu_0 \vec{m}}{4\pi a^2}$$

∴ Applying Ampere's law along close path starting from P to Q, Q to S and S to P.

$$\oint_{PQST} B.dl = \int_P^Q \vec{B}.\vec{dl} + \int_Q^S \vec{B}.\vec{dl} + \int_S^T \vec{B}.\vec{dl} + \int_T^P B.dl$$

From parts (*i*), (*ii*), (*iii*) and (*iv*) substituting the values

$$\oint_{PQST} B.dl = \frac{\mu_0 \vec{m}}{4\pi}\left(\frac{1}{a^2} - \frac{1}{R^2}\right) + \frac{\mu_0 \vec{m}}{4\pi R^2} + 0 + \frac{-\mu_0 \vec{m}}{4\pi a^2}$$

$$= \frac{\mu_0 \vec{m}}{4\pi a^2} - \frac{\mu_0 \vec{m}}{4\pi R^2} + \frac{\mu_0 \vec{m}}{4\pi R^2} - \frac{\mu_0 \vec{m}}{4\pi a^2}$$

$$\oint_{PQST} B.dl = 0$$

Proves the Ampere's law is magnetism.

Q5.22. What are the dimensions of χ, the magnetic susceptibility? Consider an H atom. Guess an expression for χ upto a constant by constructing a quantity of dimensions of χ out of parameters of atom *e*, *m*, *v*, R and μ_0. Here *m* is electronic mass, v is the electronic velocity, R is the Bohr's radius. Estimate the number so obtained and compare with the value of $|\chi| \sim 10^{-5}$ for many solid materials.

Main concept used: (*i*) $\chi = \frac{M}{H} = \frac{\text{Intensity of magnetisation}}{\text{Magnetising field}}$,

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(*ii*) Biot-Savart's law, (*iii*) dimension analysis.

Ans. As the intensity of magnetisation (M) and magnetising field (H) both has the same unit, *i.e.* ampere per metre and $\chi = \frac{M}{H}$ so χ (susceptibility) has no unit.

We have to relate χ with e, v, $\tilde{m}$, R and μ_0. We will relate these physical quantities by using dimension and Biot-Savart's law.

(*i*) By Biot-Savart's law $dB = \frac{\mu_0}{4\pi}\frac{I dl \sin\theta}{r^2}$ can be used to find out the dimension of μ_0.

$$\mu_0 = \frac{dB.4\pi r^2}{Idl \sin\theta} \quad \text{for dB}$$

for dimension of dB

$$F = Bqv \sin\theta$$

$$B = \frac{F}{qv \sin\theta} = \frac{MLT^{-2}}{QLT^{-1}} = [ML^0T^{-1}Q^{-1}]$$

$$\therefore \quad \mu_0 = \frac{[MT^{-1}Q^{-1}]L^2}{QT^{-1}L} = [MLQ^{-2}]$$

Where Q is dimension of charge.

χ depends on magnetic moment induced when H is turned on. H couples to atomic electrons through its charge e. The effect on m is via current I which involves another factor of e. The combination $\mu_0 e^2$ does not depend on the "charge" Q. Dimension $\chi = \mu_0^a e^2 m^b v^c R^d$

$$[M^0L^0T^0Q^0] = \left[MLQ^{-2}\right]^a Q^2M^b[LT^{-1}]^c [L]^d$$

$$[M^0L^0T^0Q^0] = M^{a+b} \quad L^{a+c+d} \quad T^{-c} \quad Q^{-2a+2}$$

Comparing the powers

$$c = 0, \quad a + b = 0 \qquad -2a + 2 = 0 \qquad a + c + d = 0$$
$$1 + b = 0 \qquad -2a = -2 \qquad 1 + 0 + d = 0$$
$$b = -1 \qquad a = +1 \qquad d = -1$$

$$\chi = (\mu_0)e^2 m^{-1} v^0 R^{-1}$$

$$\chi = \frac{\mu_0 e^2}{mR}$$

$\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1} $\qquad e = 1.6 \times 10^{-19}$

$m = 9.1 \times 10^{-31}$ kg $\qquad$ R $= 10^{-10}$ m

$$\chi = \frac{(4\pi \times 10^{-7})(1.6 \times 10^{-19})(1.6 \times 10^{-19})}{(9.1 \times 10^{-31}) \times 10^{-10}}$$

$$= \frac{4 \times 3.1 \times 1.6 \times 1.6}{9.1} \times 10^{-7-19-19+31+10}$$

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$$= \frac{124 \times 256 \times 10^{-45+41}}{9.1} = \frac{317.4}{91} \times 10^{-4} = 3.5 \times 10^{-4}$$

$|\chi'| = 10^{-5}$ (given)

$$\frac{\chi}{|\chi'|} = \frac{3.5 \times 10^{-4}}{10^{-5}} = \frac{3.5 \times 10^{-4}}{10^{-1} \times 10^{-4}} = 35$$

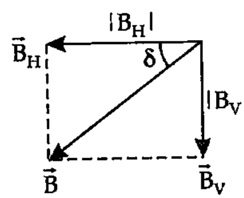
$$\chi = 35|\chi'|$$

Q5.23. Assume the dipole model for the earth's magnetic field B which is given by B_V = Vertical component of magnetic field $= \frac{\mu_0}{4\pi}\frac{2m\cos\theta}{r^3}$ and B_H = Horizontal component of magnetic field $= \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^3}$. θ is equal to 90° latitude as measured from magnetic equator.

Find the loci of the points for which (*i*) $|B|$ is minimum, (*ii*) dip angle is zero and (*iii*) dip angle is ±45°.

Main concept used: (*i*) $B^2 = B_H^2 + B_V^2$, (*ii*) angle of dip $\tan\alpha = \frac{B_V}{B_H}$.

Ans. (*i*) We know from fig.



$$B^2 = B_V^2 + B_H^2$$

$$= \left[\frac{\mu_0}{4\pi}\frac{2m\cos\theta}{r^3}\right]^2 + \left[\frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^3}\right]^2$$

(substituting the value of B_V and B_H from question)

$$= \left[\frac{\mu_0}{4\pi}\right]^2 \frac{m^2}{r^6}[4\cos^2\theta + \sin^2\theta]$$

$$B^2 = \left(\frac{\mu_0}{4\pi}\right)^2 \times \frac{m^2}{r^6}[3\cos^2\theta + 1]$$

$$B = \frac{\mu_0}{4\pi}\frac{m}{r^3}[3\cos^2\theta + 1]^{1/2} \quad ...(i)$$

From equation (*i*), the value of B will be minimum when $[3\cos^2\theta + 1]^{1/2}$ is minimum which will be at $\theta = \frac{\pi}{2}$. So magnetic equator lies at $\theta = \frac{\pi}{2}$, i.e., from magnetic dipole axis $\theta = \frac{\pi}{2}$ for magnetic equator.

(*ii*) For angle of dip δ

$$\tan\delta = \frac{B_V}{B_H} = \frac{\frac{\mu_0}{4\pi}\frac{2m\cos\theta}{r^3}}{\frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^3}}.$$

$$\tan\delta = 2\cot\theta$$

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for $\delta = 0$, $\cot\theta = 0$, $\theta = \dfrac{\pi}{2}$

So angle of dip will lie at magnetic equator.

(*iii*) $\tan\delta = \dfrac{B_V}{B_H}$ = angle of dip $\delta = \pm 45°$

or $\dfrac{B_V}{B_H} = \tan \pm 45° \Rightarrow \dfrac{B_V}{B_H} = \tan 45°$

$\dfrac{B_V}{B_H} = 1$ or $\boxed{B_V = B_H}$

$\delta = \pm 45°$

$\tan \pm 45° = 2\cot\theta$ [$\because \tan\delta = 2\cot\theta$]

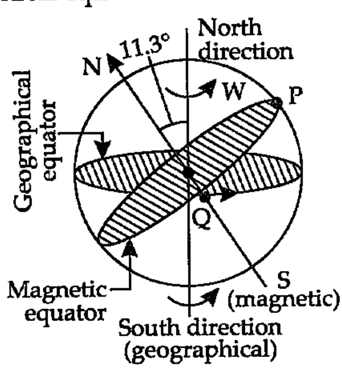
$\cot\theta = \dfrac{1}{2}$ or $\tan\theta = 2$

$\theta = \tan^{-1} 2$ is the locus of points where the angle of dip $\delta = \pm 45°$

Q5.24. Consider the plane S formed by the dipole axis and axis of earth. Let P be the point on the magnetic equator and in S. Let Q be the point of intersection of the geographical and magnetic equators. Obtain the angle of declination and dip at P and Q.

Main concept used: (*i*) Angles of magnetic axis with geographical axis, (*ii*) Angles of magnetic and geographical equator with their axis.

Ans. Q is the point of intersection between geographical meridian and magnetic meridian. So angle dip at P and Q will be zero with horizontal as magnetic middle stay horizontally.



As angle between axis of rotation of earth and magnetic axis is 11.3° and their respective equators are at 90° with their respective axis.

So angle between the plane of magnetic and geographical planes *i.e.* declination will be 11.3° at P and Q both.

Q5.25. There are two current carrying planar coils made each from identical wires of length L. Coil C_1 is circular of radius R and coil C_2 is square of side a. They are so constructed that they have same frequency of oscillation when they are placed in same uniform field $\vec{B}$ and carry the same current. Find a in terms of R.

Main concept used: (*i*) Magnetic moment $m = nIA$, (*ii*) Time period of oscillation.

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Ans. As the frequencies (ω) for both coil are given same

$\therefore$

$$\omega_1 = \omega_2$$

$$\frac{2\pi}{T_1} = \frac{2\pi}{T}$$

So the time period of both the coils C_1 and C_2 are equal so

$$T_1 = T_2$$

$$2\pi\sqrt{\frac{I_1}{m_1 B}} = 2\pi\sqrt{\frac{I_2}{m_2 B}}$$

$$\sqrt{\frac{I_1}{m_1}} = \sqrt{\frac{I_2}{m_2}}$$

as B is same in both coils so squaring both sides we get,

$$\frac{I_1}{I_2} = \frac{m_1}{m_2} \quad \text{...I,}$$

I_1, I_2 are the moment of inertia of coils C_1 and C_2 placed in same magnetic field B.

$$I_1 = \frac{mR^2}{2} \quad \text{...II,} \qquad I_2 = \frac{ma^2}{12} \quad \text{...III}$$

as the length of wire is same and identical so masses $m_1 = m_2 = m$

For circular shaped coil, magnetic moment

$$m_1 = n_1 I A_1 [\because \text{ current (I) in both are same, i.e. } I_1 = I_2 = I]$$

$$m_1 = \frac{L}{2\pi R} \cdot I \cdot \pi R^2 \qquad [\because L = 2\pi R n_1]$$

$$m_1 = \frac{LIR}{2} \quad \text{...IV}$$

For square shaped coil, magnetic moment

$$m_2 = n_2 I A_2 \text{ as current } I_1 = I_2 = I \text{ (given)} = \frac{L}{4a} I.a^2$$

$$m_2 = \frac{LIa}{4} \quad \text{...V}$$

Substitute II, III, IV, V in I

$$\frac{mR^2/2}{ma^2/12} = \frac{LIR/2}{LIa/4}$$

or

$$\frac{R^2/2}{a^2/12} = \frac{R/2}{a/4}$$

$$\frac{R^2}{2} \times \frac{a}{4} = \frac{R}{2} \cdot \frac{a^2}{12}$$

$$\frac{R}{8} = \frac{a}{24}$$

$$24R = 8a$$

$$3R = a$$

❑❑❑

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6 Electromagnetic Induction

MULTIPLE CHOICE QUESTIONS—I

Q6.1. A square of side L metres lies in X-Y plane in a region where the magnetic field is given by

$B = B_0(2i + 3j + 4k)$ Tesla, where B_0 is constant. The magnitude of flux passing through the square is

(*a*) $2B_0L^2$ Wb (*b*) $3B_0L^2$ Wb

(*c*) $4B_0L^2$ Wb (*d*) $\sqrt{29}\,B_0L^2$ Wb

Main concept used: $Q = \vec{B} . \vec{A}$ direction of A is perpendicular to the plane of square.

Ans. (*c*): Square lies in X-Y plane in $\vec{B}$ so $\vec{A} = L^2.\hat{k}$

$$\therefore \quad Q = B.A$$
$$= B_0(2i + 3j + 4k).(L^2\hat{k}) = B_0[2 \times i.k + 3 \times j.k + 4 \times k.k]$$
$$= B_0L^2[0 + 0 + 4] = 4B_0L^2 \text{ Wb.}$$

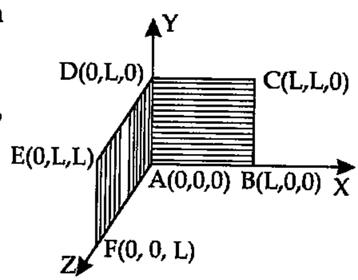
Q6.2. A loop made of straight edges has six corners at A(0, 0, 0), B(L, 0, 0), C(L, L, 0), D(0, L, 0), E(0, L, L) and F(0, 0, L). A magnetic field $B = B_0(i + k)$ Tesla is present in region. The flux passing through the loop ABCDE FA (in that order) is

(*a*) B_0L^2 Wb (*b*) $2B_0L^2$ Wb

(*c*) $\sqrt{2}\,B_0L^2$ Wb (*d*) $4B_0L^2$ Wb

Main concept used: Direction of $\vec{A}$ of loop.

Ans. (*b*): The loop can be considered in two planes.



(*i*) Plane of ABCDA is in X-Y plane

So its vector $\vec{A}$ is in Z-direction so

$$A_1 = |A|\hat{k} = L^2\hat{k}$$

(*ii*) Plane of DEFAD is in Y-Z plane

So $A_2 = |A|\, i = L^2 i$

$$\therefore \quad A = A_1 + A_2 = L^2(\hat{i} + \hat{k})$$
$$B = B_0(i + k)$$

So $Q = B.A = B_0(i + k) . L^2(i + k) = B_0L^2[i.i + i.k + k.i + k.k]$

$$= B_0L^2[1 + 0 + 0 + 1] \qquad \because \cos 90° = 0$$
$$= 2B_0L^2 \text{ Wb}$$

Verifies answer (*b*).

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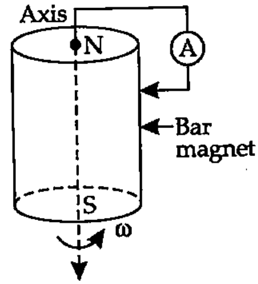
Q6.3. A cylindrical bar magnet is rotated about its axis. A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then

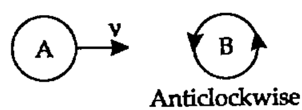


(*a*) a direct current flows in the Ammeter A.
(*b*) no current flows in ammeter A.
(*c*) an alternating sinusoidal current flows through the ammeter A with a time period $T = \frac{2\pi}{\omega}$
(*d*) a time varying non-sinusoidal current flows through the ammeter A.

Main concept used: Induced current flows only when circuit is complete and there is a variation of B about a circuit.

Ans. (*b*): Here, circuit with ammeter is complete. We know that as there is a symmetry in magnetic field of a bar magnet about its axial axis, so no change in magnetic field across the circuit when magnet is rotated, either alone or with circuit. No current flows in ammeter verifies answer (*b*).

Q6.4. There are two coils A and B as shown in figure. A current starts flowing in B as shown in figure, when A is moved towards B and stops when A stops moving. The current in A is counterclockwise. B is kept stationary when A moves. We can infer that

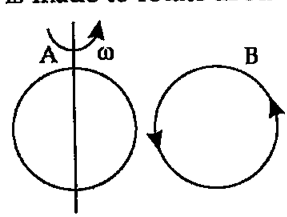


(*a*) there is a constant current in the clockwise direction in A.
(*b*) there is a varying current in A.
(*c*) there is no current in A.
(*d*) there is a constant current in the counter clockwise direction in A.

Main concept used: Due to the current in A and its motion the flux changes across B. So induced current is produced.

Ans. (*d*): When coil A moves towards coil B with constant velocity so rate of change of magnetic flux due to coil B in coil A will be constant gives constant current in A in same direction as in B by Lenz's law. Hence verifies answer (*d*).

Q6.5. Same as Question 6.4 except the coil A is made to rotate about a vertical axis as in figure. No current flows in B if A is at rest. The current in coil A when the current in B (at $t = 0$) is counter clockwise and the coil A is as shown at this instant, $t = 0$, is



(*a*) constant current clockwise.

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(*b*) varying current clockwise
(*c*) varying current counterclockwise
(*d*) constant current counterclockwise

Main concept used: Lenz's law.

Ans. (*a*): At $t = 0$ current in B is clockwise and coil A is considered above B. The counterclockwise flow of the current in B is equivalent to north pole of magnet and magnetic field lines are emanating upward to coil A. When coil A start rotating at $t = 0$, the current in A is constant along clockwise direction by Lenz's rule. As flux changes across coil A by rotating it near the N-pole formed by flowing current in B in anticlockwise. So verifies ans. (*a*).

Q6.6. The self inductance L of a long solenoid of length l and area of cross-section A, with a fixed number of turns N increases as

(*a*) l and A increases
(*b*) l decreases and A increases
(*c*) l increases and A decreases
(*d*) Both l and A decreases.

Main concept used: Self inductance $L = \mu_r \mu_0 n^2 A.l$.

Ans. (*b*): As we know $L = \mu_r \mu_0 \frac{N^2}{ll} . A.l$

$$L = \mu_r \mu_0 \frac{N^2 . A}{l}$$

as L is constt. for a coil

$$L \propto A \quad \text{and} \quad L \propto \frac{1}{l}$$

As μ_r and N are constant here so, to increase L for a coil, area A must be increased and l must be decreased so verify ans. (*b*).

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q6.7. A metal plate is getting heated. It can be because

(*a*) a direct current is passing through the plate.
(*b*) it is placed in a time varying magnetic field.
(*c*) it is placed in a space varying magnetic field but does not vary with time.
(*d*) a current (either direct or alternating) is passing through the plate.

Main concept used: Heating effect of current I^2Rt and eddy current.

Ans. (*a*) (*b*) (*d*): A plate or conductor is heated by heating effect of current ($H = I^2Rt$). Verify answers (*a*) (*d*).

When time dependent magnetic field varies across the metal plate due to production of eddy currents, heating of plate takes place verify (*b*).

Q6.8. An e.m.f. is produced in a coil, which is not connected to external voltage source. This can be due to

(*a*) the coil being in a time varying magnetic field.
(*b*) the coil moving in a time varying magnetic field.

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(*c*) the coil moving in a constant magnetic field.
(*d*) the coil is stationary in external spatially varying magnetic field which does not change with time.

Main concept used: e.m.f. produced by electromagnetic induction.

Ans. (*a*) (*b*) (*c*): e.m.f. produced in coil due to change in magnetic flux with time in options (a), (b) and (c). But in part (d) magnetic field does not change with time although varying magnetic field but rate of change of magnetic field does not change so rejects the option (d). So answers are (*a*) (*b*) and (*c*).

Q6.9. The mutual inductance M_{12} of coil 1 with respect to coil 2
(*a*) increases when they are brought nearer.
(*b*) depends on the current passing through coils.
(*c*) increases when one of them is rotated about an axis.
(*d*) is the same as M_{21} of coil 2 with respect to coil 1.

Main concept used: Mutual inductance depends on Geometry of coils which is constant for two coils.

Ans. (*a*) (*d*): As we know that $M_{12} = M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$ where $r_1 l$ is the common area of cross-section of coil so M_{12} does not depend on passing current and rotation. l is also common length. So answer (*a*) and (*d*) verified.

Q6.10. A circular coil expands radially in a region of magnetic field and no electromotive force is produced in the coil. This can be because
(*a*) the magnetic field is constant.
(*b*) the magnetic field is in the same plane as the circular coil and it may or may not vary.
(*c*) the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably.
(*d*) there is a constant magnetic field in the perpendicular (to the plane of the coil) direction.

Main concept used: Induced e.m.f. can produce only when there is change in magnetic flux with coil or inside coil.

Ans. (*b*) (*d*): When coil expands in constant magnetic field, the magnetic flux inside the coil (along its area vector) increase then induced current produce so rejects option (*a*) and (*c*).
As the component of magnetic field along area vector is zero so $\phi = B.A$ becomes zero. So, no induced current flows in coil verifies the options (b) and (d).

VERY SHORT ANSWER TYPE QUESTIONS

Q6.11. Consider a magnet surrounded by a wire with an ON/OFF switch S (in figure). If the switch is thrown from the off position (open circuit) to the on position (closed circuit) will a current flow in the circuit? Explain.

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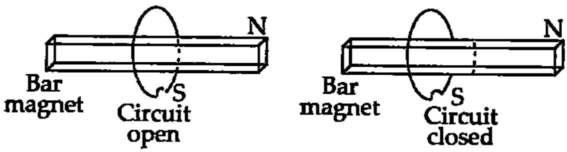


Main concept used: $\because \phi = BA\cos\theta$, so ϕ can be changed by changing either B, or A or θ.

Ans. As we know that induced e.m.f. or induced current can flow when there is change in magnetic flux with respect to time and magnetic flux $\phi = BA\cos\theta$. As there is no any change in B, *i.e.*, magnet, A, *i.e.*, area of circuit and no change in angle between $\vec{B}$ and $\vec{A}$. So no induced current will flow.

Q6.12. A wire in the form of a tightly wound solenoid is connected to a DC source and carries a current. If the coil is stretched so that there are gaps between successive elements of the spiral coil, will the current increase or decrease? Explain.

Main concept used: Lenz's law, leakage of flux and $L = \mu_r \mu_0 n^2 Al$.

Ans. When the coil is stretched the magnetic flux will leak through the gaps between two turns. According to Lenz's law, the e.m.f. is produced in nearby coil which will oppose the increase or decrease in flux in coil. So, on stretching the coil, the current will increase. When the coil is stretched then n (no. of turns per unit length) will decrease. So L decrease or reactance decrease which makes current increase.

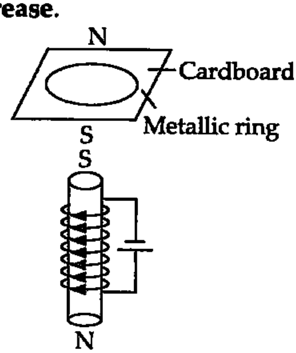
Q6.13. A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid will the current increase or decrease? Explain.

Main concept used: Ferromagnetic core increases flux in solenoid which in turn decrease the current by Lenz's law.

Ans. When the ferromagnetic iron core is inserted inside the solenoid the magnetic flux will increase.

As flux increased, then by Lenz's law, to oppose increase in flux *i.e.*, to decrease flux **the current in coil will decrease.**

Q6.14. Consider a metal ring kept on the top of a fixed solenoid (say on a cardboard) as in figure. The centre of ring coincides with axis of the solenoid. If the current is suddenly switched on, the metal ring jumps up. Explain.



Main concept used: Lenz's law.

Ans. When the current is switched on in solenoid, magnetic flux increased across the ring so induced current produced in such

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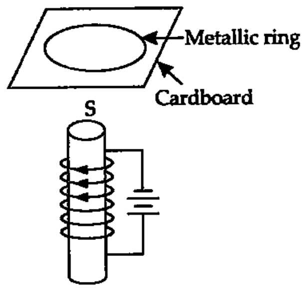
direction so that it can decrease the flux or oppose the increased flux. So, the direction of magnetic flux between solenoid will be like that ring is repelled and jumps. Or when current is switched on, the upper end becomes south pole. By Lenz's law the induced current produce south pole lower side and North pole upper side so repelled and jumps up.

Q6.15. Consider a metal ring kept (supported by a cardboard) on top of a fixed solenoid carrying current I as in figure. The centre of the ring coincides with the axis of solenoid. If the current in the solenoid is switched off, what will happen to the ring?



Main concept used: Application of Lenz's law.

Ans. As current was already flowing through the solenoid so it behaves like a magnet and let S pole is upper side as flux in ring is constant. So there is no induced current in ring.

When current is switched off, magnetic flux decreases so induced current produced in ring in such a way so that it can increase the flux. So North pole is produced in ring in lower side, and attracted by solenoid. So, downward force of attraction between ring and solenoid acts but as cardboard and solenoid are fixed ring will not be able to move downward.

Q6.16. Consider a metallic pipe with an inner radius of 1 cm. If a cylindrical bar magnet of radius 0.8 cm is dropped through the pipe it takes more time to come down, than it takes for similar unmagnetised cylindrical iron bar dropped through the metallic pipe. Explain.

Main concept used: Eddy current and Lenz's law.

Ans. When a cylindrical bar magnet of radius 0.8 cm is dropped inside a hollow cylindrical metallic pipe of radius 1 cm the magnetic flux across the pipe changes, and due to induction, eddy current will produce.

The direction of eddy current will be in such a way so that it can oppose the cause *i.e.*, motion of magnet. So upward force acts on bar magnet and decreases the acceleration, which in turn takes more time to fall magnet through the pipe, as compared to simple metallic piece in place of magnet.

SHORT ANSWER TYPE QUESTIONS

Q6.17. A magnetic field in a certain region is given by $B = B_0 \cos \omega t\, \hat{k}$, and a coil of radius *a* with resistance R is placed in X-Y plane with its

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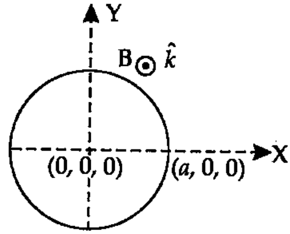
centre at the origin in the magnetic field as in figure. Find magnitude and direction of the current at $(a, 0, 0)$ at

$$t=\frac{\pi}{2\omega},\quad t=\frac{\pi}{\omega}\quad \text{and}\quad t=\frac{3\pi}{2\omega}$$



Main concept used: Faraday's law of EMF.

Ans. The direction of magnetic field is along Z-axis. So the magnetic flux passing through circular region of coil of radius a placed in X-Y plane.

$$\phi = \vec{B}.\vec{A} = BA\cos\theta$$

The direction area vector A of coil is along Z-direction and B is also in Z-direction so $\theta = 0°$

$$\phi = (B_0 \cos \omega t).(\pi a^2)\cos 0°$$

$$\phi = B_0 \pi a^2 \cos \omega t$$

By Faraday's law of electromagnetic induction,

$$\varepsilon = \frac{d\phi}{dt} = B_0 \pi a^2 \sin \omega t.\omega$$

$$\varepsilon = B_0 \pi a^2 \omega \sin \omega t$$

Flow of induced current in coil $I = \frac{\varepsilon}{R}$ where R is resistance of coil.

$$\therefore \quad I = \frac{B_0 \pi a^2 \omega}{R}\sin \omega t$$

For currents at $t = \frac{\pi}{2\omega}, \quad t = \frac{\pi}{\omega}, \quad t = \frac{3\pi}{2\omega}$

$\therefore$ Current at $t = \frac{\pi}{2\omega}$

$$\therefore \quad I = \frac{B_0 \pi a^2 \omega}{R}\sin \omega.\frac{\pi}{2\omega} = \frac{B_0 \pi a^2 \omega}{R}\sin\frac{\pi}{2} \quad \left[\sin\frac{\pi}{2}=1\right]$$

So, at $t = \frac{\pi}{2\omega}$,

$$\text{Current } I = \frac{B_0 \pi a^2 \omega}{R} \qquad (\text{along } +\hat{j})$$

at $t = \frac{\pi}{\omega}$,

$$I = \frac{B_0 \pi a^2 \omega}{R}\sin \omega \frac{\pi}{\omega}$$

$$I = \frac{B_0 \pi a^2 \omega}{R}\sin \pi = \frac{B_0 \pi a^2 \omega}{R}\times 0 = 0$$

at $t = \frac{\pi}{\omega}$, induced current $I = 0$

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at $t = \frac{3\pi}{2\omega}$, $\quad I = \frac{B_0 \pi a^2 \omega}{R} \sin \omega . \frac{3\pi}{\omega}$

$$I = \frac{B_0 \pi a^2 \omega}{R} \sin 3\pi \qquad [\because \sin(2\pi + \pi) = \sin \pi]$$

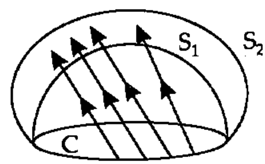
$$= \frac{B_0 \pi a^2 \omega}{R} \sin \pi$$

$$I = \frac{-B_0 \pi a^2 \omega}{R}$$

So, at $t = \frac{3\pi}{\omega}$,

Induced current $I = \frac{-B_0 \pi a^2 \omega}{R}$ (along $-\hat{j}$)

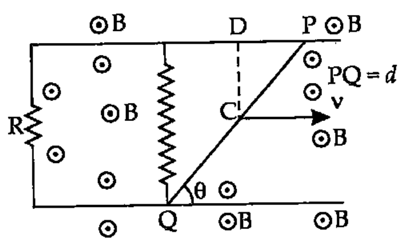
Q6.18. Consider a closed loop C in a magnetic field (as in figure). The flux passing through the loop is defined by choosing a surface whose edge coincides with loop and using the formula $\phi = B_1.dA_1 + B_2.dA_2 + B_3.dA_3 + \ldots$. Now if we chose two different surfaces S_1 and S_2 having C as their edge, would we get the same answer for flux? Justify your answer.



Main concept used: Properties of magnetic lines.

Ans. The magnetic flux linked with the surface can be considered as the number of magnetic field lines passing through the surface. So let $d\phi = \vec{B}.\vec{A}$ represents magnetic field lines in an area A to magnetic flux B. By the concept of continuity of line B cannot end or start in space, therefore, the number of lines passing through S_1 must be the same as the number of lines passing through S_2 surface. Therefore, in both the cases we get the same flux.

Q6.19. Find the current in the wire for the configuration shown in figure. Wire PQ has negligible resistance. $\vec{B}$, the magnetic field is coming out of the paper. θ is fixed angle made by PQ travelling smoothly over two conducting parallel wires separated by distance d.



Main concept used: Induced current due to change in area of loop.

Ans. The motional electric field E due to the motion along CD is $E = vB$.

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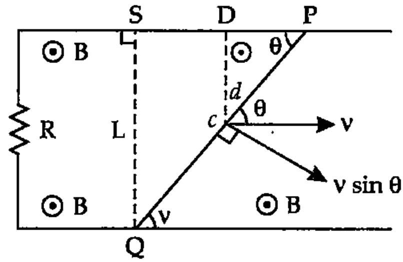


The direction of E will be ⊥ to both v and B $\because F = q \times B$
F is force on free charge particle of PQ.
So the motional e.m.f. ε = E along PQ × effective length PQ.

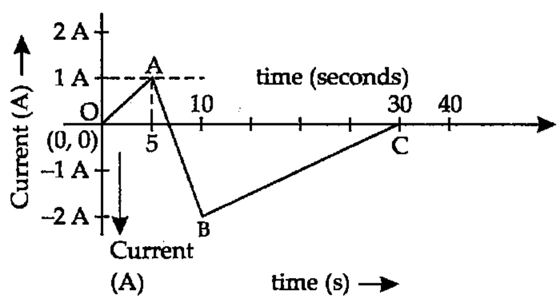
Electric field along PQ $= v \times B = v \sin\theta \cdot B = vB\sin\theta$

$$\varepsilon = (vB\sin\theta)\left(\frac{d}{\sin\theta}\right)$$
$$\varepsilon = vBd$$

∴ Induced current $= \frac{\varepsilon}{R} = \frac{vBd}{R}$

It is independent of q.

Q6.20. A (current versus time) graph of the current passing through solenoid is shown in figure. For which time is the back electromotive force (u) a maximum? If the back e.m.f. at $t = 3$s is e, find the back e.m.f. at $t = 7$s, 15 s and 40s. OA, AB, BC are straight line segments.



Main concepts used: (*i*) Current is variable so magnetic flux will change which in turns changes the emf (*ii*) Lenz's law.

Ans. The maximum back electromotive force (u) will be maximum when there is maximum rate of change of magnetic flux which is directly proportional to the rate of change of current.

Maximum change or rate of current will be where (t – I) graph makes maximum angle with time axis which is in part AB.

So the maximum back e.m.f. will occur between 5 s to 10 s. As the back e.m.f. at $t = 3$ s is 'e' (given)

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Rate of change of current at $t = 3$ s = slope of OA graph with time axis

So rate of change of current at 3 s = $\frac{1}{5}$ A/s.

So back electromotive force at $t = 3$ s = $L \times \frac{1}{5} = \frac{L}{5} = e$ given)

$\because e = L.\frac{dI}{dt}$ and L = constant for solenoid.

Similarly **back e.m.f.** u_1 between 5 to 10 sec.

$$u_1 = L.\left(\frac{-3}{5}\right) = -3\frac{L}{5} = -3e$$

back e.m.f. between 10 to 30 sec

$$u_2 = L\frac{[0-(-2)]}{(30-10)} = \frac{+2L}{20} = \frac{+1}{2}\frac{L}{5}$$

$$u_2 = +\frac{1}{2}e$$

So back e.m.f. at 7 sec = –3e

back e.m.f. at 15 sec = $+\frac{1}{2}e$

At 40 sec graph is along time axis, i.e. its slope with time axis is zero.

So, $\frac{dI}{dt} = 0$.

or back e.m.f. at 40 sec = 0

Q6.21. There are two coils A and B separated by some distance. If a current of 2 A flows through A, a magnetic flux of 10^{-2} Wb passes through B (no current through B). If no current passes through A and 1 A current passes through B what is flux through A?

Main concept used: Mutual inductance.

Ans. Let I_A current is passing through coil A having mutual inductance M_{AB} with respect to coil B.

N_A = number of turns in coil A
N_B = number of turns in coil B
ϕ_A = flux linked with coil A due to coil B
ϕ_B = flux linked with coil B due to coil A
M_{BA} = Mutual inductance of coil B with respect to coil A

Then

$$\text{Total flux through B} = M_2\phi_2 = M_{BA}I_1$$
$$10^{-2} = M_{BA} \times 2$$
$$M_{BA} = \frac{10^{-2}}{2} = 5\text{ mH}$$

Now total flux through A= $M_A\phi_A = M_{AB}\, I_2$ $[\because M_{AB} = M_{BA}]$

$= 5 \text{ mH} \times 1 \text{ Wb} = 5 \text{ mWb}$ **Ans.**

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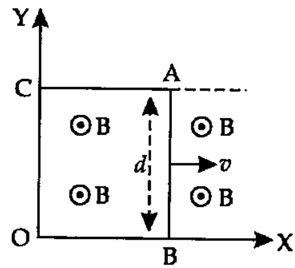
LONG ANSWER TYPE QUESTIONS

Q6.22. A magnetic field $B = B_0 \sin(\omega t)\hat{k}$ covers a large region where a wire AB slides smoothly over two parallel conductors separated by a distance d (figure). The wires are in X-Y plane. The wire AB of length d has resistance R and parallel wires have negligible resistance. If AB is moving with velocity v, what is the current in circuit? What is the force needed to keep the wire moving at constant velocity?



Main concept used: e.m.f. induced in AB will be due to change in area of loop within magnetic field and Force F = B I L sin θ, θ is angle between $\overline{B}$ and L.

Ans. In figure CA and OB are long parallel conducting wires, connected by d and conductor CO. The resistance of ACOB is negligible.

Let wire AB at $t = 0$ is at $x = 0$ *i.e.*, on Y-axis. Now AB moves with velocity $v\hat{i}$.

Let at any time t, position of conductor AB is $x(t) = v\hat{i}\,t$

Motional e.m.f. across AB

$$V_{AB} = \frac{W_{AB}}{q} = \frac{F.d}{q} = \frac{qvi \times B}{q}.d$$

$$V_{AB} = v\hat{i} \times B_0 \sin \omega t.\hat{k} \times d$$

$$\varepsilon_1 = B_0 \sin \omega t\, v(\hat{i} \times \hat{k})d = B_0 dv \sin \omega t\,(-\hat{j})$$

$$\varepsilon_1 = -B_0 vd \sin \omega t\,(\hat{j}) \text{ (along –ve Y direction in AB)}$$

e.m.f. due to change in magnetic field from $t = 0$ to at $t = 1$ when AB is at $(x, 0)$

$$\therefore \quad \varepsilon_2 = \frac{-d\phi}{dt} = \frac{-d}{dt}\vec{B}.\vec{A} = \frac{-d}{dt}[B_0 \sin \omega t\,\hat{k}.(x.d)\hat{k}]$$

Area vector A is along $\hat{k}$

$$\therefore \quad \varepsilon_2 = \frac{-d}{dt}(B_0 \sin \omega t\; x.d)$$

$$\varepsilon_2 = -B_0\, xd.\omega \cos \omega t \qquad ...(II)$$

∴ Total magnitude of e.m.f. $= -B_0\, vd \sin \omega t - B_0\, xd\omega \cos \omega t$

$$\varepsilon = -B_0 d[v \sin \omega t + x\omega \cos \omega t]$$

The direction of electric induced current by Fleming Right Hand Rule is from (A to B) *i.e.*, in clockwise in loop.

$$I = \frac{\varepsilon}{R} = \frac{B_0 d}{R}[v \sin \omega t + \omega x \cos \omega t]$$

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The force F acting on conductor AB = BI d sin θ

θ is angle between $B\hat{k}$ and $I(-\hat{j}) = 90°$

$$F = I \times B\, dl$$

$$= I(-\hat{j}) \times B(\hat{k})\, d \sin\theta \qquad [\because \theta = 90°]$$

$$= -IB(\hat{j} \times \hat{k})\, d \sin 90° = -IB(\hat{i})d$$

$$F = -\hat{i} I B d$$

it shows the direction of force on AB is towards $-x$ direction, which can be verified by Fleming's left hand rule

$\therefore$ $$|F| = IBd$$

$\therefore$ To keep wire in motion with constant velocity, it will be towards positive x direction $+\,i$ opp. to electromagnetic force

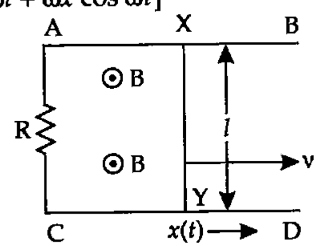
$$F = B \,.\, d\, I = |B| \cdot d \frac{\varepsilon}{R}$$

$$F = \frac{B d B_0 d}{R}[v \sin\omega t + \omega x \cos \omega t]$$

$$= \frac{B_0 \sin \omega t\, B_0 d^2}{R}[v \sin \omega t + \omega x \cos \omega t]$$

$$F = \frac{B_0^2 d^2 \sin \omega t}{R}[v \sin \omega t + \omega x \cos \omega t]$$

Q6.23. A conducting wire XY of mass m and negligible resistance slides smoothly on two parallel conducting wires as shown in figure. The closed circuit has a resistance R due to AC. AB and CD are perfect conductors. There is a magnetic field $B = B(t)\hat{k}$.



(*i*) Write down the equation for the acceleration of the wire XY.

(*ii*) If B is independent of time, obtain $v(t)$ assuming $v(0) = u_0$.

(*iii*) For (*ii*) show that the decrease in kinetic energy of XY is equal to the heat lost in R.

Main concept used: Induced e.m.f. or e.m.i. magnetic force, power consumption.

Ans. (*i*) Let wire XY at $t = 0$ is at $x = 0$

and at $t = t$ is at $x = x(t)$

Magnetic flux is a function of time $\phi(t) = B(t) \times A$

$\therefore$ $$\phi(t) = B(t)\,[l.x(t)]$$

$$\varepsilon = -\frac{d\phi(t)}{dt} = -\frac{dB(t)}{dt} l.x(t) - B(t)\, l.\frac{dx(t)}{dt}$$

$$\varepsilon = \frac{-dB(t)}{dt} l.x(t) - B(t)\, l\, v(t)$$

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The direction of induced current by Fleming's Right Hand Rule or by Lenz's law is in clockwise direction in loop XY < AX.

$$I = \frac{\varepsilon}{R} = \frac{-l}{R}\left[x(t)\frac{dB(t)}{dt} + B(t)\, v(t)\right] \quad ...(I)$$

The force acting on the conductor is $F = B(t)\, I\, l \sin 90°$

$$F = B(t)\, I.l$$

$$F = \frac{B(t)\, l\varepsilon}{R} = \frac{-B(t)\, l^2}{R}\left[\frac{-dB(t)}{dt}.x(t) - B(t).v(t)\right]$$

$$\frac{md^2x}{dt^2} = \frac{-B(t)l^2}{R}\left[x(t)\frac{dB(t)}{dt} + B(t)\, v(t)\right]$$

or
$$\frac{d^2x}{dt^2} = \frac{-l^2}{mR}B(t)\left[x(t)\frac{dB(t)}{dt} + B(t)\cdot v(t)\right] \quad ...(II)$$

(*ii*) Now B is independent of time *i.e.* B does not change with time or it is constant

$$\therefore \quad \frac{dB}{dt} = 0,\; B(t) = B \text{ and } v(t) = v \quad ...(III)$$

Put (III) in (II) we get

$$\frac{d^2x}{dt^2} = \frac{-l^2}{mR}[0 + Bv]$$

$$\frac{d^2x}{dt^2} + \frac{B^2l^2}{mR}\frac{dx}{dt} = 0 \quad (\div \text{ by } m)$$

$$\frac{dv}{dt} + \frac{B^2\, l^2}{mR}v = 0$$

Integrating using variables separable from differential equation we have

$$v = A\exp\left(\frac{-l^2\, B^2\, t}{mR}\right)$$

at $t = 0$, $v = u$

$$\therefore \quad v(t) = u\exp\left(\frac{-l^2\, B^2\, t}{mR}\right) \quad ...(IV)$$

(*iii*) Heat lost per second in (*ii*) where $\frac{dB}{dt} = 0$

$$H = I^2R$$

Magnitude of current from equation I in (*i*) part

$$I = \frac{Blv}{R} = \frac{-l}{R}[0 + Bv] \quad \left[\because \frac{dB}{dt} = 0\right]$$

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Heat produced per second $H = I^2R$

$$\therefore \quad H = \frac{B^2 l^2 v^2}{R^2}.R$$

$$H = \frac{B^2 l^2}{R} u^2 \exp\left[\frac{-2l^2 B^2 t}{mR}\right]$$

[v from eqn. (IV) in (*iii*) part]

$$\text{Power lost} = \int_0^t I^2 R dt = \frac{B^2 l^2 u^2}{R} \int_0^t e^{\frac{-2l^2B^2t}{mR}} dt$$

$$\left[\because v^2 = u^2 \exp^{\frac{-2l^2B^2t}{mR}}\right]$$

$$\text{Power lost} = \frac{B^2 l^2 u^2}{R} \frac{mR}{2l^2 B^2}\left[1 - e^{(-2l^2B^2t/mR)}\right]$$

$$= \frac{mu^2}{2}\left[1 - e^{\frac{-2l^2B^2t}{mR}}\right]$$

$$= \frac{mu^2}{2} - \frac{m}{2}u^2 e^{\frac{-2l^2B^2t}{mR}}$$

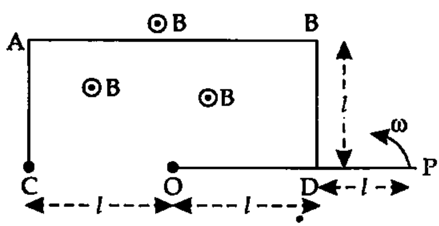
$$= \left[\frac{mu^2}{2} - \frac{mv^2(t)}{2}\right]$$

= initial K.E. – final K.E.

Power lost = decrease in kinetic energy

This proves that decrease in K.E. of XY is equal to the heat lost in R.

Q6.24. ODBAC is a fixed rectangular conductor of negligible resistance (CO is not connected) and OP is a conductor which rotates anticlockwise with an angular velocity ω (figure). The entire system is in a uniform magnetic field B whose direction is along the normal to the surface of the rectangular conductor ABDC.



The conductor OP is in electric contact with ABDC. The rotating conductor has resistance of λ per unit length. Find the current in rotating conductor as it rotates by 180°.

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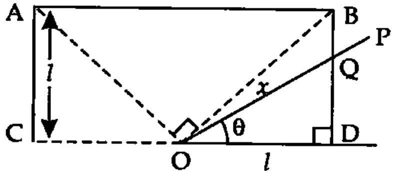
Main concept used: Induced e.m.f. produced by change in area.

Considering the position of OP between $0<\theta<\frac{\pi}{4}, \frac{\pi}{4}<\theta<\frac{3\pi}{4}$ and $\frac{3\pi}{4}<\theta<\frac{4\pi}{4}$ and O is mid point of CD and OD = BD = l



Ans. (*i*) Let rotating conductor is in contact with BD at Q making angle $0°<\theta<45°$

Magnetic flux in ΔODQ is ϕ

$$\phi = B.A$$

The direction of B and A are 0° or 180°

$$Q = B\times\frac{1}{2}l\times QD \qquad \left[\because \frac{QD}{l}=\tan\theta \quad QD = l\tan\theta\right]$$

$$\therefore \quad \phi = B.A\cos 0 = BA = B\frac{1}{2}\times l.l\tan\theta$$

$$\phi = \frac{1}{2}Bl^2\tan\theta = \frac{1}{2}Bl^2\tan\omega t \qquad (\because \theta=\omega t)$$

$$\text{Induced e.m.f. } \varepsilon = \frac{-d\phi}{dt} = \frac{d}{dt}\frac{1}{2}Bl^2\tan\omega t$$

$$\varepsilon = \frac{1}{2}Bl^2\,\omega\sec^2\omega t$$

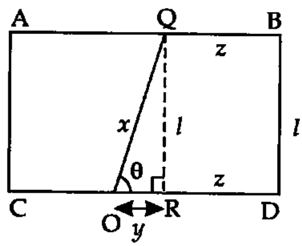
$$I = \frac{\varepsilon}{R} = \frac{Bl^2}{2R}\omega\sec^2\omega t \qquad \left[\because \cos\theta = \frac{l}{x}, x = \frac{l}{\cos\theta}\right]$$

$$\text{R of OQ} = \lambda x = \frac{\lambda l}{\cos\theta}$$

$$R = \frac{\lambda l}{\cos\omega t}$$

$$I = \frac{Bl^2}{2.\lambda l}\omega\cos\omega t\sec^2\omega t = \frac{Bl\omega}{2\lambda\cos\omega t}$$

(*ii*) Now rotating conductor rotates from B to A *i.e.*, 45° to 135° or $\frac{\pi}{4}<\theta<\frac{3\pi}{4}$.



$$\phi = B.A = B \text{ or } ODBQ$$
$$= B.$$

$$\text{Area of } \Delta ORQ = \frac{1}{2}y\times l$$

$$\tan\theta = \frac{l}{y}$$

$$y = \frac{l}{\tan\theta}$$

$\therefore$ Area of $\Delta ORQ = \frac{1}{2}\frac{l^2}{\tan\theta} = \frac{l^2}{2\tan^2\omega t}$

Flux through OQBD

$$\phi = B.\left(lz + \frac{l^2}{2\tan\theta}\right)$$

$$= Blz + \frac{1}{2}Bl^2\cot\theta = Blz + \frac{1}{2}Bl^2\cot\omega t$$

$$\therefore \quad \frac{d\phi}{dt} = \frac{d}{dt}Blz + \frac{1}{2}Bl^2(-\text{cosec}^2\,\omega t)\,\omega$$

as l B and z are constant $\therefore \frac{d(l\,B\,z)}{dt} = 0$

$$\because \quad \varepsilon = \frac{-d\theta}{dt}$$

$$\therefore \quad -\varepsilon = 0 - \frac{1}{2}\frac{Bl^2\omega}{\sin^2\omega t}$$

$$\varepsilon = \frac{1}{2}\frac{Bl^2\,\omega}{\sin^2\omega t}$$

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\lambda x} = \frac{\varepsilon\sin\omega t}{\lambda l} \qquad \left[\because \sin\theta = \frac{l}{x}, x = \frac{l}{\sin\omega t}\right]$$

$$= \frac{\sin\omega t}{\lambda l}\cdot\frac{1}{2}\frac{Bl^2\,\omega}{\sin^2\omega t}$$

$$\boxed{I = \frac{Bl\omega}{2\lambda\sin\omega t}}$$

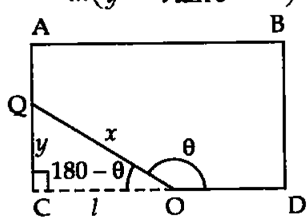
(*iii*) When $\frac{3\pi}{4} < \theta < \frac{2\pi}{2}$, the flux through OQABD = ϕ = B . A

$$\phi = B.\left(2l^2 + \frac{1}{2}ly\right)$$

$$\phi = B.\left(2l^2 + \frac{l^2\tan\omega t}{2}\right) \quad \ldots\left(\begin{array}{l}\tan(180-\theta) = \frac{y}{l}\\ y = l(-\tan\theta)\\ y = -l\tan\theta\end{array}\right)$$

$$\frac{d\phi}{dt} = \frac{d}{dt}\left[2l^2 + \frac{l^2}{2}(\tan\omega t)\right]B$$

$$-\varepsilon = 0 + \frac{Bl^2}{2}\frac{d}{dt}\tan\omega t$$



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$$-\varepsilon = +\frac{Bl^2\omega}{2}\sec^2\omega t = -\frac{Bl^2\omega}{2\cos^2\omega t}$$

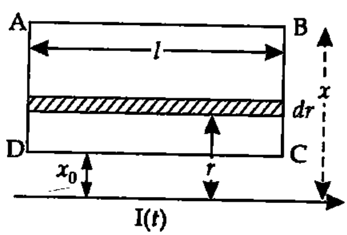
$$I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\lambda x}$$

$$I = \frac{-Bl^2\omega}{2\cos^2\omega t}\,\frac{\cos\omega t}{\lambda(-l)} \qquad \left[\because \frac{l}{x} = \cos(180-\theta)\right.$$

$$\boxed{I = \frac{Bl\omega}{2\lambda\cos\omega t}} \qquad \left.\frac{l}{x} = -\cos\theta \Rightarrow x = \frac{-l}{\cos\omega t}\right]$$

Q6.25. Consider an infinitely long wire carrying a current I(t) with $\frac{dI}{dt} = \lambda$ = constant. Find the current produced in rectangular loop of wire ABCD of resistance R.

Main concept used: Total magnetic flux across rectangle can be find out by integration.



Ans. Consider a strip of width dr and length l inside a rectangle at distance r from the surface of current carrying conductor. The magnetic field across strip of length l = $B(r) = \frac{\mu_0 I}{2\pi r}l$. B($r$) is perpendicular to the paper upward.

$\therefore$ Flux in strip $\phi = \frac{\mu_0 I}{2\pi} l \int_{x_0}^{x} \frac{dr}{r}$

$$\phi = \frac{\mu_0 Il}{2\pi}[\log_e r]_{x_0}^{x} = \frac{\mu_0 Il}{2\pi}\log_e \frac{x}{x_0}$$

$$\varepsilon = \frac{-d\phi}{dt}$$

So $$IR = \frac{d\phi}{dt}$$

$$I = \frac{1}{R}\frac{d}{dt}\left[\frac{\mu_0 Il}{2\pi}\log_e\frac{x}{x_0}\right] = \frac{\mu_0 l}{2\pi R}.\log_e\frac{x}{x_0}\frac{dI}{dt}$$

$$I = \frac{\mu_0 \lambda l}{2\pi R}\log_e\frac{x}{x_0} \qquad \left[\because \frac{dI}{dt} = \lambda \text{ (given)}\right]$$

Q6.26. A rectangular loop of wire ABCD is kept close to an infinitely long wire carrying a current $I(t) = I_0\left(1 - \frac{t}{T}\right)$ for $0 \le t \le T$ and $I(0) = 0$

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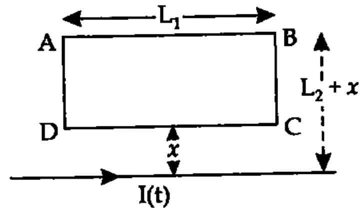
for $t > T$ (figure). Find the total charge passing through a given point in the loop in time T. The resistance of the loop is R.



Main concept used: Relation between instantaneous current and instantaneous magnetic flux.

Ans. If $I(t)$ is instantaneous current then,

$$I(t) = \frac{1}{R}\frac{d\phi}{dt} \qquad ...(I)$$

If Q is charge passing in time t

$$\therefore \qquad I(t) = \frac{dQ}{dt} \qquad ...(II)$$

From (I) and (II)

$$\frac{dQ}{dt} = \frac{1}{R}.\frac{d\phi}{dt}$$

or

$$dQ = \frac{1}{R}.d\phi \qquad ...(III)$$

Integrating both sides,

$$\int_{Q_1}^{Q_2} dQ = \frac{1}{R}\int_{\phi_1}^{\phi_2} d\phi$$

$$Q_2(t) - Q_1(t) = \frac{1}{R}[\phi_2(t) - \phi_1(t)]$$

For magnetic flux in rectangle:

Magnetic flux due to current carrying conductor at a distance x'

$$Q(t) = \frac{\mu_0 I(t)}{2\pi x'}$$

If length of strip is L_1 so total flux on strip of length L_1 at distance x' is

$$Q(t) = \frac{\mu_0 I(t)}{2\pi x'} L_1$$

x' varies from x to $(x + L_2)$ so total flux in strip

$$\phi(t) = \frac{\mu_0}{2\pi} L_1 \int_x^{x+L_2} \frac{dx}{x'} I(t) = \frac{\mu_0 L_1}{2\pi}.I(t_1) \log_e \frac{(L_2 + x)}{x}$$

The magnitude of charge is given on length L_1

$$\int dQ = \frac{1}{R}\int d\phi \qquad \text{[from (III)]}$$

$$\int_0^{Q} dQ = \frac{1}{R}.\frac{\mu_0 L_1}{2\pi} \log_e \left(\frac{L_2 + x}{x}\right) \int_0^{1} I(t_1)$$

$$Q = \frac{\mu_0 L_1}{R\,2\pi} \log_e \left(\frac{L_2 + x}{x}\right)(I - 0) = \frac{\mu_0 L_1 I_1}{2\pi R} \log\left(\frac{L_2 + x}{x}\right)$$

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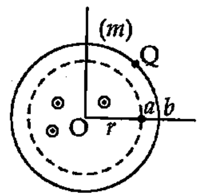
Q6.27. A magnetic field B is confined to a region $r \leq a$ and points out of the paper (in the Z-axis), $r = 0$ being the centre of circular region. A charged ring (charge Q) of radius b ($b > a$) and mass m lies in the X-Y plane with its centre at the origin. The ring is free to rotate and is at rest. The magnetic field is brought to zero in time Δt. Find the angular velocity ω of the ring after the field vanishes.



Main concept used: Change in magnetic flux causes induced e.m.f. in turn electric field around the ring. The torque experienced by the ring produces change in angular momentum.

Ans. As the magnetic field is reduced to zero in $\Delta(t)$, so magnetic flux linked with the ring reduces from maximum to zero. Change of magnetic flux across the conducting ring induces e.m.f. The induced e.m.f. causes the electric field across the ring.

The induced e.m.f. in metallic ring = $(E \times 2\pi b)$ $\quad (\because V = Ed)$ ...(I)

By Faraday's law of e.m.i.

The induced e.m.f. = rate of change of magnetic flux

$$= \frac{B\pi a^2}{\Delta t} \quad \text{...(II)}$$

From (I) and (II)

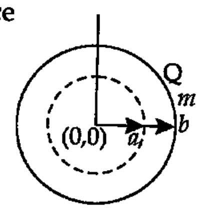
$$2\pi b E = \frac{B\pi a^2}{\Delta t}$$

Since the charged ring experiences an electric force = QE

This electric force try to rotate the ring given by

= Force × Perpendicular distance

$$= QE \times 2b = Qb\frac{B\pi a^2}{2\pi b \Delta t}$$



$$\text{Torque on ring} = Q.\frac{Ba^2}{2\Delta t}$$

Change in angular momentum = Torque × Δt

$$= Q.\frac{Ba^2}{2\Delta t}.\Delta t$$

$$\text{Change in angular momentum} = Q.\frac{Ba^2}{2}$$

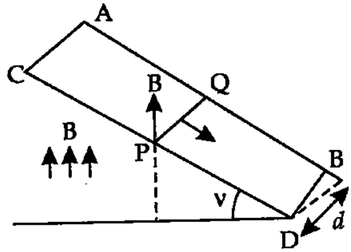
As initial momentum of ring was zero.

So, final momentum of ring = $Q.\frac{Ba^2}{2}$.

$$mb^2\omega = Q.\frac{Ba^2}{2} \quad (\because L = mr^2\omega)$$

$$\omega = \frac{Q.Ba^2}{2mb^2}$$

Q6.28. A rod of mass m and resistance R slides smoothly over two parallel perfectly conducting wires kept sloping at an angle θ with respect to horizontal (figure). The circuit is closed through a perfect conductor at the top. There is a constant magnetic field B along the vertical direction. If the rod is initially at rest, find the velocity of rod as a function of time.



Main concept used: EMI, friction, motion on slope.

Ans. From free body diagram the component of B perpendicular to wire PQ or line F_m is $\vec{B}\cos\theta$:

Angle between B cos θ and PQ = 90°

$$d\phi = \vec{B}\cdot\vec{d}A \text{ (Angle between, } \cos\theta \text{ and } ar \text{ PQCD is zero)}$$

$$d\phi = (\vec{B}\cos\theta)(d \times v \times dt)$$

$$\frac{d\phi}{dt} = Bvd\cos\theta$$

$$-\varepsilon = Bvd\cos\theta$$

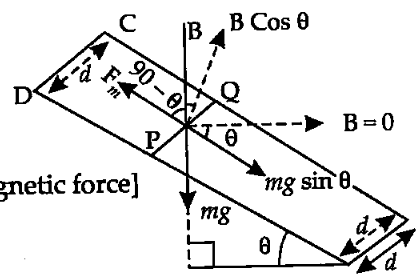
$$\varepsilon = -Bvd\cos\theta$$

$$I = \frac{-Bvd}{R}\cos\theta$$

$$P = F_m \cdot v \quad [F_m \text{ magnetic force}]$$

$$F_m = \frac{P}{v} = \frac{\varepsilon . I}{v}$$

$$F_m = \frac{-B\,v\,d\cos\theta}{v} \cdot \frac{Bvd\cos\theta}{P}$$



∴ $$F_m = \frac{vB^2d^2\cos^2\theta}{R}$$

This Lorentz force opposes the motion of sliding wire PQ

∴ Net force acting on wire PQ of mass m,

∴ By Newton's second law $m\frac{d^2x}{dt^2} = mg\sin\theta - F_m$

$$m\frac{d^2x}{dt^2} = mg\sin\theta - \frac{vB^2d^2\cos^2\theta}{R}$$

$$\frac{dv}{dt} = g\sin\theta - \frac{vB^2d^2\cos^2\theta}{mR}$$

$$\frac{dv}{dt} + \frac{vB^2d^2\cos^2\theta}{mR} = g\sin\theta$$

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It is a linear differential equation,

$$v = \frac{g \sin\theta}{\dfrac{B^2d^2\cos^2\theta}{mR}} + Ae^{\left(\frac{-B^2d^2\cos^2\theta}{mR}\right)t}$$

here, A is constant,

$$v = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta} - Ae^{\left(\frac{-B^2d^2\cos^2\theta}{mR}\right)t}$$

Initially at $t = 0$, $v = 0$

$$\therefore \quad 0 = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta} - A.e^0$$

$$A = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}$$

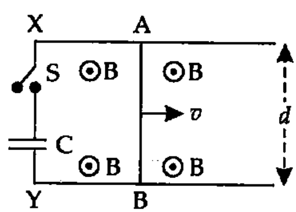
$$\therefore \quad v = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta} - \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}e^{-\left(\frac{B^2d^2\cos^2\theta}{mR}\right)t}$$

$$v = \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}\left[1 - e^{-\left(\frac{B^2d^2\cos^2\theta}{mR}\right)t}\right]$$

Let us consider a new constant $\dfrac{mR}{B^2.d^2.\cos^2\theta} = \alpha$

$$\boxed{v = \alpha g\sin\theta\left[1 - e^{-\frac{t}{\alpha}}\right]}$$

Q6.29. Find the current in the sliding rod AB of resistance R for the arrangement shown in figure. $\vec{B}$ is a constant and is out of the paper. Parallel wires have no resistance. $\vec{v}$ is constant. Switch S is closed at time $t = 0$.



Main concept used: Properties of capacitor, e.m.f.

Ans. Induced current I in loop ABYX = $I_i = \dfrac{\varepsilon}{R} = \dfrac{-1}{R}\dfrac{dQ}{dt}$

$$I_i = \frac{1}{R}.\frac{d}{dt}\vec{B}.\vec{A}$$

$$I_i = \frac{vBd}{R} \qquad \text{...(I)}$$

angle between $\vec{B}$ and $\vec{A}$ is zero. Direction of I from A to B is given by Fleming's right hand rule.

As switch S is closed at $t = 0$.

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Current of first equation will charge the capacitor. Let Q(t) be the charge on capacitor.

$$\therefore \quad Q(t) = C.V$$
$$Q(t) = C.I_c R$$
$$\therefore \quad I_c = \frac{Q(t)}{RC}$$

The capacitor opposes the flow of charge, so net current in circuit

$$I = I_i - I_c$$
$$I = \frac{Bvd}{R} - \frac{Q(t)}{RC}$$
$$\frac{dQ(t)}{dt} = \frac{Bvd}{R} - \frac{Q(t)}{RC}$$
$$\frac{Bvd}{R} = \frac{dQ(t)}{dt} + \frac{Q(t)}{RC}$$

or
$$\frac{dQ(t)}{dt} + \frac{Q(t)}{RC} = \frac{Bvd}{R}$$

So the solution of linear differential equation is

$$Q(t) = \frac{\frac{Bvd}{R}}{\frac{1}{RC}} + A e^{\frac{-t}{RC}}$$
$$Q(t) = Bvd\,C + A\,e^{-t/RC}$$

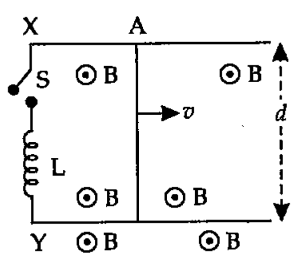
Initially at $t = 0$, Q = 0

So
$$A = -Bvd\,C$$

So
$$Q(t) = BvdC - Bvd\,C\,e^{-\frac{t}{RC}} \qquad [\because e^0 = 1]$$
$$Q(t) = BvdC\left[1 - e^{-\frac{t}{RC}}\right]$$

Current in circuit $I = \frac{dQ(t)}{dt} = \frac{Bvd}{R} e^{-\frac{t}{RC}}$ or $\frac{dQ(t)}{dt} = \frac{BvdC}{RC} e^{-\frac{t}{RC}}$

Q6.30. Find the current in the sliding rod AB of resistance R for the arrangement shown in figure. $\vec{B}$ is constant and is out of the paper. Parallel wires have no resistance, $\vec{v}$ is constant. Switch S is closed at $t = 0$.



Main concept used: Kirchhoff's law, induced current and current in inductor.

Ans. Conductor AB moves towards right with speed v and magnetic field $\vec{B}$ is perpendicularly upward so angle between B and v is 90° and an induced e.m.f., ε flows in loop AXYB.

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$$\varepsilon = \frac{d}{dt}\vec{B}\vec{A} = \frac{d}{dt}BA\cos 0 = \frac{d}{dt}B(d \times v \times t) = Bvd$$

$\because$ angle between $\vec{B}$ and $\vec{A}$ is 0°.

$\therefore \quad \varepsilon = B.v.d$

at $t = 0$, current starts increasing in loop along with inductor L due to potential difference

By Kirchhoff's law in loop ABYX,

$$-L\frac{dI(t)}{dt} + Bvd = IR$$

$$\frac{L\,dI(t)}{dt} + RI(t) = Bvd$$

It is a differential equation.

$$I(t) = \frac{Bvd}{R} + Ae^{-\frac{Rt}{2}} \qquad \text{...(I)}$$

at $t = 0$, $I = 0$

$$\therefore \quad 0 = \frac{Bvd}{R} + Ae^{0} \qquad [\because e^0 = 1]$$

$$A = \frac{-Bvd}{R}$$

$$\therefore \quad I(t) = \frac{Bvd}{R} - \frac{Bvd}{R}e^{-\frac{Rt}{2}} \qquad \text{[from I]}$$

$$\boxed{I = \frac{Bvd}{R}\left[1 - e^{-\frac{Rt}{2}}\right]}$$

This is the required current expression.

Q6.31. A metallic ring of mass m and radius l (ring being horizontal) is falling under gravity in a region having a magnetic field. If z is the vertical direction, the z component of magnetic field is $B_z = B_0(1 + \lambda z)$. If R is the resistance of the ring and if the ring falls with velocity v, find the energy lost in resistance. If the ring has reached a constant velocity, use the conservation of energy to determine v in terms of m B, λ and acceleration due to gravity g.

Main concept used: Relation between induced current, power and velocity of free falling ring.

Ans. Magnetic flux across the ring of mass 'm' and radius 'l' falling under gravity in a region of having magnetic field $B_z = B_0(1 + \lambda z)$ is

$$\phi = \vec{B}_z.\vec{A} = B_0(1 + \lambda z).\pi l^2.$$

The angle between $\vec{B}$ and $\vec{A}$ is 0°

$$\varepsilon = \frac{d}{dt}[B_0(1 + \lambda z)]\,\pi l^2$$

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$$IR = (B_0\pi l^2)\left[0 + \lambda\frac{dz}{dt}\right]$$

$$I = \frac{B_0\pi\lambda l^2}{R}\frac{dz}{dt} = \frac{B_0\pi\lambda l^2}{R}v$$

$$\text{Energy lost} = I^2R = \frac{B_0^2\pi^2\lambda^2 l^4}{R^2}v^2.R$$

$$\text{Energy lost} = \frac{B_0^2\pi^2\lambda^2 l^4 v^2}{R}$$

The energy must come from decrease in P.E $= mg\frac{dz}{dt} = mgv$

$$\therefore \quad mgv = \frac{B_0^2\pi^2\lambda^2 v^2 l^4}{R}$$

$$v = \frac{mgR}{B_0^2\pi^2\lambda^2 l^4} \quad \text{or} \quad v = \frac{mgR}{(\pi l^2\lambda B_0)^2}$$

It is the required relation.

Q6.32. A long solenoid S has n turns per metre with diameter a. At the centre of this coil, we place a smaller coil of N turns and diameter b (where $b < a$). If the current in solenoid increases linearly, with time, what is the induced e.m.f. appearing in the smaller coil. Plot graph showing nature of variation in e.m.f., if current varies as a function of $mt^2 + C$.

Main concept used: When varying current is passed through solenoid the varying magnetic field can induce the current in another coil (smaller).

Ans. Varying magnetic field B(t) in solenoid is

$$B_1(t) = \mu_0 n\, I(t)$$

This varying magnetic field changes flux in the smaller coil.

Magnetic flux in IInd coil

$$\phi_2 = B_1(t).A$$
$$= \mu_0 n\, I(t).\pi b^2$$

Induced e.m.f. in second coil due to solenoid's varying magnetic field in 1 turn

$$\varepsilon' = \frac{-d\phi_2}{dt} = \frac{-d}{dt}\mu_0 n\pi b^2 I(t)$$
$$= -\mu_0 n\pi b^2\frac{d}{dt}(mt^2 + C)$$
$$= -\mu_0 n\pi b^2 . 2mt$$

So net e.m.f. produced in N turns of smaller coil

$$\boxed{\varepsilon = -\mu_0 Nn\pi b^2\, 2mt}$$

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7 Alternating Current

MULTIPLE CHOICE QUESTIONS—I

Q7.1. If the rms current in 50 Hz a.c. circuit is 5 A, the value of current 1/300 seconds after its value becomes zero is

(*a*) $5\sqrt{2}$ A (*b*) $5\sqrt{\frac{3}{2}}$ A (*c*) $\frac{5}{6}$ A (*d*) $\frac{5}{\sqrt{2}}$ A

Main concept used: Relations $I = I_0 \sin \omega t$, $\omega = 2\pi\nu$, $I_{rms} = I = I_0/\sqrt{2}$

Ans. (*b*): $I = 5$ A, $\nu = 50$ Hz, $t = \frac{1}{300}$ s

$$I_{rms} = I = \sqrt{2} I_0 = 5\sqrt{2} \text{ A}$$

$$I = I_0 \sin \omega t$$

at $t = \frac{1}{300}$ sec

$$I = 5\sqrt{2} \sin 2\pi\nu . t = 5\sqrt{2} \sin 2\pi \times 50 \times \frac{1}{300}$$

$$= 5\sqrt{2} \sin \frac{\pi}{3} = 5\sqrt{2} \sin 60°$$

$$= 5\sqrt{2}\frac{\sqrt{3}}{2} = 5\sqrt{\frac{3}{2}} \text{ A}$$

Q7.2. An alternating current generator has an internal resistance R_g and an internal reactance X_g. It is used to supply power to a passive load consisting of a resistance R_g and reactance X_L. For maximum power to be delivered from the generator to load the value of X_L is equal to

(*a*) zero (*b*) X_g (*c*) $-X_g$ (*d*) R_g

Main concept used: To delivering maximum power from generator its total reactance (internal and external) must be zero and resistance must be equal.

Ans. (*c*): As internal resistance of generator is already equal to external resistance R_g. So to deliver maximum power to make reactance equal to zero, the reactance in external circuit will be $-X_g$.

Q7.3. When a voltage measuring device is connected to AC mains, the meter shows the steady input voltage of 220 V. This means

(*a*) input voltage cannot be AC voltage, but a DC voltage.
(*b*) maximum input voltage is 220 V.
(*c*) the meter reads not v but $\langle v^2 \rangle$ and is calibrated to read $\sqrt{\langle v^2 \rangle}$.
(*d*) the pointer of the meter stuck by some mechanical defect.

Main concept used: Voltmeter in AC reads r.m.s. value of voltage

$$\boxed{I_{rms} = \sqrt{2} I_0} \quad V_{rms} = \sqrt{2} V_0$$

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Ans. (*c*): The voltmeter in AC circuit reads value $[\langle v^2 \rangle]$ and meter is calibrated to r.m.s. value $\langle v^2 \rangle$ which is multiplied by $\sqrt{2}$ to get V_{rms}.

Q7.4. To reduce the resonant frequency in an LCR series circuit with a generator

(*a*) the generator frequency should be reduced.
(*b*) another capacitor should be added in parallel to the first.
(*c*) the iron core of the inductor should be removed.
(*d*) dielectric in the capacitor should be removed.

Main concept used: Resonant frequency.

Ans. (*b*): The resonant frequency of LCR series circuit is

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

So to reduce resonant frequency ν_0, we have either to increase L or to increase C.

To increase capacitance another capacitor must be connected in parallel with the first.

Q7.5. Which of the following combinations should be selected for better turning of LCR circuit used for communication?

(*a*) R = 20 Ω, L = 1.5 H, C = 35 μF (*b*) R = 25 Ω, L = 2.5 H, C = 45 μF
(*c*) R = 15 Ω, L = 3.5 H, C = 30 μF (*d*) R = 25 Ω, L = 1.5 H, C = 45 μF

Main concept used: Quality factor of LCR circuit must be as high as possible.

Ans. (*c*): We know that for communication, quality factor must be higher and quality factor is Q.

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

So for higher Q, L must be large, and R and C must be of smaller value.

This condition is satisfied in (*c*) part.

Q7.6. An inductor of reactance 1 Ω and resistor of 2 Ω are connected in series to the terminals of 6 V (rms) AC source. The power dissipated in the circuit is

(*a*) 8 W (*b*) 12 W (*c*) 14.4 W (*d*) 18 W

Main concept used: Impedance, average power, $I_{rms} = \frac{I}{\sqrt{2}}$ and $\cos\phi = \frac{R}{Z}$ of AC.

Ans. (*c*): We know that average power dissipated in L, R series circuit with AC source.

$$P_{av} = E_{rms} I_{rms} \cos\phi$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{E_{rms}}{Z} = \frac{6}{\sqrt{5}}$$

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$$Z = \sqrt{X_R^2 + X_L^2}$$

$$Z = \sqrt{2^2 + 1} = \sqrt{5}$$

$$\cos\phi = \frac{R}{Z} = \frac{2}{\sqrt{5}}$$

$$P_{av} = 6 \times \frac{6}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} = \frac{72}{5} = 14.4 \text{ Watt}$$

Q7.7. The output of a stepdown transformer is measured to be 24 V, when connected to a 12 W light bulb. The value of the peak current is

(*a*) $\frac{1}{\sqrt{2}}$ A (*b*) $\sqrt{2}$ A (*c*) 2 A (*d*) $2\sqrt{2}$ A

Main concept used: $V_s I_s = V_p I_p$

Ans. (*a*)

$$V_s = 24 \text{ V}$$

$$P_s = 12 \text{ W}$$

$$I_s V_s = 12$$

$$I_s = \frac{12}{V_s} = \frac{12}{24} = 0.5 \text{ Amp}$$

$$I_0 = I_s\sqrt{2} = 0.5\sqrt{2}$$

$$I_0 = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}} \text{ Amp}$$

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q7.8. As the frequency of an AC circuit increases, the current first increases and then decreases. What combination of circuit elements is most likely to comprise the circuit?

(*a*) Inductor and capacitor (*b*) Resistor and inductor

(*c*) Resistor and capacitor (*d*) Resistor, inductor and capacitor

Main concept used: Dependence of reactance on frequency.

Ans. (*a*) (*d*): In a circuit I_{max} will be when reactance is minimum. On increasing frequency in LCR circuit the reactance ($X_L = 2\pi\nu L$) due to inductance will increase and due to capacitance $\frac{1}{2\pi\nu C}$ it will decrease. So on increasing frequency X_L will be positive and X_C will be negative for minimum reactance $X_L - X_C$ must be zero. As reactance or impedance (Z) due to LCR series AC circuit is $Z = \sqrt{R^2 + (X_L \sim X_C)^2}$, Z will be minimum when $X_L \sim X_C = 0$ or $X_L = X_C$. So for given condition, X_L and X_C must be in circuit.

Q7.9. In an alternating current circuit, consisting of elements in series, the current increases on increasing frequency of supply. Which of the following elements are likely to constitute the circuit?

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(*a*) Only resistor (*b*) Resistor and an inductor
(*c*) Resistor and a capacitor (*d*) Only a capacitor

Main concept used: Reactance decreases and the current increases on increasing ν.

Ans. (*c*) and (*d*) Here in question on increasing ν the current also increases. So reactance of circuit decreases as reactance or resistance does not depend on frequency and $X_L = 2\pi\nu L$ and $X_C = \frac{1}{2\pi\nu C}$. So on increasing frequency, X_C decreases to increase current capacitors are in *c* and *d*.

Q7.10. Electrical energy is transmitted over large distances at high alternating voltages. Which of the following statements is (are) correct?

(*a*) For a given power level, there is a lower current.
(*b*) Lower current implies less power loss.
(*c*) Transmission lines can be made thinner.
(*d*) It is easy to reduce the voltage at the receiving end using step-down transformers.

Main concept used: Power loss = I^2R, $V_1I_1 = V_2I_2$ (Transformers)

Ans. (*a*) (*b*) and (*d*): As we know that Power = $I_{rms}^2 R$

So to decrease power loss I_{rms} and R must be lower for a constant power supply. To decrease I_{rms}, V_{rms} must be increased by step up transformer to get same power in step up transformer

$$\text{Output power} = \text{Input power}$$
$$V_sI_s = V_pI_p$$

$\because$ $V_s > V_p$ so $I_p \gg I_s$ so loss of power during transmission become lower.

We can again reduce voltage by using step down transformer.

Q7.11. For an LCR circuit, the power transferred from the driving source to the driven oscillator is $P = I^2Z\cos\phi$.

(*a*) Here, the power factor $\cos\phi \geq 0$, $P \geq 0$.
(*b*) The driving force can give no energy to the oscillator (P = 0) in some cases.
(*c*) The driving force cannot syphon out (P < 0), the energy out of oscillator.
(*d*) The driving force can take away energy out of the oscillator.

Main concept used: $P = I^2Z\cos\phi$

$$\cos\phi = \frac{R}{Z}$$

Ans. (*a*) (*b*) and (*c*): As per question $P = I^2Z\cos\phi$

$$\text{Power factor } \cos\phi = \frac{R}{Z}$$

as $R > 0$ and $Z > 0$

So $\cos\phi = \frac{R}{Z}$ is positive $\Rightarrow P > 0$

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Q7.12. When an ac voltage of 220 Volt is applied to the capacitor C,

(*a*) the maximum voltage between plates is 220 V.

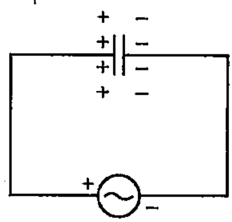
(*b*) the current is in phase with the applied voltage.

(*c*) the charge on the plates is in phase with the applied voltage.

(*d*) power delivered to the capacitor is zero.

Main concept used: Power factor for C is zero $\because$ $\phi = 90°$.

Ans. (*c*) (*d*) When capacitor is connected to ac supply the plate of capacitor will be at higher potential which is connected to positive terminal than the plate connected to the negative terminal.



Power applied to circuit is

$$P_{av} = V_{rms} I_{rms} \cos\phi$$

$$\phi = 90° \text{ for pure capacitor circuit}$$

$\therefore$ $$P_{av} = 0 \text{ as } \cos 90° = 0$$

Q7.13. The line that draws power supply to your house from street has

(*a*) zero average current.

(*b*) 220 V average voltage.

(*c*) voltage and current out of phase by 90°.

(*d*) voltage and current possibly differing in phase ϕ such that $|\phi| < \frac{\pi}{2}$.

Main concept used: Power factor $\cos\phi = \frac{R}{Z}$

Ans. (*a*) (*d*): AC supply are used in houses.

So average current over a cycle of AC is zero. In household circuit L and C are connected, so R and Z cannot be equal.

So, power factor $= \cos\phi = \frac{R}{Z} \neq 0$

as $\phi \neq \frac{\pi}{2} \Rightarrow \phi < \frac{\pi}{2}$

i.e. phase angle between voltage and current lies between 0 and $\frac{\pi}{2}$.

VERY SHORT ANSWER TYPE QUESTIONS

Q7.14. If a L-C circuit is considered analogous to a harmonically oscillating spring block system, which energy of the L-C circuit would be analogous to the potential energy and which one analogous to kinetic energy?

Main concept used: P.E. is related to electrostatic energy and K.E. to magnetic energy.

Ans. An L-C circuit is analogous to a harmonically oscillating spring block system. The electrostatic energy due to charging of capacitor $\frac{1}{2}CV^2$ is analogous to potential energy and energy due to motion of charge particle i.e., magnetic energy $\frac{1}{2}LI^2$ is analogous to kinetic energy.

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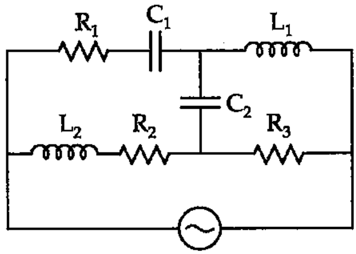
Q7.15. Draw the effective equivalent circuit of a circuit shown in figure, at very high frequencies and find the effective impedance.



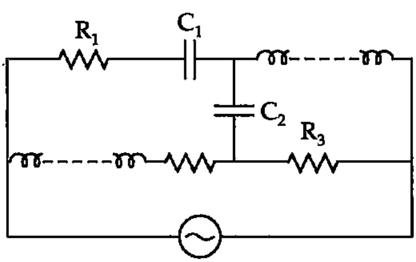
Main concept used: Reactance X_L and X_C at high frequencies.

Ans. ∵ We know that reactance due to inductance $X_L = 2\pi\nu$. As frequency is high so X_L will be high or L can be considered as open circuit for high frequency of ac.

In similar way, $X_C = \dfrac{1}{2\pi\nu C}$.

At high frequency, X_C will be low.

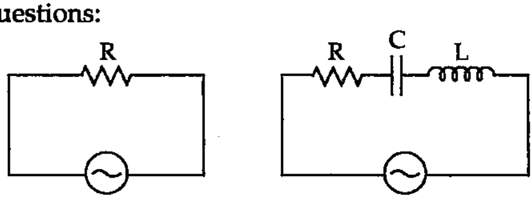
So reactance of capacitance can be considered negligible and capacitors can be considered as closed circuit.



Here the above figure is equivalent circuit to given circuit.

Total impedance = R = $R_1 + R_3$

Q7.16. Study the circuits (*a*) and (*b*) shown in figure and answer the following questions:



(*a*) (*b*)

(*a*) Under which conditions would the rms currents in two circuits be the same?

(*b*) Can the rms current in circuit (*b*) be larger than that in (*a*)?

Ans. (I_{rms}) in (*a*) $= \dfrac{V_{rms}}{R}$

$$(I_{rms}) \text{ in } b = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

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(*a*) $(I_{rms})_a = (I_{rms})_b$

$$\frac{V_{rms}}{R} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\therefore \quad R = \sqrt{R^2 + (X_L - X_C)^2}$$

Squaring both sides

$$R^2 = R^2 + (X_L - X_C)^2$$

or $(X_L - X_C)^2 = 0$

$$X_L = X_C$$

So I_{rms} in circuits (*a*) and (*b*) will be equal if $X_L = X_C$.

(*b*) For $(I_{rms})_b > (I_{rms})_a$

$$\frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} > \frac{V_{rms}}{R}$$

as $V_{rms} = V$ so,

$$\sqrt{R^2 + (X_L - X_C)^2} < R$$

Squaring both sides

$$R^2 + (X_L - X_C)^2 < R^2$$

$$(X_L - X_C)^2 < 0$$

Square of any number can never be negative. Reactance of X_L and X_C cannot be negative.

So the rms current in circuit (*b*) cannot be larger than that in (*a*).

Q7.17. Can the instantaneous power output of an ac source ever be negative? Can the average power output be negative?

Ans. Let the applied e.m.f. $= E = E_0 \sin(\omega t)$

$$I = I_0 \sin(\omega t - \phi)$$

Instantaneous power output of ac source

$$P = EI$$

$$= E_0 \sin \omega t . I_0 \sin(\omega t - \phi)$$

$$= E_0 I_0 \sin \omega t [\sin \omega t \cos \phi - \cos \omega t \sin \phi]$$

$$= E_0 I_0 [\sin^2 \omega t \cos \phi - \sin \omega t \cos \omega t \sin \phi]$$

$$= E_0 I_0 \left[\frac{(1 - \cos 2\omega t)}{2} \cos \phi - \frac{1}{2} \sin 2\omega t \sin \phi \right]$$

$$= \frac{E_0 I_0}{2} [\cos \phi - \cos 2\omega t \cos \phi - \sin 2\omega t \sin \phi]$$

$$= \frac{E_0 I_0}{2} [\cos \phi - (\cos 2\omega t \cos \phi + \sin 2\omega t \sin \phi)]$$

$$P = \frac{E_0 I_0}{2} [\cos \phi - \cos(2\omega t - \phi)]$$

Taken phase angle ϕ, ±ve.

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Instantaneous power $P = \frac{E_0 I_0}{2}[\cos\phi - \cos(2\omega t \pm \phi)]$

as $\cos\phi = \frac{R}{Z}$, R and Z can never be negative and value of $\cos\theta$ $(\theta = 2\omega t \pm \phi)$ can vary from (1 to 0 to – 1) in any case P can never be negative.

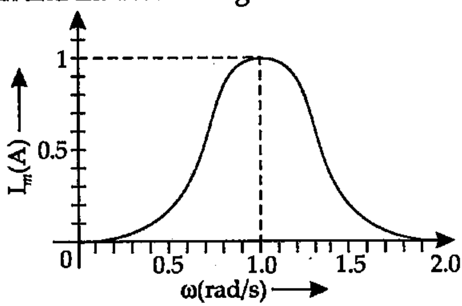
We know that average power of LCR series ac circuit is

$$P_{av} = \frac{E_0 I_0}{2}\cos\phi$$

again as $\cos\phi = \frac{R}{Z}$ is always positive, because R and Z, the reactances are always positive.

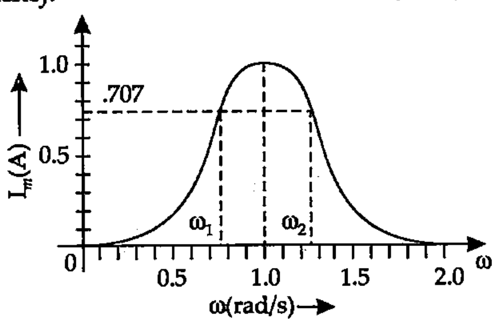
So P_{av} can never be negative.

Q7.18. In series LCR circuit, the plot of I_{max}, versus ω is shown in figure. Calculate the bandwidth and mark in the figure.



Ans. We know that bandwidth $= (\omega_2 - \omega_1)$.

Where ω_1 and ω_2 are two frequencies where the current amplitude of LCR circuit becomes $\frac{1}{\sqrt{2}}$ times (i.e., I_{rms}) the value of current is maximum at resonant frequency.



$$\boxed{I = \frac{E_0}{\sqrt{2}}} = \frac{1}{\sqrt{2}} = 0.707 \text{ Amp}$$

From graph ω_1 and ω_2 at 0.707 Amp current is 0.8 and 1.2 rad/sec.

So Bandwidth $\omega_2 - \omega_1 = 1.2 - 0.8 = 0.4$ rad/sec.

Q7.19. The alternating current in the circuit is described by the graph as shown in the figure. Show rms current in this graph.

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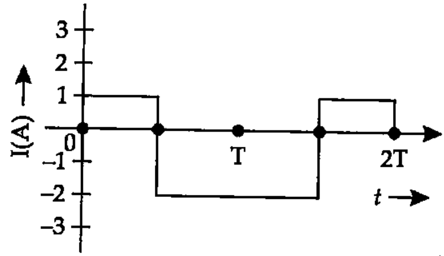
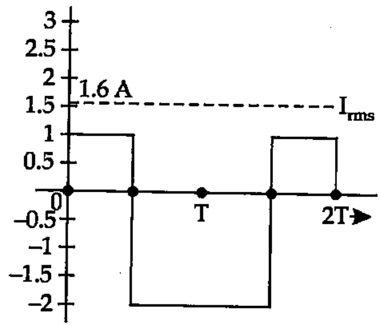


Ans. Graph of rms current is shown below by dotted line



$$I_{rms} = \sqrt{\frac{1^2 + 2^2}{2}} = \sqrt{\frac{5}{2}} \cong 1.6 \text{ A}$$

Q7.20. How does the sign of the phase angle ϕ, by which the supply voltage leads the current in an LCR series circuit, change as the supply frequency is gradually increased from very low to high values.

Ans. The phase angle (ϕ) by which voltage leads the current in LCR series circuit where $XL > X_C$.

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{2\pi\nu L - \dfrac{1}{2\pi\nu C}}{R}$$

If ν is small $X_C > X_L$ so $\left[2\pi\nu L - \dfrac{1}{2\pi\nu C}\right]$ is negative, so $\tan\phi < 0$.

For ν is large, $X_L > X_C$

So $X_L - X_C$ is positive or $\tan\phi > 0$

for $X_L = X_C$ *i.e.,* at resonant frequency

$X_L - X_C = 0$ so $\tan\phi = 0$.

So phase angle in series LCR ac circuit will change from a negative to zero and then zero to positive value.

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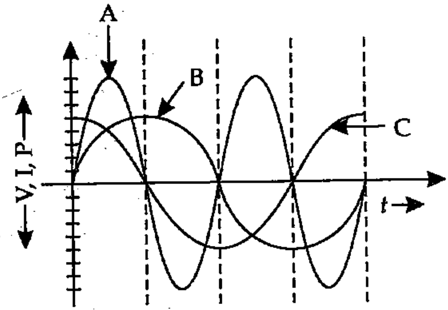
SHORT ANSWER TYPE QUESTIONS

Q7.21. A device X is connected to an AC source. The variation of voltage, current and power in one complete cycle is shown in figure.



(*a*) Which curve shows the power consumption over a full cycle?

(*b*) What is the average power consumption over a cycle?

(*c*) Identify the device X.

Ans. (*a*) We know that P = VI, the amplitude of power will be equal to multiplication of V and I so it must be maximum amplitude as compared to V and L. So curve A shows the power consumption over a cycle. The power of a circuit will become zero, when either V = 0 or I = 0 or both become zero which is clear from graph 'A is zero (on dotted lines) at these position.

(*b*) The graph of power is represented by A. The graph 'A' is symmetric with X-axis, *i.e.* area of graph on positive and negative side are equal. So net area of power graph is zero. So power consumption in circuit is zero, which tallies with that average power of A.C. circuit over a cycle is zero.

(*c*) As the average power of device is zero so power factor $\cos\phi = 0$ i.e., $\phi = 90°$. Change in phase between current and voltage is in either inductor or capacitor. So device may be either capacitor or inductor or the combination of both inductor and capacitor.

Q7.22. Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current?

Ans. For direct current 'One Ampere' is equal to one coulomb charge flowing per sec.

In an AC, current changes it's direction with time equal to the half of time period of AC. So the charge vibrate to and fro with frequency equal to the frequency of AC. So net force acting on the charge is zero.

So the AC current in ampere must be defined in terms of some property which is independent of direction of charge or current.

Joule's heating effect is such a property and hence Joule's heating law is used to define one ampere current in AC. According to Joule's heating effect, one ampere current in AC is the current which can produce same quantity of heat per second as the direct current can produce in one ohm resistance.

Q7.23. A coil of 0.01 H inductance and 1 Ω resistance is connected to 200 V, 50 Hz AC supply. Find the impedance of circuit and time lag between maximum alternating voltage and current.

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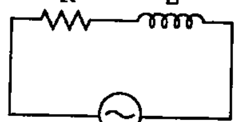
Ans. $R = 1\ \Omega$, $L = 0.01$ H, $V = 200$ V, $\nu = 50$ Hz.

Impedance of the circuit



$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi\nu L)^2}$$
$$= \sqrt{(1)^2 + (2\times 3.14\times 50\times 0.01)^2}$$
$$= \sqrt{1+9.86} = \sqrt{10.86}$$
$$= 3.3\ \Omega.$$

$\therefore$ For phase angle ϕ, $\tan\phi = \frac{Z}{R}$

$$\tan\phi = \frac{X_L}{R} = \frac{2\pi\nu L}{R} = \frac{2\times 3.14\times 50\times 0.01}{1}$$
$$\tan\phi = 3.14$$
$$\phi = \tan^{-1} 3.14 = 72^\circ$$

$$\text{Phase difference } \phi = \frac{72\times\pi}{180^\circ}\text{ rad}$$
$$\phi = 1.20 \text{ radian}$$

Time lag between alternating voltage and current

$$\boxed{\phi = \omega t}$$

$$t = \frac{\phi}{\omega} = \frac{72\times\pi}{180\times 2\times\pi\times 50} = \frac{1}{250}\text{ sec}$$

Q7.24. A 60 W load is connected to the secondary of a transformer whose primary draws line voltage. If a current of 0.54 A flows in the load, what is the current in primary coil? Comment on the type of transformer being used.

Ans. $P_s = 60$ W, $I_s = 0.54$ A, $I_p = ?$

$$P_s = V_s I_s$$
$$60 = V_s \times 0.54$$
$$\frac{60}{0.54} = V_s$$
$$V_s = 111.10 \text{ Volt}$$

In multiple of 11

$$V_s \cong 110 \text{ Volt}$$

$$\text{Ratio factor of transformer} = \frac{\text{output voltage}}{\text{input voltage}}$$

or $$r = \frac{V_s}{V_p} \Rightarrow r = \frac{110\text{ Volt}}{220\text{ Volt}} = \frac{1}{2}$$

or $r < 1$, so transformer is step down transformer.

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In transformer, output power = input power

$$I_s V_s = I_p V_p$$

$$I_p = \frac{I_s V_s}{V_p} = \frac{0.54 \times 110}{220} = \mathbf{0.27\ Ampere}$$

Q7.25. Explain why the reactance provided by a capacitor to an alternating current decreases with increasing frequency?

Ans. When AC current is applied across a capacitor plate, the plates of the capacitor get charge and discharge alternately. Current through the capacitor is a result of this charging and discharging by AC voltage or current.

A capacitor does not allow a direct current (having zero frequency) to pass through it. But as frequency of current increases capacitor will pass more current through it as on increasing frequency, the charging and discharging happens at fast rate. It implies that the reactance offered by capacitor decreases on increasing frequency.

So the reactance of capacitor can be written as $\boxed{X_C = \frac{1}{\omega C}}$.

Q7.26. Explain why the reactance offered by an inductor increases with increasing frequency of an AC voltage?

Ans. According to Lenz's law, when current in an inductor change, the direction of induced e.m.f. will oppose the change in current in inductor. The magnetic flux will be in opposite direction with the magnetic flux produced by changing e.m.f. or current in coil and vice-versa.

Since the induced e.m.f. is directly proportional to the rate of change of current, so an inductor produces greater reactance to flow of current through it.

If direct current passes through an inductor the reactance produced by inductor is zero. So on applying an AC current its reactance increases with increasing the frequency of AC.

So the reactance is directly proportional to frequency or reactance of inductor is $X_L = 2\pi\nu L = \omega L$.

LONG ANSWER TYPE QUESTIONS

Q7.27. An electrical device draws 2 kW power from AC mains [voltage 223 V (rms) = $\sqrt{50,000}$ V]. The current differs (lags) in phase by ϕ $\left(\tan\phi = \frac{-3}{4}\right)$ as compared to the voltage. Find (*i*) R (*ii*) $(X_C - X_L)$ and (*iii*) I_M. Another device has twice the values for R, X_C and X_L. How are the answers affected?

Ans. $P = 2000$ W   Current lags the voltage so

$V^2 = 50,000$ V $X_C > X_L$

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$$\tan\phi = \frac{-3}{4}$$

$$I_m = I_0$$

$$P = \frac{V^2}{Z}$$

$$2000 = \frac{50,000}{Z}$$

$$Z = \frac{50,000}{2000} = 25\ \Omega$$

$$\boxed{Z = \sqrt{R^2 + (X_C - X_L)^2}}$$

$$R^2 + (X_C - X_L)^2 = 25^2$$

$$R^2 + (X_C - X_L)^2 = 625 \qquad \text{...(I)}$$

$$\tan\phi = \frac{-3}{4}$$

$$\frac{X_C - X_L}{R} = \frac{-3}{4}$$

$$X_C - X_L = \frac{-3}{4}R$$

$$(X_C - X_L)^2 = \frac{9}{16}R^2 \qquad \text{...(II)}$$

Put the value of $(X_C - X_L)^2$ in (I)

$$R^2 + \frac{9}{16}R^2 = 625$$

$$\frac{25}{16}R^2 = 625$$

$$R^2 = 16 \times 25$$

$$R = 4 \times 5 = 20\ \Omega$$

$$\frac{X_C - X_L}{R} = \frac{-3}{4}$$

$$X_C - X_L = \frac{-3}{4} \times 20$$

$$X_C - X_L = -15\ \Omega$$

$$I_{rms} = \frac{V}{Z} = \frac{223}{25} \cong 9\ A$$

$$I_0 = \sqrt{2}\, I_{rms} = \sqrt{2} \times 9$$

$$I_0 = 12.6\ A$$

As (*i*) R, X_C, X_L all are doubled $\tan\phi = \dfrac{X_C - X_L}{R}$ will remain same.

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(*ii*) Z will become double then $I = \frac{V}{Z}$ become half as value of V does not change. (*iii*) As I become half P = VI will become again half as voltage remains same.

Q7.28. 1 MW power is to be delivered from a power station to a town 10 km away. One uses a pair of Cu wires of radius 0.5 cm for this purpose. Calculate the fraction of ohmic losses to power transmitted if.

(*i*) Power is transmitted at 220 V. Comment on the feasibility of doing this.

(*ii*) A step-up transformer is used to boost the voltage to 11000 Volt, power transmitted, then a step-down transformer is used to bring voltage to 220 V. ($\rho_{Cu} = 1.7 \times 10^{-8}\ \Omega\text{-m}$)

Ans. (*i*) When power is transmitted at 220 V.

Power lost in transmission $\boxed{P = I^2R}$

$$\boxed{P = VI}$$

$$P = 1\text{ MW} = 10^6\text{ W}$$

$$V = 220\text{ Volt}$$

$$\therefore \quad I = \frac{P}{V} = \frac{1000000}{220}$$

$$I = \frac{50000}{11}\text{ Amp}$$

$$\boxed{R = \rho\frac{l}{A}} \Rightarrow R = \frac{\rho l}{\pi r^2}$$

$$l = 10\text{ km} \times 2 = 20{,}000\text{ m}$$

$$\therefore \quad A = \pi r^2$$

$$r = 0.5\text{ cm} = 0.5 \times 10^{-2} = 5 \times 10^{-3}\text{ m}$$

$$\rho_{Cu} = 1.7 \times 10^{-8}\ \Omega\text{-m}$$

$$R = \frac{\rho l}{A}$$

$$\therefore \quad R = \frac{1.7 \times 10^{-8} \times 20{,}000}{3.14 \times 5 \times 10^{-3} \times 5 \times 10^{-3}}$$

$$= \frac{170 \times 20000 \times 10^{-8+6}}{314 \times 5 \times 5} = \frac{170 \times 20000}{314 \times 25 \times 100}\ \Omega$$

$$R = \frac{170 \times 800}{314 \times 100} = \frac{170 \times 4}{157} = \frac{680}{157} \cong 4\ \Omega$$

$$\therefore \quad \text{Power loss} = I^2R$$

$$= \frac{50000}{11} \times \frac{50000}{11} \times 4 = \frac{100 \times 10^8}{121} = 8.26 \times 10^7$$

Power loss in heating = **82.6 MW**

as 82.6 MW > 1 MW

So this method cannot be used to transmit the power.

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(*ii*) When power is transmitted at 11000 V

$$P = 10^6 \text{ W}$$
$$VI = 1000000$$
$$11000\, I = 1000000$$
$$I = \frac{1000000}{11000} = \frac{1000}{11}$$

$\therefore$ $R_{Cu} = 4\,\Omega$ [as already calculated in part (*i*)]

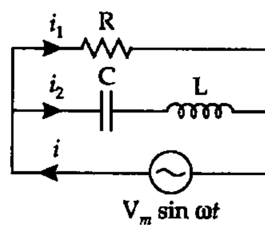
$\therefore$ Power loss = $P = I^2R$

$$P = \frac{1000}{11} \times \frac{1000}{11} \times 4 = \frac{4000}{121} \times 10^4$$
$$P = 3.3 \times 10^4 \text{ Watt}$$

Fractional power loss $= \dfrac{3.3 \times 10^4}{10^6} = \dfrac{3.3}{1000} = 0.033$

Power loss in % = 3.3%

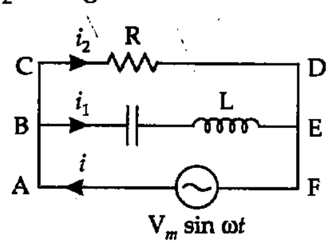
Q7.29. Consider the LCR circuit shown in figure. Find the net current i and the phase of i. Show that $i = \dfrac{V}{Z}$. Find the impedance Z for this circuit.



Main concept used: Kirchhoff's law.

Ans. Total current i from the source $V_m \sin \omega t$ is divided at B in two parts, i_1 through capacitor and inductor and part i_2 through resistance.

Potential across R = Potential of source



P.D. across R = $V_m \sin \omega t$

$$i_2 R = V_m \sin \omega t$$

$$\boxed{i_2 = \frac{V_m \sin \omega t}{R}} \qquad \text{...I}$$

q_1 is charge on the capacitor at any time t, then for series combination of C, L. applying Kirchhoff's voltage law in loop ABEFA.

$$V_C + V_L = V_m \sin \omega t$$

$$\frac{q_1}{C} + L\frac{di_1}{dt} = V_m \sin \omega t$$

$$\frac{q_1}{C} + L\frac{d^2q_1}{dt^2} = V_m \sin \omega t \qquad \text{...II}$$

Let $q_1 = q_m \sin(\omega t + \phi)$ ...III

$$i_1 = \frac{dq_1}{dt} = q_m \omega \cos(\omega t + \phi) \qquad \text{...IV}$$

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$$\frac{d^2 q_1}{dt^2} = -q_m \omega^2 \sin(\omega t + \phi) \qquad \text{...V}$$

Substitute the values of equations III and V in equation II

$$\frac{q_m \sin(\omega t + \phi)}{C} - Lq_m \omega^2 \sin(\omega t + \phi) = V_m \sin \omega t$$

$$q_m \sin(\omega t + \phi)\left[\frac{1}{C} - L\omega^2\right] = V_m \sin \omega t$$

at $\phi = 0$,

$$q_m \sin \omega t \left[\frac{1}{C} - L\omega^2\right] = V_m \sin \omega t$$

$$q_m \left[\frac{1}{C} - L\omega^2\right] \sin \omega t = V_m \sin \omega t$$

$$q_m \left[\frac{1}{C} - L\omega^2\right] = V_m$$

$$q_m = \frac{V_m}{\omega\left[\frac{1}{C\omega} - L\omega\right]} \qquad \text{...(VI)}$$

Applying Kirchhoff's junction rule at junction B, $i = i_2 + i_1$ using relation I, IV

$$i = \frac{V_m \sin \omega t}{R} + q_m \omega \cos(\omega t + \phi)$$

Now using relation VI for q_m and at $\phi = 0$

$$i = \left[\frac{V_m \sin \omega t}{R} + \frac{V_m \omega \cos \omega t}{\omega\left[\frac{1}{\omega C} - \omega L\right]}\right]$$

$$i = \frac{V_m}{R} \sin \omega t + \frac{V_m}{\left(\frac{1}{\omega C} - \omega L\right)} \cos \omega t$$

Let $$A = \frac{V_m}{R} = C \cos \phi \qquad \text{...(VII)}$$

$$B = \frac{V_m}{\frac{1}{\omega C} - \omega L} = C \sin \phi \qquad \text{...(VIII)}$$

$$i = C \cos \phi \sin \omega t + C \sin \phi \, . \cos \omega t$$

$$= C [\cos \phi \sin \omega t + \sin \phi \cos \omega t]$$

$$i = C \sin(\omega t + \phi)$$

Squaring and adding (VII), (VIII)

$$A^2 + B^2 = C^2 \cos^2 \phi + C^2 \sin^2 \phi$$

$$= C^2[\cos^2 \phi + \sin^2 \phi)$$

$$A^2 + B^2 = C^2$$

or $$C = \sqrt{A^2 + B^2}$$

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$$\phi = \tan^{-1}\frac{B}{A} = \tan^{-1}\frac{\dfrac{V_m}{\dfrac{1}{\omega C} - \omega L}}{\dfrac{V_m}{R}}$$

$\therefore$ $$\tan\phi = \frac{R}{\left(\dfrac{1}{\omega C} - \omega L\right)}$$

$\because$ $$C^2 = A^2 + B^2 = \frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\dfrac{1}{\omega C} - \omega L\right)^2}$$

$$C = \left[\frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\dfrac{1}{\omega C} - \omega L\right)^2}\right]^{\frac{1}{2}}$$

$\therefore$ $$i = \left[\frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\dfrac{1}{\omega C} - \omega L\right)^2}\right]^{\frac{1}{2}} \sin(\omega t + \phi)$$

$$i = V_m\left[\frac{1}{R^2} + \frac{1}{\left(\dfrac{1}{\omega C} - \omega L\right)^2}\right]^{\frac{1}{2}} \sin(\omega t + \phi) \quad \text{...(IX)}$$

and $$\phi = \tan^{-1}\frac{R}{\left(\dfrac{1}{\omega C} - \omega L\right)}$$

$\because$ $$I = \frac{V}{R} \quad \text{or} \quad i = \frac{V}{Z}$$

for ac $$i = \frac{V}{Z}\sin(\omega t + \phi) \quad \text{...(X)}$$

Comparing (IX) and (X)

So $$\frac{1}{Z} = \left[\frac{1}{R^2} + \frac{1}{\left(\dfrac{1}{\omega C} - \omega L\right)^2}\right]^{\frac{1}{2}}$$

This is the impedance Z for the circuit.

Q7.30. For LCR circuit driven at frequency ω, the equation reads,

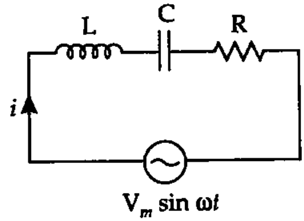
$$L\frac{di}{dt} + Ri + \frac{q}{C} = V_i = V_m \sin\omega t.$$

(*a*) Multiply the equation by *i* and simplify where possible.
(*b*) Interpret each term physically.

(*c*) Cast the equation in the form of conservation energy statement.

(*d*) Integrate the equation over one cycle to find that the phase difference between V and *i* must be acute.

Main concept used: (*i*) Voltage Kirchhoff's law (*ii*) Loss of energy through resistor to know net loss of energy.



Ans. Consider L-C-R series circuit with AC supply

$$V = V_m \sin \omega t$$

Applying voltage Kirchhoff's law over the circuit

$$\therefore \quad V_L + V_C + V_R = V_m \sin \omega t$$

$$L\frac{di}{dt} + \frac{q}{C} + iR = V_m \sin \omega t$$

(*a*) Multiply the above equation by *i* on both the sides

$$Li\frac{di}{dt} + \frac{iq}{C} + i^2R = V_m \, i \sin \omega t$$

Multiply above equation by $\frac{1}{2}$ on both sides

$$\frac{1}{2}Li\frac{di}{dt} + i\frac{q}{2C} + \frac{i^2R}{2} = \frac{1}{2}V_m i \sin \omega t \qquad \left(\because i = \frac{dq}{dt}\right)$$

$$\frac{d\left(\frac{1}{2}Li^2\right)}{dt} + \frac{1}{2C}\frac{dq^2}{dt} + \frac{i^2R}{2} = \frac{1}{2}V_m i \sin \omega t \qquad \text{...(I)}$$

(*b*) (*i*) $\dfrac{d\left(\frac{1}{2}Li^2\right)}{dt}$ represents the rate of change of potential energy in inductance L.

(*ii*) $\boxed{\dfrac{d}{dt}\dfrac{q^2}{2C}}$ represents energy stored in *dt* time in the capacitor.

(*iii*) i^2R represents Joules heating loss.

(*iv*) $\frac{1}{2}V_m i \sin \omega t$ is the rate at which driving force pours in energy. It goes into ohmic loss and increase of stored energy in capacitor and inductor.

(*c*) Here equation (I) is in the form of conservation of energy statement.

(*d*) Integrating eqn. (I) both sides with respect to *dt* over a cycle we get

$$\int_0^T \frac{d\left(-\ i\right)}{dt}dt + \int_0^T \frac{dq}{dt\,2C}dt + \int_0^T \frac{-}{2}i \quad dt = \frac{1}{2}\int_0^T V_m i \sin \omega t \, dt$$

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$$\int_0^T \frac{d}{dt}\left[\frac{1}{2}Li^2 + \frac{q^2}{2C}\right]dt + \frac{1}{2}\int_0^T i^2 R\, dt = \frac{1}{2}\int_0^T Vi\, dt \qquad [\because V = V_m \sin \omega t]$$

$$0 + \frac{1}{2}i^2 RT = \frac{1}{2}\int_0^T Vi\, dt$$

$$i^2 RT = \int_0^T Vi\, dt$$

as i^2 RT is +ve [$\because i^2$, R and T can never be negative]

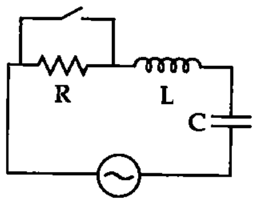
So, $\int_0^T Vi\, dt$ is positive which is only possible if phase difference ϕ is constant and the angle is acute.

Q7.31. In the LCR circuit shown in the figure below, the AC driving voltage is $V = V_m \sin \omega t$.

(*a*) Write down the equation of motion for $q(t)$.

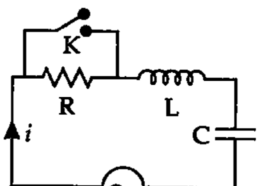
(*b*) at $t = t_0$, the voltage source stop and R is short circuited. Now write down how much energy is stored in each of L and C.

(*c*) Describe subsequent motion of charges.



Main concept used: Kirchhoff's voltage law, and get relations after differentiating either one or two with respect to time.

Ans. (*a*) Consider series LCR circuit and tapping key K to short circuit R. Let *i* be the current in circuit. Then by Kirchhoff's Voltage Law, when key K is open,



$$V_R + V_L + V_C = V_m \sin \omega t$$

$$iR + L\frac{di(t)}{dt} + \frac{q(t)}{C} - V_m \sin \omega t = 0 \qquad [\because i(t) = i = I_m \sin(\omega t + \phi)]$$

$\Rightarrow$ As charge $q(t)$ changes in circuit with time in AC,

then $$i = \frac{dq(t)}{dt}$$

$$\frac{di}{dt} = \frac{d^2q(t)}{dt^2} \quad \text{(differentiating again)}$$

$$R\frac{dq(t)}{dt} + L\frac{d^2q(t)}{dt^2} + \frac{q(t)}{C} = V_m \sin \omega t$$

$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{q(t)}{C} = V_m \sin \omega t$$

This is the equation for variation of motion of charge with respect to time.

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(*b*) Let time dependent charge in circuit is at phase angle with voltage then $q = q_m \cos(\omega t + \phi)$

$$i = \frac{dq}{dt} = \omega q_m \sin(\omega t + \phi) \qquad \text{...(I)}$$

$$i_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}} \qquad \text{...(II)}$$

$$\tan\phi = \frac{(X_C - X_L)}{R}$$

At $t = t_0$, R is short circuited, then energy stored in L and C, when K is closed will be,

$$U_L = \frac{1}{2}Li^2 \qquad \text{...(III)}$$

At $t = t_0$,

$$i = i_m \sin(\omega t_0 + \phi) \qquad \text{...(IV)}$$

From (II), $$i = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}} \sin(\omega t_0 + \phi) \qquad \text{...(V)}$$

So, from (III),

$$\therefore \quad U_L = \frac{1}{2}L\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \sin^2(\omega t_0 + \phi)$$

$$U_C = \frac{q^2}{2C} = \frac{1}{2C}\left[q_m^2 \cos^2(\omega t_0 + \phi)\right]$$

Comparing (IV) and (I) $\quad i_m = q_m\omega$

$$\therefore \quad q_m = \frac{i_m}{\omega}$$

$$\therefore \quad U_C = \frac{1}{2C}\cdot\frac{i_m^2}{\omega^2}\cos^2(\omega t_0 + \phi) = \frac{i_m^2}{2C\omega^2}\cos^2(\omega t_0 + \phi)$$

Using equation (II)

$$U_C = \frac{}{2C\ \ ^2}\left[\frac{}{\left(\sqrt{R\ \ + (X_C - X_L)}\right)^2}\right] \cos\ (\omega\ \ \ + \phi)$$

(from IV and V)

(*c*) When R is short circuited, the circuit becomes L-C oscillator. The capacitor will go on discharging and all energy will transfer to L, and back and forth. Hence there is oscillation of energy from electrostatic to magnetic and vice versa.

❑❑❑

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8 Electromagnetic Waves

MULTIPLE CHOICE QUESTIONS—I

Q8.1. One requires 11 eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of appropriate electromagnetic radiation to achieve the dissociation lies in

(*a*) visible region. (*b*) infrared region.
(*c*) ultraviolet region. (*d*) microwave region.

Main concept used: (*i*) $E = h\nu$ and (*ii*) Range of electromagnetic spectrum.

Ans. (*c*): $E = 11 \text{ eV} = 11 \times 1.6 \times 10^{-19}$ J

$h = 6.62 \times 10^{-34}$ J-S

$\therefore$ $E = h\nu$

$$\nu = \frac{E}{h} = \frac{11 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = \frac{8.8 \times 10^{-19+34}}{3.31} = \frac{880}{331} \times 10^{15} \text{ Hz}$$

Q8.2. A linearly polarized electromagnetic wave given as $E = E_0\hat{i}\cos(kz - \omega t)$ is incident normally on a perfectly reflecting infinite wall at $z = a$. Assuming that the material of the wall is optically inactive, the reflected wave will be given as

(*a*) $E_r = -E_0\hat{i}\cos(kz - \omega t)$. (*b*) $E_r = E_0\hat{i}\cos(kz + \omega t)$.
(*c*) $E_r = -E_0\hat{i}\cos(kz + \omega t)$. (*d*) $E_r = E_0\hat{i}\cos(kz - \omega t)$.

Main concept used: When a wave is reflected from a denser medium then its phase angle is changed by 180°.

Ans. (*b*): $E = E_0 i \cos(kz - \omega t)$ (given)

As E is along +ive X axis so reflected ray will be along negative X-axis. Its electric component will also be opposite to earlier, *i.e.* in –z direction, phase will change by π ($z \to -z$) and ($i \to -i$)

$$E_r = E_0(-i)\cos[k(-z) - \omega t + \pi]$$

$$E_r = -E_0\hat{i}\cos[\pi - (kz + \omega t)]$$

$$\therefore \quad E_r = +E_0\hat{i}\cos(kz + \omega t)$$

Q8.3. Light with an energy flux of 20 W/cm² falls on a non-reflecting surface at normal incidence. If the surface has an area of 30 cm², the total momentum delivered (for complete absorption) during 30 minutes is

(*a*) 36×10^{-5} kg m/s. (b) 36×10^{-4} kg m/s.
(c) 108×10^{4} kg m/s. (d) 1.08×10^{7} kg m/s.

Main concept used: Momentum of incident light $= \frac{V}{C}$ (Total energy)

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Ans. (*b*): Energy flux = ϕ = 20 W/cm^2

A = 30 cm^2, t = 30 × 60 sec

U = Total energy falling t sec = ϕ At

U = 20 × 30 × 30 × 60 J

$$\text{Momentum of the incident light} = \frac{U}{C} = \frac{20 \times 30 \times 30 \times 60}{3 \times 10^8}$$
$$= 36 \times 10^{-4} \text{ kg ms}^{-1}$$

As no reflection from the surface and for complete absorption so momentum of reflected radiation is zero.

Momentum delivered to surface = Change in momentum

$$= p_f - p_i = 0 - 36 \times 10^{-4} = -36 \times 10^{-4} \text{ kg m/s}$$

(–) sign shows the direction of momentum.

Q8.4. The electric field intensity produced by the radiations coming from 100 W bulb at a 3 m distance is E. The electric field intensity produced by the radiations coming from 50 W bulb at the same distance is

(*a*) $\frac{E}{2}$. (*b*) 2E. (*c*) $\frac{E}{\sqrt{2}}$. (*d*) $\sqrt{2}E$.

Main concept used: E.F. intensity on a surface due to incidence radiation is $I_{av} \propto E_o^2$ and $\frac{P_{av}}{A} \propto E_o^2$

$P_{av} \propto E_o^2$ (as A is constant)

Ans. (*c*): $E_o \propto \sqrt{P_{av}}$

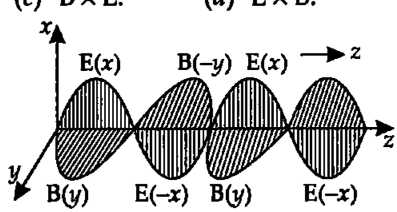
$$\frac{(E_o)_1}{(E_o)_2} = \sqrt{\frac{(P_{av})_1}{(P_{av})_2}} = \sqrt{\frac{100 \text{ W}}{50 \text{ W}}} = \frac{\sqrt{2}}{1}$$

$$\therefore \quad (E_o)_2 = \frac{(E_o)_1}{\sqrt{2}}$$

Q8.5. If E and B represent the electric and magnetic field vectors of the electromagnetic wave, the direction of propagation of electromagnetic wave is along

(*a*) $\vec{E}$. (*b*) $\vec{B}$. (*c*) $\vec{B} \times \vec{E}$. (*d*) $\vec{E} \times \vec{B}$.

Main concept used: The direction of e.m. wave can be find out by right hand grip rule or by $\vec{E} \times \vec{B}$.



Ans. (*d*): The direction of propagation of electromagnetic wave is perpendicular to both $\vec{E}$ and $\vec{B}$ and is given by $\vec{E} \times \vec{B}$ by right thumb rule.

The electric field E is along E(+*x*) and E(–*x*) axis and magnetic field B is along B(*y*) and B(–*y*) axis. So by cross product of E and B, direction is perpendicular to E and B, from $\vec{E}$ to $\vec{B}$ *i.e.* (E × B) in +z direction.

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Q8.6. The ratio of contributions made by the electric field and magnetic field components to the intensity of an E.M. wave is

(*a*) $c:1$ (*b*) $c^2:1$ (*c*) $1:1$ (*d*) $\sqrt{c}:1$

Main concept used: (*i*) $I = U_{av}.c$, (*ii*) $U_{av} = \frac{1}{2}\varepsilon_o E^2 = \frac{1}{2}\frac{B_o^2}{\mu_o}$,

(*iii*) $c = \frac{E_o}{B_o}$.

Ans. (*c*): Average energy by electric field E_o is U_{av}

$$U_{av} = \frac{1}{2}\varepsilon_o E_o^2 \quad \text{But } E_o = cB_o$$

$$(U_{av})_{\text{electric field}} = \frac{1}{2}\varepsilon_o (cB_o)^2 = \frac{1}{2}\varepsilon_o c^2 B_o^2$$

$$= \frac{1}{2}\varepsilon_o \cdot \frac{1}{\mu_o \varepsilon_o}(B_o)^2 \qquad \left[\because c^2 = \frac{1}{\mu_o \varepsilon_o}\right]$$

$$(U_{av})_{\text{electric field}} = \frac{1}{2\mu_o}B_o^2 = (U_{av})_{\text{(magnetic field)}}$$

$$\text{Ratio} = \frac{(U_{av})_{\text{electric field}}}{(U_{av})_{\text{magnetic field}}} = \frac{1}{1}, \text{ i.e. } 1:1$$

Q8.7. An E.M. wave radiates outwards from a dipole antena, with E_o as the Amplitude of its electric field vector. The electric field E_o which transports significant energy from the source falls off as

(*a*) $\frac{1}{r^3}$ (*b*) $\frac{1}{r^2}$ (*c*) $\frac{1}{r}$ (*d*) remains constant

Main concept used: $E_o \propto \frac{1}{r}$ from dipole antenna.

Ans. (*c*): As we know that electromagnetic waves are radiated from dipole antena and radiated energy $E \propto \frac{1}{r}$.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q8.8. An electromagnetic wave travels in vacuum along +Z direction

$$E = (E_1\hat{i} + E_2\hat{j})\cos(kz - \omega t).$$

Choose the correct options from the following:

(*a*) The associated magnetic field is given as

$$B = \frac{1}{C}(E_1\hat{i} - E_2\hat{j})\cos(kz - \omega t).$$

(*b*) The associated magnetic field is given as

$$B = \frac{1}{C}(E_1\hat{i} - E_2\hat{j})\cos(kz - \omega t).$$

(*c*) The given electromagnetic field is circularly polarised.

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(*d*) The given electromagnetic wave is plane polarised.

Main concept used: From Maxwell's equations $|B_o| = \frac{|E_o|}{|C|}$.

Ans. (*a*), (*b*) and (*d*): Here, in electromagnetic wave, the electric field vector is given as $E = (E_1\hat{i} + E_2\hat{j})\cos(kz - \omega t)$. In electromagnetic wave the associated magnetic field vector $\vec{B} = \frac{\vec{E}}{\vec{C}} = \frac{1}{C}(E_1 i + E_2 j)\cos(kz - \omega t)$ verify option (*a*) and (*b*) as $\vec{E}$ and $\vec{B}$ are along X- and Y-axis respectively so the direction of propagation is Z-axis and E.M. wave is plane polarised along X and Y direction $\vec{E}$ and $\vec{B}$ respectively verifies option (*d*).

Q8.9. An electromagnetic wave travelling along Z-axis is given as $E = E_o\cos(kz - \omega t)$. Choose the correct options from the following:

(*a*) The associated magnetic field is given as $B = \frac{1}{c}\hat{k} \times \vec{E} = \frac{1}{\omega}(\hat{k} \times \vec{E})$.

(*b*) The electromagnetic field can be written in terms of the associated magnetic field $\vec{E} = c(\vec{B} \times \hat{k})$.

(*c*) $\hat{k} \cdot E = 0, \quad \hat{k} \cdot B = 0$.

(*d*) $\hat{k} \times \vec{E} = 0, \quad \hat{k} \times \vec{B} = 0$.

Main concept used: $E = E_o\cos(kz - \omega t)$

Ans. (*a*), (*b*) and (*c*): E.M. wave is travelling in +Z direction. Its electric field is given by

$E = E_o\cos(kz - \omega t)$ along X direction which is perpendicular to Z-axis *i.e.*, along X direction.

The associated magnetic field B is also perpendicular to +Y *i.e.*, $\hat{k} \times \vec{E}$.

As $B = \frac{E}{c} = \frac{1}{c}(\hat{k} \times \vec{E})$ (along Y-axis).

The associated electric field can be written in terms of magnetic field as $\vec{E} = c(\vec{B} \times \hat{k})$.

Angle between $\hat{k}$ and $\vec{E}$ is 90° $\Rightarrow$ E.B = EB cos 0° = 1

As we know that B, E and direction of propagation of E.M. wave are perpendicular to each other.

Here, $E = E_o\cos(kz - \omega t)$

As the direction of propagation of E.M. wave is in +Z direction *i.e.*, $v\hat{k}$, then E is in $E\hat{i}$ and $B\hat{j}$.

For option (*a*) $\quad B = \frac{1}{c}\hat{k} \times \vec{E}$ [given in option (*a*)]

$$Bj = \frac{1}{c}\hat{k} \times \vec{E}\hat{i} \text{ is true} \quad (\because \hat{k} \times \hat{i} = \hat{j})$$

$$\frac{1}{\omega}(\hat{k} \times \vec{E}) = B$$ [given in option (*a*)]

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$$\frac{1}{\omega}(\hat{k} \times E\hat{i}) = B\hat{j} \text{ is true as } \hat{k} \times \hat{i} = \hat{j} \quad \text{verifies answer } (a).$$

For option (*b*) $E\hat{i} = c(B\hat{j} \times \hat{k})$ is true $\therefore \hat{} = \hat{j} \times \hat{k}$

For option (*c*) $\hat{k}.E(\hat{i}) = 0$ *is true* $\because \hat{k}.\hat{i} = 0$

$\hat{k}.B(\hat{j}) = 0$ *is true* $\because \hat{k}.\hat{j} = \cos 90° = 0$

For option (*d*) $\hat{k} \times E(\hat{i}) = 0$ is false as $\hat{k} \times \hat{i} = -\hat{j} \neq 0$

Q8.10. A plane electromagnetic wave propagating along X-direction can have the following pairs of E and B

(*a*) E_x, B_y (*b*) E_y, B (c) B_x, E_y (d) E_z, B_y

Main concept used: The direction of propagation $\vec{v}$, magnetic field $\vec{B}$ and electric field are perpendicular to each other.

Ans. (*b*) and (*d*): As the EM wave is plane polarised and its propagation is in +X direction. So direction of $\vec{E}$ and $\vec{B}$ will be in either Y and Z direction or Z and Y direction. So verifies answers (*b*) and (*d*).

Q8.11. A charged particle oscillates about its mean equilibrium position with a frequency 10^9 Hz. The electromagnetic waves produced:

(*a*) will have the frequency of 10^9 Hz.

(*b*) will have the frequency of 2×10^9 Hz.

(*c*) will have wavelength 0.3 m.

(*d*) fall in the region of radiowaves.

Main concept used: (*i*) $c = \nu\lambda$, (*ii*) the frequency of wave as the frequency due to which it is produced.

Ans. (*a*) (*c*) and (*d*): Vibrating particle produces electric and magnetic field, so will produce an E.M. wave of same frequency 10^9 Hz verifies answer (*a*).

$\because \nu = 10^9$ Hz, $c = 3 \times 10^8$ m/s

So, $$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{10^9} = \frac{3 \times 10^8}{10 \times 10^8} = 0.3 \text{ m} \quad \text{verifies answer } (c).$$

As the range of radiowaves are between 10 Hz to 10^{12} Hz and 10^9 Hz lies between this range verifies answer (*d*).

Q8.12. The source of electromagnetic waves can be a charge

(*a*) moving with constant velocity. (*b*) moving in a circular orbit.

(*c*) at rest. (*d*) falling in an electric field.

Main concept used: An E.M. wave can be produced either by accelerated or oscillating charge.

Ans. (*b*) and (*d*): Motion of a particle in circular orbit is accelerated motion verifies answer (*b*).

When a charge particle falls in electric field the velocity of charge particle changes so its motion becomes accelerated and can produce E.M. wave. It verifies answer (*d*).

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Q8.13. An E.M. wave of intensity I falls on a surface kept in vacuum and exerts radiation pressure p on it. Which of the following are true?

(*a*) Radiation pressure is I/c if the wave is totally absorbed.

(*b*) Radiation pressure is I/c if the wave is totally reflected.

(*c*) Radiation pressure is $\frac{2I}{c}$ if the wave is totally reflected.

(*d*) Radiation pressure is in the range $\frac{I}{c} < p < \frac{2I}{c}$ for real surface.

Main concept used: Due to dual nature of the wave, E.M. wave also has particle nature.

Ans. (*a*) (*c*) and (*d*): Radiation pressure is the force exerted by particles (dual nature of particle) on unit area, due to the change in momentum of radiated particles per unit area per sec = $\frac{I}{c}$.

I = intensity of radiation
c = velocity of radiation

Radiations are absorbed, so momentum per unit area per second = $\frac{I}{c}$ verify the answer (*a*).

When radiation is reflected back, the momentum becomes double as in earlier case, so discards answer (*b*) and verifies answer (*c*).

So variation of radiation pressure p comes between the range $\frac{I}{c} < p < \frac{2I}{c}$ verifies answer (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q8.14. Why is the orientation of the portable radio with respect to broadcasting station important?

Ans. Transmitted carrier wave signals are plane polarised and if the intensity of signal is poor then receiving antenna of radio must be parallel to the component of either electric or magnetic field. Because energy is only due to amplitudes of electric and magnetic components in EM wave, magnitude of amplitude is in particular direction perpendicular to each other and perpendicular to wave propagation.

Q8.15. Why does the microwave oven heats up a food item containing water molecule most efficiently?

Main concept used: Resonance phenomenon.

Ans. In Microwave oven, molecules of food item starts to vibrate by driven force due to microwaves with the frequency of microwave. But the natural frequency of water molecules matches with microwave frequency which causes resonance (more amplitude) which further causes increase in temperature.

Q8.16. The charge on parallel plate capacitor varies as $q = q_0 \cos 2\pi \nu t$. The plates are very large and close to each other. Separation between

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plates is d and common area of plates is A. Neglecting the edge effects, find the displacement current through the capacitor.

Ans. The displacement current I_d in capacitor is

$$I_d = I_C = \frac{dq}{dt}, \text{ where } q = q_0 \cos(2\pi\nu t) \qquad \text{(Given)}$$

$$\therefore \quad I_d = I_C = \frac{d}{dt} q_0 \cos(2\pi\nu t)$$

$$I_d = -q_0 2\pi\nu \sin 2\pi\nu t$$

$$I_d = -2\pi\nu q_0 \sin(2\pi\nu t)$$

Q8.17. A variable frequency AC source is connected to a capacitor. How will the displacement current change with decrease in frequency?

Main concept used: $X_c \propto \frac{1}{\nu}$ and $I = \frac{V}{X_C}$

Ans. As we know that $X_C = \frac{1}{2\pi\nu C}$ and $I_C = \frac{V}{X_C}$

so reactance of capacitor increases on decreasing frequency $X_c \propto \frac{1}{\nu}$.

So $$I = 2\pi\nu CV$$

As reactance of capacitor increases, the current by Ohm's law will decrease. $\left[\because I \propto \frac{1}{X_C}\right]$

So the displacement current decreases frequency decreases when the conduction current is equal to displacement current.

Q8.18. The magnetic field of a beam, emerging from a filter facing a floodlight, is given by $B = 12 \times 10^{-8} \sin(1.2 \times 10^7 z - 3.6 \times 10^{15} t)$ T.

Find the average intensity of the beam.

Main concept used:
$$I_{av} = \frac{B_0^2}{2\mu_0} C$$
$$B = B_0 \sin(kx - 2\pi\nu t) \qquad \left(\because k = \frac{2\pi}{\lambda}\right)$$

Ans. $B = 12 \times 10^{-8} \sin[1.2 \times 10^7 Z - 3.6 \times 10^{15} t]$ ...I

Standard equation $B = B_0 \sin(kx - \omega t)$ ...II

Comparing the different terms in above two (I, II) equations

$$B_0 = 12 \times 10^{-8} \text{ Tesla}$$

$$I_{av} = \frac{B_0^2}{2\mu_0} C = \frac{12 \times 10^{-8} \times 12 \times 10^{-8} \times 3 \times 10^8}{2 \times 4\pi \times 10^{-7}}$$

$$= \frac{12 \times 12 \times 3 \times 10^{-8-8+8+7}}{8 \times 3.14} = \frac{54}{3.14 \times 10} = \mathbf{1.72\ W/m^2}$$

Q8.19. Poynting vector S is defined as a vector whose magnitude is equal to the wave intensity and whose direction is along the direction

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of the wave propagation. Mathematically, it is given by $S = \frac{1}{\mu_o}(\vec{E} \times \vec{B})$. Show the nature of S versus t graph.

Ans. Consider an electromagnetic wave. Let electric field (E) of EM wave varies along Y-axis the propagation of wave is along X-axis, then $\vec{E} \times \vec{B}$ will give the direction of flow of energy in electromagnetic wave along X-axis.

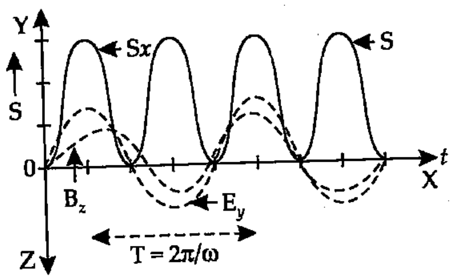
$$\vec{E} = E_o \sin(\omega t - kx)\hat{j}$$

$$\vec{B} = B_o \sin(\omega t - kx)\hat{k}$$

$$S = \frac{1}{\mu_o}(\vec{E} \times \vec{B}) = \frac{1}{\mu_o} \sin^2(\omega t - kx)\hat{j} \times \hat{k}$$

$$S = \frac{1}{\mu_o} \sin^2(\omega t - kx)\hat{i}$$

Variation of |S| with time will be as given in figure.



Q8.20. Professor CV Raman surprised his students by suspending freely a tiny light ball in transparent vacuum chamber by shining a laser beam on it. Which property of EM waves was he exhibiting? Give one more example of this property.

Ans. We know the dual nature of radiation and matter. EM wave carries energy and momentum. Due to this change in momentum (by direction or velocity of wave), EM wave, exert pressure on the surface, by reflection and refraction. This property of EM waves helped professor CV Raman to surprise his students by suspending freely a tiny light ball in transparent vacuum chamber by shining a laser beam on it. The tails of the comets are also due to radiation pressure.

Electromagnetic radiations can pass even through vacuum and has particle nature.

Mobile phone placed in evacuated transparent chamber can ring up which can be seen through transparent chamber, but sound of ring tone cannot be heard, proves the propagation of electromagnetic wave in vacuum.

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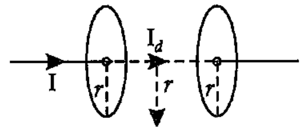
SHORT ANSWER TYPE QUESTIONS

Q8.21. Show that the magnetic field B at a point in between plates of a parallel plate capacitor during charging is $\frac{\mu_o \varepsilon_o r}{2}\frac{dE}{dt}$ (symbols having usual meaning).



Ans. Let I_d be the displacement current in the region of magnetic field between two plates of a parallel plate capacitor.

The magnetic field induction at a point in a region between two plates of a capacitor at a perpendicular distance from the axis of plate is

$$B = \frac{\mu_o I}{2\pi r} = \frac{\mu_o}{2\pi r} I_d = \frac{\mu_o}{2\pi r}\left(\varepsilon_o \frac{d\phi}{dt}\right)$$

$$B = \frac{\mu_o \varepsilon_o}{2\pi r}\frac{d}{dt}(E\pi r^2) \quad \because \boxed{\phi_E = \vec{E}\cdot\vec{A}}$$

$$B = \frac{\mu_o \varepsilon_o r}{2}\frac{dE}{dt}$$

Q8.22. Electromagnetic waves with wavelength

(*i*) λ_1 is used in satellite communication.

(*ii*) λ_2 is used to kill germs in water purifiers.

(*iii*) λ_3 is used to detect leakage of the oil in underground pipelines.

(*iv*) λ_4 is used to improve visibility in runways during fog and mist conditions.

(*a*) Identify and name the part of electromagnetic spectrum to which these radiations belong.

(*b*) Arrange these wavelengths in ascending order of their magnitude.

(*c*) Write one more application of each.

Ans. (*a*) (*i*) Microwave is used in satellite communications so, λ_1 is the wavelength of microwave. It is used in microwave oven.

(*ii*) Ultraviolet rays are used to kill germs in water purifier so, λ_2 is the wavelength of ultraviolet rays.

λ_2 are UV rays that can be focused into very narrow beam for high precision application such as LASIK (Laser—assisted in situ kerotomileusis) eye surgery.

(*iii*) X-rays are used to detect leakage of oil in underground pipelines so λ_3 is in X-ray region. It is also used to detect cracks in machinery and to detect fracture in bones of body.

(*iv*) Infrared rays λ_4 are used to improve visibility due to larger wavelength of low scattering.

Infrared rays are used in optical communication.

(*b*) The arrangement of the wavelengths in ascending order are

$$\lambda_3 < \lambda_2 < \lambda_4 < \lambda_1$$

(*c*) Uses are along with part (*a*).

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Q8.23. Show that average value of radiant flux density S over a single period T is given by $S = \frac{1}{2c\mu_o} E_o^2$.

Ans. Radiant flux density $S = \frac{1}{\mu_o}(\vec{E} \times \vec{B})$

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

or

$$c^2 = \frac{1}{\mu_o \varepsilon_o}$$

$$\frac{1}{\mu_o} = \varepsilon_o c^2$$

$$\therefore \quad S = \varepsilon_o c^2 (\vec{E} \times \vec{B}) \qquad ...(I)$$

Let electromagnetic waves be propagated along X-axis so its electric and magnetic field vectors are along Y and Z axis.

$$\therefore \quad \vec{E} = E_o \cos(kx - \omega t)\hat{j}$$

$$\vec{B} = B_o \cos(kx - \omega t)\hat{k}$$

$$\vec{E} \times \vec{B} = (E_o B_o) \cos^2(kx - \omega t)\,(\hat{j} \times \hat{k})$$

Put E × B in I

$$\therefore \quad S = \varepsilon_o c^2 E_o B_o \cos^2(kx - \omega t)\hat{i}$$

So average value of the magnitude of radiant flux density over complete cycle is

$$S_{av} = c^2 \varepsilon_o (E_o B_o) \frac{1}{T} \int_0^T \cos^2(kx - \omega t)\, dt\, \hat{i}$$

$$= \frac{c^2 \varepsilon_o E_o B_o}{T} \left[\frac{T}{2}\right] = \frac{c^2}{2} \varepsilon_o E_o \left(\frac{E_o}{c}\right) \qquad \left[\because c = \frac{E_o}{B_o} \text{ or } B_o = \frac{E_o}{c}\right]$$

$$= \frac{c \varepsilon_o E_o^2}{2} \qquad \left[c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \text{ or } \varepsilon_o = \frac{1}{\mu_o c^2}\right]$$

$$= \frac{c}{2} \cdot \frac{1}{\mu_o c^2} E_o^2$$

$$S_{av} = \frac{E_o^2}{2\mu_o c} \qquad \textbf{Hence proved}$$

Q8.24. You are given a 2 μF parallel plate capacitor. How would you establish an instantaneous displacement current of 1 mA in the space between its plates?

Ans. $C = 2\,\mu F = 2 \times 10^{-6}$ F, $I_d = 1$ mA $= 10^{-3}$ A

$$\because \quad q = CV$$

$$I_d\, dt = C.dV$$

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$$I_d = C \cdot \frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{I_d}{C} = \frac{10^{-3}\,A}{2 \times 10^{-6}} = \frac{1000}{2}\ \text{Volts}$$

$$\frac{dV}{dt} = 500\ V/s$$

Q8.25. Show that the radiation pressure exerted by an EM wave of intensity I on a surface kept in vacuum is $\frac{I}{c}$.

Ans. Pressure on surface due to particle nature of wave $P = \frac{F}{A}$

Force = Rate of change of momentum

If mass (m) of radiant particle for wave of velocity c then

$$E = mc^2$$

$$E = U \text{ (let)}$$

$$U = mc \, . \, c$$

$$U = p \, . \, c$$

where, momentum $p = mc$

Now differentiating w.r.t. time (t) we get

$$\frac{dU}{dt} = c \, . \, \frac{dp}{dt}$$

$$\frac{1}{c} \cdot \frac{dU}{dt} = \frac{dp}{dt} \qquad \left[\because \frac{dp}{dt} = F \text{ (by Newton's second law)}\right]$$

$$\therefore \quad F = \frac{1}{c}\frac{dU}{dt} \quad \text{or} \quad \frac{F}{A} = \frac{1}{A} \cdot \frac{1}{c} \cdot \frac{dU}{dt}$$

Pressure (P) on surface due to e.m. wave radiation and $P = \frac{F}{A}$

$$P = \frac{1}{c} \cdot \left[\frac{dU}{Adt}\right]$$

We know that intensity of radiation is equal to the radiant energy (U) falling on unit surface per second.

$$\therefore \quad I = \frac{1}{A} \cdot \frac{dU}{dt} \quad \text{or} \quad \boxed{P = \frac{I}{c}}$$ Hence proved.

Q8.26. What happens to the intensity of light from a bulb if the distance from the bulb is doubled? As a laser beam travels across the length of a room its intensity essentially remains constant.

What geometrical characteristic of LASER beam is responsible for the constant intensity which is missing in the case of light from the bulb?

Ans. Bulb spreads its light in all around spherically and symmetrically. So if the distance from the bulb is doubled, the surface area covered by radiations changes from $4\pi r^2$ to $4\pi(2r)^2$ *i.e.*, 2^2 or four times decreased

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in straight laser. But in a bulb it decreased by $4(4\pi)$, i.e. 16π decreased. So the intensity becomes one-fourth the initial value in straight line. Since $\left(I \propto \frac{1}{r^2}\right)$ and for spherical source $\left(I \propto \frac{1}{4\pi r^2}\right)$.

In case of laser, it does not spread in all directions. It passes only along a straight line. So its intensity remains same almost.

Geometrical characteristics of LASER beam which is responsible for the constant intensity are:

(*i*) Monochromatic, (*ii*) Coherent, (*iii*) Highly collimated from all around source, (*iv*) Unidirectional.

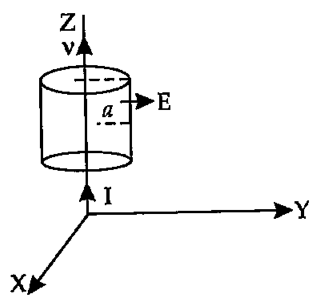
These characteristics are missing in the case of light from the bulb.

Q8.27. Even though an electric field E exerts a force qE on a charged particle, yet electric field of EM wave does not contribute to the radiation pressure (but transfers energy). Explain.

Ans. In electromagnetic wave, the electric field is oscillating, so the resultant electric force on the particle will be zero, as the direction of electric force changes every half of time period. As the electric field vibrates, the radiation of energy will takes place only due to vibrating electric and magnetic field.

LONG ANSWER TYPE QUESTIONS

Q8.28. An infinitely long thin wire carrying a uniform linear static charge density λ is placed along the +Z-axis (figure). The wire is set into motion along its length with a uniform velocity $\vec{v} = v\hat{k}_z$. Calculate the poynting vector $S = \frac{1}{\mu_o}(E \times B)$.



Ans. Consider a cylindrical Gaussian surface in such a way that the axis of cylinder lies on wire. Electric field intensity due to long straight wire at a distance a and charge density λ c/m

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_o a} = \frac{\lambda}{2\pi\varepsilon_o a}\hat{j}$$

$$\vec{B} = \frac{\mu_o I}{2\pi a}\hat{i}$$

$$I = \frac{q}{t} = \frac{\lambda l}{t} = \lambda . v$$

$$\therefore \quad \vec{B} = \frac{\mu_o \lambda v}{2\pi a}\hat{i}$$

$$S = \frac{1}{\mu_o}(E \times B) = \frac{1}{\mu_o}\left[\frac{\lambda}{2\pi\varepsilon_o a}\hat{j} \times \frac{\mu_o \lambda v}{2\pi a}\hat{i}\right]$$

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$$= \frac{\lambda^2 v}{4\pi^2 a^2 \varepsilon_o}(\hat{j} \times \hat{i}) = \frac{\lambda^2 v}{4\pi^2 a^2 \varepsilon_o}(-\hat{k})$$

$$S = \frac{-\lambda^2 v}{4\pi^2 a^2 \varepsilon_o}\hat{k}$$

Q8.29. Sea water at frequency $\nu = 4 \times 10^8$ Hz has permittivity $\varepsilon = 80\varepsilon_o$, permeability $\mu \approx \mu_o$ and resistivity $\rho = 0.25\ \Omega$-m. Imagine a parallel plate capacitor immersed in sea water and driven by an alternating voltage source $V(t) = V_o \sin(2\pi\nu t)$. What fraction of the conduction current density is the displacement current density?

Ans. Suppose distance between the parallel plates of capacitor is 'd' and the applied voltage

$$V(t) = V_o \sin(2\pi\nu t)$$

$$\therefore \quad E = \frac{V(t)}{d} = \frac{V_o \sin(2\pi\nu . t)}{d}$$

By Ohm's conduction current density

$$\vec{J}_C = \frac{1}{\rho}\vec{E} = \frac{1}{\rho}\frac{V_o \sin(2\pi\nu t)}{d}$$

Let $$J_o^C = \frac{V_o}{\rho d}$$

$$\therefore \quad \boxed{J_C = J_o^C \sin 2\pi\nu t}$$

The displacement current

$$J_d = \varepsilon\frac{dE}{dt} = \varepsilon\frac{d}{dt}\frac{V_o}{d}\sin(2\pi\nu t) = \frac{\varepsilon V_o}{d}\cdot\cos(2\pi\nu t)\,(2\pi\nu)$$

$$J_d = \frac{2\pi\nu\varepsilon V_o}{d}\cos(2\pi\nu t)$$

Let $$J_o^d = \frac{2\pi\nu\varepsilon V_o}{d}$$

Then $$J_d = J_o^d \cos(2\pi\nu t)$$

$$\frac{J_o^d}{J_o^c} = \frac{\frac{2\pi\nu\varepsilon V_o}{d}}{\frac{V_o}{\rho d}} = 2\pi\nu\varepsilon_o\rho$$

$$\frac{J_o^d}{J_o^c} = 2\pi\nu\varepsilon\rho = 2\pi \times 4 \times 10^8 \times 80\varepsilon_o \times 0.25$$

$$= 4\pi\varepsilon_o \times 2 \times 80 \times 0.25 \times 10^8$$

$$= \frac{2 \times 80 \times 0.25 \times 10^8}{9 \times 10^9} = \frac{160 \times 25}{9 \times 10 \times 100}$$

$$\frac{J_o^d}{J_o^c} = \frac{4}{9}$$

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Q8.30. A long straight cable of length l is placed symmetrically along Z-axis and has radius a ($<<< l$). The cable consists of a thin wire and a co-axial conducting tube. An alternating current $I(t) = I_o \sin(2\pi\nu t)$ flows down the central thin wire and returns along the co-axial conducting tube. The induced electric field at a distance s from the wire inside the cable is

$$\vec{E}(s, t) = \mu_o I_o \nu \cos(2\pi\nu t) \log_e\left(\frac{s}{a}\right)\hat{k}.$$

(*i*) Calculate the displacement current density inside the cable.

(*ii*) Integrate the displacement current density across the cross-section of the cable to find the total displacement current I_d.

(*iii*) Compare the conduction current I_o with the displacement current I_o^d.

Main concept used: $\boxed{I_d = \varepsilon_o \frac{dE}{dt}}$.

Ans. (*i*) Induced electric field $E(s, t)$ at distance s($s <$ radius of co-axial cable) is given as $E(s, t) = \mu_o I_o \nu \cos 2\pi\nu t \log_e\left(\frac{s}{a}\right)\hat{k}$. Displacement current density J_d is given by,

$$J_d = \varepsilon_o \frac{dE}{dt} = \varepsilon_o \mu_o I_o \nu \frac{d}{dt}\left[\cos 2\pi\nu t . \log_e \frac{s}{a}\right]\hat{k}$$

$$J_d = \varepsilon_o \mu_o I_o \nu \left[(-\sin 2\pi\nu t) \cdot 2\pi\nu . \log_e \frac{s}{a}\right]\hat{k}$$

[$\because$ s and a are constant]

$$J_d = -\varepsilon_o \mu_o I_o 2\pi\nu^2 \log_e\left(\frac{s}{a}\right)\sin(2\pi\nu t)\,\hat{k}$$

$$= -\frac{1}{C^2} I_o 2\pi\nu^2 \left[-\log_e\left(\frac{a}{s}\right)\right]\sin(2\pi\nu t)\,\hat{k} \quad \left[\because C = \frac{1}{\sqrt{\mu_o \varepsilon_o}}\right]$$

$$= +\frac{2\pi\nu^2}{C^2} I_o \log_e\left(\frac{a}{s}\right)\sin 2\pi\nu t\,\hat{k}$$

$$= \frac{2\pi\nu^2}{\nu^2\lambda^2} I_o \log_e\left(\frac{a}{s}\right)\sin 2\pi\nu t\,\hat{k}$$

$$\boxed{J_d = \frac{2\pi I_o}{\lambda^2}\log_e \frac{a}{s}\sin(2\pi\nu t)\,\hat{k}}$$

(*ii*)

$$I_d = \int J_d . s\, ds\, d\theta = \int_{s=0}^{a} J_d \cdot s\, ds \int_0^{2\pi} d\theta = \int_{s=0}^{a} J_d\, s . ds\, [2\pi]$$

$$= 2\pi \int_{s=0}^{a} \frac{2\pi I_o}{\lambda^2}\left[\log_e\left(\frac{a}{s}\right)\sin(2\pi\nu t)\hat{k}\right] . s\, ds$$

$$I_d = \left(\frac{2\pi}{\lambda}\right)^2 I_o \sin(2\pi\nu t)\hat{k} \int_{s=0}^{a} \log\left(\frac{a}{s}\right) . s\, ds$$

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Integration of $\int_{s=0}^{a} \log\left(\frac{a}{s}\right) \cdot s\, ds$

$$= \left[\log\left(\frac{a}{s}\right) \cdot \int_0^a s\, ds\right]_0^a - \int_{s=0}^{a} \left[\frac{d}{ds}\left[\log\left(\frac{s}{a}\right)\right] \cdot \int s\, ds\right] ds$$

$$= \left[\log\left(\frac{a}{s}\right) \frac{s^2}{2}\right]_0^a - \int_{s=0}^{a} \left(\frac{s}{a}\right) \cdot \frac{s^2}{2} ds$$

$$= \left[\log\left(\frac{a}{a}\right) \cdot \frac{a^2}{2} - 0\right] - \frac{1}{2a} \int_{s=0}^{a} s^3 ds$$

$$= 0 - \frac{1}{2a} \cdot \left[\frac{s^4}{4}\right]_0^a = -\frac{a^3}{8} \qquad [\because \log_e 1 = 0]$$

$$\therefore\ I_d = -\frac{a^3}{8} \cdot \left(\frac{2\pi}{\lambda}\right)^2 . I_o \sin 2\pi\nu t\, \hat{k} = -\frac{a}{2} \cdot \frac{a^2}{4}\left(\frac{2\pi}{\lambda}\right)^2 I_o \sin(2\pi\nu t)\, \hat{k}$$

The negative sign shows that the displacement current I_d is opposite to the conduction current I_C.

$$\therefore \qquad I_d = \frac{a}{2}\left(\frac{2\pi a}{2\lambda}\right)^2 I_o \sin(2\pi\nu t)\,(-\hat{k})$$

$$\boxed{I_d = \frac{a}{2}\left(\frac{\pi a}{\lambda}\right)^2 I_o \sin(2\pi\nu t)\,(-\hat{k})}$$

I_d is in –Z direction as I_C is in +Z direction.

(*iii*) $I_d = \frac{a}{2}\left(\frac{\pi a}{\lambda}\right)^2 I_o \sin(2\pi\nu t)(-\hat{k})$

$I_d = I_o^d \sin(2\pi\nu t)$

where $I_o^d = \frac{a}{2}\left(\frac{\pi a}{\lambda}\right)^2 I_o$

$$\text{Required ratio } \frac{I_o^d}{I_o} = \frac{\frac{a}{2}\left(\frac{\pi a}{\lambda}\right)^2 \cdot I_o}{I_o} = \frac{a}{2}\left(\frac{\pi a}{\lambda}\right)^2$$

$$\boxed{\frac{I_o^d}{I_o} = \frac{\pi^2 a^3}{2\lambda^2}}$$

Q8.31. A plane EM wave travelling in vacuum along Z-direction is given by $\vec{E} = E_o \sin(kz - \omega t)\hat{i}$ and $\vec{B} = B_o \sin(kz - \omega t)\hat{j}$.

(*i*) Evaluate $\oint \vec{E} . \vec{dl}$ over the rectangular loop 1234 shown in figure.

(*ii*) Evaluate $\int \vec{B} . \vec{ds}$ over the surface bounded by loop 1234.

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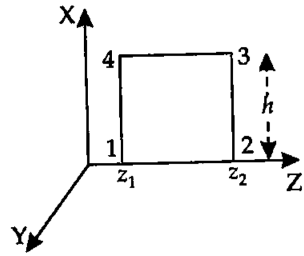
(*iii*) Use equation $\oint E.dl = \frac{-d\phi_B}{dt}$ to prove $\frac{E_o}{B_o} = C$.

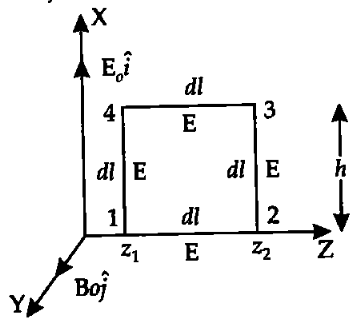
(*iv*) By using similar process and the equation $\oint B.dl = \mu_o I + \varepsilon_o \frac{d\phi_E}{dt}$.

Prove that $C = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$.



Ans. As the electromagnetic wave is propagating along Z-axis then its electric and magnetic field vectors are along X and Y axis.

(*i*) $\vec{E} = E_o\hat{i}$ and $\vec{B} = B_o\hat{j}$ as in figure below.



Line integral of E over loop 1234,

$$\oint \vec{E}.\vec{dl} = \int_1^2 \vec{E}.\vec{dl} + \int_2^3 \vec{E}.\vec{dl} + \int_3^4 \vec{E}.\vec{dl} + \int_4^1 \vec{E}.\vec{dl}$$

$$= \int_1^2 E.dl \cos 90° + \int_2^3 E.dl \cos 0° + \int_3^4 E.dl \cos 90° + \int_4^1 E.dl \cos 180°$$

$$\oint \vec{E}.\vec{dl} = \int_1^2 0 + \int_2^3 E.dl + \int_3^4 0 + \int_4^1 E dl(-1)$$

$$= 0 + [E.h]_{z_2} - [E.h]_{z_1} = E_o h \sin[kz_2 - \omega t) - E_o h \sin(kz_1 - \omega t)$$

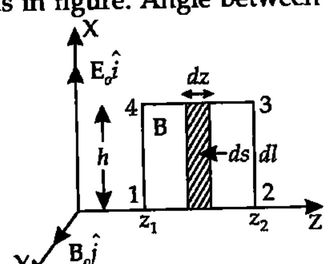
$$= hE_o[\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)]$$

(*ii*) Consider a strip of area $ds = h.dz$ as in figure. Angle between $\vec{ds}$ and $\vec{B}$ is zero.

$$\therefore \int \vec{B}.\vec{ds} = \int \vec{B}.\vec{ds} \cos 0 (\hat{j})$$

$$= \int_{z_1}^{z_2} B.ds.\hat{j}$$

$$= \int_{z_1}^{z_2} B_o \sin(kz - \omega t)\, h.dz$$



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$$= \frac{-B_o h}{k}\left[\cos(kz - \omega t)\right]_{z_1}^{z_2}$$

$$= \frac{-B_o h}{k}\left[\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)\right]$$

$$\int \vec{B}.\vec{ds} = \frac{B_o h}{k}\left[\cos(kz_1 - \omega t) - \cos(kz_2 - \omega t)\right]$$

(*iii*) Given $\oint \vec{E}.\vec{dl} = \frac{-d\phi_B}{dt} = -\frac{d}{dt}\oint B.ds$ $\quad [\because B = B_o \sin(kz - \omega t)j]$

$$= \frac{d}{dt}\left[\frac{B_o h}{k}\{\cos(kz_1 - \omega t) - \cos(kz_2 - \omega t)\}\right]\hat{j}$$

$$= \frac{B_o h}{k}[-\sin(kz_1 - \omega t)(-\omega) + \sin(kz_2 - \omega t)(-\omega)]$$

$$= \frac{B_o h}{k}[\omega \sin(kz_1 - \omega t) - \omega \sin(kz_2 - \omega t)]$$

$$E = \frac{B_o h\omega}{k}[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)]$$

$$E = E_0[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)]$$

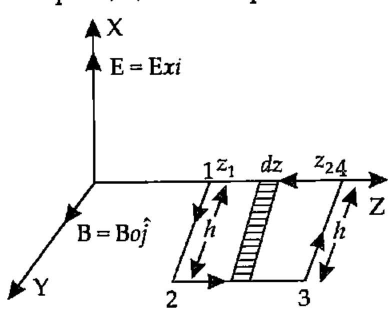
$$E_0 = \frac{B_o \omega}{k} \qquad \left[\because \frac{\omega}{k} = C\right]$$

$$\therefore \quad E_0 = B_Y C$$

$$\frac{E_0}{B_0} = \frac{E_0}{B_Y} = \frac{B_Y C}{B_Y} \qquad [\because B = B_0\hat{j} \Rightarrow B = B_Y]$$

$$\boxed{\frac{E_0}{B_0} = C}$$

(*iv*) Consider a loop 1.2, 3, 4 in Y-Z plane as in figure.



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$$\oint \vec{B}.\vec{dl} = \int_1^2 \vec{B}.\vec{dl} + \int_2^3 \vec{B}.\vec{dl} + \int_3^4 \vec{B}.\vec{dl} + \int_4^1 \vec{B}.\vec{dl} \quad ...I$$

$$= \int_1^2 \vec{B}.\vec{dl}\cos 0° + \int_2^3 \vec{B}.\vec{dl}\cos 90° + \int_3^4 \vec{B}.\vec{dl}\cos 90° + \int_4^1 \vec{B}.\vec{dl}\cos 90° \quad ...II$$

$$\oint \vec{B}.\vec{dl} = \int_1^2 B.dl + \int_4^1 \vec{B}.(-dl) = \int_1^2 B.dl - \int_4^1 B.dl$$

$$= [B\,h]_{z_1} - [B\,h]_{Z_2} \qquad [\because dl = h \text{ in figure}]$$

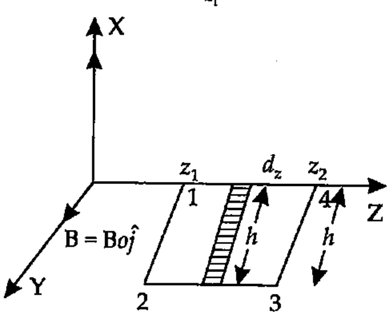
$$\oint \vec{B}.\vec{dl} = [B_0 h \sin(kz - \omega t)]_{z_1} - [B_0 h \sin(kz - \omega t)]_{z_2}$$

$$\oint B.dl = B_0 h[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)] \quad ...III$$

Now to calculate $\phi_E = \int E.ds$. Let us consider the rectangular strip of loop 1, 2, 3, 4 of area ds each $ds = hdz$.

$$\phi_E = \int \vec{E}.\vec{ds} = \int E.ds \cos 0° = \int E.ds = E_0 \int_{z_1}^{z_2} \sin(kz - \omega t).h\,dz$$

$$\phi_E = E_0 h\left[\frac{-\cos(kz - \omega t)}{k}\right]_{z_1}^{z_2}$$



$$\phi_E = \frac{E_0 h}{k}[\cos(kz_1 - \omega t) - \cos(kz_2 - \omega t)]$$

$$\frac{d\phi_E}{dt} = \frac{-E_0 h\omega}{k}[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)] \quad ...IV$$

By Ampere's circuital law

$$\int B.dl = \mu_0\left[I_C + \varepsilon_0 \frac{d\phi_E}{dt}\right] \qquad [I_C = \text{conduction current}]$$

$I_C = 0$ in vacuum

$$\therefore \qquad \int B.dl = \mu_0\,\varepsilon_0 \frac{d\phi}{dt}$$

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Using relations obtained in eqn. (III) and (IV)
$B_0 h$ [sin ($kz_1 - \omega t$)-sin ($kz_2 - \omega t$)]

$$= \mu_0 \varepsilon_0 \frac{E_0 h\omega}{k} [\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)]$$

$$B_0 h = \frac{\mu_0 \varepsilon_0 h\omega E_0}{k}$$

$$B_0 = E_0 \frac{\omega \mu_0 \varepsilon_0}{k}$$

$$\frac{E_0}{B_0} \frac{\omega}{k} = \frac{1}{\mu_0 \varepsilon_0}$$

$$\frac{C\omega}{k} = \frac{1}{\mu_0 \varepsilon_0} \qquad \left[\because \frac{E_0}{B_0} = C\right]$$

$$\frac{CCk}{k} = \frac{1}{\mu_0 \varepsilon_0} \qquad [\omega = Ck]$$

$$C^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$ Hence proved.

Q8.32. A plane EM wave travelling along Z-direction is described by $\vec{E} = E_0 \sin(kz - \omega t)\hat{i}$ and $\vec{B} = B_0 \sin(kz - \omega t)\hat{j}$. Show that

(*i*) The average energy density of the wave is given by

$$U_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2$$

(*ii*) The time averaged intensity of the wave is given by

$$I_{av} = \frac{1}{2} C \varepsilon_0 E_0^2$$

Ans. (*i*) The electromagnetic wave carry energy which is due to electric field vector and magnetic field vector. In electromagnetic wave, E and B varies with time.

The energy density due to electric field $\vec{E}$ is $U_E = \frac{1}{2}\varepsilon_o E^2$

The energy density due to magnetic field B is $U_B = \frac{1}{2\mu_0} B^2$

Total average energy density of electromagnetic wave

$$U = U_E + U_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

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$$E = E_0 \sin(kz - \omega t)\,\hat{i}$$
$$B = B_0 \sin(kz - \omega t)\,\hat{j}$$

average value of E^2 over a cycle $= \frac{E_0^2}{2}$

The average value of B^2 over a cycle $= \frac{B_0^2}{2}$

$$\therefore \quad U_{av} = \frac{1}{2}\varepsilon_0 \frac{1}{2}E_0^2 + \frac{1}{2\mu_0}\frac{(B_0^2)}{2}$$

$$U_{av} = \frac{1}{4}\left[\varepsilon_0 E_o^2 + \frac{B_0^2}{\mu_o}\right]$$

(*ii*) We know that $E_0 = CB_0$ and $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{4}\frac{E_0^2/C^2}{\mu_0} = \frac{E_0^2}{4\mu}\mu_0\varepsilon_0 = \frac{1}{4}\varepsilon_0 E_0^2$$

$$U_B = U_E$$

$$U_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0}$$

$$\because \quad \frac{E_0}{B_0} = C \quad \Rightarrow \quad \frac{E_0^2}{B_0^2} = C^2$$

$$\Rightarrow \quad \frac{E_0^2}{B_0^2} = \frac{1}{\mu_0\varepsilon_0} \quad \Rightarrow \quad B_0^2 = \mu_0\,\varepsilon_0\,E_0^2$$

$$\therefore \quad U_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{\mu_0\,\varepsilon_0\,E_0^2}{4\mu_0} = \frac{1}{4}\varepsilon_0 E_o^2 + \frac{1}{4}\varepsilon_0 E_0^2$$

$$U_{av} = \frac{1}{2}\varepsilon_0 E_0^2$$

$$U_{av} = \frac{1}{4}\varepsilon_0 \cdot \frac{B_0^2}{\mu_0\varepsilon_0} + \frac{B_0^2}{4\mu_0} \qquad \left[\because\ E_0^2 = \frac{B_0}{\mu_0\,\epsilon_0}\right]$$

$$= \frac{B_0^2}{4\mu_0} + \frac{B_0^2}{4\mu_0}$$

$$U_{av} = \frac{B_0^2}{2\mu_0}$$

$$\therefore \quad U_E = U_B$$

Time average intensity of wave

$$I_{av} = U_{av}C = \frac{1}{2}\frac{B_0^2 C}{\mu_o} = \frac{1}{2}\varepsilon_0 E_o^2 C$$

❑❑❑

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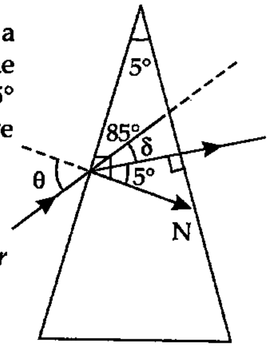
9 Ray Optics and Optical Instruments

MULTIPLE CHOICE QUESTIONS—I

Q9.1. A ray of light incident at an angle θ on a refracting surface of a prism emerges from the other face normally. If the angle of prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is



(*a*) 7.5°. (*b*) 5°.
(*c*) 15°. (*d*) 2.5°.

Main concept used: $\delta = (\mu - 1)A$ and $\delta = i - r$

Ans. (*a*): $\delta = (1.5 - 1)5° = 0.5 \times 5 = 2.5°$

$$\theta - r = \delta, \theta = \delta + r$$

$$\theta = 2.5 + 5 = 7.5°$$

Q9.2. A short pulse of white light is incident from air to glass slab at normal incidence. After travelling through the slab, the first colour to emerge is

(*a*) blue. (*b*) green. (*c*) violet. (*d*) red.

Main concept used: $c = \nu\lambda$, ν does not change during refraction.

Ans. (*d*): $\because$ $c = \nu\lambda$ and ν is constant during refraction so $c \propto \lambda$. The velocity of red colour is maximum in glass as the $\lambda_R > \lambda_V$.

Q9.3. An object approaches a convergent lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image

(*a*) moves away from the lens with uniform speed 5 m/s.
(*b*) moves away from the lens with uniform acceleration.
(*c*) moves away from the lens with non-uniform acceleration.
(*d*) moves towards the lens with a non uniform acceleration.

Main concept used: When $u = \infty$, then $v = f$ and when $u = f$, then $v = \infty$

Ans. (*c*): When an object approaches towards a lens with uniform speed, its image moves away from the lens to infinity with non uniform acceleration.

Q9.4. A passenger in an aeroplane shall

(*a*) never see a rainbow.
(*b*) may see a primary and a secondary rainbow as concentric circles.
(*c*) may see a primary and a secondary rainbow as concentric arcs.
(*d*) shall never see a secondary rainbow.

Ans. (*b*): A passenger in an aeroplane may see primary and secondary rainbow as concentric circles.

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Q9.5. You are given four sources of light each one providing a light of a single colour, red, blue, green and yellow. Suppose the angle of refraction far a beam of yellow light corresponding to a particular angle of incidence at interface of two media is 90°. Which of the following statements is correct if source of yellow light is replaced with that of other lights without changing the angle of incidence?

(*a*) The beam of red light would undergo total internal reflection.

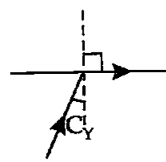
(*b*) The beam of red light would bend towards normal while it gets refracted through the second medium.

(*c*) The beam of blue light would undergo total internal reflection.

(*d*) The beam of green light would bend away from the normal as it gets refracted through the second medium.

Main concept used: Critical angle for yellow is greater than green and blue and smaller than red.

Ans. (*c*): We know that if angle of refraction is 90° for the length then incidence angle is called critical angle. So light rays are passing from denser to rarer medium.



As $\sin c = \frac{1}{\mu}$ so, $c \propto \frac{1}{\mu}$ and $\mu_v > \mu_g > \mu_Y > \mu_R$

So, critical angle for $C_v < C_g < C_Y < C_R$, *i.e.*, critical angle of blue and green light is smaller than that of yellow and it is greater for red colour light.

As the angle of refraction for yellow light is 90° for a particular incident angle. This incidence angle is critical angle for yellow let it be C_Y. As $C_R > C_v$. So it will not get total internal reflection and $C_V < C_Y$, $C_g < C_Y$.

So light of blue and green colour get total internal reflection. So correct answer is (*c*).

Q9.6. The radius of curvature of curved surface of plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will

(*a*) act as a convex lens only for the objects that lie on its curved side.

(*b*) act as a concave lens for the objects that lie on its curved side.

(*c*) act as a convex lens irrespective of side on which the object lies.

(*d*) act as a concave lens irrespective of side on which the object lies.

Main concept used: Lens Maker's formula.

Ans. (*c*): If object lies on curved side then R_1 = +20 cm and $R_2 = \infty$, $\mu_1 = 1$, $\mu_2 = 1.5$

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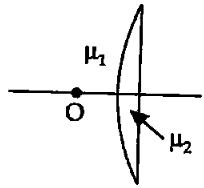
$$\frac{1}{f} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.5 - 1)\left[\frac{1}{20} - \frac{1}{\infty}\right] = \frac{0.5}{20} = \frac{5}{200} = \frac{1}{40}$$

$$f = +40 \text{ cm}$$



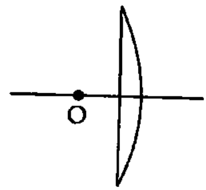
If object lies on plane side $R_1 = \infty$ and $R_2 = -20$ cm, $\mu_1 = 1, \mu_2 = 1.5$

$$\frac{1}{f} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.5 - 1)\left[\frac{1}{\infty} - \left(-\frac{1}{20}\right)\right] = 0.5 \times \left(+\frac{1}{20}\right)$$

$$\frac{1}{f} = \frac{5}{200}$$

$$f = +40 \text{ cm}$$



So, lens will always act as a convex lens irrespective of side on which objects lie. So, answer is (*c*).

Q9.7. The phenomena involved in the reflection of radiowaves by ionosphere is similar to

(*a*) reflection of light by plane mirror.
(*b*) total internal reflection of light in air during a mirage.
(*c*) dispersion of light by water molecules during the formation of rainbow.
(*d*) scattering of light by the particles of air.

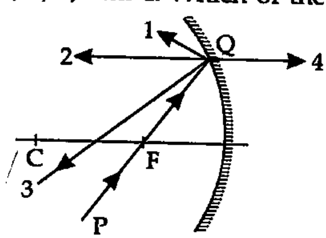
Main concept used: Refractive index of ionosphere is less than atmosphere for radiowaves. Although the refractive index of any material can never be less than one of vacuum.

Ans. (*b*): Ionosphere is transparent optical medium and radiowave is reflected back. Reflection through transparent surface is total internal reflection so that internal reflection of radiowave takes place.

Q9.8. The direction of light rays incidence on a concave mirror is shown by PQ, while the direction in which the ray would travel after reflection is shown by four rays marked 1, 2, 3, and 4. Which of the four rays correctly shows the direction of reflected ray?

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

Main concept used: Normal at incidence point in spherical mirrors passes through centre of curvature of lens.



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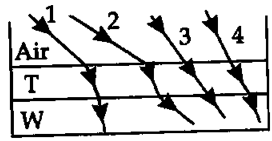
Ans. (*b*): Incidence ray PQ is coming through principal focus F so it must be parallel to principal axis, *i.e.* either 2 or 4.

As it is a concave mirror so, ray cannot go behind the mirror so ray (4) is discarded.

So ray 2 is the reflected ray. It is verifies answer (*b*)

We can verify it again by drawing normal Q_C and find that $\angle r = \angle i$. So ray (2) is the reflected ray.

Q9.9. The optical density of turpentine is higher than that of water while its mass density is lower than water. Figure below shows a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in figure, the path shown is correct?



(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

Main Concept: Laws of refraction.

Ans. (*b*): $\mu_a < \mu_T > \mu_W$. Here, incidence ray passes from air to turpentine to water, *i.e*, from rare to denser then denser to rarer so first it bends towards normal then away from normal so the path shown is correct for ray (2).

Q9.10. A car is moving with constant speed of 60 km/hr on a straight road. Looking at the rear view mirror, the driver finds that car following him is at a distance of 100 m and is approaching with a speed of 5 km/h.

In order to keep track of the car in the rear, the driver begins to glance alternatively at the rear and side mirror of his car after every 2 s till the other car overtakes. If the two cars were maintaining their speeds, which of the following statements is /are correct?

(*a*) The speed of the car in the rear is 65 km/h.

(*b*) In the side mirror, the car, in the rear would appear to approach with a speed of 5 km h^{-1} to the driver of the leading car.

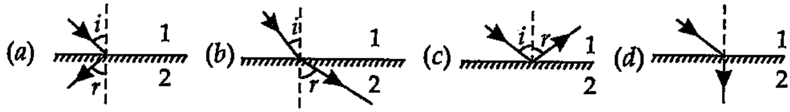
(*c*) In the rear view mirror, the speed of the approaching car would appear to decrease as the distance between the cars decreases.

(*d*) In the side mirror, the speed of the approaching car would appear to increase as the distance between the cars decreases.

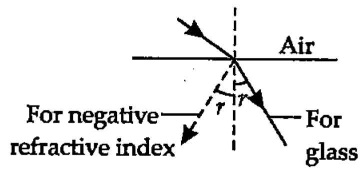
Main concept used: If object is at infinity, image in convex mirror is at focus so speed is zero when car is at infinity i.e., very far from centre of curvature of mirror.

Ans. (*d*): So when rear car approaches, initially it appear at rest as image is formed at focus. When car approaches nearer this speed will appear to increase so answer is (*d*).

Q9.11. There are certain materials developed in laboratories which have a negative refractive index. In figure below, a ray incidents from air (medium 1) into such a medium (medium 2) shall follow a path given by:



Ans. (*a*): The negative refractive index materials are those in which incident ray from air (medium 1) to them refract or bends differently or opposite and symmetric to normal to that of positive refractive index medium. So answer is (*a*).



MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q9.12. Consider an extended object immersed in water contained in a plane trough. When seen from close to the edge of the trough the object looks distorted because

(*a*) the apparent depth of the points close to the edge are nearer the surface of the water compared to the points away from the edge.

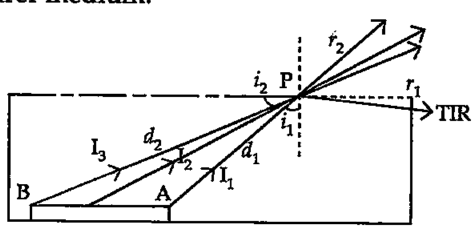
(*b*) the angle subtended by the image of the object at the eye is smaller than the actual angle subtended by the object in air.

(*c*) some of the points of the object far away from the edge may not be visible because of total internal reflection.

(*d*) water in a trough acts as a lens and magnifies the object.

Main concept used: Refraction and total internal reflection when light passes from denser to rarer medium.

Ans. (*a*), (*b*) and (*c*): We know that shifting (h) of image of an object immersed in liquid from object is directly proportional to the real distance of object from the surface of liquid



$$h = t\left(1 - \frac{1}{\mu}\right)$$

t = real depth or distance of the object from the surface of liquid of refractive index μ: If the object is seen from one edge of trough the relative differences of depth (distance) in H_2O between two ends of objects is larger than if it is seen from the top or away from edge. By above formula or distortion of nearer end is smaller than farther, verifies the option (*a*).

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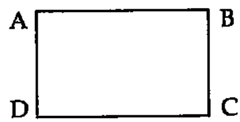
The angle subtended by an object is larger than its image in water, as its image shifts upward verifies option (*b*).

Rays coming out from object to observer passes from denser to rarer medium, and angle of incidence for rays from farther end B of object is larger than near end A. The incidence angles for rays coming from end 3 may have incidence angle more than critical angle and can cause total internal reflection, so will not reach to observer, cause nonvisible of farther end verifies option (*c*).

Q9.13. A rectangular block of glass ABCD has a refractive index 1.6. A pin is placed midway on the face AB (Fig). When observed from the face AD, the pin shall–

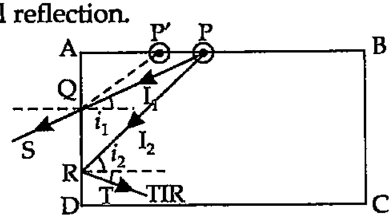


(*a*) appear to be nearer to A.
(*b*) appear to be nearer to D.
(*c*) appear to be at the centre of AD.
(*d*) not be seen at all.

Main concept used: Total internal reflection.

Ans. (*a*) (*b*): $\because \sin C = \frac{1}{\mu}$

$\therefore \sin C = \frac{1}{1.6} \Rightarrow C = 38.7°$



A point P is on the mid point of face AB. When seen through face AD, near to point A, the angle of incidence (i_1) will be smaller than critical angle $i_c = 38.7°$. So image of P will form at P' and image can be seen at P'. P' is nearer to both A and D as compared to P verifies option (*a*) and (*b*).

When seen near point D through face AD, angle of incidence $i_2 > i_c$ so total internal reflection takes place and object cannot be observed.

But object can be seen when viewed near to A so option (*d*) not verified.

Q9.14. Between the primary and secondary rainbows, there is a dark band, known as Alexandar's dark band. This is because

(*a*) light scattered into this region interfere destructively.
(*b*) there is no light scattered into this region.
(*c*) light is absorbed in this region.
(*d*) angle made at the eye by the scattered rays with respect to incident light of the sun, lies between approximately 42° and 50°.

Main concept used: Scattering and dispersion of light.

Ans. (*d*): Alexandar's dark band lies between the primary and secondary rainbow. This forms due to the light scattered into this region interfere destructively. Because the primary and secondary rainbows subtend angles (41° to 42°) and (51° to 54°) respectively at the observer's eye with respect to incident light ray, so the scattered rays with respect to the incident ray of the sun lies between approximately 42° to 50°.

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Q9.15. A magnifying glass is used, as the object to be viewed can be brought closer to the eye than the normal near point. This results in

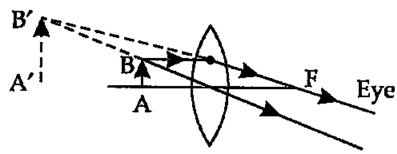
(*a*) a larger angle to be subtended by the object at the eye and hence viewed in greater detail.

(*b*) the formation of virtual erect image.

(*c*) increase in the field of view.

(*d*) Infinite magnification at the near point.

Main concept used: An object is seen more clearly if subtends larger angle to the eye. Very far object subtends very small angle to eye.



Ans. (*a*) and (*b*): In magnifying glass, the object is placed within the focal length and the image formed is magnified and erect. As (A′B′) image is magnified so it subtends larger angle at the eye than object (AB), so can be seen more clearly.

Q9.16. An astronomical refractive telescope has an objective of focal length 20 m and an eye piece of focal length 2 cm.

(*a*) The length of the telescope tube is 20.02 m.

(*b*) The magnification is 1000.

(*c*) The image formed is inverted.

(*d*) An objective of larger aperture will increase the brightness and reduce the chromatic aberration of the image.

Main concept used: (*i*) $L = fo + fe$ (*ii*) $m = \frac{fo}{fe}$ and (*iii*) ray diagram of telescope.

Ans. (*a*), (*b*) and (*c*): The length of the telescope

$$L = fo + fe = 20 + 00.02 = 20.02 \text{ m}$$

and

$$m = \frac{fo}{fe} = \frac{20 \text{ m}}{0.02 \text{ m}} = \frac{2000}{2} = 1000$$

The final image formed in telescope (Refracting) is inverted, virtual and smaller than object.

VERY SHORT ANSWER TYPE QUESTIONS

Q9.17. Will focal length of a lens for a red light be more, same or less than that for blue light?

Main concept used: $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Ans. As we know that ($\mu_V > \mu_R$), so $\frac{1}{f}$ will be large for blue light and smaller for Red coloured light. So f will be larger for red coloured light.

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Q9.18. The near vision of an average person is 25 cm. To view an object with an angular magnification of 10, what should be the power of the microscope?

Main concept used: Magnification of microscope at distance of distinct vision.

Ans. To see the final image at distinct vision D = 25 cm
for eye lens $u = -f$ and $v = -25$

$$m = \frac{+v}{u}$$

$$10 = \frac{-25}{-f}$$

$$f = +\frac{25}{10} = +2.5 \text{ cm} = 0.025 \text{ m}$$

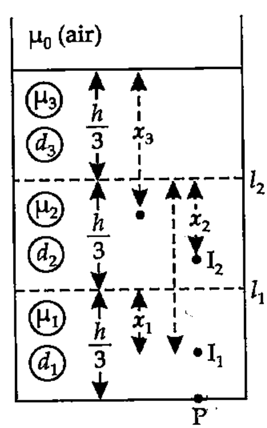
$$\therefore \quad P = \frac{1}{f} = \frac{1}{0.025} = \frac{1000}{25} = \mathbf{40\ D}$$

Q9.19. An unsymmetrical double convex, thin lens forms the image of a point object on its axis. Will the position of the image change if the lens is reversed?

Main concept used: Lens maker's formula

Ans. Position of image in convex lens changes either by changing u of f. Here, u is constant and $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. Here, μ and curvature of the lens are same so f is constant. So on reversing the lens, position and nature of the image **will not change.**

Q9.20. Three immisible liquids of densities $d_1 > d_2 > d_3$ and refractive indices $\mu_1 > \mu_2 > \mu_3$ are put in a beaker. The height of each liquid column is $\frac{h}{3}$. A dot is made at the bottom of the beaker. For near normal vision, find the apparent depth of the dot.



Main concept used: (*i*) $\mu = \frac{\text{real depth}}{\text{apparent depth}}$,

(*ii*) Image formed by one medium acts as an object for second medium.

Ans. The liquids are immisible, so liquids arranged from bottom to top d_1, d_2 and d_3 with their refractive indices μ_1, μ_2, μ_3 respectively separated by layers l_1, l_2 as shown in figure.

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Consider the layer l_1. Let object is at its bottom at P then distance (x_1) of image of P by liquid of refractive index μ_1 from layer l_1

$$x_1 = \frac{h/3}{{}_2\mu_1} \qquad \because \quad {}_2\mu_1 = \frac{\mu_1}{\mu_2}$$

$$\therefore \qquad x_1 = \frac{h}{3\frac{\mu_1}{\mu_2}} = \frac{h}{3}\frac{\mu_2}{\mu_1}$$

This image I_1 acts as object for liquid of μ_2 and form the image at I_2. The distance x_2 below level l_2

$$\text{Real depth} = \left(\frac{h}{2} + x_1\right)$$

$$x_2 = \frac{\text{Real depth}}{{}_3\mu_2} = \frac{\text{Real depth}}{\mu_2/\mu_3}$$

$$x_2 = \frac{\mu_3}{\mu_2}\left[\frac{h}{3} + x_1\right] = \frac{\mu_3}{\mu_2}\left[\frac{h}{3} + \frac{h}{3}\frac{\mu_2}{\mu_1}\right] = \frac{h}{3}\frac{\mu_3}{\mu_2}\left[1 + \frac{\mu_2}{\mu_1}\right]$$

$$= \frac{h}{3}\left[\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_2}\frac{\mu_2}{\mu_1}\right] = \frac{h}{3}\left[\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right]$$

as the point P is seen from outside

$$x_3 = \frac{\mu_0}{\mu_3}\left[\frac{h}{3} + x_2\right] = \frac{1}{\mu_3}\left[\frac{h}{3} + \frac{h}{3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right)\right] \qquad [\because \mu_0 = 1 \text{ (refractive index of air)}]$$

$$= \frac{h}{3}\left[\frac{1}{\mu_3} + \frac{1}{\mu_3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right)\right]$$

$$x_3 = \frac{h}{3}\left[\frac{1}{\mu_3} + \frac{\mu_3}{\mu_3}\left(\frac{1}{\mu_2} + \frac{1}{\mu_1}\right)\right] = \frac{h}{3}\left[\frac{1}{\mu_3} + \frac{1}{\mu_2} + \frac{1}{\mu_1}\right]$$

x_3 is apparent depth.

Q9.21. For a glass prism $(\mu = \sqrt{3})$ the angle of minimum deviation is equal to the angle of the prism. Find the angle of the prism.

Ans. The required relation for minimum angle of deviation

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}} \qquad [\because \delta_m = A \text{ (given)}]$$

$$\therefore \qquad \mu = \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}$$

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$$\sqrt{3} = 2\cos\frac{A}{2} \Rightarrow \cos\frac{A}{2} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{A}{2} = \cos 30° \Rightarrow \frac{A}{2} = 30°$$

A = 60° is angle of prism.

SHORT ANSWER TYPE QUESTIONS

Q9.22. A short object of length L is placed along the principal axis of a concave mirror away from focus. The object distance is u. If the mirror has a focal length f, what will be the length of the image? You may take $L << |v - f|$

Main concept used: The length of image is the difference between the image distance of extremities.

Ans. As the mean distance of object from mirror is u

$$\therefore \qquad u_1 = u - \frac{L}{2} \quad \text{and} \quad u_2 = \left(u + \frac{L}{2}\right)$$

Let the image of the two ends of object form at distance v_1 and v_2 $(v_1 > v_2)$. So length of image on principal axis is $L' = (v_1 - v_2)$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{u-f}{uf} \Rightarrow v = \frac{uf}{u-f}$$

$$\text{So} \qquad L' = v_1 - v_2 = \frac{\left(u - \frac{L}{2}\right)f}{\left(u - \frac{L}{2}\right) - f} - \frac{\left(u + \frac{L}{2}\right)f}{\left(u + \frac{L}{2}\right) - f}$$

$$\Rightarrow \qquad L' = f\left[\frac{u - \frac{L}{2}}{\left(u - f - \frac{L}{2}\right)} - \frac{u + \frac{L}{2}}{\left(u - f + \frac{L}{2}\right)}\right]$$

$$\Rightarrow \qquad L' = f\left[\frac{\left(u - \frac{L}{2}\right)\left(u - f + \frac{L}{2}\right) - \left(u + \frac{L}{2}\right)\left(u - f - \frac{L}{2}\right)}{\left(u - f - \frac{L}{2}\right)\left(u - f + \frac{L}{2}\right)}\right]$$

$$= \frac{f\left[u^2 - uf + \frac{uL}{2} - \frac{uL}{2} + \frac{fL}{2} - \frac{L^2}{4} - \left(u^2 - uf - \frac{uL}{2} + \frac{uL}{2} - \frac{fL}{2} - \frac{L^2}{4}\right)\right]}{(u-f)^2 - \frac{L^2}{4}}$$

$$\therefore \quad L << (u - f) \quad \therefore \quad \frac{L^2}{4} <<< (u - f)$$

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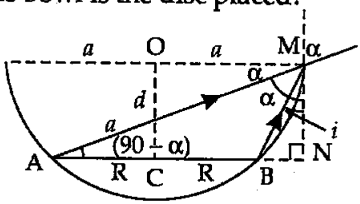
So neglecting the terms $\frac{L^2}{4}$

$$L^2 = f\left[\frac{\frac{fL}{2}+\frac{fL}{2}}{(u-f)^2}\right]$$

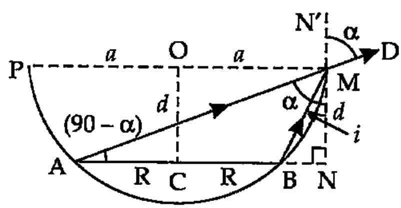
$$L' = \frac{f\,fL}{(u-f)^2} \Rightarrow L' = \frac{Lf^2}{(u-f)^2}$$

It is the length of image f.

Q9.23. A circular disc of radius R is placed co-axially and horizontally inside an opaque hemispherical bowl of radius 'a' as in figure. The far edge of the disc is just visible when viewed from the edge of the bowl. The bowl is filled with transparent liquid of refractive index μ, and the near edge of the disc becomes just visible. How far below the top of the bowl is the disc placed?



(*a*) Bowl filled with air



(*b*) Bowl filled with liquid

Main concept used: Snell's law and Geometry.

Ans. In figure AM and BM are the rays from the ends of disc AB reaching at one end of bowl at M. MN is tangent at M, so MN ⊥ AB *i.e.*, ∠N = 90°

Taking incidence ray BM and refracted ray MD

$$BN = CN - CB = OM - CB = a - R$$

$$MB = \sqrt{d^2 + (a - R^2)}$$

∴ $$\sin i = \frac{BN}{BM} = \frac{(a-R)}{\sqrt{d^2+(a-R)^2}}$$

$$\angle r = \angle \alpha = \angle AMN$$

$$\sin r = \cos(90° - \alpha) = \frac{AN}{AM} = \frac{a+R}{\sqrt{d^2+(a+R)^2}}$$

For incidence ray BM to the horizontal level of liquid MP, MN will be normal at M. ∠i and ∠r will be incidence and refracted angles when ray BM passes from liquid (μ) to air. By Snell's law, as ray passes from liquid to air

$${}_1\mu_0 = \frac{\sin i}{\sin r} \Rightarrow \frac{\mu_0}{\mu_1} = \frac{\sin i}{\sin r} \qquad [\mu_0 \text{ for air} = 1] \quad [\mu_1 = \mu \text{ for liquid}]$$

$$\frac{1}{\mu} = \frac{\sin i}{\sin r}$$

$$\frac{1}{\mu} = \frac{\dfrac{a-R}{\sqrt{d^2+(a-R)^2}}}{\dfrac{(a+R)}{\sqrt{d^2+(a+R)^2}}} = \frac{(a-R)\sqrt{d^2+(a+R)^2}}{(a+R)\sqrt{d^2+(a-R)^2}}$$

$$d = \frac{\mu(a^2-d^2)}{\sqrt{(a+r)^2-\mu(a-r)^2}}$$

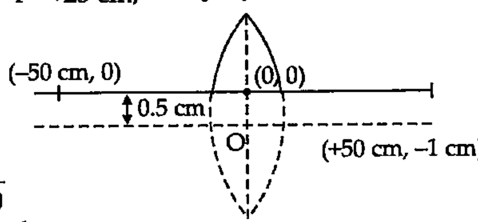
It is required expression.

Q9.24. A thin convex lens of focal length 25 cm is cut into two pieces 0.5 cm above the principal axis. The top part is placed at (0, 0) and an object is placed at (– 50 cm, 0). Find the co-ordinates of the image.

Main concept used: There is no effect on focal length if a lens is cut by plane parallel to principal axis.

Ans. Object is placed 0.5 cm above principal axis

$u = -50$ cm f = +25 cm, $v = ?$



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-50} = \frac{1}{25}$$

$$\frac{1}{v} = \frac{1}{25} - \frac{1}{50}$$

$$= \frac{2-1}{50} = \frac{1}{50}$$

$$v = 50 \text{ cm}$$

$$m = \frac{+v}{u} = \frac{+(50)}{-50} = -1$$

So the size of image is equal to that of object, m is negative so image is inverted.

So image is at (50 cm, –1 cm) and 0.5 cm below the X – X′ axix.

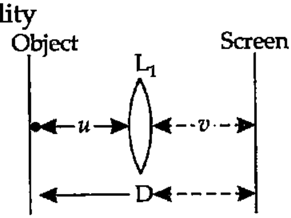
Q9.25. In many experimental set ups, the source and the screen are fixed at a distance say D and the lens is movable. Show that there are two positions for the lens for which an image is formed on the screen. Find the distance between these points and the ratio of image sizes for these two points.

Main concept used: Principle of reversibility

Ans.

$$u = -(D-v)$$

$$v = v$$



$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{v} + \frac{1}{(D-v)}$$

$$= \frac{D-v+v}{v(D-v)} = \frac{D}{vD-v^2}$$

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$$fD = vD - v^2$$

$$v^2 - vD + fD = 0$$

$$v = \frac{+D \pm \sqrt{D^2 - 4Df}}{2} = \frac{D}{2} \pm \frac{\sqrt{D^2 - 4Df}}{2} \qquad \text{...I}$$

$$u = -(D - v) = -\left[D - \left\{\frac{D}{2} \pm \frac{\sqrt{D^2 - 4Df}}{2}\right\}\right]$$

$$u = -\left[\frac{D}{2} \mp \frac{\sqrt{D^2 - 4Df}}{2}\right] \qquad \text{...II}$$

From II, when the position of object $u_2 = \frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2}$

[infront of l_2 (lower sign)]

Then from I, the position of image $v_2 = \frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}$ [lower sign]

Similarly when position of object $u_1 = \frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}$

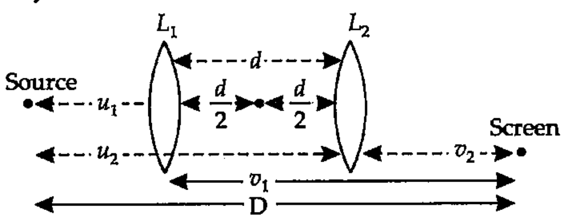
[from II upper sign]

then the position of image $v_1 = \frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2}$ [from I upper sign]

The distance between two positions of lens $d = v_1 - v_2$

$$d = \frac{D}{2} + \frac{\sqrt{D^2 - 4Df}}{2} - \left[\frac{D}{2} - \frac{\sqrt{D^2 - 4Df}}{2}\right]$$

$d = \sqrt{D^2 - 4Df}$ is the distance between two positions of lenses.



In first case of L_1

$$u_1 = \frac{D}{2} - \frac{d}{2}$$

$$v_1 = \frac{D}{2} + \frac{d}{2}$$

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$$\therefore \qquad m_1 = \frac{v_1}{u_1} = \frac{\frac{D}{2}+\frac{d}{2}}{\frac{D}{2}-\frac{d}{2}} = \frac{D+d}{D-d}$$

In second case

$$u_2 = \frac{D}{2}+\frac{d}{2}$$

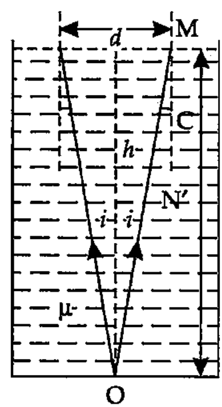
$$v_2 = \frac{D}{2}-\frac{d}{2}$$

$$\therefore \qquad m_2 = \frac{v_2}{u_2} = \frac{\frac{D}{2}-\frac{d}{2}}{\frac{D}{2}+\frac{d}{2}} = \frac{D-d}{D+d}$$

$$\frac{m_2}{m_1} = \frac{\frac{D-d}{D+d}}{\frac{D+d}{D-d}} = \left(\frac{D-d}{D+d}\right)^2$$

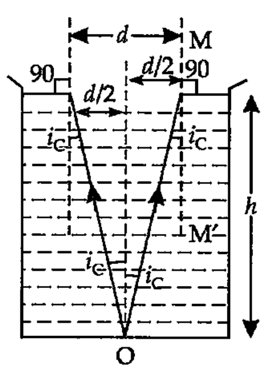
$\boxed{\frac{m_2}{m_1} = \left(\frac{D-d}{D+d}\right)^2}$ is the required ratio of size of images in two cases.

Q9.26. A jar of height h is filled with a transparent liquid of refractive index μ (figure). At the centre of the jar on the bottom surface is a dot. Find the minimum diameter of a disc, such that when placed on the top surface symmetrically about the centre, the dot is invisible.



Main concept used: Total internal reflection

Ans. The point O will be invisible if the light ray coming from O does not come out or it get graze the surface of liquid or gets total internal reflection as in figure here.



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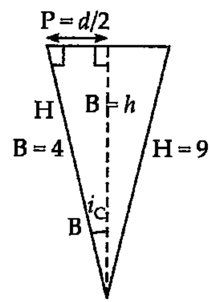
Ray OA is incident at A with critical angle i_c and it's angle of refraction will be 90°.

For other ray if incident angle is more than i_c it will get total internal reflection from the surface so will not come out from liquid.

So disc of diameter d is required to stop the rays from 'O' out of liquid.

$$\tan i_c = \frac{d/2}{h} \quad \text{or} \quad \tan i_c = \frac{d}{2h} \quad \Rightarrow \quad d = 2h \tan i_c$$

$$\sin i_c = \frac{1}{\mu} = \frac{P}{H} \quad \text{[From figure]}$$



$$B^2 = H^2 - P^2$$

$$B = \sqrt{\mu^2 - 1}$$

$$\therefore \quad \tan i = \frac{P}{B} = \frac{1}{\sqrt{\mu^2 - 1}}$$

$$\therefore \quad d = 2h \times \frac{1}{\sqrt{\mu^2 - 1}}$$

Q9.27. A myopic adult has a far point at 0.1 m. His power of accommodation is 4D.

(*i*) What power of lens required to see the distant objects?

(*ii*) What is his near point without glasses?

(*iii*) What is his near point with glasses? (Take the image distance from the lens of the eye to the retina to be 2 cm).

Main concept used: $P = P_1 + P_2$ and $P = \frac{1}{f}$

Ans. (*i*) Power of lens required to see clearly the object placed at infinity. $u = -\infty$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-10} - \frac{1}{\infty}$$

$$\frac{1}{f} = \frac{1}{-10}$$

$$f = -10 \text{ cm} = -0.1 \text{ m}$$

$$P = \frac{1}{f} \text{m}$$

$$P = \frac{1}{-0.1} \text{m}$$

$$P = -10 \text{ Diopter}$$

(*ii*) **When no corrective lens used:** Let powers of eye when object is at far point, near point are P_f and P_n respectively and power of accommodation $P_a = +4D$

$$\therefore \quad P_n = P_f + P_a$$

When object is at far point its clear image is formed at retina 2 cm from eye lens

$$\therefore \quad u = -10 \text{ cm} = -0.1 \text{ m}, \qquad v = 2 \text{ cm} = 0.02 \text{ m}$$

If f is focal length of eye lens focused at far point then

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{(-0.1)}$$

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$$\frac{1}{f} = 50 + 10 = 60$$

$$P_f = 60\text{ D}$$

$$\therefore \quad P_n = P_f + P_a = 60 + 4 = 64\text{ D}$$

Let the near point be x_n

$$u = -x_n \qquad (v = 2\text{ cm} = 0.02\text{ m})$$

$$\therefore \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{0.02} + \frac{1}{x_n} = \text{Power } (P_n)$$

$$50 + \frac{1}{x_n} = 64$$

$$\frac{1}{x_n} = 64 - 50 = 14\text{ D}$$

Near point without glass $x_n = \frac{1}{14}\text{m} = \frac{100}{14}\text{ cm} = \mathbf{7\text{ cm}}$ (Approx.)

(*iii*) **When used corrective lens:** When corrective lens is used then eye can see the object at infinity. Power of eye lens in this situation is P_∞

$u = \infty$ and $v = 2\text{ cm} = 0.02\text{ m}$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$P_\infty = \frac{1}{0.02} - \frac{1}{\infty} = 50$$

$$P_\infty = 50 + 0$$

$$P_\infty = 50\text{ D}$$

If P_n' = Power of eye at near point when corrective lens is used

$$P_n' = P_\infty + P_a = 50 + 4 = 54\text{ D}$$

Let near point in this situation is x_n'

$$u = -x_n'\text{ m}$$

$$v = +2\text{ cm} = 0.02\text{ m}$$

$$\frac{1}{f} = 54$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{0.02} + \frac{1}{x_n'} = 54 \qquad \text{(all distances are in m)}$$

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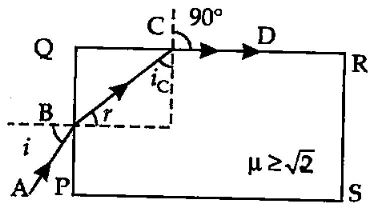
$$50 + \frac{1}{x'_n} = 54 \qquad \left(\frac{1}{x'_n} = 4\right)$$

$$x'_n = \frac{1}{4} \text{ m} = 0.25 \text{ m}$$

LONG ANSWER TYPE QUESTIONS

Q9.28. Show that for a material with refractive index $\mu \geq \sqrt{2}$, light incident at any angle shall be guided along the length perpendicular to incident face.

Main concept used: Snell's law and Total internal reflection



Ans. Consider a rectangular slab of refractive index $\mu \geq \sqrt{2}$. An incidence ray incidence at angle i on face PQ at incidence point. Refracted ray BC strike at face QR which is perpendicular to PQ with incidence angle i_c so that refracted ray CD passes normal to the face PQ as per required in question. So i_c must be critical angle

$$\mu = \frac{1}{\sin i_c} \qquad \text{(Snell's law at } c\text{)}$$

$$\sin i_c \geq \frac{1}{\mu} \qquad \left[\sin i_c = \frac{1}{\mu}\right]$$

$$\sin (90 - r) \geq \frac{1}{\mu} \qquad [\because \quad r + 90 + i_c = 180]$$

$$\cos r \geq \frac{1}{\mu}$$

$$\cos^2 r \geq \frac{1}{\mu^2} \qquad \text{(squaring both sides)}$$

$$-\cos^2 r \leq -\frac{1}{\mu^2}$$

$$1 - \cos^2 r \leq 1 - \frac{1}{\mu^2}$$

$$\sin^2 r \leq 1 - \frac{1}{\mu^2} \qquad \text{...(I)}$$

$$\frac{\sin i}{\sin r} = \mu \qquad \text{(by Snell's law)}$$

$$\sin i = \mu \sin r$$

or

$$\sin^2 i = \mu^2 \sin^2 r \qquad \text{(squaring both sides)}$$

$$\frac{1}{\mu^2} \sin^2 i = \sin^2 r \qquad \text{...(II)}$$

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Put (II) in (I)

$$\frac{1}{\mu^2}\sin^2 i \le 1-\frac{1}{\mu^2}$$

$$\sin^2 i \le \mu^2 - 1 \qquad \text{[on multiplying by } \mu^2 \text{ on both sides]}$$

For smallest angle *i.e.*, $i = 90°$

$$\therefore \quad \sin^2 90 \le \mu^2 - 1 \qquad [\because \mu \ge \sqrt{2}]$$

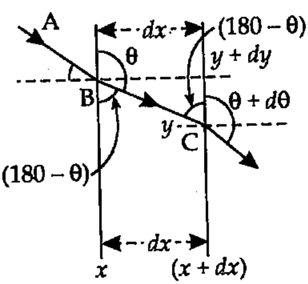
$$1 + 1 \le \mu^2$$

$$2 \le \mu^2$$

Taking square root

$$\sqrt{2} \le \mu \qquad \text{Hence proved.}$$

Q9.29. The mixture of a pure liquid and a solution in a long vertical column (*i.e.*, horizontal dimensions is very-very less than vertical dimensions) produces diffusion of solute particles and hence a refractive index gradient along the vertical dimension. A ray of light entering the column at right angles to the vertical is deviated from its original path. Find the deviation in travelling a horizontal distance $d \ll h$, the height of the column.



Ans. Consider a long vertical column of transparent liquid of infinite height (h) and thickness (dx). Consider a ray AB that enters at an angle θ into liquid of height y in column of liquid and emerges at an angle $(\theta + d\theta)$ at height $(y + dy)$. From Snell's law,

$$\mu(y)\sin\theta = \mu(y+dy)\sin(\theta + d\theta)$$

$$\mu(y)\sin\theta \cong \left[\mu(y) + \frac{d\mu}{dy}\cdot dy\right](\sin\theta\cos d\theta + \cos\theta\sin d\theta)$$

As $d\mu$, $d\theta$ are very small tends to zero

$$\therefore \quad \sin d\theta = d\theta, \quad \cos d\theta \cong 1 \qquad \mu(y) = \mu \qquad (\mu \text{ at height y constant})$$

$$\therefore \quad \mu\sin\theta \cong [\mu + d\mu](\sin\theta + \cos\theta\,.\,d\theta)$$

$$\mu\sin\theta \cong \mu\sin\theta + \mu\cos\theta\,d\theta + d\mu\sin\theta + \cos\theta\,.\,d\theta\,d\mu$$

Again $\quad d\theta\,d\mu \cong 0$

$$\mu\cos\theta\,.\,d\theta = -d\mu\sin\theta$$

$$d\theta = -\frac{1}{\mu}d\mu\frac{\sin\theta}{\cos\theta} = -\frac{1}{\mu}d\mu\tan\theta$$

From figure $\quad \tan(180-\theta) = \dfrac{dx}{dy}$

$$\tan\theta = \frac{dx}{dy}$$

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$$\therefore \qquad d\theta = -\frac{1}{\mu}\frac{d\mu}{dy}\cdot dx$$

By integration on both sides

$$\int_0^{\theta} d\theta \cong \frac{-1}{\mu}\int_0^{d}\frac{d\mu}{dy}dx = -\frac{1}{\mu}\frac{d\mu}{dy}\cdot\int_0^{d} dx$$ [∵ μ and y does not change horizontally ∵ $d\mu$, dy are constant horizontally.]

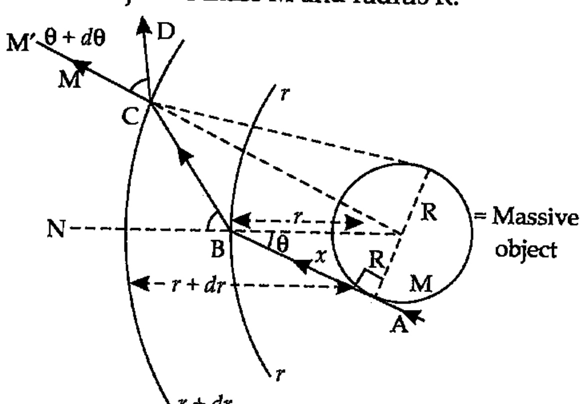
$$\theta \cong \frac{-1}{\mu}\frac{d\mu}{dy}d$$

Q9.30. If the light passes near a massive object, the gravitational interaction causes a bending of ray. This can be thought of as happening due to a change in the effective refractive index of the medium given by

$$n(r) = 1 + \frac{2GM}{rc^2}$$

where r is the distance of the point of consideration from the centre of the mass of the massive body, 'G' is the universal gravitational constant: M is the mass of the body and c is the speed of light in vacuum. Considering a spherical object find the deviation of the ray from the original path as it grazes the object.

Ans. Consider two spherical surfaces at r and $(r + dr)$ distance from the centre of massive object of mass M and radius R.



A ray ABCD incident at B and C on two surfaces at r and $(r + dr)$ then, by Snell's law

$$\mu(r)\sin\theta \cong \mu(r+dr)\sin(\theta + d\theta)$$

$$\mu(r)\sin\theta \cong \left[\mu(r) + \frac{d\mu}{dr}dr\right][\sin\theta\cdot\cos d\theta + \cos\theta\sin d\theta]$$

[∵ $d\theta \to 0$ then $\cos d\theta = 1$ and $\sin d\theta = d\theta$]

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$\therefore \quad \mu(r) \sin\theta \cong [\mu(r) + d\mu][\sin\theta + \cos\theta\, d\theta]$

$$\mu(r) \sin\theta \cong \mu(r)\sin\theta + d\mu \sin\theta + \mu(r)\cos\theta \cdot d\theta + d\mu\, d\theta \cdot \cos\theta$$

$d\mu.\, d\theta \cos\theta$ is very small as $d\mu$ and $d\theta$ are very small

$\therefore \quad d\mu\, d\theta \cos\theta \cong 0$

$\therefore \quad \mu(r)\sin\theta \cong \mu(r)\sin\theta + d\mu \sin\theta + \mu\cos\theta \cdot d\theta$

$$0 \cong d\mu \sin\theta + \mu(r)\cos\theta\, d\theta$$

$$-d\mu \sin\theta = \mu(r)\cos\theta \cdot d\theta$$

Dividing both side by dr

$$\frac{-d\mu}{dr}\sin\theta = \mu(r)\cos\theta\frac{d\theta}{dr} \quad \text{...(I)}$$

$$\mu(r) = 1 + \frac{2GM}{c^2}\cdot\frac{1}{r} \quad \text{(given)}$$

$$\frac{d\mu}{dr} = 0 + \frac{2GM}{c^2}(-1)r^{-2}$$

$$-\frac{d(\mu)}{dr} = \frac{+2GM}{c^2 r^2} \quad \text{...(II)}$$

$$\frac{2GM}{r^2c^2}\sin\theta = \left[1 + \frac{2GM}{rc^2}\right]\cos\theta \cdot \frac{d\theta}{dr}$$

As the G is very small and c^2 is very large so

$$\frac{2GM}{rc^2} \to 0$$

$$\frac{2GM}{r^2c^2}\frac{\sin\theta}{\cos\theta} = \frac{d\theta}{dr}$$

$$\frac{2GM}{r^2c^2}\tan \cong \frac{d\theta}{dr}$$

$$\frac{2GM}{c^2}\int\frac{\tan\theta}{r^2}dr = \int_0^{\theta_0} d\theta$$

$$r^2 = x^2 + R^2 \quad \text{(From figure)}$$

Differentiate w.r.t. r both sides

$$2rdr = 2x\, dx$$

$$dr = \frac{x}{r}dx$$

$$\therefore \quad \int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\alpha}^{\alpha}\frac{1}{r^2}\frac{R}{r}dx \quad \left(\tan\theta = \frac{R}{x}\right)$$

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$$= \frac{2GM}{c^2}\int_{-\alpha}^{+\alpha}\frac{R}{\cancel{x}r^3}\cancel{x}\,dx = \frac{2GM}{c^2}\int_{-\alpha}^{\alpha}\frac{R}{(x^2+R^2)^{3/2}}dx$$

$$\int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\alpha}^{\alpha} R\frac{dx}{(x^2+R^2)^{3/2}}$$

$\because \quad x = R\tan\theta$

$dx = R\sec^2\theta \cdot d\theta$

$$\theta_0 = \frac{2GM}{c^2}\int_{-\pi/2}^{\pi/2}\frac{R\,R\sec^2\theta\,d\theta}{(R^2\tan^2\theta+R^2)^{3/2}} = \frac{2GM}{c^2}\int_{-\pi/2}^{\pi/2}\frac{R^2\sec^2\theta\,d\theta}{[R^2(\tan^2\theta+1)]^{3/2}}$$

$$\int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\pi/2}^{\pi/2}\frac{R^2\sec^2\theta}{R^3\sec^3\theta}d\theta$$

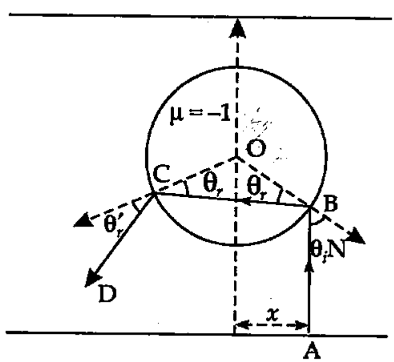
$$= \frac{2GM}{Rc^2}\int_{-\pi/2}^{\pi/2}\frac{1}{\sec\theta}d\theta = \frac{2GM}{Rc^2}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta$$

$$= \frac{2GM}{Rc^2}[\sin\theta]_{-\pi/2}^{\pi/2} = \frac{2GM}{Rc^2}\left[\sin\frac{\pi}{2} - \sin\left(\frac{-\pi}{2}\right)\right]$$

$$\theta_0 = \frac{2GM}{Rc^2}\left[1+\sin\frac{\pi}{2}\right] = \frac{2GM}{Rc^2}[1+1]$$

$\boxed{\theta_0 = \frac{4GM}{Rc^2}}$ is required rotation.

Q9.31. An infinitely long cylinder of radius R is made of an unusual exotic material with refractive index –1 (figure). The cylinder is placed between two planes whose normals are along the y direction. The centre of the cylinder 'O' lies along the y-axis. A narrow laser beam is directed along the y direction from the lower plate. The laser source is at a horizontal distance x from the diameter in the y direction. Find the range of x-such that light emitted from the lower plane does not reach the upper plane.



Main concept used: $\mu_r = -1$ and $\mu_d = 1$, when light passes from μ to $-\mu$ then reflection takes place from normal at incidence point.

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Ans. As the cylinder is made of refractive index (–1) and is placed in air of $\mu = 1$ so, when ray AB is incident at B to cylinder, θ_r will be negative *i.e.*, refracted ray will get reflection from normal. Similar thing happens at incidence point C and angle of refraction and incident at B and C will be equal (θ_r) as OB = OC = R and of refraction at C is 'θ_r''.

$$\theta_1 = |\theta_i| = |\theta_r| = |\theta_r'| \text{ as reflection takes place}$$

the total deviation of outcoming ray from the incoming ray $4\theta_1$. Rays shall not reach the receiving plane if $\frac{\pi}{2} \leq 4\theta \leq \frac{3\pi}{2}$ angles measured clockwise from the y axis.

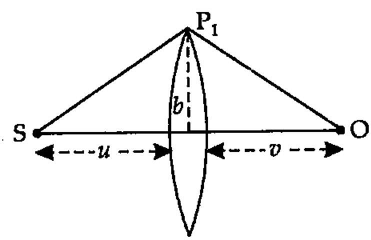
$$\text{or} \qquad \frac{\pi}{8} \leq \theta \leq \frac{3\pi}{8} \qquad \text{(on dividing by 4 to all sides)}$$

$$\sin\theta_1 = \frac{x}{R}$$

$$\frac{\pi}{8} \leq \sin^{-1}\frac{x}{R} \leq \frac{3\pi}{8} \quad \text{or} \quad \frac{\pi}{8} \leq \frac{x}{R} \leq \frac{3\pi}{8}$$

Thus for light emitted from the source shall not reach the receiving plane if $\frac{R\pi}{8} \leq x \leq \frac{3R\pi}{8}$

Q9.32. (*i*) Consider a thin lens placed between a source(S) and an observer (O) (figure). Let the thickness of the lens vary as $W(b) = W_0 - \frac{b^2}{\alpha}$, where b is the vertical distance from the pole, W_0 is a constant. Using Fermat's principle *i.e.*, the time of transit for a ray between the source and observer is an extremum, find the condition that all paraxial rays starting from the source will converge at a point 'O' on the axis. Find the focal length.



(*ii*) A gravitational lens may be assumed to have a varying width of the form

$$W(b) = K_1 \log\left(\frac{K_2}{b}\right) \qquad (b_{min} < b < b_{max})$$

$$= K_1 \log\left(\frac{K_2}{b_{min}}\right) \qquad (b < b_{min})$$

show that an observer will see an image of a point object as a ring about the centre of the lens with an angular radius

$$\beta = \sqrt{\frac{(n-1)K_1 \frac{u}{v}}{u+v}}$$

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Ans. The time taken by ray from S to $P_1 = \frac{SP_1}{c}$

$$t_1 = \frac{\sqrt{u^2+b^2}}{c} = \frac{u}{c}\left(1+\frac{b^2}{u^2}\right)^{1/2}$$

$$t_1 = \frac{u}{c}\left[1+\frac{b^2}{2u^2}\right] \qquad \text{(assuming } b \lll u\text{)}$$

Similarly, time required by ray from P_1 to O $= \frac{P_1O}{c}$

$$t_2 = \frac{v}{c}\left[1+\frac{b^2}{2v^2}\right]$$

Time required by ray to travel through the lens

$$t_3 = \frac{(\mu-1)\,W(b)}{c}$$

Thus total time by ray from S to O $= t = t_1 + t_2 + t_3$

$$t = \frac{u}{c}\left(1+\frac{b^2}{2u^2}\right)+\frac{v}{c}\left(1+\frac{b^2}{2v^2}\right)+\frac{(\mu-1)\,W(b)}{c}$$

$$= \frac{1}{c}\left[u+\frac{b^2}{2u}+v+\frac{b^2}{2v}+(\mu-1)\,W(b)\right]$$

$$= \frac{1}{c}\left[u+v+\frac{b^2}{2}\left(\frac{1}{u}+\frac{1}{v}\right)+(\mu-1)\,W(b)\right]$$

if $\qquad \frac{1}{D} = \frac{1}{u}+\frac{1}{v}$

$\therefore \qquad t = \frac{1}{c}\left[u+v+\frac{b^2}{2D}+(\mu-1)\left(W_0-\frac{b^2}{\alpha}\right)\right]$

By Fermat's Principle

$$\frac{dt}{db} = \frac{1}{c}\left[0+0\frac{2b}{2D}+(\mu-1)\left(0-\frac{2b}{\alpha}\right)\right] = \frac{b}{cD}-(\mu-1)\frac{-2b}{\alpha c}$$

But time from S to O remains constant, so, $\frac{dt}{db} = 0$

or $\qquad \frac{b}{cD}-\frac{2(\mu-1)b}{\alpha c} = 0$

$$\frac{2(\mu-1)b}{\alpha c} = \frac{b}{cD}$$

$$\alpha = 2(\mu-1)D$$

Thus, the convergence lens is formed $\alpha = 2(\mu-1)D$.

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It is independent of b and hence all paraxial rays from S will converge at O (*i.e.,* for rays $b <<< u$ and $b << v$).

Since $\frac{1}{D} = \frac{1}{u} + \frac{1}{v}$ it is equivalent to focal length f

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$\because$
$$t = \frac{1}{c}\left[u + v + \frac{b^2}{2D} + (\mu - 1)\, W(b)\right]$$

here
$$W(b) = K_1 \log_e\left(\frac{K_2}{b}\right)$$

$$t = \frac{1}{c}\left[u + v + \frac{b^2}{2D} + (\mu - 1)\, K_1 \log\frac{K_2}{b}\right]$$

$$\frac{dt}{db} = \frac{1}{c}\left[0 + 0 + \frac{2b}{2D} + (\mu - 1)\, K_1 \frac{b}{K_2}\right]\frac{d}{db}\left(\frac{K_2}{b}\right)$$

$$= \frac{1}{c}\left[\frac{b}{D} + (\mu - 1)\frac{K_1}{K_2} b \cdot K_2(-1)\frac{1}{b^2}\right]$$

as time from S to O is constant, so, $\frac{dt}{db} = 0$

so
$$0 = \frac{1}{c}\left[\frac{b}{D} - (\mu - 1)\frac{K_1}{b}\right]$$

$\therefore$
$$(\mu - 1)\frac{K_1}{b} = \frac{b}{D} \qquad \left(\because \frac{1}{c} \neq 0\right)$$

$$b^2 = K_1 D(\mu - 1)$$

$$b = \sqrt{(\mu - 1) K_1 D}$$

Thus all the rays passing at a height b shall contribute to the image. The ray path makes an angle

$$\beta = \frac{b}{v} = \frac{\sqrt{(\mu - 1) K_1 D}}{v} = \sqrt{\frac{(\mu - 1) K_1 D}{v^2}} \qquad \left(\because \frac{1}{D} = \frac{1}{v} + \frac{1}{u}\right)$$

$$= \sqrt{\frac{(\mu - 1) K_1 uv}{v^2 (u + v)}} \qquad \left(D = \frac{uv}{u + v}\right)$$

$$\beta = \sqrt{\frac{(\mu - 1) K_1 u}{v(u + v)}}$$

❑❑❑

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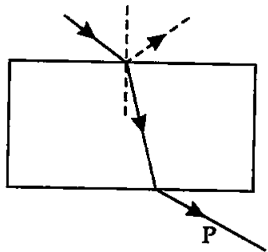
10 Wave Optics

MULTIPLE CHOICE QUESTIONS—I

Q10.1. Consider a light beam incident from air to a glass slab at Brewster's angle as shown in figure



A Polaroid is placed in the path of the emergent ray at point P and rotated about an axis passing through the centre and perpendicular to the plane of the polaroid.

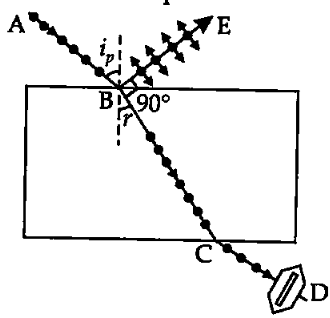
(*a*) For a particular orientation, there shall be darkness as observed through polaroid.

(*b*) The intensity of light as seen through the polaroid shall be independent of the rotation.

(*c*) The intensity of the light as seen through the polaroid shall go through a minimum but not zero for two orientations of the polaroid.

(*d*) The intensity of light as seen through the polaroid shall go through a minimum for four orientations of the polaroid.

Main concept used: Brewster's angle.



Ans. (*c*): When a ray ABCD of light passes through prism in such a way that angle between reflected ray BE and refracted ray BC is 90° then only reflected ray is plane polarised. So polaroid rotated in the way of CD the intensity will never be zero but varies in one complete rotation so, it verifies answer (*c*).

Q10.2. Consider sunlight incident on a slit of width 10^4 Å. The image seen through the slit shall

(*a*) be a fine sharp slit white in colour at the centre.

(*b*) a bright slit white at the centre diffusing to zero intensities at the edges.

(*c*) a bright slit white at the centre diffusing to regions of different colours.

(*d*) only be a diffused slit white in colour.

Ans. (*a*): Width of slit 10^4 Å = 10,000 Å.

Wavelength of visible light varies from 4000 to 8000 Å. As the width of slit 10000 Å is comparable to that of wavelength of visible light *i.e.*

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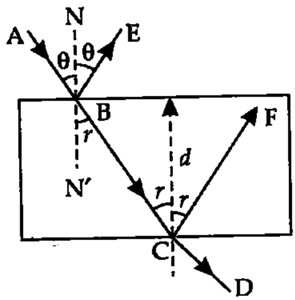
8000 Å. Hence the diffraction occurs with maxima at the centre. So at the centre all colours appear *i.e.* white colour at the centre appear.

Q10.3. Consider a ray of light incident from air onto a slab of glass (refractive index n) of width 'd' at an angle θ. The phase difference between the ray reflected by the top surface of the glass and the bottom surface is

(*a*) $\frac{2\pi\mu d}{\lambda}\left[1-\frac{1}{n^2}\sin^2\theta\right]^{\frac{1}{2}}+\pi$ (*b*) $\frac{4\pi d}{\lambda}\left[1-\frac{1}{n^2}\sin^2\theta\right]^{\frac{1}{2}}$

(c) $\frac{4\pi d}{\lambda}\left[1-\frac{1}{n^2}\sin^2\theta\right]^{\frac{1}{2}}+\frac{\pi}{2}$ (*d*) $\frac{4\pi d}{\lambda}\left[1-\frac{1}{n^2}\sin^2\theta\right]^{\frac{1}{2}}+2\pi$

Ans. (*a*): Consider a ray of light ABCD through prism, and reflected rays BE and CF from incidence points B and C as shown in figure.



The time difference between two reflected rays BE and CF is equal to the time taken by ray to travel from B to C.

$\therefore$ Time difference dt between two reflected rays BE and CF are

$$dt = \frac{BC}{v_g} \quad ...(I)$$

$$\because \quad \mu = \frac{v_a}{v_g}$$

$$v_a = c$$

$$\therefore \quad v_g = \frac{c}{\mu} \quad ...(II)$$

$$\cos r = \frac{d}{BC}$$

$$BC = \frac{d}{\cos r} \quad ...(III)$$

Substitute (II) and (III) in (I),

$$dt = \frac{d/\cos r}{c/\mu} = \frac{\mu d}{c\cos r} \quad ...(IV)$$

$$\mu = \frac{\sin\theta}{\sin r} \quad \text{or} \quad \sin r = \frac{\sin\theta}{\mu}$$

$$\cos r = \sqrt{1-\sin^2 r} = \sqrt{1-\frac{\sin^2\theta}{\mu^2}}$$

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From IV, $$dt = \frac{\mu d}{c\sqrt{1-\frac{\sin^2\theta}{\mu^2}}} = \frac{\mu d}{c}\left[1-\frac{\sin^2\theta}{\mu^2}\right]^{-1/2}$$

Phase difference $$d\phi' = \frac{2\pi}{T}\cdot dt = \frac{2\pi\mu d}{Tc}\left[1-\frac{\sin^2\theta}{\mu^2}\right]^{-1/2}$$

$$d\phi' = \frac{2\pi\mu d}{\frac{1}{v}.v\lambda}\left[1-\frac{1}{\mu^2}\sin^2\theta\right]^{-\frac{1}{2}} = \frac{2\pi\mu d}{\lambda}\left[1-\frac{1}{\mu^2}\sin^2\theta\right]^{-\frac{1}{2}}$$

The phase diff. between ray AB and BC after refraction is π

$\therefore$ Net phase difference $= d\phi' + \pi$

$$d\phi = \frac{2\pi\mu d}{\lambda}\left[1-\frac{\sin^2\theta}{\mu^2}\right]^{-1/2} + \pi$$ [it is very near to option (*a*).]

Q10.4. In a Young's double slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case

(*a*) there shall be alternate interference patterns of red and blue.
(*b*) there shall be an interference pattern for red distinct from that for blue.
(*c*) there shall be no interference fringes.
(*d*) there shall be an interference pattern for red mixing with one for blue.

Main concept used: Condition for interference is coherent soure or of same fequency.

Ans. (*c*): For sustained interference the source must be coherent and should emit the light of same frequency.

In this problem one hole is covered with red and other with blue, which has different frequency, so no interference takes place.

Q10.5. The given figure shows a standard two slit arrangement with slits S_1, S_2. P_1, P_2 are the two minima points on either side of P.

At P_2 on the screen there is a hole and behind P_2 is a second screen, 2-slit arrangement with slits S_3 and S_4 and a second screen behind them.

(*a*) There would be no interference pattern on the second screen but it would be lighted.
(*b*) The second screen would be totally dark.
(*c*) There would be a single bright point on the second screen.
(*d*) There would be a regular two slit pattern on the second screen.

Main concept used: Each point on wavefront acts as source of secondary wavelets.

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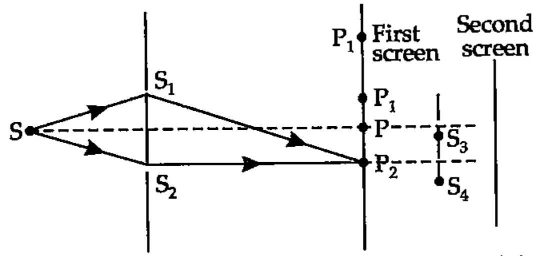
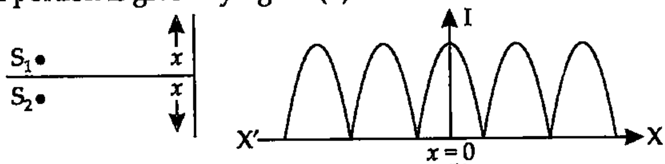


Ans. (*d*): At P_2 is minima due to two wavefronts in opposite phase coming from, two slits S_1 and S_2, but there is wavefronts from S_1, S_2 so P_2 will act as a source of secondary wavelets. Wavefront starting from P_2 reaches at S_3 and S_4 slits which will again acts as two monochromatic or coherent sources and will form pattern on second screen.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q10.6. Two sources S_1 and S_2 of intensity I_1 and I_2 are placed in front of a screen [fig. (a)]. The pattern of intensity distribution seen in the central portion is given by figure (b).



(*a*) (*b*)

In this case which of the following statements are true?

(*a*) S_1 and S_2 have the same intensities.

(*b*) S_1 and S_2 have a constant phase difference.

(*c*) S_1 and S_2 have the same phase.

(*d*) S_1 and S_2 have the same wavelength.

Main concept used: Condition for sustained interference.

Ans. (*a*), (*b*) and (*c*): (*i*) As the intensity at dark fringe is zero so intensities of S_1 and S_2 are equal.

(*ii*) As the graph of maxima and minima is symmetric. So the waves from S_1 and S_2 are at same phase difference or zero phase difference.

Q10.7. Consider sunlight incident on a pinhole of width 10^3 Å. The image of the pinhole seen on a screen shall be

(*a*) a sharp white ring.

(*b*) different from a geometrical image.

(*c*) a diffused central spot white in colour.

(*d*) a diffused coloured region around a sharp central white spot.

Ans. (*b*) and (*d*): The width of pinhole 10^3 Å = 1000 Å and wavelength of visible light is 4000 Å to 8000 Å *i.e.*, size of slit less than (or comparable) with the wavelength of light.

So light from pinhole will diffract from the hole. Due to the diffraction pattern of fringes, the shape are quite different from hole.

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Q10.8. Consider the diffraction pattern for a small pinhole. As the size of the hole is increased

(*a*) the size decreases. (*b*) the intensity increases.

(*c*) the size increases. (*d*) the intensity decreases.

Ans. (*a*) and (*b*): We know that width (B_0) of central maxima $B_0 = \frac{D\lambda}{d}$ and width of n^{th} secondary maxima $= \frac{\lambda}{d}$ here distance (D) between slit and screen, wavelength λ of source does not change.

So on increasing width of hole of pinhole, '*d*' increase. Hence the size of central maxima decreases verifies the option (*a*).

As the energy passing through hole increased on increasing the size of hole. So the intensity of pattern will increase. Hence verifies the option (*b*).

Q10.9. For the light diverging from a point source

(*a*) the wavefront is spherical.

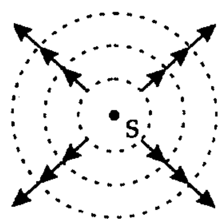
(*b*) the intensity decreases in proportion to the distance squared.

(*c*) the wavefront is parabolic.

(*d*) the intensity at wavefront does not depend on distance.

Main concept used: Properties of wavefront

Ans. (*a*) and (*b*): light from point source emits in all around the source with same speed so forms a **spherical surface of wavefront** or spherical wavefront.



As the intensity (I) always decreases as the reciprocal of square of distance.

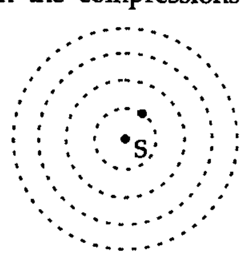
$$I = \frac{P}{4\pi r^2}$$

r = radius of spherical wavefront at any time ($r = vt$)

VERY SHORT ANSWER TYPE QUESTIONS

Q10.10. Is Huygen's principle valid for longitudinal sound waves?

Ans. Consider a source of sound formed with the compressions and rarefactions forward in all directions with same velocity. So longitudinal waves propagate with spherical symmetry in all directions as the wavefront in light waves. So Huygen's principle is valid for longitudinal sound waves also.



On a surface of sphere there will be either compression or rarefaction and that part can also behave like a source of sound but with low intensity.

Q10.11. Consider a point at the focal point of a convergent lens. Another convergent lens of short focal length is placed on the other side. What is the nature of wavefronts emerging from the final image?

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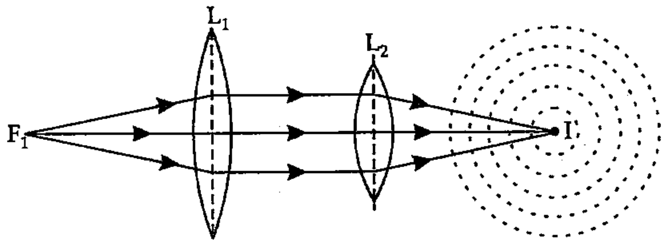
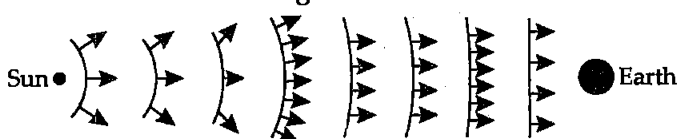
Ans. Consider a point 'F' on focus of converging lens L_1. The light rays from F, becomes parallel after refraction through L_1. When these parallel rays falls on converging lens L_2 placed co-axial on the other side of F of L_1. L_2 converges the rays at it's focus at I. It now behave like a point source of rays and form a spherical wave front.



Q10.12. What is the shape of the wavefront on earth for sunlight?

Ans. As the sun is very-very far from the earth so can be considered at infinity and sun can be considered as a point source which gives spherical wavefront. The size of the earth is very small as compared to distance of sun from earth and size of the sun so the plane wavefront reaches on earth as shown in figure here.



Q10.13. Why is the diffraction of sound waves more evident in daily experience than that of light wave?

Ans. We know that frequencies of sound waves varies from 20 Hz to 20,000 Hz, so its corresponding wavelength varies from 15 m to 15 mm respectively. The size of slit (almost) becomes comparable to wavelength of sound so diffraction of sound wave takes place easily.

But the wavelength of visible light varies from 0.4 to 0.7 micron which is very small. So the size of most of the slits is not comparable with wavelength of visible light, due to this diffraction of light cannot take place.

Q10.14. The human eye has an approximate angular resolution of $\phi = 5.8 \times 10^{-4}$ radian and a typical photoprinter prints a minimum of 300 dpi (dots per inch) (1 inch = 2.54 cm). At what minimum distance z should a printed page be held so that one does not see the individual dots?

Ans. Angular separation = 5.8×10^{-4} radian

The average distance between any two dots = $\frac{2.54}{300} = 0.85 \times 10^{-2}$ cm

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At the distance z cm, angle subtended $= \frac{\text{arc}}{\text{rad}} = \frac{0.85 \times 10^{-2}}{z}$

Resolution angle for human $= 5.8 \times 10^{-2}$ rad $= \frac{0.85 \times 10^{-2}}{z}$

Maximum distance up to which human eye cannot see

$$2 \text{ dots distinctly} = z = \frac{0.85 \times 10^{-2}}{5.8 \times 10^{-2}} = 14.5 \text{ cm}$$

which is less than distance of distinct vision.

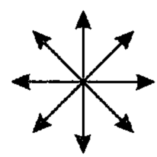
So a normal person cannot see the dots.

Q10.15. A polaroid I is placed in the front of a monochromatic source. Another polaroid II is placed in front of this polaroid I and rotated till no light passes. A third polaroid (III) is now placed in between Ist and IInd . In this case will the light emerge from II. Explain.

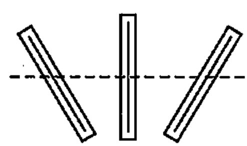
Main concept used: Sunlight unpolarised and Law of MALUS

Ans. Polaroid III is placed between two crossed polaroids I and II and no light passes through IInd polaroid.

Polaroid III is now placed between I, II and now rotates keeping I, II in no rotation. Let angle between polaroid I and III be θ and intensity of plane polarised light after Ist polaroid is I_0. Then



Unpolarised light



$I_1 = I_0$

$I_3 = I_0 \cos^2\theta$ is intensity of light after polaroid III

$$I_2 = I_3 \cos^2\theta' = I_0 \cos^2\theta . \cos^2(90 - \theta)$$

$$I_2 = I_0 \cos^2\theta . \sin^2\theta$$

$$I_2 = I_0(2\cos\theta\sin\theta)^2 = I_0 (\sin 2\theta)^2$$

$$I_2 = \frac{I_0}{4}\sin^2 2\theta$$

θ is angle between I and III Polaroid.

No light will pass through II Polaroid from above eqn. when

$$\sin^2 2\theta = 0$$

$$\sin 2\theta = 0$$

$$\sin 2\theta = \sin 0$$

$$\theta = 0°$$

$\therefore$ No light will pass when the angle between I and III Polaroid is zero *i.e.*, plane of polaroid are parallel. In all other cases when III is not parallel to either I or II, light will pass.

SHORT ANSWER TYPE QUESTIONS

Q10.16. Can reflection result in plane polarised light if the light is incident on the interface from the side with higher refractive index?

Main concept used: Brewster's angle

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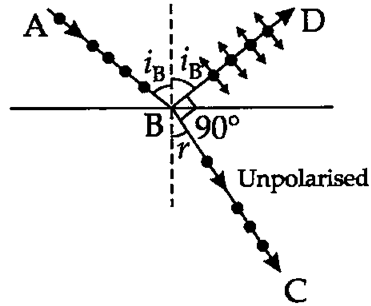
Ans. When a ray of light passes from a medium (air) of refractive index μ_1 to another medium of refractive index μ_2, more than μ_1 with incidence angle is equal to Brewster's angle (i_B), then transmitted ray BC will be unpolarised light and the reflected light BD will be plane polarised. Angle DBC between refracted and reflected ray is 90°.



$$\angle r + \angle i_B + 90 = 180°$$

So $\quad \angle r + \angle i_B = 90° \quad$ or $\quad \angle r = 90 - i_B$

By Snell's law, $\mu_2 = \dfrac{\sin i_B}{\sin r} = \dfrac{\sin i_B}{\cos i_B} = \tan i_B$

$\therefore \quad \dfrac{\mu_2}{\mu_1} = \tan i_B \quad \left(\begin{array}{c} \because \mu_2 > \mu_1 \\ \therefore \tan i_B > 1 \\ i_B > 45° \end{array}\right) \quad$ ...(I)

$\sin i_C = \dfrac{1}{\mu_2}$ when light passes from medium 2 to 1

$\sin i_C = \dfrac{\mu_1}{\mu_2} \quad (\because \mu_2 > \mu_1)$

so $\quad \sin i_C < 1 \quad$ ...(II)

or $\quad i_C < 90°$

From I $\quad \tan i_B > 1$

From II $\quad 1 > \sin i_C$

or $\quad \tan i_B > 1 > \sin i_C \quad$ or $\quad |\tan i_B| > |\sin i_C|$

Because $\quad 45 < i_B$

$\quad i_C < 90°$

So $\quad 45° < i_B < 90°$

$\quad 0 < i_C < 90°$

So $\quad i_B > i_C$

Thus, polarisation by reflection takes place.

Q10.17. For the same objective, find the ratio of the least separation between two points to be distinguished by a microscope for light of 5000 Å and electrons accelerated through 100 V used as the illuminating substance.

Main concept used: R.P. (microscope) $= \dfrac{1}{d} = \dfrac{2 \sin \beta}{1.22\,\lambda}$

and $\quad \lambda_D = \dfrac{1.22}{\sqrt{V}}$ nm

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Ans. $\lambda = 5000\ \text{Å} = 5000 \times 10^{-10}\ m$

In microscope, $$R.P. = \frac{1}{d} = \frac{2 \sin \beta}{1.22\ \lambda}$$

Limit of resolution by light of 5000 Å

$$d_{min} = \frac{1.22\ \lambda}{2 \sin \beta} \quad \text{or} \quad d_{min} = \frac{1.22 \times 5000 \times 10^{-10}}{2 \sin \beta}$$

The de Broglie wavelength λ_d of illuminated light $= \frac{1.22}{\sqrt{V}}$ nm

$$\lambda_d = \frac{1.22}{\sqrt{100}}\ \text{nm} = \frac{1.22}{10} \times 10^{-10}\ \text{m}$$

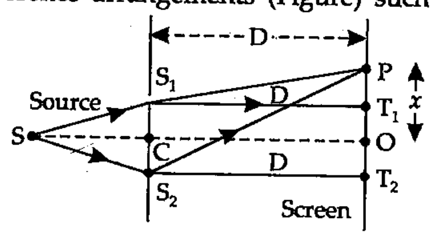
The limit of resolution by 100 V light $d'_{min} = \frac{1.22\ \lambda_d}{2 \sin \beta}$

$$d'_{min} = \frac{1.22 \times 1.22 \times 10^{-10}}{2 \sin \beta}$$

The required ratio $= \frac{d'_{min}}{d_{min}} = \frac{\frac{1.22 \times 1.22 \times 10^{-10}}{2 \sin \beta}}{\frac{1.22 \times 5000 \times 10^{-10}}{2 \sin \beta}} = \frac{1.22}{5000}$

The required ratio $= \frac{122}{500} \times 10^{-3} = \mathbf{0.244 \times 10^{-3}}$

Q10.18. Consider two slit interference arrangements (Figure) such that the distance of the screen from the slits is half the distance between the slits. Obtain the value of D in terms of λ such that the **first minima** on the screen falls at a distance D from the centre O.



Main concept used: For nth minima the path difference $= (2n-1)\frac{\lambda}{2}$ in Y D.S.E.

Ans. According to θ,

$x = \text{D}$ (Given) ...(I)

$\text{D} = \frac{1}{2}d$ (Given) ...(II)

$d = 2\text{D}$

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Path difference at $P = S_2P - S_1P$

$$\text{Path diff. } p = \sqrt{D^2 + \left(x + \frac{d}{2}\right)^2} - \sqrt{D^2 + \left(x - \frac{d}{2}\right)^2}$$

Substitute the value of d and x from I and II

$$= \sqrt{D^2 + (D + D)^2} - \sqrt{D^2 + (D - D)^2}$$

$$= \sqrt{5D^2} - \sqrt{D^2}$$

$$p = D(\sqrt{5} - 1) \quad \text{...(III)}$$

The path difference for nth dark fringe from central maxima 'O' is $(2n - 1)\frac{\lambda}{2}$.

$\therefore$ For Ist minima $p = \frac{\lambda}{2}$

Put the value of p in (III)

$$\frac{\lambda}{2} = D(\sqrt{5} - 1)$$

$$D = \frac{\lambda}{2(\sqrt{5} - 1)}$$

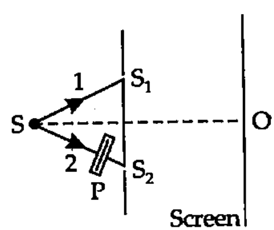
Rationalising the denominator, we get,

$$D = \frac{\lambda}{2(\sqrt{5} - 1)} \times \frac{(\sqrt{5} + 1)}{(\sqrt{5} + 1)} = \frac{(2.236 + 1)}{2 \times (5 - 1)}\lambda = \frac{3.236}{2 \times 4}\lambda$$

$$= \frac{3.236}{8}\lambda = 0.404\,\lambda$$

LONG ANSWER TYPE QUESTIONS

Q10.19. Figure shows the two slit arrangement with a source which emits unpolarised light. P is a polariser with axis whose direction is not given. If I_0 is the intensity of principal maxima, when no polariser is present, calculate in the present case, the intensity of the principal maxima as well as of the first minima.



Ans. Let the amplitudes of ray 1 and 2 are A_1 and A_2 respectively. A_2 has constant phase diff. ϕ

$$A_1 = A_\perp^1 + A_{||}^1$$

$$A_2 = A_\perp^2 + A_{||}^2$$

where $\quad A_\perp^1 = A_\perp^0 \,(\sin kx - \omega t)$

$$A_\perp^2 = A_\perp^0 \,(kx - \omega t + \phi)$$

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Maximum amplitude of $A_\perp$ and $A_{||}$ are same = $A_\perp^0$

∴ Similarly for

$$A_{||}^1 = A_{||}^0 \sin(kx - \omega t)$$

$$A_{||}^2 = A_{||}^0 \sin(kx - \omega t + \phi)$$

When ray 2 is **polarised** by P then its $A_{||}^2$ vector stopped.

Case I: When there is no polaroid.

A = Resultant amplitude

$$= A_\perp + A_{||} = [A_\perp^1 + A_\perp^2] + [A_{||}^1 + A_{||}^2]$$

$$= [A_\perp^0 \sin(\omega t - kx) + A_\perp^0 \sin(\omega t - kx + \phi)]$$

$$+ [A_{||}^0 \sin(\omega t + \phi) + A_{||}^0 \sin(\omega t - kx + \phi)] \quad \text{...I}$$

The amplitudes of S_1, S_2 when there is no polariser then perpendicular and parallel components are equal, as the wavefront is coming from same source S.

$$\therefore \quad A_\perp^1 = A_{||}^1 = A_\perp^2 = A_{||}^2 = A_0$$

Where $A_\perp^1$ and $A_{||}^1$ are the amplitudes of electric and magnetic field vector respectively and $A_\perp^2$ and $A_{||}^2$ are the amplitudes of electric and magnetic field vector, respectively.

From equation I, we have

$$A = A_0 \sin(\omega t - kx) + A_0 \sin(\omega t - kx + \phi) + A_0 \sin(\omega t - kx) + A_0 \sin(\omega t - kx + \phi)$$

$$A = 2A_0[2 \sin \omega t - kx + \sin(\omega t - kx + \phi)].$$

We know that the intensity of maxima in Young's double slit experiment

$I_1 = I_2 = I$ and the resultant intensity at a point = $\boxed{I = I_0(1 + \cos\phi)}$

$$\because \quad I = kA^2 \quad \text{or} \quad I_0 = kA_0^2$$

Intensity at a point in Young's double slit without polariser

$$\therefore \quad I = 2I_0(1 + \cos\phi) = 2 \cdot kA_0^2(1 + \cos\phi)$$

For maxima $\cos\phi = 1$

$I = 2kA_0^2(1 + 1) = 4kA_0^2$ leaving the k (constant) then

$$I = 4A_0^2$$

Case II: When polariser P is placed in the path of ray 2: Let $\perp^r$ vector of ray 2 is blocked by polariser

$$A_1 = A_\perp^1 + A_{||}^1$$

$$A_2 = A_{||}^2 \qquad (\because A_\perp^2 \text{ vector blocked})$$

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Resultant amplitude

$$A = A_1^2 + A_2^2$$

$$I = (A_{||}^1 + A_{||}^2)^2 + |A_\perp^1|^2$$

$$|A_{||}^0|_{av} = |A_\perp^0|_{av} = A_0$$

$$I = kA_0^2(1+\cos\phi) + A_0^2\frac{1}{2} \quad ...(I)$$

$$= kA_0^2\left[1+\cos\phi+\frac{1}{2}\right]$$

$$= kA_0^2\left[\frac{3}{2}+\cos\phi\right] \quad (\cos\phi = 1 \text{ for maxima})$$

$$= kA_0^2 \times \frac{5}{2} \quad ...(II)$$

$$I_0 = 4kA_0^2 \quad \text{(when no polarisation of ray 2)}$$

$$A_0^2 = \frac{I_0}{4k} \quad ...(III)$$

Intensity of principal maxima with polariser

$$I = \frac{I_0}{4}\frac{5}{2} \quad \text{[from II and III]}$$

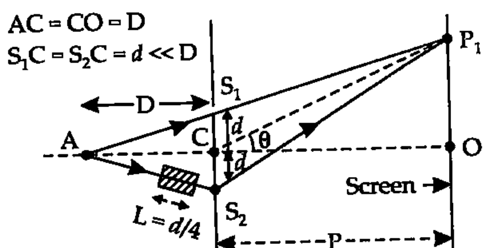
$$\boxed{I = \frac{5}{8}I_0}$$

Intensity at the Ist minima $\cos\phi = -1$

$$\therefore \quad I = |A_0|^2(1-1) + \frac{A_0^2}{2} \quad \text{[from I]}$$

$$I = \frac{I_0}{4}\cdot\frac{1}{2} = \frac{I_0}{8}$$

Q10.20. A small transparent slab containing material of $\mu = 1.5$ is placed along AS_2 (Figure). What will be the distance from 'O' of the principal maxima and of the Ist minima on either side of the principal maxima, obtained in the absence of the glass slab.



Main concept used: When ray passes through the slab it gets lateral displacement, then fringes shifts by $(\mu - 1)$.

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Ans. When ray AS_2 passes through glass slab of thickness L and refractive index μ the path difference caused by slab is $(\mu - 1)L$ and path difference caused by Young's double slit experiment is $2d \sin \theta$. Total path difference at P_2 is

$$\therefore \quad \Delta x = 2d \sin \theta + (\mu - 1) L$$

for principal maxima, path difference $\Delta x = 0$

$\therefore 2d \sin \theta_0 + (\mu - 1) L = 0$ (For central maxima $\theta = \theta_0$)

$$\sin \theta_0 = \frac{-(\mu - 1) L}{2d} = \frac{-(1.5 - 1)L}{2 \times 4L} \quad \left[\text{as } L = \frac{d}{4}, d = 4L\right]$$

$$\sin \theta_0 = \frac{-0.5 \cdot L}{2 \times 4L} = \frac{-1}{16}$$

For central maxima, $OP = D \tan \theta_0 \approx D \sin \theta_0 = \frac{-D}{16}$

For small $\angle\theta_0$, $\sin \theta_0 = \theta_0$ and $\tan \theta_0 = \theta_0$

For the first minima the path difference $= \frac{+\lambda}{2}$

$$\therefore \quad 2d \sin \theta_1 + (\mu - 1)L = \frac{\pm\lambda}{2}$$

(For both upper and lower side from O)

$$2d \sin \theta_1 + (1.5 - 1)L = \frac{\pm\lambda}{2}$$

$$2d \sin \theta_1 = \frac{\pm\lambda}{2} - 0.5\,L \quad \left(\because \; L = \frac{d}{4}\right)$$

$$\sin \theta_1 = \frac{\pm\frac{\lambda}{2} - 0.5\frac{d}{4}}{2d} = \frac{\pm\frac{\lambda}{2} - \frac{d}{8}}{2d}$$

$$\sin \theta_1 = \frac{(\pm 4\lambda - d)/8}{2d}$$

For diffraction, $\lambda = d$ (half the slit dist.)

$$\therefore \quad \sin \theta_1 = \frac{(\pm 4\lambda - \lambda)}{16\lambda} = \frac{(\pm 4 - 1)\lambda}{16\lambda}$$

$$\sin \theta_1 = \frac{\pm 4 - 1}{16}$$

For positive direction side, $\sin \theta_1^+ = \frac{3}{16}$

For negative direction side, $\sin \theta_1^- = \frac{-5}{16}$

The distance of first minima from principal maxima on either side x_1^+ and x_1^-, are

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$$x_1^+ = D\tan\theta_1^+ = D\cdot\frac{\sin\theta_1^+}{\cos\theta_1^+}$$

$$x_1^+ = D\tan\theta_1^+ = D\frac{\sin\theta_1^+}{\sqrt{[1-\sin^2\theta_1^+]}} = \frac{D3/16}{\sqrt{1-\frac{9}{256}}}$$

$$x_1^+ = D\tan\theta_1 = \frac{\frac{3D}{16}}{\frac{\sqrt{256-9}}{16}} = \frac{3D}{\sqrt{247}} \text{ [above point O on screen]}$$

$$x_1^+ = \frac{3D}{\sqrt{247}}$$

The first minima starts after the end of central maxima. So the Ist minima starts at distance of $D\tan\theta = x_1^+ = \frac{3D}{\sqrt{247}}$ above O in positive direction.

The distance of first minima on the negative side is

$$x_1^- = \frac{D\sin\theta_1^-}{\sqrt{1-\sin^2\theta_1^-}} = \frac{\frac{-5D}{16}}{\frac{\sqrt{16^2-5^2}}{16}}$$

$$x_1^- = \frac{-5D}{\sqrt{256-25}} = \frac{-5D}{\sqrt{231}}$$

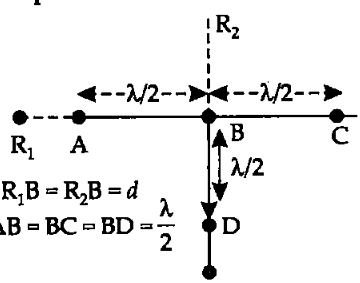
Q10.21. Four identical monochromatic sources A, B, C, D as shown in the figure produce waves of the same wavelength λ and are coherent. Two receivers R_1 and R_2 are at great but equal distances from B.

(*i*) Which of the two receivers picks up the larger signal?

(*ii*) Which of the two receivers pick up the larger signal when B is turned off?

(*iii*) Which of the two receivers pick up the larger signal when D is turned off?

(*iv*) Which of the two receivers can distinguish which of the sources B or D has been turned off?



Main concept used: The resultant disturbance at a point is equal to sum of disturbances due to individual sources.

Ans. Consider all the disturbances at R_1

Let the wave from source A has zero path difference at R_1

$\therefore$ $\qquad y_A = a\cos\omega t \qquad$ (at R1)

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Path diff. between A and B is $\lambda/2$ or π so wave of B at R_1 is

$$yB = -a \cos \omega t$$

Path difference between wave A and C is λ or 2π.
So the wave from source C is

$$y_C = a \cos (\omega t - 2\pi)$$

or

$$y_C = a \cos \omega t$$

Path difference between A and D = $R_1 D - AR_1$

$$= \sqrt{d^2 + \left(\frac{\lambda}{2}\right)^2} - \left(d - \frac{\lambda}{2}\right) = \left(d^2 + \frac{\lambda^2}{4}\right)^{1/2} - d + \frac{\lambda}{2}$$

$$= d\left[1 + \frac{\lambda^2}{4d^2}\right]^{1/2} - d + \frac{\lambda}{2} = d\left[1 + \frac{\lambda^2}{8d^2}\right] - d + \frac{\lambda}{2}$$

Neglecting the term $\frac{\lambda^2}{8d^2}$, $(\because \lambda^2 << 8d^2)$

$\therefore$ Path difference between A and D = $d + 0 - d + \frac{\lambda}{2} = \frac{\lambda}{2}$ or π

$$\therefore \quad y_D = a \cos (\omega t - \pi) = -a \cos \omega t$$

So all the signals picked up by R_1 from A, B, C and D

$$y_{R_1} = y_A + y_B + y_C + y_D$$

$$= a \cos \omega t - a \cos \omega t + a \cos \omega t - a \cos \omega t$$

$$y_{R_1} = 0 \text{ so } \langle I_{R_1} \rangle \approx 0$$

So resultant signal at R_1 due to A, B, C and D sources is zero or no signal.

(*i*) New distance $BR_2 = d$

Let the signal picked up by R_2 from B = $a_1 \cos \omega t$

The path diff. between B and D = $\frac{\lambda}{2}$ or π so

The signal picked up by R_2 from D = $a \cos (\omega t - \pi)$

$$y_D = -\cos \omega t$$

The path diff. between A and D signals at $R_2 = AR_2 - R_2B$

$$\text{Path diff.} = \sqrt{d^2 + \left(\frac{\lambda}{2}\right)^2} - d$$

$$\Delta x = \left[d^2 + \frac{\lambda^2}{4}\right]^{1/2} - d = d\left[1 + \frac{\lambda^2}{8d^2}\right] - d$$

$$\therefore \quad \Delta x = d + \frac{d\lambda^2}{8d^2} - d = \frac{\lambda^2}{8d}$$

$$\text{Phase diff} = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda} \cdot \frac{\lambda^2}{8d} = \frac{2\pi\lambda}{8d} = \phi$$

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So the signals received by R_2 from A and C are

$$y_A = a\cos(\omega t - \phi)$$
$$y_C = a\cos(\omega t - \phi)$$

Signals picked up by R_2 from A, B, C and D

$$y_{R_2} = y_A + y_B + y_C + y_D$$
$$= a\cos(\omega t - \phi) + a\cos\omega t + a\cos(\omega t - \phi) - a\cos\omega t$$
$$= 2a\cos(\omega t - \phi) \qquad \left(\because \langle I \rangle = \frac{1}{2}(a)^2\right)$$

$$\langle I_{R_2} \rangle = \frac{1}{2}[4a^2] \quad \Rightarrow \quad \langle I_{R_2} \rangle = 2a^2 \qquad \boxed{\because I = A^2}$$

Thus, R_2 picks up the larger signal than R_1.

(*ii*) If B is switched off,

(*a*) Signals picked up by

$$R_1 = y_A + y_C + y_D$$
$$R_1 = a\cos\omega t + a\cos\omega t - a\cos\omega t = a\cos\omega t$$

$$\therefore \qquad (I_{R_1})_{av} = \frac{1}{2}a^2 \qquad \text{...(I)}$$

(*b*) Signals picked up by

$$R_2 = y_A + y_C + y_D$$
$$= a\cos(\omega t - \phi) + a\cos(\omega t - \phi) - a\cos\omega t$$

Net signals at

$$R_2 = 2a\cos(\omega t - \phi) - a\cos(\omega t) = a[2\cos(\omega t - \phi) - \cos(\omega t)]$$

$$\langle I_{R_2} \rangle = \frac{a^2}{2} \qquad \text{...(II)}$$

$$\langle I_{R_1} \rangle = \langle I_{R_2} \rangle$$

So R_1 and R_2 picks up the signals of the same intensities.

(*iii*) If D is switched off

(*a*) Signals picked up by receiver R_1

$$R_1 = y_B + y_C + y_A = -a\cos\omega t + a\cos\omega t + a\cos\omega t$$
$$y_{R_1} = a\cos\omega t$$

$$\therefore \quad \langle I_{R_1} \rangle = \frac{1}{2}a^2$$

(*b*) Signals picked up by receiver R_2

$$R_2 = y_A + y_B + y_C$$
$$= a\cos(\omega t - \phi) + a\cos\omega t + a\cos(\omega t - \phi)$$
$$= 2a\cos(\omega t - \phi) + a\cos\omega t$$

$$\therefore \qquad \frac{2\pi\lambda}{a} = \phi \text{ is very small so can be neglected.}$$

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So signal received by

$$R_2 = 2a\cos\omega t + a\cos\omega t = 3a\cos\omega t$$

$$\langle I_{R_2}\rangle = \frac{9a^2}{2}$$

$$\langle I_{R_1}\rangle < \langle I_{R_2}\rangle$$

So R_2 picks up larger signal compared to R_1.

(*iv*) A signal at R_1 indicates that B has been switched off.
A signal at R_2 indicates that D has been switched off.

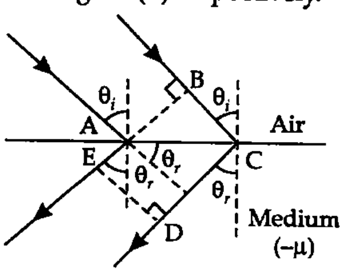
Q10.22. The optical properties of a medium are governed by the relative permittivity (ε_r) and relative permeability (μ_r). The refractive index is defined as $\sqrt{\varepsilon_r\mu_r} = \mu$. For ordinary material $\varepsilon_r > 0$ and $\mu_r > 0$ and the positive sign is taken for the square root.

In 1964, a Russian scientist V. Veselago postulated the existence of a material with $\varepsilon_r < 0$ and $\mu_r < 0$. Since then such metamaterials have been produced in the laboratories and their optical properties studied. For such materials $\mu = -\sqrt{\mu_r\varepsilon_r}$. As light enters a medium of such refractive index the phases travel away from the direction of propagation.

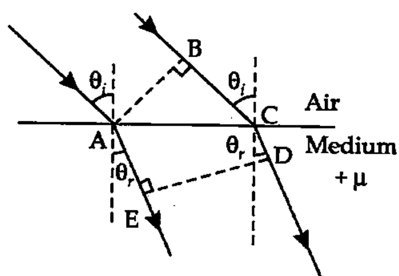
(*i*) According to the description above, show that if rays of light enter such a medium from air ($\mu = 1$) and an angle θ in 2nd quadrant then the refracted beam is in the 3rd quadrant.

(*ii*) Prove that Snell's law holds for such a medium.

Ans. (i) Let the postulates $-\sqrt{\mu_r\varepsilon_r} = \mu$ and $+\sqrt{\mu_r\varepsilon_r} = \mu$ are true then two parallel rays would proceed as shown in figure (a) and figure (b) respectively.



(a)



(b)

Let two parallel rays at incidence angle θ_i from air would proceed in medium as shown in figures above. ED shows a wavefront, then all the points on ED will remain in same phase. All the points with the same optical path length must have the same phase.

so $$-\sqrt{\mu_r\varepsilon_r}\,AE = BC - \sqrt{\mu_r\varepsilon_r}\,CD$$

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$$BC = \sqrt{\mu_r \varepsilon_r}\,(CD - AE)$$
$$BC > 0;\ CD > AE$$

As showing that the postulate is reasonable if however, the light proceeded in the sense it does for ordinary material *e.g.*, refracted rays are in IV quadrant in figure (b) then

$$-\sqrt{\varepsilon_r \mu_r}\,AE = BC - \sqrt{\mu_r \varepsilon_r}\,CD$$
$$BC = \sqrt{\mu_r \varepsilon_r}\,(CD - AE)$$
$$AE > CD \text{ hence } BC < 0$$

Figure showing that BC < 0 this is not possible. Hence the given postulate is correct.

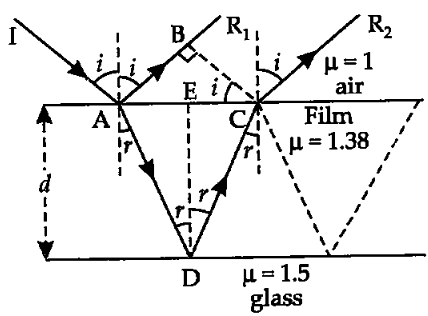
(*ii*) From fig. (a)

$$BC = AC \sin\theta_i$$
$$CD - AE = AC \sin\theta_r$$
$$BC = \sqrt{\mu_r \varepsilon_r} \qquad \text{(From Figure CD – AE = BC)}$$
$$AC \sin\theta i = \sqrt{\mu_r \varepsilon_r}\,AC \sin\theta_r$$
$$\frac{\sin\theta_i}{\sin\theta_r} = \sqrt{\mu_r \varepsilon_r}$$

which proves the Snell's law.

Q10.23. To ensure almost 100% transmittivity, photographic lenses are often coated with thin layer of dielectric material. The refractive index of this material is intermediated between that of air and glass (which makes the optical element of the lens). A typically used dielectric film is Mg F_2 ($\mu = 1.38$). What should be the thickness of the film so that at the centre of the visible spectrum (5500 Å) there is maximum transmission?

Ans. In the given figure, incidence ray IA incident at point A from air to film surface with incident angle *i*. Here at A, it gets partial reflection and refraction, passes through paths AR_1 and AD respectively. At D it again gets partial reflection (and refraction) from the glass and film interface surface. At C the interface surface of film and air and finally after refraction from C pass through path CR_2 parallel to AR_1.



The amplitude (intensity) of wave during refraction and reflection decreases.

If the interference due to two reflected rays AR_1 and CR_2 is destructive interference, then the reflected rays AR_1 and CR_2 will not dominant.

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Both reflections are from lower to higher refractive index surfaces so, optical path difference between AR_1 and CR_2 will be

$$\mu(AD + CD) - AB \quad ...(I)$$

If d is the thickness of film then,

$$AD = AC = \frac{d}{\cos r} \quad ...(II)$$

$$AB = AC \sin i$$

or $$\tan r = \frac{\frac{AC}{2}}{d}$$

∴ $$d \tan r = \frac{AC}{2}$$

$$AC = 2d \tan r$$

or $$AB = 2d \tan r \sin i \quad ...(III)$$

So the optical path difference from (I)

$$= \mu\, AD + \mu\, AD - AB \quad (\because AD = CD)$$

$$= 2\,\mu\, AD - AB$$

$$= \frac{2\mu d}{\cos r} - 2d \tan r \sin i \quad \text{(From II and III)}$$

$$= 2 \cdot \frac{\sin i}{\sin r} \cdot \frac{d}{\cos r} - 2d \cdot \frac{\sin r}{\cos r} \cdot \sin i = \frac{2d \sin i}{\cos r}\left[\frac{1}{\sin r} - \frac{\sin r}{1}\right]$$

$$= \frac{2d \sin i\,(1 - \sin^2 r)}{\cos r \sin r} = \frac{2d \sin i \cos^2 r}{\sin r \cos r}$$

optical path difference between AR_1 and AR_2

$$= 2d\,\mu \cdot \cos r \quad \left(\because \mu = \frac{\sin i}{\sin r}\right)$$

For two rays AR_1 and CR_2 to interfere destructively, path difference should be $\frac{\lambda}{2}$.

∴ $$2d\,\mu \cos r = \frac{\lambda}{2}$$

$$\mu d \cos r = \frac{\lambda}{4}$$

For photographic lenses the sources are vertical planes *i.e.*, rays incident at very small angle.

so $$i = r \approx 0$$

∴ $$\mu d \approx \frac{\lambda}{4} \quad (\because \cos 0 = 1)$$

$$d = \frac{\lambda}{4\mu} = \frac{5500\ \text{Å}}{1.38 \times 4}$$

$$d = 1000\ \text{Å}$$

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11 Dual Nature of Radiation and Matter

MULTIPLE CHOICE QUESTIONS—I

Q11.1. A particle is dropped from a height H. The de-Broglie wavelength of the particle as a function of height is proportional to

(*a*) H (*b*) $H^{1/2}$ (*c*) H^0 (*d*) $H^{-1/2}$

Main concept used: de-Broglie wavelength $\lambda = \frac{h}{p}$

Ans. (*d*): Velocity of freeely falling body after falling from a height

$H = v = \sqrt{2gH}$

We know that de-Broglie wavelength $\lambda = \frac{h}{p}$

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2gH}}$$

h, *m*, and *g* are constants

$$\therefore \quad \frac{h}{m\sqrt{2g}} \text{ is canstant} \Rightarrow \lambda \propto \frac{1}{\sqrt{H}} \Rightarrow \lambda \propto H^{-1/2}$$

Q11.2. The wavelength of a photon needed to remove a proton from a nucleus which is bound to the nucleus with 1 MeV energy is nearly

(*a*) 1.2 nm (*b*) 1.2×10^{-3} nm (*c*) 1.2×10^{-6} nm (*d*) 1.2×10^{1} nm

Main concept used: Energy of photon = $\frac{hc}{\lambda}$

Ans. (*b*): Energy of the photon must be equal to the binding energy of proton

so Energy of photon = 1 MeV = $10^6 \times 1.6 \times 10^{-19}$ J

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-13}} = \frac{6.63 \times 3}{1.60} \times 10^{-26+13}$$

$$= \frac{19.89}{1.60} \times 10^{-13} = 12.4 \times 10^{-13} = 1.24 \times 10^1 \times 10^{-13}$$

$$= 1.24 \times 10^{-9} \times 10^{-3} = \mathbf{1.24 \times 10^{-3}\ nm}$$

Q11.3. Consider a beam of electrons (each electron with energy E_0) incident on a metal surface kept in an evacuated chamber. Then

(*a*) no electrons will be emitted as only photons can emit the electrons.

(*b*) electrons can be emitted but all with an energy E_0.

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(c) electrons can be emitted with any energy with a maximum of $E_0 - \phi$ (ϕ is work function).

(d) electrons can be emitted with any energy with a maximum of E_0.

Main concept used: Elastic collision and work function.

Ans. (d): When a beam of electrons of energy E_0 of each electron incident on a metal surface kept in vacuum, then due to elastic collisions with electrons on surface, energy of incident electrons will be transferred to the emitted electrons. To emit the electrons below the surface a part of E_0 of incident electrons is consumed against work function so energy of emitted electrons becomes less than E_0.

So, the maximum energy of emitted electrons can be E_0.

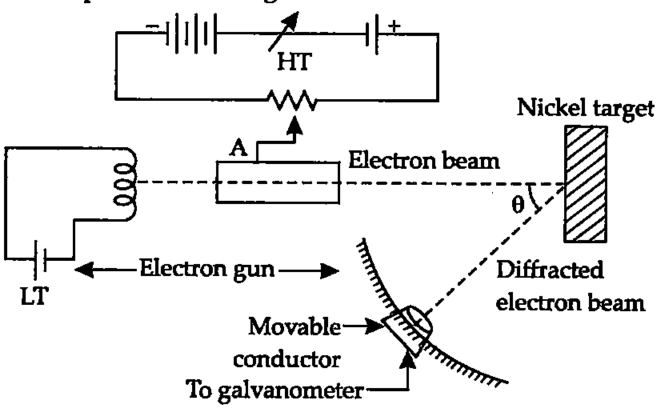
Q11.4. Consider figure given below. Suppose the voltage applied to A is increased. The diffracted beam will have the maxima at a value of θ that

(a) will be larger than the earlier value.

(b) will be the same as the earlier value.

(c) will be less than the earlier value.

(d) will depend on the target.



Main concept used: Wave nature of cathode rays in Davisson-Germer experiment, $\lambda_d = \frac{12.27}{\sqrt{V}}$ Å, diffraction of waves. In interference of electrons from the crystalline layers

$$2d \sin\theta = \lambda$$

Ans. (c): In Davisson-Germer experiment, the de-Broglie wavelength of diffracted beam of electrons

$$\lambda_d = \frac{12.27}{\sqrt{V}} \text{ Å} \quad ...(i)$$

V is the applied voltage. If there is a maxima of the diffracted electrons at an angle θ, then

$$2d \sin\theta = \lambda \quad ...(ii)$$

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From equation (i), as the applied voltage in this experiment increases, the wave length λ_d decreases in turn sin θ or θ decreases by relation (ii). Hence verifies the option (*c*).

Q11.5. A proton, a neutron, an electron and an α-particle have the same energy. Then their de-Broglie wavelengths compared as

(*a*) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$ (*b*) $\lambda_\alpha < \lambda_p = \lambda_n > \lambda_e$
(*c*) $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$ (*d*) $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$

Main concept used: $\lambda_d = \frac{h}{\sqrt{2mk}}$, $K = \frac{1}{2}mv^2$, $\lambda_d = \frac{h}{p}$

Ans. (*b*): de-Broglie wavelength $\lambda_d = \frac{h}{p}$

$$E_p = E_n = E_e = E_\alpha$$

$$\text{K.E.} = K = \frac{1}{2}mv^2 \qquad \therefore \quad \lambda_d = \frac{h}{p}$$

$$2K = mv^2 \qquad \lambda d = \frac{h}{\sqrt{2mK}}$$

$$2Km = m^2v^2$$

$$2mK = p^2$$

$$\sqrt{2mK} = p$$

or $$\lambda = \frac{h}{\sqrt{2mK}}$$ [as h and E (K.E.) is constt.]

$$\therefore \quad \lambda \propto \frac{1}{\sqrt{m}}$$

$$m_\alpha > m_p = m_n > m_e$$

$$\therefore \quad \lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$$

Q11.6. An electron is moving with an initial velocity $v = v_0\hat{i}$ and is in a magnetic field $B = B_0\hat{j}$. Then its de-Broglie wavelength

(*a*) remains constant. (*b*) increases with time.
(*c*) decreases with time. (*d*) increases and decreases periodically.

Main concept used: $F = q(v \times B)$ and $\lambda = \frac{h}{p}$

Ans. (*a*): Given $\vec{v} = v_0\hat{i}$ and $B = B_0\hat{j}$

Force on moving electron in perpendicular magnetic field B is

$$F = -e(\vec{v} \times \vec{B})$$
$$= -e[v_0 i \times B_0\hat{j}]$$
$$= -ev_0B_0\, \hat{i} \times \hat{j}$$
$$F = -ev_0B_0\hat{k}$$

So the force is perpendicular to v and B both as the force is perpendicular to the velocity. So will not change v or mv so the de-Broglie wavelength remains same.

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Q11.7. An electron (mass m) with an initial velocity $v = v_0\hat{i}$ $(v_0 > 0)$ is in an electric field $E = -E_0\hat{i}$ (E_0 = constant > 0). It's de-Broglie wavelength at time t is given by

(*a*) $\dfrac{\lambda_0}{\left[1+\dfrac{eE_0}{m}\dfrac{t}{v_0}\right]}$ (*b*) $\lambda_0\left[1+\dfrac{eE_0}{mv_0}t\right]$ (*c*) λ_0 (*d*) $\lambda_0 t$

Main concept used: $\lambda_0 = \dfrac{h}{mv}$, $F = qE$, $a_e = \dfrac{qE}{m}$

Ans. (*a*): Initial de-Broglie wavelength $\lambda_0 = \dfrac{h}{mv_0}$

Force on electron = $F = qE \Rightarrow F = (-e)(-E_0 i)$

$$ma = eE_0\hat{i}$$

$$a = \frac{eE_0}{m}\hat{i}$$

Velocity of electron after time t is $v = v_0 + at$

$$v = v_0 i + \frac{eE_0}{m} i \cdot t$$

$$v = \left[v_0 + \frac{eE_0 t}{m}\right]\hat{i}$$

$\therefore$ New de-Broglie wavelength $\lambda = \dfrac{h}{mv}$

$$\lambda = \frac{h}{m\left[v_0 + \dfrac{eE_0 t}{m}\right]i} = \frac{h}{mv_0\left[1+\dfrac{eE_0 t}{mv_0}\right]}$$

$$\lambda = \frac{\lambda_0}{\left[1+\dfrac{eE_0 t}{mv_0}\right]} \qquad \left(\because \frac{h}{mv_0} = \lambda_0 \text{ from eqn. I}\right)$$

Q11.8. An electron (mass m) with an initial velocity $v = v_0\hat{i}$ is in an electric field $E = E_0\hat{j}$. If $\lambda_0 = \dfrac{h}{mv_0}$, its de-Broglie wavelength at time t is given by

(*a*) λ_0 (*b*) $\lambda_0\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}$ (*c*) $\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}}$ (*d*) $\dfrac{\lambda_0}{\left[1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}\right]}$

Main concept used: $\lambda_0 = \dfrac{h}{mv}$, $F = qE$, $a_e = \dfrac{eE}{m}$

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Ans. (*c*): Initial de-Broglie wavelength $\lambda_0 = \frac{h}{mv_0}$. Force on moving electron due to electric field E = $F = -eE = -eE_0\hat{j}$. Acceleration in electron due to force by electric field,

$$ma = -eE_0\hat{j}$$

$$a = \frac{-eE_0}{m}\hat{j}$$

Acceleration on electron acts in $-Y$ direction the initial velocity of electron along X-axis,

$$v_{x_0} = v_0\hat{i}$$

Initial velocity of electron in Y direction = 0

$$\therefore \quad v_{y_0} = 0$$

Velocity of electron after time t along X-axis

$$v_x = v_0\hat{i}$$

So velocity of electron after time *t* along Y-axis

$$v = u + at$$

$$v_y = 0 + \left(\frac{-eE_0}{m}\hat{j}\right)t$$

$$v_y = \frac{-eE_0}{m}\hat{j}\,t$$

Magnitude of velocity of electron after time *t* is

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\vec{v} = \sqrt{v_0^2 + \left(\frac{-eE_0\hat{j}t}{m}\right)^2}$$

$$|v| = v_0\sqrt{1 + e^2E_0^2t^2}$$

$$\therefore \quad \lambda' = \frac{h}{mv} = \frac{h}{mv_0\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}}$$

$$\lambda' = \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}} \qquad \left(\because \lambda_0 = \frac{h}{mv_0}\right)$$

Hence, option (*c*) is verified.

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MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q11.9. Relativistic corrections become necessary when the expression for the kinetic energy $\frac{1}{2}mv^2$, becomes comparable with mc^2, where m is the mass of the particle. At what de Broglie wavelength, will relativistic corrections becomes important for an electron?

(*a*) $\lambda = 10$ nm (*b*) $\lambda = 10^{-1}$ nm

(*c*) $\lambda = 10^{-4}$ nm (*d*) $\lambda = 10^{-6}$ nm

Main concept used: Relativistic correction become necessary when speed of particle is either more than speed of light or comparable to it.

Ans. (*c*) and (*d*): de-Broglie wavelength $\lambda = \frac{h}{mv}$

$$v = \frac{h}{m\lambda} \qquad \begin{pmatrix} h = 6.6 \times 10^{-34} \\ m = 9 \times 10^{-31} \text{kg} \end{pmatrix}$$

$$= \frac{6.6 \times 10^{-34}}{9 \times 10^{-31} \lambda} = \frac{6.6 \times 10^{-34+31}}{9\lambda} = \frac{0.73 \times 10^{-3}}{\lambda}$$

$$= \frac{7.3 \times 10^{-4}}{\lambda}$$

For option

(*a*) $\lambda = 10 \text{ nm} = 10 \times 10^{-9} \text{ m} = 10^{-8} \text{ m}$

$\therefore \quad v = 7.3 \times 10^{-4} \times 10^{+8} = 7.3 \times 10^4 < 3 \times 10^8$ (Speed of light)

(*b*) $\lambda = 10^{-1} \text{ nm} = 10^{-1} \times 10^{-9} \text{ m} = 10^{-10} \text{ m}$

$\therefore \quad v = \frac{7.3 \times 10^{-4}}{10^{-10}} = 7.3 \times 10^{-4+10} = 7.3 \times 10^6 \approx 10^7 < 10^8$ (Speed of light)

(*c*) $\lambda = 10^{-4} \text{ nm} = 10^{-4} \times 10^{-9} \text{ m} = 10^{-13} \text{ m}$

$v = \frac{7.3 \times 10^{-4}}{10^{-13}} = 7.3 \times 10^{-4+13} = 7.3 \times 10^9 \approx 10^{10} > 10^8$ (Speed of light)

(*d*) $\lambda = 10^{-6} \text{ nm} = 10^{-15} \text{ m}$

$v = 7.3 \times 10^{-4} \times 10^{15} = 7.3 \times 10^{11} \approx 10^{12} > 10^8$ (Speed of light)

So the velocity of electron is more for option (*c*) and (*d*) where the relativistic correction become necessary although the speed of electron is 7.3×10^6 m/s is comparable with (c) speed of light, there must be relativistic correction. Options are (*c*) and (*d*).

Q11.10. Two particles A_1 and A_2 of masses m_1 and m_2 ($m_1 > m_2$) have the same de-Broglie wavelength. Then

(*a*) their momenta are the same.

(*b*) their energies are the same.

(*c*) energy of A_1 is less than the energy of A_2.

(*d*) energy of A_1 is more than the energy of A_2.

Main concept used: $\lambda_d = \frac{h}{p}$

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Ans. (*a*) and (*c*): $\lambda = \frac{h}{p}$

$$p = \frac{h}{\lambda} \quad \text{or} \quad p \propto \frac{1}{\lambda}$$

$$\frac{p_1}{p_2} = \frac{\lambda_2}{\lambda_1}$$

$\because$ $\lambda_1 = \lambda_2 = \lambda$ (given)

$\therefore$ $p_1 = p_2$ verifies Ans. (*a*)

$$E_a = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2\,\text{m}}$$

$E \propto \frac{1}{m}$ [as $p_1 = p_2$ = const. (as proved above)]

$$\frac{E_1}{E_2} = \frac{m_2}{m_1}$$

$\frac{E_1}{E_2} < 1$ [$\because m_1 > m_2$ (given)]

$\therefore$ $E_2 > E_1$ verifies answer (*c*)

Q11.11. The de-Broglie wavelength of a photon is twice the de-Broglie wavelength of an electron. The speed of the electron $v_e = \frac{c}{100}$. Then

(*a*) $\frac{E_e}{E_p} = 10^{-4}$ (*b*) $\frac{E_e}{E_p} = 10^{-2}$ (*c*) $\frac{p_e}{m_e c} = 10^{-2}$ (*d*) $\frac{p_e}{m_e c} = 10^{-4}$

Main concept used: $\lambda_d = \frac{h}{p}$ and $p^2 = 2mK$

Ans. (*b*) and (*c*): de-Broglie wavelength $\lambda = \frac{h}{p}$

$\therefore$ $$\lambda_e = \frac{h}{m_e v_e} = \frac{h}{m_e \frac{c}{100}}$$

$$= \frac{100\,h}{m_e c} \quad ...(I)$$

Now K.E. $= \frac{1}{2}mv^2 \Rightarrow E = \frac{m^2v^2}{2m}$ ...(II)

$\Rightarrow$ $m_e v_e = \sqrt{2m_e E_e}$

Now $\lambda_e = \frac{h}{m_e v_e} = \frac{h}{\sqrt{2m_e E_e}}$ [from (II)]

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$\Rightarrow$ $$E_e = \frac{h^2}{2m_e\lambda_e^2} \quad ...(III)$$

For proton $\lambda_p = 2\lambda_e$ [given]

$\therefore$ $$E_p = \frac{hc}{\lambda_p} = \frac{hc}{2\lambda_e}$$

Now $$\frac{E_p}{E_e} = \frac{hc}{2\lambda_e} \times \frac{2m_e\lambda_e^2}{h^2} = \frac{\lambda_e m_e c}{h} \quad \text{[from (III)]}$$

$$= \frac{100\,h}{m_e c} \times \frac{m_e c}{h} = 100$$

$\Rightarrow$ $$\frac{E_e}{E_p} = 10^{-2} \quad \text{[verifies Ans. (b)]}$$

Now $$p_e = m_e v_e = m_e \times \frac{c}{100} \quad \left(\because v_e = \frac{c}{100}\right)$$

$\Rightarrow$ $$\frac{p_e}{m_e c} = \frac{1}{100} = 10^{-2} \quad \text{[verifies Ans. (c)]}$$

Q11.12. Photons absorbed in matter are converted to heat. A source emitting n photons per-second of frequency ν is used to convert 1 kg of ice at 0°C to water at 0°C. Then the time T taken for the conversion

(*a*) decreases with increasing n with ν fixed.
(*b*) decreases with n fixed ν increasing.
(*c*) remains constant with n and ν changing such that nν is a constant.
(*d*) increases when the product nν increases.

Main concept used: $E = nh\nu$ and $mL = E$ (Heat)

Ans. (*a*), (*b*) and (*c*): Heat energy required to convert 1 kg (1000 gm) of ice at 0°C to water at 0°C is

$E = mL$

$E' = nh\nu$

n = no. of photons incident per second.

Let time T is taken by radiation to melt the ice at 0°C. Then,

$$E = n\,h\nu\,t$$

$$mL = n\,h\nu\,t$$

$$t = \frac{mL}{nh\nu}$$

$\therefore$ $$t \propto \frac{1}{n} \quad \text{and} \quad T \propto \frac{1}{\nu}$$

or $$t \propto \frac{1}{n\nu} \quad [\because m,\ L \text{ and } h \text{ are constants}]$$

[verifies Ans. (*a*), (*b*), and (*c*)]

$\therefore$ if nν, increases then, T decreases so does not verify answer (*d*).

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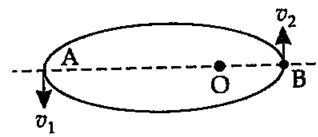
Q11.13. A particle moves in a closed orbit around the origin, due to a force which is directed towards the origin. The de-Broglie wavelength of the particle varies cyclically between two values λ_1 and λ_2 ($\lambda_1 > \lambda_2$). Which of the following statement are true?

(*a*) The particle could be moving in a circular orbit with origin as centre.

(*b*) The particle could be moving in an elliptical orbit with origin as its focus.

(*c*) When the de-Broglie wavelength is λ_1, the particle is nearer the origin than when its value is λ_2.

(*d*) When the de-Broglie wavelength is λ_2 the particle is nearer the origin than when its value is λ_1.



Ans. (*b*) and (*d*): As the de-Broglie wavelength of particle varies cyclically between two values λ_1 and λ_2. It is possible when the particle is moving in elliptical orbit with origin as one of its focus. Because if λ_1 and λ_2 are equal, their speed must be equal and particle must move in circular orbit. Hence it verifies answer (*b*).

Let v_1 and v_2 be the speeds of particle at A and B respectively and origin is at O. If λ_1 and λ_2 are the de-Broglie wavelengths associated with particle at A and B respectively, then

$$\lambda_1 = \frac{h}{mv_1} \quad \text{and} \quad \lambda_2 = \frac{h}{mv_2}$$

so $$\frac{\lambda_1}{\lambda_2} = \frac{v_2}{v_1} \qquad [\because \lambda_1 > \lambda_2] \quad \text{(Given)}$$

so $$v_1 < v_2 \qquad \text{...(I)}$$

By the law of conservation of angular momentum the speed of the particle will be more when it is closer to focus. It is verified by I.

∴ So the object is close to B than A or the particle is nearer to the origin when wavelength is λ_2 than when wavelength is λ_1. Hence verifies answer (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q11.14. A proton and an α-particle are accelerated using the same potential difference. How are the de-Broglie wavelength. λ_p and λ_α related to each other?

Main concept used: As both proton and α-particles are accelerated at same potential so their K.Es. will be same *i.e.*, $K_1 = K_2 = K = qV$ by relation ($E = eV$) and

$$\lambda = \frac{h}{\sqrt{2mK}}$$

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Ans. As we know that both the particles are accelerated at the same potential difference so their K.Es. will be equal *i.e.,*

$$K_1 = K_2 = K = qV \qquad \left(\because \; V = \frac{W}{q}\right)$$

So
$$\lambda = \frac{h}{\sqrt{2mK}} \quad \text{or} \quad \lambda_d = \frac{h}{\sqrt{2mqV}}$$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \frac{h}{\sqrt{2m_p q_p V_p}} \times \frac{\sqrt{2m_\alpha q_\alpha V_a}}{h}$$

$$m_\alpha = 4m_p; \quad q_\alpha = 2e; \quad q_p = e; \quad V_p = V_\alpha = V \text{ (P.D. applied)}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{2 \times 4\, m_p\, 2e\, V}{2m_p\, eV}}$$

$$\lambda_p = \sqrt{8}\, \lambda_\alpha$$

So, the de-Broglie wavelength of proton is $\sqrt{8}$ times of alpha (α) particle.

Q11.15. (*i*) In an explanation of photoelectric effect, we assume one photon of frequency ν collides with an electron and transfers its energy. This leads to the equation for maximum energy $E_{max.}$ of the emitted electron as $E_{max} = h\nu - \phi_0$ where ϕ_0 is the work function of the metal. If an electron absorbs two photons (each of frequency ν) what will be the maximum energy for the emitted electron?

(*ii*) Why is this fact (two photons absorption) not taken into consideration in our discussion of stopping potential ?

Ans. (*i*) Here, 2 photons transfer its energy to one electron as $E = h\nu$

$$\therefore \quad E_e = E_p$$
$$h\nu_e = 2h\nu$$
$$\therefore \quad \nu_e = 2\nu$$

Maximum energy of emitted electron is

$$E_{max} = h\nu_e - \phi_0 = h(2\nu) - \phi_0 = 2h\nu - \phi_0$$

(*ii*) The probability of absorbing 2 photons by electron is very low due to their mass difference. So possibilities of such emission of electrons is negligible.

Q11.16. There are materials which absorb photons of shorter wavelength and emit photons of longer wavelength. Can there be stable substances which absorb photons of longer wavelength and emit light of shorter wavelength?

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Ans. We know that as the wavelength of photon increases, its frequency decreases or energy increases.

Case I: Photons of shorter wavelength *i.e.*, of larger energy emits the photons of smaller energy here some energy, is consumed against work function. So, it is possible by law of conservation of energy.

Case II: Photons of longer wavelength always emits photons of shorter wavelength and photons of smaller energy can never emits photons of larger energy as some part of energy ($h\nu$) is consumed in work function (ϕ) of metal. Otherwise it will discard the law of conservation of energy (universal law) so, it cannot be possible in stable materials.

Q11.17. Do all the electrons that absorb a photon come out as photoelectrons?

Ans. In photoelectric effect, we can observe that most of the electrons knocked by photons are scattered into the metal by absorbing photons.

So number of emitted electrons are very small than number of photons absorbed.

One photon cannot emit one electron generally. Some energy of photons is consumed against work function of metal.

So, not all the electrons that absorb a photon comes out from metal surface.

Q11.18. There are two sources of light each emitting with a power of 100 W. One emits X-rays of wavelength 1 nm and the other visible light at 500 nm. Find the ratio of number of photons of X-rays to the photons of visible light of the given wavelength.

Ans. Let E_x and E_v are the energies given by one photon in X-rays and visible rays then,

$$E_x = h\nu_x \quad \text{and} \quad E_v = h\nu_v$$

Let n_x and n_v are number of photons in X-rays and visible light to give equal energies as both sources (X-rays and visible) emitting same power 100 W each so

$$n_x h\nu_x = n_v h\nu_v$$

$$\frac{n_x}{n_v} = \frac{\nu_v}{\nu_x} = \frac{\frac{1}{\lambda_v}}{\frac{1}{\lambda_x}} = \frac{\lambda_x}{\lambda_v} \qquad \left(\because \nu = \frac{c}{\lambda} \text{ or } \nu = \frac{1}{\lambda}\right)$$

$$\frac{n_x}{n_v} = \frac{1\text{ nm}}{500\text{ nm}} \qquad \left(\begin{matrix}\because \lambda_x = 1\text{ nm} \\ \text{and } \lambda_v = 500\text{ nm}\end{matrix}\right) \quad \text{[Given]}$$

$$\therefore \qquad n_x : n_v = 1 : 500$$

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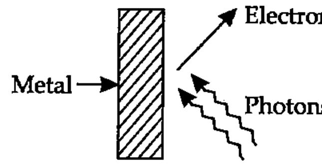
SHORT ANSWER TYPE QUESTIONS

Q11.19. Consider figure for photo-emission. How would you reconcile with momentum conservation? Note light (Photons) have momentum in a different direction, than the emitted electrons.



Main concept used: Momentum is vector quantity.

Ans. When photons strike to metal surface, photons transfer their momentum to atoms of metal by decreasing its own speed upto zero. This momentum of photons transfer to nucleus and electrons of the metal.

The exited electrons emit approximately opposite to the direction of photons. But total momentum transferred by the photons will be equal to the momentum of all electrons and nucleus.

Q11.20. Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. Find the work function in eV.

Main concept used: $E_{max} = h\nu - \phi$

Ans. Let the maximum energies of emitted electrons are K_1 and K_2 when 600 nm and 400 nm visible light are used according to question

$$K_2 = 2K_1$$

$$K_{max} = h\nu - \phi = \frac{hc}{\lambda} - \phi$$

$$K_1 = \frac{hc}{\lambda_1} - \phi$$

$$K_2 = \frac{hc}{\lambda_2} - \phi = 2K_1$$

$$\frac{hc}{\lambda_2} - \phi = 2\left[\frac{hc}{\lambda_1} - \phi\right] = \frac{2hc}{\lambda_1} - 2\phi$$

$$\phi = hc\left[\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right] \qquad (\because \; hc = 1240 \text{ eV nm})$$

$$\therefore \quad \phi = 1240\left[\frac{2}{600} - \frac{1}{400}\right]\text{eV} = \frac{1240}{200}\left[\frac{2}{3} - \frac{1}{2}\right] = 6.2\frac{(4-3)}{6}$$

Work function $\phi = \frac{6.2}{6} = 1.03$ eV.

Q11.21. Assuming an electron is confined to a 1 nm wide region, find the uncertainty in momentum using Heisenberg uncertainty principle. ($\Delta x \times \Delta p \cong h$). You can assume the uncertainty in position Δx as 1 nm. Assuming $p \approx \Delta p$, find the energy of the electron in electron volt.

Ans. As electron revolves in circular path so $\Delta r = 1 \text{ nm} = 10^{-9}$ m

$$\Delta x \times \Delta p \cong h \qquad \text{[Given]}$$

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$$\Delta p = \frac{h}{\Delta x} = \frac{6.62 \times 10^{-34}}{2\pi\, \Delta r} \text{JS}$$

$$\therefore \quad \Delta p = \frac{6.62 \times 10^{-34}}{2 \times 3.14 \times 10^{-9}} \text{ kg m/s}$$

$$= \frac{331}{314} \times 10^{-34+9} \quad \text{or} \quad \Delta p = \frac{331}{314} \times 10^{-25}$$

$$\therefore \quad E = \frac{1}{2} mv^2 = \frac{m^2v^2}{2m} = \frac{\Delta p^2}{2m}$$

$$= \frac{331}{314} \times \frac{331}{314} \times \frac{10^{-25} \times 10^{-25}}{2 \times 9.1 \times 10^{-31}} = \frac{331 \times 331 \times 10^{-50+31}}{314 \times 314 \times 18.2} \text{J}$$

$$= \frac{331 \times 331 \times 10^{-19} \times 1.6 \times 10^{-19}}{314 \times 314 \times 18.2} e$$

$$= \frac{331 \times 331 \times 16}{314 \times 314 \times 182} = 3.8 \times 10^{-2} \text{ eV}$$

Q11.22. Two monochromatic beams of light A and B of equal intensity I, hit a screen. The number of photons hitting the screen by beam A is twice that by beam B. Then, what inference can you make about their frequencies?

Main concept used: Intensity or Energy of photons $E = nh\nu$

Ans. $I_A = I_B$ [Given]

$$n_A h\nu_A = n_B h\nu_B$$

$\because \quad n_A = 2n_B$ [Given]

$$\therefore \quad 2n_B \nu_A = n_B \nu_B$$

$$2\nu_A = \nu_B$$

So the frequency of source B is twice the frequency of source A.

Q11.23. Two particles A and B of de-Broglie wavelengths λ_1 and λ_2 combine to form a particle C. The process conserves the momentum. Find the de-Broglie wavelength of the particle C (The motion is one dimensional).

Ans. de-Broglie wavelengths

$$\lambda = \frac{h}{p} \quad \text{or} \quad p = \frac{h}{\lambda} \Rightarrow p_1 = \frac{h}{\lambda_1}, \quad p_2 = \frac{h}{\lambda_2} \quad \text{and} \quad p_3 = \frac{h}{\lambda_3}$$

$$p_1 + p_2 = p_3$$

$$\frac{h}{\lambda_1} + \frac{h}{\lambda_2} = \frac{h}{\lambda_3}$$ (λ_3 = the wavelength of particle C)

$$\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$$

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$$\frac{\lambda_2 + \lambda_1}{\lambda_1\lambda_2} = \frac{1}{\lambda_3}$$

$$\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$$

Case I: When p_1 and p_2 are positive then $\lambda_3 = \dfrac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$

Case II: When p_1 and p_2 both are negative

$$\lambda_3 = \left|\frac{-\lambda_1\lambda_2}{\lambda_1 + \lambda_2}\right| = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$$

Case III: $p_A > 0, p_B < 0$ or

$$\frac{h}{\lambda_3} = \frac{h}{\lambda_1} - \frac{h}{\lambda_2} \quad \text{or} \quad \frac{1}{\lambda_3} = \frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}$$

$$\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}$$

Case IV: $p_A < 0, p_B > 0$

$$\frac{h}{\lambda_3} = \frac{h}{\lambda_2} - \frac{h}{\lambda_1} = \frac{(\lambda_1 - \lambda_2)h}{\lambda_1\lambda_2}$$

$$\therefore \quad \lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

Q11.24. A neutron beam of energy E scatters from atoms on a surface, with a spacing $d = 0.1$ nm. The first maxima of intensity in the reflected beam occur at $\theta = 30°$. What is the kinetic energy E of the beam in eV?

Main concept used: Bragg's law of diffraction $2d \sin\theta = n\lambda$ (maxima)

$$E = \frac{p^2}{2m}, \quad p = \frac{h}{\lambda}$$

Ans. By Bragg's law of diffraction, condition for nth maxima is

$$2d \sin\theta = n\lambda$$

$n = 1$ so $\lambda = 2d \sin\theta$ [$\theta = 30°$ (Given)]

$= 2 \times 0.1 \times 10^{-9} \sin 30°$ ($\because$ $d = 0.1$ nm)

$= 0.1 \times 10^{-9}$ m $= 10^{-10}$ m

$$p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{10^{-10}} = 6.6 \times 10^{-24} \text{ kg m/s}$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$$

$$\therefore \quad E = \frac{6.6 \times 6.6 \times 10^{-2.4} \times 10^{-24}}{2 \times 1.6 \times 10^{-27}} \text{ J}$$

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$$= \frac{6.6 \times 6.6 \times 10^{-48}}{2 \times 1.6 \times 10^{-27} \times 1.6 \times 10^{-19}} \text{ eV} = \frac{66 \times 66 \times 10^{-48+46}}{2 \times 16 \times 16}$$

$$E = \frac{33 \times 33}{128} \times 10^{-2} = 8.5 \times 10^{-2} = \mathbf{0.085 \text{ eV}}$$

LONG ANSWER TYPE QUESTIONS

Q11.25. Consider a thin target [sq. of side 10^{-2} m and 10^{-3} m thickness] of sodium, which produces a photocurrent of 100 μA, when a light of intensity 100 W/m^2 (λ = 660 nm) falls on it. Find the probability that a photoelectron is produced when a photon strikes a sodium atom. (Take density of Na = 0.97 kg/m^3)

Ans. Area of square sheet $= (A) = 10^{-2} \times 10^{-2} = 10^{-4}$ m^2

Thickness $(d) = 10^{-3}$ m

Current $(i) = 100\ \mu A = 10^{-4}$ A

Intensity $(I) = 100$ W/m^2

Mass of target (m) = Vol. × density

m = Area of sheet × thickness × density

$= (10^{-4} \times 10^{-3}) \times 0.97 \text{ kg} = 0.97 \times 10^{-7}$ kg

$m = 0.97 \times 10^{-4}$ gm

$$\therefore \text{No. of Na Atoms in target} = \frac{6.023 \times 10^{23}}{23} \times 0.97 \times 10^{-4} = 0.254 \times 10^{19}$$

Number of Na atoms in Na target $= 2.54 \times 10^{18}$ Atoms

Total energy falling per second on target $= nh\nu$

$$\text{Intensity} \times \text{Area} = n \times h \times \frac{c}{\lambda}$$

$$I \times A = \frac{nhc}{\lambda}$$

$$n = \frac{IA\lambda}{hc} = \frac{100 \times 10^{-4} \times 660 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}} = \frac{1000 \times 660}{66 \times 3} \times 10^{-13-8+34}$$

$$= \frac{10000}{3} \times 10^{-21+34} = \frac{10}{3} \times 10^{3} \times 10^{13}$$

Number of photons (n) incident per second on Na-metal

$$\therefore \quad n = 3.3 \times 10^{16}$$

Let P is the probability of emission of photoelectrons per atom per photon.

Number of photoelectrons emitted per second

$N = P \cdot n.$ (No. of sodium atom)

$N = P \times 3.3 \times 10^{16} \times 2.54 \times 10^{18}$ ...(I)

$i = 100\ \mu A = 10^{-4} A = N e$

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$\Rightarrow \quad N = \dfrac{i}{e}$

$\therefore \quad P = \dfrac{i}{e \times 3.3 \times 10^{16} \times 2.54 \times 10^{18}}$

$= \dfrac{10^{-4}}{1.6 \times 10^{-19} \times 3.3 \times 10^{16} \times 2.54 \times 10^{18}}$

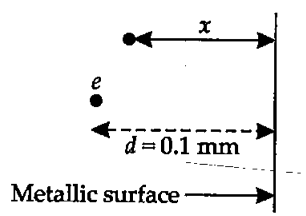
$= \dfrac{10^{-4}}{1.6 \times 3.3 \times 2.54 \times 10^{-19+34}} = \dfrac{10^{-4-34+19}}{13.4} = 0.075 \times 10^{-19}$

$\mathbf{P = 7.5 \times 10^{-21}}$

Q11.26. Consider an electron in front of a metallic surface at a distance d (treated as an infinite plane surface). Assume the force of attraction by the plate is given as $\dfrac{1}{4} \cdot \dfrac{q^2}{4\pi\varepsilon_0 d^2}$. Calculate work in taking the charge to an infinite distance from the plate. Taking d = 0.1 nm, find the work done in electron volts. (such a force law is not valid for $d < 0.1$ nm)

Main concept used: $W.D. = \int_0^{\alpha} F \cdot dr$

Ans. As per question $\quad F = \dfrac{1}{4}\dfrac{q^2}{4\pi\varepsilon_0 d^2}$



Let at any instant electron is at distance x from the metal surface. Force of attraction between metal surface and electron is F

$$F = \frac{1}{4}\frac{q^2}{4\pi\varepsilon_0 x^2}$$

Work done by external agency in taking the electron from distance d to ∞ is $W.D. = \int_d^{\infty} F \cdot dx$

$$W.D. = \int_d^{\infty} \frac{q^2 dx}{4 \times 4\pi\varepsilon_0 x^2} = \frac{q^2}{4 \times 4\pi\varepsilon_0}\int_d^{\infty} x^{-2} dx = \frac{q^2}{4 \times 4\pi\varepsilon_0}\frac{x^{-1}}{-1}$$

$$W.D. = \frac{-q^2}{4 \cdot 4\pi\varepsilon_0}\left[\frac{1}{x}\right]_d^{\infty} = \frac{-q^2 k}{4}\left[\frac{1}{\infty} - \frac{1}{d}\right]$$

$$W.D. = \frac{+kq^2}{4d} \qquad [d = 0.1 \text{ nm} = 10^{-10} \text{ m}]$$

Work done is positive

$$\text{So} \quad W.D. = \frac{(1.6 \times 10^{-19})^2 \times 9 \times 10^9}{4 \times 10^{-10}} \text{J} = \frac{1.6 \times 9 \times 1.6 \times 10^{-38+9+10}}{4 \times 1.6 \times 10^{-19}} \text{eV}$$

$$= \frac{9 \times 1.6 \times 10^{-19+19}}{4} = 3.6 \text{ eV}$$

W.D. = **3.6 eV**

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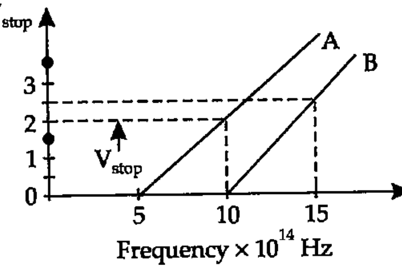
Q11.27. A student performs an experiment on photoelectric effect, using two materials A and B. A plot of V_{stop} versus ν is given in figure. (*i*) Which material A or B has higher work function? (*ii*) Given the electric charge of an electron = 1.6×10^{-19} C, find the value of h obtained from the experiment for both A and B. Comment on whether it is consistent with Einstein's theory.



Ans. (*i*) $\because$ $\phi_0 = h\nu_0$

ν_0 = Threshold frequency

For metal A $\nu_{0A} = 5 \times 10^4$ Hz

For metal B $\nu_{0B} = 10 \times 10^4$ Hz

$$\frac{\phi_{0A}}{\phi_{0B}} = \frac{h\nu_{0A}}{h\nu_{0B}} = \frac{5 \times 10^{14}}{10 \times 10^{14}}$$

$\therefore$ $\phi_{0B} = 2\phi_{0A}$

So work function of material B is twice of material A.

(*ii*) By the differentiation of potential $V = \dfrac{W}{Q}$

if W = E and Q = e (charge on an electron)

then $V = \dfrac{E}{e}$ or $E = eV$

$h\nu = eV$ $(\because E = h\nu)$

Differentiating both sides we get

$$h \,.\, d\nu = edV$$

$$h = e \cdot \frac{dV}{d\nu}$$

For metal A, $h = \dfrac{1.6 \times 10^{-19}[2-0]}{(10-5) \times 10^{14}} = \dfrac{3.2}{5} \times 10^{-19-14}$

$= 0.64 \times 10^{-33}$ JS $= 6.4 \times 10^{-33}$ JS

$h = 6 \times 10^{-34}$ JS ...(I)

For metal B, $h = \dfrac{e \times (2.5-0)}{(15-10) \times 10^{14}} = \dfrac{1.6 \times 10^{-19} \times 2.5 \times 10^{-14}}{5}$

$= \dfrac{4.00}{5} \times 10^{-33} = 0.8 \times 10^{-33}$

$h = 8 \times 10^{-34}$ JS ...(II)

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The value of planks constant (h) for both experimental graphs are not equal, so the experiment is not consistent with Einstein's theory.

But due to experimental limitation values are very near to 6.6×10^{-34} JS, so can be considered consistent with Einstein theory.

Q11.28. A particle A with mass m_A is moving with a velocity v and hits a particle B (mass m_B) at rest (one dimensional motion). Find the change in de-Broglie wavelength of particle A. Treat the collision as elastic.

Ans. As collision is elastic so law of conservation of momentum and Kinetic energy are obeyed.

$$m_A v + m_B(0) = m_A v_1 + m_B v_2$$

$$m_A(v - v_1) = m_B v_2 \quad \text{...(I)}$$

and

$$\frac{1}{2} m_A v^2 + \frac{1}{2} m_B (0)^2 = \frac{1}{2} m_A v_1^2 + \frac{1}{2} m_B v_2^2$$

$$m_A (v^2 - v_1^2) = m_B v_2^2$$

$$m_A (v - v_1)(v + v_1) = m_B v_2^2 \quad \text{...(II)}$$

Dividing (II) by (I) we get,

$$v + v_1 = v_2 \quad \text{...(III)}$$

$$v = v_2 - v_1$$

Put (III) in (I)

$$m_A v - m_A v_1 = m_B(v + v_1)$$

$$m_A v - m_A v_1 = m_B v + m_B v_1$$

$$(m_A - m_B)v = v_1(m_A + m_B)$$

$$v_1 = \frac{(m_A - m_B)}{(m_A + m_B)} v$$

From (II)

$$v_2 = v + \frac{(m_A - m_B)}{(m_A + m_B)} v = v\left[1 + \frac{(m_A - m_B)}{(m_A + m_B)}\right]$$

$$= \frac{(m_A + m_B + m_A - m_B)v}{(m_A + m_B)} = \frac{2 m_A v}{m_A + m_B}$$

$$\lambda_{A\,\text{initial}} = \frac{h}{m_A v} \quad \text{and} \quad \lambda_{A\,\text{final}} = \frac{h}{m_A v}$$

$$\lambda_{A\,\text{final}} = \frac{h(m_A + m_B)}{m_A(m_A - m_B)v}$$

$$\Delta\lambda = \lambda_{A\,\text{final}} - \lambda_{A\,\text{initial}} = \frac{h}{m_A v}\left[\frac{m_A + m_B}{(m_A - m_B)} - 1\right]$$

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$$= \frac{h}{m_A v}\left[\frac{m_A + m_B - (m_A - m_B)}{(m_A - m_B)}\right]$$

Change in de-Broglie wavelength $\Delta\lambda = \dfrac{2m_B h}{m_A(m_A - m_B)v}$

Q11.29. Consider a 20 W bulb emitting light of wavelength 5000 Å and shining on a metal surface kept at a distance 2 m. Assume that the metal surface has work function of 2eV and that each atom on the metal surface can be treated as a circular disk of radius 1.5 Å.

(*i*) Estimate number of photons emitted by the bulb per second. (assume no other losses)

(*ii*) Will there be photoelectric emission?

(*iii*) How much time would be required by atomic disk to receive energy equal to work function 2eV?

(*iv*) How many photons would atomic disk receive within time duration calculated in (*iii*) above ?

(*v*) Can you explain how photoelectric effect was observed instantaneously?

Ans. (*i*) $P = 20$ W, $\lambda = 5000$ Å $= 5000 \times 10^{-10}$ m, $d = 2$ m, $\phi = 2$ eV,
$r = 1.5$ Å $= 1.5 \times 10^{-10}$ m (Atomic radius)

Let number of photons emitted by bulb per second is n_1 then Power P is

$$P = n_1 h\nu \quad \text{or} \quad P = n_1 \frac{c}{\lambda}$$

$$\therefore \quad n_1 = \frac{P\lambda}{hc} = \frac{20 \times 5000 \times 10^{-10}}{6.62 \times 10^{-34} \times 3 \times 10^{8}}$$

$$n_1 = \frac{100000 \times 10^{-10-8+34}}{19.86} \approx \frac{100000}{20} \times 10^{-18+34}$$

$$n_1 = 5 \times 10^{16+3} = 5 \times 10^{19} \text{ no. of photons per sec.}$$

Number of photons emitted by bulb per second

$$n_1 = 5 \times 10^{19}$$

(*ii*) Energy of incident photon $E = h\nu = \dfrac{hc}{\lambda}$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{5000 \times 10^{-10}} \text{J} = \frac{19.86 \times 10^{-34+10+8}}{5000 \times 1.6 \times 10^{-19}} \text{eV}$$

$$= 2.48 \text{ eV}$$

Energy of photon $= h\nu = \dfrac{hc}{\lambda} = \dfrac{(6.62 \times 10^{-34})(3 \times 10^{8})}{5000 \times 10^{-10}}$ J

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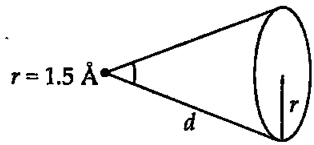
$$= \frac{6.62 \times 3 \times 10^{-24+8}}{5000 \times 1.6 \times 10^{-19}} \text{eV} = \frac{19.86 \times 10^{-16+19}}{8000}$$

$$\text{Energy of photon} = \frac{20 \times 10^3}{8 \times 10^3} = \frac{5}{2} \text{eV} = 2.5 \text{ eV}$$

As the energy of an incident photon is more than 2 eV *i.e.*, the work function of metal surface hence the photoelectric emission takes place.

(*iii*) Let Δt be the time spent in getting the energy ϕ (work function of metal)



Energy received by atomic disk in Δt time $E = P \times A \cdot \Delta t$

$$E = P \times \pi r^2 \cdot \Delta t$$

energy transferred by bulb in full solid angle $4\pi d^2$ to atoms = $4\pi d^2 \phi$

$$\therefore \quad P \times \pi r^2 \Delta t = 4\pi d^2 \phi$$

$$\Delta t = \frac{4d^2\phi}{Pr^2} = \frac{4 \times 2 \times 2 \times 2 \times 1.6 \times 10^{-19}}{20 \times 1.5 \times 1.5 \times 10^{-10} \times 10^{-10}} \text{ sec}$$

$$\Delta t = \frac{12.8 \times 10^{-19+20}}{5 \times 2.25} = \frac{128}{12.25} = 11.4 \text{ sec}$$

(*iv*) Number of photons received by one atomic disk in time Δt is

$$N = \frac{n_1 \pi r^2 \Delta t}{4\pi d^2} = \frac{n_1 r^2 \Delta t}{4d^2} = \frac{5 \times 10^{19} \times 1.5 \times 1.5 \times 10^{-20} \times 11.4}{4 \times 2 \times 2}$$

[n_1 from part (*i*) and Δt from part (*iii*)]

$$N = \frac{12.25 \times 11.4 \times 10^{-1}}{8 \times 2 \times 2} \approx 0.80 \cong 1 \text{ photon per atom}$$

N = 1 photon per atom.

(*v*) Time of emission of electrons is 11.4 sec. So the photoelectric emission is not instantaneous in the problem. It takes about 11.4 sec.

In photoelectric emission there is a collision between incident photon and free electron and nucleus, which lasts for very-very short interval of time (10^{-9} sec) hence we say photoelectric emission is instantaneous.

❑❑❑

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12 Atoms

MULTIPLE CHOICE QUESTIONS—I

Q12.1. Taking Bohr's radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about

(*a*) 53 pm (*b*) 27 pm (*c*) 18 pm (*d*) 13 pm

Main concept used: Bohr's model, $r \propto \frac{1}{Z}$ or $r = \frac{r_0}{Z}$

Ans. (*c*): According to Bohr's model of atom, radius of an atom in its ground state is $r = r_0/Z$ where r_0 is Bohr's radius, and Z is atomic number. As $r_0 = 53$ pm and atomic number of Lithium atom is 3 so,

$r = \frac{53}{3} = 17.67$ pm ≈ 18 pm verifies option (*c*).

Q12.2. The binding energy of a Hydrogen atom, considering an electron moving around a fixed nuclei (proton) is

$$B = -\frac{me^4}{8n^2\varepsilon_0^2h^2} \qquad (m = \text{mass of electron})$$

If one decides to work in a frame of reference, where the electron is at rest, the proton would be moving around it. By similar arguments, the binding energy would be

$$B = \frac{-Me^4}{8n^2\varepsilon_0^2h^2} \qquad (\text{M} = \text{mass of proton})$$

The last expression is not correct because

(*a*) *n* would not be integral.
(*b*) Bohr-quantisation applies only to electron.
(*c*) the frame in which the electron is at rest is not inertial.
(*d*) the motion of the proton would not be in circular orbits, even approximately.

Main concept used: $m_p > m_e$ need more centripetal force for revolution.

Ans. (*c*): As the mass of an electron is negligible as compared to proton. So the centripetal force cannot provide the electrostatic force, $F_p = \frac{m_p v^2}{r}$. So the given expression is not true, as it form non-inertial frame of reference due to $m_e <<< m_p$ or centripetal force on $F_e <<< F_p$. So verifies answer (*c*).

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Q12.3. The simple Bohr's model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because

(*a*) of the electrons not being subject to central force.
(*b*) of the electrons colliding with each other.
(*c*) of screening effects.
(*d*) the force between the nucleus and an electron will no longer be given by Coulomb's law.

Main concept used: How a centripetal force can be increased.

Ans. (*a*): As the mass of an electron is negligible as compared to a nucleon, so electron cannot being subject to central force. Verifies answer (*a*).

Q12.4. For the ground state, the electron in an H-atom has an angular momentum *h*, according to the simple Bohr's model. Angular momentum is a vector and hence there will be infinitely many orbits with the vector pointing in all possible directions. In actuality, this is not true,

(*a*) because Bohr's model gives incorrect values of angular momentum.
(*b*) because only one of these would have a minimum energy.
(*c*) angular momentum must be in the direction of spin of electron.
(*d*) because electrons go around only in horizontal orbits.

Main concept used: Bohr's second postulate on atomic model.

Ans. (*a*): According to Bohr's second postulate of atomic model, angular momentum of revolving electron must be some integral multiple of $\frac{h}{2\pi}$ so the Bohr's model does not give correct value of angular momentum. Hence verified answer (*a*).

Q12.5. O_2 molecule consists of two oxygen atoms. In the molecule, nuclear force between the nuclei of two atoms

(*a*) is not important because nuclear forces are short ranged.
(*b*) is as important as electrostatic force for binding the two atoms.
(*c*) cancels repulsive electrostatic force between the nuclei.
(*d*) is not important because oxygen nucleus have equal number of neutrons and protons.

Main concept used: Properties of Nuclear and Coulombian forces.

Ans. (*a*): Nuclear forces is too much stronger. Only attractive force as compared to electrostatic repulsive force and nuclear force decreases to zero on increasing distance. So in case of oxygen molecule, the distance between atoms of oxygen is larger as compared to the distances between nucleons in a nucleus. So force between the nuclei of two oxygen atoms is not important as nuclear forces are short ranged forces. Hence verified answer (*a*).

Q12.6. Two H-atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced is

(*a*) 10.20 eV (*b*) 20.40 eV (*c*) 13.6 eV (*d*) 27.2 eV

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Main concept used: Electrons of K(n = 1) energy level are called ground state electrons. It has minimum energy *i.e.*, – 13.6 eV.
Ans. (*a*): Total energy of two H-atoms in ground state

$$= 2\,(-13.6) = -27.2 \text{ eV}$$

The maximum amount by which their combined kinetic energy is reduced when any one H-atom goes into first excited state after the inelastic collision *i.e.*, the total energy of two H-atom after inelastic collision

$$E = \frac{13.6}{n^2} + 13.6$$

$$= \frac{13.6}{2^2} + 13.6 \qquad [\because \text{ for excited state } (n = 2)]$$

$$= 3.4 + 13.6 = 17.0 \text{ eV}$$

So loss in KE due to inelastic collision

$$= 27.2 - 17.0 = 10.2 \text{ eV}$$

Q12.7. A set of atoms in an excited state decays
(*a*) in general to any of the states with lower energy.
(*b*) into a lower state only when excited by an external electric field.
(*c*) all together simultaneously into a lower state.
(*d*) to emit photons only when they collide.

Main concept used: By emitted a photon by an electron, it comes back in its lower energy.
Ans. (*a*): A set of atoms in an excited state decays in general to any of the states with lower energy.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q12.8. An ionised H-molecule consists of an electron and two protons. The protons are separated by a small distance of the order of Å. In the ground state,
(*a*) the electron would not move in circular orbits.
(*b*) the energy would be $(2)^4$ times that of h-atom.
(*c*) the electrons, orbit would go around the protons.
(*d*) the molecule will soon decay in a proton and a H-atom.

Main concept used: In H-molecule, 2 proton or 2 nucleus and 1 electron.
Ans. (*a*) and (*c*): In H-molecule two nucleus of 2 H-atoms are separated of the order of angstrom, *i.e.* of order of nuclear forces. One electron will revolve around the protons. Hence verifies answer (*a*) and (*c*).

Q12.9. Consider aiming a beam of free electrons towards free protons. When they scatter, an electron and a proton cannot combine to produce a H-atom,
(*a*) because of energy conservation.
(*b*) without simultaneously releasing energy in the form of radiation.

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(c) because of momentum conservation.
(d) because of angular momentum conservation.

Ans. (a) and (b): The binding energy of H-atom is larger as compared to the energy of free electron. Large amount of energy is required so that electron can reach near the proton which is possible when nuclear force (attractive) acts between *e* and *p* become of order of nuclear force, it is not possible without energy conservation and releasing energy simultaneously. Hence verifies answers (a) and (b).

Q12.10. The Bohr's model for spectra of a H-atom
(a) will not be applicable to hydrogen in molecular form.
(b) will not be applicable as it is for a He atom.
(c) is valid only at room temperature.
(d) predicts continuous as well as discrete spectral lines.

Main concept used: Niel's and Bohr's atomic model is valid for H-atom only and explains the line of spectrum.

Ans. (a) and (b): Not applicable for H molecule or He atom. It does not depend on minor change in temperature.

Q12.11. The Balmer series for the H-atom can be observed
(a) if we measure the frequencies of light emitted when an excited atom falls to the ground state.
(b) if we measure the frequencies of light emitted due to transitions between excited states and the first excited state.
(c) in any transition in a H-atom.
(d) as a sequence of frequencies with the higher frequencies getting closely packed.

Main concept used: When an electron is excited and jumps from higher to lower energy level it emits photons for spectrum.

Ans. (b) and (d): When electron jumps from higher energy level to first energy level, the spectrum of light is called Balmer series. Spectrum lines become closer if electron jumps from higher energy level to ground level. Hence verifies answers (b) and (d).

Q12.12. Let $E_n = \dfrac{-1me^4}{8\varepsilon_0^2 n^2 h^2}$ be the energy of *n*th level of H-atom. If all the H-atoms are in the ground state and radiation of frequency $\dfrac{(E_2 - E_1)}{h}$ falls on it,

(a) it will not be absorbed at all.
(b) some of the atom will move to the first excited state.
(c) all atoms will be excited to the $n = 2$ state.
(d) no atoms will make a transition to the $n = 3$ state.

Main concept used: An electron of an atom absorbs photons of same energy as required by electron to reach in next higher orbit only.

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Ans. (*b*) and (*d*): When the energy of radiation of photons is $\frac{(E_2 - E_1)n}{h}$, then electron jumps in next energy level (n = 2) after receiving this energy equal to the $E_2 - E_1$ energy. The new state is its unstable state. Electron jumps from E_2 to E_1 by radiating the energy of same frequency *i.e.*, to $(E_2 - E_1)$. Electron can jump in next orbit. So electron from ground state will jump at $n = 2$ not $n = 3$.

Q12.13. The simple Bohr's atomic model is not applicable to He^4 atom because

(*a*) He^4 is an inert gas.
(*b*) He^4 atom has neutrons in the nucleus.
(*c*) He^4 has one more electron.
(*d*) electrons are not subject to central forces.

Main concept used: Bohr's model is valid for H atom only *i.e.* for 1 electron only.

Ans. (*c*) and (*d*): Bohr's atomic model is applicable only for one electron and in He^4 there are two electrons. Electrons are not subject to central forces due to longer distances than nuclear size verifies answers (*c*) and (*d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q12.14. The mass of H atom is less than the sum of the masses of a proton and electron. Why is this?

Main concept used: Mass defect

Ans. During formation of an atom with neucleon and electrons, it need the energy to bind the nucleons together in nucleus. This energy comes from Einstein mass-energy relation

$$E = \Delta m C^2$$

where $\Delta m = [Zm_p + (A - Z)\, m_n] - M$

So the mass of a H atom is

$$m_p + m_p - \frac{B.E.}{C^2}$$

where B.E. = 13.6 eV

Q12.15. Imagine removing one electron from ${}_2He^4$ and ${}_2He^3$. Their energy levels, as worked out on the basis of Bohr's atomic model will be very close. Explain why.

Main concept used: Bohr's model explains only the stability of H-atom.

Ans. If we remove one electron from the isotopes of $2He^4$ and $2He^3$, both atoms will have 1 electron as in H-atom and the nucleus is much (four times) heavier than H-atom. So stability can remain and these atoms are very close to the H atom so the energy levels are as of Hydrogen and these atoms will be very close.

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Q12.16. When an electron falls from a higher energy to a lower energy level, the difference in the energies appears in the form of e.m. radiations. Why cannot it be emitted as other forms of energy?

Main concept used: When electron jumps from one higher energy level to another lower energy level then due to motion of charge an e.m. wave is produced.

Ans. When charge *i.e.,* electron jumps from higher to lower energy level; there is acceleration in charge particle. The accelerated charge particle can produce electromagnetic wave only.

Q12.17. Would the Bohr's formula for the H-atom remains unchanged if proton had a charge $\left(\frac{+4}{3}\right)e$ and electron a charge $\left(\frac{-3}{4}\right)e$, where $e = 1.6 \times 10^{-19}$ C. Give reasons for your answer.

Main concept used: Electrostatic force remains same as m_p and m_e does not change. Position of e and p does not change *i.e.,* p in nucleus and e revalue.

Ans. As there is no interchange in the position of proton and electron only the magnitude of charge changes as Coulombian force will be same as product of both charges $\frac{+4}{3}e \times \frac{-3}{4}e = -e^2$ is same as earlier. $p = +1e$ and $e = -1$, $+1e \times -1e = -e^2$. So electrostatic force does not change. So Bohr's formula for the new H-atom remain same.

Q12.18. Consider two different H-atoms. The electron in each atom is in an excited state. It is possible for the electrons to have different energies but the same orbital angular momentum according to Bohr's model?

Main concept used: Bohr's atomic model, $L = \frac{nh}{2\pi}$

Ans. In excited state of electrons of two H-atoms, electrons may be in orbit or energy level either $n = 2, 3, \ldots$ and can have same energy but angular momentum by Bohr's model is $L = \frac{nh}{2\pi}$. As n for both may be different so both H-atom will have different angular momentum.

SHORT ANSWER TYPE QUESTIONS

Q12.19. Positronium is just like a H-atom with the proton replaced by the positively charged antiparticle of the electron (called the positron) which is as massive as electron. What would be the ground state energy of positronium?

Main concept used: $E_n = -\frac{m_e e^4}{8\varepsilon_0 n^2 h^2} = -13.6 \text{ eV}$ for H atom in ground state.

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Ans. As in the new H-atom (positronium) proton is replaced by positron of mass $m = m_e/2$ as under

$$\text{Mass of positronium} = m = m_e^- + m_e^+$$

$$m = m_e + m_e = 2m_e$$

$$m_e^+ = m_e^{-1} = \frac{m_e}{2}$$

as $E_n = -13.6$ and so energy of positron

$$E'_n = \frac{-m_e^+ e^4}{8\varepsilon_0 n^2 h^2} = \frac{-\left(\frac{m_e}{2}\right)e^4}{8\varepsilon_0 n^2 h^2} = \frac{-13.6}{2}$$

So $\quad E'_n = \dfrac{-13.6}{2} \qquad \left(\because\ m_e = \dfrac{m}{2}\right)$

$E'_n = -6.8\text{ eV}$

Q12.20. Assume that there is no repulsive force between the electrons in an atom, but force between positive and negative charges is given by Coulomb's law as usual. Under such circumstances calculate the ground state energy of a He-atom.

Main concept used: Energy of an electron revolving in stable nth orbit $E_n = Z\dfrac{m_e e^4}{8\varepsilon_0^2 n^2 h^2}$, m_e is mass of electron.

Ans. For H atom $\qquad Z = 1$ and $n = 1$

$$\therefore \qquad E_n = \frac{-m_e e^4}{8\varepsilon_0^2 1^2 h^2} = -13.6\text{ eV}$$

For He-atom, $\qquad Z = 4$ and $n = 1$

$$\therefore \qquad E_n = \frac{-4m_e e^4}{8\varepsilon_0^2 1^2 h^2} = -4 \times 13.6 = -54.4\text{ eV}$$

Q12.21. Using Bohr's model, calculate the electric current created by the electron when the H-atom is in the ground state.

Main concept used: $I = \dfrac{-e}{T} = -e\text{v}$

Ans. Let for an electron of H atom velocity in orbit $= v$ m/s

Radius of orbit $= a_0 =$ Bohr's radius

So the number of revolutions per second $\text{v} = \dfrac{2\pi a_0}{v}$

$$\therefore \qquad I = -e\text{v} \qquad \left(\because\ \text{v} = \frac{2\pi a_0}{v}\right)$$

$$= \frac{-e2\pi a_0}{v}$$

(–) sign shows that the direction of current is opposite to the direction of motion of electron.

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Q12.22. Show that first few frequencies of light that is emitted when electrons fall to nth level from levels higher than n, are approximate harmonics. (*i.e.*, in the ratio 1 : 2 : 3) when $n >> 1$.

Main concept used: Spectrum of H atom.

Ans. When an electron falls from $(n + n')^{th}$ to nth energy level, the frequency of radiations in spectrum of H-atom like atoms is given as

$$\nu = CRZ^2\left[\frac{1}{(n+n')^2} - \frac{1}{n^2}\right]$$

here $n >> n'$

$n' = 1, 2, 3, \ldots$

R = Rydberg's constant

$$\nu = CRZ^2\left[\frac{1}{n^2\left[1+\frac{n'}{n^2}\right]^2} - \frac{1}{n^2}\right] = CRZ^2\left[\frac{1}{n^2}\left[1+\frac{n'}{n}\right]^{-2} - \frac{1}{n^2}\right]$$

Neglecting the higher terms as $n >> n'$

$$= CRZ^2\left[\frac{1}{n^2}\left(1-\frac{2n'}{n}\right) - \frac{1}{n^2}\right] = CRZ^2\left[\frac{1}{n^2} - \frac{2n'}{n^3} - \frac{1}{n^2}\right]$$

$$= \frac{-CRZ^2 2n'}{n^3} = \left(\frac{2CRZ^2}{n^3}\right)n'$$

So the first few frequencies of light that is emitted when electrons fall from $(n + n')$ to nth energy level are in the ratio of $n' = 1 : 2 : 3, \ldots$ when $n >>> 1$.

Q12.23. What is the minimum energy, that must be given to a H atom in ground state so that it can emit an H_γ line in Balmer series? If the angular momentum of the system is conserved, what would be the angular momentum of such H_γ photon?

Main concept used: (*i*) H_γ line in Balmer series corresponds to $n = 5$ to $n = 2$ (*ii*) Energy of electron in ground state of H atom = –13.6 eV, (*iii*) Energy of electron in nth energy level = $\frac{-13.6}{n^2}$.

Ans. We know that H_γ spectral line in Balmer series formed when electron falls from $n = 5$ to $n = 1$.

Here the electron is in ground state *i.e.*, $n = 1$ and must be taken to $n = 5$ for H_γ line. So the energy of

$$H_\gamma = E_5 - E_1 = \left(-\frac{13.6}{5^2}\right) - (-13.6) = -0.54 + 13.6 = 13.06 \text{ eV}$$

Since angular momentum is conserved, so the angular momentum of

H_γ = change in angular momentum of electron

$= L_5 - L_2 = 5h - 2h$

$= 3h = 3 \times 6.63 \times 10^{-34} = 19.89 \times 10^{-34}$ kg m²/s

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LONG ANSWER TYPE QUESTIONS

Q12.24. The first four spectral lines in the Lyman series of a H-atom are λ = 1218Å, 1028Å, 974.3Å and 951.4Å. If instead of Hydrogen, we consider Deuterium, calculate the shift in the wavelength of these lines.

Main concept used: Reduced mass of 2 particle system $\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2}$.

Ans. Reduced mass of H atom (mass defect) = μ_H then

$$\frac{1}{\mu_H} = \frac{1}{me} + \frac{1}{M} \qquad \text{(M is mass of H atom)}$$

$$\frac{1}{\mu_H} = \frac{M + m_e}{M \cdot m_e} = \frac{M\left[1 + \frac{m_e}{M}\right]}{M \cdot m_e} \qquad (\because \quad M >>> m_e)$$

$$\therefore \qquad \mu_H = m_e\left[1 + \frac{m_e}{M}\right]^{-1} = m_e\left[1 - \frac{m_e}{M}\right]$$

For Deuterium M = 2 M

reduced mass of Deuterium = μ_D

$$\mu_D = m_e\left[1 - \frac{m_e}{M}\right]\left[1 + \frac{m_e}{2M}\right]$$

Total energy E_n of the electron revolving in an n^{th} stationary orbit,

$$E_n = \frac{-me^4}{8n^2\varepsilon_0^2h^2}$$

m is the reduced mass of the electron and proton in H atom.

So $$h\nu = En_i - En_f$$

$$\nu = \frac{me^4}{8\varepsilon_0^2h^2}\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right] = \frac{c}{\lambda}$$

$\nu \propto m$ (reduced mass)

$$\frac{1}{\lambda} \propto \mu \quad \Rightarrow \quad \lambda \propto \frac{1}{\mu}$$

For hydrogen and deuterium = $\frac{\lambda_H}{\lambda_D}$

So $$\frac{\lambda_D}{\lambda_H} = \frac{\mu_H}{\mu_D} = \frac{m_e\left[1 - \frac{m_e}{M}\right]}{m_e\left[1 - \frac{m_e}{M}\right]\left[1 + \frac{m_e}{2M}\right]}$$

$$\lambda_D = \left[1 + \frac{m_e}{2M}\right]^{-1}\lambda_H = \left(1 - \frac{m_e}{M}\right)\lambda_H$$

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$$\lambda_D = \lambda_H\,(0.99973)$$
$$\lambda_H = 1218\text{Å}, 1028\text{ Å}, 974\text{Å and } 954\text{Å} \qquad \text{(Given)}$$
$$\lambda_{D1} = 0.9973 \times 1218\text{Å} = 1214\text{Å}$$
$$\lambda_{D2} = 0.9973 \times 1028 = 1025\text{Å}$$
$$\lambda_{D3} = 0.9973 \times 974 = 971\text{Å}$$
$$\lambda_{D4} = 0.9973 \times 954 = 951\text{Å}$$

Q12.25. Deuterium was discovered in 1932 by Harold Urey by measuring the small change in wavelength for the particular transition in $_1H^1$ and $_1H^2$. This is because, the wavelength of transition depend to a certain extent on the nuclear mass. If nuclear motion is taken into account, then the electrons and nucleus revolve around their common centre of mass.

Such a system is equivalent to a single particle with a reduced mass μ revolving around the nucleus at a distance equal to the electron-nucleus separation. Here

$$\mu = \frac{m_e M}{(M + m_e)}$$

where M is the nuclear mass and m_e is the electronic mass.

Estimate the percentage difference in wavelength for the first line of the Lyman series in $_1H^1$ and $_1H^2$ (mass of $_1H^1$ nucleus = 1.6725×10^{-27} kg and mass of $_1H^2$ nucleus is 3.3374×10^{-27} kg. Mass of electron = 9.109×10^{-31} kg)

Main concept used: Percent wavelength difference

$$\frac{\Delta\lambda}{\Delta H} \times 100 = \frac{(\lambda_D - \lambda_H)}{\lambda_H} \times 100$$

Ans. Total energy of electron in n^{th} stable orbit in H or like atom

$$E_n = \frac{\mu Z^2 e^4}{8\varepsilon_0^2 h^2 n^2}$$

μ = reduced mass of electron, proton and neutron (mass defect)

$$E_H = \frac{\mu_H (1)^2 e^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = \frac{\mu_H e^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{1} - \frac{1}{2^2}\right] = \frac{\mu_H e^4}{8\varepsilon_0^2 h^2}\left[\frac{3}{4}\right]$$

$$E = h\nu = \frac{h}{\lambda} \quad \text{or} \quad \lambda_H = \frac{h}{E_H}$$

$$\therefore \qquad h\nu_H = \frac{\mu_H e^4}{8\varepsilon_0^2 h^2}\,\frac{3}{4}$$

$$\nu_H = \frac{\mu_H e^4}{8\varepsilon_0^2 h^3}\cdot\frac{3}{4}$$

The percentage difference in the wavelength = $\dfrac{(\lambda_D - \lambda_H)}{\lambda_H} \times 100$

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Percent change in wavelength

$$\% \text{ change} \quad \Delta E = \left[\frac{\lambda_D}{\lambda_H} - 1\right] \times 100 \qquad (\because \Delta E = E_1 - E_2) \quad ...(I)$$

$$h\nu = \frac{\mu e^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_2^2} - \frac{1}{n_2^2}\right]$$

$$\nu = \frac{\mu e^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{c}{\lambda} = \frac{\mu e^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda} = \frac{\mu e^4}{8\varepsilon_0^2 c h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

as μ = mass defect, e, ε_0 c, and h are constants for an atom.

$$\therefore \qquad \lambda \propto \frac{1}{h}$$

So eqn. Ist can be written as percentage change in the wavelength

$$= \left[\frac{\mu_H}{\mu_D} - 1\right] \times 100$$

$$\because \qquad \mu = \frac{m_e M}{(M + m_e)} \qquad \text{(Given)}$$

$$\therefore \quad \text{Percentage change in wavelength} = \left[\frac{\dfrac{m_e M_H}{(M_H + m_e)}}{\dfrac{m_e M_D}{(M_D + m_e)}} - 1\right] 100$$

$$\frac{\Delta\lambda}{\lambda_H} \times 100 = \left[\frac{M_H}{M_D}\frac{(M_D + m_e)}{(M_H + m_e)} - 1\right] \times 100$$

$$= \left[\frac{M_H}{M_D}\frac{M_D\left(1 + \dfrac{m_e}{M_D}\right)}{M_H\left(1 + \dfrac{m_e}{M_H}\right)} - 1\right] \times 100$$

$$= \left[\left(1 + \frac{m_e}{M_D}\right)\left(1 + \frac{m_e}{M_H}\right)^{-1} - 1\right] \times 100$$

$$= \left[\left(1 + \frac{m_e}{M_D}\right)\left(1 - \frac{m_e}{M_H}\right) - 1\right] \times 100$$

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me << M_D so neglecting the higher degree term

$$\frac{\Delta\lambda}{\lambda_H}\times 100 = \left[1-\frac{m_e}{M_H}+\frac{m_e}{M_D}-\frac{m_e\, m_e}{M_D\cdot M_H}-1\right]\times 100$$

$$= m_e\left[\frac{1}{M_D}-\frac{1}{M_H}\right]\times 100$$

$$= 9.1\times 10^{-31}\left[\frac{1}{3.3374\times 10^{-27}}-\frac{1}{1.6725\times 10^{-27}}\right]\times 100$$

$$= \frac{9.1\times 10^{-31+2}}{10^{-27}}\left[\frac{1.6725-3.3374}{3.3374\times 1.6725}\right]$$

$$\frac{\Delta\lambda\times 100}{\lambda_H} = \frac{-9.1\times 10^{-29+27}\times 0.6649}{3.3374\times 1.6725} = \frac{-6.05059\times 10^{-2}}{5.5180}$$

$$\frac{\Delta\lambda\times 100}{\lambda_H} = -1.084\times 10^{-2}\ \%\ \text{decrease in wavelength.}$$

(–) sign shows that $\lambda_D < \lambda_H$.

Q12.26. If a proton had a radius R and the charge was uniformly distributed, calculate using Bohr theory, the ground state energy of a H-atom when (*i*) R = 0.1Å (*ii*) R = 10Å.

Main concept used: Energy of H atom when (*i*) point nucleus (*ii*) spherical nucleus of radius R.

Ans. (*i*) Consider in H atom nucleus as a point charge electron is revolving around nucleus with speed v and radius r_A. The Coulombian force provides centripetal force to revolve around nucleus.

$$\therefore \qquad \frac{m_e v^2}{r_A} = \frac{-Ke^2}{r_A^2} \qquad \text{...(I)}$$

Here $\qquad K = \dfrac{1}{4\pi\varepsilon_0}$

(–) sign shows the force of attraction.

By Bohr's postulate, angular momentum = $\dfrac{nh}{2\pi}$

$$mv\, r_A = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi m r_A}$$

$$\frac{m\, n^2h^2}{4\pi^2 m^2 r_A^2 r_A} = \frac{Ke^2}{r_A^2} \qquad \text{[From I]}$$

$$r_A = \frac{n^2h^2}{4\pi^2 m\, Ke^2} \qquad \text{...(II)}$$

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For ground state $n = 1$

$$r_A = \frac{h^2}{4\pi^2 mKe^2}$$

$$= \frac{6.63 \times 10^{-34} \times 6.63 \times 10^{-34}}{(2 \times 3.14)^2 \times 9.1 \times 10^{-31} \times 9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}$$

$$= \frac{6.63 \times 6.63 \times 10^{-68+38+31-9}}{9.1 \times 9 \times 1.6 \times 1.6 \times 4 \times 3.14 \times 4 \times 3.14}$$

$$= r_A = 0.53\ \text{Å} \quad \Rightarrow \quad 0.53 \times 10^{-10}\ \text{m}$$

$$\text{P.E.} = \frac{-Ke^2}{r_A} = \frac{-9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{0.53 \times 10^{-10}}\ \text{J}$$

$$= \frac{-9 \times 1.6 \times 1.6 \times 10^{-19} \times 10^{-19} \times 10^9}{0.53 \times 10^{-10} \times 1.6 \times 10^{-19}}\ \text{eV} = \frac{-9 \times 1.6}{0.53} \times 10^{-19+9+10}$$

$$\text{P.E.} = \frac{14.4}{0.53} = 27.17 = 27.2\ \text{eV}$$

$$\text{K.E} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \frac{m \cdot n^2 h^2}{4\pi^2 m^2 r^2} = \frac{1}{2} \frac{h^2}{4\pi^2 mr^2} \qquad (n = 1 \text{ for ground state})$$

$$= \frac{1}{2} \frac{6.62 \times 10^{-34} \times 6.62 \times 10^{-34}}{4 \times 3.14 \times 3.14 \times 9 \times 10^{-31} \times 0.53 \times 10^{-10} \times 0.53 \times 10^{-10}}\ \text{J}$$

$$= \frac{6.62 \times 6.62 \times 10^{-68+31+10+10}}{4 \times 3.14 \times 3.14 \times 18 \times 0.53 \times 0.53 \times 1.6 \times 10^{-19}}\ \text{eV}$$

$$= \frac{6.62 \times 6.62 \times 10^{-68+51+19}}{4 \times 3.14 \times 3.14 \times 18 \times 0.53 \times 0.53}$$

$$\text{K.E.} = 0.1373 \times 10^2\ \text{eV} = 13.7\ \text{eV}$$

$$\text{P.E.} = 27.2\ \text{eV}$$

(*ii*) Now for spherical nucleus of radius, R, electron moves charge inside the nucleus $R >> r_b$ then electron moves inside the nucleus. Then (r_b is radius of new Bohr's orbit of revolving electron)

$$\text{Charge} = \frac{e \cdot \left(\frac{4}{3}\pi r_b^3\right)}{\frac{4}{3}\pi R^3}$$

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$$e' = q_2 = \frac{er_b^3}{R^3}$$

$$q_1 = e$$

$$\frac{mv^2}{r_b} = \frac{K ee'}{r_b^2} \qquad \text{(By Coulomb's law)}$$

$$mvr_b = \frac{nh}{2\pi} \Rightarrow v_b = \frac{nh}{2\pi m r_b} \qquad \text{(By Bohr's postulate)}$$

$$\therefore \quad \frac{m}{r_b}\frac{n^2h^2}{4\pi^2 m^2 r_b^2} = \frac{kee'}{r_b^2}$$

$$r_b = \frac{n^2h^2}{4\pi^2 mkee'}$$

Now for ground state of H, n = 1 and $e' = \frac{er_A^3}{R^3}$, then

$$\therefore \quad r_b = \frac{h^2}{4\pi^2 mK \cdot e \cdot e \cdot \frac{r_b^3}{R^3}} = \left(\frac{h^2}{4\pi^2 mKe^2}\right) \times \frac{R^3}{r_b^3} = r_A \times \frac{R^3}{r_b^3}$$

$$\left[\because r_A = \frac{h^2}{4\pi^2 mKe^2} = 0.53 \text{ Å calculated in part } (i)\right]$$

$$r_b = r_A \left(\frac{R}{r_b}\right)^3$$

$$r_b^4 = r_A R^3 = 0.53\text{Å} \times (10\text{Å})^3$$

$$r_b^4 = 0.53 \times 1000 \,(\text{Å})^4 \qquad (\because r_A = 0.53 \text{ Å})$$

$$r_b = [530 \text{ A}^4]^{1/4} = 4.8 \text{ Å} < R = 10 \text{ Å}$$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{m}{2} \cdot \frac{h^2}{4\pi^2 m^2 r_b^2} = \frac{h^2}{8\pi^2 m r_b^2} \qquad \left[\because v = \frac{n^2h^2}{4\pi^2 m^2 r_b^2}\right]$$

$$= \frac{6.62 \times 6.62 \times 10^{-34} \times 10^{-34}}{8 \times 3.14 \times 3.14 \times 9.1 \times 10^{-31} \times 4.8 \times 4.8 \times 10^{-20} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= \frac{6.62 \times 6.62 \times 10^{-68+31+20+19}}{8 \times 3.14 \times 3.14 \times 9.1 \times 4.8 \times 4.8 \times 1.6} \text{ eV}$$

$$= \frac{43.8244}{26460.2} 10^{-68+70} = 0.001656 \times 10^2 \text{ eV}$$

$$\text{K.E.} = 0.167 \text{ eV}$$

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$$\text{P.E.} = \frac{e^2}{4\pi\varepsilon_0} \frac{(r_b^2 - 3R^2)}{R^3} \qquad \left(\text{P.E.} = \frac{Kq_1q_2}{r}\right)$$

$$\text{P.E.} = \left[\frac{e^2}{4\pi\varepsilon_0 r_A}\right] \frac{r_A (r_b^2 - 3R^2)}{R^3} \qquad \left[\text{multyplying by } \frac{r_A}{r_A}\right]$$

From part (*i*)

$$\text{P.E.} = \frac{e^2}{4\pi\varepsilon_0 r_A} = 27.2 \text{ eV}$$

$$\therefore \text{ P.E.} = 27.2\left[\frac{0.53(\sqrt{530} - 300)}{1000}\right] \qquad [\because r_b = (530)^{1/4} \text{ Å and } R = 10\text{Å}]$$

$$= \frac{27.2 \text{ eV} \times 0.53(23.02 - 300) \text{ Å}^3}{1000 \text{ Å}^3} = 27.2 \times \frac{0.53(-276.9)}{1000} \text{eV}$$

$$\text{P.E.} = \frac{3992.9}{1000} = \mathbf{-3.99 \text{ eV}}$$

$$\text{K.E.} = \mathbf{0.167 \text{ eV}}$$

Q12.27. In the Auger process, an atom makes a transition to a lower state without emitting a photon. The excess energy is transferred to outer electron, which may be ejected by the atom (this is called an Auger electron). Assuming the nucleus to be massive, calculate the Kinetic energy of an $n = 4$ Auger electron emitted by chromium by absorbing the energy from a $n = 2$ to $n = 1$ transition.

Main concept used: As chromium nucleus is massive recoil of the atom by emitted electron is negligible and the entire energy of transition may be considered to be ejected (Auger) electron. As there is a single valence electron in Cr the energy states may be thought of as given by the Bohr's model.

Ans. The energy E_n of the nth state

$$E_n = +Z^2R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = Z^2R\left(\frac{1}{1} - \frac{1}{4}\right) \qquad (\text{for } n_1 = 1, \quad n_2 = 2)$$

$Z = 24$

R = Rydberg constt.

$$\therefore \qquad E_n = \frac{3}{4}Z^2R$$

The energy required to eject an electron from $n = 4$ state is

$$E_4 = Z^2R\frac{1}{4^2} = \frac{1}{16}Z^2R$$

Energy given to electron is converted into K.E. of ejected electron.

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Hence, the K.E. of Auger (ejected) electron = $E_n - E_4$

$$K.E. = Z^2R\frac{3}{4} - \frac{1}{16}Z^2R = \frac{11}{16}Z^2R = \frac{11}{16} \times 24 \times 24 \times 13.6 \text{ eV}$$

$$K.E. = 11 \times 36 \times 13.6 = \mathbf{5385.6\ eV}$$

Q12.28. The inverse square law of electrostatics is $|F| = \frac{e^2}{(4\pi\varepsilon_0)r^2}$ for the force between an electron and a proton. The $1/r$ dependence of $|F|$ can be understood in quantum theory as being due to the fact that the particle of light (photon) is massless. If the photon had a mass m_p, force would be modified to $|F| = \frac{e^2}{(4\pi\varepsilon_0)r^2}\left[\frac{1}{r^2} + \frac{\lambda}{r}\right]e^{-\lambda r}$ where $\lambda = \frac{m_p c}{\hbar}$ and $\hbar = \frac{h}{2\pi}$.

Estimate the change in the ground state energy of a H-atom if $m_p = 10^{-6}$ times the mass of an electron.

Ans. Mass of photon $= 9.1 \times 10^{-31} \times 10^{-6}$ kg

$= 9.1 \times 10^{-37}$ kg

Wavelength associated with a photon $= \frac{h}{m_p c}$

$$\lambda = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-37} \times 3 \times 10^8}$$

$$= \frac{6.62}{9.1 \times 3} \times 10^{-34+37-8} = 2.4 \times 10^{-7} >> r_A \qquad \text{(see Q.26)}$$

$$\lambda << \frac{1}{r_A} < e.\lambda_{r_A} << 1$$

$$U(r) = \frac{-e^2 e^{-\lambda_r}}{\pi\varepsilon_0 r}$$

$$mvr = \frac{h}{2\pi} = \hbar \quad \text{or} \quad v = \frac{\hbar}{mr} \qquad \text{...(I)}$$

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0}\left[\frac{1}{r^2} + \frac{\lambda}{r}\right] \qquad \left[\because F = \frac{e^2}{(4\pi\varepsilon_0)r^2}\left[\frac{1}{r^2} + \frac{\lambda}{r}\right]e^{-\lambda r} \text{ given}\right]$$

$$\frac{m}{r} \cdot \frac{\hbar^2}{m^2r^2} = \frac{e^2}{4\pi\varepsilon_0}\left[\frac{1+\lambda r}{r^2}\right]$$

$$\frac{\hbar^2}{mr} = \frac{e^2}{4\pi\varepsilon_0}(1 + \lambda r)$$

$$\frac{\hbar^2 4\pi\varepsilon_0}{me^2} = (r + \lambda r^2)$$

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If $\lambda = 0, r = r_A = \dfrac{\hbar^2 4\pi\varepsilon_0}{me^2}$ [neglecting r^2]

$$\frac{\hbar^2}{m} = \frac{e^2}{4\pi\varepsilon_0} r_A \qquad (r_A \approx r + \lambda r^2)$$

$\because \quad \lambda_A >> r_B$ and $r = r_A + \delta$

taking $r_A = r + \lambda r^2$ $[\because r = (r_a + \delta)]$

(put $r = r_A + \delta$)

$$r_A = (r_A + \delta) + \lambda(r_A + \delta)$$

$$= r_A + \delta + \lambda(r_A^2 + \delta^2 + 2r_A\delta)$$

$$0 = \delta + \lambda r_A^2 + 2r_A\delta\lambda \quad \text{(neglecting small term } \delta^2\text{)}$$

$$0 = \delta + 2r_A\delta\lambda + \lambda r_A^2$$

$$\Rightarrow \quad \delta[1 + 2r_A\lambda] = -\lambda r_A^2$$

$$\delta = \frac{-\lambda r_A^2}{(1 + 2r_A\lambda)} = -\lambda r_A^2 (1 + 2r_A\lambda)^{-1}$$

$$\delta = -\lambda r_A^2[1 - 2r_A\lambda] = -\lambda r_A^2 + 2r_A^3\lambda^2$$

$\therefore$ λ and $r_A << 1$ so $r_A^3\lambda^2$ is very small so by neglecting it we get,

$$\boxed{\delta = -\lambda r_A^2}$$

$$V(r) = \frac{-e^2}{4\pi\varepsilon_0} = \frac{e^{(-\lambda\delta - \lambda r_A)}}{(r_A + \delta)} = \frac{-e^2}{4\pi\varepsilon_0} \cdot \frac{e^{-\lambda(\delta + r)}}{r_A\left(1 + \dfrac{\delta}{r_A}\right)}$$

$$= \frac{-e^2}{4\pi\varepsilon_0 r_A} e^{-\lambda r}\left(1 + \frac{\delta}{r_A}\right)^{-1} \qquad (\because \quad r = r_A + \delta)$$

$$= \frac{-e^2 e^{-\lambda r}}{4\pi\varepsilon_0 r_A}\left(1 - \frac{\delta}{r_A}\right)$$

$V(r) = -27.2$ eV remains unchanged

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{\hbar}{mr}\right)^2 = \frac{1}{2}\frac{\hbar^2}{mr^2} \qquad \left(\text{From I, } v = \frac{\hbar}{mr}\right)$$

$$= \frac{-\hbar^2}{2m(r_A + \delta)^2} = \frac{-\hbar^2}{2mr_A^2\left(1 + \dfrac{\delta}{r_A}\right)^2} = \frac{\hbar^2}{2mr_A^2}\left(1 - \frac{2\delta}{r_A}\right)$$

$$= \frac{\hbar^2}{2mr_A}\left(1 - \frac{-\lambda r_A^2}{r_A}\right) = \frac{\hbar^2}{2mr_A}(1 + 2\lambda r_A)$$

$$= 13.6 \text{ eV } (1 + 2\lambda r_A)$$

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$$\text{Total energy} = \frac{-e^2}{4\pi\varepsilon_0 r_A} + \frac{\hbar^2}{2mr_A^2}(1 + 2\lambda r_A)$$
$$= [-27.2 + 13.6\,(1 + 2\lambda r_A)]\ \text{eV}$$
$$= -27.2 + 13.6 + 27.2\ \lambda r_A\ \text{eV}$$
$$\text{Total E} = -13.6 + 27.2\ \lambda r_A$$
$$\text{Change in energy} = -13.6 + 27.2\ \lambda r_A - (-13.6) = \mathbf{27.2\ \lambda r_A\ eV}$$

Q12.29. The Bohr's model for H-atom relies on the Coulomb's law of electrostatics. Coulomb's law has not directly been verified for very short distances of the order of angstroms. Supposing Coulomb's law between two opposite charges $+q_1, -q_2$ is modified to

$$|F| = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \qquad (r \geq R_0)$$

$$|F| = \frac{q_1 q_2}{4\pi\varepsilon_0 R_0^2}\left[\frac{R_0}{r}\right]^{\varepsilon} \qquad (r \leq R_0)$$

Calculate in such a case, the ground state energy of a H-atom if $\varepsilon = 0.1$, $R_0 = 1$ Å

Ans. Case I: when $r \leq R_0 = 1$ Å

Let $\varepsilon = 2 + \delta$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 R_0^2}\frac{R_0^{2+\delta}}{r^{2+\delta}} = \left(\frac{e\cdot(-e)}{4\pi\varepsilon_0}\right)\frac{R^{\delta}}{r^{2+\delta}} \qquad [q_1 = e, q_2 = -e]$$

$$|F| = -(1.6\times10^{-19})^2 \times 9\times10^9\ \frac{R^{\delta}}{r^{2+\delta}}$$

[(–)ve sign shows force of attraction]

$$|F| = 23.04\times10^{-29}\ \frac{R^{\delta}}{r^{2+\delta}}$$

The electrostatic force of attraction between positively charged nucleus and negatively charged electron provides necessary centripetal force.

$$\frac{mv^2}{r} = (23.04\times10^{-29})\ \frac{R_0^{\delta}}{r^{2+\delta}}$$

Let $\qquad 23.04\times10^{-29} = \Lambda$

$$\therefore \qquad v^2 = \Lambda\frac{R_0^{\delta}}{r^{2+\delta}}\cdot\frac{r}{m} = \frac{\Lambda R_0^{\delta}}{mr^{1+\delta}}$$

By Bohr's IInd postulate, angular momentum

$$L = \frac{nh}{2\pi} \quad \text{and} \quad \hbar = \frac{h}{2\pi} \qquad \text{[Given in last Q.]}$$

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$\therefore$

$$L = n\hbar$$

$$mvr = n\hbar$$

$$r = \frac{n\hbar}{mv} = \frac{n\hbar}{m}\sqrt{\frac{mr^{1+\delta}}{\Lambda R_0^\delta}} = \frac{n\hbar}{m}\left[\frac{m}{\Lambda R_0^\delta}\right]^{1/2} r^{1/2+\delta/2}$$

$$r^{1-\frac{1}{2}-\frac{\delta}{2}} = \frac{n\hbar}{m}\left[\frac{m}{\Lambda R_0^\delta}\right]^{1/2}$$

$$r^{\frac{1}{2}(1-\delta)} = \frac{n\hbar}{m}\left[\frac{m}{\Lambda R_0^\delta}\right]^{1/2}$$

$$r_n = \left[\frac{n^2\hbar^2}{m\Lambda R_0^\delta}\right]^{\frac{1}{1-\delta}}.$$

$$= \left[\frac{\hbar^2}{m\Lambda R_0^\delta}\right]^{\frac{1}{1-\delta}} \qquad (n = 1 \text{ for ground state})$$

$$r_1 = \left[\frac{(1.05\times10^{-34})^2}{9.1\times10^{-31}\times23.04\times10^{-29}\times10^{19}}\right]^{\frac{1}{2.9}}$$

$$\left[\because \hbar = \frac{h}{2\pi} = \frac{6.6\times10^{-34}}{2\times3.14} = (1.05\times10^{-34})\right]$$

where $\hbar = 1.05 \times 10^{-34}$ JS^{-1}

$R_0 = 10^{19}$ and $1 - \delta = 2.9$

$\mathbf{r_1 = 8 \times 10^{-11}}$ **m = 0.08 nm**

This is a radius of orbit of electron in ground state of hydrogen atom.

Velocity of electron in ground state

By IInd postulate of Bohr's Atomic model

$$mv_n r_n = \frac{nh}{2\pi} \qquad \left(\hbar = \frac{h}{2\pi}\right)$$

$$v_n = \frac{n\hbar}{mr_n} = \frac{n\hbar}{m}\left[\frac{m\Lambda R_0^\delta}{n^2\hbar^2}\right]^{\frac{1}{1-\delta}}$$

Case II: $n = 1,\ v_1 = \dfrac{\hbar}{mr_1} = \dfrac{1.05\times10^{-34}}{9.1\times10^{-31}\times0.08\times10^{-9}} = \dfrac{1.05}{0.728}\times10^{-34+31+9}$

$$= 1.44 \times 10^{-34+40} = 1.44 \times 10^6 \text{ m/s}$$

$$v_1 = 1.44\times10^6 \text{ m/s}$$

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$$KE = \frac{1}{2}mv_1^2 - 9.43\times10^{-19}$$

$$= \frac{1}{2}\times9.1\times10^{-31}\times(1.44\times10^{6})^2 - 9.43\times10^{-19}\,J$$

$$= 5.9\,eV$$

P.E. from R_0 to $\frac{-\Lambda}{R_0}$

$$\text{P.E. from } R_0 \text{ to } r = +\Lambda R_0^{\delta}\int_{R_0}^{r}\frac{dr}{r^{2+\delta}}$$

$$\text{P.E.} = \frac{\Lambda R_0^{\delta}}{-(1+\delta)}\left[\frac{1}{r^{1+\delta}}\right]_{R_0}^{r}$$

This is the P.E. of electron in ground state R_0 to $r = \frac{-\Lambda}{R_0}$

$$\text{P.E.} = -\frac{\Lambda R_0^{\delta}}{(1+\delta)}\left[\frac{1}{r^{1+\delta}} - \frac{1}{R_0^{1+\delta}}\right]$$

$$= \frac{-\Lambda}{(1+\delta)}\left[\frac{R_0^{\delta}}{r^{1+\delta}} - \frac{1}{R_0}\right] = \frac{-\Lambda}{(1+\delta)}\left[\frac{R_0^{\delta}}{r^{1+\delta}} - \frac{1}{R_0} + \frac{1+\delta}{R_0}\right]$$

Put $\delta = -1.9$

$$\text{P.E.} = \frac{-\Lambda}{(1+(-1.9))}\left[\frac{R_0^{-1.9}}{r^{-0.9}} - \frac{1}{R_0} - \frac{0.9}{R_0}\right]$$

$$= \frac{-\Lambda}{-0.9}\left[\frac{R_0^{-1.9}}{r^{-0.9}} - \frac{1.9}{R_0}\right] = \frac{23.04\times10^{-2.9}}{0.9}[(0.8)^{0.9} - 1.9]\,J$$

$$= -17.3\,eV$$

$$\text{Total E} = \text{P.E.} + \text{K.E.}$$

$$= (-17.3 + 5.9)\,eV$$

Total energy $= -11.4$ eV

This is the required total energy of electron in ground state of H-atom.

❑❑❑

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13 Nuclei

MULTIPLE CHOICE QUESTIONS—I

Q13.1. Suppose we consider a large number of containers each containing initially 10,000 atoms of a radioactive material with a half life of 1 year. After 1 year,

(*a*) all the containers will have 5000 atoms of the material.

(*b*) all the container, will contain the same number of atoms of the material but that number will only be approximately 5000.

(*c*) the containers will in general have different number of atoms of the material but their average will be close to 5000.

(*d*) none of the containers can have more than 5000 atoms.

Ans. (*c*): Half life time for a radioactive substance is defined as the time in which a radioactive atomic substance remains half of its original value of radioactive atom. So after one year means one half life *i.e.*, average atoms of radioactive substance remain after 1 year in each container is equal to 1/2 of 10,000 = 5000 atoms (average).

Q13.2. The gravitational force between a H-atom and another particle of mass m will be given by Newton's law: $F = G\frac{M \cdot m}{r^2}$, where r is in km and

(*a*) $M = m_{proton} + m_{electron}$.

(*b*) $M = m_{proton} + m_{electron} - \frac{B}{c^2}$ $(B = 13.6 \text{ eV})$.

(*c*) M is not related to mass of H-atom.

(*d*) $M = m_{proton} + m_{electron} - \frac{|V|}{c^2}$ [$|V|$ = magnitude of potential energy of electron in the H-atom.]

Ans. (*b*): During formation of H-atom some mass of nucleons convert into energy by $E = mc^2$, this energy is used to bind the nucleons along with nucleus. So mass of atom becomes slightly less than sum of actual masses of nucleons and electrons.

$$\text{Actual mass of H atom} = M_p + M_e - \frac{B.E.}{c^2} \qquad \left(\frac{B}{c^2} \text{ is binding energy}\right)$$

B.E. (B) of H atom is 13.6 eV per atom.

Q13.3. When a nucleus in an atom undergoes a radioactive decay, the electronic energy levels of atom

(*a*) do not change for any type of radioactivity.

(*b*) change for α and β-radioactivity but not for γ-radioactivity.

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(*c*) change for α-radioactivity but not for others.
(*d*) change for β-radioactivity but not for others.

Ans. (*b*): β-particles carries one unit of negative charge, and α-particle carries 2 units of positive charge, and γ-particle carries no charge. So the electronic energy level of the atom changes in emission of α and β particle, but not in γ decay.

Q13.4. M_x and M_y denote the atomic masses of parent and the daughter nuclei respectively in a radioactive decay. The Q_1-value for a β^- decay is Q_1 and for a β^+ decay is Q_2. If m_e denotes the mass of an electron, then which of the following statements is correct?

(*a*) $Q_1 = (M_x - M_y)c^2$ and $Q_2 = (M_x - M_y - 2m_e)c^2$
(*b*) $Q_1 = (M_x - M_y)c^2$ and $Q_2 = (M_x - M_y)c^2$
(*c*) $Q_1 = (M_x - M_y - 2m_e)c^2$ and $Q_2 = (M_x - M_y + 2m_e)c^2$
(*d*) $Q_1 = (M_x - M_y + 2m_e)c^2$ and $Q_2 = (M_x - M_y + 2m_e)c^2$

Ans. (*a*): Let the parent nuclei $_ZX^A$ is radioactive atom and decay β^- as under, where Z and A have usual meanings.

$$_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu} + Q_1$$

$$Q_1 = \left[m_n\left({}_ZX^A\right) - m_n\left({}_{Z+1}Y^A\right) - m_e\right]c^2$$

$$= \left[m_n({}_ZX^A) + m_eZ - m_n({}_{Z+1}Y^A) - (Z+1)m_e\right]c^2$$

$$= [m({}_ZX^A) - m({}_{Z+1}Y^A)]C^2$$

Let the nucleus $= {}_ZX^A$ radiate β decay $_ZX^A \rightarrow {}_{Z-1}Y^A + {}_1e^0 + \nu + Q_2$

$$Q_2 = [m({}_ZX^A) - m({}_{Z-1}Y^A) - 2m_e]c^2$$

$$= [m_n({}_ZX^A) + m_eZ - m_n({}_{Z-1}Y^A) - (Z-1)m_e - 2m_e]c^2$$

$$= [m({}_ZX^A) - m({}_{Z-1}Y^A) - 2m_e]c^2$$

$$Q_2 = (M_x - M_y - 2m_e)c^2$$

Q13.5. Tritium is an isotope of hydrogen whose nucleus triton contains 2 neutrons and 1 proton. Free neutrons decay into $p + \bar{e} + \bar{\nu}$, if one of the neutrons in triton decays, it would transform into $_2He^3$ nucleus. This does not happen. This is because

(*a*) Triton energy is less than that of He^3 nucleus.
(*b*) yhe electron created in the beta decay process cannot remain in the nucleus.
(*c*) both the neutrons in triton have to decay simultaneously, resulting in a nucleus with 3 protons which is not a He^3 nucleus.
(*d*) free neutrons decay due to external perturbation which is absent in triton nucleus.

Ans. (*a*): Triton ($_1H^3$) has 1 proton and 2 neutrons. If a neutron decays as $n \rightarrow p + \bar{e} + \bar{\nu}$, then nucleus will have 2 proton and 1 neutron, *i.e.* triton atom converts in $_2He^3$ (2 proton and 1 neutron).

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Binding energy of $_1H^3$ is much smaller than $_2He^3$ so transformation is not possible energetically.

Q13.6. Heavy stable nuclei have more neutrons than protons. This is because of the fact that

(*a*) neutrons are heavier than protons.
(*b*) electrostatic force between protons are repulsive.
(*c*) neutrons decay into protons through beta decay.
(*d*) nuclear forces between neutrons are weaker than that between protons.

Ans. (*b*): Electrostatic force between proton-proton is repulsive which causes the unstability of nucleus. So nutrons are larger than protons.

Q13.7. In a nuclear reactor, moderators slow down the speed of neutrons which come out in the fission process. The moderator used have light nuclei. Heavy nuclei will not serve the purpose because

(*a*) they will break up.
(*b*) elastic collision of neutrons with heavy nuclei will not slow them down.
(*c*) the net weight of reactor would be unbearably high.
(*d*) substances with heavy nuclei do not occur in liquid or gaseous state at room temperature.

Main concept used: Mass of moderator must not be too much large for elastic collision.

Ans. (*b*): For elastic collision masses of both must be equal so that they can exchange the velocities. To slow down the speed of neutron substance should be made up of 1 proton for perfectly elastic *i.e.*, we need light nuclei not heavy. In heavy nuclei only direction will change not the speed.

MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q13.8. Fusion processes, like combining two deuterons to form a He nucleus are impossible at ordinary temperature and pressure. The reasons for this can be traced to the fact:

(*a*) nuclear forces have short range.
(*b*) nuclei are positively charged.
(*c*) the original nuclei must be completely ionised before fusion can take place.
(*d*) the original nuclei must first break up before combining with each other.

Ans. (*a*) and (*b*): Two deuteron can combine to form He atom when their nuclei come close to nuclear range where electrostatic repulsive force between positively charged deuterons does not act. Electrostatic force increases very high on decreasing their distance $\left(\because F \propto \frac{1}{r^2}\right)$.

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To overcome this electrostatic repulsive force nuclei need very high temperature and pressure. Hence to combine two nuclei, they must reach closer of the range of where nuclear force acts and electrostatic repulsive force does not act verifies the answers (*a*) and (*b*).

Q13.9. Samples of two radioactive nuclides A and B are taken. λ_A and λ_B are the disintegration constants of A and B respectively. In which of the following cases the two samples can simultaneously have the same decay rate at any time?

(*a*) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A = \lambda_B$.

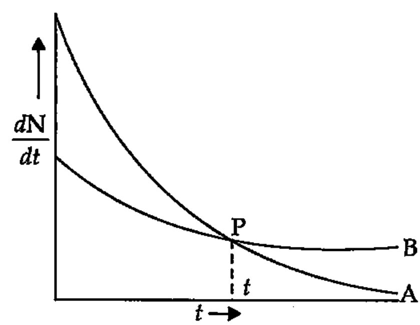
(*b*) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A > \lambda_B$.

(*c*) Initial rate of decay of B is twice the initial rate of decay of A and $\lambda_A > \lambda_B$.

(*d*) Initial rate of decay of B is same as the rate of decay of A at $t = 2h$ and $\lambda_B < \lambda_A$.

Ans. (*b*) and (*d*): Both radioactive samples can have same rate of decay at any time from initial time, if the initial rate of decay of A is equal to the twice of B and $\lambda_A > \lambda_B$. Decay rate can be same if $\lambda_A > \lambda_B$ and initial rate of decay of both are equal at $t = 2h$.

Q13.10. The variation of decay rate of two radioactive samples A and B with time is shown in figure. Which of the following statements are true?



(*a*) Decay constant of A is greater than that of B hence A always decays faster than B.

(*b*) Decay constant of B is greater than that of A but its decay rate is always smaller than that of A.

(*c*) Decay constant of A is greater than that of B, but it does not always decay faster than B.

(*d*) Decay constant of B is smaller than that of A but still its decay rate become equal to that of A at later instant.

Ans. (*c*) and (*d*): From the given graph slope of A is greater than of B so rate of decay of A is greater than of B. $\frac{dN}{dt} = -\lambda t$ or at instant t or for a particular time t, $\frac{dN}{dt} \propto \lambda$ so $\lambda_A > \lambda_B$ at point P the intersecting point of two graphs at time t is same.

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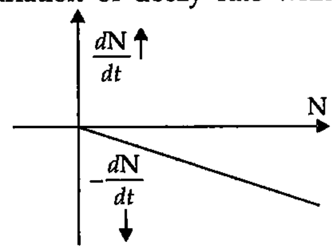
VERY SHORT ANSWER TYPE QUESTIONS

Q13.11. $_1He^3$ and $_2He^3$ nuclei have the same mass number. Do they have the same binding energy?

Ans. (*c*): In $_2He^3$ there are two protons which give electrostatic force of repulsion but in $_1He^3$ between nucleons there is only nuclear attractive force. So binding energy of $_1He^3$ is larger than $_2He^3$.

Q13.12. Draw a graph showing the variation of decay rate with number of active nuclei.



Ans. By law of radioactive decay

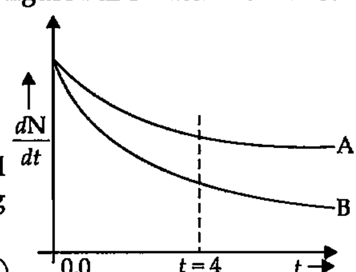
$$-\frac{dN}{dt} \propto \lambda N$$

N is the number of radioactive nuclei in the sample.

So, $\frac{dN}{dt}$ can be negative.

The variation of decay rate with number of active nuclei is shown by the above graph.

Q13.13. Which sample A or B shown in figure has shorter mean life?



Ans. Initially at $t = 0$ from figure given

$$\left(\frac{dN_0}{dt}\right)_A = \left(\frac{dN_0}{dt}\right)_B$$

so $$(N_o)_A = (N_o)_B$$

i.e., initially both samples has equal number of radioactive atoms. Considering at any instant $t = t$ from figure,

$$\left(\frac{dN}{dt}\right)_A > \left(\frac{dN}{dt}\right)_B$$

$$\lambda_A N_A > \lambda_B N_B \qquad \left(\frac{-dN}{dt} = \lambda N\right)$$

$$N_A > N_B$$

$$\lambda_A < \lambda_B$$

$\therefore$ $$\tau = \frac{1}{\lambda}$$

$\therefore$ $$\frac{1}{\tau_A} < \frac{1}{\tau_B}$$

$$\tau_A > \tau_B$$

So mean life time of sample A is greater than of B.

Q13.14. Which one of the following cannot emit radiation and why? Excited nucleus, excited electron.

Ans. Excited electron has energy in eV while excited nucleus in M eV. Energy of γ-rays have energy of the order of M eV. So excited *e* cannot emit radiation when nucleus is excited.

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Q13.15. In pair annihilation, an electron and a positron destroy each other to produce gamma radiations. How is the momentum conserved?

Ans. When an electron and positron combine together coming from opposite directions they destroy each other by the emission of two γ-rays in opposite direction to conserve linear momentum as below

$$_0e^{-1} + _0e^{+1} \rightarrow 2\gamma\text{-ray photons.}$$

SHORT ANSWER TYPE QUESTIONS

Q13.16. Why do stable nuclei never have more protons than neutrons?

Ans. If in a stable nucleus number of protons are larger than neutrons the repulsive electrostatic force between proton-proton becomes larger in place of nuclear force of attraction between nucleons.

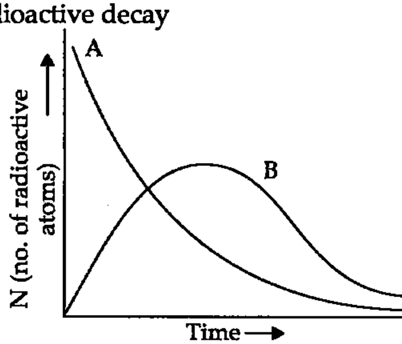
So for stability, repulsive force between proton-proton must be smaller than nuclear attractive force between nucleons.

Q13.17. Consider a radioactive nucleus A which decays to a stable nucleus C, through the following sequence

$$A \rightarrow B \rightarrow C$$

Here B is an intermediate nuclei which is also radioactive. Considering that there are N_0 atoms of A initially, plot the graph showing the variation of number of atoms of A and B versus time.

Main concept used: Law of radioactive decay



Ans. Let us consider that in sample A initially at $t = 0$, there are N_0 atoms of AB and atoms of B are zero initially.

When A decay to B the number of radioactive atoms of A decreases and of B increases.

When rate of decay of A decreases to a lower value, then the number of radioactive atoms of B becomes maximum. After this the radio-active atoms and rate of decay of B decreases. Finally, the number of radioactive atoms of sample A and B becomes very low near to zero.

Q13.18. A piece of wood from the ruins of an ancient building was found to have a C^{14} activity of 12 disintegrations per minute per gram of its carbon content. The carbon C^{14} activity of the living wood is 16 disintegrations per minute per gram. How long ago did the tree, from which the wooden sample came die? Given half life of C^{14} is 5760 years.

Main concept used: Carbon dating.

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Ans. Rate of disintegration in old wood sample of C-14 radioactive atoms is 12 atoms per min per gm. Initially rate of disintegration of C-14 when the tree was live = 16 atoms per min per gm.

$$T_{1/2} \text{ of C–14 nuclei} = 5760 \text{ years}$$

According to radioactive decay law,

$$N = N_0 e^{-\lambda t} \quad \text{or} \quad R = R_0 e^{-\lambda t}$$

$$12 = 16e^{-\lambda t}$$

$$e^{\lambda t} = \frac{16}{12}$$

$$\log_e e^{\lambda t} = \log_e \frac{4}{3}$$

$$\lambda t = \log_e \frac{4}{3}$$

$$t = \frac{2.303 \times \log_{10}\left(\frac{4}{3}\right)}{\lambda} \text{ half life} \qquad \left(\because \quad \lambda = \frac{0.6931}{T_{1/2}}\right)$$

$$= \frac{2.303(\log 4 - \log 3) \times 5760}{0.6931} \text{ years}$$

$$\therefore \quad t = \frac{2.303\,(0.6020 - 0.4771)\,5760}{0.6931} = \mathbf{2391.20 \text{ years}}$$

Q13.19. Are the nucleons fundamental particles, or do they consist of still smaller parts? One way to find out is to probe a nucleon just as Rutherford probed an atom. What should be the kinetic energy of an electron for it to be able to probe a nucleon? Assume the diameter of a nucleon to be approximately 10^{-15} m.

Main concept used: de-Broglie wavelength

Ans. To detect the properties of nucleons inside the nucleus the wavelength of particle which may detect nucleons that must be of size of nucleons (10^{-15} m). So the wavelength of particle which can detect the nucleons must be equal to or less than 10^{-15}.

$$\lambda = 10^{-15}\, m$$

$$\lambda = \frac{h}{p}$$

$$\because \quad E = h\nu = \frac{hc}{\lambda} \qquad [\because\ c = \nu\lambda]$$

$$\text{K.E.} = \text{P.E. of (electron)} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{10^{-15}} \text{ J}$$

$$\text{K.E.} = \frac{6.6 \times 3 \times 10^{-34+8+15}}{1.6 \times 10^{-19}} \text{ eV} = \frac{99 \times 10^{-34+23+19}}{8}$$

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$$= 12.4 \times 10^{-34+42} = 1.24 \times 10^1 \times 10^{+8}$$
$$\text{K.E.} = 1.24 \times 10^9 \text{ eV}$$

So the K.E. of particle which may detect nucleon inside the nucleus must be of 1.24×10^9 eV per particle.

Q13.20. A nuclide 1 is said to be the mirror isobar of nuclide 2, if $Z_1 = N_2$ and $Z_2 = N_1$.

(*a*) What nuclide is a mirror isobar of $_{11}Na^{23}$?

(*b*) Which nuclide out of the two mirror isobars have greater binding energy and Why?

Main concept used: Mirror isobar and Binding energy difference in neutrons and protons.

Ans. (*a*): Here Z is atomic number and N is no. of neutron in $_{11}Na^{23}$

$$Z_1 = 11$$
$$N_1 = 23 - 11 = 12$$

Mirror isobar of $_{11}Na^{23}$ is

$$Z_2 = N_1 = 12$$

So Mg is isobar of $_{11}Na^{23}$

So $_{12}Mg^{23}$ is the mirror isobar of $_{11}Na^{23}$.

(*b*) As the neutrons in $_{12}Mg^{23}$ are '11' and in $_{11}Na^{23}$ are '12' so, the number of neutrons in Na is larger than Mg and hence nuclear short range attractive forces in Na will be larger than repulsive electrostatic forces between proton-proton.

So, $_{11}Na^{23}$ has more binding energy than $_{12}Mg^{23}$.

LONG ANSWER TYPE QUESTIONS

Q13.21. Sometimes a radioactive nucleus decays into a nucleus which itself is radioactive. An example is:

$$^{38}\text{Sulphur} \xrightarrow[= 2.48 \text{ hrs.}]{\text{half life}} {}^{38}\text{Cl} \xrightarrow[= 0.62 \text{ h}]{\text{Half life}} {}^{38}\text{Ar (stable)}$$

Assume that we start with 1000 S^{38} nuclei at time $t = 0$. The number of Cl^{38} is of count zero at $t = 0$ and will again be zero at $t = \infty$. At what value of t, would the number of counts be a maximum?

Main concept used: Radioactive law of decay.

Ans. $$S^{38} \xrightarrow[2.48\text{h}]{\text{H.Life}} Cl^{38} \xrightarrow[0.62\text{h}]{\text{H.Life}} Ar^{38}$$

Initially at $t = 0$, number of radioactive atoms of $S^{38} = N_1$ and of Cl^{38} are zero.

At any time t, $$\frac{dN_1}{dt} = -\lambda_1 N_1$$

and $$N_1 = N_0 e^{-\lambda_1 t}$$

It is the rate of formation of Cl^{38} from S^{38}. Let N_2 is the number of Cl^{38} atoms (radioactive):

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$$\frac{dN_2}{dt} = -\lambda_2 N_2 + \lambda_1 N_1$$
$$= \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_2 N_2 \qquad \text{...(I)}$$

Multiplying both sides by $e^{+\lambda_2 t} dt$

$$e^{\lambda_2 t} dN_2 = \lambda_1 N_0 e^{-\lambda_1 t + \lambda_2 t} dt - \lambda_2 N_2 e^{+\lambda_2 t} dt$$
$$e^{\lambda_2 t} dN_2\, dt + \lambda_2 N_2 e^{+\lambda_2 t} dt = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t} dt$$

Integrating both sides

$$N_2 e^{\lambda_2 t} = \frac{N_0 \lambda_1}{(\lambda_2 - \lambda_1)} e^{(\lambda_2 - \lambda_1)t} + C \qquad \left[\because e^{\lambda_2 t} dN_2 \cdot dt = 0\right]$$

$\therefore$ Cl^{38} atom is formed after disintegration of S^{38}, so initially number of Cl^{38} atoms are $N_2 = 0$.
at $t = 0$, $N_2 = 0$,

$$0 \times e^0 = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1} e^0 + C \qquad \text{...(II)}$$

$$\therefore \quad 0 \times 1 = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1}(1) + C$$

$$\text{or} \quad C = \frac{-N_0 \lambda_1}{\lambda_2 - \lambda_1}$$

$$\therefore \quad N_2 e^{\lambda_2 t} = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1} e^{(\lambda_2 - \lambda_1)t} - \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1}$$

$$\frac{N_2}{e^{-\lambda_2 t}} = \frac{N_0 \lambda_1}{(\lambda_2 - \lambda_1)}\left[e^{(\lambda_2 - \lambda_1)t} - 1\right] \qquad \text{...(III)}$$

Multiplying $e^{-\lambda_2 t}$ to both sides we get

$$N_2 = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1}\left[e^{(\lambda_2 - \lambda_1 - \lambda_2)t} - e^{-\lambda_2 t}\right] \qquad [\because e^0 = 1]$$

$$N_2 = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$$

$$N_2 \lambda_2 - N_2 \lambda_1 = \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_1 N_0 e^{-\lambda_2 t}$$

N_o are the number of S^{38} atoms

No. of Cl^{38} atoms after time t will be $N_2 = N_0 e^{-\lambda_2 t}$

For $N_{2\,max} \cdot \frac{dN_2}{dt} = 0$

$$\therefore \quad N_2 \lambda_2 - N_2 \lambda_1 = \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_1 N_2$$
$$N_2 \lambda_2 - \lambda_1 N_2 + \lambda_1 N_2 = \lambda_1 e^{-\lambda_1 t}$$

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$$N_2\lambda_2 = \lambda_1 e^{-\lambda_1 t}$$
$$N_2\lambda_2 = \lambda_1 N_0 e^{-\lambda_1 t}$$
$$N_2 = \frac{\lambda_1}{\lambda_2} N_0 e^{-\lambda_1 t}$$

Put the value of N_2 in (III)

$$\frac{\lambda_1}{\lambda_2} N_0 \, e^{-\lambda_1 t} \, e^{\lambda_2 t} = \frac{N_0\lambda_1}{(\lambda_2 - \lambda_1)}\left[e^{(\lambda_2-\lambda_1)t} - 1\right]$$
$$e^{(\lambda_2-\lambda_1)t} = \frac{\lambda_2}{(\lambda_2 - \lambda_1)}\left[e^{(\lambda_2-\lambda_1)t} - 1\right]$$

By cross multiplication and multiplying both sides by $e^{-(\lambda_2-\lambda_1)t}$

$$\frac{\lambda_2 - \lambda_1}{\lambda_2} = 1 - e^{-(\lambda_2-\lambda_1)t}$$
$$1 - \frac{\lambda_1}{\lambda_2} = 1 - e^{(\lambda_1-\lambda_2)t}$$
$$\frac{\lambda_1}{\lambda_2} = e^{(\lambda_1-\lambda_2)t}$$
$$\log_e\left(\frac{\lambda_1}{\lambda_2}\right) = \log_e e^{(\lambda_1-\lambda_2)t}$$

or
$$\log_e\left(\frac{\lambda_1}{\lambda_2}\right) = (\lambda_1 - \lambda_2)\, t$$

$$t = \frac{\log_e\left(\frac{\lambda_1}{\lambda_2}\right)}{\lambda_1 - \lambda_2} \qquad \left(\because \ \lambda = \frac{0.6931}{T/2}\right) \quad ...(IV)$$

$$\therefore \quad \left(\frac{\lambda_1}{\lambda_2}\right) = \frac{\frac{0.6931}{2.48}}{\frac{0.6931}{0.62}} = \frac{0.62}{2.48} = \frac{1}{4}$$

$$(\lambda_1 - \lambda_2) = \frac{0.6931}{2.48} - \frac{0.6931}{0.62} = \frac{0.6931(-1.86)}{2.48 \times 0.62} = \frac{-0.6931 \times 1.86}{2.48 \times 0.62}$$

$$\therefore \quad t = \frac{\log_e\left(\frac{1}{4}\right) \times 2.48 \times 0.62}{-0.6931 \times 1.86} = \frac{-\log_e 4 \times 2.48 \times 0.62}{-0.6931 \times 1.86}$$
$$= \frac{2.303 \times 0.3010 \times 2.48 \times 0.62}{0.6931 \times 1.86} = \frac{1.06586}{1.2892} = \mathbf{0.8267\ hrs.}$$

Number of Cl^{38} radioactive atoms will be maximum at $N_2 = 0.8267$ hrs.

Q13.22. Deuteron is a bound state of a neutron and a proton with a binding energy B = 2.2 MeV. A γ-ray of energy E is aimed at a deuteron

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nucleus to try to break it into a (neutron + proton) such that the n and p move in the direction of the incident γ-ray. If E = B, show that this cannot happen. Hence calculate how much bigger than B must E be for such a process to happen.

Main concept used: Laws of conservation of energy and momentum.

Ans. Binding energy (B) of deuteron = 2.2 MeV

Some part of energy of γ-ray is used up against binding energy B = 2.2 MeV and the rest part will impart K.E. to neutron and proton.

$$E - B = K_n + K_p \qquad \left(K.E. = \frac{1}{2}mv^2 \times \frac{m}{m} \quad K.E. = \frac{p^2}{2m}\right)$$

$$E - B = \frac{p_n^2}{2m} + \frac{p_p^2}{2m} \qquad ...(I)$$

By law of conservation of momentum,

$p_n + p_p$ = momentum of γ-ray of Energy E

$$\left[\lambda = \frac{h}{p} \quad \text{or} \quad p = \frac{h\nu}{\lambda\nu} = \frac{h\nu}{c} = \frac{E}{c}\right]$$

$$\therefore \qquad p_n + p_p = \frac{E}{c} \qquad ...(II)$$

Case I: If E = B then from

$$\therefore \qquad \frac{p_n^2}{2m} + \frac{p_p^2}{2m} = 0 \quad \text{or} \quad p_n^2 + p_p^2 = 0 \qquad ...(III)$$

It can be possible if $p_n = p_p = 0$ because square of non zero number can never be zero.

If $p_n = p_p = 0$ then equation IInd cannot be satisfied and the process cannot take place.

From II, $0 + 0 = \frac{E}{c}$ or E = 0 but energy E of γ ray cannot be zero.

Case II: If E > B or E = B + λ where λ will be very small than B then from (I),

$$(B + \lambda) - B = \frac{p_n^2}{2m} + \frac{p_p^2}{2m}$$

$$\lambda = \frac{1}{2m}\left(p_n^2 + p_p^2\right)$$

$$\lambda = \frac{1}{2m}\left[\left(\frac{E}{c} - p_p\right)^2 + p_p^2\right] \qquad \left[\because p_n = \frac{E}{c} - p_p\right]$$

$$2m\lambda = \frac{E^2}{c^2} + p_p^2 - \frac{2E}{c}p_p + p^2$$

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$$2p_p^2 - \frac{2E}{c}p_p + \left(\frac{E^2}{c^2} - 2m\lambda\right) = 0$$

It is a quadratic equation so its solution by quadratic formula

$$a = 2, b = \frac{-2E}{c}, c = \left(\frac{E^2}{c^2} - 2m\lambda\right)$$

$$p_p = \frac{+\frac{2E}{c} \pm \sqrt{\frac{4E^2}{c^2} - 4\times 2\left(\frac{E^2}{c^2} - 2m\lambda\right)}}{4}$$

For a real and equal value of p_p discriminant must be zero as the value of p_p must be one.

$$\therefore \qquad \frac{4E^2}{c^2} - 8\left[\frac{E^2}{c^2} - 2m\lambda\right] = 0$$

$$\frac{4}{c^2}[E^2 - 2E^2 + 4mc^2\lambda] = 0$$

$$\therefore \qquad -E^2 + 4mc^2\lambda = 0$$

$$\lambda = \frac{E^2}{4mc^2}$$

$\because$ λ is very small

so $$E = B$$

$$\boxed{\lambda \cong \frac{B^2}{4mc^2}}$$

Q13.23. The deuteron is bound by nuclear forces just as H-atom is made up of p and e bound by electrostatic forces. If we consider the force between neutron and proton in deuteron as given in the form of a Coulomb potential but with an effective charge e'

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e'^2}{r}$$

estimate the value of $\left(\frac{e'}{e}\right)$ given that the binding energy of a deuteron is 2.2 MeV.

Ans. The binding energy of H atom in ground state

$$E = \frac{m e^4}{8\pi\varepsilon_0^2 h^2} = 13.6\,\text{eV} \qquad \text{...(I)}$$

If proton and neutron had charge e' each and governed by the same electrostatic force, then in the above equation we would need to

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replace electronic mass m by the reduced mass m' of proton-neutron (as some mass of proton and neutron is used by binding energy) and electronic charge e is replaced by e′.

$$\frac{1}{m'} = \frac{1}{M} + \frac{1}{N} \qquad \begin{pmatrix} M = \text{mass of proton} \\ N = \text{mass of neutron} \end{pmatrix}$$

$$m' = \frac{M \cdot N}{M + N} \qquad \text{(take } M = N\text{)}$$

$$= \frac{M}{2} \qquad \text{(if } m = \text{mass of electron)}$$

$$m' = \frac{1836\, m}{2} = 918\, m$$

$$\therefore \quad \text{Binding energy (E)} = \frac{918\, m\, e'^4}{8\pi\varepsilon_0^2 h^2} \qquad \text{...(II)}$$

Dividing (II) by (I) we get,

$$\frac{E'}{E} = \frac{918\, e'^4}{e^4} = \frac{2.2\,\text{MeV}}{13.6\text{eV}}$$

$$\left(\frac{e'}{e}\right)^4 = \frac{2.2 \times 10^6\,\text{eV}}{918 \times 13.6\ \text{eV}}$$

$$\frac{e'}{e} = \frac{2200000}{1248.48} = (176.21)^{1/4}$$

Required ratio $\frac{e'}{e}$ = **3.64**

Q13.24. Before the neutrino hypothesis, the β-decay process was thought to be the transition $n \to p + \overline{e}$. If this was true show that if the neutron was at rest, the proton and electron would emerge with fixed energies and calculate them. Experimentally the electron energy was found to have a large range.

Ans. Neutron was at rest before β decay from neutron. Hence energy of neutron = $E_n = m_n c^2$ and momentum of neutron $p_n = 0$ as its velocity is zero.

By the law of conservation of momentum,

$$p_n = p_p + p_e \text{ (Beta)}$$
$$0 = p_p + p_e$$

Let $p_e = p_p$ then

$$\Rightarrow \qquad |p_p| = |p_e| = p\text{(eV)}$$

$$\text{Energy of proton} = E_p = \sqrt{(m_p^2 c^4 + p_p^2 c^2)}$$

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Energy of electron (β) = $E_e = \sqrt{(m_e^2c^4 + p_e^2c^2)}$ $(\because \ |p_e| = p_p)$

From conservation,

$\therefore$
$$E_p = \sqrt{m_p^2c^4 + p^2c^2}$$
$$E_e = \sqrt{m^2c^4 \quad p^2c^2}$$

$\therefore$ By the law of conservation of energy,

$$(m_p^2c^4 + p^2c^2)^{1/2} + (m_e^2c^4 + p^2c^2)^{1/2} = m_nc^2$$

$\because$ $m_pc^2 = 936$ MeV

and $m_nc^2 = 938$ MeV

and $m_ec^2 = 0.5$ MeV

As the energy difference in neutron and proton is very small, pc will be small $pc << m_pc^2$ while pc may be greater than m_ec^2 so by neglecting $(m_ec^2)^2 = (0.5)^2$ (Given)

$$\Rightarrow \quad m_pc^2 + \frac{p^2c^2}{2m_p^2c^4} = m_nc^2 - pc$$

$$m_pc^2 + \frac{p^2c^2}{2m_p^2c^4} + pc = m_nc^2$$

Again $pc << m_pc^2$ so neglecting $\frac{p^2c^2}{2m_p^2c^4}$ we get

$$pc = m_nc^2 - m_pc^2 = 938 \text{ MeV} - 936 \text{ MeV}$$

$pc = 2$ MeV is the momentum

$\because$ $E = mc^2$

$E^2 = m^2c^4$

E is the energy of either proton or neutron then

$$E_p = \sqrt{m_p^2c^4 + p^2c^2} = \sqrt{(936)^2 + 2^2} = 936 \text{ MeV}$$

$$E_e = \sqrt{m_e^2c^4 + p^2c^2} = \sqrt{(a)^2 + 2^2} = \mathbf{2.06\ MeV}$$

Q13.25. The activity R of an unknown radioactive nuclide is measured at hourly intervals. The results found are tabulated as follows

t (hours)	0	1	2	3	4
R (mega Bq)	100	35.36	12.51	4.42	1.56

(*i*) Plot the graph of R versus *t* and calculate half-life from the graph.

(*ii*) Plot the graph of $\log \frac{R}{R_0}$ versus *t* and obtain the value of half life from the graph.

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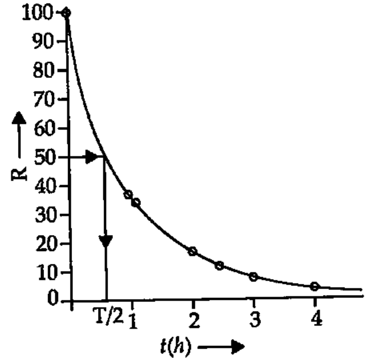
Ans. (*i*) Graph between R versus t is exponential curve. From the graph at slightly more than $t = \frac{1}{2}h$ the R should be 50% so at R = 50% the

$$t(h) = 0.7\,h = 0.7 \times 60 \text{ min} = 42 \text{ min}$$



(*ii*) For Graph between $\log\left(\frac{R}{R_0}\right)$ versus $t(h)$

at $t = 0$, $\log_e \frac{R}{R_0} = \log \frac{100}{100} = \log 1 = 0$

at $t = 1$ hour, $\log_e \frac{35.36}{100} = \log_e 0.3536 = -1.04$
$= 2.302 \log_{10} 0.3536 = -1.04$

at $t = 2$ hours, $\log_e \frac{12.5}{100} = \log_e 0.125$
$= 2.303 \log_{10} 0.125 = -2.08$

at $t = 3$ hour, $\log_e \frac{4.42}{100} = -3.11$

at $t = 4$ hour, $\log_e \frac{1.56}{100} = -4.16$

t (hours)	1	2	3	4
$\log_e \frac{R}{R_0}$	–1.04	–2.08	–3.11	–4.16

The graph showing the variation of $\log_e \frac{R}{R_0}$ versus $t(h)$ as follows:

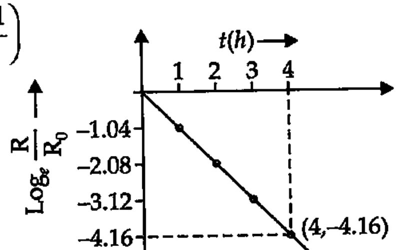
We know that disintegration constant

$$\lambda = \frac{\log_e \frac{R}{R_0}}{t_{1/2}} = -\left(\frac{4.16 - 3.11}{1}\right)$$

$\lambda = -1.05$ per hour

$$t_{1/2} = \frac{0.6931}{\lambda} = \frac{0.6931}{1.05}$$

$t_{1/2} = 42$ min



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Q13.26. Nuclei with magic number of proton Z = 2, 8, 20, 28, 50, 52 and magic number of neutron N = 2, 8, 20, 28, 50, 82, and 126 are found to be very stable.

(*i*) Verify this by calculating the proton separation energy S_p, for $_{50}Sn^{120}$ and $_{51}Sb^{121}$.

The proton separation energy for a nuclide is the minimum energy required to separate the least tightly bound proton from a nucleus of that nuclide. It is given by

$$S_p = [M_{Z-1,N} + M_H - M_{ZN}]\, c^2$$

Given $\quad ^{119}In = 118.9058\, u;\ _{50}Sn^{120} = 199.902199\, u;$

$$_{51}Sb^{121} = 120.903824\, u \text{ and } _1H^1 = 1.0078252\, u$$

(*ii*) What does the existance of magic number indicate?

Ans. (*i*) $S_p = [M_{Z-1,\,N} + M_H - M_{Z,\,N}]c^2$

Here in this formula M_{Z-1} is the mass of atom of Z – 1 atomic number.

M_Z is the mass of atom of mass number Z

$\therefore \quad M_{Z-1}$ = Mass of atom whose atomic number is 50 – 1 = 49.

It is $_{49}In^{119}$ in this case $M_{Z-1} = {_{49}In^{119}} = 118.9058$ and N = 119 – 49 = 70.

$$S_p \text{ for } _{50}Sn^{120} = c^2[118.9058 + 1.0078252 - 199.902199]$$

$$S_p \text{ for } _{50}Sn^{120} = 0.0114362\, c^2$$

Now for S_p of $_{51}Sb^{121}$

$$S_p = [M_{Z-1,\,N} + M_H - M_{Z,N}]c^2$$

$\Rightarrow \quad Z = 51, Z - 1 = 50$ for S_n

$$M_{Z-1} = \text{mass of } _{50}Sn = 199.902199\, u$$

$$\therefore \quad S_p \text{ for } _{51}Sb^{121} = [199.902199 + 1.0078252 - 120.903824]c^2$$

$$= 0.0059912\, c^2$$

$$\therefore \quad S_p(_{50}Sn^{120}) > S_p(_{51}Sb^{121})$$

(*ii*) The existence of magic numbers indicates that the shell structure of nucleus is similar to the shell structure of atom. This also explains the peaks in binding energy per nucleon curve.

❑❑❑

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14 Semiconductor Electronics: Materials, Devices and Simple Circuits

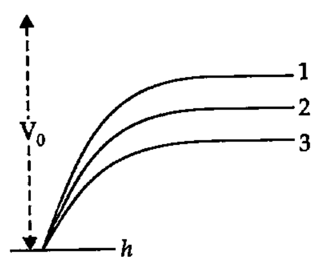
MULTIPLE CHOICE QUESTIONS—I

Q14.1. The conductivity of a semiconductor increases with increase in temperature, because

(*a*) number density of free current carriers increases.
(*b*) relaxation time increases.
(*c*) both number density of free current carriers and relaxation time increases.
(*d*) number density of current carriers increases and relaxation time decreases, but effect of decrease in relaxation time is much less than increase in number density.

Ans. (*d*): In semiconductor, the density of charge carriers(electron, holes) are very small, so its resistance is high. When temperature increases the charge carriers (density) increases which increases the conductivity. As temperature of semiconductor increases, the speed of free electrons increases which decrease the relaxation time. As the density of charge carrier is small so there is small effect on decrease of relaxation time.

Q14.2. In given figure, V_0 is the potential barrier across a p-n junction, when no battery is connected across the junction



(*a*) 1 and 3 both corresponds to forward bias of junction.
(*b*) 3 corresponds to forward bias of junction and 1 corresponds to reverse bias of junction.
(*c*) 1 corresponds to forward bias and 3 corresponds to reverse bias of junction.
(*d*) 3 and 1 both corresponds to reverse bias of junction.

Main concept used: Thickness of depletion layer in p-n junction.

Ans. (*b*): When p-n junction is in forward bias, it compresses or decreases the depletion layer so potential barrier in forward bias decreases and in reverse bias potential barrier increases.

Q14.3. In given figure below, assuming the diodes to be ideal,

(*a*) D_1 is forward biased and D_2 in reverse biased and hence current flows from A to B.

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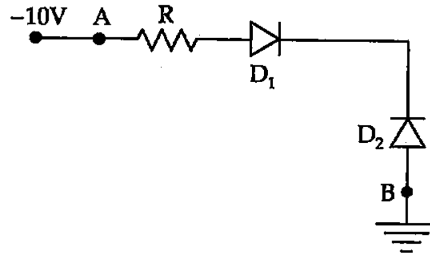
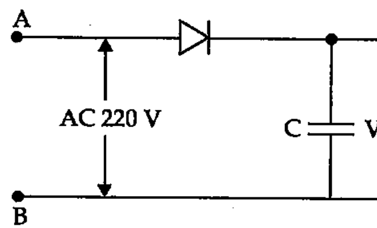


(*b*) D_2 is forward biased and D_1 is reverse biased and hence no current flows from B to A and vice versa.
(*c*) D_1 and D_2 both are forward biased and hence current flows from A to B.
(*d*) D_1 and D_2 are both in reverse bias hence no current flows from A to B and vice-versa.

Main concept used: Potential at the ends of diode.

Ans. (*b*): In circuit, A is at –10V and B is at 0V. So B is positive than A. So D_2 is in forward bias and D1 is in reverse bias so no current flows from A to B or B to A.

Q14.4. A 220 V AC supply is connected between points A and B (figure). What will be the potential difference V across the capacitor?



(*a*) 220 V (*b*) 110 V
(*c*) 0V (*d*) $220\sqrt{2}$ V

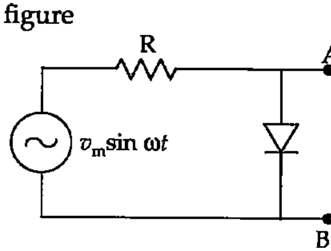
Main concept used: p-n junction conducts current in forward bias *i.e.*, in positive cycle.

Ans. (*d*): Potential difference across capacitor will be peak voltage when diode is in forward bias. Diode will be in forward bias when end A is at positive potential of cycle. So potential at C = peak value of $V = V_{rms}\sqrt{2} = 220\sqrt{2}\,V$.

Q14.5. Hole in semiconductor is
(*a*) an antiparticle of electron.
(*b*) a vacancy created when an electron leaves a covalent bond.
(*c*) absence of free electrons.
(*d*) an artificially created particle.

Ans. (*b*): Atoms of semiconductor are binded by covalent bonds between the atoms of same or different type. Due to thermal agitation when an electron leaves its position and become free, then it leaves a vacancy of electron, this vacancy in the bond (covalent) is called hole.

Q14.6. The output of the given circuit in figure



(*a*) would be zero at all times.
(*b*) would be like a half wave rectifier with positive cycles in output.
(*c*) would be like a half wave rectifier with negative cycles in output.
(*d*) would be like that of a full wave rectifier.

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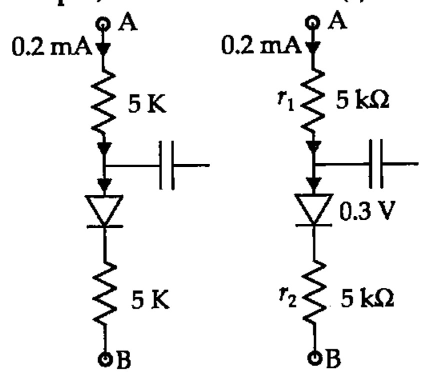
Ans. (*c*): When positive cycle is at A, diode will be in forward bias and resistance due to diode is approximately zero so potential across diode will be about zero.

Similarly, when there is negative half cycle at A, diode will be in reverse bias and resistance will be maximum so potential difference across diode is $V_m \sin \omega t$ with negative at A.

So we get only negative output at A so it behaves like a half wave rectifier with negative cycle at A in output, verifies the answer (*c*).

Q14.7. In the circuit shown in figure below, if the diode forward voltage drop is 0.3V, the voltage difference between A and B is

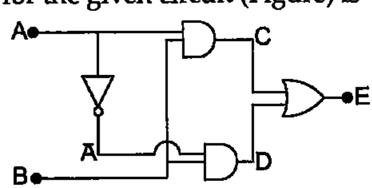


(*a*) 1.3 V (*b*) 2.3 V
(*c*) 0 V (*d*) 0.5 V

Ans. (*b*): In the middle right of the circuit the capacitor behaves like an open circuit for d.c 0.2 mA current so current will flow from A to B only. Let potential across A and B is V, so by Kirchhoff's loop law

$$V_{AB} = (5000 \times 0.2 \times 10^{-3}) + 0.3 + 5000 \times 0.2 \times 10^{-3}$$
$$V_{AB} = 1\text{ V} + 0.3\text{ V} + 1\text{ V}$$
$$V_{AB} = 2.3\text{ Volt}$$

Q14.8. Truth table for the given circuit (Figure) is



(*a*)

A	B	E
0	0	1
0	1	0
1	0	1
1	1	0

(*b*)

A	B	E
0	0	1
0	1	0
1	0	0
1	1	1

(*c*)

A	B	E
0	0	0
0	1	1
1	0	0
1	1	1

(*d*)

A	B	E
0	0	0
0	1	1
1	0	1
1	1	0

Ans. (*c*)

A	B	$\bar{A}$	C	D	E
0	0	1	0	0	0
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	0	1

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MULTIPLE CHOICE QUESTIONS—II MORE THAN ONE OPTION

Q14.9. When an electric field is applied across a semiconductor.

(*a*) electrons move from lower energy level to higher energy level in the conduction band.

(*b*) electrons move from higher energy level to lower energy level in the conduction band.

(*c*) holes in the valence band move from higher energy level to lower energy level.

(*d*) holes in the valence band move from lower energy level to higher energy level.

Main concept used: Motion of a charged particle in electric field.

Ans. (*a*) and (*c*): In the direction of electric field the P.E. or energy of electric field decreases. So electrons always move opposite to direction of E.F., *i.e.* from lower energy level to higher energy level and holes move from higher energy level to lower energy level.

In semiconductors, electrons are in conduction band and holes are in valence band.

Q14.10. Consider an *n-p-n* transistor with its base-emitter junction forward biased and collector-base junction reversed biased. Which of the following statements are true?

(*a*) Electrons crossover from emitter to collector.

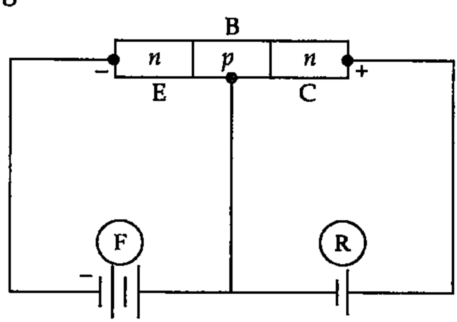
(*b*) Holes move from base to collector.

(*c*) Electrons move from emitter to base.

(*d*) Electrons from emitter move out of base without going to collector.

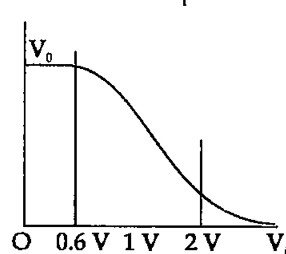
Main concept used: Working of transistor

Ans. (*a*) and (*c*): Electrons are repelled by forward bias from emitter to collector ≅ 5% of electrons combine with holes of base and rest 95% electrons of emitter are attracted by reverse bias of collector-base junction.



So electrons move from emitter to collector through base *i.e.*, emitter to base verifies (a) and (c).

Q14.11. Figure given alongside shows the transfer characteristics of a base biased CE transistor. Which of the following statements are true?



(*a*) At $V_i = 0.4$ V, transistor is in active state.

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(*b*) At $V_i = 1$ V, it can be used as an amplifier.
(*c*) At $V_i = 0.5$ V, it can be used as a switch turned off.
(*d*) At $V_i = 2.5$ V, it can be used as a switch turned on.

Ans. (*a*) At $V_i = 0.4$ V there is no collector current only output voltage is V_0 due to V_{ce} battery. So at 0.4 V transistor is not in active mode.

(*b*) When at $V_i = 1$ volt, transistor is between active region (0.6 V to 2 V). So it can be used as an amplifier.

(*c*) At $V_i = 0.5$ V, the transistor is in cut-off state and it can be used as a switch turned off.

(*d*) At $V_i = 2.5$ V, the transistor is beyond active region. The collector current is in saturated state and there is no effect on changing input voltage and the transistor can be used as a switch turned on.

So, the statements (*b*), (*c*) and (*d*) are true.

Q14.12. In a *n-p-n* transistor circuit, the collector current is 10 mA. If 95% of the electrons emitted (by emitter) reach the collector, which of the following statements are true?

(*a*) The emitter current will be 8 mA.
(*b*) The emitter current will be 10.53 mA.
(*c*) The base current will be 0.53 mA.
(*d*) The base current will be 2 mA.

Ans. (*b*) and (*c*): $I_c = 10$ mA

$$I_c = 95\% \text{ of } I_e \quad \text{or} \quad I_c = \frac{95}{100} I_e$$

$$I_e = \frac{10 \text{ mA} \times 100}{95} = 10.53 \text{ mA}$$

$$I_b + I_c = I_e$$

So $$I_b = I_e - I_c = 10.53 - 10 = 0.53 \text{ mA}$$

Q14.13. In the depletion region of a diode

(*a*) there are no mobile charges.
(*b*) equal number of holes and electrons exist, making the region neutral.
(*c*) the recombination of holes and electrons has taken place.
(*d*) immobile charged ions exist.

Ans. (*a*), (*b*), (*c*) and (*d*): During formation of *p-n* junction electrons from *n* side and holes from *p* side move towards each other and form a potential barrier across the junction. Within the barrier there are holes and electrons or ions which cannot move.

So in depletion layer no mobile charges but there are negative and positive ions; barrier layer was formed by recombination of electrons and holes when *pn* junction formed.

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Q14.14. What happens during the regulation action of a Zener diode?
(*a*) The current and voltage across the Zener diode remains fixed.
(*b*) The current through the series resistance (R_S) changes.
(*c*) The Zener resistance is constant.
(*d*) The resistance offered by the Zener diode changes.

Ans. (*b*) and (*d*): During action of regulation, current in R_s changes and resistance offered by Zener diode changes. The current through the Zener diode changes but voltage across the Zener remains constant.

Q14.15. To reduce the ripples in a rectifier circuit with capacitor filter
(*a*) R_L should be increased.
(*b*) input frequency should be decreased.
(*c*) input frequency should be increased.
(*d*) capacitors with high capacitance should be used.

Main Concept: The Ripple factor (r) of full-wave rectifier is given by $r = \frac{1}{4\sqrt{3} \cdot R_L C_V}$ or $r \propto \frac{1}{R_r} \Rightarrow r \propto \frac{1}{C}$ and $r \propto \frac{1}{V}$

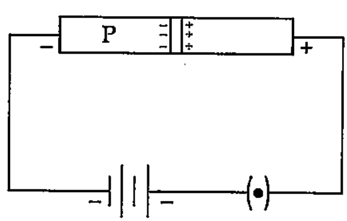
Ans. (*a*), (*c*) and (*d*): As the ripple factor

$$r = \frac{1}{4\sqrt{3}R_L C_V}$$

So to decrease r, R_L and C must be increased.

Input frequency should be increased.

Q14.16. The breakdown in a reverse biased *p-n* junction diode is more likely to occur due to
(*a*) large velocity of the minority charge carriers if the dopping concentration is small.
(*b*) large velocity of the minority charge carriers if the dopping concentration is large.
(*c*) strong electric field in a depletion region if the dopping concentration is small.
(*d*) strong electric field in a depletion region if the dopping concentration is large.



Ans. (*a*) and (*d*): In reverse biasing, minority charge carriers accelerate due to high electric field applied (due to reverse P.D.). So the minority charge carrier in both junction accelerated toward depletion layer, which on striking with atoms causes ionisation of atoms resulting the secondary electron and so more number of charge carriers constitute more current in reverse direction.

More current in thin depletion layer is due to the heavy dopping and large no. of ions in the depletion layer which give rise to strong electric field across the junction.

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VERY SHORT ANSWER TYPE QUESTIONS

Q14.17. Why are elemental dopants for silicon or Germanium usually chosen from group XIII or XV?

Ans. The size of the dopant atom should be equivalent to the size of Si or Ge. So that the symmetry of pure Si or Ge, does not disturb and dopants can contribute the charge carrier on forming covalent bonds with Si or Germanium atoms. As the silicon and germanium belongs to XIV th group so similar size of atom will be in XIII and XV grp of modern periodic table.

Q14.18. Sn, C, Si, Ge are all group XIV elements. Yet, Sn is a conductor, C is an insulator, while Si and Ge are semiconductors. Why?

Main concept used: Conduction level of an element depends on the energy gap between valence and conduction band of element.

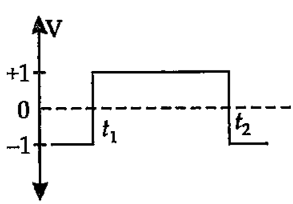
Ans. A material will conduct current if there is no energy gap between conduction and valence band in energy band diagram of atom. This energy gap decreases from insulator to semiconductor and from semiconductor to conductor.

The energy gaps in Sn, C, Si and Ge are 0 eV, 0.54 eV, 1.1 eV and 0.7 eV respectively related to their atomic size. So the Sn is a conductor, C is an insulator while Si and Ge are semiconductors.

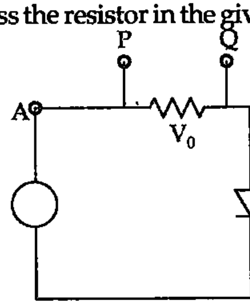
Q14.19. Can the potential barrier across p-n junction be measured by simply connecting a voltmeter across the junction?

Ans. We cannot measure the potential barrier across p-n junction by voltmeter, because the resistance across depletion layer is smaller than resistance of voltmeter so current to measure the potential will not flow in voltmeter, and current passes through the junction.

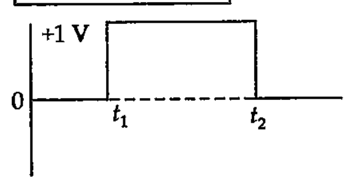
Q14.20. Draw the output waveform across the resistor in the given figure.



Input waveform at A



Ans. In given circuit waveform is connected at A and waveform obtained across resistance diode conducts when diode is in forward biased so output will be only when input is +1V is between t_1 to t_2. So output waveform will be only t_1 to t_2 which, is in given figure.



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Q14.21. The amplifiers X, Y and Z are connected in series. If the voltage gains of X, Y and Z are 10, 20, 30 respectively and input signal is 1 mV peak value, then what is the output signal voltage (peak value)

(*i*) If DC supply voltage is 10 V?

(*ii*) If DC supply voltage is 5 V?

Main concept used: Voltage gain $= \dfrac{\text{Output voltage}}{\text{Input Voltage}}$

Ans. $A_{V_x} = 10, \quad A_{V_y} = 20 \quad \text{and} \quad A_{V_z} = 30$

Signal $A_{V_i} = 1\text{ mV} = 10^{-3}\text{ V}$

$$A_V = \frac{V_0}{V_i}$$

$$A_{V_x} \times A_{V_y} \times A_{V_z} = \frac{V_0}{V_i}$$

$$V_0 = V_i \times A_{V_x} \times A_{V_y} \times A_{V_z} = 10^{-3} \times 10 \times 20 \times 30$$

$$= 6\text{V (output signal)}$$

(*i*) If DC supply voltage is 10 V and output voltage 6 V as the theoretical gain and practical gains are equal so output will be 6 V.

(*ii*) When DC supply voltage is 5 V or $V_{ce} = 5$ V then output peak cannot exceed by 5 V. So output will be 5 V.

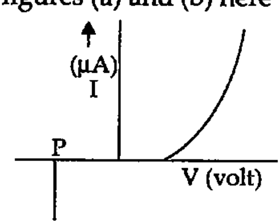
Q14.22. In a CE transistor amplifier, there is a current and voltage gain associated with the circuit. In other words there is a power gain. Considering power a measure of energy, does the circuit violate conservation of energy?

Ans. In CE transistor amplifier the DC supply is connected to give energy to signal. Due to this fact there is a large power gain in CE configuration amplifier.

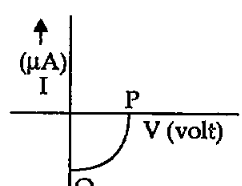
So energy of output signal is equal to the sum of energy of signal (low) and energy supplied by DC source in output CE circuit (V_{ce}).

SHORT ANSWER TYPE QUESTIONS

Q14.23. (*i*) Name the type of a diode whose characteristics are shown in figures (a) and (b) here



(*a*)



(*b*)

(*ii*) What does the point P in figure (*a*) represent?

(*iii*) What does the points P and Q in figure (*b*) represent?

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Ans. (*i*) Diode in figure (a) is Zener diode and figure (*b*) shows solar cell.

(*ii*) Zener breakdown voltage is represented in figure (*a*).

(*iii*) In figure (*b*), Q represents zero voltage and negative current. It means that light energy falling on solar cell with at least minimum threshold frequency gives the current in opposite direction to that due to a battery connected to solar cell. But for point P the battery is short-circuited and hence represents the short-circuited current.

In figure (*b*), Point P represents some positive voltage on solar cell with zero current through solar cell.

So there is a battery connected to a solar cell which gives rise to the equal and opposite current to that in solar cell by virtue of light falling on it.

As current is zero for point P, hence we say P represents open circuit voltage.

Q14.24. Three photodiodes D_1, D_2 and D_3 are made of semiconductors having band gaps of 2.5 eV, 2 eV and 3 eV respectively. Which one will be able to detect light of wavelength 6000 Å?

Ans. $\lambda = 6000\ \text{Å} = 6000 \times 10^{-10}$ m

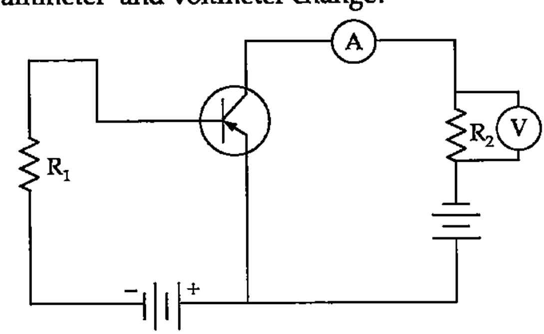
$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{6000 \times 10^{-10} \times 1.6 \times 10^{-19}} \text{eV}$$

$$= \frac{6.62 \times 3 \times 10^{-34+8+10+19}}{6 \times 1.6 \times 10^{3}} \text{eV} = \frac{3.31 \times 10^{-34+37}}{1.60 \times 10^{3}} \text{eV}$$

$$= \frac{331}{160} \times \frac{10^3}{10^3} \text{eV} = 2.06 \text{ eV}$$

The incident radiation is detected by the photodiode D_2 has band gap or knee voltage 2 eV only which is less than incident radiation of 2.06 eV.

Q14.25. If the resistance R_1 is increased (see figure) how will the readings of the ammeter and voltmeter change?



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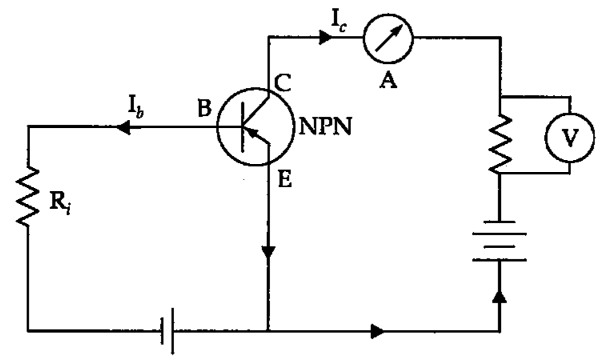
Ans. Consider the figure given below to find out the change in reading of output E-C circuit



$$I_B = \frac{V_{BB} - V_{BE}}{R_i}$$

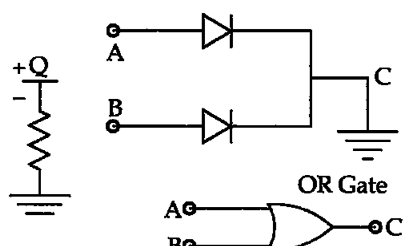
As R_i increases the P.D. across R_i also increases so potential across BE decreases so the Ic will also decrease in turn reading of ammeter and voltmeter will decrease because $Ic = \beta I_B$.

Q14.26. Two car garages have a common gate which needs to open automatically when a car enters either of the garages or cars enter both. Device a circuit that resembles this situation using diodes for this situation.

Ans. When a car enters the gate, any one or both are open.

The device is shown in figure.

A	B	C
0	0	0
0	1	1
1	0	1
1	1	1



So OR gate gives the desired output.

Q14.27. How would you set up a circuit to obtain NOT gate using a transistor?

Ans. The NOT gate is a device which has only one input and one output *i.e.*, $\overline{A} = Y$ means, Y equals to NOT A. Thus,

A	$Y = \overline{A}$
0	1
1	0

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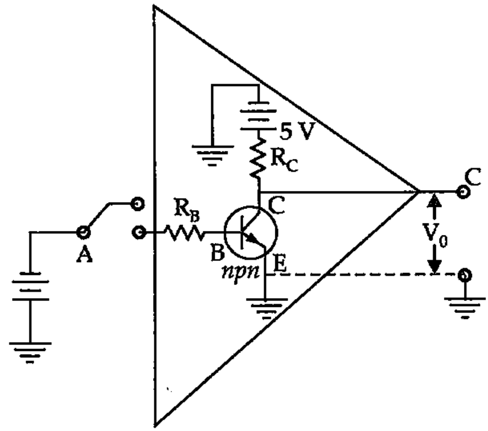
Here the base B of the transistor is connected to the input A through the input resistance R_B and the emitter is earthed. The collector is connected to 5 V battery. The output Y is the voltage at C w.r.t. earth.

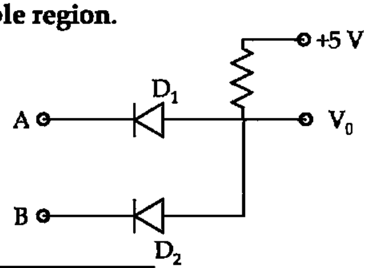


The resistance R_B and R_C are so chosen that if emitter-base junction is unbiased, the transistor is in cut-off region and if emitter-base junction is in forward bias by 5 V input at A, the transistor is in saturation state.

Q14.28. Explain why elemental semiconductor cannot be used to make visible LEDs.

Ans. In elemental conductor, the band gap is such that the **emissions are in infrared region** and **not in visible region.**

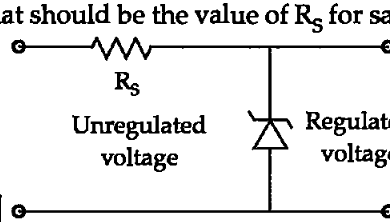
Q14.29. Write the truth table for the circuit shown in figure given alongside. Name the gate that the circuit resembles.



Ans. The circuit resembles AND gate. The boolean expression of this circuit is V_0 = A.B *i.e.,* V_0 equals A AND B. The truth table of this gate is as:

A	B	$V_0 = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

Q14.30. A Zener of power rating 1 W is to be used as a voltage regulator. If Zener has a breakdown of 5 V and it has to regulate voltage which fluctuated between 3 V and 7 V what should be the value of R_S for safe operation (see figure)?



Ans. Given P = 1 Watt

Zener breakdown voltage = 5 V

Minimum voltage V_{min} = 3 V

Maximum voltage V_{max} = 7 V

$$\text{Current} = \boxed{I_{max} = \frac{P}{V_Z}}$$

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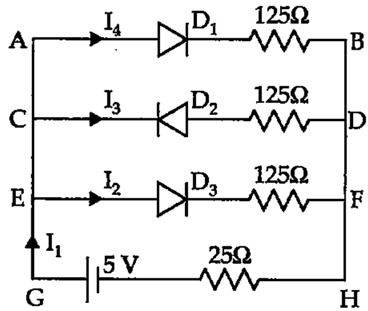
$$I_{Z\,max} = \frac{1\text{ Watt}}{5} = 0.2\text{ A}$$

$$\boxed{R_S = \frac{V_{max} - V_{min}}{I_{Z\,max}}} = \frac{7-3}{0.2} = \frac{4}{0.2} = 20\,\Omega$$

LONG ANSWER TYPE QUESTIONS

Q14.31. If each diode in figure has a forward bias resistance of 125 Ω and infinite resistance in reverse bias, what will be the values of the currents I_1, I_2, I_3 and I_4?



Ans. Diode D_1 and D_3 are in forward bias and D_2 is in reverse bias so resistance in arm A B and EF = 125 + 25 = 150 Ω and Resistance in CD is infinite.

$$\frac{1}{R_{24}} = \frac{1}{150} + \frac{1}{150} = \frac{2}{150} = \frac{1}{75}$$

$$R_{24} = 75\,\Omega$$

Total resistance in circuit = R = 75 + 25 = 100 Ω

So $\quad V = IR$

$$5 = I_1\,100$$

$$I_1 = \frac{5}{100} = 0.05\text{ Amp}$$

$$I_1 = I_2 + I_3 + I_4$$

∴ Resistance R of CD = ∞

so $\quad I_3 = 0$

so $\quad I_1 = I_2 + I_4$

Here resistances are equal

∴ $\quad I_2 = I_4$

or $\quad I_1 = 2I_2$

∴ $\quad I_2 = \frac{I_1}{2}$

$$\frac{0.05}{2} = I_2$$

$$I_2 = 0.025$$

I_1 = 0.05 Amp, I_2 = 0.025 Amp, I_4 = 0.025 Amp and I_3 = 0 Amp.

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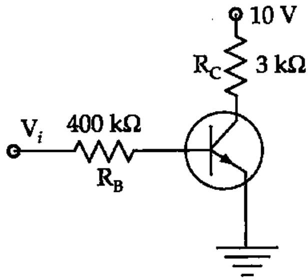
Q14.32. In the circuit shown in figure, when the input voltage of the base resistance is 10 V, V_{BE} is zero and V_{CE} is also zero. Find the values of I_B, I_c and β.



Ans. Voltage across R_B = 10V

$$V_B = R_B I_B$$

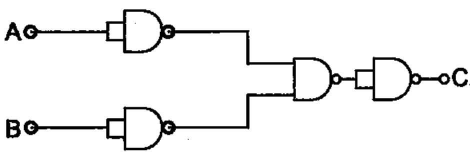
$$I_B = \frac{V_B \text{ (Voltage across } R_B)}{R_B}$$

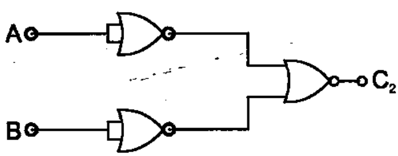
$$= \frac{10}{400 \times 1000} \text{ A} = 2.5 \times 10^{-5} \text{ A} = 25 \times 10^{-6} \text{ A}$$

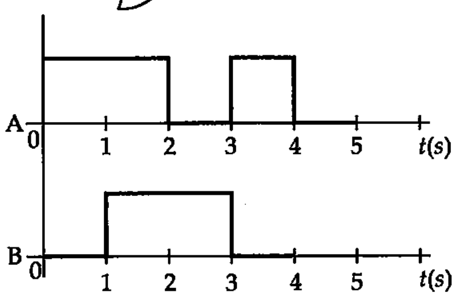
$$I_C = \frac{V_C \text{(Voltage across } R_C)}{R_C} = \frac{10}{3 \times 1000} = 3.3 \times 10^{-3} \text{ A}$$

$$\text{Current gain } (\beta) = \frac{I_C}{I_B} = \frac{3.3 \times 10^{-3}}{25 \times 10^{-6}} = 133$$

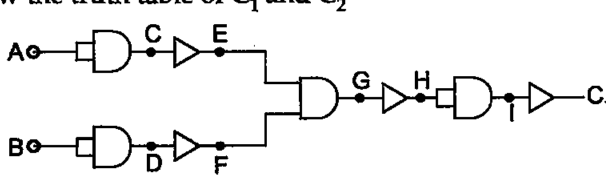
Q14.33. Draw the output signals C_1 and C_2 in the given combination of gates.







Ans. To draw the truth table of C_1 and C_2



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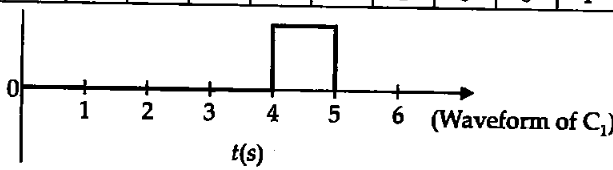
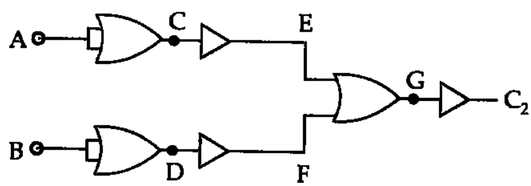
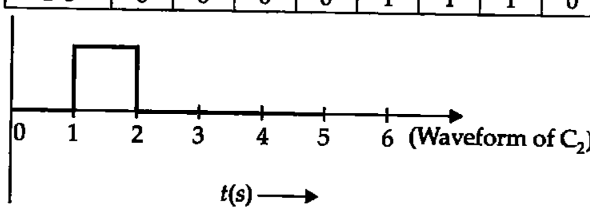
time/sec	A	B	C	D	E	F	G	H	I	C_1
0–1	1	0	1	0	0	1	0	1	1	0
1–2	1	1	1	1	0	0	0	1	1	0
2–3	0	1	0	1	1	0	0	1	1	0
3–4	1	0	1	0	0	1	0	1	1	0
4–5	0	0	0	0	1	1	1	0	0	1

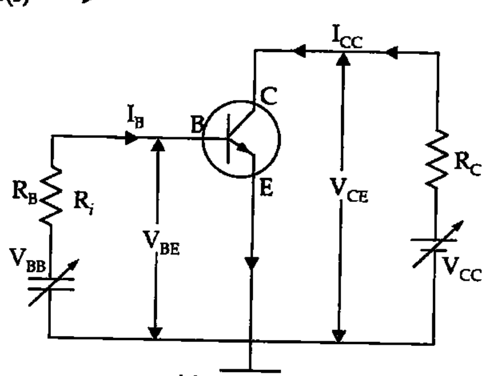




time/sec	A	B	C	D	E	F	G	C_2
0–1	1	0	1	0	0	1	1	0
1–2	1	1	1	1	0	0	0	1
2–3	0	1	0	1	1	0	1	0
3–4	1	0	1	0	0	1	1	0
4–5	0	0	0	0	1	1	1	0



Q14.34. Consider the circuit arrangement shown in figure (*a*) for studying input and output characteristics of *n-p-n* transistor in CE configuration. Select the values of R_B and R_C for a transistor whose V_{BE} = 0.7 Volt, so that the transistor is operating at a point Q as shown in characteristics [figure (*b*)].



(*a*)

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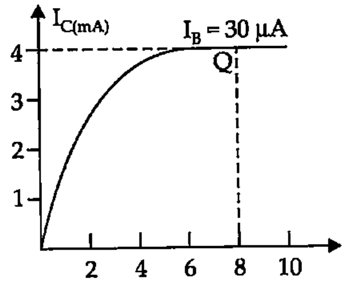

(b)

Given that the input impedance of the transistor is very small and $V_{CC} = V_{BB} = 16$ V, also find the voltage gain and power gain of circuit making appropriate assumptions.

Ans. For output characteristics at point Q

$V_{CE} = 8$ V, $I_B = 30\ \mu A$, $I_C = 4$ mA, $V_{BE} = 0.7$ V

Applying Kirchhoff's law in collector-emitter loop

$$V_{CC} = V_{CE} + R_C I_C$$

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} = \frac{16-8}{4 \times 10^{-3}} = \frac{8}{4} \times 10^3 = 2000\ \Omega$$

$$R_C = 2000\ \Omega$$

Now applying Kirchhoff's loop law in base-emitter circuit,

$$V_{BB} = I_B R_B + V_{BE}$$

$$R_B = \frac{V_{BB} - V_{BE}}{I_B} = \frac{16-0.7}{30 \times 10^{-6}} = \frac{15.3 \times 10^{-6}}{30} = 530\ k\Omega$$

Voltage gain $= A_V = -\beta \frac{R_C}{R_B}$ and $\beta = \frac{I_C}{I_B}$

$$\beta = \frac{4 \times 10^{-3}}{30 \times 10^{-6}} = \frac{40}{30} \times 10^3 = \frac{4000}{30} = \frac{400}{3}$$

Average Voltage A_V

$$A_V = -\beta \frac{R_C}{R_B}$$

(–) sign shows change in phase angle of output is 180° by input voltage.

$$A_V = \frac{+400}{3} \times \frac{2000}{530,000} = \frac{80}{15.6} = 0.52$$

Power gain = I.V

Power gain $= \beta \times A_V = 0.52 \times \frac{400}{3}$

Power gain $= \frac{208}{3} = 69.33$ or $P_{gain} = \mathbf{69.33}$

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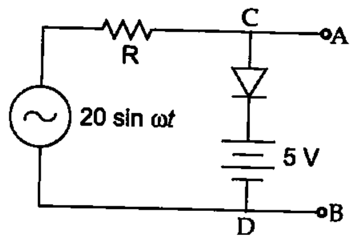
Q14.35. Assuming the ideal diode, draw the output waveform for the circuit given in figure. Explain the waveform.



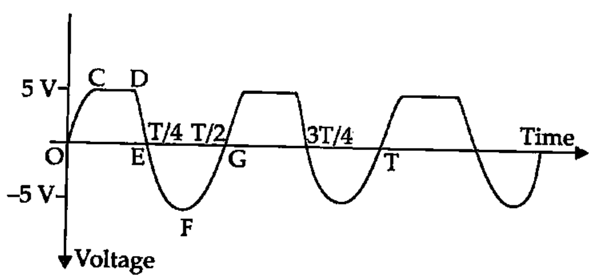
Main concept used: When a diode is in forward bias its resistance is zero and when it is in reverse bias its resistance is infinite.

Ans. When signal 20 sin ωt gives input voltage less than 5 volt (because after 5V diode will get positive voltage at its P-junction) then diode will be in reverse bias so resistance of diode remain infinity so input signal will not pass through diode and battery path so output across A and B will increase from 0 – 5 V (graph OC).

Now when the input voltage 20 sin ωt increase beyond 5 V then path of diode and 5 V battery will offer very low resistance, so the current passes through diode and battery and output (across A and B) remain 5V (graph CD).

Now when the voltage decreases the diode will be in reverse bias and output will again fall from 5 V to 0 V as input changes (graph DE). When input voltage becomes negative (there is opposite of 5 V battery in p-n junction input voltage becomes more than 5 V now) the diode is in reversed bias it will not conduct current through CD, and in output across AB will get same as input AC *i.e.* for negative cycle diode offer infinite resistance as compared to R in series graph E, F, G.

Same repetition of input and output continues—graph showing the output waveform.



Q14.36. Suppose a *n*-type wafer is created by doping Si crystal having 5×10^{28} atoms/m^3 with one ppm concentration of As. On the surface 200 ppm Boron is added to create P region in this wafer. Considering $n_i = 1.5 \times 10^{16}$ m^{-3} (*i*) Calculate the densities of charge carriers in the *n* and *p* regions. (*ii*) Comment which charge carriers would contribute largely for reverse saturation current when diode is reverse biased.

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Ans. When As (pentavalent) is added to Si the n-type wafer is created. So the number of majority carriers in *n*-type wafer,

$$n_e = (N_D \text{ and } D_{Si}) = \frac{1}{10^6} \times 5 \times 10^{28} = 5 \times 10^{22} /m^3$$

For number of minority carriers n_h

$$n_e \cdot n_h = n_i^2$$

$$n_h = \frac{n_i^2}{n_e} = \frac{1.5 \times 10^{16} \times 1.5 \times 10^{16}}{5 \times 10^{22}} \qquad (n_i = 1.5 \times 10^{16}/m^3) \text{ (Given)}$$

$$= 0.3 \times 1.5 \times 10^{32-22} = 0.45 \times 10^{+10} \text{ per } m^3$$

When Boron (Trivalent) is implanted in Si crystal, *p*-type wafer is formed with number of holes,

$$n_h = (N_D \times n \text{ of Si})$$

$$= \frac{200}{10^6} \times 5 \times 10^{28} = 1000 \times 10^{28-6}$$

$$n_h = 1 \times 10^{25} \text{ per } m^3$$

Minority carrier in p-type wafer

$$n_e \cdot n_h = n_i^2$$

$$n_e = \frac{n_i^2}{n_h} = \frac{1.5 \times 10^{16} \times 1.5 \times 10^{16}}{10^{25}} = 2.25 \times 10^{32-25}$$

$$= 2.25 \times 10^7 \text{ electrons per } m^3$$

When reversed bias is applied on *p-n* junction then the minority charge carrier moves toward depletion layer *i.e.,* holes n_h = $(0.45 \times 10^{10}$ per $m^3)$ from *n* side and $n_e = 2.25 \times 10^7/m^3$ from *p* side moves towards junction and make the depletion layer thicker.

Q14.37. An X-OR gate has following truth table. It is represented by following logic relation $Y = \overline{A} \cdot B + A \cdot \overline{B}$. Build this gate using AND, OR and NOT gates.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

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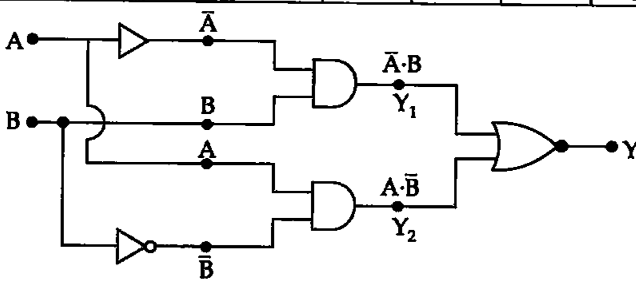
Ans. In given logic relation $Y = \bar{A}B + A\bar{B} = Y_1 Y_2$

Y	$A.\bar{B}$	$\bar{B}$	A	B	Y	$\bar{A}$	$\bar{A}.B$
0	0	1	0	0	0	1	0
1	0	0	0	1	1	1	1
1	1	1	1	0	1	0	0
0	0	0	1	1	0	0	0



logic relation in given table is

$$Y = \bar{A} \cdot B + A \cdot \bar{B}$$
$$= Y_1 + Y_2$$

$Y_1 = \bar{A} \cdot B$ and $Y_2 = A \cdot \bar{B}$

Y_1 can be obtained by $\bar{A}$ through NOT and B direct to AND gate. So, $Y_1 = \bar{A} \cdot B$.

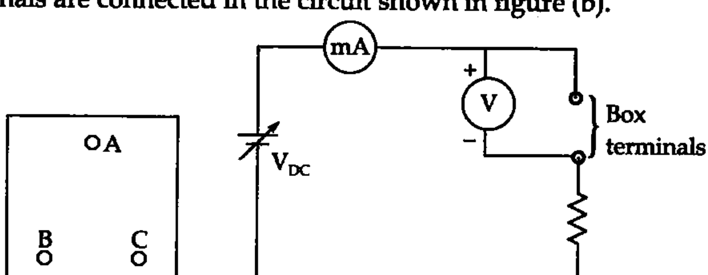
Y_2 can be obtained by $\bar{B}$ through NOT and A direct to AND gate. So, $Y_2 = A \cdot \bar{B}$.

Now Y_1 and Y_2 are feed into the two terminals of OR gate to get

$$Y = Y_1 + Y_2 \quad \text{or} \quad Y = \bar{A}B + A\bar{B}$$

Q14.38. Consider a box with three terminals on the top of it as shown in figure (a). Three components namely, two germanium diodes and one resistor are connected across these three terminals in some arrangement.

A student performs an experiment in which any two of these three terminals are connected in the circuit shown in figure (b).



(a) (b)

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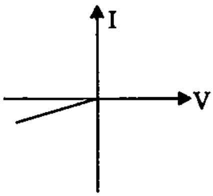
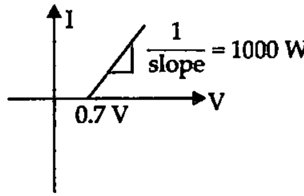
The student obtains graphs of current-voltage characteristics for unknown combination of the components between the two terminals connected in the circuit. The graphs are:

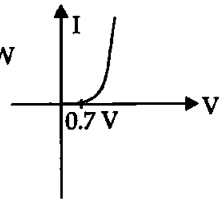
(*i*) When A is positive and B is negative figure (c)
(*ii*) When A is negative and B is positive figure (d)
(*iii*) When B is negative and C is positive figure (e)



(c)

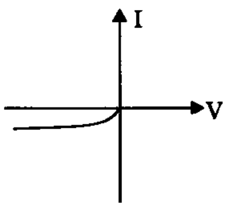


(d)

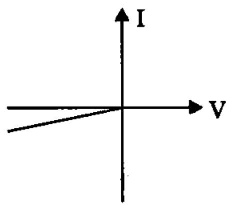


(e)

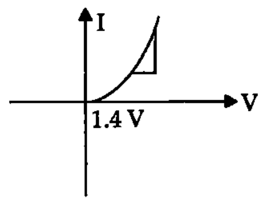
(*iv*) When B is positive and C is negative figure (f).
(*v*) When A is positive and C is negative figure (g).
(*vi*) When A is negative and C is positive figure (h).



(f)



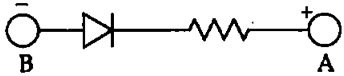
(g)



(h)

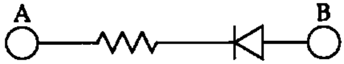
From these graphs of current-voltage characteristics shown in Figure (*c*) to (*h*), determine the arrangement of components between A, B and C.

Ans. (*i*) Figure (*c*) shows the reverse bias characteristics of *p-n* junction. It is possible when *p-n* junction first is in a series combination with *p* side of diode towards B and *n* side towards A resistance. Its resistance is linear or ohmic resistance.



(*ii*) Figure (*d*) shows the forward bias characteristics of a transistor where 0.7 V is the knee voltage of a *p-n* junction first

$$\frac{1}{\text{slope}} \Rightarrow \frac{1}{V/I} = V\left(\frac{1}{1000}\right)\Omega$$



p side of *p-n* junction first is connected to B and *n* side to A terminal. Resistance is in series with diode.

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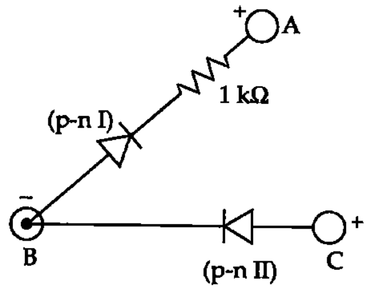
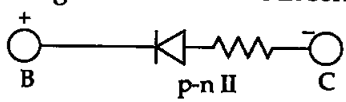
(*iii*) Figure (*e*) also shows the forward bias characteristics of *p-n* junction second having non ohmic as graph is not straight line. It also has knee voltage 0.7 V.

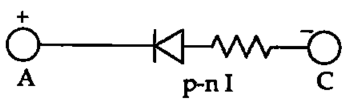
Here *p* side of *p-n* junction is connected to C and n side with B terminal as shown in figure.



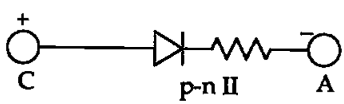
(*iv*) In figure (*f*) graph shows reverse bias characteristics of *p-n* junction II. *n* side with B and p side with C along with a resistance in series.



(*v*) Figure (*g*) shows the reverse bias characteristics of *p-n* junction I.



(*vi*) Figure (*h*) shows forward biased characteristics of *p-n* junction II.



Q14.39. For the transistor circuit shown in figure evaluate V_E, R_B, R_E given that

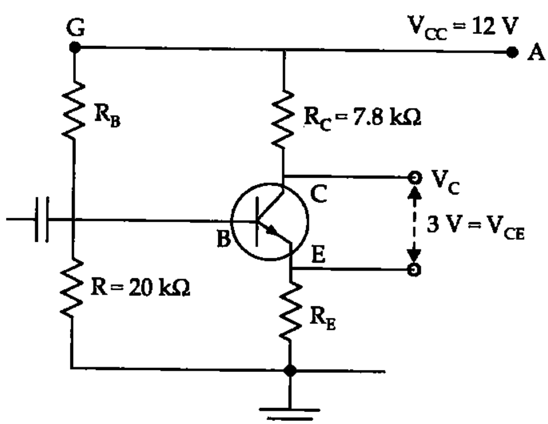
$I_C = 1\text{mA}$

$V_{CE} = 3\text{V}$

$V_{BE} = 0.5\text{V}$

$V_{CC} = 12\text{ V}$

$\beta = 100$



Ans. $I_C = I_B + I_E$

$\because\ I_B <<< I_C$

$\therefore\ I_C = I_E$

$I_C = 1\text{ mA}$ [Given]

$\therefore\ I_E = I_C = 10^{-3}\text{ A}$

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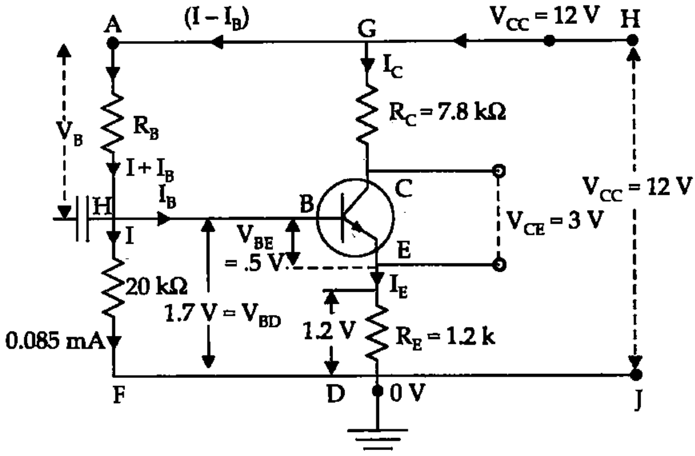

$\therefore \qquad I_C = I_E = 1 \text{ mA}$

By Kirchhoff's loop law, in loop DGHJ

$$I_C R_C + I_E R_E + V_{CE} = V_{CC}$$

$$I_C(R_C + R_E) + V_{CE} = 12$$

$$(R_C + R_E)\, 1 \times 10^{-3} + 3 = 12$$

$$R_C + R_E = \frac{9}{10^{-3}}$$

$$7.8 \times 10^3 + R_E = 9000$$

$$R_E = 9000 - 7800 = 1200\ \Omega = 1.2\ \text{k}\Omega$$

$$V_E = I_E \times R_E = I_C \times R_E$$

$$= 10^{-3} \times 1200 = 1.2\ \text{V}$$

$$V_{BD} = V_E + V_{BE} = 1.2 + 0.5\text{V} = 1.7 \text{ volt}$$

$$V_{BE} = 0.5 \text{ V}, \text{ and } V_{BE} = 1.2 \text{ V}$$

$$I = \frac{V_B}{20000} = \frac{1.7}{20000} \text{ A} = 0.085 \text{ mA}$$

$$R_B = \frac{V_{BD}}{(I + I_B)} = \frac{V_{CC} - V_{BD}}{\left[I + \left(\frac{I_C}{\beta}\right)\right]} \qquad \left(\because I_B = \frac{I_C}{\beta}\right)$$

$$R_B = \frac{12 - 1.7}{\left[0.085 + \frac{1}{100}\right] \times 10^{-3}} = \frac{10.3 \times 10^3}{[0.085 + 0.01]} = \frac{10.3 \times 10^3}{0.095}$$

$$= 108 \times 10^3\ \Omega = 108\ \text{k}\Omega$$

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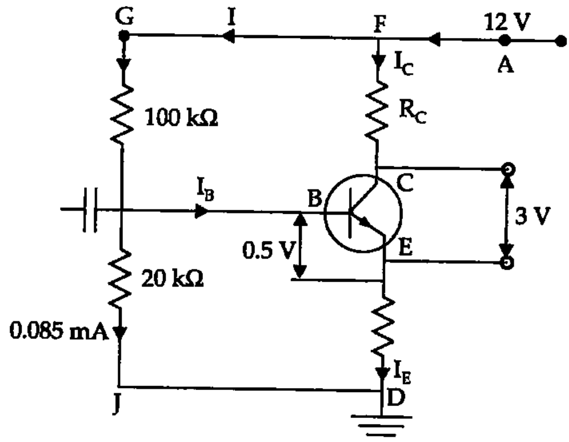
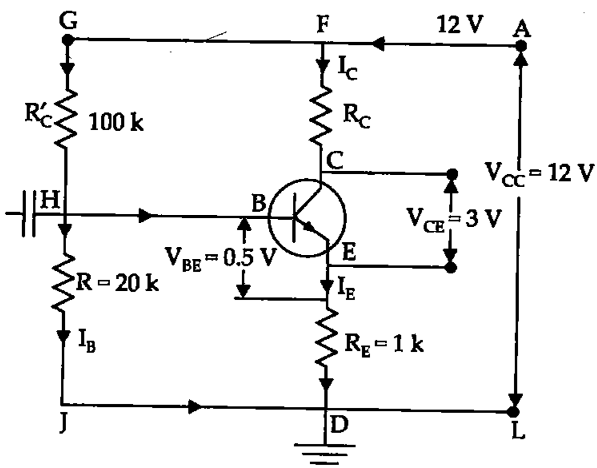
Q14.40. In the circuit shown in figure find the value of R_C



$$\beta = 100$$
$$V_{BE} = 0.5\ V$$
$$V_{CE} = 3\ V$$

Ans. From figure applying Kirchhoff's junction rule at transistor

$$I_e = I_C + I_B \quad \text{...(i)}$$
$$I_C = \beta I_B \quad \text{...(ii)}$$



Applying Kirchhoff's loop law to the loop AFCEDLA

$$I_C R_C + V_{CE} + V_E = V_{JA}$$
$$I_C R_C + 3 + I_E R_E = 12\ V \quad \text{...(i)}$$

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$$I_C = I_B + I_E$$
$$I_B <<< I_C$$
$$I_E \cong I_C = \beta I_B \qquad \left(\beta = \frac{I_C}{I_B} \quad I_C = I_E\right)$$

From (i)
$$I_C R_C + I_C R_E = 12 - 3$$
$$R_C \beta I_B + \beta I_B R_E = 9$$
$$\beta I_B (R_C + R_E) = 9 \qquad ...(ii)$$

Applying Kirchhoff's loop law to the loop JHBEDJ
$$RI_B + V_{BE} + R_E I_E = V_{CC}$$
$$I_B R_B + R_E I_E = V_{CC} - V_{BE}$$
$$I_B R + \beta I_B R_E = V_{CC} - V_{BE}$$
$$I_B (R + R_E \beta) = V_{CC} - V_{BE}$$
$$I_B = \frac{V_{CC} - V_{BE}}{R + R_E \beta}$$
$$= \frac{12 - 0.5}{(20 + 1 \times 100)K} = \frac{11.5}{120 \times 10^3}$$
$$I_B = 0.096 \text{ mA}$$

From eqn. (ii),
$$R_C + R_E = \frac{9}{\beta I_B} = \frac{9}{(100 \times 0.096) \text{ mA}}$$
$$R_C + 1000 = \frac{9}{9.6 \times 10^{-3}} = 0.938 \text{ k}\Omega$$
$$R_C + 1000 = 938 \ \Omega$$
$$R_C = 938 - 1000$$
$$R_C = 62 \ \Omega$$

□□□