CHAPTER 1

Real Number

TOPIC 1 : EUCLIDS DIVISION LEMMA AND FUNDAMENTAL THEOREM OF ARITHMETIC

VERY SHORT ANSWER TYPE QUESTIONS

1. Explain why 13233343563715 is a composite number?

Ans : [Board Term-1, 2016 LGRKEGO]

The given number ends in 5. Hence it is a multiple of 5. Therefore it is a composite number.

2. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then calculate the least prime factor of $(a+b)$.

Ans : [Board Term-1, 2014]

a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then least prime factor of $(a+b)$ is 2.

3. What is the HCF of the smallest composite number and the smallest prime number?

Ans :

The smallest prime number is 2 and the smallest composite number is $4 = 2^2$.
Hence, required HCF $(2^2,\ 2) = 2$.

4. Calculate the HCF of $3^3\times5$ and $3^2\times5^2$.

Ans :

We have
$$\begin{aligned} 3^3\times5 &= 3^2\times5\times3 \\ 3^2\times5^2 &= 3^2\times5\times5 \\ \text{HCF }(3^3\times5,\ 3^2\times5^2) &= 3^2\times5 \\ &= 9\times5 = 45 \end{aligned}$$

5. If HCF $(a,b) = 12$ and $a\times b = 1{,}800$, then find LCM (a,b).

Ans :

We know that
$$\text{HCF}\,(a,b)\times\text{LCM}\,(a,b) = a\times b$$
Substituting the values we have
$$12\times\text{LCM}\,(a,b) = 1800$$
or,
$$\text{LCM}\,(a,\ b) = \frac{1{,}800}{12} = 150$$

SHORT ANSWER TYPE QUESTIONS - I

6. Find HCF of the numbers given below:
$k, 2k, 3k, 4k$ and $5k$,, where k is a Positive integer.

Ans : [Board Term-1, 2015, Set-FHN8MGD]

Here we can see easily that k is common factor between all and this is highest factor Thus HCF of $k, 2k, 3k, 4k$ and $5k$, is k.

7. Find the HCF and LCM of 90 and 144 by the method of prime factorization.

Ans : [Board Term-1, 2012, Set-69]

$$\begin{aligned} \text{We have}\qquad 90 &= 9\times10 \\ &= 2\times3^2\times5 \\ \text{and}\qquad 144 &= 16\times9 \\ &= 2^4\times3^2 \\ \text{HCF} &= 2\times3^2 = 18 \\ \text{LCM} &= 2^4\times3^2\times5 = 720 \end{aligned}$$

8. Using Euclid's algorithm, find the HCF of 240 and 288.

Ans : [Board Term-1, 2012, Set-35]

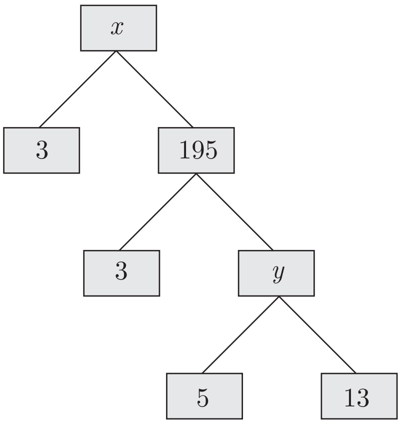
$$\begin{aligned} \text{We have}\qquad 240 &= 228\times1+12 \\ \text{and}\qquad 288 &= 12\times19+0 \end{aligned}$$
Hence, HCF of 240 and 228 = 12

9. Given that HCF $(306,\ 1314) = 18$. Find LCM $(306,\ 1314)$

Ans : [Board Term-1, 2013, FFC]

$$\begin{aligned} \text{We have}\qquad \text{HCF }(306,\ 314) &= 18 \\ \text{LCM }(306,\ 1314) &= ? \end{aligned}$$
Let $a = 306$ and $b = 1314$, then we have
$$\text{LCM}\,(a,b)\times\text{HCF}\,(a,b) = a\times b$$
Substituting values we have
$$\begin{aligned} \text{or,}\qquad \text{LCM}\,(a,b)\times18 &= 306\times1314 \\ \text{or,}\qquad LCM\,(a,b) &= \frac{306\times1314}{18} \\ LCM(306,1,314) &= 22{,}338 \end{aligned}$$

10. Complete the following factor tree and find the composite number x.



Ans : [Board Term-1, 2015, Set - WJQZQBN

We have $y = 5 \times 13 = 65$

and $x = 3 \times 195 = 585$

11. Explain why $(7 \times 13 \times 11) + 11$ and $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3$ are composite numbers.

Ans : [Board Term-1, 2012, Set-64]

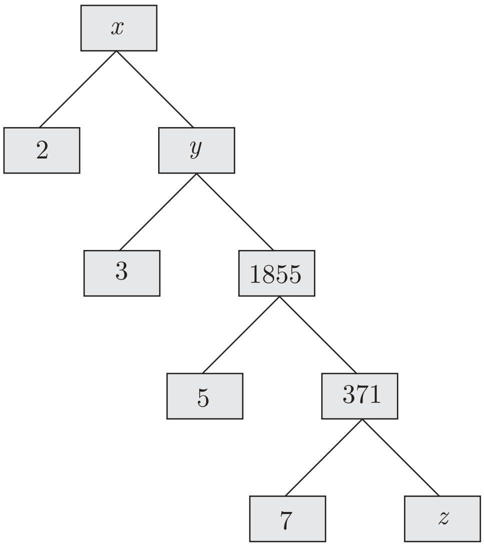
$$\begin{aligned}(7 \times 13 \times 11) + 11 &= 11 \times (7 \times 13 + 1) \\ &= 11 \times (91 + 1) \\ &= 11 \times 92\end{aligned}$$

and

$$\begin{aligned}(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3 &= 3(7 \times 6 \times 5 \times 4 \times 2 \times 1 + 1) \\ &= 3 \times (1681) = 3 \times 41 \times 41\end{aligned}$$

Since given numbers have more than two prime factors, both number are composite.

12. Complete the following factor tree and find the composite number x



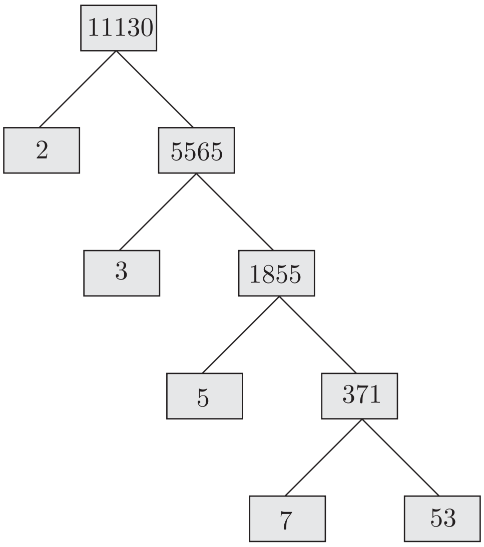
Ans : [Board Term-1, 2015, Set-DDE-M]

We have $z = \frac{371}{7} = 53$

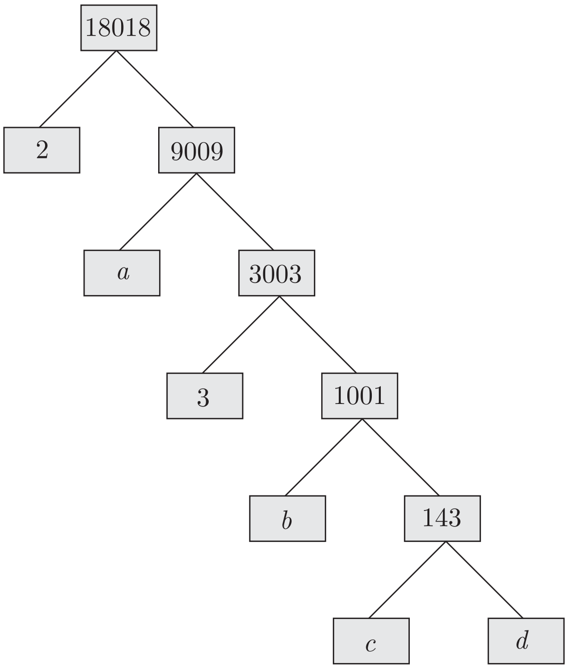
$y = 1855 \times 3 = 5565$

$x = 2 \times y = 2 \times 5565 = 11130$

Thus complete factor three is as given below.



13. Find the missing numbers a, b, c and d in the given factor tree:



Ans : [Board Term-1, 2012, Set-52]

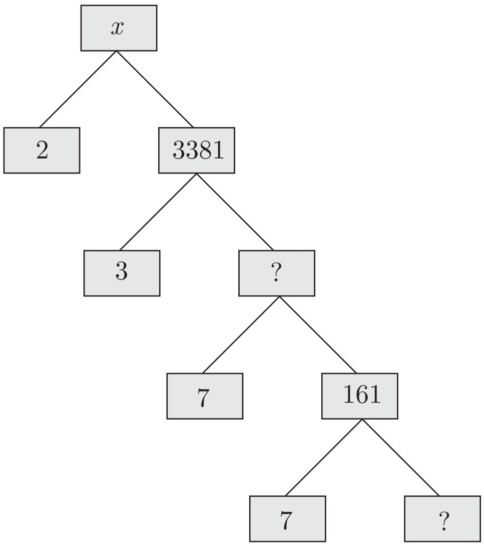
We have $a = \frac{9009}{3003} = 3$

$b = \frac{1001}{143} = 7$

Since $143 = 11 \times 13$,

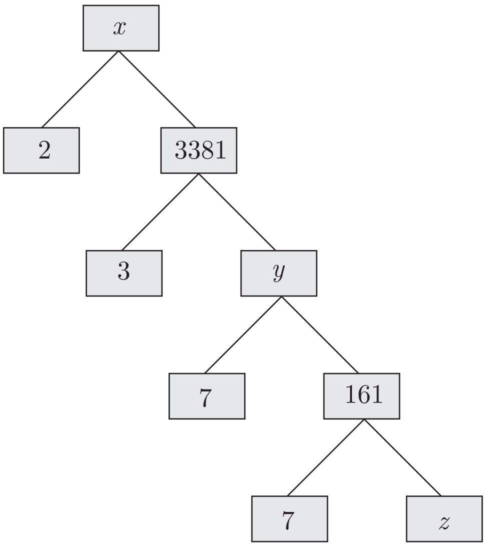
Thus $c = 11$ and $d = 13$ or $c = 13$ and $d = 11$

14. Complete the following factor tree and find the composite number x.



Ans : [Term-1, 2015, Set - 44,], [Board term-2012, Set - 44]

We complete the given factor tree writing variable y and z as following.



We have $$z = \frac{161}{7} = 23$$

$$y = 7 \times 161 = 1127$$

Composite number, $x = 2 \times 3381 = 6762$

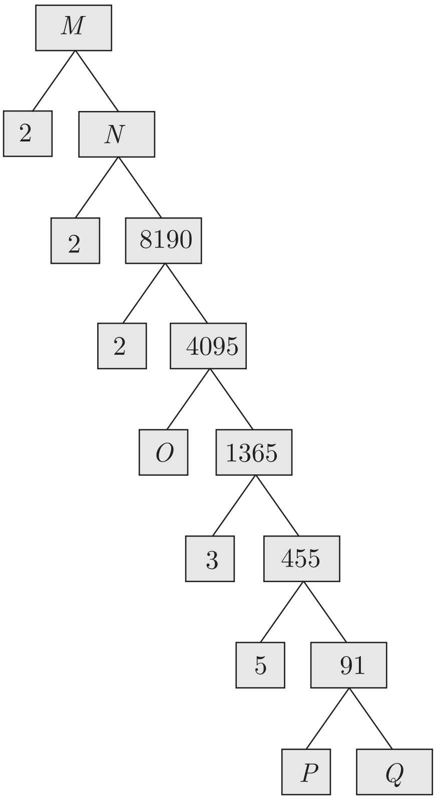
15. Explain whether $3 \times 12 \times 101 + 4$ is a prime number or a composite number.

Ans : [Board Term-1, 2016-17 Set; 193RQTQ, 2015, DDE-E]

A prime number (or a prime) is a natural number greater than 1 that cannot be formed by multiplying two smaller natural numbers. A natural number greater than 1 that is not prime is called a composite number. For example, 5 is prime because the only ways of writing it as a product, 1×5 or 5×1, involve 5 itself. However, 6 is composite because it is the product of two numbers (2×3) that are both smaller than 6. Every composite number can be written as the product of two or more (not necessarily distinct) primes.

$$\begin{aligned} 3 \times 12 \times 101 + 4 &= 4(3 \times 3 \times 101 + 1) \\ &= 4(909 + 1) \\ &= 4(910) \\ &= 2 \times 2 \times (10 \times 7 \times 13) \\ &= 2 \times 2 \times 2 \times 5 \times 7 \times 13 \\ &= \text{a composite number} \end{aligned}$$

16. Complete the factor-tree and find the composite number M.



Ans : [NCERT]

We have $$91 = P \times Q = 7 \times 13$$

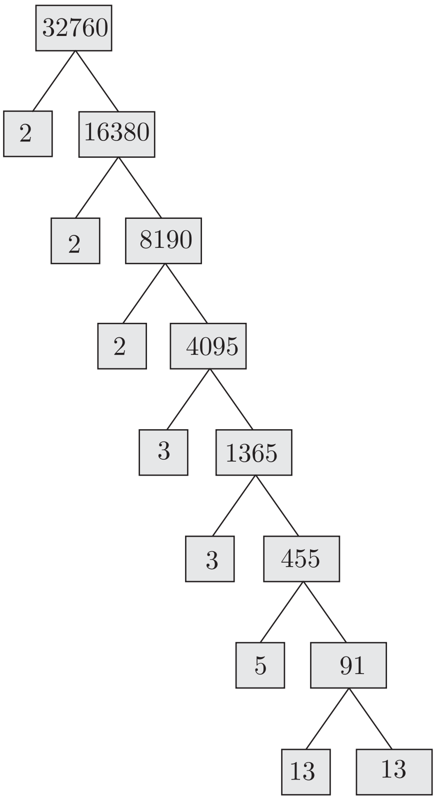
So $P = 7, Q = 13$ or $P = 13, Q = 7$

$$O = \frac{4095}{1365} = 3$$

$$N = 2 \times 8190 = 16380$$

Composite number $$M = 16380 \times 2 = 32760$$

Thus complete fact tree is shown below.



17. Find the smallest natural number by which 1200 should be multiplied so that the square root of the product is a rational number.
[Term-1, 2015, WJQZQBN, [Term-1, 2016, WV98HN3]

Ans : [Term-1, 2015, WJQZQBN, [Term-1, 2016, WV98HN3]

We have
$$\begin{aligned} 1200 &= 12 \times 100 \\ &= 4 \times 3 \times 4 \times 25 \\ &= 4^2 \times 3 \times 5^2 \end{aligned}$$

Here if we multiply by 3, then its square root will be a rational number because all power will be 2. Thus the required smallest natural number is 3. 2

18. Show that any positive even integer can be written in the form $6q, 6q+2$ or $6q+4$, where q is an integer.

Ans : [Board Term1, 2016 Set ORDAWEZ]

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq + r$$

Take $b = 6$, then $0 \leq r < 6$ because $0 \leq r < b$,

Thus $a = 6q, 6q+1, 6q+2, 6q+3, 6q+4, 6q+5$

Here $6q$, , $6q+2$ and $6q+4$ are divisible by 2 and so $6q$, $6q+2$ and $6q+4$ are even positive integers.

Hence a is always an even integer if

$$a = 6q,\ 6q+2,\ 6q+4$$

19. Show that any positive odd integer is of the form $4q+1$ or $4q+3$, where q is some integer.

Ans : [Board Term-1, Set-70,55][NCERT]

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq + r$$

Take $b = 4$, then $0 \leq r < 4$ because $0 \leq r < b$,

Thus $a = 4q, 4q+1, 4q+2, 4q+3$

Here we can see easily that $a = 4q$, $4q+2$ are even, as they are divisible by 2. Also $4q+1, 4q+3$ are odd, as they are not divisible by 2.

Thus any positive integer which has the form of $(4q+1)$ or $(4q+3)$ is odd.

20. Can two numbers have 15 as their HCF and 175 as their LCM? Give reasons.

Ans : [Board Term-1, 2012, Set-50]

LCM of two numbers should be exactly divisible by their HCF. Since, 15 does not divide 175, two numbers cannot have their HCF as 15 and LCM as 175.

21. Check whether 4^n can end with the digit 0 for any natural number n.

Ans : [Board Term-1, 2015, Set-FHN8MGD; NCERT]

If the number 4^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of 4^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $4^n = 2^{2n}$ is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with the digit zero. Hence 4^n cannot end with the digit zero.

22. Show that 7^n cannot end with the digit zero, for any natural number n.

Ans : [Board Term-1, 2012, Set-63]

If the number 7^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of 7^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $7^n = (1 \times 7)^n$ is 7. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 7^n. So, there is no natural number n for which 7^n ends with the digit zero. Hence 7^n cannot end with the digit zero.

23. Check whether $(15)^n$ can end with digit 0 for any $n \in N$.

Ans : [Board Term-1, 2012, Set-71]

If the number $(15)^n$, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of $(15)^n$ would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $(15)^n = (3 \times 5)^n$ are 3 and 5. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of $(15)^n$. Since there is no prime factor 2, $(15)^n$ cannot end with the digit zero.

24. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.

Ans : [Board Term-1, 2016 Set ORDAWEZ]

Here we have to determine the HCF of all length which can measure all dimension.

$$\begin{aligned} \text{Length,} \qquad l &= 8m\,50cm = 850cm \text{ cm} \\ &= 50 \times 17 = 2 \times 5^2 \times 17 \\ \text{Breadth} \qquad b &= 6m\,25\,cm = 625\,cm \\ &= 25 \times 25 = 5^2 \times 5^2 \\ \text{Height} \qquad h &= 4m\,75cm = 475cm \\ &= 25 \times 19 = 5^2 \times 19 \\ \text{HCF}(l, b, h) &= \text{HCF }(850, 625, 475) \\ &= 5^2 = 25 \end{aligned}$$

25. If two positive integers p and q are written as $p = a^2 b^3$ and $q = a^3 b$, where a and b are prime numbers than verify $LCM(p, q) \times HCF(q, q) = p.q$

Ans : [Sample Paper 2017]

$$\begin{aligned} \text{We have} \qquad p &= a^2 b^3 = a \times a \times b \times b \times b \\ \text{and} \qquad q &= a^3 b = a \times a \times a \times b \\ \text{Now} \qquad LCM(p, q) &= a \times a \times a \times b \times b \times b \\ &= a^3 b^2 \\ \text{and} \qquad HCF(p, q) &= a \times a \times b \\ &= a^2 b \\ LCM(p, q) \times HCF(p, q) &= a^3 b^3 \times a^2 b \\ &= a^5 b^4 \\ &= a^2 b^3 \times a^3 b \\ &= pq \end{aligned}$$

SHORT ANSWER TYPE QUESTIONS - II

26. Find the HCF of 180, 252 and 324 by Euclid's Division algorithm.

Ans : [Board Term-1, 2016 Set MV98HN3]

$$\begin{aligned} \text{We have} \qquad 324 &= 252 \times 1 + 72 \\ 252 &= 72 \times 3 + 36 \\ 72 &= 36 \times 2 + 0 \\ \text{Thus } HCF(324, 252) &= 36 \\ \text{Now} \qquad 180 &= 36 \times 5 + 0 \\ \text{Thus} \quad HCF(36, 180) &= 36 \end{aligned}$$

Thus HCF of 180, 252, and 324 is 36.
Hence required number $= 999999 - 63 = 999936$ 1

27. Use Euclid division lemma to show that the square of any positive integer cannot be of the form $5m + 2$ or $5m + 3$ for some integer m.

Ans : [Board Term-1, 2015, Set-FHN8MGD]

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq + r, \; 0 \le r < b \text{ and } q \in \omega$$

Take $b = 5$, then $0 \le r < 5$ because $0 \le r < b$

$$\begin{aligned} \text{Thus} \qquad a &= 5q,\ 5q + 1,\ 5q + 2,\ 5q + 3 \text{ and } 5q + 4, \\ \text{Now} \qquad a^2 &= (5q)^2 = 25q^2 = 5(5q^2) = 5m \\ a^2 &= (5q + 1)^2 = 25q^2 + 10q + 1 = 5m + 1 \\ a^2 &= (5q + 2)^2 = 25q^2 + 20q + 4 = 5m + 4 \\ \text{Similarly} \quad a^2 &= (5q + 3)^2 = 5m + 4 \\ \text{and} \qquad a^2 &= (5q + 4)^2 = 5m + 1 \end{aligned}$$

Thus square of any positive integer cannot be of the form $5m + 2$ or $5m + 3$. 3

28. Show that numbers 8^n can never end with digit 0 of any natural number n.

Ans : [Board Term-1, 2015, Set-DDE-E][NCERT]

If the number 8^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2. That is, the prime factorization of 8^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $(8)^n = (2^3)^n = 2^{3n}$ is 2. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of $(8)^n$. Since there is no prime factor 5, $(8)^n$ cannot end with the digit zero.

29. Find the HCF, by Euclid's division algorithm of the numbers 92690, 7378 and 7161.

Ans : [Board Term-1, 2013, Set LK-59]

By using Euclid's Division Lemma, we have

$$\begin{aligned} 92690 &= 7378 \times 12 + 4154 \\ 7378 &= 4154 \times 1 + 3224 \\ 4154 &= 3224 \times 1 + 930 \\ 3224 &= 930 \times 3 + 434 \\ 930 &= 434 \times 2 + 62 \\ 434 &= 62 \times 7 + 0 \\ \text{HCF }(92690, 7378) &= 62 \end{aligned}$$

Now, using Euclid's Division Lemma on 7161 and 62, we have

$$\begin{aligned} 7161 &= 62 \times 115 + 31 \\ 62 &= 31 \times 2 + 0 \end{aligned}$$

Thus HCF $(7161, 62) = 31$

Hence, HCF of 92690, 7378 and 7161 is 31.

30. 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and if it equal contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Ans : [Board Term-1, 2011, Set-66]

The required answer will be HCF of 144 and 90.

$$\begin{aligned} 144 &= 2^4 \times 3^2 \\ 90 &= 2 \times 3^2 \times 5 \\ \text{HCF}(144,\ 90) &= 2 \times 3^2 = 18 \end{aligned}$$

Thus each stack would have 18 cartons.

31. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together?

Ans : [Board Term-1, 2011, Set-44]

The required answer is the LCM of 9, 12, and 15 minutes.

Finding prime factor of given number we have,

$$\begin{aligned} 9 &= 3\times3 = 3^2 \\ 12 &= 2\times2\times3 = 2^2\times3 \\ 15 &= 3 \times 5 \\ \text{LCM}(9,\ 12,\ 15) &= 2^2 \times 3^2 \times 5 \\ &= 150 \text{ minutes} \end{aligned}$$

The bells will toll next together after 180 minutes.

32. Find HCF and LCM of 16 and 36 by prime factorization and check your answer.

Ans :

Finding prime factor of given number we have,

$$\begin{aligned} 16 &= 2\times2\times2\times2 = 2^4 \\ 36 &= 2\times2\times3\times3 = 2^2\times3^2 \\ \text{HCF}(16,\ 36) &= 2\times2\ = 4 \\ \text{LCM}\ (16,\ 36) &= 2^4\times3^2 \\ &= 16\times9\ = 144 \end{aligned}$$

To check HCF and LCM by using formula

$$\begin{aligned} \text{HCF}(a,b)\times\text{LCM}(a,b) &= a\times b \\ \text{or,} \qquad 4\times144 &= 16\times36 \\ 576 &= 576 \\ \text{Thus} \qquad \text{LHS} &= \text{RHS} \end{aligned}$$

33. Find the HCF and LCM of 510 and 92 and verify that HCF × LCM = Product of two given numbers.

Ans : [Board Term-1, 2011, Set-39]

Finding prime factor of given number we have,

$$\begin{aligned} 92 &= 2^2\times23 \\ 510 &= 30\times17\ = 2\times3\times5\times17 \\ \text{HCF}\ (510,\ 92) &= 2 \\ \text{LCM}\ (510,\ 92 &= 2^2\times23\times3\times5\times14 \\ &= 23460 \\ \text{HCF}\ (510,\ 92) &\times \text{LCM}\ (510,\ 92) \\ &= 2\times23460 = 46920 \end{aligned}$$

Product of two numbers $= 510\times92 = 46920$

Hence, $\text{HCF}\times\text{LCM} = \text{Product of two numbers}$

34. The HCF of 65 and 117 is expressible in the form $65m–117$. Find the value of m. Also find the LCM of 65 and 117 using prime factorization method.

Ans : [Board Term-1, 2011, Set-40]

Finding prime factor of given number we have,

$$\begin{aligned} 117 &= 13\times2\times3 \\ 65 &= 13\times5 \\ \text{HCF}(117,\ 65) &= 13 \\ \text{LCM}(117,\ 65) &= 13\times5\times3\times3 = 585 \\ \text{HCF} &= 65m-117 \\ 13 &= 65m–117 \\ 65m &= 117+13 = 130 \\ m &= \frac{130}{6.5} = 2 \end{aligned}$$

35. Sho that any positive odd integer is of the form $6q+1, 6q+3$ or $6q+5$, where q is some integer.

Ans : [Board Term-1, 2011, Set-60]

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq+r$$

Take $b=6$, then $0\le r<6$ because $0\le r<b$,

Thus $\qquad a = 6q, 6q+1, 6q+2, 6q+3, 6q+4, 6q+5$

Here $6q$, , $6q+2$ and $6q+4$ are divisible by 2 and so

$6q$, $6q+2$ and $6q+4$ are even positive integers.

But $6q+1, 6q+3, 6q+5$ are odd, as they are not divisible by 2.

Thus any positive odd integer is of the form $6q+1, 6q+3$ or $6q+5$.

36. Show that exactly one of the number $n, n+2$ or $n+4$ is divisible by 3.

Ans : [Sample Paper 2017]

If n is divisible by 3, clearly $n+2$ and$n+4$ is not divisible by 3.

If n is not divisible by 3, then two case arise as given below.

Case 1: $n = 3k+1$

$$\begin{aligned} n+2 &= 3k+1+2 = 3k+3 = 3(k+1) \\ \text{and} \qquad n+4 &= 3k+1+4 = 3k+5 = 3(k+1)+2 \end{aligned}$$

We can clearly see that in this case $n+2$ is divisible by 3 and $n+4$ is not divisible by 3. Thus in this case only $n+2$ is divisible by 3.

Case 1: $n = 3k+2$

$$\begin{aligned} n+2 &= 3k+2+2 = 3k+4 = 3(k+1)+1 \\ \text{and} \qquad n+4 &= 3k+2+4 = 3k+6 = 3(k+2) \end{aligned}$$

We can clearly see that in this case $n+4$ is divisible by 3 and $n+2$ is not divisible by 3. Thus in this case only $n+4$ is divisible by 3.

Hence, exactly one of the numbers $n, n+2, n+4$, is divisible by 3.

LONG ANSWER TYPE QUESTIONS

37. Find HCF and LCM of 378, 180 and 420 by prime factorization method. Is HCF × LCM of these numbers equal to the product of the given three numbers?

Ans :

Finding prime factor of given number we have,

$$\begin{aligned} 378 &= 2\times3^3\times7 \\ 180 &= 2^2\times3^2\times5 \\ 420 &= 2^2\times3\times7\times5 \\ \text{HCF}(378,\ 180,\ 420) &= 2\times3\ = 6 \\ \text{LCM}(378,\ 180,\ 420) &= 2^2\times3^3\times5\times7 \\ &= 2^2\times3^3\times5\times7\ = 3780 \\ \text{HCF}\times\text{LCM} &= 6\times3780 = 22680 \end{aligned}$$

Product of given numbers

$$= 378\times180\times420$$

$$= 28576800$$

Hence, HCF × LCM ≠ Product of three numbers.

38. State Fundamental theorem of Arithmetic. Find LCM of numbers 2520 and 10530 by prime factorization by 3.

Ans : [Board Term-1, 2016 Set-ORDAWEZ]

The fundamental theorem of arithmetic (FTA), also called the unique factorization theorem or the unique-prime-factorization theorem, states that every integer greater than 1 either is prime itself or is the product of a unique combination of prime numbers.

OR

Every composite number can be expressed as the product powers of primes and this factorization is unique.

Finding prime factor of given number we have,

$$\begin{aligned} 2520 &= 20 \times 126 = 20 \times 6 \times 21 \\ &= 2^3 \times 3^2 \times 5 \times 7 \\ 10530 &= 30 \times 351 = 30 \times 9 \times 39 \\ &= 30 \times 9 \times 3 \times 13 \\ &= 2 \times 3^4 \times 5 \times 13 \\ \text{LCM}(2520, 10530) &= 2^3 \times 3^4 \times 5 \times 7 \times 13 \\ &= 294840 \end{aligned}$$

39. Can the number 6^n, n being a natural number, end with the digit 5 ? Give reasons.

Ans : [Board Term-1, 2015, Set-WJQZBN]

If the number 6^n for any n, were to end with the digit five, then it would be divisible by 5.

That is, the prime factorization of 6^n would contain the prime 5. This is not possible because the only prime in the factorization of $6^n = (2 \times 3)^n$ are 2 and 3. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 6^n. Since there is no prime factor 5, 6^n cannot end with the digit five.

40. State Fundamental theorem of Arithmetic. Is it possible that HCF and LCM of two numbers be 24 and 540 respectively. Justify your answer.

Ans : [Board Term-1, 2015, Set-WJQZBN]

Fundamental theorem of Arithmetic : Every integer greater than one ither is prime itself or is the product of prime numbers and that this product is unique. Up to the order of the factors.

LCM of two numbers should be exactly divisible by their HCF. In other words LCM is always a multiple of HCF. Since, 24 does not divide 540 two numbers cannot have their HCF as 24 and LCM as 540.

$$\begin{aligned} HCF &= 24 \\ LCM &= 540 \\ \frac{LCM}{HCF} &= \frac{540}{24} = 22.5 \text{ not an integer} \end{aligned}$$

41. Find the HCF of 256 and 36 using Euclid's Division Algorithm. Also, find their LCM and verify that HCF × LCM = Product of the two numbers.

Ans : [Board Term-1, 2015, Set-DDE-E]

By using Euclid's Division Lemma, we have

$$\begin{aligned} 256 &= 36 \times 7 + 4 \\ 36 &= 4 \times 9 + 0 \end{aligned}$$

Hence, the HCF of 256 and 36 is 4.

$$\begin{aligned} \text{LCM :} \quad 256 &= 2^8 \\ 36 &= 2^2 \times 3^2 \\ \text{LCM }(36, 256) &= 2^8 \times 3^2 = 256 \times 9 \\ &= 2304 \\ \text{HCF} \times \text{LCM} &= \text{Product of the two number} \\ 4 \times 2,304 &= 256 \times 36 \\ 9216 &= 9,216 \end{aligned}$$

Hence verified.

42. A fruit vendor has 990 apples and 945 oranges. He packs them into baskets. Each basket contains only one of the two fruits but in equal number. Find the number of fruits to be put in each basket in order to have minimum number of baskets.

Ans : [Board Term-1, 2016 Set-O4YP6G7]

Required answer is the HCF of 990 and 945.
By using Euclid's Division Lemma, we have

$$\begin{aligned} 990 &= 945 \times 1 + 45 \\ 945 &= 45 \times 21 + 0 \end{aligned}$$

Thus HCF of 990 and 945 is 45. The fruit vendor should put 45 fruits in each basket to have minimum number of baskets.

43. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

Ans : [Board Term-1, 2015, 2012, Set-48]

$$\begin{aligned} \text{We have} \quad n^3 - n &= n(n^2 - 1) \\ &= (n-1)n(n+1) \\ &= (n-1)n(n+1) \end{aligned}$$

Thus $n^3 - n$ is product of three consecutive positive integers.

Since, any positive integers a is of the form $3q, 3q+1$ or $3q+2$ for some integer q.

Let $a, a+1, a+2$ be any three consecutive integers.

Case I : $a = 3q$

If $a = 3q$ then,

$$a(a+1)(a+2) = 3q(3q+1)(3q+2)$$

Product of two consecutive integers $(3q+1)$ and $(3q+2)$ is an even integer, say $2r$.

$$\begin{aligned} \text{Thus} \quad a(a+1)(a+2) &= 3q(2r) \\ &= 6qr, \text{ which is divisible by 6.} \end{aligned}$$

Case II : $a = 3q+1$

If $a = 3q+1$ then

$$\begin{aligned} a(a+1)(a+2) &= (3q+1)(3q+2)(3q+3) \\ &= (2r)(3)(q+1) \\ &= 6r(q+1) \text{which is divisible by} \end{aligned}$$

6.

Case III : $a = 3q+2$

If $a = 3q+2$ then

$$a(a+1)(a+2) = (3q+2)(3q+3)(3q+4)$$
$$= 3(3q+2)(q+1)(3q+4)$$

Here $(3q+2)$ and $= 3(3q+2)(q+1)(3q+4)$

$= $ multiple of 6 every q

$= 6r$ (say)

which is divisible by 6. Hence, the product of three consecutive integers is divisible by 6 and $n^3 - n$ is also divisible by 3.

44. Prove that $n^2 - n$ is divisible by 2 for every positive integer n.

Ans : [Board Term-1, 2012 Set-25]

We have $n^2 - n = n(n-1)$

Thus $n^2 - n$ is product of two consecutive positive integers.

Any positive integer is of the form $2q$ or $2q+1$, for some integer q.

Case 1 : $n = 2q$

If $n = 2q$ we have

$$n(n-1) = 2q(2q-1)$$
$$= 2m,$$

where $m = q(2q-1)$ which is divisible by 2.

Case 1 : $n = 2q+1$

If $n = 2q+1$, we have

$$n(n-1) = (2q+1)(2q+1-1)$$
$$= 2q(2q+1)$$
$$= 2m$$

where $m = q(2q+1)$ which is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n.

45. Find HCF of 81 and 237 and express it as a linear combination of 81 and 237 i.e. HCF $(81, 237) = 81x + 237y$ for some x and y.

Ans : [Board Term-1, 2012 Set-35] [NCERT]

By using Euclid's Division Lemma, we have

$$237 = 81 \times 2 + 75 \qquad ...(1)$$
$$81 = 75 \times 1 + 6 \qquad ...(2)$$
$$75 = 6 \times 12 + 3 \qquad ...(3)$$
$$6 = 3 \times 2 + 0 \qquad ...(4)$$

Hence,HCF $(81, 237) = 3$.

In order to write 3 in the form of $81x + 237y$,

$$3 = 75 - 6\times12$$
$$= 75 - (81 - 75\times1)\times12 \qquad \text{Replace 6 from (2)}$$
$$= 75 - 81\times12 + 75\times12$$
$$= 75 + 75\times12 - 81\times12$$
$$= 75(1+12) - 81\times12$$
$$= 75\times13 - 81\times12$$
$$= 13(237 - 81\times2) - 81\times12 \qquad \text{Replace 75 from (1)}$$
$$= 13\times237 - 81\times2\times13 - 81\times12$$
$$= 237\times13 - 81(26+12)$$
$$= 237\times13 - 81\times38$$
$$= 81\times(-38) + 237\times(13)$$
$$= 81x + 237y]$$

Hence $x = -38$ and $y = 13$. These values of x and y are not unique.

46. Show that the square of any positive integer is of the forms $4m$ or $4m+1$, where m is any integer.

Ans : [Board Term-1, 2012 Set-39]

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq + r$$

Take $b = 4$, then $0 \le r < 4$ because $0 \le r < b$,

Thus $a = 4q, 4q+1, 4q+2, 4q+3$

Case 1 : $a = 4q$

$$a^2 = (4q)^2 = 16q^2 = 4(4q^2)$$
$$= 4m$$

where $m = 4q^2$

Case 2 : $a = 4q+1$

$$a^2 = (4q+1)^2 = 16q^2 + 8q + 1$$
$$= 4(4q^2 + 2q) + 1$$
$$= 4m + 1$$

where $m = 4q^2 + 2q$

Case 3 : $a = 4q+2$

$$a^2 = (4q+2)^2$$
$$= 16q^2 + 16q + 4$$
$$= 4(4q^2 + 4q + 1)$$
$$= 4m$$

where $m = 4q^2 + 4q + 1$

Case 4 : $a^2 = (4q+3)^2 = 16q^2 + 24q + 9$

$$= 16q^2 + 24q + 8 + 1$$
$$= 4(4q^2 + 6q + 2) + 1$$
$$= 4m + 1$$

where $m = 4q^2 + 6q + 2$

From cases 1, 2, 3 and 4 we conclude that the square of any +ve integer is of the form $4m$ or $4m+1$.

47. Use Euclid's Division Lemma to show that the cube of any positive integer is of the form $9m$, $9m+1$, or $9m+8$, for some integer m.

Ans : [KVS, NCERT]

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq + r$$

Take $b = 3$, then $0 \le r < 3$ because $0 \le r < b$,

Thus $a = 3q, 3q+1$, and $3q+2$

Case 1 : $a = 3q$

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3)$$
$$= 9m \text{ where } m = 3q^3$$

Case 2 : $a = 3q+1$

$$a^3 = (3q+1)^3$$
$$= 27q^3 + 9q(3q+1) + 1$$
$$= 9(3q^3 + 3q^2 + 1) + 1$$

or $a^3 = 9m + 1$ where $m = 3q^3 + 3q^2 + 1$

Case 3 : $a = 3q + 2$

$$\begin{aligned} a^3 &= (3q+2)^3 \\ &= 27d^3 + 18d(3d+2) + 8 \\ &= 9(3q^3 + 6q^2 + 4q) + 8 \end{aligned}$$

or $a^3 = 9m + 8$ where $m = 3q^2 + 6q^2 + 4q$

From Case 1, 2 and 3, we conclude that the cube of any positive integer is of the form $9m, 9m+1$ or $9m+8$ for some integer m.

TOPIC 2 : IRRATIONAL NUMBERS, TERMINATING AND NON-TERMINATING, RECURRING DECIMALS

VERY SHORT ANSWER TYPE QUESTIONS

1. What is the condition for the decimal expansion of a rational number to terminate? Explain with the help of an example.

Ans : [Board Term-1, 2016 Set-O4YP6G7]

The decimal expansion of a rational number terminates, if the denominator of rational number can be expressed as $2^m 5^n$ where m and n are non negative integers and p and q both co-primes.

e.g. $$\frac{3}{10} = \frac{3}{2^1 \times 5^1} = 0.3$$

2. Find the smallest positive rational number by which $\frac{1}{7}$ should be multiplied so that its decimal expansion terminates after 2 places of decimal.

Ans : [Board Term-1, 2016 Set LGRKEGO]

Since $$\frac{1}{7} \times \frac{7}{100} = \frac{1}{100} = 0.01.$$

Thus smallest rational number is $\frac{7}{100}$

3. What type of decimal expansion does a rational number has? How can you distinguish it from decimal expansion of irrational numbers?

Ans : [Board Term-1, 2016 Set-ORDAWEZ]

A rational number has its decimal expansion either terminating or non-terminating, repeating An irrational numbers has its decimal expansion non-repeating and non-terminating.

4. Calculate $\frac{3}{8}$ in the decimal form.

Ans :

We have $$\begin{aligned} \frac{3}{8} &= \frac{3}{2^3} = \frac{2 \times 5^3}{2^3 \times 5^3} \\ &= \frac{375}{10^3} = \frac{375}{1,000} \\ &= 0.375 \end{aligned}$$

5. The decimal representation of $\frac{6}{1250}$ will terminate after how many places of decimal?

Ans :

We have $$\begin{aligned} \frac{6}{1250} &= \frac{6}{2 \times 5^4} = \frac{6 \times 2^3}{2 \times 2^3 \times 5^4} \\ &= \frac{6 \times 2^3}{2^4 \times 5^4} = \frac{6 \times 2^3}{(10)^4} \\ &= \frac{48}{10000} = 0.0048 \end{aligned}$$

Thus $\frac{6}{1250}$ will terminate after 4 decimal places.

6. Write whether rational number $\frac{7}{75}$ will have terminating decimal expansion or a non-terminating decimal.

Ans : [Sample Paper 2017]

We have $$\frac{7}{75} = \frac{7}{3 \times 5^2}$$

Since denominator of given rational number is not of form $2^m \times 5^n$, Hence, It is non-terminating decimal expansion.

SHORT ANSWER TYPE QUESTIONS - I

7. Show that $5\sqrt{6}$ is an irrational number.

Ans : [Board Term-1 2015, Set-CJEOQ]

Let $5\sqrt{6}$ be a rational number, which can be expressed as $\frac{a}{b}$, where $b \neq 0$; a and b are co-primes.

Now $$5\sqrt{6} = \frac{a}{b}$$

$$\sqrt{6} = \frac{a}{5b}$$

or, $$\sqrt{6} = \text{rational}$$

But, $\sqrt{6}$ is an irrational number. Thus, our assumption is wrong. Hence, $5\sqrt{6}$ is an irrational number. 1

8. Write the denominator of the rational number $\frac{257}{500}$ in the form $2^m \times 5^n$, where m and n are non-negative integers. Hence write its decimal expansion without actual division.

Ans : [Board Term-1, 2012, Set-67, NCERT Exemplar]

We have $$\begin{aligned} 500 &= 25 \times 20 \\ &= 5^2 \times 5 \times 4 \\ &= 5^3 \times 2^2 \end{aligned}$$

Here denominator is 500 which can be written as $2^2 \times 5^3$.

Now decimal expansion,

$$\begin{aligned} \frac{257}{500} &= \frac{257 \times 2}{2 \times 2^2 \times 5^3} = \frac{514}{10^3} \\ &= 0.514 \end{aligned}$$

9. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Ans : [K.V.S.]

We have $\sqrt{2} = \sqrt{\frac{200}{100}}$ and $\sqrt{3} = \sqrt{\frac{300}{100}}$

We need to find a rational number x such that

$$\frac{1}{10}\sqrt{200} < x < \frac{1}{10}\sqrt{300}$$

Choosing any perfect square such as 225 or 256 in between 200 and 300, we have

$$x = \sqrt{\frac{225}{100}} = \frac{15}{10} = \frac{5}{3}$$

Similarly if we choose 256, then we have

$$x = \sqrt{\frac{256}{100}} = \frac{16}{10} = \frac{8}{5}$$

10. Write the rational number $\frac{7}{75}$ will have a terminating decimal expansion. or a non-terminating repeating decimal.

Ans : [Sample Question Paper 2017,-18]

We have $$\frac{7}{75} = \frac{7}{3 \times 5^2}$$

The denominator of rational number $\frac{7}{75}$ can not be written in form $2^m 5^n$ So it is non-terminating repeating decimal expansion.

SHORT ANSWER TYPE QUESTIONS - II

11. Express $\left(\frac{15}{4}+\frac{5}{40}\right)$ as a decimal fraction without actual division.

Ans : [Board Term-1, 2011, Set-74]

We have $$\frac{15}{4}+\frac{5}{40} = \frac{15}{4}\times\frac{25}{25}+\frac{5}{40}\times\frac{25}{25}$$

$$= \frac{375}{100}+\frac{125}{1000}$$

$$= 3.75 + 0.125 = 3.875$$

12. Express the number $0.3\overline{178}$ in the form of rational number $\frac{a}{b}$.

Ans : [Board Term-1, 2011, Set-A1][NCERT]

Let $x = 0.3\overline{178}$

$x = 0.3178178178$

$10,000x = 3178.178178...$

$10x = 3.178178....$

Subtracting, $9990x = 3175$

or, $$x = \frac{3175}{9990} = \frac{635}{1998}$$

13. Prove that $\sqrt{2}$ is an irrational number.

Ans : [Board Term-1, 2011, Set-A1. NCERT]

Let $\sqrt{2}$ be a rational number.

Then $$\sqrt{2} = \frac{p}{q},$$

where p and q are co-prime integers and $q \neq 0$ On squaring both the sides we have,

$$2 = \frac{p^2}{q^2}$$

or, $$p^2 = 2p^2$$

Since p^2 is divisible by 2, thus p is also divisible by 2.

Let $p = 2r$ for some positive integer r, then we have

$$p^2 = 4r^2$$

$$2q^2 = 4r^2$$

or, $$q^2 = 2r^2$$

Since q^2 is divisible by 2, thus q is also divisible by 2. We have seen that p and q are divisible by 2, which contradicts the fact that p and q are co-primes. Hence, our assumption is false and $\sqrt{2}$ is irrational.

14. If p is prime number, then prove that $\sqrt{p}$ is an irrational.

Ans :

Let p be a prime number and if possible, let $\sqrt{p}$ be rational

Thus $$\sqrt{p} = \frac{m}{n},$$

where m and n are co-primes and $n \neq 0$.
Squaring on both sides, we get

$$p = \frac{m^2}{n^2}$$

or, $$pn^2 = m^2 \qquad ...(1)$$

Here p divides pn^2. Thus p divides m^2 and in result p also divides m.
Let $m = pq$ for some integer q and putting $m = pq$ in eq. (1), we have

$$pn^2 = p^2q^2$$

or, $$n^2 = pq^2$$

Here p divides pq^2.Thus p divides n^2 and in result p also divides n.

[$\because$ p is prime and p divides $n^2 \Rightarrow p$ divides n]
Thus p is a common factor of m and n but this contradicts the fact that m and n are primes. The contradiction arises by assuming that $\sqrt{p}$ is rational. Hence, $\sqrt{p}$ is irrational.

15. Prove that $3+\sqrt{5}$ is an irrational number.

Ans :

Assume that $3+\sqrt{5}$ is a rational number, then we have

$$3+\sqrt{5} = \frac{p}{q}, \quad q \neq 0$$

$$\sqrt{5} = \frac{p}{q} - 3$$

$$\sqrt{5} = \frac{p-3q}{q}$$

Here $\sqrt{5}$ is irrational and $\frac{p-3q}{q}$ is rational. But rational number cannot be equal to an irrational number. Hence $3+\sqrt{5}$ is an irrational number.

16. Prove that $\sqrt{5}$ is an irrational number and hence show that $2-\sqrt{5}$ is also an irrational number.

Ans : [Board Term-1, 2011, Set-60]

Assume that $\sqrt{5}$ be a rational number then we have

$$\sqrt{5} = \frac{a}{b},$$

$(a,b$ are co-primes and $b \neq 0)$

$$a = b\sqrt{5}$$

Squaring both the sides, we have

$$a^2 = 5b^2$$

Thus 5 is a factor of a^2 and in result 5 is also a factor of a.

Let $a = 5c$ where c is some integer, then we have

$$a^2 = 25c^2$$

Substituting $a^2 = 5b^2$ we have

$$5b^2 = 25c^2$$

$$b^2 = 5c^2$$

Thus 5 is a factor of b^2 and in result 5 is also a factor of b.

Thus 5 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{5}$ is rational number is wrong. Hence $\sqrt{5}$ is irrational.

Let us assume that $2-\sqrt{5}$ be rational equal to a, then we have

$$2-\sqrt{5} = a$$

$$2-a = \sqrt{5}$$

Since we have assume $2-a$ is rational, but $\sqrt{5}$ is not rational. Rational number cannot be equal to an irrational number. Thus $2-\sqrt{5}$ is irrational.

17. If two positive integers p and q are written as $p = a^2b^3$ and $q = a^3b$, a and b are prime number then. Verify.

LCM $\times(p.q.)\times$ HCF $(p.q.) = pq$.

Ans : [Sample Question Paper 2017-18]

We have $p = a^2b^3 = a\times a\times b\times b\times b$

and $q = a^3b = a\times a\times a\times b$

Now $LCM(p,q) = a\times a\times a\times b\times b\times b$

$= a^3b^2$

and $HCF(p,q) = a\times a\times b$

$= a^2b$

$$\begin{aligned} LCM(p,q)\times HCF(p,q) &= a^3b^3\times a^2b \\ &= a^5b^4 \\ &= a^2b^3\times a^3b \\ &= pq \end{aligned}$$

LONG ANSWER TYPE QUESTIONS

18. Prove that $\sqrt{3}$ is an irrational number. Hence, show that $7+2\sqrt{3}$ is also an irrational number.

Ans : [Board Term-1, 2012, Set-DDE-M]

Assume that $\sqrt{3}$ be a rational number then we have

$$\sqrt{3} = \frac{a}{b},$$

$(a,b$ are co-primes and $b\neq 0)$

$$a = b\sqrt{3}$$

Squaring both the sides, we have

$$a^2 = 3b^2$$

Thus 3 is a factor of a^2 and in result 3 is also a factor of a.

Let $a = 3c$ where c is some integer, then we have

$$a^2 = 9c^2$$

Substituting $a^2 = 9b^2$ we have

$$3b^2 = 9c^2$$

$$b^2 = 3c^2$$

Thus 3 is a factor of b^2 and in result 3 is also a factor of b.

Thus 3 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{3}$ is rational number is wrong. Hence $\sqrt{3}$ is irrational.

Let us assume that $7+2\sqrt{3}$ be rational equal to a, then we have

$$7+2\sqrt{3} = \frac{p}{q}$$

$q\neq 0$ and p and q are co-primes

$$2\sqrt{3} = \frac{p}{q}-7 = \frac{p-7q}{q}$$

or $$\sqrt{3} = \frac{p-7q}{2q}$$

Here $p-7q$ and $2q$ both are integers, hence $\sqrt{3}$ should be a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. Hence our assumption is not correct and $7+2\sqrt{3}$ is irrational.

19. Show that there is no positive integer n, for which $\sqrt{n-1}+\sqrt{n-1}$ is rational.

Ans : [Board Term-1, 2012, Set-48]

Let us assume that there is a positive integer n for which $\sqrt{n-1}+\sqrt{n-1}$ is rational and equal to $\frac{p}{q}$, where p and q are positive integers and $(q\neq 0)$.

$$\sqrt{n-1}+\sqrt{n-1} = \frac{p}{q} \qquad ...(1)$$

or, $$\frac{q}{p} = \frac{1}{\sqrt{n-1}+\sqrt{n+1}}$$

$$= \frac{\sqrt{n-1}-\sqrt{n+1}}{(\sqrt{n-1}+\sqrt{n+1})(\sqrt{n-1}-\sqrt{n+1})}$$

$$= \frac{\sqrt{n-1}-\sqrt{n+1}}{(n-1)-(n+1)}$$

or $$\frac{q}{p} = \frac{\sqrt{n-1}-\sqrt{n+1}}{-2}$$

$$\sqrt{n+1}-\sqrt{n-1} = \frac{2q}{p} \qquad ...(2)$$

Adding (1) and (2), we get

$$2\sqrt{n+1} = \frac{p}{q}+\frac{2q}{p} = \frac{p^2+2q^2}{pq}$$

...(3)

Subtracting (2) from (1) we have

$$2\sqrt{n-1} = \frac{p^2-2q^2}{pq}$$

...(4)

From (3) and (4), we observe that $\sqrt{n+1}$ and $\sqrt{n-1}$ both are rational because p and q both are rational. But it possible only when $(n+1)$ and $(n-1)$ both are perfect squares. But they differ by 2 and two perfect squares never differ by 2. So both $(n+1)$ and $(n-1)$ cannot be perfect squares, hence there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational.

HOTS QUESTIONS

20. Show that 571 is a prime number.

Ans :

$$\begin{aligned} \text{Let} \qquad x &= 571 \\ \sqrt{x} &= \sqrt{571} \end{aligned}$$

Now 571 lies between the perfect squares of $(23)^2 = 529$ and $(24)^2 = 576$. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Here 571 is not divisible by any of the above numbers, thus 571 is a prime number.

21. Find the least number that is divisible by all numbers between 1 and 10 (both inclusive).

Ans :

The required number is the LCM of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,

$$\begin{aligned} \text{LCM} &= 2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 7 \\ &= 2520 \end{aligned}$$

22. An army contingent of 104 members is to march behind an army band of 96 members in a parade. The two groups are to march in the same number of columns in which they can march ?

Ans : [Board Term-1, 2012, Set-52]

Let the number of columns be x which is the largest number, which should divide both 104 and 96. It means x should be HCF of 104 and 96.

By using Euclid's Division Lemma, we have

$$\begin{aligned} 104 &= 96 \times 1 + 8 \\ 96 &= 8 \times 12 + 0 \end{aligned}$$

Thus HCF of 104 and 96 is 8 and columns are required.

23. If d is the HCF of 30 and 72, find the value of x and y satisfying $d = 30x + 72y$.

Ans :

Using Euclid's Division Lemma, we have

$$\begin{aligned} 72 &= 30 \times 2 + 12 & ...(1) \\ 30 &= 12 \times 2 + 6 & ...(2) \\ 12 &= 6 \times 2 + 0 & ...(3) \end{aligned}$$

$$\text{Thus} \quad \text{HCF}(30,\, 72) = 6$$

$$\text{Now} \quad 6 = 30 - 12 \times 2$$

From (2)

$$6 = 30 - (72 - 30 \times 2) \times 2$$

From (1)

$$\begin{aligned} 6 &= 30 - 72 \times 2 + 30 \times 4 \\ 6 &= 30(1 - 4) - 72 \times 2 \\ 6 &= 30 \times 5 + 72 \times (-2) \\ 6 &= 30x + 72y \end{aligned}$$

Thus $x = 5$ and $y = -2$ .Here x and y are not unique.

24. If HCF of 657 and 963 is expressible in the form of $657x + 963 \times (-15)$, find the value of x.

Ans :

Using Euclid's Division Lemma we have

$$\begin{aligned} 963 &= 657 \times 1 + 306 \\ 657 &= 306 \times 2 + 45 \\ 306 &= 45 \times 6 + 36 \\ 45 &= 36 \times 1 + 9 \\ 36 &= 9 \times 4 + 0 \\ \text{HCF}(657, 963) &= 9 \end{aligned}$$

$$\begin{aligned} \text{Now} \qquad 9 &= 657x + 963 \times (-15) \\ \text{or,} \qquad 657x &= 9 + 963 \times 15 \\ &= 9 + 14445 \\ \text{or,} \qquad 657x &= 14454 \\ \text{or,} \qquad x &= \frac{14454}{657} = 22 \end{aligned}$$

25. Express the HCF/LCM of 48 and 18 as a linear combination.

Ans :

Using Euclid's Division Lemma, we have

$$\begin{aligned} 48 &= 18 \times 2 + 12 & (1) \\ 18 &= 12 \times 1 + 6 & (2) \\ 12 &= 6 \times 2 + 0 \end{aligned}$$

$$\text{Thus} \quad \text{HCF}(18, 48) = 6$$

$$\text{Now} \quad 6 = 18 - 12 \times 1$$

From (2)

$$6 = 18 - (48 - 18 \times 2)$$

From (1)

$$\begin{aligned} 6 &= 18 - 48 \times 1 + 18 \times 2 \\ 6 &= 18 \times (2 + 1) - 48 \times 1 \\ 6 &= 18 \times 3 - 48 \times 1 \\ 6 &= 18 \times 3 + 48 \times (-1) \end{aligned}$$

$$\text{Thus} \quad 6 = 18x + 48y,$$

where $x = 3, y = -1$

Here x and y are not unique.

$$\begin{aligned} 6 &= 18 \times 3 + 48 \times (-1) \\ &= 18 \times 3 + 48 \times (-1) + 18 \times 48 - 18 \times 48 \\ &= 18(3 + 48) + 48(-1 - 18) \\ &= 18 \times 51 + 48 \times (-19) \\ 6 &= 18x + 48y, \end{aligned}$$

where $x = 51, y = -19$

CHAPTER 2

Polynomials

VERY SHORT ANSWER TYPE QUESTIONS

1. If α and β are the roots of $ax^2-bx+c=0\,(a\neq 0)$, then calculate $\alpha+\beta$.

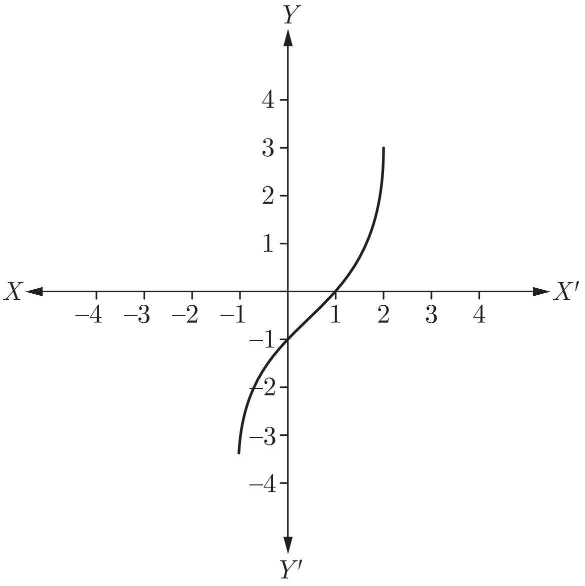
Ans : [Board Term-1, 2014]

We know that

$$\text{Sum of the roots } = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Thus $$\alpha+\beta = -\left(\frac{-b}{a}\right) = \frac{b}{a}$$

2. In given figure, the graph of a polynomial $p(x)$ is shown. Calculate the number of zeroes of $p(x)$.



Ans :

The graph intersects x-axis at one point $x=1$. Thus the number of zeroes of $p(x)$ is 1.

3. Calculate the zeroes of the polynomial $p(x)=4x^2-12x+9$.

Ans :

We have
$$\begin{aligned} p(x) &= 4x^2-12x+9 \\ &= 4x^2-6x-6x+9 \\ &= 2x(2x-3)-3(2x-3) \\ &= (2x-3)(2x-3) \end{aligned}$$

Substituting $p(x)=0$, and solving we get$x=\frac{3}{2},\frac{3}{2}$

$x=\frac{3}{2},\frac{3}{2}$

Hence, zeroes of the polynomial are $\frac{3}{2}$, $\frac{3}{2}$. 1

4. If sum of the zeroes of the quadratic polynomial $3x^2-kx+6$ is 3, then find the value of k.

Ans :

We have $$p(x) = 3x^2-kx-6$$

$$\text{Sum of the zeroes } = 3 = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Thus $$3 = -\frac{(-k)}{3} \Rightarrow k = 9$$

5. If -1 is a zero of the polynomial $f(x)=x^2-7x-8$, then calculate the other zero.

Ans :

We have $$f(x) = x^2-7x-8$$

Let other zero be k, then we have

Sum of zeroes, $$-1+k = -\left(\frac{-7}{1}\right) = 7$$

or $$k = 8$$

SHORT ANSWER TYPE QUESTIONS - I

1. If zeroes of the polynomial $x^2+4x+2a$ are a and $\frac{2}{a}$, then find the value of a.

Ans : [Board Term-1, 2016 Set-O4YP6G7]

Product of (zeroes) roots,

$$\frac{c}{a} = \frac{2a}{1} = \alpha\times\frac{2}{\alpha} = 2$$

or, $$2a = 2$$

Thus $$a = 1$$

2. Find all the zeroes of $f(x)=x^2-2x$.

Ans : [Board Term-1, 2013, LK-59]

We have
$$\begin{aligned} f(x) &= x^2-2x \\ &= x(x-2) \end{aligned}$$

Substituting $f(x)=0$, and solving we get$x=0,2$

Hence, zeroes are 0 and 2.

3. Find the zeroes of the quadratic polynomial $\sqrt{3}\,x^2 - 8x + 4\sqrt{3}$.

Ans : [Board Term-1, 2013, LK-59]

$$\begin{aligned} p(x) &= \sqrt{3}\,x^2 - 8x + 4\sqrt{3} = 0 \\ &= \sqrt{3}\,x^2 - 6x - 2x + 4\sqrt{3} = 0 \\ &= \sqrt{3}\,x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3}) = 0 \\ &= (\sqrt{3}\,x - 2)(x - 2\sqrt{3}) = 0 \end{aligned}$$

Substituting $p(x) = 0$, and solving we get $x = \frac{2}{\sqrt{3}},\ 2\sqrt{3}$

Hence, zeroes are $\frac{2}{\sqrt{3}}$ and $2\sqrt{3}$.

4. Find a quadratic polynomial, the sum and product of whose zeroes are 6 and 9 respectively. Hence find the zeroes.

Ans : [Board Term-1, 2016 Set- LGRKEGO]

Sum of zeroes, $\alpha + \beta = 6$

Product of zeroes $\alpha\beta = 9$

Now $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

Thus $= x^2 - 6x + 9$

Thus quadratic polynomial is $x^2 - 6x + 9$.

Now $p(x) = x^2 - 6x + 9$

$= (x - 3)(x - 3)$

Substituting $p(x) = 0$, we get $x = 3, 3$

Hence zeroes are 3, 3

5. Find the quadratic polynomial whose sum and product of the zeroes are $\frac{21}{8}$ and $\frac{5}{16}$ respectively.

Ans : [Board Term-1, 2012, Set-35]

Sum of zeroes, $\alpha + \beta = \frac{21}{8}$

Product of zeroes $\alpha\beta = \frac{5}{16}$

Now $p(x)\ x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - \frac{21}{8}x + \frac{5}{16}$

or $p(x) = \frac{1}{16}(16x^2 - 42x + 5)$

6. Form a quadratic polynomial $p(x)$ with 3 and $-\frac{2}{5}$ as sum and product of its zeroes, respectively.

Ans : [Board Term-1, 2012, Set-64]

Sum of zeroes, $\alpha + \beta = 3$

Product of zeroes $\alpha\beta = -\frac{2}{5}$

Now $p(x)\ x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 3x - \frac{2}{5}$

$= \frac{1}{5}(5x^2 - 15x - 2)$

The required quadratic polynomial is $\frac{1}{5}(5x^2 - 15x - 2)$

7. What should added to the polynomial $x^3 - 3x^2 + 6x - 15$ so that it is completely divisible by $x - 3$.

Ans : [Board Term-1, 2016 Set-ORDAWEZ]

We divide $x^3 - 3x^2 + 6x - 15$ by $x - 3$ as follows.

$$\begin{array}{r|l} & \quad\quad\quad\quad x^2 + 6 \\ x - 3 & x^3 - 3x^2 + 6x - 15 \\ & \underline{x^3 - 3x^2 \quad\quad\quad\quad} \\ & \quad\quad\quad 6x - 15 \\ & \quad\quad\quad \underline{6x - 18} \\ & \quad\quad\quad\quad\quad 3 \end{array}$$

Here remainder is 3, hence -3 must be added so that there is no remainder.

8. If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

Ans : [Board Term-1, 2012, Set-40]

We have $\frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{mn} = \frac{(m+n)^2 - 2mn}{mn}$ (1)

Sum of zeroes $m + n = -\frac{11}{3}$

Product of zeroes $mn = \frac{-4}{3}$

Substituting in (1) we have

$$\begin{aligned} \frac{m}{n} + \frac{n}{m} &= \frac{(m+n)^2 - 2mn}{mn} \\ &= \frac{(-\frac{11}{3})^2 - \frac{-4}{3} \times 2}{\frac{-4}{3}} \\ &= \frac{121 + 4 \times 3 \times 2}{-4 \times 3} \end{aligned}$$

or $\frac{m}{n} + \frac{n}{m} = \frac{-145}{12}$

9. If p and q are the zeroes of polynomial $f(x) = 2x^2 - 7x + 3$, find the value of $p^2 + q^2$.

Ans : [Board Term-1, 2012, Set-21]

We have $f(x) = 2x^2 - 7x + 3$

Sum of zeroes $p + q = -\frac{b}{a} = -\left(\frac{-7}{2}\right) = \frac{7}{2}$

Product of zeroes $pq = \frac{c}{a} = \frac{3}{2}$

Since, $(p + q)^2 = p^2 + q^2 + 2pq$

so, $p^2 + q^2 = (p + q)^2 - 2pq$

$= \left(\frac{7}{2}\right)^2 - 3 = \frac{49}{4} - \frac{3}{1} = \frac{37}{4}$

Hence $p^2 + q^2 = \frac{37}{4}$.

10. Find the condition that zeroes of polynomial $p(x) = ax^2 + bx + c$ are reciprocal of each other.

Ans : [Board Term-1, 2012, Set-50]

We have $p(x) = ax^2 + bx + c$

Let α and $\frac{1}{\alpha}$ be the zeroes of $p(x)$, then

Product of zeroes,

$$\frac{c}{a} = \alpha \times \frac{1}{\alpha} = 1 \text{ or } \frac{c}{a} = 1$$

So, required condition is, $c = a$

11. Find the value of k if -1 is a zero of the polynomial $p(x) = kx^2 - 4x + k$.

Ans : [Board Term-1, 2012, Set-62]

We have $\quad p(x) = kx^2 - 4x + k$

Since, -1 is a zero of the polynomial, then

$$p(-1) = 0$$

$$k(-1)^2 - 4(-1) + k = 0$$

or, $\quad k + 4 + k = 0$

or, $\quad 2k + 4 = 0$

or, $\quad 2k = -4$

Hence, $\quad k = -2$

12. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}\,x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

Ans : [Board Term-1, 2015, Set-DDE-M]

We have $\quad p(x) = x^2 - 4\sqrt{3}\,x + 3$

If α and β are the zeroes of $x^2 - 4\sqrt{3}\,x + 3$, then

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = -\frac{(-4\sqrt{3})}{1}$

or, $\quad \alpha + \beta = 4\sqrt{3}$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{3}{1}$

or, $\quad \alpha\beta = 3$

Now $\quad \alpha + \beta - \alpha\beta = 4\sqrt{3} - 3.$

13. Find the values of a and b, if they are the zeroes of polynomial $x^2 + ax + b$.

Ans : [Board Term-1, 2013, FFC],

We have $\quad p(x) = x^2 + ax + b$

Since a and b, are the zeroes of polynomial, we get,

Product of zeroes, $ab = b \Rightarrow a = 1$

Sum of zeroes, $a + b = -a \Rightarrow b = -2a = -2$

14. If α and β are the zeroes of the polynomial $f(x) = x^2 - 6x + k$, find the value of k, such that $\alpha^2 + \beta^2 = 40$.

Ans : [Board Term-1, 2015, Set-WJQZQBN]

We have $\quad f(x) = x^2 - 6x + k$

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = \frac{-(-6)}{1} = 6$

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$

Now $\quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 40$

$$(6)^2 - 2k = 40$$

$$36 - 2k = 40$$

$$-2k = 4$$

Thus $\quad k = -2$

15. If one of the zeroes of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of the other, find the value of 'k'.

Ans : [Board Term-1, 2012, Set-48]

We have $\quad f(x) = 14x^2 - 42k^2x - 9$

Let one zero be α, then other zero will be $-\alpha$.

Sum of zeroes $= \alpha + (-\alpha) = 0$.

Thus sum of zero will be 0.

Sum of zeroes $\quad 0 = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$0 = -\frac{42k^2}{14} = -3k^2$$

Thus $k = 0$.

16. If one zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and the other zero.

Ans : [Board Term-1, 2012, Set-71]

Let, the zero of $2x^2 + 3x + \lambda$ be $\frac{1}{2}$ and β.

Product of zeroes $\quad \frac{1}{2}\beta = \frac{\lambda}{2} \qquad \frac{c}{a}$

or, $\quad \beta = \lambda$

and sum of zeroes, $\quad \frac{1}{2} + \beta = -\frac{3}{2} \qquad -\frac{b}{a}$

or $\quad \beta = -\frac{3}{2} - \frac{1}{2} = -2$

Hence $\quad \lambda = \beta = -2$

Thus other zero is -2.

17. If α and β are zeroes of the polynomial $f(x) = x^2 - x - k$, such that $\alpha - \beta = 9$, find k.

Ans : [Board Term-1, 2013, Set FFC]

We have $\quad f(x) = x^2 - x - k$

Since α and β are the zeroes of the polynomial, then

Sum of zeroes, $\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$= -\left(\frac{-1}{1}\right) = 1$$

$$\alpha + \beta = 1 \qquad \text{...(i)}$$

Given $\quad \alpha - \beta = 9 \qquad$ ...(ii) 1

Solving (i) and (ii) we get $\alpha = 5$ and $\beta = -4$

$$\alpha\beta = \frac{\text{Constan term}}{\text{Coefficient of } x^2}$$

or $\quad \alpha\beta = -k$

Substituting $\alpha = 5$ and $\beta = -4$ we have

$$(5)(-4) = -k$$

Thus $\quad k = 20$

18. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q.

Ans : [Board Term-1, 2012, Set-39]

We have $f(x) = 2x^2 - 5x - 3$

Let the zeroes of polynomial be α and β, then

Sum of zeroes $\alpha + \beta = \frac{5}{2}$

Product of zeroes $\alpha\beta = -\frac{3}{2}$

According to the question, zeroes of $x^2 + px + q$ are 2α and 2β.

Sum of zeros, $2\alpha + 2\beta = \frac{-p}{1}$

$$2(\alpha + \beta) = -p$$

Substituting $\alpha + \beta = \frac{5}{2}$ we have

$$2 \times \frac{5}{2} = -p$$

or $p = -5$

Product of zeroes, $2\alpha 2\beta = \frac{q}{1}$

$$4\alpha\beta = q$$

Substituting $\alpha\beta = -\frac{3}{2}$ we have

$$4 \times \frac{-3}{2} = q$$

$$-6 = q$$

Thus $p = -5$ and $q = -6$.

19. If α and β are zeroes of $x^2 - (k-6)x + 2(2k-1)$, find the value of k if $\alpha + \beta = \frac{1}{2}\alpha\beta$.

Ans : [KVS Practice Test 2017]

We have $p(x) = x^2 - (k-6)x + 2(2k-1)$

Since α, β are the zeroes of polynomial $p(x)$, we get

$$\alpha + \beta = -[-(k-6)] = k - 6$$

$$\alpha\beta = 2(2k-1)$$

Now $\alpha + \beta = \frac{1}{2}\alpha\beta$

Thus $k + 6 = \frac{2(2k-1)}{2}$

or, $k - 6 = 2k - 1$

$$k = -5$$

Hence the value of k is -5.

SHORT ANSWER TYPE QUESTIONS - II

1. Verify whether 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$.

Ans : [Board Term-1, 2013, LK-59]

If 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x) = 0$

(1) 2, $p(x) = 2x^2 - 11x^2 + 17x - 6$

$$\begin{aligned} p(2) &= 2(2)^3 - 11(2)^2 + 17(2) - 6 \\ &= 16 - 44 + 34 - 6 \\ &= 50 - 50 \end{aligned}$$

or $p(2) = 0$

(2) 3, $p(3) = 2(3)^3 - 11(3)^2 + 17(3) - 6$

$$\begin{aligned} &= 54 - 99 + 51 - 6 \\ &= 105 - 105 \end{aligned}$$

or $p(3) = 0$

(3) $\frac{1}{2}$ $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6$

$$= \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6$$

or $p\left(\frac{1}{2}\right) = 0$

Hence, 2, 3, and $\frac{1}{2}$ are the zeroes of $p(x)$.

2. If the sum and product of the zeroes of the polynomial $ax^2 - 5x + c$ are equal to 10 each, find the value of 'a' and 'c'.

Ans : [Board Term-1, 2011, Set-25]

We have $f(x) = ax^2 - 5x + c$

Let the zeroes of $f(x)$ be α and β, then,

Sum of zeroes $\alpha + \beta = -\frac{-5}{a} = \frac{5}{a}$

Product of zeroes $\alpha\beta = \frac{c}{a}$

According to question, the sum and product of the zeroes of the polynomial $f(x)$ are equal to 10 each.

Thus $\frac{5}{a} = 10$ ...(1)

and $\frac{c}{a} = 10$ ...(2)

Dividing (2) by eq. (1) we have

$$\frac{c}{5} = 1 \Rightarrow c = 5$$

Substituting $c = 5$ in (2) we get $a = \frac{1}{2}$

Hence $a = \frac{1}{2}$ and $c = 5$.

3. If one the zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Ans : [Board Term-1, 2011, Set-40]

We have $f(x) = 3x^2 - 8x + 2k + 1$

Let α and β be the zeroes of the polynomial, then

$$\beta = 7\alpha$$

Sum of zeroes, $\alpha + \beta = -\left(-\frac{8}{3}\right)$

$$\alpha + 7\alpha = 8\alpha = \frac{8}{3}$$

So $\alpha = \frac{1}{3}$

Product of zeroes, $\alpha \times 7\alpha = \frac{2k+1}{3}$

$$7\alpha^2 = \frac{2k+1}{3}$$

$$7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$$

$$7\times\frac{1}{9} = \frac{2k+1}{1}$$

$$\frac{7}{3}-1 = 2k$$

$$\frac{4}{3} = 2k \Rightarrow k=\frac{2}{3}$$

4. Quadratic polynomial $2x^2-3x+1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Ans : [Board Term-2, 2015, Set-DDE-E]

We have $f(x) = 2x^2-3x+1$

If α and β are the zeroes of $2x^2-3x+1$, then

Sum of zeroes $\alpha+\beta = \frac{-b}{a} = \frac{3}{2}$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{1}{2}$

New quadratic polynomial whose zeroes are 3α and 3β is,

$$\begin{aligned} p(x) &= x^2-(3\alpha+3\beta)x+3\alpha\times3\beta \\ &= x^2-3(\alpha+\beta)x+9\alpha\beta \\ &= x^2-3\left(\frac{3}{2}\right)x+9\left(\frac{1}{2}\right) \\ &= x^2-\frac{9}{2}x+\frac{9}{2} \\ &= \frac{1}{2}(2x^2-9x+9) \end{aligned}$$

Hence, required quadratic polynomial is $\frac{1}{2}(2x^2-9x+9)$

5. If α and β are the zeroes of the polynomial $6y^2-7y+2$, find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Ans : [Board Term-1, 2011, Set-39]

We have $p(y) = 6y^2-7y+2$

Sum of zeroes $\alpha+\beta = -\left(-\frac{7}{6}\right) = \frac{7}{6}$

Product of zeroes $\alpha\beta = \frac{2}{6} = \frac{1}{3}$

Sum of zeroes of new polynomial $g(y)$

$$\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{7/6}{2/6} = \frac{7}{2}$$

and product of zeroes of new polynomial $g(y)$,

$$\frac{1}{\alpha}\times\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{1/3} = 3$$

The required polynomial is

$$\begin{aligned} g(x) &= y^2-\frac{7}{2}y+3 \\ &= \frac{1}{2}[2y^2-7y+6] \end{aligned}$$

6. Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2+4x-3$ and verify relationship between zeroes and coefficients of the polynomial.

Ans : [Board Term-1, 2011, Set-21]

We have $p(x) = 4x^2+4x-3$

If $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x)=0$

$$\begin{aligned} p\left(\frac{1}{2}\right) &= 4\left(\frac{1}{4}\right)+4\left(\frac{1}{2}\right)-3 \\ &= 1+2-3=0 \end{aligned}$$

and

$$\begin{aligned} p\left(-\frac{3}{2}\right) &= 4\left(\frac{9}{2}\right)+4\left(-\frac{3}{2}\right)-3 \\ &= 9-6-3=0 \end{aligned}$$

Thus $\frac{1}{2},-\frac{3}{2}$ are zeroes of polynomial $4x^2+4x-3$.

$$\text{Sum of zeroes } = \frac{1}{2}-\frac{3}{2} = -1 = \frac{-4}{4}$$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes } = \left(\frac{1}{2}\right)\left(-\frac{3}{2}\right) = \frac{-3}{4}$$

$$= \frac{\text{Constan term}}{\text{Coefficient of } x^2}$$

Verified

7. Find the zeroes of the quadratic polynomial $x^2-2\sqrt{2}\,x$ and verify the relationship between the zeroes and the coefficients.

Ans : [Board Term-1, 2015, Set-FHN8MG0]

We have $p(x)x^2-2\sqrt{2}\,x = 0$

$$x(x-2\sqrt{2}) = 0$$

Thus zeroes are 0 and $2\sqrt{2}$.

Sum of zeroes $2\sqrt{2} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and product of zeroes $0 = \frac{\text{Constan term}}{\text{Coefficient of } x^2}$

Hence verified

8. Find the zeroes of the quadratic polynomial $5x^2+8x-4$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Ans : [Board Term-1, 2013, Set LK-59]

We have

$$\begin{aligned} p(x) &= 5x^2+8x-4=0 \\ &= 5x^2+10x-2x-4=0 \end{aligned}$$

$$\begin{aligned} &= 5x(x+2)-2(x+2)=0 \\ &= (x+2)(5x-2) \end{aligned}$$

Substituting $p(x)=0$ we get zeroes as -2 and $\frac{2}{5}$.

Verification :

$$\text{Sum of zeroes } = -2+\frac{2}{5} = \frac{-8}{5}$$

$$\text{Product of zeroes } = (-2)\times\left(\frac{2}{5}\right) = \frac{-4}{5}$$

Now from polynomial we have

$$\text{Sum of zeroes} \quad -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-8}{5}$$

$$\text{Product of zeroes} \quad \frac{c}{a} = \frac{\text{Constan term}}{\text{Coefficient of } x^2} = -\frac{4}{5}$$

Hence Verified.

9. If α and β are the zeroes of a quadratic polynomial such that $\alpha+\beta=0$ and $\alpha-\beta=8$. Find the quadratic polynomial having α and β as its zeroes.

Ans : [Board Term-1, 2011, Set-44]

We have
$$\alpha+\beta = 24 \qquad \text{...(1)}$$
$$\alpha-\beta = 8 \qquad \text{...(2)}$$

Adding equations (1) and (2) we have

$$2\alpha = 32 \Rightarrow \alpha = 16$$

Subtracting (1) from (2) we have

$$2\beta = 24 \Rightarrow \beta = 12$$

Hence, the quadratic polynomial

$$\begin{aligned} p(x) &= x^2-(\alpha+\beta)x+\alpha\beta \\ &= x^2-(16+8)x+(16)(8) \\ &= x^2-24x+128 \end{aligned}$$

10. If α,β and γ are zeroes of the polynomial $6x^3+3x^2-5x+1$, then find the value of $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$.

Ans : [KVS practice Test 2017, CBSE Board 2010]

We have $\quad p(x) = 6x^3+3x^2-5x+1$

Since α,β and γ are zeroes polynomial $p(x)$, we have

$$\alpha+\beta+\gamma = -\frac{b}{c} = -\frac{3}{6} = -\frac{1}{2}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a} = -\frac{5}{6}$$

and
$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{1}{6}$$

Now
$$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}$$

$$= \frac{-5/6}{-1/6} = \frac{-5}{6}\times\frac{6}{-1} = 5$$

Hence $\alpha^{-1}+\beta^{-1}+\gamma^{-1}=5$.

LONG ANSWER TYPE QUESTIONS

1. Polynomial $x^4+7x^3+7x^2+px+q$ is exactly divisible by $x^2+7x+12$, then find the value of p and q.

Ans : [Board Term-1, 2015, Set-DDE-M]

We have $\quad f(x) = x^4+7x^3+7x^2+px+q$

Now
$$\begin{aligned} x^2+7x+12 &= 0 \\ x^2+4x+3x+12 &= 0 \\ x(x+4)+3(x+4) &= 0 \\ (x+4)(x+3) &= 0 \\ x &= -4, -3 \qquad \text{...(i)} \end{aligned}$$

Since $f(x)=x^4+7x^3+7x^2+px+q$ is exactly divisible by $x^2+7x+12$, then $x=-4$ and $x=-3$ must be its zeroes and these must satisfy $f(x)=0$

So putting $x=-4$ and $x=-3$ in $f(x)$ and equating to zero we get

$$\begin{aligned} f(-4):\ (-4)^4+7(-4)^3+7(-4)^2+p(-4)+q &= 0 \\ 256-448+112-4p+q &= 0 \\ -4p+q-80 &= 0 \\ 4p-q &= -80 \text{...(1)} \end{aligned}$$

$$\begin{aligned} p(-3):\ (-3)^4+7(-3)^3+7(-3)^2+p(-3)+q &= 0 \\ 81-189+63-3p+q &= 0 \\ -3p+q-45 &= 0 \\ 3p-q &= -45 \text{ ...(2)} \end{aligned}$$

Subtracting eq, (2) from (1) we have

$$p = -35$$

On putting the value of p in eq. (1) we have

$$\begin{aligned} 4(-35)-q &= -80 \\ -140-q &= -80 \\ -q &= 140-80 \\ \text{or} \qquad -q &= 60 \\ q &= -60 \end{aligned}$$

Hence, $p=-35$ and $q=-60$.

2. If α and β are the zeroes of the polynomial $p(x)=2x^2+5x+k$ satisfying the relation, $\alpha^2+\beta^2+\alpha\beta=\frac{21}{4}$, then find the value of k.

Ans : [Board Term-1, 2012, Set-50]

We have $\quad p(x) = 2x^2+5x+k$

Sum of zeroes, $\quad \alpha+\beta = -\frac{b}{a} = -\left(\frac{5}{2}\right)$

Product of zeroes $\quad \alpha\beta = \frac{c}{a} = \frac{k}{2}$

According to the question,

$$\alpha^2+\beta^2+\alpha\beta = \frac{21}{4}$$

$$\alpha^2+\beta^2+2\alpha\beta-\alpha\beta = \frac{21}{4}$$

$$(\alpha+b)^2-\alpha\beta = \frac{21}{4}$$

Substituting values we have

$$\left(\frac{-5}{2}\right)^2-\frac{k}{2} = \frac{21}{4}$$

$$\frac{k}{2} = \frac{25}{4}-\frac{21}{4}$$

$$\frac{k}{2} = \frac{4}{4} = 1$$

Hence, $k = 2$

3. If α and β are the zeroes of polynomial $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

Ans : [Board Term-1, 2012, Set-45, 62, 2010, Set-15]

We have $p(x) = 3x^2 + 2x + 1$

Since α and β are the zeroes of polynomial $3x^2 + 2x + 1$, we have

$$\alpha + \beta = -\frac{2}{3}$$

and $$\alpha\beta = \frac{1}{3}$$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\begin{aligned}\alpha_1 + \beta_1 &= \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta} \\ &= \frac{(1-\alpha+\beta-\alpha\beta)+(1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)} \\ &= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}} \\ &= \frac{\frac{4}{3}}{\frac{2}{3}} = 2\end{aligned}$$

For $q(x)$, product of the zeroes,

$$\begin{aligned}\alpha_1\beta_1 &= \left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right] \\ &= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)} \\ &= \frac{1-\alpha-\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta} \\ &= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta} \\ &= \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{2}{3}} = 3\end{aligned}$$

Hence, Required polynomial

$$\begin{aligned}q(x) &= x^2 - (\alpha_1+\beta_1)2x + \alpha_1\beta_1 \\ &= x^2 - 2x + 3\end{aligned}$$

4. If α and β are the zeroes of the polynomial $x^2 + 4x + 3$, find the polynomial whose zeroes are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$.

Ans : [Board Term-1, 2013 LK-59]

We have $p(x) = x^2 + 4x + 3$

Since α and β are the zeroes of the quadratic polynomial $x^2 + 4x + 3$,

So, $$\alpha + \beta = -4$$

and $$\alpha\beta = 3$$ 1

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\begin{aligned}\alpha_1 + \beta_1 &= 1 + \frac{\alpha}{\beta} + 1 + \frac{\alpha}{\beta} \\ &= \frac{\alpha\beta + \beta^2 + \alpha\beta + \alpha^2}{\alpha\beta} \\ &= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta} \\ &= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)}{3} = \frac{16}{3}\end{aligned}$$ 1

For $q(x)$, product of the zeroes,

$$\begin{aligned}\alpha_1\beta_1 &= \left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right) \\ &= \left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\beta+\alpha}{\beta}\right) \\ &= \frac{(\alpha+\beta)^2}{\alpha\beta} \\ &= \frac{(-4)^2}{3} = \frac{16}{3}\end{aligned}$$

Hence, Required polynomial

$$\begin{aligned}q(x) &= x^2 - (\alpha_1+\beta_1)2x + \alpha_1\beta_1 \\ &= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3} \\ &= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right) \\ &= \frac{1}{3}(3x^2 - 16x + 16)\end{aligned}$$

5. If α and β are zeroes of the polynomial $p(x) = 6x - 5x + k$ such that $\alpha - \beta = \frac{1}{6}$, Find the value of k.

Ans :

We have $p(x) = 6x - 5x + k$

Since α and β are zeroes of $p(x) = 6x - 5x + k$,

Sum of zeroes, $$\alpha + \beta = -\left(\frac{-5}{6}\right) = \frac{5}{6} \qquad ...(1)$$

Product of zeroes $$\alpha\beta = \frac{k}{6} \qquad ...(2)$$

Given $$\alpha - \beta = \frac{1}{6} \qquad ...(3)$$

Solving (1) and (3) we get $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ and substituting the values of (2) we have

$$\alpha\beta = \frac{k}{6} = \frac{1}{2} \times \frac{1}{3}$$

Hence, $k = 1$.

6. If β and $\frac{1}{\beta}$ are zeroes of the polynomial $(a^2 + a)x^2 + 61x + 6a$. Find the value of β and α.

Ans :

We have $p(x) = (a^2 + a)x^2 + 61x + 6$

Since β and $\frac{1}{\beta}$ are the zeroes of polynomial, $p(x)$

Sum of zeroes, $\beta + \frac{1}{\beta} = -\frac{61}{a^2 + a}$

or, $\frac{\beta^2 + 1}{\beta} = \frac{-61}{a^2 + a}$ ...(1)

Product of zeroes $\beta\frac{1}{\beta} = \frac{6a}{a^2 + a}$

or, $1 = \frac{6}{a + 1}$

$$a + 1 = 6$$

$$a = 5$$

Substituting this value of a in (1) we get

$$\frac{\beta^2 + 1}{\beta} = \frac{-61}{5^2 + 5} = -\frac{61}{30}$$

$$30\beta^2 + 30 = -61\beta$$

$$30\beta^2 + 61\beta + 30 = 0$$

Now $\beta \; \frac{-61 \pm \sqrt{(-61)^2 \times 4 \times 30 \times 30}}{2 \times 30}$

$$= \frac{-61 \pm \sqrt{3721 - 3600}}{60}$$

$$\frac{-61 \mp 11}{60}$$

Thus $\beta = \frac{-5}{6}$ or $\frac{-6}{5}$

Hence, $\alpha = 5, \beta = \frac{-5}{6}, \frac{-6}{5}$

SHORT ANSWER TYPE QUESTIONS - I

1. On dividing $x^3 - 5x^2 + 6x + 4$ by a polynomial $g(x)$, the quotient and the remainder were $x - 3$ and 4 respectively. Find $g(x)$.

Ans : [Board Term-1, 2012, Set-55]

We have $x^3 - 5x^2 + 6x + 4 = g(x)(x - 3) + 4$

$$g(x) = \frac{x^3 - 5x^2 + 6x + 4 - 4}{x - 3}$$

or, $g(x) = \frac{x^3 - 5x^2 + 6x}{x - 3}$

Now we divide $x^3 - 5x^2 + 6x$ by $x - 3$ as follows.

$$\begin{array}{r l}
 & \;\; x^2 - 2x \\
x - 3 & \overline{\big)\, x^3 - 5x^2 + 6x} \\
 & \underline{\;\; x^3 - 3x^2} \\
 & \;\; -2x^2 + 6x \\
 & \;\; \underline{2x^2 + 6x} \\
 & \;\; 0
\end{array}$$

Hence $g(x) = x^2 - 2x$.

2. Find the quotient and remainder on dividing $p(x)$ by $g(x)$:

$p(x) = 4x^3 + 8x^2 + 8x + 7; g(x) = 2x^2 - x + 1$

Ans : [Board Term-1, 2012, Set-55]

$$\begin{array}{r l}
 & \;\; 2x + 5 \\
2x^2 - x + 1 & \overline{\big)\, 4x^3 + 8x^2 + 8x + 7} \\
 & \underline{\;\; 4x^3 - 2x^2 + 2x} \\
 & \;\; 10x^2 + 6x + 7 \\
 & \underline{\;\; 10x^2 - 5x + 7} \\
 & \;\; 11x + 2
\end{array}$$

Thus, Quotient $= 2x + 5$

and Remainder $= 11x + 2$

3. Check whether the polynomial $g(x) = x^2 + 3x + 1$ is a factor of the polynomial $f(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 4$.

Ans : [Board Term-1, 2012, Set-48]

$$\begin{array}{r l}
 & \;\; 3x^2 - 4x + 2 \\
x^2 + 3x + 1 & \overline{\big)\, 3x^4 + 5x^3 - 7x^2 + 2x + 4} \\
 & \underline{\;\; 3x^4 + 9x^3 + 3x^2} \\
 & \;\; -4x^3 - 10x^2 + 2x \\
 & \underline{\;\; -4x^3 - 12x^2 - 4x} \\
 & \;\; 2x^2 + 6x + 4 \\
 & \;\; \underline{2x^2 + 6x + 2} \\
 & \;\; 2
\end{array}$$

Since remainder is not zero, polynomial $g(x) = x^2 + 3x + 1$ is not a factor of the polynomial $f(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 4$.

4. What should be added in the polynomial $x^3 - 6x^2 + 11x + 8$ so that is completely divisible by $x^2 - 3x + 2$?

Ans : [Board Term-1, Set, 2015]

$$\begin{array}{r l}
 & \;\; x - 3 \\
x^2 - 3x + 2 & \overline{\big)\, x^3 - 6x^2 + 11x + 8} \\
 & \underline{\;\; x^3 - 3x^2 + 2x} \\
 & \;\; -3x^2 + 9x + 8 \\
 & \underline{\;\; -3x^2 + 9x - 6} \\
 & \;\; 14
\end{array}$$

Since Remainder $=$ 14 to make it$0 - 14$ should be added.

5. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$, find the values of a and b.

Ans : [Board Term-1, Set FHN8MGI, 2015]

$$\begin{array}{r l}
 & \;\; 2x^2 + 5 \\
3x^2 + 4x + 1 & \overline{\big)\, 6x^4 + 8x^3 + 17x^2 + 21x + 7} \\
 & \underline{\;\; 6x^4 + 8x^3 + 2x^2} \\
 & \;\; 15x^2 + 21x + 7 \\
 & \;\; \underline{15x^2 + 20x + 5} \\
 & \;\; x + 2
\end{array}$$

Comparing both the sides we get $a = 1$ and $b = 2$

6. If x^3-6x^2+6x+k is completely divisible by $x-3$, then find the value of k.

Ans : [Board Term-1, Set-WJQZQBN]

$$\begin{array}{r}
x^2-3x-3 \\
x-3\overline{\smash{\big)}\,x^3-6x^2+6x+k} \\
\underline{x^3-3x^2} \\
-3x^2+6x+k \\
\underline{-3x^2+9x} \\
-3x+k \\
\underline{-3x+9} \\
k-9
\end{array}$$

Remainder should be zero

$$\begin{aligned} k-9 &= 0 \\ \text{So,}\qquad k &= 0 \end{aligned}$$

7. Divide the polynomial $p(x)=x^3-4x+6$ by the polynomial $g(x)=2-x^2$ and find the quotient and the remainder.

Ans : [Board Term-1, 2015, Set-1E]

$$\begin{array}{r}
-x \\
-x^2-2\overline{\smash{\big)}\,x^3-4x+6} \\
\underline{x^3-2x} \\
-2x+6
\end{array}$$

Thus, Quotient $=-x$

and Remainder $=6-2x$

8. Divide the polynomial $p(x)=x^2-5x+16$ by the polynomial $g(x)=x-2$ and find the quotient and the remainder.

Ans : [Board Term-1, 2015, Set-WJQZQBN]

$$\begin{array}{r}
x-3 \\
x-2\overline{\smash{\big)}\,x^2-5x+16} \\
\underline{x^2-2x} \\
-3x+16 \\
\underline{-3x+6} \\
10
\end{array}$$

Quotient $=x-3$, Remainder $=10$

SHORT ANSWER TYPE QUESTIONS - II

1. What should be added to x^3+5x^2+7x+3 so that it is completely divisible by x^2+2x.

Ans : [Board Term-1, 2016 Set-MV98HN3]

$$\begin{array}{r}
x+3 \\
x^2+2x\overline{\smash{\big)}\,x^3+5x^2+7x+3} \\
\underline{x^3+2x^2} \\
3x^2+7x+3 \\
\underline{3x^2+6x} \\
x+3
\end{array}$$

2. Divided $6x^3+2x^2-4x+3$ by $3x^2-2x+1$ and verify the division algorithm.

Ans : [Board Term-1, 2011, Set-74]

$$\begin{array}{r}
2x+2 \\
3x^2-2x+1\overline{\smash{\big)}\,6x^3+2x^2-4x+3} \\
\underline{6x^3-4x^2+2x} \\
6x^2-6x+3 \\
\underline{6x^2-4x+2} \\
-2x+1
\end{array}$$

Quotient $=2x+2$; Remainder $=-2x+1$

$$\begin{aligned}
p(x) &= g(x)q(x)+r(x) \\
&= (3x^2-2x+1)(2x+2)+(-2x+1) \\
&= 6x^3-4x^2+2x+6x^2-4x+2-2x+1 \\
&= 6x^3+2x^2-4x+3 \qquad \text{Verified}
\end{aligned}$$

3. Find the value of a and b so that $8x^2+14x^3-2x^2+ax+b$ is exactly divisible by $4x^2+3x-2$.

Ans : [Board Term-1, 2011, Set-66]

$$\begin{array}{r}
2x^2+2x-1 \\
4x^2+3x-2\overline{\smash{\big)}\,8x^4+14x^3-2x^2+ax+b} \\
\underline{8x^4+6x^3-4x^2} \\
8x^3+2x^2+ax \\
\underline{8x^3+6x^2-4x} \\
-4x^2+(a+4)x+b \\
\underline{-4x^2-3x+2} \\
(a+7)x+b
\end{array}$$

For exact division, remainder is zero, so

$$\begin{aligned}
(a+7)x+b-2 &= 0 \\
\text{or}\qquad a+7 &= 0, b-2=0 \\
\Rightarrow\qquad a &= -7, b=2
\end{aligned}$$

4. On dividing a polynomial $3x^3+4x^2+5x-13$ by a polynomial $g(x)$, the quotient and the remainder are $(3x+10)$ and $(16x-43)$ respectively. Find $g(x)$.

Ans : [Board Term-1, 2011, Set-40]

$$\begin{array}{r}
x^2-2x+3 \\
3x+10\overline{\smash{\big)}\,3x^3+4x^2-11x+30} \\
\underline{3^3+10x^2} \\
-6x^2-11x \\
\underline{-6x^2-20x} \\
9x+30 \\
\underline{9x+30} \\
0
\end{array}$$

$$3x^3+4x^2+5x-13=(3x+10)g(x)+(16x-43)$$
$$g(x)(3x+10)=(3x^3+4x^2+5x-13)-(16x-43)$$

Hence, $g(x)=x^2-2x+3$

5. When $p(x)=x^2+7x+9$ is divisible by $g(x)$, we get $(x+2)$ and -1 as the quotient and remainder respectively, find $g(x)$.

Ans : [Board Term-1, 2011, Set-74]

$$\begin{aligned}
\text{We have}\qquad p(x) &= x^2+7x+9 \\
q(x) &= x+2 \\
r(x) &= -1 \\
\text{Now}\qquad p(x) &= g(x)q(x)+r(x)
\end{aligned}$$

$$x^2+7x+9 = g(x)(x+2)-1$$

or, $$g(x) = \frac{x^2+7x+10}{x+2}$$

$$= \frac{(x+2)(x+5)}{(x+2)} = x+5$$

Thus $g(x) = x+5$

6. Check by divisible, algorithm whether x^2-2 is a factor of $x^4+x^3+x^2-2x-3$.

Ans : [Board Term-1, 2011, Set-39]

$$\begin{array}{r|l} & x^2+x+3 \\ x^2-2 & x^4+x^3+x^2-2x-3 \\ & \underline{x^4 \qquad -2x^2} \\ & \quad x^3+3x^2-2x \\ & \quad \underline{x^3 \qquad -2x} \\ & \qquad 3x^2 \quad -3 \\ & \qquad \underline{3x^2 \quad -6} \\ & \qquad\qquad 3 \end{array}$$

Since Remainder $= 3 \neq 0$, hence x^2-2 is not a factor of the given polynomial.

7. On dividing $x^4-x^3-3x^2+3x+2$ by a polynomial $g(x)$, the quotient and the remainder are x^2-x-2 and $2x$ respectively. Find $g(x)$.

Ans : [Board Term-1, 2015, Set-CJTOQ]

$$\begin{array}{r|l} & x^2-1 \\ x^2-x-2 & x^4-x^3-3x^2+x+2 \\ & \underline{x^4-x^3-2x^2} \\ & \qquad -x^2+x+2 \\ & \qquad \underline{-x^2+x+2} \\ & \qquad\qquad 0 \end{array}$$

$$x^4-x^3-3x^2+3x+2 = (x^2-x-2)g(x)+2x$$

$$g(x)(x^2-x-2) = (x^4-x^3-3x^2+3x+2)-2x$$

$$g(x) = \frac{x^4-x^3-3x^2+x+2}{x^2-x-2}$$

Hence, $$g(x) = x^2-1$$

8. What should be added in the polynomial x^3+2x^2-9x+1 so that it is completely divisible by $x+4$7.

Ans : [Board Term-1, 2015, Set-DDE-M]

Let k be added.

$$\begin{array}{r|l} & x^2-2x-1 \\ x+4 & x^3+2x^2-9x+1 \\ & \underline{x^3+4x^2} \\ & -2x^2-9x+1 \\ & \underline{-2x^2-8x} \\ & \qquad -x+1 \\ & \qquad \underline{-x-4} \\ & \qquad\quad 5 \end{array}$$

Remainder should be zero

$$5+k = 0$$

Hence –5 should be added.

9. If the polynomial $f(x) = 3x^4+3x^3-11x^2-5x+10$ is completely divisible by $3x^2-5$, find all its zeroes.

Ans : [Board Term-1, 2013, FFC; 2011, Set-13]

Since $3x^2-5$ divides $f(x)$ completely, $(3x^2-5)$ is a factor of $f(x)$.

Thus $$3x^2-5 = 0$$

$$x^2 = \frac{5}{3}$$

$$x = \pm\sqrt{\frac{5}{3}}$$

$$\begin{array}{r|l} & x^2+x-2 \\ 3x^2-5 & 3x^4+3x^3-11x^2-5x+10 \\ & \underline{3x^4 \qquad -5x^2} \\ & \qquad 3x^3-6x^2-5x+10 \\ & \qquad \underline{3x^3 \qquad -5x} \\ & \qquad\quad -6x^2 \quad +10 \\ & \qquad\quad \underline{-6x^2 \quad +10} \\ & \qquad\qquad 0 \end{array}$$

Since (x^2+x-2) is a factor of $p(x.)$

$$x^2+2-2 = 0$$

Factorising it, we get

$$x = -2 \text{ and } 1$$

Thus -2 and 1 are zeroes or $p(x)$.

All the zeroes of $p(x)$ are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -2 and 1.

LONG ANSWER TYPE QUESTIONS

1. If two zeroes of a polynomial x^3+5x^2+7x+3 are -1 and -3, then find the third zero.

Ans : [Board Term-1, 2016 Set MV98HN3]

$$\begin{array}{r|l} & x+1 \\ x^2+4x+3 & x^3+5x^2+7x+3 \\ & \underline{x^3+4x^2+3x} \\ & \qquad x^2+4x+3 \\ & \qquad \underline{x^2+4x+3} \\ & \qquad\quad 0 \end{array}$$

$x = -1$ and $x = -3$ are zeroes.

2. Given that $x-\sqrt{5}$ is a factor of the polynomial $x^3-3\sqrt{5}\,x^2-5x+15\sqrt{5}$, find all the zeroes of the polynomial.

Ans : [Board Term-1, 2014] [Board Term-1, 2012, Set-39]

$$
\begin{array}{rl}
 & x^2-2\sqrt{5}\,x-15 \\
x-\sqrt{5} & \overline{\big)\,x^3-3\sqrt{5}\,x^2-5x+15\sqrt{5}} \\
 & \underline{x^3-\sqrt{5}\,x^2} \\
 & -2\sqrt{5}\,x^2-5x+15\sqrt{5} \\
 & \underline{-2\sqrt{5}\,x^2+10x} \\
 & \qquad -15x+15\sqrt{5} \\
 & \qquad \underline{-15x+15\sqrt{5}} \\
 & \qquad\qquad 0
\end{array}
$$

Factorising the quotient we get

$$
\begin{aligned}
x^2-2\sqrt{5}\,x-15 &= x^2-3\sqrt{5}\,x+\sqrt{5}\,x-15 \\
&= x(x-3\sqrt{5})+\sqrt{5}(x-3\sqrt{5}) \\
&= (x+\sqrt{5})(x-3\sqrt{5})
\end{aligned}
$$

$$(x+\sqrt{5})(x-3\sqrt{5}) = 0 \quad \Rightarrow \quad x = \sqrt{5},\ 3\sqrt{5}$$

All the zeroes are $\sqrt{5}$, $-\sqrt{5}$ and $3\sqrt{5}$.

3. If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by (x^2-2x+k), the remainder comes out to be $x+a$, find k and a.

Ans : [Board Term-1, 2012, Set-35]

$$
\begin{array}{rl}
 & x^2-4x+(8-k) \\
x^2-2x+k & \overline{\big)\,x^4-6x^3+16x^2-25x+10} \\
 & \underline{x^4-2x^3+kx^2} \\
 & -4x^3+(16-k)\,x^2-25x+10 \\
 & \underline{-4x^3 \qquad +8x^2-4kx} \\
 & \qquad (8-k)\,x^2-(25-4k)\,x+10 \\
 & \qquad \underline{(8-k)\,x-(16-2k)\,x+(8k-k^2)} \\
 & \qquad\qquad (2k-9)\,x+(10-8k+k^2)
\end{array}
$$

Given, remainder $= x+a$

Comparing the multiples of x

$$
\begin{aligned}
(2k-9)x &= 1\times x \\
2k-9 &= 1 \\
k &= \frac{10}{2} = 5
\end{aligned}
$$

Substituting this value of k into other portion of remainder, we get

and $\quad a = 10-8k+k^2 = 10-40+25 = -5$

4. Find the other zeroes of the polynomial $x^4-5x^3+2x^2+10x-8$ if it is given that two zeroes are $-\sqrt{2}$ and $\sqrt{2}$.

Ans : [Board Term-1, 2012, Set-35]

We have two zeroes $\sqrt{2}$ and $-\sqrt{2}$.

Two factors are $(x+\sqrt{2})$ and $(x-\sqrt{2})$

$g(x) = (x+\sqrt{2})(x-\sqrt{2}) = x^2-2$ is a factor of the given polynomial

$$
\begin{array}{rl}
 & x^2-5x+4 \\
x^2-2 & \overline{\big)\,x^4-5x^3+2x^2+10x-8} \\
 & \underline{x^4 \qquad -2x^2} \\
 & -5x^3+4x^2+10x-8 \\
 & \underline{-5x^3 \qquad -10x} \\
 & \qquad 4x^2 \qquad -8 \\
 & \qquad \underline{4x^2 \qquad -8} \\
 & \qquad\qquad 0
\end{array}
$$

Quotient $= x^2-5x+4 = (x-4)(x-1)$

Hence other zeroes are 4 and 1.

5. Show that 3 is a zero of the polynomial $2x^2-x^2-13x-6$. Hence find all the zeroes of this polynomial.

Ans :

$$
\begin{array}{rl}
 & 2x^2+5x+2 \\
x-3 & \overline{\big)\,2x^3-x^2-13x-6} \\
 & \underline{2x^3-6x^2} \\
 & \quad 5x^2-13x-6 \\
 & \quad \underline{5x^2-15x} \\
 & \qquad\quad 2x-6 \\
 & \qquad\quad \underline{2x-6} \\
 & \qquad\quad 0
\end{array}
$$

$$
\begin{aligned}
p(x) &= 2x^2-x^2-13x-6 \\
&= 2(3)^3-(3)^2-13(3)-6 \\
&= 2(27)-9-39-6 \\
&= 54-54 = 0
\end{aligned}
$$

So, $x-3$ is a factor of $p(x)$.

by long division

Factorising the quotient, we get

$$
\begin{aligned}
&= 3x^2+4x+x+2 \\
&= (2x+1)(x+2) \\
x &= -\frac{1}{2},\,-2
\end{aligned}
$$

Hence, All the zeroes of $p(x)$ are $-\frac{1}{2}, -2, 3$

6. Obtain all other zeroes of the polynomial $x^4+6x^3+x^2-24x-20$, if two of its zeroes are $+2$ and -5.

Ans : [Board Term-1, 2015, Set-DDE-E] [NCERT]

$$
\begin{array}{rl}
 & x^2+3x+2 \\
x^2+3x-10 & \overline{\big)\,x^4+6x^3+x^2-24x-20} \\
 & \underline{x^4+3x^3-10x^2} \\
 & \quad 3x^3+11x^2-24x-20 \\
 & \quad \underline{3x^3+9x^2-30x} \\
 & \qquad\quad 2x^2+6x-20 \\
 & \qquad\quad \underline{2x^2+6x-20} \\
 & \qquad\qquad 0
\end{array}
$$

As $x=2$ and -5 are the zeroes of $x^4+6x^3+x^2-24x-20$.

So $(x-2)$ and $(x+5)$ are two factors of $x^4+6x^3+x^2-24x-20$ and the product of factors is

$$(x-2)(x+5) = x^2+3x-10 = 0$$

Dividing $x^4+6x^3+x^2-24x-20$ by $x^2+3x-10$

$$
\begin{aligned}
&x^4+6x^3+x^2-24x-20 \\
&\quad = (x^2+3x-10)(x^2+3x+2) \\
&\quad = (x-2)(x+5)(x+2)(x+1)
\end{aligned}
$$

Hence other two zeroes are -2 and 1.

7. Obtain all other zeroes of the polynomial $4x^4+x^3-72x^2-18x$, if two of its zeroes are $3\sqrt{2}$ and $-3\sqrt{2}$.

Ans : [Board Term-1, 2015, Set-C3TOQ]

Ans :

As $3\sqrt{2}$ and $-3\sqrt{2}$ are the zeroes of $4x^4+x^3-72x^2-18x$, So $(x-3\sqrt{2})$ and $(x+3\sqrt{2})$ are its two factors

Now,$(x-3\sqrt{2})(x+3\sqrt{2}) = 0$

or, $x^2-18 = 0$

On Factorising quotient $4x^2+2$

We get, $x = 0$ and $\frac{1}{4}$

$$=(x^2-18)x(4x+1)$$
$$=(x-3\sqrt{2})(x+3\sqrt{2})(x)(4x+1)$$

Hence, other two zeroes are 0 and $\frac{-1}{4}$.

8. Obtain all other zeroes of the polynomial $9x^4-6x^3-35x^2+24x-4$, if two of its zeroes are 2 and -2.

Ans : [Board Term-1, 2015, Set -DDE -M]

As 2 and -2 are the zeroes of $9x^4-6x^3-35x^2+24x-4$, So $(x-2)$ and $(x+2)$ are its two factors

and $(x-2)(x+2) = x^2-4$

Dividing $9x^4-6x^3-35x^2+24x-4$ by x^2-4

$$\begin{array}{r|l} & \quad 9x^2-6x+1 \\ \hline x^2-4 & 9x^4-6x^3-35x^2+24x-4 \\ & \underline{9x^4 \qquad -36x^2 \qquad\qquad} \\ & \quad -6x^3+x^2+24x-4 \\ & \quad \underline{-6x^3 \qquad +24x \qquad} \\ & \qquad\qquad x^2 \qquad -4 \\ & \qquad\qquad \underline{x^2 \qquad -4} \\ & \qquad\qquad\qquad 0 \end{array}$$

Factorising this quotient

$$=[9x^2-3x-3x+1]$$
$$=[3x(3x-1)-1(3x-1)]$$
$$=[(3x-1)(3x-1)]$$
$$=(3x-1)(3x-1)$$

Hence, other two zeroes are $\frac{1}{3},\frac{1}{3}$. 1

9. Find all the zeros of the polynomial $3x^4+6x^3-2x^2-10x-5$ it two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Ans : [Sample Paper 2017]

$$\begin{array}{r|l} & \quad x^2+2x+1 \\ \hline 3x^2-5 & 3x^4+6x^3-2x^2-10x-5 \\ & \underline{3x^4 \qquad -5x^2 \qquad\qquad} \\ & \qquad 6x^3+3x^2-10x-5 \\ & \quad \underline{-6x^3 \qquad -10x} \\ & \qquad\qquad 3x^2 \qquad -5 \\ & \qquad\qquad \underline{3x^2 \qquad -5} \\ & \qquad\qquad\qquad 0 \end{array}$$

Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are two zeroes of the given polynomial.

So, $\left(x-\sqrt{\frac{5}{3}}\right),\left(x+\sqrt{\frac{5}{3}}\right)$ will be its two factors

$\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right) = \frac{1}{3}(3x^2-5)$

is a factor of given polynomial

Now, dividing it by $3x^2-5$.

$$x^2+2x+1 = (x+1)^2 = (x+1)(x+1)$$

two other zeroes $= -1$ and -1

Hence all the zeroes of given polynomial

$$= \sqrt{\frac{5}{3}},\sqrt{\frac{5}{3}},-1 \text{ and } -1$$

HOTS QUESTIONS

1. Find the value for k for which $x^4+10x^3+25x^2+15x+k$ is exactly divisible by $x+7$.

Ans :

We have $f(x) = x^4+10x^3+25x^2+15x+k$

If $x+7$ is a factor then -7 is a zero of $f(x)$ and $x=-7$ satisfy $f(x)=0$.

Thus substituting $x=-7$ in $f(x)$ and equating to zero we have,

$$(-7)^4+10(-7)^3+25(-7)^2+15(-7)+k=0$$
$$2401-3430+1225-105+k = 0$$
$$3626-3535+k = 0$$
$$91+k = 0$$
$$k = -91$$

2. If two zeroes of the polynomial $p(x)=x^4-6x^3-26x^2+138x-35$ are $2\pm\sqrt{3}$. Find the other zeroes.

Ans :

We have

As $2\pm\sqrt{3}$ are the zeroes of $p(x)$, so $x-(2\pm\sqrt{3})$ are the factor of $p(x)$. 1

and the product of zeros,

$$\{x-(2+\sqrt{3})\}\{x-(2-\sqrt{3})\}$$
$$=\{(x-2)-\sqrt{3}\}\{(x-2)+\sqrt{3}\}$$
$$=(x-2)^2-(\sqrt{3})^2$$
$$=x^2-4x+1$$

Dividing $p(x)$ by x^2-4x+1

$$\begin{array}{r|l} & \quad x^2-2x-35 \\ \hline x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\ & \underline{x^4-4x^3+x^2 \qquad\qquad\qquad} \\ & \quad -2x^3-27x^2+138x-35 \\ & \quad \underline{-2x^3+8x^2-2x \qquad} \\ & \qquad\quad -35x^2+140x-35 \\ & \qquad\quad \underline{-35x^2+140x-35} \\ & \qquad\qquad\qquad 0 \end{array}$$

Factorising $(x^2-2x-35)$ we get

$$=(x+5)(x-7)$$

$$x = -5, 7$$

Hence, other two zeroes of $p(x)$ are -5 and 7. 1

3. If α and β are the zeroes the polynomial $2x^2 - 4x + 5$, find the values of

(i) $\alpha^2 + \beta^2$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

(iii) $(\alpha - \beta)^2$

(iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(v) $\alpha^2 + \beta^2$

Ans :

We have $p(x) = 2x^2 - 4x + 5$

If α and β are then zeroes of $p(x) = 2x^2 - 4x + 5$, then

$$\alpha + \beta = -\frac{a}{b} = \frac{-(-4)}{2} = 2$$

and $$\alpha\beta = \frac{c}{a} = \frac{5}{2}$$

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= 2^2 - 2 \times \frac{5}{2}$$

$$= 4 - 5 = -1$$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{2}{\frac{5}{2}} = \frac{4}{5}$

(iii) $(\alpha - \beta)^2 = (\alpha - \beta)^2 - 4\alpha\beta$

$$= 2^2 - \frac{4 \times 5}{2}$$

$$4 - 10 = -6$$

(iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{-1}{(\frac{5}{2})^2} = \frac{-4}{25}$

(v) $(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$$= 2^2 - 3 \times \frac{5}{2} \times 2 = 8 - 15 = -7$$

4. On dividing the polynomial $4x^2 - 5x^3 - 39x^2 - 46x - 2$ by the polynomial $g(x)$, the quotient is $x^2 - 3x - 5$ and the remainder is $-5x + 8$. Find the polynomial $g(x)$.

Ans :

$$Dividend = (Divisor \times Quotient) + Remainder$$

$$4x^4 - 5x^3 - 39x^3 - 46x - 2$$

$$= g(x)(x^2 + 3x - 5) + (-5x + 8)$$

or, $4x^2 - 5x^3 - 39x^2 - 46x - 2 + 5x - 8$

$$= g(x)(x^2 - 3x - 5)$$

or, $4x^4 - 5x^3 - 39x^2 - 41x - 10$

$$= g(x)(x^2 - 3x - 5)$$

$$g(x) = \frac{4x^4 - 5x^3 - 39x^2 - 41x - 10}{(x^2 - 3x - 5)}$$

Hence, $g(x) = 4x^2 + 7x + 2$

5. If the squared difference of the zeroes of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

Ans :

We have $f(x) = x^2 + px + 45$

Let α and β be the zeroes of the given quadratic polynomial.

Sum of zeroes, $\alpha + \beta = -p$

Product of zeroes $\alpha\beta = 45$

Given, $(\alpha - \beta)^2 = 144$

$$(\alpha + \beta)^2 - 4\alpha\beta = 144$$

Substituting value of $\alpha + \beta$ and $\alpha\beta$ we get

$$(-p)^2 - 4 \times 45 = 144$$

$$p^2 - 180 = 144$$

$$p^2 = 144 + 180 = 324$$

Thus $p = \pm\sqrt{324} = \pm 18$

Hence, the value of p is ± 18.

CHAPTER 3

Pair of Linear Equation in Two Variables

VERY SHORT ANSWER TYPE QUESTIONS

1. Find whether the pair of linear equations $y = 0$ and $y = -5$ has no solution, unique solution or infinitely many solutions.

Ans :

The given variable y has different values. Therefore the pair of equations $y = 0$ and $y = -5$ has no solution.

2. If $am = bl$, then find whether the pair of linear equations $ax + by = c$ and $lx + my = n$ has no solution, unique solution or infinitely many solutions.

Ans :

Since, $am = bl$, we have

$$\frac{a}{1} \quad \frac{b}{m} \neq \frac{c}{n}$$

Thus, $ax + by = c$ and $lx + my = n$ has no solution.

3. If $ad \neq bc$, then find whether the pair of linear equations $ax + by = p$ and $cx + dy = q$ has no solution, unique solution or infinitely many solutions.

Ans :

Since $\qquad ad \neq bc$ or $\frac{a}{c} \neq \frac{b}{d}$

Hence, the pair of given linear equations has unique solution.

4. Two lines are given to be parallel. The equation of one of the lines is $4x + 3y = 14$, then find the equation of the second line.

Ans :

The equation of one line is $4x + 3y = 14$. We know that if two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel, then

$$\frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or $$\frac{4}{a_2} = \frac{3}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{a_2}{b_2} = \frac{4}{3} = \frac{12}{9}$$

Hence, one of the possible, second parallel line is $12x + 9y = 5$.

SHORT ANSWER TYPE QUESTIONS - I

1. Find whether the lines represented by $2x + y = 3$ and $4x + 2y = 6$ are parallel, coincident or intersecting.

Ans : [Board Term-1, 2016, MV98HN3]

Ans :

Here $a_1 = 2, b_1 = 1, c_1 = -3$ and $a_1 = 4, b_2 = 2, c_2 = -6$

If $$\frac{a_1}{a_1} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

then the lines are parallel.

Clearly $$\frac{2}{4} = \frac{1}{2} = \frac{3}{6}$$

Hence lines are coincident.

2. Find whether the following pair of linear equation is consistent or inconsistent:
$3x + 2y = 8, \; 6x - 4y = 9$

Ans : [Board Term-1, 2016 ORDAWEZ]

We have $$\frac{3}{6} \neq \frac{2}{-4}$$

i.e., $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence, the pair of linear equation is consistent.

3. Is the system of linear equations $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ consistent? Justify your answer.

Ans : [Board Term-1, 2012 set-66]

For the equation $2x + 3y - 9 = 0$ we have

$$a_2 = 2, b_1 = 3 \text{ and } c_1 = -9$$

and for the equation, $4x + 6y - 18 = 0$ we have

$$a_2 = 4, b_2 = 6 \text{ and } c_2 = -18$$

Here $$\frac{a_1}{a_1} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} \; \frac{3}{6} = \frac{1}{2}$$

and $$\frac{c_1}{c_2} = \frac{-9}{-18} = \frac{1}{2}$$

Thus $$\frac{c_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, system is consistent and dependent.

4. Given the linear equation $3x + 4y = 9$. Write another linear equation in these two variables such that the geometrical representation of the pair so formed is:
(1) intersecting lines
(2) coincident lines.

Ans : [Board Term-1, 2016, Set-O4YP6G7]

(1) For intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, one of the possible equation $3x - 5y = 10$

(2) For coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, one of the possible equation $6x + 8y = 18$

5. For what value of p does the pair of linear equations given below has unique solution ?
$4x+py+8=0$ and $2x+2y+2=0$.

Ans : [Board Term-1, 2012, Set-44]

$$\begin{aligned}\text{We have}\qquad 4x+py+8 &= 0\\ 2x+2y+2 &= 0\end{aligned}$$

The condition of unique solution, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

Hence, $\quad\frac{4}{2}\neq\frac{p}{2}$ or $\frac{2}{1}\neq\frac{p}{2}$

Thus $p\neq 4$. The value of p is other than 4 it may be $1,2,3,-4$.....etc.

6. For what value of k, the pair of linear equations $kx-4y=3,\ 6x-12y=9$ has an infinite number of solutions ?

Ans : [Board Term-1, 2012, Set-25]

$$\begin{aligned}\text{We have}\qquad kx-4y-3 &= 0\\ \text{and}\qquad 6x-12y-9 &= 0\\ \text{where,}\qquad a_1=k,b_1 &= 4,c_1=-3\\ a_2=6,b_2 &= -12,c_2-9\end{aligned}$$

Condition for infinite solutions:

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{k}{6}=\frac{-4}{-12}=\frac{3}{9}$$

Hence, $\quad k=2$

7. For what value of k, $2x+3y=4$ and $(k+2)x+6y=3k+2$ will have infinitely many solutions ?

Ans : [Board Term-1, 2012, Set-68]

$$\begin{aligned}\text{We have}\qquad 2x+3y-4 &= 0\\ \text{and}\quad (k+2)x+6y-(3k+2) &= 0\end{aligned}$$

Here $a_1=2,b_1=3,c_1=-4$

and $a_2=k+2,b_2=6,c_3=-(3k+2)$

For infinitely many solutions

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

or, $$\frac{2}{k+2}=\frac{3}{6}=\frac{4}{3k+2}$$

From $\frac{2}{k+2}=\frac{3}{6}$ we have

$3(k+2)=2\times 6\Rightarrow (k+2)=4\Rightarrow k=2$

From $\frac{3}{6}=\frac{4}{3k+2}$ we have

$3(3k+2)=4\times 6\Rightarrow (3k+2)=8\Rightarrow k=2$

Thus $k=2$

8. For what value of 'k', the system of equations $kx+3y=1,\ 12x+ky=2$ has no solution.

Ans : [Board Term-1, 2011, Set-A2 NCERT]

The given equations can be written as
$kx+3y-1=0$ and $12x+ky-2=0$

$$\begin{aligned}\text{Here,}\qquad a_1 &= k,b_1=3,c_1=-1\\ \text{and}\qquad a_2 &= 12,b_2=k,c_2=-2\end{aligned}$$

The equation for no solution if

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

or, $$\frac{k}{12}=\frac{3}{k}\neq\frac{-1}{-2}$$

From $\frac{k}{12}=\frac{3}{k}$ we have $k^2=36\Rightarrow k\pm 6$

From $\frac{3}{k}\neq\frac{-1}{-2}$ we have $k\neq 6$

Thus $k=-6$

SHORT ANSWER TYPE QUESTIONS - II

1. Which of the following pairs of linear equations are consistent/inconsistent ? If consistent, obtain the solution graphically.

Ans : [NCERT]

$$\begin{aligned}\text{(a)}\qquad x+y &= 5\\ 2x+2y &= 10\\ \text{(b)}\qquad x-y &= 8\\ 3x-3y &= 16\\ \text{(c)}\qquad 2x+y-y &= 0\\ 4x-2y-4 &= 0\\ \text{(d)}\qquad 2x-2y-2 &= 0\\ 4x-4y-5 &= 0\end{aligned}$$

Ans :

(a) The pair of linear equations is:

$$\begin{aligned}x+y &= 5\\ \text{or,}\qquad x+y-5 &= 0\\ \text{and}\qquad 2x+2y &= 10 &&\text{...(1)}\\ \text{or,}\qquad 2x+|\,2y-10 &= 0 &&\text{...(2)}\end{aligned}$$

$$\begin{aligned}\text{We have,}\qquad a &= 1,b_1=1,c_1=-5\\ a_2 &= 2,b_2=2,c_2=-10\end{aligned}$$

Now $\quad\frac{a_1}{a_2}=\frac{1}{2},\frac{b_1}{b_2}=\frac{1}{2}$, and $\frac{c_1}{c_2}=\frac{-5}{-10}=\frac{1}{2}$

Since $$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

So, the pair of linear equations is coincident having many, solutions. Thus, the equation is consistent to solve it graphically.

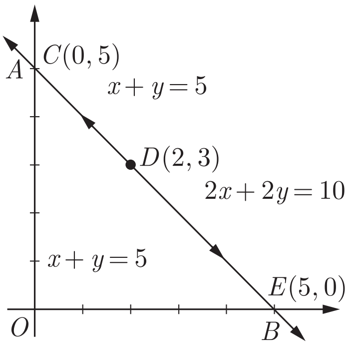
$$\begin{aligned}\text{We have}\qquad x+y &= 5\\ \text{or,}\qquad y &= 5-x\end{aligned}$$

x	0	5
y	5	0

$$\begin{aligned}\text{and}\qquad 2x+2y &= 10\\ \text{or,}\qquad y &= \frac{10-2x}{2}\end{aligned}$$

x	0	2	5
y	5	3	0

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly lines are coincident having infinite many solutions.

(b) The pair of linear equation is:

$$x+y = 8$$

or $\quad x+y-8 = 0 \qquad ...(1)$

and $\quad 3x-3y = 16$

or $\quad 3x-3y-16 = 0 \qquad ...(2)$

We have $\quad a_1 = 1, b_1 = -1, c_1 = -8$

$$a_2 = 3, b_2 = -3, c_2 = -16$$

Now $\quad \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{1}{2}$

As $\quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

The lines are parallel having no solution. So, the pair of linear equations is inconsistent.

(c) The pair of linear equations are :

$2x+y-6 = 0 \qquad ...(1)$

and $\quad 4x-2y-4 = 0 \qquad ...(2)$

where, $\quad a_1 = 2, b_1 = 1, c_1 = -6$

$$a_2 = 4, b_2 = -2, c_2 = -4$$

Now $\quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{1}{-2}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of linear equations is consistent.

We have, $2x+y-6=0$

or, $\quad y = 6-2x$

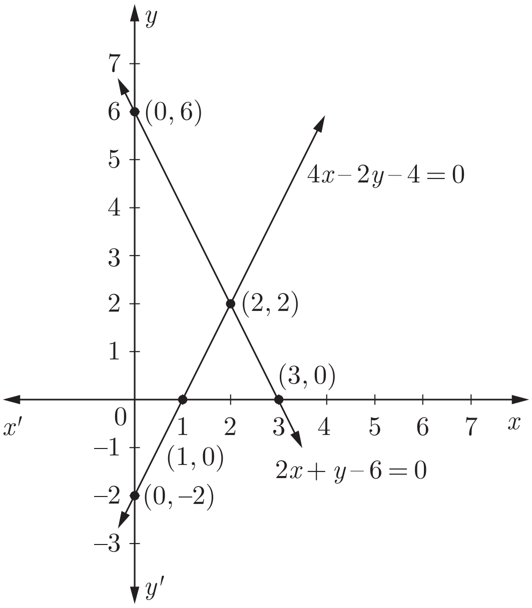
x	0	3	2
y	6	0	2

and $\quad 4x-2y-4 = 0$

or, $\quad y = 2x-2$

x	0	1	2
y	-2	0	2

Plotting the above points and drawing lines joining them, we get the following graph. Two obtained lines intersect each other at (2, 2).



(d) $\quad 2x-2y-2 = 0$

$$4x-4y-5 = 0$$

The pair of linear equations is

$2x-2y-2 = 0 \qquad ...(1)$

and $\quad 4x-4y-5 = 0 \qquad ...(2)$

where, $\quad a_1 = 2, b_1 = -2, c_1 = -2$

$$a_2 = 4, b_2 = -4, c_2 = -5$$

Now $\quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}$, and $\frac{c_1}{c_2} = \frac{-2}{-5}$

Since $\quad \left[\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\right]$

Therefore, the pair of linear equations is inconsistent.

2. Half the perimeter of a rectangular garden, whose length is 4 m more then its width, is 36 m. Find the dimensions of garden.

Ans : [NCERT]

Let the length of the garden be x m and its width be y m.

Perimeter of rectangular garden

$$p = 2(x+y)$$

Since half perimeter is given as 36 m,

$(x+y) = 36 \qquad ...(1)$

Also, $\quad x = y+4$

or $\quad x-y = 4 \qquad ...(2)$

For $\quad x+y = 36$

$$y = 36-x$$

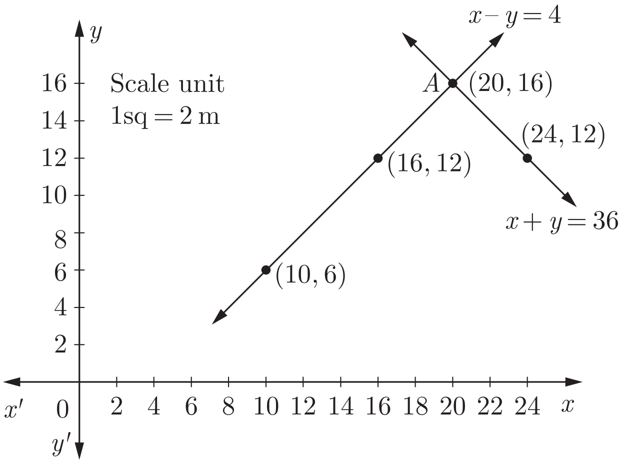
x	20	24
y	16	12

For $\quad x-y = 4$

or, $\quad y = x-4$

x	10	16	20
y	6	12	16

Plotting the above points and drawing lines joining them, we get the following graph. we get two lines intersecting each other at (20, 16)



Hence, length is 20 m and width is 16 m.

3. Given the linear equation $2x+3y-8=0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is :
(a) intersecting lines
(b) parallel lines
(c) coincident lines.

Ans : [NCERT]

Given, linear equation is $2x+3y-8=0$ ...(1)

(a) For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

To get its parallel line one of the possible equation may be taken as

$$5x+2y-9 = 0 \qquad (2)$$

(b) For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

One of the possible line parallel to equation (1) may be taken as

$$6x+9y+7 = 0$$

(c) For coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

To get its coincident line, one of the possible equation may be taken as

$$4x+6y-16 = 0$$

4. Solve the pair of equations graphically :
$4x-y=4$ and $3x+2y=14$

[Board Term-1, 2014, Set-A]

Ans :

We have $4x-y = 4$

or, $y = 4x-4$

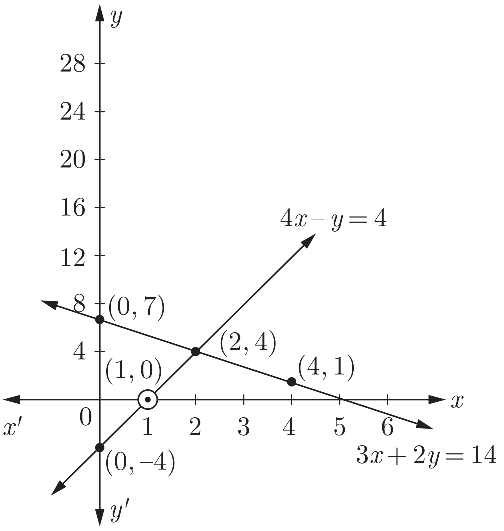
x	0	1	2
y	-4	0	4

and $3x+2y = 14$

or, $y \quad \frac{14-3x}{2}$

x	0	2	4
y	7	4	1

Plotting the above points and drawing lines joining them, we get the following graph. We get two obtained lines intersect each other at (2, 4).



Hence, $x=2$ and $y=4$.

5. Determine the values of m and n so that the following system of linear equation have infinite number of solutions :

$$(2m-1)x+3y-5 = 0$$
$$3x+(n-1)y-2 = 0$$

Ans : [Board Term-1, 2013, VKH6FFC; 2011, Set-66

We have $(2m-1)x+3y-5 = 0$ ...(1)

Here $a_1 = 2m-1, b_1 = 3, c_1 = -5$

$$3x+(n-1)y-2 = 0 \qquad ...(2)$$

Here $a_2 = 3, b_2 = (n-1), c_2 = -2$

For a pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2}$$

or $\quad 2(2m-1) = 15$ and $5(n-1) = 6$

Hence, $\quad m = \frac{17}{4}, n = \frac{11}{5}$

6. Find the values of α and β for which the following pair of linear equations has infinite number of solutions : $2x+3y=7; 2\alpha x+(\alpha+\beta)\ y=28$.

Ans : [Board Term-1, 2011, Set-25]

We have $2x+3y=7$ and $2\alpha x+(\alpha+\beta)y=28$.

For a pair of linear equations to be consistent and having infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{2\alpha} = \frac{3}{\alpha+\beta} = \frac{7}{28}$$

$$\frac{2}{2\alpha} = \frac{7}{28}$$

$$2\alpha \times 7 = 28 \times 2 \Rightarrow \alpha = 4$$

$$\frac{3}{\alpha+\beta} = \frac{7}{28}$$

$$7(\alpha+\beta) = 28\times 3$$

$$\alpha+\beta = 12$$

$$\beta = 12-\alpha = 12-4 = 8$$

Hence $\alpha = 4$, and $\beta = 8$

7. Represent the following pair of linear equations graphically and hence comment on the condition of consistency of this pair.

$$x-5y = 6, 2x-10y = 12.$$

Ans : [Board Term-1, 2011, Set-5]

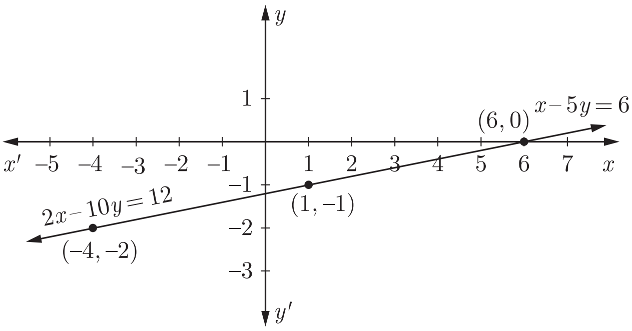
We have $x-5y = 6$ or $x = 5y+6$

x	6	1	-4
y	0	-1	-2

and $2x-10y = 12$ or $x = 5y+6$

x	6	1	-4
y	0	-1	-2

Plotting the above points and drawing lines joining them, we get the following graph.



Since the lines are coincident, so the system of linear equations is consistent with infinite many solutions.

8. For what value of p will the following system of equations have no solution ?

$$(2p-1)x+(p-1)y = 2p+1;\ y+3x-1 = 0$$

Ans : [Board Term-1, 2011, Set-28]

We have $(2p-1)x+(p-1)y-(2p+1) = 0$

Here $a_1 = 2p-1, b_1 = p-1$ and $c_1 = -(2p+1)$

Also $3x+y-1 = 0$

Here $a_2 = 3, b_2 = 1$ and $c_2 = -1$

The condition for no solution is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{2p-1}{3} = \frac{p-1}{1} \neq \frac{2p+1}{1}$$

From $\frac{2p-1}{3} = \frac{p-1}{1}$ we have

$$3p-3 = 2p-1$$

$$3p-2p = 3-1$$

$$p = 2$$

From $\frac{p-1}{1} \neq \frac{2p+1}{1}$ we have

$$p-1 \neq 2p+1 \text{ or } 2p-p \neq -1-1$$

$$p \neq -2$$

From $\frac{2p-1}{3} \neq \frac{2p+1}{1}$ we have

$$2p-1 \neq 6p+3$$

$$4p \neq -4$$

$$p \neq -1$$

Hence, system has no solution when $p = 2$

9. Find the value of k for which the following pair of equations has no solution :
$x+2y = 3, (k-1)x+(k+1)y = (k+2)$.

Ans : [Board Term-1, 2011, Set-52]

For $x+2y = 3$ or $x+2y-3 = 0$
$a_1 = 1, b_1 = 2, c_1 = -3$
For $(k-1)x+(k+1)y = (k+2)$
or $(k-1)x+(k+1)y-(k-2) = 0$
$a_2 = (k-1), b_2 = (k+1), c_2 = -(k+2)$

For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{1}{k-1} = \frac{2}{k+1} \neq \frac{3}{k+2}$$

From $\frac{1}{k-1} = \frac{2}{k+1}$ we have

$$k+1 = 2k-2$$

$$3 = k$$

Thus $k = 3$.

LONG ANSWER TYPE QUESTIONS

1. For Uttarakhand flood victims two sections A and B of class contributed Rs. 1,500. If the contribution of X-A was Rs. 100 less than that of X-B, find graphically the amounts contributed by both the sections.

Ans : [Board Term-1, 2016, Set-MV98HN3]

Let amount contributed by two sections X-A and X-B be Rs. x and Rs. y.

$$x+y = 1,500 \quad ...(1)$$

$$y-x = 100 \quad ...(2)$$

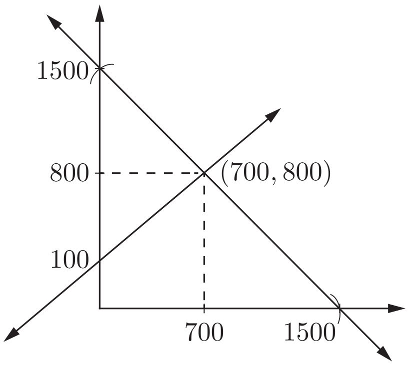
From (1) $y = 15000-x$

x	0	700	1,500
y	1,500	800	0

From (2) $y = 100+x$

x	0	700
y	100	800

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point (700, 800) Hence X-A contributes 700 Rs and X-B contributes 800 Rs.

2. Determine graphically whether the following pair of linear equations :

$$\begin{aligned} 3x - y &= 7 \\ 2x + 5y + 1 &= 0 \text{ has :} \end{aligned}$$

(1) a unique solution

(b) infinitely many solutions or

(c) no solution.

Ans : [Board Term-1, 2015, Set-DDE-E]

We have $3x - y = 7$

or $3x - y - 7 = 0$ (1)

Here $a_1 = 3, b_1 = 1, c_1 = -7$

$2x + 5y + 1 = 0$ (2)

Here $a_2 = 2, b_2 = 5, c_2 = 1$

Now $\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{-1}{5}$

Since $\frac{3}{2} \neq \frac{-1}{5}$, thus $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, given pair of linear equations has a unique solution.

Now line (1) $y = 3x - 7$

x	0	2	3
y	-7	-1	2

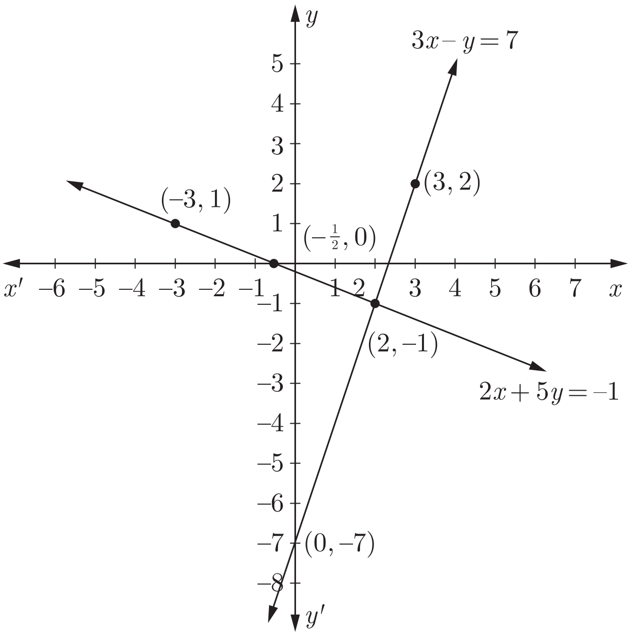
and line (2)

$$2x + 5y + 1 = 0$$

or, $y = \frac{-1 - 2x}{5}$

x	2	-3
y	-1	1

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $(2, -1)$. Hence $x = 2$ and $y = -1$

3. Draw the graphs of the pair of linear equations : $x + 2y = 5$ and $2x - 3y = -4$

Also find the points where the lines meet the $x-$axis.

Ans : [Board Term-1, 2015, Set-FHNQMGD]

We have $x + 2y = 5$

or, $y = \frac{5 - x}{2}$

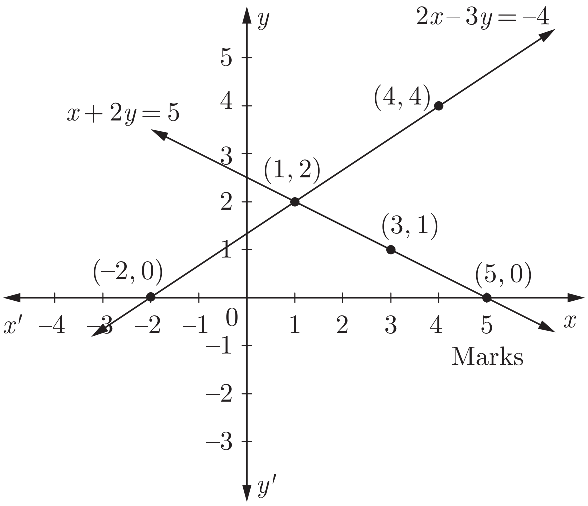
x	1	3	5
y	2	1	0

and $2x - 3y = -4$

or, $y = \frac{2x + 4}{3}$

x	1	4	-2
y	2	4	0

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly two lines meet x-axis at $(5, 0)$ and (-20)

respectively.

4. Solve graphically the pair of linear equations : $3x-4y+3=0$ and $3x+4y-21=0$
Find the co-ordinates of the vertices of the triangular region formed by these lines and x-axis. Also, calculate the area of this triangle.

Ans : [Board Term-1, 2015, Set-DDE-E]

We have $3x-4y+3 = 0$

or, $y = \dfrac{3x+3}{4}$

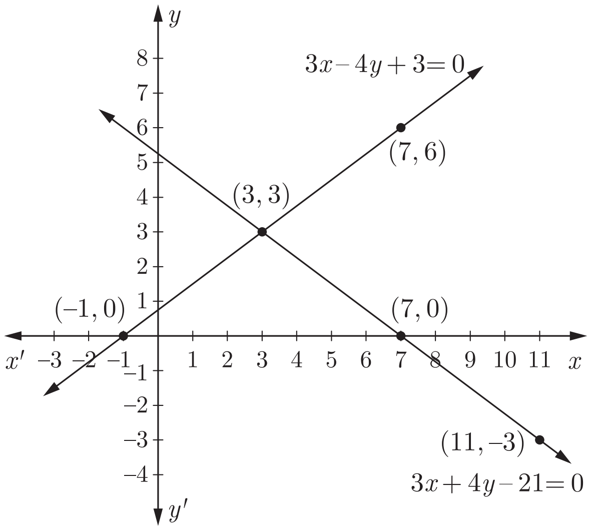
x	3	7	-1
y	3	6	0

and $3x+4y-21 = 0$

or, $y = \dfrac{21-3x}{4}$

x	3	7	11
y	3	0	-3

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $(3,3)$.

(a) These lines intersect each other at point $(3,3)$. Hence $x=3$ and $y=3$

(b) The vertices of triangular region are $(3,3)$, $(-1,0)$ and $(7,0)$.

(c) Area of $\Delta = \frac{1}{2}\times 8\times 3 = 12$

Hence, Area of obtained Δ is 12 sq unit.

5. Aftab tells his daughter, '7 years ago, I was seven times as old as you were then. Also, 3 years from now, I shall be three times as old as you will be.' Represent this situation algebraically and graphically.

Ans : [NCERT]

Let the present age of Aftab be x years and the age of daughter be y years.

7 years ago father's(Aftab) age $= (x-7)$years

7 years ago daughter's age $= (y-7)$ years

According to the question,

$$(x-7) = 7(y-7)$$

or, $$(x-7y) = -42 \qquad (1)$$

After 3 years father's(Aftab) age $= (x+3)$ years

After 3 years daughter's age $= (y+3)$ years

According to the condition,

$$x+3 = 3(y+3)$$

or, $$x-3y = 6 \qquad (2)$$

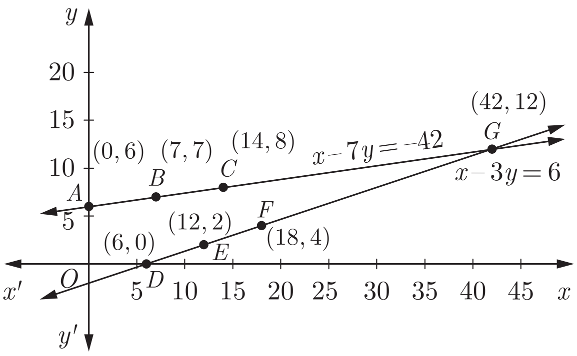
From equation(1) $x-7y = -42$

x	0	7	14
$y = \dfrac{x+42}{7}$	3	0	-3

From equation (2) $x-3y = 6$

x	6	12	18
$y = \dfrac{x-6}{3}$	0	2	4

Plotting the above points and drawing lines joining them, we get the following graph.



Two lines obtained intersect each other at $(42,12)$

Hence, father's age = 42 years

and daughter's age = 12 years

6. The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs. 160. After a month, the cost of 4kg of apples and 2kg of grapes is Rs. 300. Represent the situations algebraically and geometrically.

Ans : [NCERT]

Let the cost of 1 kg of apples be Rs.x and cost of 1 kg of grapes be Rs. y.

The given conditions can be represented given by the following equations :

$$2x+y = 160 \qquad ...(1)$$

$$4x+2y = 300 \qquad ...(2)$$

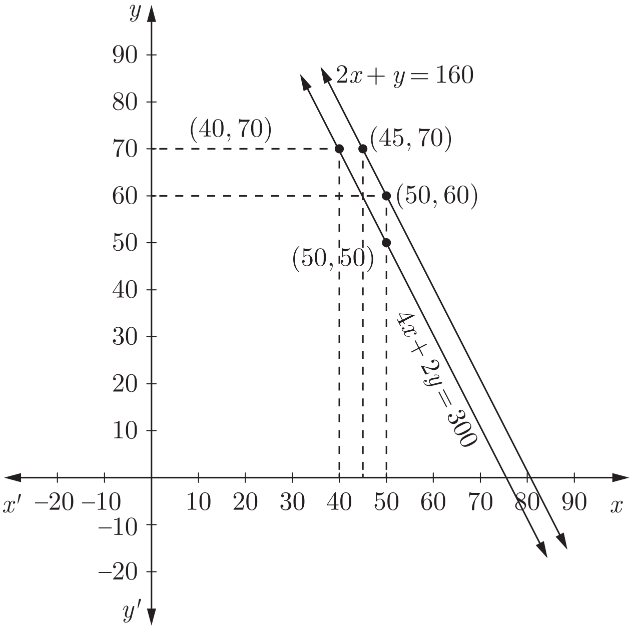
From equation (1) $y = 160-2x$

x	50	45
y	60	70

From equation (2) $y = 150-2x$

x	50	40
y	50	70

Plotting these points on graph, we get two parallel line as shown below.



7. Form the pair of linear equations in the following problems find their solutions graphically :

(a) 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(b) 5 pencils and 7 pens together cost Rs. 50 whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.

Ans : [NCERT]

(a) Let the number of boys be x and the number of girls be y.

The given conditions can be represented given by the following equations :

$$x + y = 10 \qquad \text{...(1)}$$

and
$$y - x = 4 \qquad \text{...(2)}$$

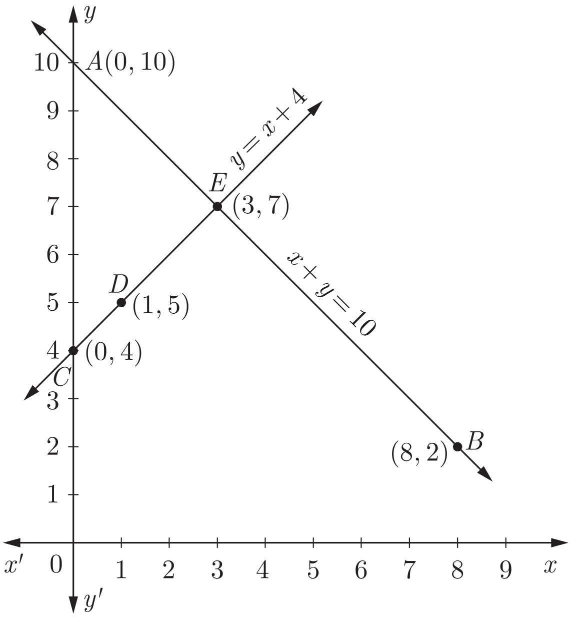
From equation (2) $y = 10 - x$

x	0	8	3
$y = 10 - x$	10	2	7

From equation (2) $y = x + 4$

x	0	1	3
$y = x + 4$	4	5	7

Plotting co-ordinates on graph paper, two lines intersect each other at point $E(3,7)$. So, $x = 3$ and $y = 7$.



Hence, number of boys is 3 and number of girls is 7.

(b) Let the cost of one pencil be Rs. x and one pen be Rs. y.

Then, the equations formed are

$$5x + 7y = 50 \qquad \text{...(1)}$$

and
$$7x + 5y = 46 \qquad \text{...(2)}$$

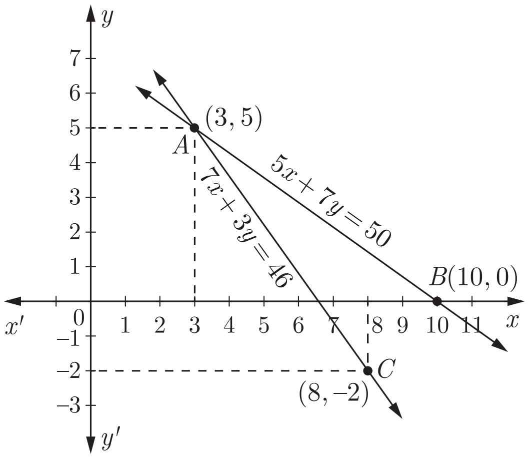
From equation (1) $y = \dfrac{50 - 5x}{7}$

x	10	3
y	0	5

From equation (2) $y = \dfrac{46 - 7x}{5}$

x	8	3
y	−2	5

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $A(3,5)$., Hence $x=3$ and $y=5$ is the required solution. Cost of one pencil is Rs. 3 and cost of one pen is Rs.5.

8. Draw the graphs of the equations $x-y+1=0$ and $3x+2y-12=0$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the X-axis and shade the triangular region.

Ans : [NCERT]

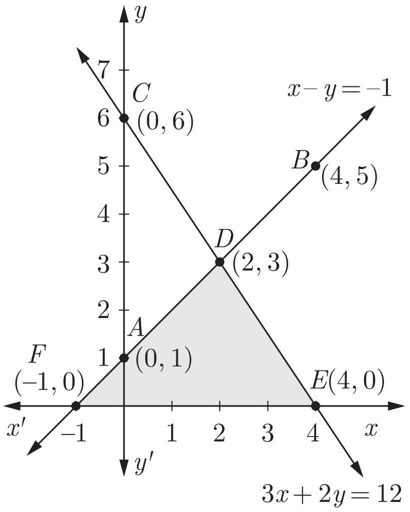
We have $x-y+1 \;=0$ ...(1)

x	0	4	2
$y=x+1$	1	5	3

and $3x+2y-12 \;=0$ ...(2)

x	0	2	4
$y=\dfrac{12-3x}{2}$	6	3	0

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $D(2,3)$. Hence, $x=2$ and $y=3$ is the solution of the given pair of equations. The line CD intersects the x-axis at the point $E(4,0)$ and the line AB intersects the x-axis at the points $F(-1,0)$. Hence, the co-ordinates of the vertices of the triangle are $D(2,3)$, $E(4,0)$ and $F(-1,0)$.

9. Solve the following pair of linear equations graphically: $2x+3y\;=12$ and $x-y=1$
Find the area of the region bounded by the two lines representing the above equations and y-axis.

Ans : [Board Term-1, 2012, Set-58]

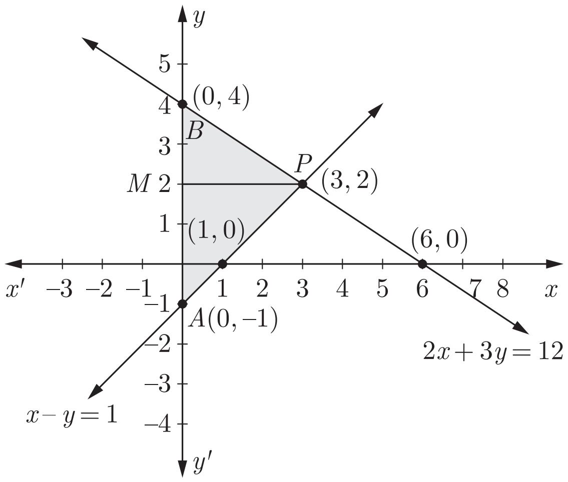
We have $2x+3y \;=12 \Rightarrow y \;=\dfrac{12-2x}{3}$

x	0	6	3
y	4	0	2

We have $x-y \;=1 \Rightarrow y \;=x-1$

x	0	1	3
y	1	0	2

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $p(3,2)$.
Hence, $x=3$ and $y=2$
Area of shaded triangle region,

$$\text{Area of } \Delta PAB \;= \frac{1}{2} \times \textit{base} \times \textit{height}$$

$$= \frac{1}{2} \times AB \times PM$$

$$= \frac{1}{2} \times 5 \times 3$$

$$= 7.5 \text{ square unit.}$$

10. Solve the following pair of linear equations graphically: $x+3y=12$, $2x-3y=12$
Also shade the region bounded by the line $2x-3y=2$ and both the co-ordinate axes.

Ans : [Board Term-1, 2013 FFC, 2012, Set-35, 48]

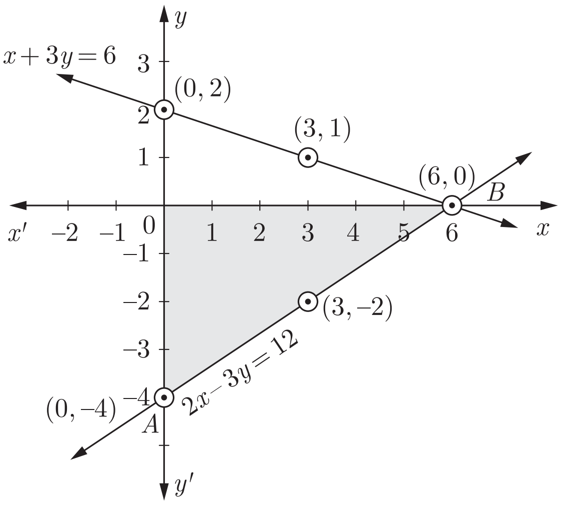
We have $x+3y \;=6 \Rightarrow y=\dfrac{6-x}{3}$ ...(1)

x	3	6	0
y	1	0	2

and $2x-3y \;=12 \Rightarrow y \;=\dfrac{23-12}{3}$

x	0	6	3
y	-4	0	-2

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $B(6,0)$. Hence, $x=6$ and $y=0$

Again ΔOAB is the region bounded by the line $2x-3y=12$ and both the co-ordinate ases. 1

11. Solve the following pair of linear equations graphically: $x-y=1,\ 2x+y=8$

Also find the co-ordinates of the points where the lines represented by the above equation intersect $y-axis$.

Ans : [Board Term-1, 2012, Set-56]

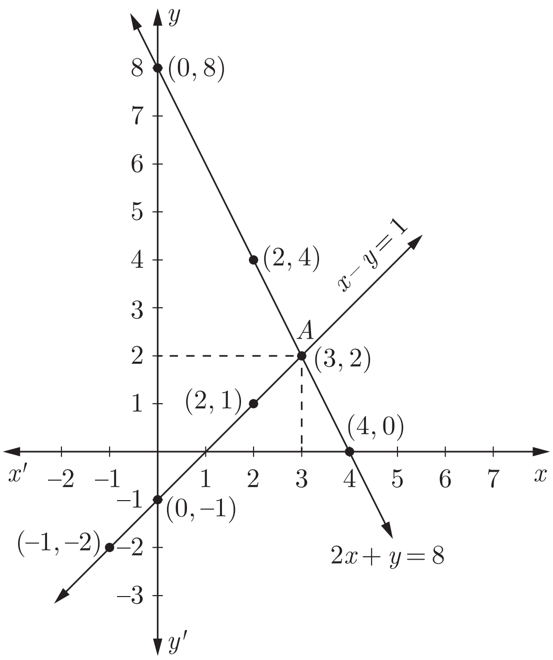
We have $x-y=1 \Rightarrow y=x-1$

x	2	3	-1
y	1	2	-2

and $2x+y=8 \Rightarrow y=8-2x$

x	2	4	0
y	4	0	8

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $A(3,2)$. Thus solution of given equations is $x=3, y=2$.

Again, $x-y=1$ intersects $y-axis$ at $(0,-1)$

and $2x+y=8$ $y-axis$ at $(0,8)$.

12. Draw the graph of the following equations:

$$2x-y=1,\ x+2y=13$$

Find the solution of the equations from the graph and shade the triangular region formed by the lines and the y-axis.

Ans : [Board Term-1, 2012 Set-52]

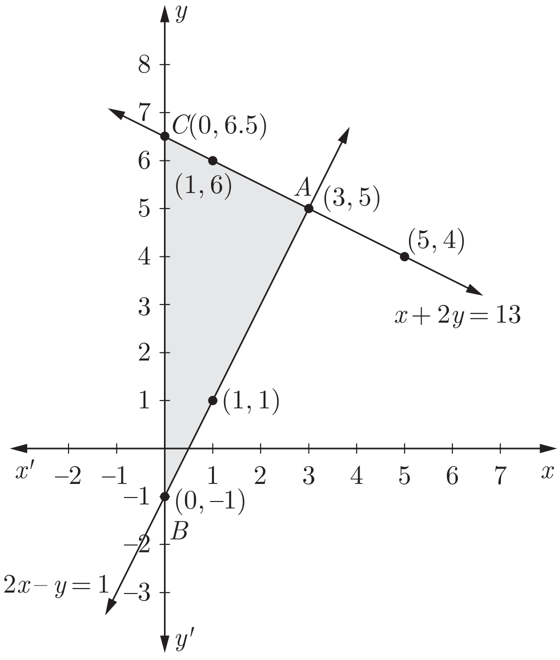
We have $2x-y=1 \Rightarrow y=2x-1$

x	0	1	3
y	-1	1	5

and $x+2y=13 \Rightarrow y=\dfrac{13-x}{2}$

x	1	3	5
y	6	5	4

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly two obtained lines intersect at point $A(3,5)$.

Hence, $x=3$ and $y=5$

ABC is the triangular shaded region formed by the obtained lines with the y-axis. 1

13. Solve the following pair of equations graphically:

$$2x+3y=12,\ x-y-1=0.$$

Shade the region between the two lines represented by the above equations and the X-axis.

Ans : [Board Term-1, 2012, Set-48]

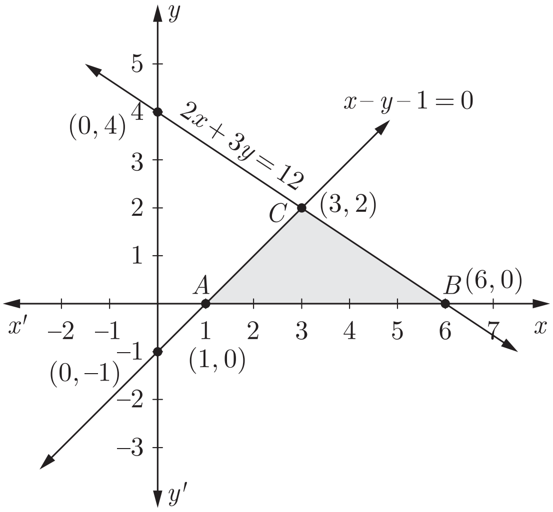
We have $2x+3y=12 \Rightarrow y=\dfrac{12-2x}{3}$

x	0	6	3
y	4	0	2

also $x-y=1 \Rightarrow y=x-1$

x	0	1	3
y	-1	0	2

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $(3,2)$, Hence, $x=3$ and $y=2$.
ΔABC is the region between the two lines represented by the given equations and the X-axis.

SHORT ANSWER TYPE QUESTIONS - I

1. Solve the following pair of linear equations by cross multiplication method:

$$x+2y=2$$
$$x-3y=7$$

Ans : [Board Term-1, 2016, Set-O4YP6G7]

We have $x+2y-2=0$

$$x-3y-7=0$$

Using the formula

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

we have $\dfrac{x}{-14-6}=\dfrac{y}{-2+7}=\dfrac{1}{-3-2}$

$$\frac{x}{-20}=\frac{y}{5}=\frac{-1}{5}$$

$$\frac{x}{-20}=\frac{-1}{5} \Rightarrow x=4$$

$$\frac{y}{5}=\frac{-1}{5} \Rightarrow y=-1$$

2. Solve the following pair of linear equations by substitution method:

$$3x+2y-7=0$$
$$4x+y-6=0$$

Ans : [Board Term-1, 2015, CJTOQ]

We have $3x+2y-7=0$ ...(1)

$4x+y-6=0$ ...(2)

From equation (2), $y\ 6-4x$ ...(3)

Putting this value of y in equation (1) we have

$$3x+2(6-4x)-7=0$$
$$3x+12-8x-7=0$$
$$5-5x=0$$
$$5x=5$$

Thus $x=1$

Substituting this value of x in (2), we obtain,

$$y=6-4\times 1=2$$

Hence, values of x and y are 1 and 2 respectively.

3. Solve the following pairs of linear equations by the substitution method:

(a) $x+y=14$

$x-y=4$

(b) $s-t=3$

$\dfrac{s}{3}+\dfrac{t}{2}=6$

(c) $3x-y=3$

$9x-3y=9$

(d) $0.2x+0.3y=2.3$

$0.4x+0.5y=2.3$

(e) $\sqrt{2x}+\sqrt{3y}=0$

$\sqrt{3x}-\sqrt{8y}=0$

(f) $\dfrac{3x}{2}-\dfrac{5y}{3}=-2$

$\dfrac{x}{3}+\dfrac{y}{2}=\dfrac{13}{6}$

Ans : [NCERT]

Given,

(a) We have $x+y=14$ ...(1)

and $x-y=4$ ...(2)

or, $y=x-4$ ...(3)

Substituting the value of y from equation (3) in equation (1), we get

$$x+x-4=14$$
$$2x=18$$

Thus $x=9$

Substituting this value of x in equation (3), we get

$$y=9-4$$

i.e., $y=5$

Hence, $s=9$ and $y=5$

(b) We have, $s-t=3$ ...(1)

and $\dfrac{s}{3}+\dfrac{t}{2}=6$ ...(2)

From equation (1), $x=t+3$ ...(3)

Substituting $x=t+3$ in equation (2), we get

$$\frac{t+3}{3}+\frac{t}{2}=6$$

$$2(t+3)+3t = 36$$
$$5t+6 = 36$$
$$5t = 30$$
$$t = 6$$

Substituting this value of t in (1) we get

$$s = 6+3 = 9$$

Hence, $s = 9, t = 6$

(c) We have $3x-y = 3$ ...(1)

and $9x-3y = 9$ ...(2)

From equation (1) $y = 3x-3$ ...(3)

Substituting this value of y in equation (3),

$$9x-3(3x-3) = 9$$
$$9 = 9$$

Hence, x and y both have infinitely many solutions

(d)We have $0.2x+0.3y = 1.3$ ...(1)

and $0.4x+0.5y = 2.3$ ...(2)

From equation (1) we have

$$3y = 13-2x$$
$$y = \frac{13-2x}{3} \quad ...(3)$$

Substituting this value of y in equation (2),

$$\frac{4}{10}x+\frac{5}{10}\times\frac{(13-2x)}{3} = \frac{23}{10}$$
$$4x+\frac{5}{3}(13-2x) = 23$$
$$12x+5(13-2x) = 3\times 23$$
$$12x+65-10x = 69$$
$$2x = 69-65 = 4$$
$$x = 2$$

Substituting $x = 2$ in equation (3), we get

$$y = \frac{13-2\times 2}{3} = \frac{9}{3}$$
$$y = 3$$

Hence, $x = 2,\ y = 3$

(d) We have $\sqrt{2}\,x+\sqrt{3}\,y = 0$ ...(1)

and $\sqrt{3}\,x-\sqrt{8}\,y = 0$ ...(2)

or $y = \frac{\sqrt{3}\,x}{\sqrt{8}}$ ...(3)

Substituting y from equation (3) in equation (1),

$$\sqrt{2}\,x+\sqrt{3}\times\left[\frac{\sqrt{3}\,x}{\sqrt{8}}\right] = 0$$
$$\sqrt{2}\,x+\frac{3x}{\sqrt{8}} = 0$$
$$\sqrt{2}\,x\times\sqrt{8}+3x = 0$$
$$\sqrt{16x}+3x = 0$$
$$4x+3x = 0$$
$$7x = 0$$

Thus $x = 0$

Substituting $x = 0$ in equation (3), we have

$$y = \frac{\sqrt{3}\times 0}{\sqrt{8}} = 0$$
$$y = 0$$

Hence, $x = 0,\ y = 0$

(e) We have $\frac{3x}{2}-\frac{5y}{3} = -2$ ...(1)

and $\frac{x}{3}+\frac{y}{3} = \frac{13}{6}$ ...(2)

From equation (2), $\frac{y}{52} = \frac{13}{6}-\frac{x}{3} = \frac{13-2x}{6}$

$$y = 2\times\frac{(13-2x)}{6} = \frac{(13-2x)}{3} \quad (3)$$

Substituting this value of y in equation (1),

$$\frac{3x}{2}-\frac{5}{3}\times\frac{(13-2x)}{3} = -2$$
$$\frac{3x}{2}-\frac{5}{9}(13-2x) = -2$$
$$27x-10(13-2x) = -36$$
$$27x-130+20x = -36$$
$$27x+20x = 130-36$$
$$47x = 94$$
$$x = 2$$

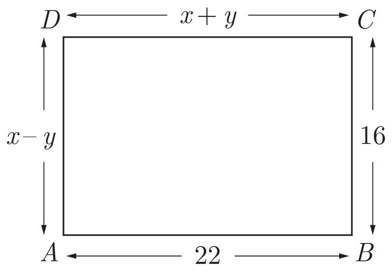
Substituting $x = 2$ in equation (3), we have

$$y = \frac{13-2\times 2}{3} = \frac{9}{3}$$
$$y = 3$$

Hence, $x = 2, y = 3$

4. In the figure given below, $ABCD$ is a rectangle. Find the values of x and y.

Ans : [Board Term-1, 2012, Set-30]



From given figure we have

$$x+y = 22 \quad ...(1)$$

and $x-y = 16$ ...(2)

Adding (1) and (2), we have

$$2x = 38$$
$$x = 19$$

Substituting the value of x in equation (1), we get

$$19+y = 22$$
$$y = 22-19 = 3$$

Hence, $x = 19$ and $y = 3$.

5. Solve : $99x+101y = 499, \quad 101x+99y = 501$

Ans : [Board Term-1, 2012, Set-55]

We have
$$99x+101y = 499 \qquad ...(1)$$
$$101x+99y = 501 \qquad ...(2)$$

Adding equation (1) and (3), we have
$$200x+200y = 1000$$
or,
$$x+y = 5 \qquad ...(3)$$

Subtracting equation (2) from equation (3), we get
$$-2x+2y = -2$$
or,
$$x-y = 1 \qquad ...(4)$$

Adding equations (3) and (4), we have
$$2x = 6$$
$$x = 3$$

Substituting the value of x in equation (3), we get
$$3+x = 5$$
$$y = 2$$

6. Solve the following system of linear equations by substitution method:
$$2x-y = 2$$
$$x+3y = 15$$

Ans : [Board Term-1, 2012, Set-50]

We have
$$2x-y = 2 \qquad ...(1)$$
$$x+3y = 15 \qquad ...(2)$$

From equation (1), we get $y = 2x-2$ ...(3)

Substituting the value of y in equation (2),
$$x+6x-6 = 15$$
or,
$$7x = 21$$
$$x = 3$$

Substituting this value of x in (3), we get

From equation (1), we have
$$y = 2\times 3-2 = 4$$
$$x = 3 \text{ and } y = 4$$

7. Find the value(s) of k for which the pair of Linear equations $kx+y = d^2$ and $x+ky = 1$ have infinitely many solutions.

Ans : [Sample Paper 2017]

We have
$$kx+y = k^2$$
and
$$x+ky = 1$$
$$\frac{a_1}{a_2} = \frac{k}{1}, \frac{b_1}{b_2} = \frac{1}{k}, \frac{c_1}{c_2} = \frac{k^2}{1}$$

For infinitely many solution
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\frac{k}{1} = \frac{1}{k} = \frac{k^2}{1} = k^2 = 1$$
$$k = \pm 1$$

SHORT ANSWER TYPE QUESTIONS - II

1. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their respective ages.

Ans : [CBSE Marking Scheme, 2015]

Let age of father and son be x and y respectively.
$$x = y = 40 \qquad ...(1)$$
$$x = 3y \qquad ...(2)$$

By solving equations (1) and (2), we get
$$x = 30 \text{ and } y = 10$$

Ages are 30 years and 10 years.

2. Solve using cross multiplication method:
$$5x+4y-4 = 0$$
$$x-12y-20 = 0$$

Ans : Board Term-1, 2015, Set-FHN8MGD]

We have
$$5x+4y-4 = 0 \qquad ...(1)$$
$$x-12y-20 = 0 \qquad ...(2)$$

By cross-multiplication method,
$$\frac{x}{b_2c_1-b_1c_2} = \frac{y}{c_1a_2-c_2a_1} = \frac{1}{b_1b_2-a_2b_1}$$
$$\frac{x}{-80-48} = \frac{y}{-4+100} = \frac{1}{-60-4}$$
$$\frac{x}{-128} = \frac{y}{96} = \frac{1}{64}$$
$$\frac{x}{-128} = \frac{1}{-64} \Rightarrow x = 2$$
and
$$\frac{y}{96} = \frac{1}{-64} \Rightarrow y = \frac{-3}{2}$$

Hence, $x = 2$ and $y = \frac{-3}{2}$

3. A part of monthly hostel charge is fixed and the remaining depends on the number of days on e has taken food in the mess. When Swati takes food for 20 days, she has to pay Rs. 3,000 as hostel charges whereas Mansi who takes food for 25 days Rs. 3,500 as hostel charges. Find the fixed charges and the cost of food per day.

Ans : [Term-1, 2016, MV98HN3, Term-1, 2015, FHN8MGD]

Ans :

Let fixed charge be x and per day food cost be y
$$x+20y = 3000 \qquad ...(1)$$
$$x+25y = 3500 \qquad ...(2)$$

Subtracting (1) from (2) we have
$$5y = 500 \Rightarrow y = 100$$

Substituting this value of y in (1), we get
$$x+20(100) = 3000$$
$$x = 1000$$

Thus $x = 1000$ and $y = 100$

Fixed charge and cost of food per day are Rs. 1,000 and Rs. 100.

4. Solve for x and y :

$$\frac{x}{2}+\frac{2y}{3} = -1$$

$$x-\frac{y}{3} = 3$$

Ans : [Board Term-1, 2015, CJTOQ, NCERT]

We have $\frac{x}{2}+\frac{2y}{3} = -1$

or $3x+4y = -6$ ...(1)

and $\frac{x}{1}-\frac{y}{3} = 3$

or $3x+y = 9$ ...(2)

Subtracting equation (2) from equation (1), we have

$$5y = -15 \Rightarrow y = -1$$

Substituting $y = -3$ in eq (1), we get

$$3x+4(-3) = -6$$

$$3x-12 = -6$$

$$3x = 12-6$$

Thus $x = 2$

Hence $x = -2$ and $y = -3$.

5. From the pair of linear equations for the following problems and find their solution by substitution method:

(a) The difference between two numbers is 26 and one number is three times the other. Find the numbers.

(b) The larger of two supplementary angles exceeds the smaller by 18 degree. Find the angles.

(c) The coach of cricket team buys 7 bats and 6 balls for Rs. 3,800. Later, she buys 3 bats and 5 balls for Rs. 1750. Find the cost of each bat and each ball.

(d) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10km, the charge paid is Rs. 105 and for a journey of 15 km the charge paid is Rs. 155. What are the fixed charges and the charge per km?

How much does a person have to pay for travelling a distance of 25 km?

(e) A fraction becomes $\frac{9}{11}$ if 2 is added to both the numerator and the denominator. If 3 is added to bothe the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

(f) Five year hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Ans : [NCERT]

(a) Let the two number be x and $y(x > y)$

We have $x-y = 26$ ...(1)

and $x = 3y$ ...(2)

Substituting the value x from equation (2) in equation (1), we get

$$3y-y = 26$$

$$2y = 26$$

Thus $y = 13$

Substituting $y = 13$ in equation (2), we get

$$x = 3\times 13 = 39$$

Hence, the two number are 39 and 13.

(b) Let the supplementary angles be x and $y(x > y)$

Now, $x+y = 180°$ ...(1)

and $x-y = 18°$ ...(2)

From equation (2) we have

, $y = x-18°$ ...(3)

Substituting the value y from equation (3) in equation (1),

$$x+x-18° = 180°$$

$$2x = 198°$$

$$x = 99°$$

Substituting $x = 99°$ in equation (3) in equation(1),

$$y = 99°-18° = 81°$$

Thus $y = 81°$

Hence, the angles are 99° and 81°

(c) Let, cost one bat and a ball be Rs. x and Rs. y respectively

$7x+6y = 3{,}800$ ...(1)

and $3x+5y = 1750$ ...(2)

From equation (2), we have

$$5y = 1750-3x$$

$y = \frac{1750-3x}{5}$ ...(3)

Substituting y from equation (3) in equation (1),

$$7x+6\times\frac{1750-3x}{5} = 3{,}800$$

$$35x+6\times(1750-3x) = 5\times 3{,}800$$

$$35x+10500-18x = 19000$$

$$35-18x = 19000-10500$$

$$17x = 8{,}500$$

Thus $x = 500$

Substituting $x = 500$ in equation (3) we have,

$$y = \frac{1750-3\times 500}{5}$$

$$= \frac{1750-1500}{5} = \frac{250}{5} = 50$$

Thus $y = 50$

Hence, cost of one bat $= 500$ Rs

and cost of one ball $= 50$ Rs

(d) Let fixed charge be Rs. x and charge per km be Rs. y.

$x+10y = 105$ ...(1)

$x+15y = 155$ ...(2)

From equation (1) we have

$x = 105-10Y$ ...(3)

Substituting x from equation (3) in equation (2),

$$\begin{aligned} 105-10y+15y &= 155 \\ 5y &= 155-105=50 \\ y &= 10 \end{aligned}$$

Substituting $= 10y$ in equation (3),

$$\begin{aligned} x &= 105-10\times 10 \\ &= 105-100=5 \end{aligned}$$

Thus $x = 5$

$$\begin{aligned} \text{Hence, fixed charges} &= Rs.5 \\ \text{Rate per km} &= Rs.10 \end{aligned}$$

Amount to be paid for travelling 25km

$$\begin{aligned} &= 5+10\times 25 \\ &= 5+250=255 \text{ Rs} \end{aligned}$$

(e) Let $\frac{x}{y}$ be the fraction, where x and y are positive integers.

We have $\dfrac{x+2}{y+2}=\dfrac{9}{11}$ and $\dfrac{x+3}{y+3}=\dfrac{5}{6}$

From 1st equation we have

$$\begin{aligned} 11\times(x+2) &= 9\times(y+2) \\ 11x+22 &= 9y+18 \\ 11-9y+4 &= 0 \end{aligned}$$

From 2nd equation we get

$$\begin{aligned} 6\times(x+3) &= 5\times(y+3) \\ 6x+18 &= 5y+15 \\ 6x-5y+3 &= 0 \\ 11x-9y+4 &= 0 \qquad \ldots(1) \\ \text{and} \quad 6x-5y+3 &= 0 \qquad \ldots(2) \end{aligned}$$

From equation (2),

$$\begin{aligned} 5y &= 6x+3 \\ \text{or,} \quad y &= \frac{6x+3}{5} \qquad \ldots(3) \end{aligned}$$

Substituting y from equation (3) in equation (1),

$$\begin{aligned} 11x-9\times\left(\frac{6x+3}{5}\right)+4 &= 0 \\ 55x-9\times(6x+3)+20 &= 0 \end{aligned}$$

Thus $x = 7$

Substituting $x=7$ in equation (3),

$$\begin{aligned} y &= \frac{6\times 7+3}{5} \\ y &= 9 \end{aligned}$$

Hence, the required fraction$=\frac{7}{9}$

(f) Let x (in years) be the present age of Jacob's son and y (in years) be the present age of Jacob. 5years hence, it has relation :

$$\begin{aligned} (y+5) &= 3(x+5) \\ y+5 &= 3x+15 \\ 3x-y+10 &= 0 \qquad \ldots(1) \end{aligned}$$

5 years ago, it has relation,

$$\begin{aligned} (y-5) &= 7(x-5) \\ 7x-y-30 &= 0 \qquad \ldots(2) \end{aligned}$$

From equation (1), we have

$$y = 3x+10 \qquad \ldots(3)$$

Substituting the value of y in equation (2) we get

$$\begin{aligned} 7x-(3x+10)-30 &= 0 \\ 4x-40 &= 0 \\ x &= 10 \end{aligned}$$

Substituting $x=10$ in equation (3), we get

$$\begin{aligned} y &= 3\times 10+10 \\ \text{Thus} \quad y &= 40 \end{aligned}$$

Hence, the present age of Jacob = 40 years and son's age = 10 years

6. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(a) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. it becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction ?

(b) Five years ago, Nuri was thrice as old as Sonu. Ten years leter, Juri will bw twice as old as Sonu. How old are Nuri and sonu ?

(c) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(d) Meena went to a bank to withdraw Rs. 2,000. She asked the cashier to give her Rs. 50 and Rs. 100 notes only. Meena got 25 notes in all. Find how many notes of Rs. 50 and Rs 100 she received.

(e) A lending library has fixed charge for the first three days and as additional charge for each day thereafter. Saritha paid Rs. 27 for a book kept for seven days, while susy paid Rs. 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Ans : [NCERT]

(a) Let the fraction be $\frac{x}{y}$

According to the given conditions we have

$$\begin{aligned} \frac{x+1}{y-1} &= 1 \text{ and } \frac{x}{y+1}=\frac{1}{2} \\ x+1 &= y-1 \text{ and } 2x=y+1 \\ \text{or,} \quad x-y &= -2 \qquad \ldots(1) \\ \text{and} \quad 2x-y &= 1 \qquad \ldots(2) \end{aligned}$$

Subtracting equation (1) from equation (2), we have

$$\begin{aligned} (2x-y)-(x-y) &= 1+2 \\ x &= 3 \end{aligned}$$

Substituting $x=3$ in equation (1), we have

$$\begin{aligned} 3-y &= -2 \\ y &= 5 \end{aligned}$$

Hence, the fraction is $\frac{3}{5}$.

(b) Let the present age of Nuri be x and Present age of Sonu be y.

According to the given conditions,

5 years ago, $\quad x-5 = 3(y-5)$

$$x - 3y = -10 \qquad ...(1)$$

10 year later, $x + 10 = 2(y + 10)$

$$x - 2y = 10 \qquad ...(2)$$

Subtracting equation (1) from equation (2), we have

$$(x - 2y)(x - 3y) = 10 + 10$$

or, $-2y + 3y = 20$

$$y = 20$$

Substituting $y = 20$ in equation (2), we have

$$x = 2 \times 20 + 10 = 50$$

Thus $x = 50$

Hence, Present age of Nuri is 50 years and present age of Sonu is 20 years.

(c) Let x be the digit at unit's place and y be the digit at ten's place.

According to the given condition

$$x + y = 9 \qquad ...(1)$$

Value of the number $= x + 10y$

When the order of the digits is reversed, the value of the new number $= y + 10x$

We have,

$$9 \times (x + 10y) = 2 \times (y + 10x)$$

$$9x + 90y = 2y + 20x$$

$$88y = 11x$$

$$x = 8y \qquad ...(2)$$

Substituting this value of x in equation (1), we have

$$8y + y = 9$$

$$9y = 9$$

$$y = \frac{9}{9} = 1$$

and $x = 8 + 1 = 9$

Hence, the number.

$$10y + x = 10 \times 1 + 8 = 18$$

(d) let number of Rs. 50 notes be x

Number of Rs. 100 notes be y

According to the given condition,

$$x + y = 25$$

and $50 \times x + 100 \times y = 2,000$

or, $x + 2y = 40 \qquad ...(2)$

Substituting equation (1) from equation (2),

$$(x + 2y) - (x + y) = 40 - 25$$

$$y = 15$$

Substituting $y = 15$ in equation (1),

$$x + 15 = 25$$

$$x = 10$$

Hence number of Rs.50 notes = 10

and number of Rs. 100 notes= 15

(e) Let the fixed charges for the first three days be Rs. x. Let the additional change per day be Rs. y.

According to the given conditions,

Sarita paid for 7 days = Rs.27

$$x + 4 \times y = 27$$

$$x + 4y = 27 \qquad ...(1)$$

[∴ Rs. $4y$ are to be paid for extra 4 days]

In the case of Susy,

$$x + 2y = 21 \qquad ...(2)$$

[∵ Rs. $2y$ are to be paid for extra 2 days]

Subtracting equation (2) from equation (1),

$$2y = 27 - 21$$

or, $2y = 6$

Thus $y = 3$

Substituting this value of y in equation (1),

$$x + 4 \times 3 = 27$$

$$x = 27 - 12$$

$$x = 15$$

Hence, fixed charges for first three dauys = Rs 15

Additional charges per extra day = Rs 3. 1

7. Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions ? In case there is a unique solution, find it by using cross-multiplication method.

(a) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

(b) $2x + y = 5$

$3x + 2y = 8$

(c) $3x - 5y = 20$

$6x - 10y = 40$

(d) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

Ans : [NCERT]

(a) We have $x - 3y - 3 = 0 \qquad ...(1)$

and $3x - 3y - 2 = 0 \qquad ...(2)$

Comparing equation (1) and (2) with $ax + by + c = 0$

$$a_1 = 1, b_1 = -3, c_1 = -3$$

and $a_2 = 3, b_2 = -9, c_2 = -2$

Thus $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{3}{2}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, no solution exists.

(b) We have $2x + y = 5$

or, $2x + y - 5 = 0 \qquad ...(1)$

and $3x + 2y = 8$

or, $3x + 2y - 8 = 0 \qquad (2)$

Comparing equation (1) and (2) with $ax + by + c = 0$,

$$a_1 = 2, b_1 = 1, c_1 = -5$$

and $a_2 = 3, b_2 = 2, c_2 = -8$

Now, $\frac{a_1}{a_2} = \frac{2}{3}$ and $\frac{b_1}{b_2} = \frac{1}{2}$

Since $\frac{2}{3} \neq \frac{1}{2}$ $\quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, a unique solution exists.

By cross multiplication method,

$$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{\{(1)(-8)-(2)(-5)\}} = \frac{x}{\{(-5)(3)-(-8)(2)\}}$$

$$= \frac{1}{\{(2)(2)-(3)(1)\}}$$

$$\frac{x}{(-8+10)} = \frac{y}{(-15+16)} = \frac{1}{(4-3)}$$

$$\frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

Thus $\frac{x}{2} = \frac{1}{1}$ and $\frac{y}{1} = \frac{1}{1}$

Hence $x = 2$ and $y = 1$ 1

(c) We have $3x - 5y = 20$

or, $3x - 5y - 20 = 0$ ...(1)

and $6x - 10y = 40$

or, $6x - 10y - 40 = 0$ ...(2)

Comparing equation (1) and (2) with $ax + by + c = 0$,

$$a_1 = 3, b_1 = -5, c_1 = -20$$

and $a_2 = 6, b_2 = -10, c_2 = -40$

Now, $\frac{a_1}{a_2} = \frac{3}{6}, \frac{b_1}{b_2} = \frac{-5}{-10}$ and $\frac{c_1}{c_2} = \frac{-20}{-40}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$$

Hence, infinitely many solutions exist.

(d) We have $x - 3y - 7 = 0$ ...(1)

and $3x - 3y - 15 = 0$ ...(2)

Comparing equation(1) and (2) with $ax + by + c = 0$,

$$a_1 = 1, b_1 = -3, c_1 = -7$$

and $a_2 = 3, b_2 = -3, c_2 = -15$

Here $\frac{a_1}{a_2} = \frac{1}{3}$ and $\frac{b_1}{b_2} = \frac{1}{3} = 1$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, a unique solution exists.

By cross multiplication,

$$\frac{x}{\{(-3)(-15)-(-3)(-7)\}} = \frac{y}{\{(-7)(3)-(-15)(1)\}}$$

$$= \frac{1}{\{(1)(-3)-(3)(-3)\}}$$

$$\frac{x}{(45-21)} = \frac{y}{(-21+15)} = \frac{1}{(-3+9)}$$

$$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

$$\frac{x}{24} = \frac{1}{6} \text{ and } \frac{y}{-6} = \frac{1}{6}$$

$$x = \frac{24}{6} \text{ and } y = \frac{-6}{6}$$

Hence, $x = 4$ and $y = -1$

8. Solve the following pair of linear equations by the substitution and cross - multiplication method :

$$8x + 5y = 9$$
$$3x + 2y = 4$$

Ans : [NCERT]

We have $8x + 5y = 9$

or, $8x + 5y - 9 = 0$ ...(1)

and $3x + 2y = 4$

or, $3x + 2y - 4 = 0$ ...(2)

Comparing equation (1) and (2) with $ax + by + c = 0$,

$$a_1 = 8, b_1 = 5, c_1 = -9$$

and $a_2 = 3, b_2 = 2, c_2 = -4$

By cross-multiplication method,

$$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - b_2b_1}$$

$$\frac{x}{\{(5)(-4)-(2)(-9)\}} = \frac{y}{\{(-9)(3)-(-4)(8)\}}$$

$$= \frac{1}{\{8 \times 2 - 3 \times 5\}}$$

or, $\frac{x}{-2} = \frac{1}{1}$ and $\frac{y}{5} = \frac{1}{1}$

$$x = -2 \text{ and } y = 5$$

We use substitution method.

From equation (2), we have

$$3x = 4 - 2y$$

or, $x = \frac{4-2y}{3}$ ...(3)

Substituting this value of y in equation (3) in (1), we get

$$8\left(\frac{4-2y}{3}\right) + 5y = 9$$

$$32 - 16y + 15y = 27$$

$$-y = 27 - 32$$

Thus $y = 5$

Substituting this value of y in equation (3)

$$x = \frac{4-2(5)}{3} = \frac{4-10}{3} = -2$$

Hence, $x = -2$ and $y = 5$.

9. From the pair of linear equations in the following problems and find their solutions (if they exist) any algebraic method :

(a) A part of monthly lostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1,000 as hostel charges whereas a student B, who takes food for 26 days, pay Rs 1,180 as hostel sharges. Find the fixed charges and the cost of food per day.

(b) A fraction becomes $\frac{1}{3}$,when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$, when 8 is added to its denomiantor. find the fraction.

(c) Yesh scored 40 marks in a test, getting 3 marks for each right answer and losing 1 marks for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test ?

(d) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the some time. If the cars travel in the same direction at different speeds, they meet in 5 hr. If they travel towards each other, they meet in 1 hr. What are the speeds of the two cars ?

Ans :

(a) Let fixed part of monthly hostel charges be y and cost of food for one day be x.

In the case of student A,

$$20x + y = 1{,}000 \qquad \text{...(1) 1}$$

In the case of student B,

$$26x + y = 1{,}180 \qquad \text{...(2)}$$

Subtracting equation (1) from equation (2),

$$x = 30$$

Substituting $x = 30$ in equation (1),

$$y = 1{,}000 - 20 \times 30$$

$$= 1{,}000 - 600 = 400$$

Hence, monthly dixed charges = Rs 400

Cost of food per day = Rs 30

(2) Let the fraction be $\frac{x}{y}$,

According to the given conditions,

$$\frac{x-1}{y} = \frac{1}{3} \text{ and } \frac{x}{y+8} = \frac{1}{4}$$

We get, $3x - y = 3$ ...(1)

and $4x - y = 8$ ...(2)

Subtracting equation (1) from equation (2), we get

$$x = 5$$

Substituting $x = 5$ in equation (1), we have

$$3 \times 5 - y = 3$$

$$y = 12$$

Hence, the fraction $= \frac{5}{12}$

(c) Let the number of right answer be x and the number of wrong answers be y.

Total number of question $= x + y$

In first case,

Marks awarded for x right answer $= 3x$

Marks lost for y wrong answer $= y \times 1 = y$

$$3x - y = 40 \qquad \text{...(1)}$$

In second case,

Marks awarded for x right answers $= 4x$

Marks lost for y wrong answers $= 2y$

$$4x - 2y = 50 \qquad \text{...(2)}$$

From equation (1), $y = 3x - 40$ ...(3)

Substituting the value of y from equation (3) in equation (2),

$$4x - 2(3x - 40) = 50$$

$$4x - 6x + 80 = 50$$

$$2x = 30$$

$$x = 15$$

Substituting the value of x in equation (3),

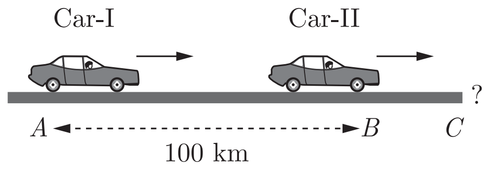
$$y = 3 \times 15 - 40 = 5$$

Total number of questions $= x + y = 15 = 5 = 20$ 1

(d) Let speed of car I be x km/hr and speed of car II be y km/hr.

Car I starts from point A and car II starts from point B

First Case :



Two cars meet at C after 5 hr.

AC = Distance travelled by car I in 5 hr = $5x$ km

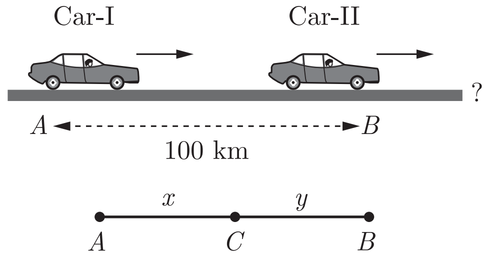
BC = Distance travelled by car II in 5 hr = $5y$ km

Since, $AC - BC = AB$

So, $5x - 5y = 100$

or, $x - y = 20$

Second Case :



Two cars meet at C after one hour, thus

$$x + y = 100 \qquad \text{...(2)}$$

Adding equation (1) and (2), we have

$$2x = 120$$

$$x = 60$$

Substituting $x = 60$ in equation (2), we have

$$60 + y = 100$$

$$y = 40$$

10. 2 man and 7 boys can do a piece of work in 4 days. It is done by 4 men and 4 boys in 3 days. How long would it take for one man or one boy to do it ?

Ans : [Board Term-1, 2013, LK-59]

Let the man can finish the work in x days and the boy can finish work in y days.

Work done by one man in one day $= \frac{1}{x}$

And work done by one boy in one day $= \frac{1}{y}$

$$\frac{2}{x} + \frac{7}{y} = \frac{1}{4} \qquad \text{...(1)}$$

and $$\frac{4}{x}+\frac{4}{y} = \frac{1}{3} \quad ...(2)$$

Let $\frac{1}{x}$ be a and $\frac{1}{y}$ be b, then we have

$$2a+7b = \frac{1}{4} \quad ...(3)$$

and $$4a+4b = \frac{1}{3} \quad ...(4)$$

Multiplying equation (3) by 2 and subtract equation (4) from it

$$10b = \frac{1}{6}$$

$$b = \frac{1}{60} = \frac{1}{y}$$

Thus $y = 60$ days.

Substituting $b = \frac{1}{60}$ in equation (3), we have

$$2a+\frac{7}{60} = \frac{1}{4}$$

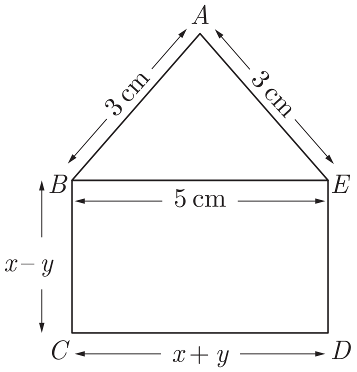
$$2a = \frac{1}{4}-\frac{7}{60}$$

$$a = \frac{1}{15}$$

Now $$\frac{1}{15} = \frac{1}{x}$$

Thus $x = 15$ days.

11. In the figure below $ABCDE$ is a pentagon with $BE || CD$ and $BC || DE$. BC is perpendicular to DC. If the perimeter of $ABCDE$ is 21 cm, find the values of x and y.



Ans : [Board Term-I, 2011, Set-45, 53]

Since $BC || DE$ and $BE || CD$ with $BC \perp DC$, $BCDE$ is a rectangle.

$$BE = CD,$$

$$x+y = 5 \quad ...(1)$$

and $$DE = BE = x-y$$

Since perimeter of $ABCDE$ is 21,

$$AB+BC+CD+DE+EA = 21$$

$$3+x-y+x+y+x-y+3 = 21$$

$$6+3x-y = 21$$

$$3x-y = 15$$

Adding equations (1) and (2), we get

$$4x = 20 \quad ...(2)$$

$$x = 5$$

Substituting the value of x in (1), we get

$$y = 0$$

Thus $x = 5$ and $y = 0$.

12. Solve for x and y :

$$\frac{x+1}{2}+\frac{y-1}{3} = 9; \ \frac{x-1}{3}+\frac{y+1}{2} = 8.$$

Ans : [Board Term-1, 2011, Set-52]

We have $$\frac{x+1}{2}+\frac{y-1}{3} = 9$$

$$3(x+1)+2(y-1) = 54$$

$$3x+2y = 53 \quad (1)$$

and $$\frac{x+1}{3}+\frac{y+1}{2} = 8$$

$$2(x-1)+3(y+1) = 48$$

$$2x+3y = 47 \quad (2)$$

Multiplying equation (1)by 3 we have

$$9x+6y = 159 \quad (3)$$

Multiplying equation (2)by 2 we have

$$4x+6y = 94 \quad (4)$$

Subtracting equation (4) from (3) we have

$$5x = 65$$

or $$x = 13$$

Substitute the value of x in equation (2),

$$2(13)+3y = 47$$

$$3y = 47-26 = 21$$

$$y = \frac{21}{3} = 7$$

Hence, $x = 13$ and $y = 7$

13. Solve for x and y :

$$\frac{6}{x-1}-\frac{3}{y-2} = 1$$

$$\frac{5}{x-1}-\frac{1}{y-2} = 2, \text{ where } x \neq 1, \ y \neq 2.$$

Ans : [Board Term-1, 2011, Set-21]

We have $$\frac{6}{x-1}-\frac{3}{y-2} = 1 \quad (1)$$

$$\frac{5}{x-1}-\frac{1}{y-2} = 2, \quad (2)$$

Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$. then given equations become

$$6p-3q = 1 \quad ...(3)$$

and $$5p+q = 2 \quad ...(4)$$

Multiplying equation (2) by 3 and adding in equation (1),

$$21p = 7$$

$$p = \frac{7}{21} = \frac{1}{3}$$

Substituting this value of p in equation (1),

$$6\left(\frac{1}{3}\right)-3q = 1$$

$$2-3q = 1 \Rightarrow q = \frac{1}{3}$$

Now, $\frac{1}{x-1} = p = \frac{1}{3}$

or, $x-1 = 3 \Rightarrow x = 4$

and $\frac{1}{y-2} = q = \frac{1}{3}$

or, $y-2 = 3 \Rightarrow y = 5$

Hence $x = 4$ and, $y = 5$.

14. Solve the following pair of equations for x and y :

$$\frac{a^2}{x}-\frac{b^2}{y} = 0, \frac{a^2b}{x}+\frac{b^2a}{y} = a+b, \qquad x \neq 0; y \neq 0.$$

Ans : [Board Term-1, Set-39]

We have $\frac{a^2}{x}-\frac{b^2}{y} = 0$ (1)

$$\frac{a^2b}{x}+\frac{b^2a}{y} = a+b = a+b \qquad (2)$$

Substituting $p = \frac{1}{x}$ and $q = \frac{1}{y}$ in the given equations,

$$a^2p-b^2q = 0 \qquad ...(1)$$

$$a^2bp+b^2aq = a+b \qquad ...(2)$$

Multiplying equation (1), by a

$$a^3p-b^2aq = 0 \qquad ...(3)$$

Adding equation (2) and equation (3),

$$(a^3+a^2b)p = a+b$$

or, $p = \frac{(a+b)}{a^2(a+b)} = \frac{1}{a^2}$

Substituting the value of p in equation (1),

$$a^2\left(\frac{1}{a^2}\right)-b^2q = 0 \Rightarrow q = \frac{1}{b^2}$$

Now, $p = \frac{1}{x} = \frac{1}{a^2} \Rightarrow x = a^2$

and $q = \frac{1}{y} = \frac{1}{b^2} \Rightarrow y = b^2$

Hence, $x = a^2$ and $y = b^2$

15. Solve for x and y :

$$ax+by = \frac{a+b}{2}$$

$$3x+5y = 4$$

Ans : [Board Term-1, 2011, Set-44]

We have $ax+by = \frac{a+b}{2}$

or $2ax+2by = a+b$ ...(1)

and $3x+5y = 4$ ...(2)

Multiplying equation (1) by 5 we have

$$10ax+10by = 5a+5b \qquad (3)$$

Multiplying equation (2) by $2b$, we have

$$6bx+10by \ 4b \qquad (4)$$

Subtracting (4) from (3) we have

$$(10a-6b)x = 5a-3b$$

or $x = \frac{5a-3b}{10a-6b} = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ in equation (2), we get

$$3\times\frac{1}{2}+5y = 4$$

$$5y = 4-\frac{3}{2} = \frac{5}{2}$$

$$y = \frac{5}{2\times 5} = \frac{1}{2}$$

Hence $x = \frac{1}{2}$ and $y = \frac{1}{2}$.

16. Solve the following pair of equations for x and y :

$4x+\frac{6}{y} = 15, 6x-\frac{8}{y} = 14$

and also find the value of p such that $y = px-2$.

Ans : [Board Term-1, 2011, Set-60]

We have $4x+\frac{6}{y} = 15$ (1)

$$6x-\frac{8}{y} = 14, \qquad (2)$$

Let $\frac{1}{y} = z$, the given equations become

$$4x+6z = 15 \qquad ...(3)$$

$$6x-8z = 14 \qquad ...(4)$$

Multiply equation (3) by 4 we have

$$16x+24z = 60 \qquad (5)$$

Multiply equation (4) by 3 we have

$$18x-24z = 24 \qquad (6)$$

Adding equation (5) and (6) we have

$$34x = 102$$

$$x = \frac{102}{34} = 3$$

Substitute the value of x in equation (3),

$$4(3)+6z = 15$$

$$6z = 15-12 = 3$$

$$z = \frac{3}{6} = \frac{1}{2}$$

Now $z = \frac{1}{y} = \frac{1}{2} \Rightarrow y = 2$

Hence $x = 3$ and $y = 2$.

Again $y = px-2$

$$2 = p(3)-2$$

$$3p = 4$$

Thus $p = \frac{4}{3}$

17. Find whether the following pair of linear equations has a unique solutions. If yes, find the solution :

$$7x-4y = 49, 5x-6y = 57.$$

Ans : [Board Term-1, 2011, Set-39]

We have $7x-4y = 49$ (1)

$$5x-6y = 57 \qquad (2)$$

Comparing with the equation $a_1x+b_1y = c_1$,

$$a_1 = 7, b_1 = -4, c_1 = 49$$

$$a_2 = 5, b_2 = -6, c_2 = 57$$

Since, $\frac{a_1}{a_2} = \frac{7}{5}$ and $\frac{b_1}{b_2} = \frac{4}{6}$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, system has a unique solution.

Multiply equation (1) by 5 we get

$$35x - 20y = 245 \qquad (3)$$

Multiply equation (2) by 7 we get

$$35x - 42y = 399 \qquad (4)$$

Subtracting (4) by (3) we have

$$22y = -154$$
$$y = -7$$

Putting the value of y in equation (2),

$$5x - 6(-7) = 57$$
$$5x = 57 - 42 = 15$$
$$x = 3$$

Hence $x = 3$ and $y = -7$

LONG ANSWER TYPE QUESTIONS

1. 4 chairs and 3 tables cost Rs 2100 and 5 chairs and 2 tables cost Rs 1750. Find the cost of none chair and one tabel separetely.

Ans : [Boared Term-1,2015, Set -WJQWQBN]

Let cost of 1 chair be Rs x and cost of 1 table be Rs y According to the question,

$$4x + 3y = 2100 \qquad ...(1)$$
$$5x + 2y = 1750 \qquad ...(2)$$

Multiplying equation (1) by 2 and equation (2) by 3,

$$8x + 6y = 4200 \qquad ...(3)$$
$$15x + 6y = 5250 \qquad ...(iv)$$

Subtracting equation (3) from (4) we have

$$7x = 1050$$
$$x = 150$$

Substituting the value of x in (1), $y = 500$

Thus cost of chair and tabel is Rs 150, Rs 500 respectively.

2. Solve the following pair of equations :

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \text{ and } \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Ans : [Board Term-1, Set-WJQZQBN]

We have $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Substitute $\frac{1}{\sqrt{x}} = X$ and $\frac{1}{\sqrt{y}} = Y$

$$2X + 3Y = 2 \qquad ...(1)$$
$$4X - 9Y = -1 \qquad ...(2)$$

Multiplying equation (1) by 3, and adding in (2) we get

$$10X = 5 \Rightarrow X = \frac{5}{10} = \frac{1}{2}$$

Thus $\frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow x = 4$

Putting the value of X in equation (1), we get

$$2 \times \frac{1}{2} + 3y = 2$$
$$3Y = 2 - 1$$
$$Y = \frac{1}{3}$$

Now $Y = \frac{1}{3} \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow y = 9$

Hence $x = 4$, $y = 9$.

3. Solve for x and y :

$$2x - y + 3 = 0$$
$$3x - 5y + 1 = 0$$

Ans : [Board Term-1, DDE-M]

We have $2x - y + 3 = 0 \qquad ...(1)$

$$3x - 5y + 1 = 0 \qquad ...(2)$$

Multiplying equation (1) by 5,and subtracting (2) from it we have

$$7x = -14$$
$$x = \frac{-14}{7} = -2$$

Substituting the value of x in equation (1) we get

$$2x - y + 3 = 0$$
$$2(-2)y + 3 = 0$$
$$-4 - y + 3 = 0$$
$$-y - 1 = 0$$
$$y = -1$$

Hence, $x = -2$ and $y = -1$.

4. Solve the following pairs of linear equations by elimination method and the substitution method.

(a) $x + y = 5$ and $2x - 3y = 4$

(b) $3x + 4y = 10$ and $2x - 2y = 2$

(c) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

Ans : [NCERT]

(a) By Elimination Method :

We have, $x + y = 5 \qquad ...(1)$

and $2x - 3y = 4 \qquad ...(2)$

Multiplying equation (1) by 3 and adding in (2) we have

$$3(x + y) + (2x - 3y) = 3 \times 5 + 4$$

or, $3x + 3y + 2x - 3y = 15 + 4$

$$5x = 19 \Rightarrow x = \frac{19}{5}$$

Substituting $x = \frac{19}{5}$ in equation (1),

$$\frac{19}{5} + y = 5$$
$$y = 5 - \frac{19}{5} = \frac{25 - 19}{5} = \frac{6}{5}$$

Hence, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

By Substituting Method :

We have, $x+y=5$ ...(1)

and $2x-3y=4$ ...(2)

From equation (1), $y=5-x$ ...(3)

Substituting the value of y from equation (3) in equation (2),

$$\begin{aligned} 2x-3y(5-x) &= 4 \\ 2x-15+3x &= 4 \\ 5x &= 19 \\ x &= \frac{19}{5} \end{aligned}$$

Substituting this value of x in equation (3), we get

$$y = 5-\frac{19}{5} = \frac{6}{5}$$

Hence $x=\frac{19}{5}$ and $y=\frac{6}{5}$

(b) By Elimination Method :

We have, $3x+4y=10$ ...(1)

and $2x-2y=2$ ...(2)

Multiplying equation (2) by 2 and adding in (1),

$$\begin{aligned} (3x+4y)+2(2x-2y) &= 10+2\times 2 \\ \text{or,}\quad 3x+4y+4x-4y &= 10+4 \\ \text{or,}\quad 7x &= 14 \\ y &= 1 \end{aligned}$$

Hence, $x=2$ and $y=1$.

By Substitution Method :

We have $3x+4y=10$ ...(1)

and $2x-2y=2$ ...(2)

From equation (2) $2y=2x-2$

or, $y=x-1$ ...(3)

Substituting this value of y in equation (1),

$$\begin{aligned} 3x+4(x-1) &= 10 \\ 7x &= 14 \\ x &= 2 \end{aligned}$$

From equation (3), $y=2-1=1$

Hence, $x=2$ and $y=1$ 1

(3) By Elimination Method :

We have, $3x-5y=4$ ...(1)

and $9x=2y+7$ ...(2)

Multiplying equation (1) by 3 and rewriting equation (2) we have

$9x-15y=12$ ...(3)

$9x-2y=7$ ...(4)

Subtracting equation (4) from equation (3),

$$\begin{aligned} -13y &= 5 \\ y &= -\frac{5}{13} \end{aligned}$$

Substituting value of y in equation (1),

$$\begin{aligned} 3x-5\left(\frac{-5}{13}\right) &= 4 \\ 3x &= 4-\frac{25}{13} \\ x &= \frac{27}{13\times 3}=\frac{9}{13} \end{aligned}$$

Hence $x=\frac{9}{13}$ and $y=-\frac{5}{13}$

By Substituting Method :

We have $3x-5y=4$ ...(1)

and $9x=2y+7$ ...(2)

$y=\frac{9x-7}{2}$ ...(3)

Substituting this value of y (3) in equation (1),

$$\begin{aligned} 3x-5\times\left(\frac{9x-7}{2}\right) &= 4 \\ 6x-45x+35 &= 8 \\ -39x &= -27 \\ x &= \frac{9}{13} \end{aligned}$$

Substituting $x=\frac{9}{13}$ in equation (3),

$$\begin{aligned} y &= \frac{9\times\frac{9}{13}-7}{2} = \frac{81-91}{2\times 13} \\ &= -\frac{10}{26} = -\frac{5}{13} \end{aligned}$$

Hence, $x=\frac{9}{13}$ and $y=\frac{-5}{13}$

5. A train covered a certain distance at a uniform speed. If the train would have been 10 km/hr scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hr more than the scheduled time. Find the distance covered by the train.

Ans : [NCERT]

Let the actual speed of the train be x km/hr and actual time taken y hr.

$$\begin{aligned} \text{Distance} &= \text{Speed}\times\text{Time} \\ &= xy\text{ km} \end{aligned}$$

According to the given condition, we have

$$\begin{aligned} xy &= (x+10)(y-2) \\ xy &= xy-2x+10y-20 \\ 2x-10+20 &= 0 \\ x-5y &= -10 \qquad (1) \end{aligned}$$

and

$$\begin{aligned} xy &= (x-10)(y+3) \\ xy &= xy+3x-10y-30 \\ 3x-10y &= 30 \qquad ...(2) \end{aligned}$$

Multiplying equation (1) by 3 and subtracting equation (2) from equation (1),

$$\begin{aligned} 3\times(x-5y)-(3x-10y) &= -3\times 10-30 \\ -5y &= -60 \\ y &= 12 \end{aligned}$$

Substituting value of y equation (1),

$$\begin{aligned} x-5\times 12 &= -10 \\ \text{or,}\quad x &= -10+60 \\ \text{or,}\quad x &= 50 \end{aligned}$$

Hence, the distance covered by the train

$$= 50\times 12 = 600\text{ km.}$$

6. The ratio of incomes of two persons is 11:7 and the ratio of their expenditures is 9:5. If each of them manages to save Rs 400 per month, find their monthly incomes.

Ans : [Board Term-1, 2012, Set-38]

Let the incomes of two persons be $11x$ and $7x$.
Also the expenditures of two persons be $9y$ and $5y$.

$$11x - 9y = 400 \quad ...(1)$$

and
$$7x - 5y = 400 \quad ...(2)$$

Multiplying equation (1) by 5 and equation (2) by 9 we have

$$55x - 45y = 2000 \quad ...(3)$$

and
$$63x - 45y = 3600 \quad ...(4)$$

Subtracting, above equation we have

$$-8x = -1600$$

or,
$$x = \frac{-1,600}{-8} = 200$$

Hence Their monthly incomes are $11 \times 200 =$ Rs 2200 and $7 \times 200 =$ Rs 1400.

7. A and B are two points 150 km apart on a highway. Two cars start A and B at the same time. If they move in the same direction they meet in 15 hours. But if they move in the opposite direction, they meet in 1 hours. Find their speeds.

Ans : [Board Term-1, 2012, Set-62, 44]

Let the speed of the car I from A be x km/hr. Speed of the car II from B be y km/hr.

Same Direction :

Distance covered by car I$= 150 +$(distance covered by car II)

$$15x = 150 + 15y$$
$$15x - 15y = 150$$
$$x - y = 10 \quad ...(1)$$

Opposite Direction :

Distance covered by car I + distance covered by car II

$$= 150 \text{ km}$$
$$x + y = 150 \quad ...(2)$$

Adding equation (1) and (2), we have

$$y = 70$$

Speed of the car I from $A = 80$ km/hr and speed of the car II from $B = 70$ km/hr.

8. If 2 is subtracted from the numerator and 1 is added to the denominator, a fraction becomes $\frac{1}{2}$, but when 4 is added to the numerator and 3 is subtracted from the denominator, it becomes $\frac{3}{2}$. Find the fraction.

Ans : [Board Term-1, 2012, Set-48]

Let the fraction be $\frac{x}{y}$ then we have

$$\frac{x-2}{y+1} = \frac{1}{2}$$
$$2x - 4 = y + 1$$
$$2x - y = 5 \quad ...(1)1$$

Also,
$$\frac{x+4}{y-3} = \frac{3}{2}$$
$$2x + 8 = 3y - 9]$$
$$2x - 3y = -17 \quad ...(2)\ 1$$

Subtracting equation (2) from equation (1),

$$2y = 22 \Rightarrow y = 11$$

Substituting this value of y in equation (1) we have,

$$2x - 11 = 5$$
$$x = 8$$

Hence,
$$\text{Fraction} = \frac{8}{11}$$

9. If a bag containing red and white balls, half the number of white balls is equal to one-third the number of red balls. Thrice the total number of balls exceeds seven times the number of white balls by 6. How many balls of each colour does the bag contain ?

Ans : [Board Term-1, 2012, Set-55]

Let the number of red balls be x and white balls be y. According to the question,

$$\frac{1}{2y} = \frac{1}{3}x \text{ or } 2x - 3y = 0 \quad ...(1)$$

and
$$3(x+y) - 7y = 6$$

or
$$3x - 4y = 6 \quad ...(2)$$

Multiplying equation (1) by 3 and equation (2) we have

$$6x - 9y = 0 \quad ...(3)$$
$$6x - 8y = 12 \quad ...(4)$$

Subtracting equation (3) from (4) we have

$$y = 12$$

Substituting $y = 12$ in equation (1),

$$2x - 36 = 0$$
$$x = 18$$

Hence, number of red balls = 18

and number of white balls = 12

10. A two digit number is obtained by either multiplying the sum of digits by 8 and then subtracting 5 or by multiplying the difference of digits by 16 and adding 3. Find the number.

Ans : [Board Term-1, 2012, Set-48]

Let the digits of number be x and y, then number will $10x + y$
According to the question, we have

$$8(x+y) - 5 = 10x + y$$
$$2x - 7y + 5 = 0 \quad ...(1)$$

also
$$16(x-y) + 3 = 10x + y$$
$$6x - 17y + 3 = 0 \quad ...(2)$$

Comparing the equation with $ax + by + c = 0$ we get

$$a_1 = 2, b_1 = -1, c_1 = 5$$
$$a_2 = 6, b_2 = -17, c_2 = 3$$

Now
$$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{c_1b_2 - a_2b_1}$$

$$\frac{x}{(-7)(3) - (-17)(5)} = \frac{y}{(5)(6) - (2)(3)}$$
$$= \frac{1}{(2)(-17) - (6)(-7)}$$

$$\frac{x}{-21+85} = \frac{y}{30-6} = \frac{1}{-34+42}$$

$$\frac{x}{64} = \frac{y}{24} = \frac{1}{8}$$

$$\frac{x}{8} = \frac{y}{3} = 1$$

Hence, $x = 8, y = 3$

So required number $= 10 \times 8 + 3 = 83$.

11. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and the breadth is increased by 3 units. The area is increased by 67 square units if length is increased by 3 units and breadth is increased by 2 units. find the perimeter of the rectangle.

Ans : [Board Term-1, 2012, Set-48]

Let length of given rectangle be x and breadth be y, then area of rectangle will be xy

According to the first condition we have

$$(x-5)(y+3) = xy-9$$

or, $3x - 5y = 6$ ...(1)

According to the second condition, we have

$$(x+3)(y+3) = xy-67$$

or, $2x+5y = 61$ ...(2)

Multiplying equation (1) by 3 and equation (2) by 5 and then adding,

$$9x - 15y = 18$$

$$10x+15y = 305$$

$$x = \frac{323}{19} = 17$$

Substituting this value of x in equation (1),

$$3(17) - 5y = 6$$

$$5y = 51-6$$

$$y = 9$$

Hence, perimeter $= 2(x+y) = 2(17+9) = 52$ units.

12. Solve for x and y : $2(3x-y) = 5xy, 2(x+3y) = 5xy$.

Ans : [Board Term-1, 2012, Set-25]

We have $2(3x-y) = 5xy$ ...(1)

$2(x+3y) = 5xy$ ...(2)

Divide equation (1) and (2) by xy,

$$\frac{6}{y} - \frac{2}{x} = 5 \quad ...(3)$$

and $$\frac{2}{y} + \frac{6}{x} = 5 \quad ...(4)$$

Let $\frac{1}{y} = a$ and $\frac{1}{x} = b$, then equations (3) and (4) become

$6a - 2b = 5$ ...(5)

$2a + 6b = 5$ ...(6)

Multiplying equation (5) by 3 and then adding with equation (6),

$$20a = 20$$

$$a = 1$$

Substituting this value of a in equation (5),

$$b = \frac{1}{2}$$

Now $\frac{1}{y} = a = 1 \Rightarrow y = 1$

and $\frac{1}{x} = b = \frac{1}{2} \Rightarrow x = 2$

Hence, $x = 2, y = 1$

13. The Present age of the father is twice the sum of the ages of his 2 children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Ans : [Board Term-1, 2012, Set-39]

Let the sum of the ages of the 2 children be x and the age of the father be y years.

Now $y = 2x$

$2x - y = 0$ ...(1)

and $20 + y = x + 40$

$x - y = -20$ ...(2)

Subtracting (2) from (1), we get

$$x = 20$$

From(1), $y = 2x = 2 \times 20 = 40$

Hence, the age of the father is 40 years.

14. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Ans : [NCERT]

Let the number of students in a row be x and the number of rows be y. Thus total will be xy.

Now $(x+3)(y-1) = xy$

$xy + 3y - x - 3 = xy$

$-x + 3y - 3 = 0$ ...(1)

and $(x-3)(y+2) = xy$

$xy - 3y + 2x - 6 = xy$

$2x - 3y - 6 = 0$ ...(2)

Multiply equation (1) 2 we have

$-2x + 6y - 6 = 0$ ...(3)

Adding equation (2) and (3) we have

$$3y - 12 = 0$$

$$y = 4$$

Substitute $y = 4$ in equation (1)

$$-x + 12 - 3 = 0$$

$$x = 9$$

Total students $xy = 9 \times 4 = 36$

Total students in the class is 36.

15. The ages of two friends ani and Biju differ by 3 years. Ani's father Dharam is twice as old as ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 year. Find the ages of Ani and Biju.

Ans : [NCERT]

Let the ages of Ani and Biju be x and y, respectively. According to the given condition,

$$x - y = \pm 3 \qquad ...(1)$$

Also, age of Ani's father Dharam $= 2x$ years

And age of Biju's sister $= \frac{y}{2}$ years

According to the given condition,

$$2x - \frac{y}{2} = 30$$

$$4x - y = 60 \qquad ...(2)$$

Case I : When $x - y = 3$ ...(3)

Subtracting equation (3) from equation (2),

$$\begin{aligned} 3x &= 57 \\ x &= 19 \text{ years} \end{aligned}$$

Putting $x = 19$ in equation (3),

$$\begin{aligned} 19 - y &= 3 \\ y &= 16 \text{ years} \end{aligned}$$

Case II : When $x - y = -3$ ...(4)

Subtracting equation (iv) from equation (2),

$$\begin{aligned} 3x &= 60 + 3 \\ 3x &= 63 \\ x &= 21 \text{ years} \end{aligned}$$

Subtracting equation (4), we get

$$\begin{aligned} 21 - y &= -3 \\ y &= 24 \text{ years} \end{aligned}$$

Hence, Ani's age $=$ 19 years or 21 years Biju age $=$ 16 years or 24 years.

16. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital.

Ans : [NCERT]

Let the amount of their respective capitals be x and y.

According to the given condition,

$$\begin{aligned} x + 100 &= 2(y - 100) \\ x - 2y &= -300 \qquad ...(1) \end{aligned}$$

and

$$\begin{aligned} 6(x - 10) &= y + 10 \\ 6x - y &= 70 \qquad ...(2) \end{aligned}$$

Multiplying equation (2) by 2 we have

$$12x - 2y = 140 \qquad ...(3)$$

Subtracting (1) from equation (3) we have

$$\begin{aligned} 11x &= 440 \\ x &= 40 \end{aligned}$$

Substituting $x = 40$ in equation (1),

$$40 - 2y = -300$$

or,

$$\begin{aligned} 2y &= 340 \\ y &= 170 \end{aligned}$$

Hence, the amount of their respective capitals are 40 and 170.

17. A fraction become $\frac{9}{11}$ if 2 is added to both numerator and denominator. If 3 is added to both numerator and denominator it becomes $\frac{5}{6}$. Find the fraction.

Ans : [Board Term-1, 2012, Set-60]

Let the fraction be $\frac{x}{y}$, then according to the question,

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$$11x + 22 = 9y + 18$$

or,

$$11x - 9y + 4 = 0 \qquad ...(1)$$

and

$$\frac{x+3}{y+3} = \frac{5}{6}$$

or,

$$6x - 5y + 3 = 0 \qquad ...(2)$$

Comparing with $ax + by + c = 0$

we get

$$\begin{aligned} a_1 &= 11, b_1 = 9, c_1 = 4, \\ a_2 &= 6, b_2 = -5, \text{ and } c_2 = 3 \end{aligned}$$

Now,

$$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - b_2b_1}$$

$$\frac{x}{(-9)(3) - (-5)(4)} = \frac{y}{(4)(6) - (11)(3)}$$

$$= \frac{1}{(11)(-5) - (9)(-9)}$$

1

or,

$$\frac{x}{-27 + 20} = \frac{y}{24 - 33} = \frac{1}{-55 + 54}$$

$$\frac{x}{-7} = \frac{y}{-9} = \frac{1}{-1}$$

Hence, $x = 7, y = 9$

Thus fraction is $\frac{7}{9}$

18. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of speed of the boat in still water and the speed of the stream.

Ans : [Board Term-1, 2012, Set-48]

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.
Speed of boat up stream $= (x - y)$ km/hr.
Speed of boat down stream $= (x + y)$ km/hr.

$$\frac{30}{x-y} + \frac{28}{x+y} = 7$$

and

$$\frac{21}{x-y} + \frac{21}{x+y} = 5$$

Let $\frac{1}{x-y}$ be a and $\frac{1}{x+y}$ be b, then we have

$$30a + 28b = 7 \qquad ...(1)$$

$$21a + 21b = 5 \qquad ...(2)$$

Multiplying equation (1) by 3 and equation (2) by 4 we have

$$90a + 84b = 21 \qquad ...(3)$$

$$84a + 84b = 20 \qquad ...(4)$$

Subtracting (4) from (3) we have,

$$6a = 1$$

$$a = \frac{1}{6}$$

Putting this value of a in equation (1),

$$30 \times \frac{1}{6} + 28b = 7$$

$$28b = 7 - 30 \times \frac{1}{6} = 2$$

$$b = \frac{1}{14}$$

Thus $x + y = 14$ ...(5)

Now, $a = \frac{1}{x - y} = \frac{1}{6}$

or, $x - y = 6$ ...(6)

and $x + y = 14$

Solving equation (5) and (6), we get

$$x = 10, y = 4$$

Hence, speed of the boat in still water = 10km/hr

and speed of the stream = 4 km/hr.

19. A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

Ans : [Board Term-1, 2012, Set-48]

Let the speed of the boat be x km/hr and the speed of the stream be y km/hr.
According to the question,

$$\frac{32}{x - y} + \frac{36}{x + y} = 7$$

and $\frac{40}{x - y} + \frac{48}{x + y} = 9$

Let $\frac{1}{x - y} = A, \frac{1}{x + y} = B$, then we have

$32A + 36B = 7$ ...(1)

and $40A + 48B = 9$ ...(2)

Multiplying equation (1) by 5 and (2) by 4, we have

$160A + 180B = 35$ ...(3)

and $160A + 192B = 36$ ...(4

Subtracting (4) from (3) we have

$$-12B = -1$$

$$B = \frac{1}{12}$$

Substituting the value of B in (2) we get

$$40A + 48\left(\frac{1}{12}\right) = 9$$

$$40A + 4 = 9$$

$$40A = 5$$

$$A = \frac{1}{8}$$

Thus $A = \frac{1}{8}$ and $B = \frac{1}{12}$

Hence $A = \frac{1}{8} = \frac{1}{x - y}$

$x - y = 8$ ...(5)

and $B = \frac{1}{12} = \frac{1}{x + y}$

$x + y = 12$ ...(6)

Adding equations (5) and (6) we have,

$$2x = 20$$

$$x = 10$$

Substituting this value of x in equation (1),

$$y = x - 8 = 10 - 8 = 2$$

Hence, the speed of the boat in still water = 10 km/hr and speed of the stream = 2 km/hr.

20. For what values of a and b does the following pair of linear equations have infinite number of solution ?

$2x + 3y = 7, a(x + y) - b(x - y) = 3a + b - 2$

Ans : [Board Term-1, 2015, CJTOQ]

We have $2x + 3y - 7 = 0$

Here $a_1 = 2, b_1 = 3, c_1 = -7$

and $a(x + y) - b(x - y) = 3a + b - 2$

$$ax + ay - bx + by = 3a + b - 2$$

$$(a - b)x + (a + b)y - (3a + b - 2) = 0$$

Here $a_2 = a - b, b_2 = a + b, c_2 = -(3a + b - 2)$

For infinite many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a - b} = \frac{3}{a + b} = \frac{-7}{(3a + b - 2)}$$

From $\frac{2}{a - b} = \frac{7}{3a + b - 2}$ we have

$$2(3a + b - 2) = 7(a - b)$$

$$6a + 2a - 4 = 7a - 7b$$

$a - 9b = -4$ ...(1)

From $\frac{3}{a + b} = \frac{7}{3a + b - 2}$ we have

$$3(3a + b - 2) = 7(a + b)$$

$$9a + 3b - 6 = 7a + 7b$$

$$2a - 4b = 6$$

$a - 2b = 3$ ...(2)

Subtracting equation (1) from (2),

$$-7b = -7$$

$$b = 1$$

Substituting the value of b in equation (1),

$$a = 5$$

Hence, $a = 5, b = 1$.

21. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3, determine the number.

Ans : [Sample Question Paper 2017]

Let the ten's and unit digit by y and x respectively,
So the number is $10y + x$

The number when digits are reversed becomes $10x + y$

Thus
$$\begin{aligned} 7(10y + x) &= 4(10x + y) \\ 70y + 7x &= 40x + 4y \\ 70y - 4y &= 40x - 7x \\ 2y &= x \qquad ...(1) \end{aligned}$$

or
$$x - y = 3 \qquad ...(2)$$

From (1) and (2) we get

$$y = 3 \text{ and } x = 6$$

Hence the number is 36.

HOTS QUESTIONS

1. At a certain time in a deer, the number of heads and the number of legs of deer and human visitors were counted and it was found that there were 39 heads and 132 legs.

Find the number of deer and human visitors in the park.

Ans :

Let the no. of deer be x and no. of human be y.
According to the question,

$$x + y = 39 \qquad ...(1)$$

and
$$4x + 2y = 132 \qquad ...(2)$$

Multiply equation (1) from by 2,

$$2x + 2y = 78 \qquad ...(3)$$

Subtract equation (3) from (2),

$$\begin{aligned} 2x &= 54 \\ x &= 27 \end{aligned}$$

Substituting this value of x in equation (1)

$$\begin{aligned} 27 + y &= 39 \\ y &= 12 \end{aligned}$$

So, No. of deer $= 27$ and No. of human $= 12$

2. Find the value of p and q for which the system of equations represent coincident lines $2x + 3y = 7$, $(p + q + 1)x + (p + 2q + 2)y = 4(p + q) + 1$

Ans :

We have
$$2x + 3y = 7$$
$$(p + q + 1)x + (p + 2q + 2)y = 4(p + q) + 1$$

Comparing given equation to $ab + by + c = 0$ we have
$a_1 = 2, b_1 = 3, c_1 = -7$
$a_2 = p + q + 1, b_2 = p + 2q + 2, c_2 = -4(p + q) - 1$
For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{p + q + 1} = \frac{3}{p + 2q + 2} = \frac{7}{4(p + q) + 1}$$

From $\dfrac{3}{p + 2q + 2} = \dfrac{7}{4(p + q) + 1}$ we have

$$\begin{aligned} 7p + 14q + 14 &= 12p + 12q + 3 \\ 5p - 2q - 11 &= 0 \qquad ...(1) \end{aligned}$$

From $\dfrac{2}{p + q + 1} = \dfrac{7}{4(p + q) + 1}$ we have

$$\begin{aligned} 8(p + q) + 2 &= 7p + 7q + 7 \\ 8p + 8q + 2 &= 7p + 7q + 7 \\ p + q - 5 &= 0 \qquad ...(2) \end{aligned}$$

Multiplying equation (2) by 5 we have

$$5p + 5q - 25 = 0 \qquad ...(3)$$

Subtracting equation (1) from (3) we get

$$\begin{aligned} 7q &= 14 \\ q &= 2 \end{aligned}$$

Hence, $p = 3$ and $q = 2$.

3. A chemist has one solution which is 50 % acid and a second which is 25 % acid. How much of each should be mixed to make 10 litre of 40 % acid solution.

Ans :

Let 50 % acids in the solution be x and 25 % of other solution be y.
Total volume in the mixture

$$x + y = 10 \qquad ...(1)\ 1$$

and
$$\frac{50}{100}x + \frac{25}{100}y = \frac{40}{100} \times 10$$

$$2x + y = 16 \qquad ...(2)\ 1$$

Subtracting equation (1) from (2) we have

$$x = 6$$

Substituting this value of x in equation (1) we get

$$\begin{aligned} 6 + y &= 10 \\ y &= 4 \end{aligned}$$

Hence, $x = 6$ and $y = 4$.

4. The length of the sides of a triangle are $2x + \frac{y}{2}$, $\frac{5x}{3} + y + \frac{1}{2}$ and $\frac{2}{3}x + 2y + \frac{5}{2}$. If the triangle is equilateral , find its perimeter.

Ans :

For an equilateral Δ,

$$2x + \frac{y}{2} = \frac{5x}{3} + y + \frac{1}{2} = \frac{1}{2}x + 2y + \frac{5}{2}$$

Now
$$\begin{aligned} \frac{4x + y}{2} &= \frac{10x + 6y + 3}{6} \\ 12x + 3y &= 10x + 6y + 3 \\ 2x - 3y &= 3 \qquad ...(1) \end{aligned}$$

Again,
$$\begin{aligned} 2x + \frac{y}{2} &= \frac{2}{3}x + 2y + \frac{5}{2} \\ \frac{4x + y}{2} &= \frac{4x + 12y + 15}{6} \\ 12x + 3y &= 4x + 12y + 15 \\ 8x - 9y &= 15 \qquad ...(2) \end{aligned}$$

Multiplying equation (1) by 3 we have

$$6x - 9y = 9 \qquad ...(1)$$

Subtracting it from (2) we get

$$2x = 6 \Rightarrow x = 3$$

Substituting this value of x into (1), we get

$$2\times 3-3y = 3$$

or, $$3y = 3 \Rightarrow y = 1$$

Now substituting these value of x and y

$$2x+\frac{y}{2} = 2\times 3+\frac{1}{2} = 6.5$$

The perimeter of equilateral triangle = side × 3

$$= 6.5\times 3 = 19.5 \text{ cm}$$

Hence, the perimeter of $\Delta = 19.5$ m

5. In an election contested between A and B, A obtained votes equal to twice the no. of persons on the electoral roll who did not cast their votes and this later number was equal to twice his majority over B. If there were 1,8000 persons on the electoral roll. How many votes for B.

Ans :

Let x and y be the no. of votes for A and B respectively.
The no. of persons who did not vote

$$= (18000-x-y)$$

We have $$x = 2(18000-x-y)$$

or $$3x+2y = 36000 \quad ...(1)$$

and $$(18000-x-y) = 2(x-y)$$

or $$3x-y = 18000 \quad ...(2)$$

Subtracting equation (2) from equation (1),

$$3y = 18000$$

$$y = 6000$$

Hence vote for $B = 6000$

6. When 6 boys were admitted and 6 girls left, the percentage of boys increased from 60% to 75%. Find the original no. of boys and girls in the class. 2

Ans :

Let the no. of boys be x and no. of girls be y.
No. of students $= x+y$

Now $$\frac{x}{x+y} = \frac{60}{100} \quad ...(1)$$

and $$\frac{x+6}{(x+6)+(y-6)} = \frac{75}{100} \quad ...(2)$$

From (1), we have

$$100x = 60x+60y$$

$$40x-60y = 0$$

$$2x-3y = 0$$

$$2x = 3y \quad (3)$$

From (2) we have

$$100x+600 = 75x+75y$$

$$25x-75y = -600$$

$$x-3y = -24 \quad ...(4)$$

Substituting the value of $3y$ from (3) in to (4) we have,

$$x-2x = -24 \Rightarrow x = 24$$

$$3y = 24\times 2$$

$$y = 16$$

Hence, no. of boys is 24 and no. of girls is 16.

7. A cyclist, after riding a certain distance, stopped for half an hour to repair his bicycle, after which he completes the whole journey of 30 km at half speed in 5 hours. If the breakdown had occurred 10 km farther off, he would have done the whole journey in 4 hours. Find where the breakdown occurred and his original speed.

Ans :

Let x km be the distance of the place where breakdown occurred and y be the original speed

$$\frac{x}{y}+\frac{30-x}{\frac{y}{2}} = 5$$

or $$\frac{x}{y}+\frac{60-2x}{y} = 5 \quad ...(1)$$

and $$\frac{x+10}{y}+\frac{30-(x+10)y}{} = 4 \quad ...(2)$$

From (1), we have

$$x+60-2x = 5y$$

$$x+5y = 60 \quad ...(3)$$

From (3) we have

$$x+10+60-2x-20 = 4y$$

$$x+4y = 50 \quad (4)$$

Subtract equation (4) from (3), $y = 10$ km/hr.

Now from (4), $$x+40 = 50$$

$$x = 10 \text{ km}$$

Break down occurred at 10 km. Hence original speed was 10 km/hr.

8. The population of a village is 5000. If in a year, the number of males were to increase by 5% and that of a female by 3% annually, the population would grow to 5202 at the end of the year. Find the number of males and females in the village.

Ans :

Let the number of males be x and females be y

Now $$x+y = 5,000 \quad ...(1)$$

and $$x+\frac{5}{100}x+y+\frac{3y}{100} = 5202$$

$$\frac{5x+3y}{100}+5000 = 5202$$

$$5x+3y = (5202-5000)\times 100$$

$$5x+3y = 20200 \quad (2)$$

Multiply (1) by 3 we have

$$3x+3y = 15,000 \quad ...(3)$$

Subtracting (2) from (3) we have

$$2x = 5200 \Rightarrow x = 2600$$

Substituting value of x in (1) we have

$$2600-y = 5000$$

$$y = 2400$$

Thus no. of males is 2600 and no. of females is 2400..

CHAPTER 4

Quadratic Equation

TOPIC 1 : SOLUTION OF QUADRATIC EQUATIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the positive root of $\sqrt{3x^2+6}=9$.

Ans : [Board Term-2, 2015, Set UDICCY2]

We have
$$\begin{aligned}\sqrt{3x^2+6} &= 9\\ 3x^2+6 &= 81\\ 3x^2 &= 81-6=75\\ x^2 &= \frac{75}{3}=25\end{aligned}$$

Thus $x = \pm 5$

Hence 5 is positive root.

2. If $x=-\frac{1}{2}$, is a solution of the quadratic equation $3x^2+2kx-3=0$, find the value of k.

[**Board Term-2, 2015, Delhi CBSE (Set, I, II, III)**]

Ans : [CBSE Marking Scheme, 2015]

We have $3x^2+2kx-3 = 0$

Putting $x=-\dfrac{1}{2}$ we get

$$\begin{aligned}3\left(-\frac{1}{2}\right)^2+2k\left(-\frac{1}{2}\right)-3 &= 0\\ \frac{3}{4}-k-3 &= 0\\ k &= \frac{3}{4}-3\\ &= \frac{3-12}{4}=\frac{-9}{4}\end{aligned}$$

Hence $k = \dfrac{-9}{4}$

3. Find the roots of the quadratic equation $\sqrt{3}\,x^2-2x-\sqrt{3}$.

Ans : [Board Term-2, 2012, (35)2011 (A1)]

We have
$$\begin{aligned}\sqrt{3}\,x^2-2x-\sqrt{3} &= 0\\ \sqrt{3}\,x^2-3x+x-\sqrt{3} &= 0\\ \sqrt{3}\,x(x-\sqrt{3})+1(x-\sqrt{3}) &= 0\\ (x-\sqrt{3})(\sqrt{3}+1) &= 0\end{aligned}$$

Thus $x = \sqrt{3},\dfrac{-1}{\sqrt{3}}$

4. Find the value of k, for which one root of the quadratic equation $kx^2-14x+8=0$ is six times the other.

Ans : [Board Term-2, 201]

[**Board Sample Paper 2016**]

We have $kx^2-14x+8=0$

Let one root be α and other root be 6α.

Sum of roots $\alpha+6\alpha = \dfrac{14}{k}$

$$7\alpha = \frac{14}{k} \text{ or } \alpha=\frac{2}{k} \qquad ...(1)$$

Product of roots $\alpha(6\alpha) = \dfrac{8}{k}$

or, $$6\alpha^2 = \frac{8}{k} \qquad ...(2)$$

Solving (1) and (2), we obtain

$$\begin{aligned}6\left(\frac{2}{k}\right)^2 &= \frac{8}{k}\\ 6\times\frac{4}{k^2} &= \frac{8}{k}\\ \frac{3}{k^2} &= \frac{1}{k}\\ 3k &= k^2\\ 3k-k^2 &= 0\\ k[3-k] &= 0\\ k &= 0 \text{ or } k=3\end{aligned}$$

Since $k=0$ is not possible, therefore $k=3$.

5. If one root of the quadratic equation $6x^2-x-k=0$ is $\frac{2}{3}$, then find the value of k.

Ans : [Board Term-II foreign-2, 2017]

We have $6x^2-x-k=0$

Substituting $x=\dfrac{2}{3}$, we get

$$\begin{aligned}6\left(\frac{2}{3}\right)^2-\frac{2}{3}-k &= 0\\ 6\times\frac{4}{3}-\frac{2}{3}-k &= 0\\ k &= 6\times\frac{4}{9}-\frac{2}{3}=\frac{24-6}{9}=2\end{aligned}$$

Thus $k=2$.

6. Find the value (s) of k if the quadratic equation $3x^2-k\sqrt{3}\,x+4=0$ has real roots.

Ans : [Sample Question Paper 2017]

If discriminant of quadratic equation is equal to zero, or more than zero, then roots are real.

We have $3x^2-k\sqrt{3}\,x+4 = 0$

Compare with $ax^2+bx+c = 0$

$$D = b^2-4ac$$

For real roots $b^2-4ac \geq 0$

$$(-k\sqrt{3})^2-4\times3\times4 \geq 0$$

$$3k^2-48 \geq 0$$
$$k^2-16 \geq 0$$
$$(k-4)(k+4) \geq 0$$

Thus $k \leq -4$ and $k \geq 4$

SHORT ANSWER TYPE QUESTIONS - I

1. Find the roots the quadratic equation $6x^2-x-2=0$.

Ans : [Board Term-2, 2012, Set (13)]

We have
$$6x^2-x-2 = 0$$
$$6x^2+3x-4x-2 = 0$$
$$3x(2x+1)-2(2x+1) = 0$$
$$(2x+1)(3x-2) = 0$$
$$3x-2 = 0 \text{ or } 2x+1=0$$
$$x = \frac{2}{3} \text{ or } x=-\frac{1}{2}$$

Hence roots of equation are $\frac{2}{3}$ and $-\frac{1}{2}$.

2. Find the roots of the following quadratic equation : $15x^2-10\sqrt{6}x+10 = 0$

Ans : [Board Term-2, 2012 Set (1)]

We have
$$15x^2-10\sqrt{6}x+10 = 0$$
$$3x^2-2\sqrt{6}x+2$$
$$= 0$$
$$3x^2-\sqrt{6}x-\sqrt{6}x+2$$
$$= 0$$
$$\sqrt{3}x(\sqrt{3}x-\sqrt{2})-\sqrt{2}(\sqrt{3}x-\sqrt{2})$$
$$= 0$$
$$(\sqrt{3}x-\sqrt{2})(\sqrt{3}x-\sqrt{2}) = 0$$

Thus $x = \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$

3. Solve the following quadratic equation for x : $4\sqrt{3}x^2+5x-2\sqrt{3} = 0$

Ans : [Board term-2, 2013, 2012, Set (22)]

We have
$$4\sqrt{3}x^2+5x-2\sqrt{3} = 0$$
$$4\sqrt{3}x^2+8x-3x-2\sqrt{3}$$
$$= 0$$
$$4x(\sqrt{3}x+2)-\sqrt{3}(\sqrt{3}x+2)$$
$$= 0$$
$$(\sqrt{3}x+2)(4x-\sqrt{3})$$
$$= 0$$

Thus $x = -\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

4. Solve for x : $x^2-(\sqrt{3}+1)x+\sqrt{3} = 0$

Ans : [Foreign Set, II, III, 2015

We have
$$x^2-(\sqrt{3}+1)x+\sqrt{3} = 0$$
$$x^2-\sqrt{3}x-1x+\sqrt{3} = 0$$
$$x(x-\sqrt{3})-1(x-\sqrt{3}) = 0$$
$$(x-\sqrt{3})(x-1) = 0$$

Thus $x = \sqrt{3}, x=1$

5. Find the roots of the following quadratic equation : $(x+3)(x-1) = 3\left(x-\frac{1}{3}\right)$

Ans : [Board Term-2, 2012, Set (52), 2011 Set (A1)]

We have
$$(x+3)(x-1) = 3\left(x-\frac{1}{3}\right)$$
$$x^2+2x-3 = 3x-1$$
$$x^2-x-2 = 0$$
$$x^2-2x+x-2 = 0$$
$$x(x-2)+1(x-2) = 0$$
$$(x-2)(x+1) = 0$$

Thus $x = 2, -1$

6. Find the roots of the following quadratic equation : $\frac{2}{5}x^2-x-\frac{3}{5} = 0$

Ans : [Board Term-2, 2012 Set (40)]

We have
$$\frac{2}{5}x^2-x-\frac{3}{5} = 0$$
$$\frac{2x^2-5x-3}{5} = 0$$
$$2x^2-5x-3 = 0$$
$$2x^2-6x+x-3 = 0$$
$$2x(x-3)+1(x-3) = 0$$
$$(2x+1)(x-3) = 0$$

Thus $x = -\frac{1}{2}, 3$

7. Solve the following quadratic equation for x :
$$4x^2-4a^2x+(a^4-b^4) = 0$$

Ans : [Delhi CBSE Term-2, 2015 (Set I, II)]

We have $4x^2-4a^2x+(a^4-b^4) = 0$

Compare with $Ax^2+Bx+C = 0$ we have
$$A = 4, B=-4a^2, C=(a^4-b^4)$$
$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$
$$= \frac{4a^2 \pm \sqrt{(-4a^2)^2-4 \times 4(a^4-b^4)}}{2 \times 4}$$

$$= \frac{4a^2 \pm \sqrt{16a^2 - 16a^4 + 16b^4}}{8}$$

$$= \frac{4a^2 \pm \sqrt{16b^4}}{8}$$

or, $$x = \frac{4a^2 \pm 4b^2}{8} = \frac{a^2 \pm b^2}{2}$$

Thus either $x = \frac{a^2+b^2}{2}$ or $x = \frac{a^2-b^2}{2}$

8. Solve the following quadratic equation for x :

$$9x^2 - 6b^2x - (a^4 - b^4) = 0$$

Ans : [Delhi CBSE Term-2, 2015 (Set III)]

We have $$9x^2 - 6b^2x - (a^4 - b^4) = 0$$

Compare with $ax^2 + bx + c = 0$ we have

$$a = 9, b = -6b^2, c = -(a^4 - b^4)$$

$$\begin{aligned} x &= \frac{6b^2 \pm \sqrt{(-6b^2)^2 - 4 \times 9 \times \{(a^4 - b^4)\}}}{2 \times 9} \\ &= \frac{6b^2 \pm \sqrt{36b^4 + 36a^4 - 36b^4}}{18} \\ &= \frac{6b^2 \pm \sqrt{36a^4}}{18} \\ &= \frac{6b^2 \pm 6a^2}{18} \end{aligned}$$

Thus $x = \frac{a^2+b^2}{3}, \frac{b^2-a^2}{3}$

9. Solve the following equation for x :

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

Ans : [Outside Delhi CBSE, 2015 Set I,II,III]

We have $$\begin{aligned} 4x^2 + 4bx + b^2 - a^2 &= 0 \\ (2x + b)^2 - a^2 &= 0 \\ (2x + b + a)(2x + b - a) &= 0 \\ x &= \frac{-(a+b)}{2}, x = \frac{a-b}{2} \end{aligned}$$

10. Solve the following quadratic equation for x :

$$x^2 - 2ax - (4b^2 - a^2) = 0$$

Ans : [Outside Delhi CBSE, 2015 Set III]

We have $$\begin{aligned} x^2 - 2ax - (4b^2 - a^2) &= 0 \\ x^2 - 2ax + a^2 - 4b^2 &= 0 \\ (x - a)^2 - (2b)^2 &= 0 \\ (x - a + 2b)(x - a - 2b) \end{aligned}$$

$= 0$

Thus $x = a - 2b, x = a + 2b$

11. Solve the quadratic equation, $2x^2 + ax - a^2 = 0$ for x. [Delhi CBSE, Term-2, 2014]

We have $$2x^2 + ax - a^2 = 0$$

Compare with $Ax^2 + Bx + C = 0$ we have

$$A = 2, B = a, C = -a^2$$

Now $$\begin{aligned} x &= \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \\ &= \frac{-a \pm \sqrt{a^2 - 4 \times 2 \times (-a^2)}}{2 \times 2} \\ &= \frac{-a \pm \sqrt{a^2 + 8a^2}}{4} \\ &= \frac{-a \pm \sqrt{9a^2}}{4} \\ &= \frac{-a \pm 3a}{4} \\ x &= \frac{-4 + 3a}{4}, \frac{-a - 3a}{4} \end{aligned}$$

Thus $x = \frac{a}{2}, -a$

12. Find the roots of the quadratic equation $4x^2 - 4px + (p^2 - q^2) = 0$

Ans : [Board Term-2, 2014]

We have $$4x^2 - 4px + (p^2 - q^2) = 0$$

Compare with $ax^2 + bx + c = 0$ we get

$$a = 4, b = -4p, c = (p^2 - q^2)$$

The roots are given by the quadratic formula

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{4p \pm \sqrt{16p^2 - 4 \times 4 \times (p^2 - q^2)}}{2 \times 4} \\ &= \frac{4p \pm \sqrt{16p^2 - 16p^2 + 16q^2}}{8} \\ &= \frac{4p \pm 4q}{8} \end{aligned}$$

Thus roots are $\frac{p+q}{2}, \frac{p-q}{2}$.

13. Sum of the areas of two squares is 468 m^2.If the difference of their perimeter is 24 m, find the sides of the squares.

Ans : [Board Term-2, 2012 Set (28), 2011 Set (A1)]

Let the side of the smaller square be y and be the side of the longer square be x, then we have

$$\begin{aligned} 4x - 4y &= 24 \\ x - y &= 6 \\ x &= y + 6 \end{aligned}$$

According to the question

$$\begin{aligned} x^2 + y^2 &= 468 \\ (y + 6)^2 + y^2 &= 468 \\ 2y^2 + 12y + 36 &= 468 \\ 2y^2 + 12y - 432 &= 0 \\ y^2 + 6y - 216 &= 0 \\ (y + 18)(y - 12) &= 0 \\ y &= -18, 12 \end{aligned}$$

As side can not be negative, $y = 12$ and $x = 12 + 6 = 18$ Hence, the side of larger square 18 m and that of smaller square 12 m.

14. Solve for x (in ferms of a and b) :

$$\frac{a}{x-b}+\frac{b}{x-a}=2, x \neq a, b$$

Ans : [Board Term-2 Foreign Set II, 2016]

We have $\frac{a(x-a)+b(x-b)}{(x-b)(x-a)}=2$

$$\begin{aligned} a(x-a)+b(x-b) &= 2\left[x^2-(a+b)x+ab\right] \\ ax-a^2+bx-b^2 &= 2x^2-2(a+b)x+2ab \\ 2x^2-3(a+b)x+(a+b)^2 &= 0 \\ 2x^2-2(a+b)x-(a-b)x+(a+b)^2 &= 0 \\ \left[2x-(a+b)\right]\left[x-(a+b)\right] &= 0 \end{aligned}$$

Thus $x = a+b, \frac{a+b}{2}$

15. Solve for x : $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

Ans : [Board Term-2 Foreign Set II, 2016]

We have $\sqrt{3}x^2-3\sqrt{2}x+\sqrt{2}x-2\sqrt{3} = 0$

$$\begin{aligned} \sqrt{3}x[x-\sqrt{6}]+\sqrt{2}[x-\sqrt{6}] &= 0 \\ (x-\sqrt{6})(\sqrt{3}x+\sqrt{2}) &= 0 \end{aligned}$$

Thus $x \quad \sqrt{6}, -\sqrt{\frac{2}{3}}$

16. If $x=\frac{2}{3}$ and $x=-3$ are roots of the quadratic equation $ax^2+7x+b=0$, find the values of a and b.

Ans : [Board Term-2 Delhi Set I, II, III, 2016]

We have $ax^2+7x+b=0$

Substituting $x=\frac{2}{3}$ in above equation we obtain

$$\begin{aligned} \frac{4}{9}a+\frac{14}{3}+b &= 0 \\ 4a+42+9b &= 0 \\ 4a+9b &= -42 \quad (1) \end{aligned}$$

and substituting $x=-3$ we obtain

$$\begin{aligned} 9a-21+b &= 0 \\ 9a+b &= 21 \quad (2) \end{aligned}$$

Solving (1) and (2), we get $a = 3$ and $b=-6$

17. Solve for x : $\sqrt{6x+7}-(2x-7)=0$

Ans : [O. D. Set III, 2016]

We have $\sqrt{6x+7}-(2x-7) = 0$

or, $\sqrt{6x+7} = (2x-7)$

Squaring both sides we get

$$\begin{aligned} 6x+7 &= (2x-7)^2 \\ 6x+7 &= 4x^2-28x+49 \\ 4x^2-34x+42 &= 0 \\ 2x^2-17x+21 &= 0 \\ 2x^2-14x-3x+21 &= 0 \\ 2x(x-7)-3(x-7) &= 0 \\ (x-7)(2x-3) &= 0 \end{aligned}$$

Thus $x = 7$ and $x=\frac{2}{3}$

18. Find the roots of $x^2-4x-8=0$ by the method of completing square.

Ans : [Board Term-2, 2015]

We have $x^2-4x-8 = 0$

Squaring both side we have

$$\begin{aligned} (x-2)^2-8-4 &= 0 \\ (x-2)^2-12 &= 0 \\ (x-2)^2 &= 12 \\ (x-2)^2 &= (2\sqrt{3})^2 \\ x-2 &= \pm 2\sqrt{3} \\ x &= 2 \pm 2\sqrt{3} \end{aligned}$$

Thus $x = 2+2\sqrt{3},\ 2-2\sqrt{3}$

19. A two digit number is four times the sum of the digits. It also equal to 3 times the product of digits. Find the number.

Ans : [Board Term-2, Foreign Set I, 2016]

Let units digit and tens digit of the two digit number be x and y respectively.

Thus number is $10y+x$

According to question, we have

$$\begin{aligned} 10y+x &= 4(y+x) \\ 10y+x &= 4y+4x \\ 10y-4y &= 4x-x \\ 6y &= 3x \\ 2y &= x \end{aligned}$$

Also,

$$\begin{aligned} 10y+x &= 3xy \\ 10y+2y &= 3(2y)y \\ 12y &= 6y^2 \\ 6y^2-12y &= 0 \\ 6y(y-2) &= 0 \\ y &= 0 \text{ or } y=2 \end{aligned}$$

As the number can not be zero $x = 4$ and $x=2y=4$.

Thus required number is 24.

20. In a cricket match, Harbhajan took three wickets less than twice the number of wickets taken by Zahir. The Product of the number of wickets taken by these two is 20. Represent the above situation in the form of quadratic equation.

[Board Term-2, 2015]

Ans : [CBSE Marking Scheme, 2015]

Let the number of wickets is taken by Zahir be x, then number of wickets taken by Harbhajan will be $2x-3$.

According to question,

$$\begin{aligned} x(2x-3) &= 20 \\ 2x^2-3x &= 20 \end{aligned}$$

Thus required quadratic equation,

$$2x^2-3x-20 = 0$$

21. Solve for x : $\sqrt{2x+9}+x=13$

Ans : [Board Term-2 Outside Delhi Set II 2016]

We have
$$\begin{aligned} \sqrt{2x+9}+x &= 13 \\ \sqrt{2x+9} &= 13-x \end{aligned}$$
Squaring both side we have
$$\begin{aligned} 2x+9 &= (13-x)^2 \\ 2x+9 &= 169+x^2-26x \\ 0 \quad & x^2+169-26x-9-2x \\ x^2-28x+160 &= 0 \\ x^2-20x-8x+160 &= 0 \\ x(x-20)-8(x-20) &= 0 \\ (x-8)(x-20) &= 0 \end{aligned}$$
Thus $x = 8$ and $x=20$.

22. Find the roots of the quadratic equation $\sqrt{2}\,x^2+7x+5\sqrt{2}=0$

Ans : [Board Term-II Outside Delhi, 2017]

We have
$$\begin{aligned} \sqrt{2}\,x^2+7x+5\sqrt{2} &= 0 \\ \sqrt{2}\,x^2+2x+5x+5\sqrt{2} &= 0 \\ \sqrt{2}\,x(x+\sqrt{2})+5(x+\sqrt{2}) &= 0 \\ (x+\sqrt{2})(\sqrt{2}\,x+5) &= 0 \end{aligned}$$
Thus $x=-\sqrt{2}$ and $=-\dfrac{5}{\sqrt{2}} =-\dfrac{5}{\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}} =-\dfrac{5}{2}\sqrt{2}$

23. Find the value of k for which the roots of the quadratic equation $2x^2+kx+8=0$ will have the equal roots ?

[Board Term-II Outside Delhi Compt., 2017]

Ans :

We have $\quad 2x^2+kx+8=0$

Compare with $ax^2+bx+c = 0$ we get
$$a = 2, b=k, \text{ and } c=8$$
For equal roots, $D=0$
$$\begin{aligned} b^2-4ac &= 0 \\ k^2-4\times2\times8 &= 0 \\ k^2 &= 64 \\ k &= \pm\sqrt{64} \end{aligned}$$
Thus $k = \pm 8$

24. Solve for x : $\sqrt{3}\,x^2+10x+7\sqrt{3}=0$

Ans : [Board Term-II Foreign 2017 Set-2]

We have
$$\begin{aligned} \sqrt{3}\,x^2+10x+7\sqrt{3} &= 0 \\ \sqrt{3}\,x^2+3x+7x+7\sqrt{3} &= 0 \\ \sqrt{3x}(x+\sqrt{3})+7(x+\sqrt{3}) &= 0 \\ (x+\sqrt{3})(\sqrt{3}\,x+7) &= 0 \end{aligned}$$
Thus $x = -\sqrt{3}$ and $x=-\dfrac{7}{\sqrt{3}}$

SHORT ANSWER TYPE QUESTIONS - II

1. Solve for x :

$$\frac{x+1}{x-1}+\frac{x-2}{x+2}=4-\frac{2x+3}{x-2}; x\neq 1,-2,2$$

Ans : [Board Term-2 Delhi Set II, 2016]

We have
$$\begin{aligned} \frac{x+1}{x-1}+\frac{x-2}{x+2} &= 4-\frac{2x+3}{x-2} \\ \frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} &= \frac{4x-8-2x-3}{x-2} \\ \frac{2x^2+4}{x^2+x-2} &= \frac{2x-11}{x-2} \\ (2x^2+4)(x-2) &= (2x-11)(x^2+x-2) \\ 5x^2+19x-30 &= 0 \\ (5x-6)(x+5) &= 0 \\ x &= -5, \frac{6}{5} \end{aligned}$$

2. Solve for x :

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0, x\neq 3, -\frac{3}{2}$$

Ans : [Board Term-2, Delhi Set I, 2016]

We have
$$\begin{aligned} 2x(2x+3)+(x-3)+(3x+9) &= 0 \\ 2x^2+5x+3 &= 0 \\ (x+1)(2x+3) &= 0 \end{aligned}$$
Thus $x = -1, x=-\dfrac{3}{2}$

3. Solve for x : $\dfrac{1}{x}+\dfrac{2}{2x-3}=\dfrac{1}{x-2}, x\neq 0,\dfrac{2}{3},2.$

Ans : [Board Term-2, Foreign Set II, 2016]

We have
$$\begin{aligned} \frac{1}{x}+\frac{2}{2x-3} &= \frac{1}{x-2} \\ \frac{2x-3+2x}{x(2x-3)} &= \frac{1}{x-2} \\ \frac{4x-3}{x(2x-3)} &= \frac{1}{x-2} \\ (x-2)(4x-3) &= 2x^2-3x \\ 4x^2-11x+6 &= 2x^2-3x \\ 2x^2-8x+6 &= 0 \\ x^2-4x+3 &= 0 \\ (x-1)(x-3) &= 0 \end{aligned}$$
Thus $x = 1,3$

4. Solve the following quadratic equation for x :

$$x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$$

Ans : [Board Term-2, Delhi Set III, 2016]

We have
$$x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$$
$$x^2+\frac{a}{a+b}x+\frac{a+b}{a}x+1=0$$
$$x\left(x+\frac{a}{a+b}\right)+\frac{a+b}{a}\left(x+\frac{a}{a+b}\right)=0$$
$$\left(x+\frac{a}{a+b}\right)\left(x+\frac{a+b}{a}\right)=0$$

Thus
$$x=\frac{-a}{a+b},\frac{-(a+b)}{a}$$

5. Solve for x :
$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3};x\neq 1,2,3$$

Ans : [Board Term-2, O.D. Set I, 2016]

We have
$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}$$
$$\frac{x-3+x-1}{(x-1)(x-2)(x-3)}=\frac{2}{3}$$
$$\frac{2x=4}{(x-1)(x-2)(x-3)}=\frac{2}{3}$$
$$\frac{2(x-2)}{(x-1)(x-2)(x-3)}=\frac{2}{3}$$
$$\frac{2}{(x-1)(x-3)}=\frac{2}{3}$$
$$3=(x-1)(x-3)$$
$$x^2-4x+3=3$$
$$x^2-4x=0$$
$$x(x-4)=0$$

Thus $x=0$ or $x=4$

6. Solve for x : $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

Ans : [Board Term-2, Outside Delhi CBSE, 2015 Set I, III, Foreign Set I, II, II, 2014]

We have
$$\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$$
$$\sqrt{3}x^2-[3\sqrt{2}-\sqrt{2}]x-2\sqrt{3}=0$$
$$\sqrt{3}x^2-3\sqrt{2}x+\sqrt{2}x-2\sqrt{3}=0$$
$$\sqrt{3}x^2-\sqrt{3}\sqrt{3}\sqrt{2}x+\sqrt{2}x-\sqrt{2}\sqrt{2}\sqrt{3}=0$$
$$\sqrt{3}x[x-\sqrt{3}.\sqrt{2}]+\sqrt{2}[x-\sqrt{2}\sqrt{3}]=0$$
$$\sqrt{3}x[x-\sqrt{6}]+\sqrt{2}[x-\sqrt{6}]=0$$
$$(x-\sqrt{6})(\sqrt{3}x+\sqrt{2})=0$$

Thus $x=\sqrt{6}=-\sqrt{\frac{2}{3}}$

7. Solve for x : $2x^2+6\sqrt{3}x-60=0$

Ans : [Board Term-2, O.D. CBSE, 2015, Set II]

We have
$$2x^2+6\sqrt{3}x-60=0$$
$$x^2+x\sqrt{3}x-30=0$$
$$x^2+5\sqrt{3}x-2\sqrt{3}x-30=0$$
$$x(x+5\sqrt{3})-2\sqrt{3}(x+5\sqrt{3})=0$$
$$(x+5\sqrt{3})(x-2\sqrt{3})=0$$

Thus $x=-5\sqrt{3},2\sqrt{3}$

8. Solve for x : $x^2+5x-(a^2+a-6)=0$

Ans : [Board Term-2 Foreign Set I, 2015]

$$x^3+5x-(a^2+a-6)=0$$
$$x=\frac{-5\pm\sqrt{25+4(a^2+a-6)}}{2}$$
$$\left[\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\right]$$
$$=\frac{-5\pm(2a+1)}{2}$$
$$=\frac{2a-4}{2},\frac{-2a-6}{2}$$
$$x=a-2,x=-(a+3)$$

9. Solve for x : $x^2-(2b-1)x+(b^2-b-20)=0$

Ans : [Board Term-2 Foreign Set II, 2015]

We have $x^2-(2b-1)x+(b^2-b-20)=0$

Compare with $Ax^2+Bx+C=0$ we have
$$A=1,B=-(2b-1),C=(b^2-b-20)$$
$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$
$$x=\frac{(2b-1)\pm\sqrt{(2b-1)^2-4(b^2-b-20)}}{2}$$
$$=\frac{(2b-1)\pm 9}{2}$$
$$=\frac{2b+8}{2},\frac{2b-10}{2}$$
$$=b+4,\ b-5$$

Thus $x=b+4$ and $x=b-5$

10. Solve for x : $\frac{16}{x}-1=\frac{15}{x+1};x\neq 0,-1$

Ans : [Board Term-2, OD 2014]

We have
$$\frac{16}{x}-1=\frac{15}{x+1}$$
$$\frac{16}{x}-\frac{15}{x+1}=1$$
$$16(x+1)-15x=x^2+x$$
$$16x+16-15x=x^2+x$$
$$x+16=x^2+x$$
$$x^2-16=0$$
$$x^2=16$$
$$x=\pm 4$$

Thus $x=-4$ and $x=+4$

11. Solve the quadratic equation $(x-1)^2-5(x-1)-6=0$

Ans : [Board Term-2, 2015]

We have
$$\begin{aligned}(x-1)^2-5(x-1)-6 &= 0\\ x^2-2x+1-5x+5-6 &= 0\\ x^2-7x+6-6 &= 0\\ x^2-7x &= 0\\ x(x-7) &= 0\end{aligned}$$

Thus $x = 0, 7$

12. Solve the equation for x : $\frac{4}{3}-3 = \frac{5}{2x+3}; x \neq 0, \frac{-3}{2}$

Ans : [Board Term-2 Delhi CBSE, 2014]

We have
$$\begin{aligned}\frac{4}{x}-3 &= \frac{5}{2x+3}\\ \frac{4}{x}-\frac{5}{2x+3} &= 3\\ \frac{4(2x+3)-5x}{x(2x+3)} &= 3\\ 8x+12-5x &= 3x(2x+3)\\ 3x+12 &= 6x^2+9x\\ 6x^2+6x-12 &= 0\\ x^2+x-2 &= 0\\ x^2+2x-x-2 &= 0\\ x(x+2)-(x+2) &= 0\\ (x+2)(x-1) &= 0\end{aligned}$$

Thus $x = -1, -2$

13. Find the roots of the equation $2x^2+x-4=0$, by the method of completing the squares.

Ans : [KVS, 2014]

We have
$$\begin{aligned}2x^2+x-4 &= 0\\ x^2+\frac{x}{2}-2 &= 0\\ x^2+2x\left(\frac{1}{4}\right)-2 &= 0\end{aligned}$$

Adding and subtracting $\left(\frac{1}{4}\right)^2$, we get
$$\begin{aligned}x^2+2x\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-2 &= 0\\ \left(x+\frac{1}{4}\right)^2-\left(\frac{1}{16}+2\right) &= 0\\ \left(x+\frac{1}{4}\right)^2-\left(\frac{1+32}{16}\right) &= 0\\ \left(x+\frac{1}{4}\right)^2-\frac{33}{16} &= 0\\ \left(x+\frac{1}{4}\right)^2 &= \frac{33}{16}\\ \left(x+\frac{1}{4}\right) &= \pm\frac{\sqrt{33}}{4}\end{aligned}$$

Thud toots are $x = \frac{-1\pm\sqrt{33}}{4}, -\left(\frac{1+\sqrt{33}}{4}\right)$

14. Solve for x : $9x^2-6ax+(a^2-b^2) = 0$

Ans : [Delhi CBSE Term-2, Board Term-2, 2012, Set (40)]

We have $9x^2-6ax+a^2-b^2=0$

Compare with $Ax^2+Bx+C = 0$ we have
$$\begin{aligned}A &= 9, B=-6a, C=(a^2-b^2)\\ x &= \frac{-B\pm\sqrt{B^2-4AC}}{2A}\\ x &= \frac{6a\pm\sqrt{(-6a)^2-4\times 9kt(a^2-b^2)}}{2\times 9}\\ &= \frac{6a\pm\sqrt{36a^2-36a^2+36b^2}}{18}\\ &= \frac{6a+6b}{18}, x=\frac{6a-6b}{18}\\ &= \frac{6(a+b)}{18}, x=\frac{6(a-b)}{18}\\ &\frac{a+b}{3}, x=\frac{a-b}{3}\end{aligned}$$

15. Solve the equation $\frac{1}{x+4}-\frac{1}{x-7} = \frac{11}{30}, x\neq -4, 7$ for x.

Ans : [Board Term-2, 2012 Set (1)]

We have,
$$\begin{aligned}\frac{1}{x+4}-\frac{1}{x-7} &= \frac{11}{30}\\ \frac{x-7-x-4}{(x+4)(x-7)} &= \frac{11}{30}\\ \frac{-11}{(x+4)(x-7)} &= \frac{11}{30}\\ \frac{-1}{(x+4)(x-7)} &= \frac{1}{30}\\ (x+4)(x-7) &= -30\\ x^2-3x-28 &= -30\\ x^2-3x+2 &= 0\\ x^2-2x-x+2 &= 0\\ (x-1)(x-2) &= 0\end{aligned}$$

Thus $x = 1, 2$.

16. Find the roots of the quadratic equation :
$$a^2b^2x^2+b^2x-a^2x-1 = 0$$

Ans : [Board Term-2, 2012 (31)]

We have
$$\begin{aligned}a^2b^2x^2+b^2x-a^2x-1 &= 0\\ b^2x(a^2x+1)-1(a^2x+1) &= 0\\ (b^2x-1)(a^2x+1) &= 0\\ x &= \frac{1}{b^2}, \text{ or } x=-\frac{1}{a^2}\end{aligned}$$

Hence, roots are $\frac{1}{b^2}$ and $-\frac{1}{a^2}$.

17. Solve the following quadratic equation for x :
$p^2x^2+(p^2-q^2)x-q^2 = 0$

Ans : [Board Term-2, 2012 Set (A1)]

We have $p^2x^2+(p^2-q^2)x-q^2 = 0$

Compare with $ax^2+bx+c = 0$ we get
$$a = p^2, b=p^2-q^2, c=-q^2$$

The roots are given by the quadratic formula
$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(p^2-q^2) - \sqrt{(p^2-q^2)^2 - 4(p^2)(-q^2)}}{2p^2}$$

$$= \frac{-(p^2-q^2) - \sqrt{p^4+q^4-2p^2q^2+4p^2q^2}}{2p^2}$$

$$= \frac{-(p^2-q^2) - \sqrt{p^4+q^4+2p^2q^2}}{2p^2}$$

$$= \frac{-(p^2-q^2) - \sqrt{(p^2+q^2)^2}}{2p^2}$$

$$= \frac{-(p^2-q^2) \pm (p^2+q^2)}{2p^2}$$

Thus $\quad x = \dfrac{-(p^2-q^2)+(p^2+q^2)}{2p^2} = \dfrac{2q^2}{2p^2} = \dfrac{q^2}{p^2}$

and $\quad x = \dfrac{-(p^2-q^2)-(p^2+q^2)}{2p^2} = \dfrac{-2p^2}{2p^2} = -1$

Hence, roots are $\dfrac{q^2}{p^2}, -1$

18. Solve the following quadratic equation for x :

$$9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$$

Ans : [Board Term-2, Foreign Set I, 2016]

We have $\quad 9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$

Now $\quad 2a^2 + 5ab + 2b^2 = 2a^2 + 4ab + ab + 2b^2$

$$= 2a[a+2b] + b[a+2b]$$

$$= (a+2b)(2a+b)$$

Hence the equation becomes

$$9x^2 - 9(a+b)x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3[3a+3b]x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3\big[(a+2b)+(2a+b)\big]x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3(a+2b)x - 3(2a+b)x + (a+2b)(2a+b) = 0$$

$$3x\big[3x-(a+2b)\big] - (2a+b)\big[3x-(a+2b)\big] = 0$$

$$\big[3x-(a+2b)\big]\big[3x-(2a+b)\big] = 0$$

$$3x-(2a+b) = 0$$

$$x = \frac{a+2b}{3}$$

$$3x-(2a+b) = 0$$

$$x = \frac{2a+b}{3}$$

Hence, roots are $\dfrac{a+2b}{3}$ and $\dfrac{2a+b}{3}$.

19. Solve for x : $x^2 + 6x - (a^2 + 2a - 8)$

Ans : [Board Term-2, Foreign Set III, 2015]

We have $\quad x^2 + 6x - (a^2 + 2a - 8) = 0$

Compare with $Ax^2 + Bx + C = 0$ we get

$$A = 1, B = 6, C = (a^2 + 2a - 8)$$

The roots are given by the quadratic formula

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-6 \pm \sqrt{36 + 4(a^2 + 2a - 8)}}{2}$$

$$= \frac{-6 \pm (2a+2)}{2}$$

Thus $\quad x = \dfrac{-6+(2a+2)}{2} = a-2$

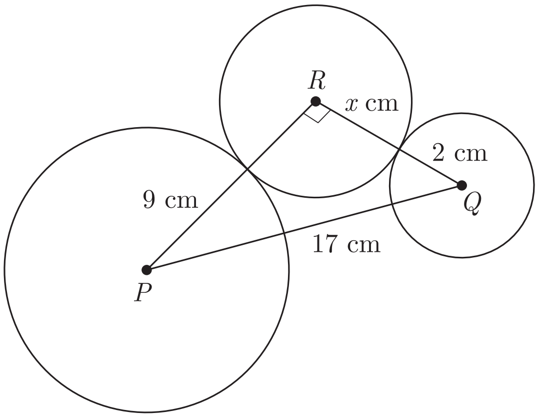
and $\quad x = \dfrac{-6-(2a+2)}{2} = -a-4$

Thus $x = a-2, -a-4$

20. p and Q are centres of circles of radii 9 cm and 2 cm respectively. $PQ = 17$ cm. R is the centre of the circle of radius x cm which touch given circles externally. Given that angle PRQ is 90°. Write an equation in x and solve it.

Ans : [Board Term-2, SQP, 2016]

As per question statement figure is given below.



In right ΔPQR, by Pythagoras theorem

$$PQ^2 = PR^2 + RQ^2$$

or, $\quad 17^2 = (x+9)^2 + (x+2)^2$

$$289 = x^2 + 18x + 81 + x^2 + 4x + 4$$

$$289 = 2x^2 + 22x + 85$$

$$0 = 2x^2 + 22x + 85 - 289$$

$$0 = 2x^2 + 22x - 204$$

$$x^2 + 11x - 102 = 0$$

$$x^2 + 17x - 6x - 102 = 0$$

$$x(x+17) - 6(x+17) = 0$$

$$(x-6)(x+17) = 0$$

$$x = 6 \text{ or } x = -17$$

As x can't be negative, $x = 6$.

21. Three consecutive natural number are such that the squre of the middle number exceeds the difference of the squares of the other two by 60. Find the number.

Ans : [Board Term-2, O.D. Set III, 2016]

Let the three consecutive natural numbers be $x, x+1$ and $x+2$.

$$\begin{aligned} \text{Now} \qquad (x+1)^2 &= (x+2)^2 - (x)^2 + 60 \\ x^2 + 2x + 1 &= x^2 + 4x + 4 - x^2 + 60 \\ x^2 - 2x - 63 &= 0 \\ x^2 - 9x + 7x - 63 &= 0 \\ x(x-9) + 7(x-9) &= 0 \\ (x-9)(x+7) &= 0 \\ x &= 9 \text{ or } x = -7 \end{aligned}$$

As x can't be negative, $x = 9$.

Hence three numbers are 9, 10, 11.

22. If $(x^2 + y^2)(a^2 + b^2) = (ax + by)^2$. Prove that $\frac{x}{a} = \frac{y}{b}$

Ans : [Board Term-2, 2014]

$$\begin{aligned} \text{We have} \qquad (x^2 + y^2)(a^2 + b^2) &= (ax + by)^2 \\ x^2 a^2 + x^2 b^2 + y^2 a^2 + y^2 b^2 &= a^2 x^2 + b^2 y^2 + 2abxy \\ x^2 b^2 + y^2 a^2 - 2abxy &= 0 \\ (xb - ya)^2 &= 0 \\ xb &= ya \end{aligned}$$

$$\text{Thus} \qquad \frac{x}{a} = \frac{y}{b} \qquad \text{Hence Proved.}$$

23. The sum of ages (in years) of a son and his father is 35 years and product of theirs ages is 150 years, find their ages.

Ans : [Delhi Term-2, 2014, Term-2, 2012 Set (40)]

Let the age of father be x years and age of son be y years

$$\text{Now} \qquad x + y = 35 \qquad (1)$$

$$\text{and} \qquad xy = 150 \qquad (2)$$

Putting the value of y, from (1) we have

$$\begin{aligned} x(35 - x) &= 150 \\ x^2 - 35x + 150 &= 0 \\ (x-30)(x-5) &= 0 \\ x &= 30, x = 5 \text{ (Rejected)} \end{aligned}$$

Age of father cant be 5 years, so we reject $x = 5$ and take $x = 30$.

$$\text{Now} \qquad y = 5$$

Hence the age of father is 30 years and the age of son is 5 years.

24. One fourth of a herd of camels was seen in forest. Twice of square root of the herd had gone to mountain and remaining 15 camels were seen on the bank of a river, find the total number of camels.

Ans : [Board Term-2, 2012 Set (1)]

Let the total number of camels be x.

According to the question,

$$\begin{aligned} \frac{x}{4} + 2\sqrt{x} + 15 &= x \\ 3x - 8\sqrt{x} - 60 &= 0 \end{aligned}$$

Let $\sqrt{x} = y$, then we have

$$\begin{aligned} 3y^2 - 8y - 60 &= 0 \\ 3y^2 - 18y + 10y - 60 &= 0 \\ 3y(y-6) + 10(y-6) &= 0 \\ (3y+10)(y-6) &= 0 \\ y &= 6 \text{ or } y = -\frac{10}{3} \end{aligned}$$

Here $y = -\frac{10}{3}$ is not possible.

Thus $y = 6$ or $y^2 = 36$, $\quad x = y^2 = 36$

Hence the number of camels is 36.

25. The sum of the squares of two consecutive naturals is 421. Find the numbers.

[Board Term-2, 2012 Set (12)]

Ans : [CBSE Marking Scheme, 2012]

Let the first natural number be x. The second consecutive natural will be $.x + 1$

According to the question,

$$\begin{aligned} x^2 + (x+1)^2 &= 421 \\ x^2 + x^2 + 2x + 1 &= 421 \\ x^2 + x - 210 &= 0 \\ x^2 + 15x - 14x - 210 &= 0 \\ x(x+15) - 14(x+15) &= 0 \\ (x+15)(x-14) &= 0 \\ x + 15 &= 0 \text{ or } x - 14 = 0 \\ x &= -15 \text{ or } x = 14 \end{aligned}$$

Rejecting negative value $x = 14$.

Therefore first number is 14 and consecutive number is 15.

26. In a class test, the sum of the marks obtained by a student in mathematics and science is 28. Had he got 3 marks more in mathematics and 4 marks less in science, the product of the marks would have been 180. Find his marks in two subjects.

Ans : [Board Term-2 2012, Set (21)]

Let marks obtained in maths be x, the marks obtained in science will be $\quad 28 - x$

$$\begin{aligned} \text{Now} \qquad (x+3)(28 - x - 4) &= 180 \\ (x+3)(24 - x) &= 180 \\ 24x - x^2 + 72 - 3x &= 180 \\ x^2 - 21x + 108 &= 0 \\ (x-9)(x-12) &= 0 \\ x &= 9 \text{ or } x = 12 \end{aligned}$$

Case I : $x = 9$

Marks obtained in maths $= 9$

Marks obtained in science $= 28 - 9 = 19$

Case II : $x = 12$

Marks obtained in maths $= 12$

Marks obtained in science $= 28 - 12 = 16$

27. If the roots of the equation $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$ are equal, prove that $\frac{a}{b},\frac{c}{d}$.

Ans : [Board 2016]

We have $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$

Compare with $Ax^2 + Bx + C = 0$ we get

$A = (a^2+b^2), B = -2(ac+bd), C = (c^2+d^2)$

If roots are equal, $D = B^2 - 4AC = 0$

or $B^2 = 4AC$

$$\begin{aligned} \text{Now } \left[-2(ac+bd)\right]^2 &= 4(a^2+b^2)(c^2+d^2) \\ (a^2c^2 + 2abcd + b^2d^2) &= 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) \\ 2abcd &= a^2d^2 + b^2c^2 \\ 0 &= a^2d^2 - 2abcd + b^2c^2 \\ 0 &= (ad-bc)^2 \\ 0 &= ad - bc \\ ad &= bc \\ \frac{a}{b} &= \frac{c}{d} \end{aligned}$$

Hence Proved

LONG ANSWER TYPE QUESTIONS

1. Solve for x : $\left(\frac{2x}{x-5}\right)^2 + \left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$

Ans : [CBSE S.A.-2, 2016, Set-HODM4OL]

We have $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$

Let $\frac{2x}{x-5} = y$ then we have

$$\begin{aligned} y^2 + 5y - 24 &= 0 \\ (y+8)(y-3) &= 0 \\ y &= 3, -8 \end{aligned}$$

Taking $y = 3$ we have

$$\begin{aligned} \frac{2x}{x-5} &= 3 \\ 2x &= 3x - 15 \\ x &= 15 \end{aligned}$$

Taking $y = -8$ we have

$$\begin{aligned} \frac{2x}{x-5} &= -8 \\ 2x &= -8x + 40 \\ 10x &= 40 \\ x &= 4 \end{aligned}$$

Hence, $x = 15, 4$

2. Solve for x : $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4} \quad x \neq -1, -2, -4$

Ans : [Bord Term-2 O.D. Set I, 2016]

We have $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$

$$\begin{aligned} \frac{x+2+2(x+1)}{(x+1)(x+2)} &= \frac{4}{x+4} \\ \frac{3x+4}{x^2+3x+2} &= \frac{4}{x+4} \\ (3x+4)(x+4) &= 4(x^2+3x+2) \\ 3x^2 + 16x + 16 &= 4x^2 + 12x + 8 \\ x^2 - 4x - 8 &= 0 \end{aligned}$$

$$\begin{aligned} \text{Now} \qquad x &= \frac{-b\sqrt{b^2+4ac}}{2a} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2 \times 1} \\ &= \frac{4 \pm \sqrt{16+32}}{2} \\ &= \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2} \\ &= 2 \pm 2\sqrt{3} \end{aligned}$$

Hence, $x = 2 + 2\sqrt{3}$ and $2 - 2\sqrt{3}$

3. Find x in terms of a, b and c :

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a, b, c$$

Ans : [Board Term-2, Delhi Set 1, 2016]

We have $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$

$a(x-b)(x-c) + b(x-a)(x-c) = 2c(x-a)(x-b)$

$ax^2 - abx - acx + abc + bx^2 - bax - bcx + abc$

$= 2cx^2 - 2cxb - 2cxa + 2abc$

$ax^2 + bx^2 - 2cx^2 - abx - acx - bax - bcx + 2cbx + 2acx$

$= 0$

$$\begin{aligned} x^2(a+b-2c) - 2abx + acx + bcx &= 0 \\ x^2(a+b-2c) + x(-2ab + ac + bc) &= 0 \end{aligned}$$

Thus $x = -\left(\frac{ac+bc-2ab}{a+b-2c}\right)$

4. Solve for x : $\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}; x =, -1, \frac{1}{4}$

Ans : [Delhi CBSE Board, 2015 (Set 3)]

We have $\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}$

$$\frac{3x-3+4x+4}{x^2-1} = \frac{29}{4x-1}$$

$$\begin{aligned}\frac{7x+1}{x^2-1} &= \frac{29}{4x-1}\\ (7x+1)(4x-1) &= 29x^2-29\\ 28x^2-7x+4x-1 &= 29x^2-29\\ -3x &= x^2-28\\ x^2+3x-28 &= 0\\ x^2+7x-4x-28 &= 0\\ x(x+7)-4(x+7) &= 0\\ (x+7)(x-4) &= 0\end{aligned}$$

Hence, $x = 4, -7$

5. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank, find the time in which each pipe would fill the tank separately.

Ans : [O. D. Set III, 2016]

Let time taken by pipe A be x minutes and time taken by pipe B be $x+5$ minutes.

In one minute pipe A will fill $\frac{1}{x}$ tank.

In one minute pipe B will fill $\frac{1}{x+5}$ tank.

Thus pipes $A+B$ will fill $\frac{1}{x}+\frac{1}{x+5}$ tank in one minute.

As per question, two pipes running together can fill a tank in $11\frac{1}{9}=\frac{100}{9}$ minutes, in one minute $\frac{9}{100}$ tank will be filled.

Now according to the question we have

$$\begin{aligned}\frac{1}{x}+\frac{1}{x+5} &= \frac{9}{100}\\ \frac{x+5+x}{x(x+5)} &= \frac{9}{100}\\ 100(2x+5) &= 9x(x+5)\\ 200x+500 &= 9x^2+45x\\ 9x^2-155x-500 &= 0\\ 9x^2-180x+25x-500 &= 0\\ 9x(x-20)+25(x-20) &= 0\\ (x-20)(9x+25) &= 0\\ x &= 20, \frac{-25}{9}\end{aligned}$$

As time can't be negative we take $x = 20$ minutes

and $x+5 = 25$ minutes

Hence pipe A will fill the tank in 20 minutes and pipe B will fill it in 25 minutes.

6. The time taken by a person to cover 150 km was $2\frac{1}{2}$ hours more than the time taken in the return journey. If he returned at a speed of 10 km/hour more than the speed while going, find the speed per hour in each direction.

Ans : [Delhi Set III, 2016]

Let the speed while going be x km/h

Speed while returning $=(x+10)$ km/h

According to question we have

$$\frac{150}{x}-\frac{150}{x+10} = \frac{5}{2}$$

$$\begin{aligned}x^2+10x-600 &= 0\\ (x+30)(x-20) &= 0\\ x &= 20\end{aligned}$$

Speed while going is 20 km/h and speed while returning will be $= 20+10 = 30$ km/h

7. The denominator of a fraction is one more than twice its numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

Ans : [Foreign Set III, 2016]

Let numerator be x then fraction will be $\frac{x}{2x+1}$

As per the question we have

$$\frac{x}{2x+1}+\frac{2x+1}{x} = 2\frac{16}{21} = \frac{58}{21}$$

$$21\left[x^2+(2x+1)^2\right] = 58(2x^2+x)$$

or, $$11x^2-26x-21 = 0$$

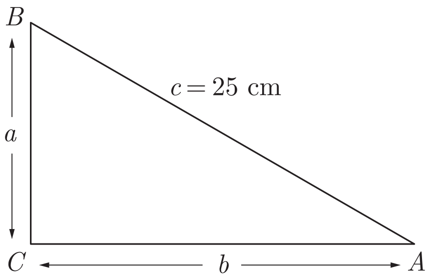
$$\begin{aligned}11x^2+33x+7x-20 &= 0\\ x &= 3, -\frac{7}{11} \text{ (rejected)}\end{aligned}$$

We reject $x=-\frac{7}{3}$, thus $x=3$ and fraction will be $\frac{3}{6+1}=\frac{3}{7}$

8. The perimeter of a right triangle is 60 cm. Its hypotenuse is 25 cm. Find the area of the triangle.

Ans : [Delhi Set II, 2016]

As per question statement figure is given below.



Here $$a+b+c = 60, c = 25$$

$$a+b = 60-c = 60-25 = 35$$

Using Pythagoras theorem

$$a^2+b^2 = 25^2 = 625$$

Substituting the values in $(a+b)^2 = a^2+b^2+2ab$,

$$\begin{aligned}35^2 &= 625+2ab\\ 1225-625 &= 2ab\end{aligned}$$

or, $$ab = 300$$

Hence, Area of ΔABC

$$\frac{1}{2}ab = 150 \text{ cm}^2.$$

9. Two water taps together can fill a tank in 9 hours 36 minutes. The tap of larger diameter takes 8 hours less than the smaller one to fill the tank. Find the time in which each tap can separately fill the tank.

Ans : [Foreign Set III, 2016]

Let the tap with smaller diameter fills the tank in x hours, then the other tap fills the tank in $(x-8)$

hours

In one hour small tap will fill $\frac{1}{x}$ tank.

In one hour large tap will fill $\frac{1}{x-8}$ tank.

Thus both tap will fill $\frac{1}{x}+\frac{1}{x-8}$ tank in one hour.

9 hours 36 minutes$= 9+\frac{36}{60} = 9+\frac{3}{5} = \frac{48}{3}$

Since two water taps together can fill a tank in $\frac{48}{3}$ hour, tank fill by both pipe in one hour is $\frac{1}{\frac{48}{5}} = \frac{5}{48}$.

Thus
$$\begin{aligned}\frac{1}{x}+\frac{1}{x-8} &= \frac{5}{48}\\ \frac{x-8+x}{x(x-8)} &= \frac{5}{48}\\ 5x(x-8) &= (2x-8)48\\ 5x^2-136+384 &= 0\\ x &= \frac{136 \pm \sqrt{(136)^2-4\times 5\times 384}}{2.5}\\ &= \frac{136 \pm \sqrt{18496-7680}}{10}\\ x &= \frac{136 \pm 104}{10} = 24, \frac{16}{5}\end{aligned}$$

There is no possibility of $x = \frac{16}{5}$ because it is less than 8 Hours.

Thus smaller tap can fill the tank in 24 hours and larger tap can fill in 24 hrs.

10. The denominator of a fraction is two more than its numerator. If the sum of the fraction and its reciprocal is $\frac{34}{15}$, find the fraction.

Ans : [Board Term-2, 2012 Set (1)]

Let numerator be x, then denominator will be $x+2$.

and
$$\text{fraction} = \frac{x}{x+2}$$

Now
$$\begin{aligned}\frac{x}{x+2}+\frac{x+2}{x} &= \frac{34}{15}\\ 15(x^2+x^2+4x+4) &= 34(x^2+2x)\\ 30x^2+60x+60 &= 34x^2+68x\\ 4x^2+8x-60 &= 0\\ x^2+2x-15 &= 0\\ x^2+5x-3x-15 &= 0\\ x(x+5)-3(x+5) &= 0\\ (x+5)(x-3) &= 0\end{aligned}$$

We reject the $x = -5$. Thus $x = 3$ and fracrtion $= \frac{3}{5}$

11. Solve for x : $\frac{2}{x+1}+\frac{3}{2(x-2)} = \frac{33}{5x}; x \neq 0, -1, 2$

Ans : [Board Term-2, Delhi 2015, Set I, II]

We have
$$\begin{aligned}\frac{2}{x+1}+\frac{3}{2(x-2)} &= \frac{23}{5x}\\ \frac{2x}{x+1}+\frac{3x}{2(x-2)} &= \frac{23}{5}\\ \frac{2\times 2x(x-2)+3x(x+1)}{2(x+1)(x-2)} &= \frac{23}{5}\\ \frac{4x^2-8x+3x^2+3x}{2(x^2-x-2)} &= \frac{23}{5}\\ \frac{7x^2-5x}{2(x^2-x-2)} &= \frac{23}{5}\\ 35x^2-25x &= 46x^2-46x-92\\ 11x^2-21x-92 &= 0\\ x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a}\\ &= \frac{21 \pm \sqrt{(-21)^2-4(11)(-92)}}{2\times 11}\\ &= \frac{21 \pm \sqrt{441+4048}}{22}\\ &= \frac{21 \pm 67}{22}\\ x &= \frac{21+67}{22} \text{ or } x = \frac{21-67}{22}\end{aligned}$$

Thus
$$x = 4, \frac{-23}{11}$$

12. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\frac{29}{20}$. Find the original fraction.

Ans : [Board Term-2, Delhi, 2015 Set I, III]

Let the denominator be x, then numerator will $x-3$

So the fraction will be $\frac{x-3}{x}$

By the given condition, new fraction will
$$\frac{x-3+2}{x+2} = \frac{x-1}{x+2}$$

Now
$$\begin{aligned}\frac{x-3}{x}+\frac{x-1}{x+2} &= \frac{29}{20}\\ 20[(x-3)(x+2)+x(x-1)] &= 29(x^2+2x)\\ 20(x^2-x-6+x^2-x) &= 29x^2+58x\\ 20(2x^2-2x-6) &= 29x^2+58x\\ 40x^2-40x-240 &= 29x^2+58x\\ 11x^2-98x-120 &= 0\\ 11x^2-110x+12x-120 &= 0\\ (11x+20)(x-10) &= 0,\end{aligned}$$

We take $x = 10$ and fraction will be is $\frac{10-3}{10} = \frac{7}{10}$.

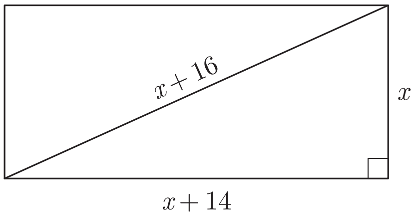
13. The diagonal of a rectangular field is 16 metre more than the shorter side. If the longer side is 14 metre more than shorter side, then find the length of the field.

Ans : [Board Term, O.D., 2015 Set I, II, III]

Let the length of shorter side be x m.

$$\text{Length of diagonal} = (x+16) \text{ m}$$

and, $\quad \text{Length of longer side} = (x+14) \text{ m}$



Now as per question we have

$$\begin{aligned} x^2+(x+14)^2 &= (x+16)^2 \\ x^2+x^2+28x+196 &= x^2+32x+256 \\ x^2-4x-60 &= 0 \\ x^2+6x-10x-60 &= 0 \\ x(x+6)-10(x+6) &= 0 \\ x &= -6, x=10 \end{aligned}$$

As lenght can't be negative $x = 10$ m. Therefore length of sides are 10 m and 24 m.

14. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed ?

Ans : [Board Term-2, O.D., 2015 Set I, II]

Let the speed of the train be x km/hr. for first 54 km. and for next 63 km, speed $= (x+6)$ km/hr.
According to the question

$$\begin{aligned} \frac{54}{x}+\frac{63}{x+6} &= 3 \\ \frac{54(x+6)+63x}{x(x+6)} &= 3 \\ 54x+324+63x &= 3x(x+6) \\ 117x+324 &= 3x^2+18x \\ 3x^2-99x-324 &= 0 \\ x^2-33x-108 &= 0 \\ x^2-36x+3x-108 &= 0 \\ x(x-36)+3(x-36) &= 0 \\ (x-36)(x+3) &= 0 \\ x &= -3, 36 \end{aligned}$$

Negative value is rejected, thus first speed of train is 36 km/h.

15. A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck.

Ans : [Board Term-2, O.D., 2015, Set II]

Let the average speed of the truck be x km/hr. for first 150 km and for next 200 km, speed $(x+20)$ km/hr.

Now $$\frac{150}{x}+\frac{200}{x+20} = 5$$

$$\begin{aligned} 150x+3000+200x &= 5x(x+20) \\ x^2-50x-600 &= 0 \\ x^2-60x+10x-600 &= 0 \\ x(x-60)+10(x-60) &= 0 \\ (x-60)(x+10) &= 0 \end{aligned}$$

Negative value is rejected, thus first speed of truck is 60 km/h.

16. The total cost of a certain length of cloth is Rs 200. If the piece was 5 m longer and each metre of cloth coast Rs 2 less, the cost of the piece would have remained unchaged. How longer is the piece and what is its original rate per metre ?

Ans : [Foreign Set I, II, 2015]

Let the length of the cloth be x m.

$$\begin{aligned} \text{cost per metre} &= \frac{200}{x} \\ \text{New length of the cloth} &= (x+5) \text{ m} \\ \text{New cost per metre} &= \left(\frac{200}{x}-2\right) \end{aligned}$$

Since cost of the piece have remained unchanged,

$$\begin{aligned} (x+5)\left(\frac{200}{x}-2\right) &= 200 \\ \frac{(x+5)(200-2x)}{x} &= 200 \\ 200x-2x^2+1000-10x &= 200x \\ x^2+5x-500 &= 0 \\ (x+25)(x-20) &= 0 \\ x &= 20, 25 \end{aligned}$$

Negative value is rejected, thus length of the piece is 20 m.

Original cost per metre is $\frac{200}{20} = 10$ Rs.

17. A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans : [CBSE O.D. 2014]

Let the speed of stream be x km/h
Then the speed of boat upstream $= (18-x)$ km/h
Speed of boat downstream $= (18+x)$ km/h
According to the question,

$$\begin{aligned} \frac{24}{18-x}-\frac{24}{18+x} &= 1 \\ \frac{24\,[(18+x)-(18-x)]}{18^2-x^2} &= 1 \\ 48x &= 324-x^2 \\ x^2+48x-324 &= 0 \\ x^2+54x-6x-324 &= 0 \\ x(x+54)-6(x+54) &= 0 \\ (x+54)(x-6) &= 0 \\ x+54 &= 0, x-6=0 \\ x &= -54, x=6 \end{aligned}$$

Since speed cannot be negative, we reject $x = -54$.

The speed of steam is 6 km/h.

18. Solve for x : $\frac{x-3}{x-4}+\frac{x-5}{x-6}=\frac{10}{3}; x \neq 4,6$

Ans :

We have $$\frac{x-3}{x-4}+\frac{x-5}{x-6} = \frac{10}{3}$$

$$\frac{(x-3)(x-6)+(x-4)(x-5)}{(x-4)(x-6)} = \frac{10}{3}$$

$$\begin{aligned}
\frac{x^2-9x+18+x^2-9x+20}{x^2-10x+24} &= \frac{10}{3}\\
3(2x^2-18x+38) &= 10x^2-100x+240\\
6x^2-54x+114 &= 10x^2-100x+240\\
4x^2-46x+126 &= 0\\
2x^2-23x+63 &= 0\\
2x^2-14x-9x+63 &= 0\\
2x(x-7)-9(x-7) &= 0\\
(2x-9)(x-7) &= 0\\
2x-9=0, x-7 &= 0\\
x &= \tfrac{9}{2}, x=7
\end{aligned}$$

19. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans : [Board Term-2, O.D., Set II 2016]

Ans :

Let the speed of stream be x km/h
Then the speed of boat upstream $=(24-x)$ km/h
Speed of boat downstream $=(24+x)$ km/h
According to the question,

$$\begin{aligned}
\frac{32}{24-x}-\frac{32}{24+x} &= 1\\
32\left[\frac{1}{24-x}-\frac{1}{24+x}\right] &= 1\\
32\left[\frac{24+x-24+x}{576-x^2}\right] &= 1\\
32(24+x-24+x) &= 576-x^2\\
64x &= 576-x^2\\
x^2+64x-576 &= 0\\
x^2+72x-8x-576 &= 0\\
x(x+72)-8(x+72) &= 0\\
(x-8)(x+72) &= 0\\
x &= 8,-72
\end{aligned}$$

Since speed cannot be negative, we reject $x=-72$.

The speed of steam is 8 km/h.

20. A student scored a total of 32 marks in class tests in mathematics ans science. Had he scored 2 marks less in science and 4 more in mathematics, the product of his marks would have been 253. Find his marks in two subjects.

Ans : [Board Term-2, 2012 Set (50)]

Let marks in Mathematics be x, then marks in science will be $32-x$

According to question,

$$\begin{aligned}
(32-x-2)(x+4) &= 253\\
(30-x)(x+4) &= 253\\
26x-x^2+120 &= 253\\
x^2-26x+133 &= 0]\text{or},\\
x^2-19x-7x+133 &= 0\\
x(x-19)-7(x-19) &= 0
\end{aligned}$$

or, $x = 7$ or $x=19$

If $x=7$ then marks in mathematics $=7$, and marks in science $=25$

If $x=19$, then marks in mathematics $=19$ and marks in science $=13$.

21. The sum of squares of two consecutive multiples of 7 is 637. Find the multiples.

Ans : [Foreign Set II, 2014]

Let $7x$ and $7x+7$ be two consecutive multiples of 7.

According to question,

$$\begin{aligned}
(7x)^2+(7x+7)^2 &= 637\\
49x^2+49x^2+49+98x &= 637\\
98x^2+98x-588 &= 0\\
x^2+x-6 &= 0\\
(x+3)(x-2) &= 0\\
x &= -3,2
\end{aligned}$$

Neglecting negative value, $x = 2$

Therefore multiples are 14 and 21.

22. Solve for x : $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2$ where $x \neq -\frac{1}{2},1$

Ans : [Out Side Delhi Set-III 2017]

We have $$\frac{x-1}{2x+1}+\frac{2x+1}{x-1} = 0$$

Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1}=\frac{1}{y}$
Substituting this value we obtain

$$\begin{aligned}
y+\frac{1}{y} &= 2\\
y^2+1 &= 2y\\
y^2-2y+1 &= 0\\
(y-1)^2 &= 0\\
y &= 1
\end{aligned}$$

Putting $y = \frac{x-1}{2x+1}$ we have

$$\frac{x-1}{2x+1} = 1 \text{ or } x-1=2x+1$$

or $x = -2$

23. Find for x : $\frac{1}{x-2}+\frac{2}{x-1} = \frac{6}{x}; x \neq 0,1,2$

Ans : [Board Outside Delhi Compt. Set-I, II 2017]

We have $$\frac{1}{x-2}+\frac{2}{x-1} = \frac{6}{x}$$

$$\frac{x-1+2x-4}{(x-2)(x-1)} = \frac{6}{x}$$

$$\begin{aligned} 3x^2-5x &= 6x^2-18x+12 \\ 3x^2-13x+12 &= 0 \\ 3x^2-4x-9x+12 &= 0 \\ x(3x-4)-3(3x-4) &= 0 \\ (3x-4)(x-3) &= 0 \\ x &= \frac{4}{3} \text{ and } 3 \end{aligned}$$

Hence, $x = 3, \frac{4}{3}$

24. Solve, for x : $\sqrt{3}\,x^2+10x+7\sqrt{3}=0$

Ans : [Board Foreign II, III 2017]

We have
$$\begin{aligned} \sqrt{3}\,x^2+10x+7\sqrt{3} &= 0 \\ \sqrt{3}\,x^2+3x+7x=7\sqrt{3} &= 0 \\ (x+\sqrt{3})(\sqrt{3}\,x+7) &= 0 \\ (x+\sqrt{3})(\sqrt{3}\,x+7) &= 0 \\ x &= -\sqrt{3} \text{ and } x=\frac{-7}{\sqrt{3}} \end{aligned}$$

Hence roots $x=-\sqrt{3}$ or $x=\frac{-7}{\sqrt{3}}$

25. The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers

Ans : [Board Term-2, 2014, Delhi]

Let the first number be x, then second number will be $x+5$

Now according to the question

$$\begin{aligned} \frac{1}{x}-\frac{1}{x+5} &= \frac{1}{10} \\ \frac{x+5-x}{x(x+5)} &= \frac{1}{10} \\ 50 &= x^2+5x \\ x^2+5x-50 &= 0 \\ x^2+10x-5x-50 &= 0 \\ x(x+10)-5(x+10) &= 0 \\ (x+10)(x-5) &= 0 \\ x &= 5, -10 \end{aligned}$$

Rejecting the negative value, numbers are 5 and 10.

26. The sum of squares of two consecutive even numbers is 340. Find the numbers.

Ans : [Foreign Set I, 2014]

Let the number be x and $x+2$

Now
$$\begin{aligned} (x)^2+(x+2)^2 &= 340 \\ x^2+x^2+4+4x &= 340 \\ 2x^2+4x-336 &= 0 \\ x^2+2x-168 &= 0 \\ (x+14)(x-12) &= 0 \\ x &= 12 \end{aligned}$$

Thus numbers are 12 and 14.

27. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

Ans : [Foreign Set I, 2014] [Board Term-2, 2012 Set(12)]

Let the odd number be $2x+1$, then consecutive odd number will be $2x+1+2=2x+3$

Now, according to question

$$\begin{aligned} (2x+1)^2+(2x+3)^2 &= 394 \\ 4x^2+4x+1+4x^2+12x+9 &= 394 \\ 8x^2+16x-384 &= 0 \\ x^2+2x-48 &= 0 \\ x^2+8x-6x-48 &= 0 \\ x(x+8)-6(x+8) &= 0 \\ x &= -8, 6 \end{aligned}$$

Rejecting the negative value,

$$\begin{aligned} \text{Ist number} &= 2\times 6+1 = 13 \\ \text{and second odd number} &= 15 \end{aligned}$$

28. Sum of the areas of two squares is 400 cm². If the difference of their perimeters is 16 cm, find the sides of the two squares.

Ans : [Board Term-2, 2013]

Let the sides of two squares be a and b,

then
$$a^2+b^2 = 400 \quad (1)$$

and
$$\begin{aligned} 4(a-b) &= 16 \\ a-b &= 4 \\ a &= 4+b \end{aligned}$$

From equations (1) and (2), we obtain

$$\begin{aligned} (4+b)^2+b^2 &= 400 \\ 16+b^2+8b=b^2 &= 400 \\ 2b^2+8b-384 &= 0 \\ b^2=4b-192 &= 0 \\ b^2+16b-12b-192 &= 0 \\ b(b+16)-12(b+16) &= 0 \\ (b+16)(b-12) &= 0 \\ b &= -16, 12 \end{aligned}$$

Rejecting the negative value, $b = 12$ cm

then $a = 4+12 = 16$ cm.

29. A train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train.

Ans : [Board Term-2, 2012, Set(22)]

Let the usual speed of train be x km/hr.

According to question we have

$$\begin{aligned} \frac{300}{x}-\frac{300}{x+5} &= 2 \\ x^2+5x-750 &= 0 \\ x^2=30x-25x-750 &= 0 \\ (x=30)(x-25) &= 0 \\ x=-30 \text{ or } x &= 25 \end{aligned}$$

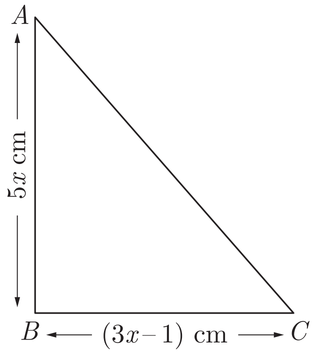
Since, speed cannot be negative $x \neq -30$.

Thus speed of train is 25 km/hr.

30. The length of the sides forming right angle of a right triangle are $5x$ cm and $(3x-1)$ cm. If the area of the triangle is 60 cm^2. Find its hypotenuse.

Ans : [Board Term-2, 2012 Set (44)]

According to the question we have drawn figure below.



Now

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2}\times \text{base} \times \text{height} \\ 60 &= \frac{1}{2}\times 5x\times(3x-1) \\ 15x^2-5x &= 120 \\ 3x^2-x-24 &= 0 \\ 3x^2-9x+8x-24 &= 0 \\ 3x(x-3)+8(x-3) &= 0 \\ (x-3)(3x+8) &= 0 \\ x=3, x &= -\frac{8}{3} \end{aligned}$$

Length can't be negative, so $x=3$

Now $AB = 5\times 3 = 15$ cm,

$BC = 3x-1 = 9-1 = 8$ cm

Now

$$\begin{aligned} AC &= \sqrt{15^2+8^2} \\ &= \sqrt{225+64} \\ &= \sqrt{289} = 17 \text{ cm.} \end{aligned}$$

Hence hypotenuse $= 17$ cm.

31. A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work.

Ans : [Board Term-2, 2012 Set (5)]

Suppose B alone finish the work in x days and A alone takes $(x-6)$ days.

$$\begin{aligned} \text{B's one day work} &= \frac{1}{x} \\ \text{A' one day work} &= \frac{1}{x-6} \\ \text{and (A+B)'s one day work} &= \frac{1}{4} \end{aligned}$$

According to the question,

$$\begin{aligned} \frac{1}{x}+\frac{1}{x-6} &= \frac{1}{4} \\ x^2-14x+24 &= 0 \\ x^2-12x-2x+24 &= 0 \\ x(x-12)-2(x-12) &= 0 \\ (x-12)(x-2) &= 0 \\ , \qquad x &= 12 \text{ or } x=2 \end{aligned}$$

But x cannot be less than 6. So $x=12$

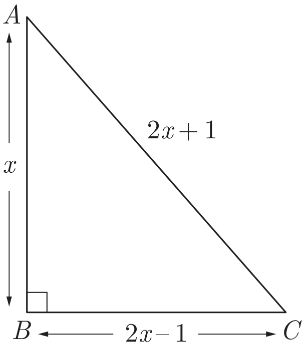
Hence B can finish the work in 12 days.

32. The length of the hypotenuse of a right triangle exceeds the length of its base by 2 cm and exceeds twice the length of altitude by 1 cm. Find the length of each side of the triangle.

Ans : [Board Term-2, 2012 Set (12)]

et altitude of triangle be x.

$$\begin{aligned} \text{Hypotenuse of triangle} &= 2x+1 \\ \text{and} \qquad \text{base of triangle} &= 2x-1 \end{aligned}$$



Using Pythagoras theorem,

$$\begin{aligned} (2x+1)^2 &= x^2+(2x-1)^2 \\ 4x^2+1+4x &= x^2+4x^2+1-4x \\ x^2-8x &= 0 \\ x(x-8) &= 0 \end{aligned}$$

Rejecting $x=0$, we get $x = 8$

Thus altitude of triangle is 8 cm

Hypotenuse of triangle is $2\times 8+1 = 17$ cm

and base of triangle is $2\times 8-1 = 15$ cm

33. The perimeter of a rectangular field is 82 m and its area is 400 square metre. Find the length and breadth of the rectangle.

Ans : [Board Term-2, 2012 Set (21)]

We have

$$\begin{aligned} \text{Perimeter} &= 2(l+b) = 82 \text{ m} \\ \text{or,} \qquad l+b &= 41 \text{ m} \end{aligned}$$

Let length be x m, then breadth $= (41-x)$ m.

$$\begin{aligned} \text{Area} = l \times b &= 400 \text{ m}^2 \\ x(41-x) &= 400 \\ 41x - x^2 &= 400 \\ x^2 - 41x + 400 &= 0 \\ (x-16)(x-25) &= 0 \\ x = 16 \text{ or } x &= 25 \end{aligned}$$

If length is 16 m, then breadth will be 25 m.
If length is 25 m, then breadth will be 16 m.

34. The product of Tanay's age (in years) five years ago and his age ten years later is 16. Determine Tanay's present age.

Ans : [Board Term-2, 2012 Set (31)]

Let the present age of Tanay's be x years.

According to question we have

$$\begin{aligned} (x-5)(x+10) &= 16 \\ x^2 + 5x - 50 &= 16 \\ x^2 + 5x - 66 &= 0 \\ x^2 + 11x - 6x - 66 &= -66 \\ x(x+11) - 16(x-11) &= 0 \\ (x+11)(x-6) &= 0 \\ x &= -11, 6. \end{aligned}$$

As age cannot be negative, we reject $x = -11$. Thus present age of Tanay is 6 years.

35. Solve for x : $\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}; x \neq 0.2$

Ans : [Delhi Compt. Set-I 2017]

We have $\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$

$$\begin{aligned} \frac{x(x+3) - (1-x)(x-2)}{x(x-2)} &= \frac{17}{4} \\ \frac{(x^2+3x) - (-x^2+3x-2)}{x^2-2x} &= \frac{17}{4} \\ \frac{2x^2+2}{x^2-2x} &= \frac{17}{4} \\ 8x^2 + 8 &= 17x^2 - 34x \\ 9x^2 - 34x - 8 &= 0 \\ 9x^2 - 36x + 2x - 8 &= 0 \\ 9x(x-4) + 2(x-4) &= 0 \\ (x-4)(9x+2) &= 0 \\ x &= 4 \text{ or } x = -\frac{2}{9} \end{aligned}$$

Hence, $x = 4, -\frac{2}{9}$

36. Solve for x : $4x^2 + 4bx - (a^2 - b^2) = 0$

Ans : [Board Foreign Set- III 2017]

We have $4x^2 + 4bx - (a^2 - b^2) = 0$

Compare with $Ax^2 + Bx + C = 0$ we get

$$A = 4, B = 4b \text{ and } C = b^2 - a^2$$

$$\begin{aligned} x &= \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \\ &= \frac{-4b \pm \sqrt{(4b)^2 - 4.4(b^2 - a^2)}}{2.4} \\ &= \frac{-4b \pm \sqrt{16b^2 - 16b^2 + 16a^2}}{8} \\ &= \frac{-4b \pm 4a}{8} \\ &= -\frac{(a+b)}{2}, \frac{(a-b)}{2} \end{aligned}$$

Hence the roots are $-\frac{(a+b)}{2}$ and $\frac{(a-b)}{2}$

SHORT ANSWER TYPE QUESTIONS - I

1. Find k so that the quadratic equation $(k+1)x^2 - 2(k+1)x + 1 = 0$ has equal roots.

Ans : [Board Term-2, 2016 Set HODM40L]

We have $(k+1)x^2 - 2(k+1)x + 1 = 0$

Compare with $Ax^2 + Bx + C = 0$ we get
$A = (k+1), B = -2(k+1), C = 1$
If roots are equal, $D = 0$, i.e.

$$\begin{aligned} B^2 &= 4AC \\ 4(k+1)^2 &= 4(k+1) \\ k^2 + 2k + 1 &= k + 1 \\ k^2 + k &= 0 \\ k(k+1) &= 0 \\ k &= 0, -1 \end{aligned}$$

$k = -1$ does not satisfy the equation, thus $k = 0$

2. If 2 is a root or the equation $x^2 + kx + 12 = 0$ and the equation $x^2 + kx + q = 0$ has equal roots, find the value of q.

Ans : [Board Sample Paper 2016]

We have $x^2 + kx + 12 = 0$

If 2 is the root of above equation, it must satisfy it.

$$\begin{aligned} (2)^2 + 2k + 12 &= 0 \\ , \quad 2k + 16 &= 0 \\ k &= -8 \end{aligned}$$

Substituting $k = -8$ in $x^2 + kx + q = 0$ we have

$$x^2 - 8x + q = 0$$

For equal roots,

$$\begin{aligned} (-8)^2 - 4(1)q &= 0 \\ 64 - 4q &= 0 \\ 4q &= 64 \\ q &= 16 \end{aligned}$$

3. Find the values of k for which the quadratic equation $9x^2 - 3kx + k = 0$ has euqal roots.

Ans : [CBSE Delhi, O.D. 2014]

We have $9x^2 - 3kx + k = 0$

Compare with $ax^2 + bx + c = 0$ we get

$$a = 9, b = -3k, c = k$$

Since roots of the equation are equal, $b^2 - 4ac = 0$

$$\begin{aligned}(-3k)^2-(4\times 9\times k)&=0\\ 9k^2-36k&=0\\ k^2-4k&=0\\ k(k-4)&=0\\ k&=0 \text{ or } k=4\end{aligned}$$

Hence, $k=4$.

4. If the equation $kx^2-2kx+6=0$ has equal roots, then find the value of k.

Ans : [Board Term-2, 2012 Set (22)]

We have $kx^2-2kx+6=0$

Compare with $ax^2+bx+c=0$ we get

$$a=k, b=-2k, c=6$$

Since roots of the equation are equal, $b^2-4ac=0$

$$\begin{aligned}(-2k)^2-4(k)(6)&=0\\ 4k^2-24k&=0\\ 4k(k-6)&=0\\ k&=0,6\end{aligned}$$

But $k\neq 0$, as coefficient of x^2 can not be zero

$$k=6$$

5. Find the values of p for which the quadratic equation $4x^2+px+3=0$ has equal roots.

Ans : [Board Term-2, 2014]

We have $4x^2+px+3=0$

Compare with $ax^2+bx+c=0$ we get

$$a=4, b=p, c=3$$

Since roots of the equation are equal, $b^2-4ac=0$

$$\begin{aligned}p^2-4\times 4\times 3&=0\\ p^2-48&=0\\ p^2&=48\\ p&=\pm 4\sqrt{3}\end{aligned}$$

6. Find the nature of the roots of the quadratic equation :

$13\sqrt{3}x^2+10x+\sqrt{3}=0$

Ans : [Board Term-2, 2012, (12)]

We have $13\sqrt{3}x^2+10x+\sqrt{3}=0$

Compare with $ax^2+bx+c=0$ we get

$$\begin{aligned}a&=13\sqrt{3}, b=10, c=\sqrt{3}\\ b^2-4ac&=(10)^2-4(13\sqrt{3})(\sqrt{3})\\ &=100-156\\ &=-56\end{aligned}$$

As $D<0$, the equation has not real roots.

SHORT ANSWER TYPE QUESTIONS - II

1. If 2 is a root of the quadratic equation $3x^2+px-8=0$ and the quadratic equation $4x^2-2px+k=0$ has equal roots, find k.

Ans : [Foreign Set II, 2014]

We have $3x^2+px-8=0$

Since 2 is a root of above equation, it must satisfy it. Substituting $x=2$ in $3x^2+px-8=0$ we have

$$\begin{aligned}12+2p-8&=0\\ p&=-2\end{aligned}$$

Since $4x^2-2px+k=0$ has equal roots,

or $4x^2+4x+k=0$ has equal roots,

$$\begin{aligned}D&=b^2-4ac=0\\ 4^2-4(4)(k)&=0\\ 16-16k&=0\\ 16k&=16\end{aligned}$$

Thus $k=1$

2. For what value of k, the roots of the quadratic equation $kx(x-2\sqrt{5})+10=0$ are equal ?

Ans : [Delhi CBSE, Term-2, 2014] [Delhi 2013]

We have $kx(x-2\sqrt{5})+10=0$

or, $kx^2-2\sqrt{5}kx+10=0$

Compare with $ax^2+bx+c=0$ we get

$$a=k, b=-2\sqrt{5}, c=10$$

Since, roots are equal, $D=b^2-4ac=0$

$$\begin{aligned}(-2\sqrt{5}k)^2-4\times k\times 10&=0\\ 20k^2-40k&=0\\ 20k(k-2)&=0\\ k(k-2)&=0\end{aligned}$$

Since $k\neq 0$, we get $k=2$

3. Find the nature of the roots of the following quadratic equation. If the real roots exist, find them : $3x^2-4\sqrt{3}x+4=0$

Ans : [Board Term-2, 2012 Set (5)]

We have $3x^2-4\sqrt{3}x+4=0$

Compare with $ax^2+bx+c=0$ we get

$$\begin{aligned}a=3k, b&=-4\sqrt{3}, c=4\\ b^2-4ac&=(-4\sqrt{3})^2-4(3)(4)\\ &=48-48=0\end{aligned}$$

Thus roots are real and equal.

Roots are $\left(-\frac{b}{2a}\right),\left(-\frac{b}{2a}\right)$ or $\frac{2\sqrt{3}}{3},\frac{2\sqrt{3}}{3}$

4. Determine the positive value of 'k' for which the equation $x^2+kx+64=0$ and $x^2-8x+k=0$ will both have real and equal roots.

Ans : [Board Term-2, 2012 Set (44)] [Delhi CBSE Term-2, 2014]

We have $x^2+kx+64=0$

Compare with $ax^2+bx+c=0$ we get

$$a=1, b=k, c=64$$

For real and equal roots, $b^2-4ac=0$

Thus

$$\begin{aligned}k^2-4\times 1\times 64&=0\\ k^2-256&=0\\ k&=\pm 16\end{aligned}$$

(1)

Now for equation $x^2 - 8x + k = 0$ we have

$$\begin{aligned} b^2 - 4ac &= 0 \\ (-8)^2 - 4 \times 1 \times k &= 0 \\ 64 &= 4k \\ k &= \frac{64}{4} = 16 \end{aligned}$$

(2)

From (1) and (2), we get $k = 16$. Thus for $k = 16$, given equations have equal roots.

5. Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k-1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation.

Ans : [Delhi CBSE Board Term-2, 2015, Set I, III]

We have $\quad kx^2 + 1 - 2(k-1)x + x^2 = 0$

$$(k+1)^2 - 2(k-1)x + 1 = 0$$

Compare with $ax^2 + bx + c = 0$ we get

$$a = k+1, b = -2(k-1), c = 1$$

For real and equal roots, $b^2 - 4ac = 0$

$$\begin{aligned} 4(k-1)^2 - 4(k+1) \times 1 &= 0 \\ 4k^2 - 8k + 4 - 4k - 4 &= 0 \\ 4k^2 - 12k &= 0 \\ 4k(k-3) &= 0 \end{aligned}$$

As k can't be zero, thus $k = 3$.

6. Find the value of k for which the quadratic equation $(k-2)x^2 + 2(2k-3)x + (5k-6) = 0$ has equal roots.

Ans : Board Term-2, 2015]

We have $\quad (k-2)x^2 + 2(2k-3)x + (5k-6) = 0$

Compare with $ax^2 + bx + c = 0$ we get

$$a = k-2, b = 2(2k-3), c = (5k-6)$$

For real and equal roots, $b^2 - 4ac = 0$

$$\begin{aligned} \{2(2k-3)\}^2 - 4(k-2)(5k-6) &= 0 \\ 4(4k^2 - 12k + 9) - 4(k-2)(5k-6) &= 0 \\ 4k^2 - 12k + 9 - 5k^2 + 6k + 10k - 12 &= 0 \\ k^2 - 4k + 3 &= 0 \\ k^2 - 3k - k + 3 &= 0 \\ k(k-3) - 1(k-3) &= 0 \\ (k-3)(k-1) &= 0 \end{aligned}$$

Thus $\qquad k = 1, 3$

7. If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b + c$.

Ans : [Outside Delhi, Set-II, 2016]

We have $(a-b)x^2 + (b-c)x + (c-a) = 0$

Compare with $ax^2 + bx + c = 0$ we get

$$a = (a-b,), b = (b-c), c = c-a$$

For real and equal roots, $b^2 - 4ac = 0$

$$\begin{aligned} (b-c)^2 - 4(a-b)(c-a) &= 0 \\ b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) &= 0 \\ b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab &= 0 \\ 4a^2 + b^2 + c^2 + 2bc - 4ab - 4ac &= 0 \end{aligned}$$

Using $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$ we have

$$\begin{aligned} (-2a + b + c)^2 &= 0 \\ -2a + b + c &= 0 \end{aligned}$$

Hence, $\qquad b + c = 2a$

8. If the quadratic equation, $(1+a^2)b^2x^2 + 2abcx + (c^2 - m^2) = 0$ in x has equal roots, prove that $c^2 = m^2(1+a^2)$

Ans : [Board Term-2, 2014]

We have $\quad (1+a^2)b^2x^2 + 2abcx + (c^2 - m^2) = 0$

Compare with $Ax^2 + Bx + C = 0$ we get

$$A = (1+a^2), B = 2abc, C = (c^2 - m^2)$$

If roots are equal, $B^2 - 4AC = 0$

$$\begin{aligned} (2abc)^2 - 4(1+a^2)b^2(c^2 - m^2) &= 0 \\ 4a^2b^2c^2 - (4b^2 + 4a^2b^2)(c^2 - m^2) &= 0 \\ 4a^2b^2c^2 - [4b^2c^2 - 4b^2m^2 + 4a^2b^2c^2 - 4a^2b^2m^2] &= 0 \\ 4a^2b^2c^2 - 4b^2c^2 + 4b^2m^2 - 4a^2b^2c^2 + 4a^2b^2m^2 &= 0 \\ 4b^2[a^2m^2 + m^2 - c^2] &= 0 \\ c^2 &= a^2m^2 + m^2 \\ c^2 &= m^2(1+a^2) \end{aligned}$$

9. If -3 is a root of quadratic equation $2x^2 + px - 15 = 0$, while the quadratic equation $x^2 - 4px + k = 0$ has equal roots. Find the value of k.

Ans : [Outside Delhi Compt. Set II, III 2017]

Given -3 is a root of quadratic equation

We have $\qquad 2x^2 + px - 15 = 0$

Since 3 is a root of above equation, it must satisfy it. Substituting $x = 3$ in above equation we have

$$\begin{aligned} 2(-3)^2 + p(-3) - 15 &= 0 \\ 2 \times 9 - 3p - 15 &= 0 \\ p &= 1 \end{aligned}$$

Since $\quad x^2 - 4px + k = 0$ has equal roots,

or $\quad x^2 - 4x + k = 0$ has equal roots,

$$\begin{aligned} b^2 - 4ac &= 0 \\ 4^2 - 4k &= 0 \\ 16 - 4k &= 0 \\ 4k &= 16 \\ k &= 4 \end{aligned}$$

10. If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ hs no real roots.

Ans : [Board outside Delhi Set-I 2017]

We have $\quad (a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$

Compare with $Ax^2 + Bx + C = 0$ we get

$$A = (a^2 + b^2), B = 2(ac + bd) \text{ and } C = (c^2 + d^2)$$

For no real roots, $D = B^2 - 4AC < 0$

$$\begin{aligned} D &= B^2 - 4AC \\ &= [2(ac+bd)]^2 - 4(a^2+b^2)(c^2+d^2) \\ &= 4[a^2c^2+2abcd+b^2d^2] - 4[a^2c^2+a^2d^2+b^2c^2+b^2d^2] \\ &= 4[a^2c^2+2abcd+b^2d^2-a^2c^2-a^2d^2-b^2c^2-b^2d^2] \\ &= -4[a^2d^2+b^2c^2-2abcd] \\ &= -4(ad-bc)^2 \end{aligned}$$

Since $ad \neq bc$, therefore $D \neq 0$ and always negative. Hence the equation has no real roots.

11. Find the value of c for which the quadratic equation $4x^2-2(c+1)x+(x+1)=0$ has equal roots.

Ans : [Delhi Comp. Set-III 2017]

We have $4x^2-2(c+1)x+(x+1)=0$

Compare with $Ax^2+Bx+C=0$ we get

$$A=4, B=2(c+1), C=(c+1)$$

If roots are equal, $B^2-4AC=0$

$$\begin{aligned} [2(c+1)]^2-4\times 4(c+1) &= 0 \\ 4(c^2+2c+1)-4(4c+4) &= 0 \\ 4(c^2+2c+1-4c-4) &= 0 \\ c^2-2c-3 &= 0 \\ c^2-3c+c-3 &= 0 \\ c(c-3)+1(c-3) &= 0 \\ (c-3)(c+1) &= 0 \\ c &= 3,-1 \end{aligned}$$

Hence for equal roots $c=3,-1$.

12. Show that if the roots of the following equation are equal that $ad=bc$ or $\frac{a}{b}=\frac{c}{d}$.

$$x^2(a^2+b^2)+2(ac+bd)x+c^2+d^2=0$$

Ans : [Delhi Compt. Set I, II 2017 Outside Delhi Set II]

We have $x^2(a^2+b^2)+2(ac+bd)x+c^2+d^2=0$

Compare with $Ax^2+Bx+C=0$ we get

$$A=a^2+b^2, B=2(ac+bd), C=c^2+d^2$$

If roots are equal, $B^2-4AC=0$

$$\begin{aligned} [2(ac+bd)]^2-4(a^2+b^2)(c^2+d^2) &= 0 \\ 4(a^2c^2+2abcd+b^2d^2)-4(a^2c^2+a^2b^2+b^2c^2+b^2d^2) &= 0 \\ 4(a^2c^2+2abcd+b^2d^2-a^2c^2-a^2d^2-b^2c^2-b^2d^2) &= 0 \\ -4(a^2d^2+b^2c^2-2abcd) &= 0 \\ (ad-bc)^2 &= 0 \\ ad &= bc \\ \frac{a}{b} &= \frac{c}{d} \end{aligned}$$

Hence Proved.

LONG ANSWER TYPE QUESTIONS

1. If roots of the quadratic equation $x^2+2px+mn=0$ are real and equal, show that the roots of the quadratic equation $x^2-2(m+n)x+(m^2+n^2+2p^2)=0$ are also equal.

Ans : [Foreign Set II, 2016]

We have $x^2+2px+mn=0$

Compare with $Ax^2+Bx+C=0$ we get

$$A=1, B=2p, C=mn$$

If roots are equal, $B^2-4AC=0$

$$4p^2-4mn=0$$

or, $$p^2 = mn \qquad (1)$$

Now we have

$$x^2-2(m+n)x+(m^2+n^2+2p^2)=0$$

Compare with $Ax^2+Bx+C=0$ we get

$$A=1, B=-2(m+n), C=(m^2+n^2+2p^2)$$

If roots are equal, $B^2-4AC=0$

$$\begin{aligned} 4(m+n)^2-4(m^2+n^2+2p^2) &= 0 \\ m^2+n^2+2mn-m^2-n^2-2p^2 &= 0 \\ 2mn-2p^2 &= 0 \\ p^2 & \end{aligned}$$

$= mn$

Thus if roots of $x^2+2px+mn=0$ are equal then those of $x^2-2a(m+n)x+(m^2n^2+2p^2)=0$ are also equal.

2. Find the positive values of k for which quadratic equations $x^2+kx+64=0$ and $x^2-8x+k=0$ both will have the real roots.

Ans : [Foreign Set I-2016]

(i) For $x^2+kx+64=0$ to have real roots

$$\begin{aligned} k^2-256 &\geq 0 \\ k^2 &\geq 256 \\ k &\geq 16 \text{ or } k<-16 \end{aligned}$$

(ii) For $x^2-8x+k=0$ to have real roots

$$\begin{aligned} 64-4k &\geq 0 \\ 16-k &\geq 0 \\ 16 &\geq k \end{aligned}$$

For (i) and (ii) to hold simultaneously

$$k = 16$$

3. Find the positive value of k for which $x^2-8x+k=0$, will have real roots.

Ans : [Board Term-2, 2014]

We have $x^2-8x+k=0$

Compare with $Ax^2+Bx+C=0$ we get

$$A=1, B=-8, C=k$$

If roots are equal, $B^2-4AC=0$

Since the given equation has real roots, $B^2-4AC>0$

$$\begin{aligned} (-8)^2-4(1)(k) &\geq 0 \\ 64-4k &\geq 0 \\ 16-k &\geq 0 \\ 16 &\geq k \end{aligned}$$

Thus $k \leq 16$

4. Find the values of k for which the equation $(3k+1)^2+2(k+1)x+1$ has equal roots. Also find the roots.

Ans : [Board Term-2, 2014]

We have $(3k+1)^2+2(k+1)x+1$

Compare with $Ax^2+Bx+C=0$ we get

$$A=(3k+1), B=2(k+1), C=1$$

If roots are equal, $B^2 - 4AC = 0$

$$\begin{aligned} [2(k+1)]^2 - 4(3k+1)(1) &= 0 \\ 4(k^2+2k+1) - (12k+4) &= 0 \\ 4k^2 + 8k + 4 - 12k - 4 &= 0 \\ 4k^2 - 4k &= 0 \\ 4k(k-1) &= 0 \\ k &= 0, 1. \end{aligned}$$

Substituting $k = 0$, in the given equation,

$$\begin{aligned} x^2 + 2x + 1 &= 0 \\ (x+1)^2 &= 0 \\ x &= -1 \end{aligned}$$

Again substituting $k = 1$, in the given equation,

$$\begin{aligned} 4x^2 = 4x + 1 &= 0 \\ (2x+1)^2 &= 0 \end{aligned}$$

or, $$x = -\frac{1}{2}$$

Hence, roots $= -1, -\frac{1}{2}$

5. Find the values of k for which the quadratic equations $(k+4)x^2 + (k+1)x + 1 = 0$ has equal roots. Also, find the roots.

Ans : [Delhi CBSE, Term-2, 2014]

We have $\quad (k+4)x^2 + (k+1)x + 1 = 0$

Compare with $Ax^2 + Bx + C = 0$ we get

$$A = (k+4), B = (k+1), C = 1$$

If roots are equal, $B^2 - 4AC = 0$

$$\begin{aligned} (k+1)^2 - 4(k+4)(1) &= 0 \\ k^2 + 1 + 2k - 4k - 16 &= 0 \\ k^2 - 2k - 15 &= 0 \\ (k-5)(k+3) &= 0 \\ k &= 5, -3 \end{aligned}$$

For $k = 5$, equation becomes

$$\begin{aligned} 9x^2 + 6x + 1 &= 0 \\ (3x+1)^2 &= 0 \end{aligned}$$

or $$x = -\frac{1}{3}$$

For $k = -3$, equation becomes

$$\begin{aligned} x^2 - 2x = 1 &= 0 \\ (x-1)^2 &= 0 \\ x &= 1 \end{aligned}$$

Hence roots are 1 and $-\frac{1}{3}$.

6. If $x = -2$ is a root of the equation $3x^2 + 7x + p = 0$, find the value of k so that the roots of the equation $x^2 + k(4x + k - 1) + p = 0$ are equal.

Ans : [Foreign Set I, II, 2015]

We have $\quad 3x^2 + 7x + p = 0$

Since $x = -2$ is the root of above equation. It must satisfy it.

Thus $$\begin{aligned} 3(-2) + 7(-2) + p &= 0 \\ p &= 2 \end{aligned}$$

Since roots of the equation $x^2 + 4kx + k^2 - k + 2 = 0$ are equal.

$$\begin{aligned} 16k^2 - 4(k^2 - k + 2) &= 0 \\ 16k^2 - 4k^2 + 4k - 8 &= 0 \\ 12k^2 + 4k - 8 &= 0 \\ 3k^2 + k - 2 &= 0 \\ (3k-2)(k+1) &= 0 \\ k &= \frac{2}{3}, -1 \end{aligned}$$

Hence, roots $= \frac{2}{3}, -1$

7. If $x = -4$ is a root of the equation $x^2 + 2x + 4p = 0$, find the values of k for which the equation $x^2 + px(1+3k) + 7(3+2k) = 0$ has equal roots.

Ans : [Foreign Set III, 2015]

We have $\quad x^2 + 2x + 4p = 0$

Since $x = -4$ is the root of above equation. It must satisfy it.

$$\begin{aligned} (-4)^2 + (2 \times -4) + 4p &= 0 \\ p &= -2 \end{aligned}$$

Since equation $x^2 - 2(1+3k)x + 7(3+2k) = 0$ has equal roots.

$$\begin{aligned} 4(1+3k)^2 - 28(3+2k) &= 0 \\ 9k^2 - 8k - 20 &= 0 \\ (9k+10)(k-2) &= 0 \\ k & \frac{-10}{9}, 2 \end{aligned}$$

Hence, the value of k are $-\frac{10}{9}$ and 2.

8. Find the value of p for which the quadratic equation $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0$, $p \neq -1$ has equal roots. Hence find the roots of the equation.

Ans : [Board Term-2, 2015 Set II]

We have $\quad (p+1)x^2 - 6(p+1)x + 3(p+9) = 0$

Compare with $ax^2 + bx + c = 0$ we get

$$a = p+1, b = -6(p+1), c = 3(p+9)$$

For real and equal roots, $b^2 - 4ac = 0$

$$\begin{aligned} 36(p+1)^2 - 4(p+1) \times 3(p+9) &= 0 \\ 3(p^2 + 2p + 1) - (p+1)(p+9) &= 0 \\ 3p^2 + 6p + 3 - (p^2 + 9p + p + 9) &= 0 \\ 2p^2 - 4p - 6 &= 0 \\ p^2 - 2p - 3 &= 0 \\ p^2 - 3p + p - 3 &= 0 \\ p(p-3) + 1(p-3) &= 0 \\ (p-3)(p+1) &= 0 \\ p &= -1, 3 \end{aligned}$$

Neglecting $p \neq -1$ we get $p = 3$

Now the equation becomes

$$4x^2 - 24x + 36 = 0$$

or $$x^2 - 6x + 9 = 0$$

or, $$\begin{aligned} (x-3)(x-3) &= 0 \\ x &= 3, 3 \end{aligned}$$

Thus roots are 3 and 3.

9. If the equation $(1+m^2)x^2+2mcx+(c^2-a^2)=0$ has equal roots, prove that $c^2=a^2(1+m^2)$

Ans : [Delhi CBSE Board, 2015]

We have $(1+m^2)x^2+2mcx+(c^2-a^2)=0$

Compare with $Ax^2+Bx+C=0$ we get

$$A=1+m^2, B=2mc, C=(c^2-a^2)$$

If roots are equal, $B^2-4AC=0$

$$(2mc)^2-4(1+m^2(c^2-a^2))=0$$
$$4m^2c^2-4(1+m^2)(c^2-a^2)=0$$
$$m^2c^2-(c^2-a^2+m^2c^2-m^2a^2)=0$$
$$m^2c^2-c^2+a^2-m^2c^2+m^2a^2=0$$
$$-c^2+a^2+m^2a^2=0$$
$$, \qquad c^2=a^2(1+m^2)$$

Hence Proved.

10. If (-5) is a root of the quadratic equation $2x^2+px+15=0$ and the quadratic equation $p(x^2+x)+k=0$ has equal roots, then find the values of p and k.

Ans : [Delhi CBSE Board, 2015 (Set II)]

We have $2x^2+px-15=0$

Since $x=-5$ is the root of above equation. It must satisfy it.

$$2(-5)^2+p(-5)-15=0$$
$$50-5p-15=0$$
$$5p=35 \Rightarrow p=7$$

Now $p(x^2+x)+k=0$ has equal roots

or $7x^2+7x+k=0$

Taking $b^2-4ac=0$ we have

$$7^2-4\times 7\times k=0$$
$$7-4k=0$$
$$k=\frac{7}{4}$$

Hence $p=7$ and $k=\frac{7}{4}$.

11. If the roots of the quadratic equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are equal. Then show that $a=b=c$.

Ans : [Delhi CBSE Board, 2015 (Set II)]

We have

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$$
$$x^2-ax-bx+ab+$$
$$+x^2-bx-cx+bc+$$
$$+x^2-cx-ax+ac=0$$
$$3x^2-2ac-2bx-2cx+ab+bc+ca=0$$

For equal roots $B^2-4AC=0$

$$\{-2(a+b+c)\}^2-4\times 3(ab+bc+ca)=0$$
$$4(a+b+c)^2-12(ab+bc+ca)=0$$
$$a^2+b^2+c^2-3(ab+bc+ca)=0$$
$$a^2+b^2+c^2+2ab+2bc+2ac-3ab-3bc-3ac=0$$
$$a^2+b^2+c^2-ab-ac-bc=0$$
$$\tfrac{1}{2}[2a^2+2b^2+2c^2-2ab-2ac-2bc]=0$$
$$\tfrac{1}{2}[(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ac)]=0$$
$$\tfrac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]=0$$

or, $(a-b)^2+(b-c)^2+(c-a)^2=0$

If $a\neq b\neq c$

$(a-b)^2>0,\ (b-c)^2>0,\ (c-a)^2>0$

If $(a-b)^2=0\Rightarrow a=b$

$(a-c)^2=0\Rightarrow b=c$

$(c-a)^2=0\Rightarrow c=a$

Thus $a=b=c$

Hence Proved

12. If the roots of the quadratic equation $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$ in x are equal then show that either $a=0$ or $a^3+b^3+c^3=3abc$

Ans : [Board Outside Delhi Set II, III 2017]

Ans :

We have $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$

Compare with $Ax^2+Bx+C=0$ we get

$A=(c^2-ab), B=(a^2-bc), C=(b^2-ac)$

If roots are equal, $B^2-4AC=0$

$$[2(a^2-bc)]^2-4(c^2-ab)(b^2-ac)=0$$
$$4[a^4+b^2c^2-2a^2bc]-4(b^2c^2-c^3a-ab^3-a^2bc)=0$$
$$4[a^4+b^2c^2-2a^2bc-b^2c^2+c^3a+ab^3-a^2bc]=0$$
$$4[a^4+ac^3+ab^3-3a^2bc]=0$$
$$a(a^3+c^3+b^3-3abc)=0$$
$$a=0 \text{ or } a^3+b^3+c^3=3abc$$

13. Solve for x : $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$ where $a+b+x\neq 0$ and $a,b,x\neq 0$

Ans : [Board Foreign Set II, III 2017]

We have $\frac{1}{a+b+c}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$

$$\frac{-(a+b)}{x^2+(a+b)x}=\frac{b+a}{ab}$$
$$x^2+(a+b)x+ab=0$$
$$(x+a)(x+b)=0$$
$$x=-a, x=-b$$

Hence $x=-a,-b$

14. The difference between the radii of the smaller circle and the larger circle is 7 cm and the difference between their areas of the two circles in 1078 sq. cm. Find the radius of the smaller circle.

Ans : [Board Comp. I, II, III 2017]

We have $$r_2 - r_1 = 7 \text{ cm}, \; r_2 > r_1 \qquad (1)$$

and $$\pi(r_2^2 - r_1^2) = 1078 \text{ cm}^2$$

$$\pi(r_2 - r_1)(r_2 + r_1) = 1078$$

$$\frac{22}{7} \times 7(r_2 + r_1) = 1078$$

$$r_2 + r_1 = \frac{1078}{22} = 49 \qquad (2)$$

Adding (1) and (2) we get

$$2r_2 = 56$$

$$r_2 = 28 \text{ cm}$$

and $$r_1 = 49 - 28 = 21$$

Hence radii of two circles are 28 cm and 21 cm.

15. A train travelling at a uniform speed for 360 km have taken 48 minutes less to travel the same distance if its speed were 5 km/hour more. Find the original speed of the train

Ans : [Sample Paper 2017]

Ans :

Let the original speed of the train be x km/hr.

$$\text{Time taken} = \frac{Distan\,ce}{Speed} = \frac{360}{x} \text{ hours}$$

$$\text{Time taken at increase speed} = \frac{360}{x+5} \text{ hours.}$$

According to the question

$$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$$

$$360\left[\frac{1}{x} - \frac{1}{x+5}\right] = \frac{4}{5}$$

$$\frac{1800}{x^2 + 5x} = \frac{4}{5}$$

$$x^2 + 5x - 2250 = 0$$

$$x^2 + (50 - 45)x - 2250 = 0$$

$$x^2 - 50x - 45x - 2250 = 0$$

$$(x + 50)(x - 45) = 0$$

$$x = -50 \text{ or } x = 45$$

As speed can not be negative, original speed of train is 45 km/hr.

16. Check whether the equation $5x^2 - 6x - 2 = 0$ has real roots if it has, find them by the method of completing the square. Also verify that roots obtained satisfy the given equation.

Ans : [Sample Question Paper 2017]

We have $5x^2 - 6x - 2 = 0$

Compare with $ax^2 + bx + c = 0$ we get

$$a = 5, b = (-6) \text{ and } c = (-2)$$

$$b^2 - 4ac = (-6)^2 - 4 \times 5 \times -2$$

$$= 36 + 40 = 36 > 0$$

So the equation has real and two distinct roots.

$$5x^2 - 6x = 2$$

Dividing both the sides by 5 we get

$$x^2 - \frac{6}{5}x = \frac{2}{5}$$

$$x^2 - 2x\left(\frac{3}{5}\right) = \frac{2}{5}$$

Adding square of the half of coefficient of x

$$x^2 - 2x\left(\frac{3}{5}\right) + \frac{9}{25} = \frac{2}{5} + \frac{9}{25}$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{19}{25}$$

$$x - \frac{3}{5} = \pm\frac{\sqrt{19}}{5}$$

$$x = \frac{3 + \sqrt{19}}{5} \text{ or } \frac{3 - \sqrt{19}}{5}$$

Verification :

$$5\left[\frac{3 + \sqrt{19}}{5}\right]^2 - 6\left[\frac{3 + \sqrt{19}}{5}\right] - 2$$

$$= \frac{9 + 6\sqrt{19} + 19}{5} - \left(\frac{18 + 6\sqrt{19}}{5}\right) - 2$$

$$= \frac{28 + 6\sqrt{19}}{5} - \frac{18 + 6\sqrt{19}}{5} - 2$$

$$= \frac{28 + 6\sqrt{19} - 18 - 6\sqrt{19} - 10}{5}$$

$$= 0$$

Similarly

$$5\left[\frac{3 - \sqrt{19}}{5}\right]^2 - 6\left[\frac{3 - \sqrt{19}}{5}\right] - 2 = 0$$

Hence verified.

HOTS QUESTIONS

1. Solve $\dfrac{1}{(a+b+x)} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}, a + b \neq 0.$

Ans : [Sample Paper, 2016]

We have $$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$$

$$\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{x - (a+b+x)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\frac{x - a - b - x}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$x(a+b+x) = -ab$$

$$x^2 + (a+b)x + ab = 0$$

$$(x+a)(x+b) = 0$$

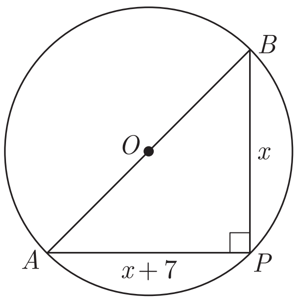
$$x = -a \text{ or } x = -b$$

2. A pole has to erected at a point on the boundary of a circular park of diameter 17 m in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 meters. Find the distances from the two gates where the pole is to be erected.

[Foreign Set I, II, 2016]

Ans : [CBSE Marking Scheme, 2011, 2012]

As per question the figure is shown below.



Let p be the location of the pole such that its distance from gate B, x metres.

Thus $AP = x + 7$
Here AP is diameter or, $\angle APB = 90^\circ$ and $AB = 17$ m

$$\begin{aligned} x^2 + (x+7)^2 &= (17)^2 \\ x^2 + x^2 + 14x - 240 &= 0 \\ x^2 + 7x - 120 &= 0 \\ x &= \frac{-7 \pm \sqrt{49 + 480}}{2} \\ &= \frac{-2 \pm 23}{2} = 8, -15 \end{aligned}$$

Thus $x = 8$ m and $x + 7 = 15$ m
Hence distance between two gates are 8 m and 15 m.

3. Find the value of k for which the distance between $(9, 2)$ and $(3, k)$ is 10 units.

Ans : [Board Term-2, 2012 set (43); Set 2011 set (B1)]

Ans : [CBSE Marking Scheme, 2011, 2012]

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Thus
$$\begin{aligned} \sqrt{(3-9)^2 + (k-2)^2} &= 10 \\ (-6)^2 + k^2 - 4k + 4 &= 100 \\ k^2 - 4k + 40 &= 100 \\ k^2 - 4k - 60 &= 0 \\ k^2 - 10k + 6k - 60 &= 0 \\ k(k-10) + 6(k-10) &= 0 \\ (k-10)(k+6) &= 0 \\ k &= 10, -6 \end{aligned}$$

4. A shopkeeper buys a number of books for Rs. 1200. If he had bought 10 more books for the same amount, each book would have cost him Rs. 20 less. How many books did he buy?

Ans : [Board Term-2, 2012 Set (22)]

Let the number of books bought be x.

As per question we have

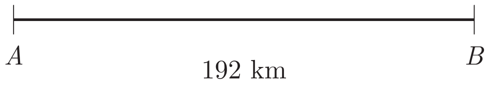
$$\frac{1200}{x} - \frac{1200}{x+10} = 20$$

$$\begin{aligned} x^2 + 10x - 600 &= 0 \\ (x+30)(x-20) &= 0 \\ , \qquad x &= -30 \text{ or } x = 20 \end{aligned}$$

Since number of books cannot be negative, $x = 20$
Thus number of books bought is 20.

5. A journey of 192 km from a town A to town B takes 2 hours more by an ordinary passenger train than a super fast train. If the speed of the faster train is 16 km/h more, find the speed of the faster and the passanger train.

Ans : [Board Term-2, 2012 Set (28)]



Thus speed of super-fast train $= (x + 16)$ km/h

Now $\quad T_{passenger} = \frac{192}{x}$

and $\quad T_{\sup erfast} = \frac{192}{(x+16)}$

As per question we have

$$\frac{192}{x} - \frac{192}{x+16} = 2$$

$$\begin{aligned} 192(x+16) - 192x &= 2(x^2 + 16x) \\ 192x + 192 \times 16 - 192x &= 2(x^2 + 16x) \\ x^2 + 16x - 1536 &= 0 \\ x^2 + 48x - 32x - 1536 &= 0 \\ x(x+48) - 32(x+48) &= 0 \\ (x-32)(x+48) &= 0 \\ x &= 32 \text{ or } -48 \end{aligned}$$

Since speed can't be negative, therefore -48 is not possible. Speed of passenger train is 32 km/h

6. If the price of a book is reduced by Rs. 5, a person can buy 5 more book for Rs.300. Find the original list price of the book.

Ans : [Board Term-2, 2012 sEt (17)]

Let the original list price be Rs. x

No. of books bought for Rs. 300 $= \frac{300}{x}$

Reduced list price of the book $= (x - 5)$

No. of books brough for Rs. 300 $= \frac{300}{x-5}$

According to questions, we have

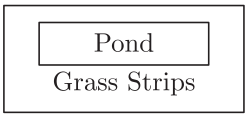
$$\frac{300}{x-5} - \frac{300}{x} = 5$$

$$\begin{aligned} x^2 - 5x - 300 &= 0 \\ (x-20)(x+15) &= 0 \end{aligned}$$

Since price cannot be negative, $x = 20$

Thus original list price is 20 rs.

7. In a rectangular part of dimension $50m \times 40m$ a rectangular pond is constructed so that the area of grass strip of uniform breadth surrounding the pond would be $1184 m^2$. Find the length and breadth of the pond.



Ans : [Board Foregin Set-I, III, 2017]

Let width of grass strip be x mts.

$$\begin{aligned} \text{Length of pond} &= (50-2x)\text{mt} \\ \text{and breadth of pond} &= (40-2x)\text{mt} \end{aligned}$$

and area of park – area of pond = area of grass strip

$$\begin{aligned} (50\times 40)-(50-2x)(40-2x) &= 1184 \\ 2000-2000+180x-4x^2 &= 1184 \\ x^2-45x+296 &= 0 \\ x^2-37x-8x+296 &= 0 \\ x(x-37)-8(x-37) &= 0 \\ x &= 8, 37 \end{aligned}$$

Here 37 is not possible, thus $x=8$

$$\begin{aligned} \text{Length of pond} &= 50-2\times 8 \\ &= 34 \text{ m} \\ \text{and breadth of pond} &= 40-2\times 8 \\ &= 24 \text{ m.} \end{aligned}$$

8. A car covers a distance of 2592 km with a uniform speed. The number of hours taken for journey is one half the number representing the speed in km/hour. Find the time taken to ever the distances.

Ans : [Delhi Compt Set-I, III 2017]

Let the speed of the car be x km/hr.

Therefore time taken $= \frac{x}{2}$ hour

Now
$$\begin{aligned} \text{Speed} &= \frac{Distan\ ce}{Time} \\ x &= \frac{2592}{\frac{x}{2}} \\ x^2 &= 2592\times 2 = 5184 \\ x &= \sqrt{5184} = 72 \end{aligned}$$

Hence the time taken $\frac{72}{2} = 36$ hours.

9. Speed of a boat in still water is 15 km/hour. It goes 30 km up stream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream.

Ans : [Board Delhi Set-I, III, 2017]

Let the speed of the Stream be x km/hr.
Speed of boat up stream $= 15-x$
and speed of boat down stream $= 15+x$
According to the question

$$\begin{aligned} \frac{30}{10-x}+\frac{30}{15+x} &= 4\frac{1}{2} \\ \frac{30(15+x)+30(15-x)}{15^2-x^2} &= \frac{9}{2} \\ 900\times 2 &= 9(15^2-x^2) \\ 9x^2 &= 2025-1800 = 225 \\ x^2 &= 25 \\ x &= \pm 5 \end{aligned}$$

Hence, the speed of the stream = 5 km/hr

10. A takes 6 days less than B to do a work. If both A and B working together can do it in 4 days, how many days will B take to finish it ?

Ans : [Board 2016]

Let B complete a work in x days.
Then A takes $x-6$ days to complete it.
Together they complete it in 4 days.
According to work done per day,

$$\begin{aligned} \frac{1}{x-6}+\frac{1}{x} &= \frac{1}{4} \\ \frac{x+x-6}{x(x-6)} &= \frac{1}{4} \\ 4(2x-6) &= x(x-6) \\ 8x-24 &= x^2-6x \\ x^2-14x+24 &= 0 \\ x^2-12x-2x+24 &= 0 \\ x(x-12)-2(x-12) &= 0 \\ (x-2)(x-12) &= 0 \\ x = 2 \text{ or } 12 \end{aligned}$$

Here $x=2$ is not possible because $x-6$ is (-4) which is not possible.

Thus $x=12$ and B takes 12 days to finish the work.

11. In a class test Raveena got a total of 30 mark in English and Mathematics. Had she got 2 more marks in Mathematics and 3 marks less in English then the product of her marks obtained would have be 210. Find the individual marks obtained in two subjects.

Ans : [Outside Delhi Compt. I, II, III 2017]

Let Raveena got marks in English be x.
Marks in Mathematics $= (30-x)$
Marks in English $= (x-3)$
According to problem

$$\begin{aligned} (x-3)(30-x+2) &= 210 \\ 35x-96-x^2 &= 210 \\ x^2-35x+306 &= 0 \\ x^2-18x-17x+306 &= 0 \\ x(x-18)-17(x-18) &= 0 \\ (x-18)(x-17) &= 0 \\ x &= 18, 17 \end{aligned}$$

Hence, if she got 18 marks in English, then she got 12 in mathematics.
If she got 17 marks in English, then she got 13 marks in mathematics.

12. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank?

Ans : [Outside Delhi I 2017]

Tow tap running together fill the tank in $3\frac{1}{13}$ hr.

$$= \frac{40}{13} \text{ hours}$$

Thus It will fill in 1 hour $= \frac{13}{40}$ tank

If first tap alone fills the tank in x hrs, then second tap alone fills it in $(x+3)$ hr.

Now,
$$\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$$
$$\frac{x+3+x}{x(x+3)} = \frac{13}{40}$$
$$80x+120 = 13x^2+39x$$
or,
$$13x^2-41x-120 = 0$$
$$13x^2-(65-24)x+120 = 0$$
$$(x-5)(13x+24) = 0$$
$$x = 5,\ x = -\frac{24}{13}$$

Here $x = -\frac{24}{13}$ is not possible. Hence, 1st tap takes 5 hours and 2nd tap takes $= 5+3 = 8$ hours

13. Two taps running together can fill a cistern in $2\frac{8}{11}$ minutes. If one tap takes 1 minute more than the other to fill the cistern, find the time in which each tap separately can fill the cistern.

Ans : [Outside Delhi Set-III 2017]

Two taps together fill the cistern in $2\frac{8}{11}$ minutes

$$= \frac{30}{11} \text{ minutes}$$

Thus It will fill in 1 minute $= \frac{11}{30}$ cistern

Let first tap fills the same cistern in x minutes

and 2nd tap will take $= (x+1)$ minutes

Thus
$$\frac{1}{x} + \frac{1}{(x+1)} = \frac{11}{30}$$
$$\frac{x+1+x}{x(x+1)} = \frac{11}{30}$$
$$60x+30 = 11x^2+11x$$
$$11x^2-49x-30 = 0$$
$$11x^2-(55-6)x-30 = 0$$
$$11x^2-55x+6x-30 = 0$$
$$11x(x-5)+6(x-5) = 0$$
$$(x-5)(11x+6) = 0$$
$$x = 5,\ x = -\frac{6}{11}$$

Here $x = -\frac{6}{11}$ is not possible. Hence, 1st tap takes 5 minute and 2nd tap takes 6 minute.

14. A and B working together can do a work in 6 days. If A takes 5 days less than B to finish the work, in how many days B alone can do it alone?

Ans : [Outside Delhi Compt. Set-I]

Since $A+B$ finish the work in 6 days.

They will finish in one day $= \frac{1}{6}$ work

Let B alone does the same work in x days, then A alone will finish in $(x-5)$ days.

Now,
$$\frac{1}{x-5} + \frac{1}{x} = \frac{1}{6}$$
$$\frac{x+x-5}{x(x-5)} = \frac{1}{6}$$
$$12x-30 = x^2-5x$$
$$x^2-17x+30 = 0$$
$$x^2-(15+2)x+30 = 0$$
$$x^2-15x-2x+30 = 0$$
$$x(x-15)-(x-15) = 0$$
$$(x-15)(x-2) = 0$$
$$x = 15, x = 2$$

Here $x = 2$ is not possible. Hence, B finishes the work in 15 days.

15. Ram takes 6 days less than Bhagat to finish a place of work. If both of them together can finish the work in 4 days, in how many days Bhagat alone can finish the work ?

Ans : [Delhi Compt. Set-III 2017, Outside Delhi Set-II2017]

Ram and Bhagat together do the work in 4 days

Ram and Bhagat will do in one days $= \frac{1}{4}$ work

Let Bhagat alone does the same work in x days.

Ram will take $= (x-6)$ days

Now
$$\frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$$
$$\frac{x-6+x}{x(x-6)} = \frac{1}{4}$$
$$8x-24 = x^2-6x$$
$$x^2-14x+24 = 0$$
$$x^2-(12+2)x+24 = 0$$
$$x^2-12x-2x+24 = 0$$
$$x(x-12)-2(x-12) = 0$$
$$(x-12)(x-12) = 0$$
$$x = 12, x = 2$$

If Bhagat complete the work in 2 days, Ram will take $= 2-6 = -4$ days that is impossible. Hence, Bhagat can finish in 12 days.

CHAPTER 5

Arithmetic Progression

TOPIC 1 : TO FIND N^{TH} TERM OF THE ARITHMETIC PROGRESSION

VERY SHORT ANSWER TYPE QUESTIONS

1. Is -150 a term of the A.P. 11, 8, 5, 2, ...?

Ans : [CBSE S.A2 2016 Set-HODM40L]

Let the first term of an A.P. be a and common difference be d.

We have $a = 11,\ d = -3,\ a_n = -150$

$$\begin{aligned} \text{Now} \qquad a_n &= a+(n-1)d \\ -150 &= 11+(n-1)(-3) \\ -150 &= 11-3n+3 \\ 3n &= 164 \\ \text{or,} \qquad n &= \frac{164}{3} = 54.66 \end{aligned}$$

Since, 54.66 is not a whole number, -150 is not a term of the given A.P.

2. Which of the term of $A.P. 5, 2, -1,......$ is -49?

Ans : [CBSE Marking Scheme, 2012]

Let the first term of an A.P. be a and common difference d.

We have $a = 5,\ d = -3$

Now $a_n = a+(n-1)d$

Substituting all values we have

$$\begin{aligned} -49 &= 5+(n-1)(-3) \\ -49 &= 5-3n+3 \\ 3n &= 49+5+3 \\ n &= \frac{57}{3} = 19^{th} \text{ term.} \end{aligned}$$

3. Find the first four terms of an A.P. Whose first term is -2 and common difference is -2.

Ans : [Board Term-2, 2012 Set (17)]

We have $a_1 = -2,$

$$\begin{aligned} a_2 = a_1+d &= -2+(-2) = -4 \\ a_3 = a_1+d &= -4+(-2) = -6 \\ a_4 = a_3+d & \ -6+(-2) = -8 \end{aligned}$$

Hence first four terms are $-2, -4, -6, -8$

4. Find the tenth term of the sequence $\sqrt{2}, \sqrt{8}, \sqrt{18}, ...$

Ans : [Board Sample paper, 2016]

Let the first term of an A.P. be a and common difference be d.

Given AP is $\sqrt{2}, \sqrt{8}, \sqrt{18}$ or $\sqrt{2},\ 2\sqrt{2},\ 3\sqrt{2}...$

where, $a = \sqrt{2}, d = \sqrt{2}, n = 10$

$$\begin{aligned} \text{Now} \qquad a_n &= a+(n-1)d \\ a_{10} &= \sqrt{2}+(10-1)\sqrt{2} \\ &= \sqrt{2}+9\sqrt{2} \\ &= 10\sqrt{2} \end{aligned}$$

Therefore tenth term of the given sequence $\sqrt{200}$.

5. Find the next term of the series $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ ....

Ans : [Board Term-2, 2012 Set (22)]

Let the first term of an A.P. be a and common difference d.

$$\begin{aligned} \text{Here,} \qquad a &= \sqrt{2},\ a+d = \sqrt{8} = 2\sqrt{2} \\ d &= 2\sqrt{2}-\sqrt{2} = \sqrt{2} \\ \text{Next term} &= \sqrt{32}+\sqrt{2} \\ &= 4\sqrt{2}+\sqrt{2} \\ &= 5\sqrt{2} \\ &= \sqrt{50} \end{aligned}$$

6. Is series $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$ an A.P.? Give reason.

Ans : [Board Term-2, 2015]

Let common difference be d then we have

$$\begin{aligned} d = a_2-a_1 &= \sqrt{6}-\sqrt{3} = \sqrt{3}(\sqrt{2}-1) \\ d = a_3-a_2 &= \sqrt{9}-\sqrt{6} = 3-\sqrt{6} \\ d = a_4-a_3 &= \sqrt{12}-\sqrt{9} = 2\sqrt{3}-3 \end{aligned}$$

As common difference are not equal, the given series is not in A.P.

7. What is the next term of an A.P. $\sqrt{7},\ \sqrt{28},\ \sqrt{63}, ...$?

Ans : [Foreign Set I, II, III, 2014]

Let the first term of an A.P. be a and common difference be d.

Here, $a = \sqrt{7},\ a+d = \sqrt{28}$

$$\begin{aligned} d &= \sqrt{28}-\sqrt{7} = 2\sqrt{7}-\sqrt{7} \\ &= 7 \\ \text{Next term} &= \sqrt{63}+\sqrt{7} \\ &= 3\sqrt{7}+\sqrt{7} = 4\sqrt{7} \\ &= \sqrt{7\times 16} \\ &= \sqrt{112} \end{aligned}$$

8. If the common difference of an A.P. is -6, find $a_{16}-a_{12}$.

Ans : [KVS 2014]

Let the first term of an A.P. be a and common difference be d.

Now $d = -6$

$$\begin{aligned} a_{16} &= a+(16-1)(-6) = a-90 \\ a_{12} &= a+(12-1)(-6) = a-66 \\ a_{16}-a_{12} &= (a-90)-(a-66) = a-90-n+66 \\ &= -24 \end{aligned}$$

9. For what value of k will the consecutive terms $2k+1$, $3k+3$ and $5k-1$ from an A.P.?

Ans : [Foreign Set I, II, III, 2016]

If x, y and z are in AP the we have

$$y-x = z-y$$

Thus if $2k+1$, $3k+3$, $5k-1$ are in A.P. then

$$\begin{aligned} (5k-1)-3k+3 &= (3k+3)-(2k+1) \\ 5k-1-3k-3 & \;\; 3k+3-2k-1 \\ 2k-4 &= k+2 \\ 2k-k &= 4+2 \\ k &= 6 \end{aligned}$$

10. Find the 25th term of the A.P. $-5, \frac{-5}{2}, \frac{5}{2}, \ldots..$

Ans : [Foreign Set I, II, III, 2015]

Let the first term of an A.P. be a and common difference be d.

Here, $a = -5, d = -\frac{5}{2}-(-5) = \frac{5}{2}$

$$\begin{aligned} a_n &= a+(n-1)d \\ a_{25} &= 5+(25-1)\times\left(\frac{5}{2}\right) \\ &= -5+60 \\ &= 55 \end{aligned}$$

11. The first three terms of an A.P. are $3y-1$, $3y+5$ and $5y+1$ respectively then find y.

Ans : [Delhi CBSE Term-2, 2015]

If x, y and z are in AP then we have

$$y-x = z-y$$

Therefore if $3y-1, 3y+5$ and $5y+1$ in A.P.

$$\begin{aligned} (3y+5)-(3y-1) &= (5y+1)-(3y+5) \\ 3y+5-3y+1 &= 5y+1-3y-5 \\ 6 &= 2y-4 \\ 2y &= 6+4 \\ y &= \frac{10}{2} = 5 \end{aligned}$$

12. For what value of k will $k+9$, $2k-1$ and $2k+7$ are the consecutive terms of an A.P.

Ans : [Outsidde Delhi Set II, 2016]

If x, y and z are consecutive terms of an A.P. then we have

$$y-x = z-y$$

Thus if $k+9, 2k-1$, and $2k+7$ are consecutive terms of an A.P. then we have

$$\begin{aligned} (2k-1)-(k+9) &= (2k+7)-(2k-1) \\ 2k-1-k-9 &= 2k+7-2k+1 \\ k-10 &= 8 \\ k &= 10+8 = 18 \end{aligned}$$

13. What is the common difference of an A.P. in which $a_{21}-a_7 = 84$?

Ans : 2016

Let the first term of an A.P. be a and common difference be d.

$$\begin{aligned} a_{21}-a_7 &= 84 \\ a+(21-1)d-\left[a+(7-1)d\right] &= 84 \\ a+20d-a-6d &= 84 \\ 14d &= 84 \\ d &= 6 \end{aligned}$$

14. In the A.P. $2, x, 26$ find the value of x.

Ans : [Board Term-2, 2012(13)]

If x, y and z are in AP then we have

$$y-x = z-y$$

Since $2, x$ and 26 are in A.P. we have

$$\begin{aligned} x-2 &= 26-x \\ 2x &= 26+2 \\ x &= \frac{28}{2} = 14 \end{aligned}$$

15. For what value of $k; k+2$, $4k-6$, $3k-2$ are three consecutive terms of an A.P.

Ans : [Board, Term-2, Delhi 2014], [Board Term-2, 2012 Set (1)]

If x, y and z are three consecutive terms of an A.P. then we have

$$y-x = z-y$$

Since $k+2$, $4k-6$ and $3k-2$ are three consecutive terms of an AP, we obtain

$$\begin{aligned} (4k-6)-(k+2) &= (3k-2)-(4k-6) \\ 4k-6-k-2 &= 3k-2-4k+6 \\ 3k-8 &= -k+4 \\ 4k &= 4+8 \\ k &= \frac{12}{4} = 3 \end{aligned}$$

16. If $18, a, b, -3$ are in AP, then find $a+b$.

Ans : [Board Term-2, 2012 Set (34)]

If $18, a, b, -3$ are in AP, then,

$$\begin{aligned} a-18 &= -3-b \\ a+b &= -3+18 \end{aligned}$$

$$a+b = 15$$

17. Find the common difference of the A.P. $\frac{1}{3q}$, $\frac{1-6q}{3q}$, $\frac{1-12q}{3q}$, ...

Ans : [Board Term-2, Delhi 2013]

Let common difference be d then we have

$$d = \frac{1-6q}{3q} - \frac{1}{3q}$$

$$= \frac{1-6q-1}{3q} = \frac{-6q}{3q} = -2$$

18. Find the first four terms of an A.P. whose first term is $3x+y$ and common difference is $x-y$.

Ans : [Board Term-2, 2012 Set(25)]

Let the first term of an A.P. be a and common difference be d.

Now $$a_1 = 3x+y$$

$$a_2 = a_1 + d = 3x+y+x-y = 4x$$

$$a_3 = a_2 + d = 4x+x-y = 5x-y$$

$$a_4 = a_3 + d = 5x-y+x-y$$

$$= 6x-2y$$

So, the four terms are $3x+y$, $4x, 5x-y$ and $6x-2y$.

19. Find the 37^{th} term of the A.P. $\sqrt{x}$, $3\sqrt{x}$, $5\sqrt{x}$

Ans : [Board Term-2, 2012 Set (50)]

Let the nth term of an A.P. be a_n and common difference be d.

Here, $$a_1 = \sqrt{x}$$

$$a_2 = 3\sqrt{x}$$

$$d = a_2 - a_1 = 3\sqrt{x} - \sqrt{x} = 2\sqrt{x}$$

$$a_n = a + (n-1)d$$

$$a_{37} = \sqrt{x} + (37-1)2\sqrt{x}$$

$$= \sqrt{x} + 36 \times 2\sqrt{x}$$

$$= 73\sqrt{x}$$

20. For an A.P., if $a_{25} - a_{20} = 45$, then find the value of d.

Ans : [Board Term-2, 2011, Set B1]

Let the first term of an A.P. be a and common difference be d.

Now $$a_{25} - a_{20} = \{a+(25-1)d\} - \{a+(20-1)d\}$$

$$45 = a + 24d - a - 19d$$

$$45 = 5d$$

$$d \frac{45}{5} = 9$$

SHORT ANSWER TYPE QUESTIONS - I

1. Find, 100 is a term of the A.P. 25, 28, 31, ... or not.

Ans : [Board Term-2, 2012(12)]

Let the first term of an A.P. be a, common difference be d and number of terms be n.

Let $a_n = 100$

Here $a = 25, d = 28-25 = 31-28 = 3$

Now $$a_n = a + (n-1)d,$$

$$100 = 25 + (n-1) \times 3$$

$$100 - 25 = 75 = (n-1) \times 3$$

$$25 = n-1$$

$$n = 26$$

Hence, 100 is a term of the given A.P.

2. Is 184 a term of the sequence 3, 7, 11, ...?

Ans : [Board Term-2, 2012(44)]

Let the first term of an A.P. be a, common difference be d and number of terms be n.

Let $a_n = 184$

Here, $a = 3, d = 7-3 = 11-7 = 4$

Now $$a_n = a + (n-1)d,$$

$$184 = 3 + (n-1)4$$

$$\frac{181}{4} = n-1$$

$$45.25 = n-1$$

$$46.25 = n$$

Since 46.25 is not an whole number, thus 184 is not a term of given A.P.

3. Find the 7^{th} term from the end of A.P. 7, 10, 13, ... 184.

Ans : [Delhi Set 2014]

[Board Term-2, 1012 Set (34)]

Let us write A.P. in reverse order i.e., 184, ... 13, 10, 7

Let the first term of an A.P. be a and common difference be d.

Now $$d = 7-10 = -3$$

$$a = 184, n = 7$$

7^{th} term from the end,

$$a_7 = a + 6d$$

$$a_7 = 184 + 6(-3)$$

$$= 184 - 18 = 166.$$

Hence, 166 is the 7^{th} term from the end.

4. Which term of an A.P. 150, 147, 144, ... is its first negative term?

Ans : [KVS 2014]

Let the first term of an A.P. be a, common difference be d and nth term be a_n.

For first negative term $a_n < 0$

$$a + (n-1)d < 0$$

$$150+(n-1)(-3) < 0$$
$$150-3n+3 < 0$$
$$-3n < -153$$
$$n > 51$$

Therefore, the first negative term is 52^{nd} term.

5. In a certain A.P. 32^{th} term is twice the 12^{th} term. Prove that 70^{th} term is twice the 31^{st} term.

Ans : [Board Term-2, 2015, 2012, Set-28]

Let the first term of an A.P. be a, common difference be d and nth term be a_n.

Now we have $a_{32} = 2a_{12}$

$$a+31d = 2(a+11d)$$
$$a+31d = 2a+22d$$
$$a = 9d$$
$$a_{70} = a+69d$$
$$= 9d+69d = 78d$$
$$a_{31} = a+30d$$
$$= 9d+30d = 39d$$
$$a_{70} = 2a_{31}$$

Hence Proved.

6. The 8^{th} term of an A.P. is zero. Prove that its 38^{th} term is triple of its 18^{th} term.

Ans : [Board Term-2, 2012(28)]

Let the first term of an A.P. be a, common difference be d and nth term be a_n.
We have, $a_8 = 0$ or, $a+7d = 0$ or, $a = -7d$

Now $a_{38} = a+37d$

$$a_{38} = -7d+37d = 30d$$
$$a_{18} = a+17d$$
$$= -7d+17d = 10d$$
$$a_{38} = 30d = 3\times 10d = 3\times a_{18}$$
$$a_{38} = 3a_{18}$$

Hence Proved

7. If five times the fifth term of an A.P. is equal to eight times its eighth term, show that its 13^{th} term is zero.

Ans : [Board Term-2, 2012(13)]

Let the first term of an A.P. be a, common difference be d and nth term be a_n.

Now $5a_5 = 8a_8$

$$5(a+4d) = 8(a+7d)$$
$$5a+20d = 8a+56d$$
$$3a+36d = 0$$
$$3(a+12d) = 0$$
$$a+12d = 0$$
$$a_{13} = 0$$

Hence Proved

8. The fifth term of an A.P. is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference.

Ans : [Foreign Set II, 2015]

Let the first term be a and common difference be d.

$$a+4d = 20 \quad ...(1)$$
$$a+6d+a+10d = 64$$
$$a+8d = 32 \quad ...(2)$$

Solving equations (1) and (2), we have

$$d = 3$$

9. The ninth term of an A.P. is -32 and the sum of its eleventh and thirteenth term is -94. Find the common difference of the A.P.

Ans : [Foreign Set III, 2015]

Let the first term be a and common difference be d.

Now $a+8d = a_9$

$$a+8d = -32 \quad ...(1)$$

and $a_{11} - a_{13} = -94$

$$a+10d+a+12d = -94$$
$$a+11d = -47 \quad ...(2)$$

Solving equation (1) and (2), we have

$$d = -5$$

10. The seventeenth term of an A.P. exceeds its 10^{th} term by 7. Find the common difference.

Ans : [Board Term-2, 2015, 14]

Let the first term be a and common difference be d.

Now $a_{17} = a_{10}+7$

$$a+16d = a+9d+7$$
$$16d-9d = 7$$
$$7d = 7$$
$$d = 1$$

Thus common difference is 1.

11. The fourth term of an A.P. is 11. The sum of the fifth and seventh terms of the A.P. is 34. Find the common difference.

Ans : [Foreign set I, 2015]

Let the first term be a and common difference be d.

Now $a_4 = 11$

$$a+3d = 11 \quad ...(1)$$

and $a_5 + a_7 = 34$

$$a+4d+a+6d = 34$$

, $2a+10d = 34$

, $a+5d = 17 \quad ...(2)$

Solving equations (1) and (2) we have

$$d = 3$$

12. Find the middle term of the A.P. 213, 205, 197, ... 37.

Ans : [Board Term-2, Delhi 2015 (Set II)]

Let the first term of an A.P. be a, common difference be d and number of terms be m.
Here, $a = 213, d = 205-213 = -8, a_m = 37$

$$a_m = a+(m-1)d$$
$$37 = 213+(m-1)(-8)$$
$$37-213 = -8(m-1)$$
$$m-1 = \frac{-176}{-8} = 22$$

$$m = 22+1 = 23$$

The middle term will be $= \dfrac{23+1}{2} = 12^{th}$

$$\begin{aligned} a_{12} &= a+(12-1)d \\ &= 213+(12-1)(-8) \\ &= 213-88 = 125 \end{aligned}$$

Middle term will be 125.

13. Find the middle term of the A.P. 6, 13, 20, ... 216.

Ans : [board Term-2, Delhi 2015 (Set I, III)]

Let the first term of an A.P. be a, common difference be d and number of terms be m.
Here, $a = 6, a_m = 216, d = 13-6 = 7$

$$\begin{aligned} a_m &= a+(m-1)d \\ 216 &= 6+(m-1)(7) \\ 216-6 &= 7(m-1) \\ m-1 &= \frac{210}{7} = 30 \\ m &= 30+1 = 31 \end{aligned}$$

The middle term will be $= \dfrac{31+1}{2} = 16^{th}$

$$\begin{aligned} a_{16} &= a+(16-1)d \\ &= 6+(16-1)(7) \\ &= 6+15\times 7 \\ &= 6+105 = 111 \end{aligned}$$

Middle term will be 111.

14. If the 2^{nd} term of an A.P. is 8 and the 5^{th} term is 17, find its 19^{th} term.

Ans : [board Term-2, 2016 Set HoDM40L]

Let the first term be a and common difference be d.

$$\begin{aligned} \text{Now} \qquad a_2 &= a+d \\ 8 &= a+d \qquad (1) \\ \text{and} \qquad a_5 &= a+4d \\ 17 &= a+4d \qquad (2) \end{aligned}$$

Solving (1) and (2), we have

$$\begin{aligned} a &= 5, d = 3, \\ a_{19} &= a+18d \\ &= 5+54 = 59 \end{aligned}$$

15. If the number $x+3, 2x+1$ and $x-7$ are in A.P. find the value of x.

Ans : [Board Term-2 2012(5)]

If x, y and z are three consecutive terms of an A.P. then we have

$$\begin{aligned} y-x &= z-y \\ (2x+1)-(x+3) &= (x-7)-(2x+1) \\ 2x+1-x-3 &= x-7-2x-1 \\ x-2 &= -x-8 \\ 2x &= -6 \\ x &= -3 \end{aligned}$$

16. Find the values of a, b and c, such that the numbers $a, 10, b, c, 31$ are in A.P.

Ans : [Board Term-2, 2012 (21)]

Let the first term be a and common difference be d.
Since $a, 10, b, c, 31$ are in A.P.

$$\begin{aligned} \text{Now} \qquad a+d &= 10 \qquad (1) \\ a+4d &= a_5 \\ a+4d &= 31 \qquad (2) \end{aligned}$$

Solving (1) and (2) we have

$$d = 7 \text{ and } a = 3$$

Now $a = 3, b = 3+14 = 17, c = 3+21 = 24$

Thus $a = 3, b = 17, c = 24$.

17. For A.P. show that $a_p + a_{p+2q} = 2a_{p+q}$.

Ans : [Board Term-2, 2012(1)]

Let the first term be a and the common difference be d. Let a_n be the nth term.

$$\begin{aligned} a_p &= a+(p-1)d \\ a_{p+2q} &= a+(p+2q-1)d \\ a_p + a_{p+2q} &= a+(p-1)d+a+(p+2q-1)d \\ &= a+pd-d+a+pd+2qd-d \\ &= 2a+2pd+2qd-2d \\ \text{or } a_p + a_{p+2q} &= 2\left[a+(p+q-1)d\right] \qquad ...(1) \\ \text{But} \quad 2a_{p+q} &= 2\left[a+(p+q-1)d\right] \qquad ...(2) \end{aligned}$$

From (1) and (2), we get $a_p + a_{p+2q} = 2a_{p+q}$

18. The sum of first terms of an A.P. is give by $S_n = 2n^2 + 8n$. Find the sixteenth term of the A.P.

Ans : [Sample Question Paper 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$\begin{aligned} \text{Now} \qquad S_n &= 2n^2+3n \\ S_1 &= 2\times 1^2+3\times 1 = 2+3 = 5 \end{aligned}$$

Since $S_1 = a_1$,

$$\begin{aligned} a_1 &= 5 \\ S_2 &= 2\times 2^2+3\times 2 = 8+6 = 14 \\ a_1+a_2 &= 14 \\ a_2 &= 14-a_1 = 14-5 = 9 \\ d &= a_2-a_1 = 9-5 = 4 \\ a_{16} &= a+(16-1)d \\ &= 5+15\times 4 = 65 \end{aligned}$$

19. The 4^{th} term of an A.P. is zero. Prove that the 25^{th} term of the A.P. is three times its 11^{th} term.

Ans : [Outside Delhi Set, II 2016]

Let the first term be a, common difference be d and nth term be a_n.

$$\begin{aligned} \text{We have,} \quad a_4 &= 0 \\ a+3d &= 0 \qquad [\,a+(n-1)d = a_n] \\ 3d &= -a \\ -3d &= a \qquad ...(1) \\ \text{Now,} \quad a_{25} &= a+24d = -3d+24d = 21d \qquad ...(2) \end{aligned}$$

$$a_{11} = a+10d = -3d+10d = 7d \qquad ...(3)$$

From eqn (2) and eq (3) we have

$$a_{25} = 3a_{11} \qquad \text{Hence Proved.}$$

SHORT ANSWER TYPE QUESTIONS - I

1. Find the 20th term of an A.P. whose 3rd term is 7 and the seventh term exceeds three times the 3rd term by 2. Also find its n^{th} term (a_n).

Ans : [Board Term-2, 2012 (31)]

Let the first term be a, common difference be d and nth term be a_n.

We have

$$\begin{aligned} a_3 &= a+2d = 7 && (1)\\ a_7 &= 3a_3+2 \\ a+6d &= 3\times 7+2 = 23 && (2)\end{aligned}$$

Solving (1) and (2) we have

$$\begin{aligned} 4d &= 16 \Rightarrow d = 4\\ a+8 &= 7 \Rightarrow a = -1\\ a_{20} &= a+19d = -1+19\times 4 = 75\\ a_1 &= a+(n-1)d\\ &= -1+4n-4\\ &= 4n-5.\end{aligned}$$

Hence n^{th} term is $4n-5$

2. If 7^{th} term of an A.P. is $\frac{1}{9}$ and 9^{th} term is $\frac{1}{7}$, find 63^{rd} term.

Ans : [Board Term-2, Delhi, 2014]

Let the first term be a, common difference be d and nth term be a_n.

We have

$$\begin{aligned} a_7 &= \frac{1}{9} \Rightarrow a+6d = \frac{1}{9} && (1)\\ a_9 &= \frac{1}{7} \Rightarrow a+8d = \frac{1}{7} && (2)\end{aligned}$$

Subtracting equation (1) from (2) we get

$$2d = \frac{1}{7}-\frac{1}{9} = \frac{2}{63} = \frac{1}{63}$$

Substituting the value of d in (2) we get

$$\begin{aligned} a+8\times\frac{1}{63} &= \frac{1}{7}\\ a &= \frac{1}{7}-\frac{8}{63} = \frac{9-8}{63} = \frac{1}{63}\end{aligned}$$

Thus

$$\begin{aligned} a_{63} &= a+(63-1)\,d\\ &= \frac{1}{63}+62\times\frac{1}{63} = \frac{1+62}{63}\\ &= \frac{63}{63} = 1\end{aligned}$$

Hence, $a_{63} = 1$

3. The ninth term of an A.P. is equal to seven times the second term and twelfth term exceeds five times the third term by 2. Find the first term and the common difference.

Ans : [Board Sample Paper, 2016]

Let the first term be a, common difference be d and nth term be a_n.

Now

$$\begin{aligned} a_9 &= 7a_2\\ a+8d &= 7(a+d)\\ a+8d &= 7a+7d\\ -6a+d &= 0 && (1)\end{aligned}$$

and

$$\begin{aligned} a_{12} &= 5a_3+2\\ a+11d &= 5(a+2d)+2\\ a+11d &= 5a+10d+2\\ -4a+d &= 2 && ...(2)\end{aligned}$$

Subtracting (2) from (1), we get

$$\begin{aligned} -2a &= -2\\ a &= 1\end{aligned}$$

Substituting this value of a in (1) we get

$$\begin{aligned} -6+d &= 0\\ d &= 6\end{aligned}$$

Hence first term is 1 and common difference is 6.

4. Determine an A.P. whose third term is 9 and when fifth term is subtracted from 8^{th} term, we get 6.

Ans : [Board Term-2, 2015]

Let the first term be a, common difference be d and nth term be a_n.

We have

$$\begin{aligned} a_3 &= 9\\ a+2d &= 9 && ...(1)\end{aligned}$$

and

$$\begin{aligned} a_8-a_5 &= 6\\ (a+7d)-(a+4d) &= 6\\ 3d &= 6\\ d &= 2\end{aligned}$$

Substituting this value of d in (1), we get

$$\begin{aligned} a+2(2) &= 9\\ a &= 5\end{aligned}$$

So, A.P. is 5, 7, 9, 11, ...

5. Divide 56 in four parts in A.P. such that the ratio of the product of their extremes (1^{st} and 4^{rd}) to the product of means (2^{nd} and 3^{rd}) is 5:6.

Ans : [Foreign Set I, 2016]

Let the four numbers be $a-3d, a-d, a+d, a+3d$

Now $a-3d+a-d+a+d+a+3d = 56$

$$4a = 56 \Rightarrow a = 14$$

Hence numbers are $14-3d, 14-d, 14+d, 14+3d$

Now, according to question,

$$\begin{aligned} \frac{(14-3d)(14+3d)}{(14-d)(14+d)} &= \frac{5}{6}\\ \frac{196-9d^2}{196-d^2} &= \frac{5}{6}\\ 6(196-9d^2) &= 5(196-d^2)\\ 6\times 196-54d^2 &= 5\times 196-5d^2\\ (6-5)\times 196 &= 49d^2\\ d^2 &= \frac{196}{49} = 4\end{aligned}$$

$$d = \pm 2$$

Thus numbers are

$$\begin{aligned} a - 3d &= 14 - 3 \times 2 = 8 \\ a - d &= 14 - 2 = 12 \\ a + d &= 14 + 2 = 16 \\ a + 3d &= 14 + 3 \times 2 = 20 \end{aligned}$$

Thus required AP is $8, 12, 16, 20$.

6. The $p^{th,}$ q^{th} and r^{th} terms of an A.P. are a, b and c respectively, Show that $a(q - r) + b(r - p) + c(p - q) = 0$.

Ans : [Foreign Set II, 2016]

Let the first term be A and the common difference be D.

$$\begin{aligned} a &= A + (p - 1)D \\ b &= A + (q - 1)D \\ c &= A + (r - 1)D \end{aligned}$$

Now

$$\begin{aligned} a(q - r) &= [A + (p - 1)D][q - r] \\ b(r - p) &= [A + (q - 1)D][r - p] \end{aligned}$$

and

$$c[p - q] = [A + (r - 1)D][p - q]$$

$$\begin{aligned} & a(q - r) + b(r - p) + c(p - q) \\ &= [A + (p - 1)D][q - r] + \\ &\quad + [A + (q - 1)D][r - p] + \\ &\quad + [A + (r - 1)D][p - q] + \\ &= A[p - q + q - p + q - r] + \\ &\quad + D(p - 1)(q - r) + \\ &\quad + D(q - 1)(r - p) + \\ &\quad + D(r - 1)(p - q) \\ &= A[0] + \\ &\quad + D[p(q - r) - (q - r)] \\ &\quad + D[q(r - p) - (r - p)] \\ &\quad + D[r(p - q) - (p - q)] \\ &= D[p(q - r) + q(r - p) + r(p - q)] + \\ &\quad - D[(q - r) + (r - p) + (p - q)] \\ &= D[pq - pr + qr - qp + rp - rq] + 0 \\ &= D[0] = 0 \end{aligned}$$

7. The sum of n terms of an A.P. is $3n^2 + 5n$. Find the A.P. Hence find its 15^{th} term.

Ans : [Board Term-2, 2013], [Board Term-2, 2012 Set (38, 39)]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now

$$\begin{aligned} S_n &= 3n^2 + 5m \\ S_{n-1} &= 3(n - 1)^2 + 5(n - 1) \\ &= 3(n^2 + 1 - 2n) + 5n - 5 \\ &= 3n^2 + 3 - 6n + 5n - 5 \\ &= 3n^2 - n - 2 \\ a_n &= S_n - S_{n-1} \\ &= 3n^2 + 5n - (3n^2 - n - 2) \\ &= 6n + 2 \end{aligned}$$

Thus A.P. is 8, 14, 20, ...

Now

$$a_{15} = a + 14d = 8 + 14(6) = 92$$

8. The digit of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less then the original number. Find the number.

Ans : [Outside Delhi Set II, 2016]

Let these digit of 3 digit no be $- a - d, a, a + d$

Since their sum is 15,

$$\begin{aligned} a - d + a + a + d &= 15 \\ 3a &= 15 \Rightarrow a = 5 \end{aligned}$$

Required 3 digit no

$$\begin{aligned} &= 100(a - d) + 10a + a + d \\ &= 100a - 100d + 10a + a + d \\ &= 111a - 99d \end{aligned}$$

No obtained by reversing digit

$$\begin{aligned} &= 100(a + d) + 100 + a - d \\ &= 100a + 100d + 10a + a - d \\ &= 111a + 99d \end{aligned}$$

According the question,

$$\begin{aligned} 111a + 99d &= 111a - 99d - 594 \\ 2 \times 99d &= 594 \Rightarrow d = - 8 \end{aligned}$$

Thus number is

$$\begin{aligned} 111a - 99d &= 111 \times 5 - 99 \times - 3 \\ &= 555 + 297 = 852 \end{aligned}$$

9. For what value of n, are the n^{th} terms of two A.Ps 63, 65, 67, ... and 3, 10, 17, ... equal?

Ans :

Let a, d and A, D be the 1^{st} term and common difference of the 2 APs respectively.

n is same

For 1st AP, $a = 63$, $d = 2$

For 2nd AP, $A = 3$, $D = 7$

Since nth term is same,

$$\begin{aligned} an &= An \\ a + (n - 1)d &= A + (n - 1)D \\ 63 + (n - 1)2 &= 3 + (n - 1)7 \\ 63 + 2n - 2 &= 3 + 7n - 7 \\ 61 + 2n &= 7n - 4 \\ 65 &= 5n \Rightarrow n = 13 \end{aligned}$$

When n is 13, the n^{th} terms are equal i.e., $a_{13} = A_{13}$

LONG ANSWER TYPE QUESTIONS

1. The sum of three numbers in A.P. is 12 and sum of their cubes is 288. Find the numbers.

Ans : [delhi Set III, 2016]

Let the three numbers in A.P. be $a - d, a, a + d$.

$$\begin{aligned} a - d + a + a + d &= 12 \\ 3a &= 12 \\ a &= 4 \end{aligned}$$

Also, $(4 - d)^3 + 4^3 + (4 + d)^3 = 288$

$$\begin{aligned} 64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 & \\ &= 288 \\ 24d^2 + 192 &= 288 \end{aligned}$$

$$d^2 = 4$$

$$d = \pm 2$$

The numbers are 2, 4, 6 or 6, 4,2

2. Find the value of a, b and c such that the numbers $a, 7, b, 23$ and c are in A.P.

Ans : [Board Term-2, 2015]

Let the common difference be d.
Since $a, 7, b, 23$ and c are in AP, we have

$$a + d = 7 \quad ..(1)$$

$$a + 3d = 23 \quad ...(2)$$

Form (1) and (2), we get

$$a = -1, d = 8$$

$$b = a + 2d = -1 + 2 \times 8 = -1 + 16 = 15$$

$$c = a + 4d = -1 + 4 \times 8 = -1 + 32 = 31$$

Thus $a = -1, b = 15, c = 31$

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the sum of first ten multiple of 5.

Ans : [Board Term-2, Delhi, 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n
Here, $a = 5, n = 10, d = 5$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2 \times 5 + (10-1)5]$$

$$= 5[10 + 9 \times 5]$$

$$= 5[10 + 45]$$

$$= 5 \times 55 = 275$$

Hence the sum of first ten multiple of 5 is 275.

2. Find the sum of first five multiples of 2.

Ans : [Board Term-2, 2012 st (05)]

Let the first term be a, common difference be d, nth term be a_n and sum of $n$the term be S_n
Here, $a = 2, d = 2, n = 5$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_5 = \frac{5}{2}[2 \times 2 + (5-1)2]$$

$$= \frac{5}{2}[4 + 4 \times 2] = \frac{5}{2}[4 + 8]$$

$$= \frac{5}{2} \times 12 = 5 \times 6 = 30$$

3. Find the sum of first 16 terms of the A.P. 10, 6, 2, ...

Ans : [Board Term-2, 2012, Set (32)

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n
Here, $a = 10, d = 6 - 1 = -4, n = 16$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{16} = \frac{16}{2}[2 \times 10 + (16-1)(-4)]$$

$$= 8[20 + 15 \times (-4)]$$

$$= 8[20 - 60]$$

$$= 8 \times (-40)$$

$$= -320$$

4. What is the sum of five positive integer divisible by 6.

Ans : [Board Term-2, 2012 Set (23)]

Let the first term be a, common difference be d, nth term be a_n and sum of $n$the term be S_n
Here, $a = 6, d = 6, n = 5$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_5 = \frac{5}{2}[2 \times 6 + (5-1)(6)]$$

$$= \frac{5}{2}[12 + 4 \times 6]$$

$$= \frac{5}{2}[12 + 24] = \frac{5}{2}[36]$$

$$= 5 \times 18 = 90$$

5. If the sum of n terms of an A.P. is $2n^2 + 5n$, then find the 4^{th} term.

Ans : [Board Term-2, 2012, Set (12)]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now, $S_n = 2n^2 + 5n$

n^{th} term of A.P.

$$a_n = S_n - S_{n-1}$$

$$a_n = (2n^2 + 5n) - [2(n-1)^2 + 5(n-1)]$$

$$= 2n^2 + 5n - [2n^2 - 4n + 2 + 5n - 5]$$

$$= 2n^2 + 5n - 2n^2 - n + 3$$

$$= 4n + 3$$

Thus 4^{th} term $a_4 = 4 \times 4 + 3 = 19$

6. If the sum of first k terms of an A.P. is $3k^2 - k$ and its common difference is 6. What is the first term?

Ans : [Board Term-2, 2012, Set (44)]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n
Let the sum of k terms of A.P. is $S_n = 3k^2 - k$

We have $S_k = 3k^2 - k$
Now k^{th} term of A.P.

$$a_k = S_n - S_{n-1}$$

$$a_k = (3k^2 - k) - [3(k-1)^2 - (k-1)]$$

$$= 3k^2 - k - [3k^2 - 6k + 3 - k + 1]$$

$$= 3k^2 - k - 3k^2 + 7k - 4$$

$$= 6k - 4$$

First term $a = 6 \times 1 - 4 = 2$

7. Which term of the A.P. $8, 14, 20, 26, ...$ will be 72 more than its 41^{st} term.

Ans : [Board Outside Delhi Set-II, 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 8, d = 6$.

Since n^{th} term is 72 more than 41^{st} term. we get

$$\begin{aligned} a_n &= a_{41} + 72 \\ 8 + (n-1)6 &= 8 + 40 \times 6 + 72 \\ 6n - 6 &= 240 + 72 \\ 6n &= 312 + 6 = 318 \\ n &= 53 \end{aligned}$$

8. If the n^{th} term of an A.P. $-1, 4, 9, 14, ...$ is 129. Find the value of n.

Ans : [Board Outside Delhi Compt. Set I, II, III 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = -1$ and $d = 4 - (-1) = 5$

$$\begin{aligned} -1 + (n-1) \times 5 &= a_n \\ -1 + 5n - 5 &= 129 \\ 5n &= 135 \\ n &= 27 \end{aligned}$$

Hence 27^{th} term is 129.

9. Write the n^{th} term of the A.P. $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, ...$

Ans : [Board Outside Delhi Compt. Set-I, II, III 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $$a = \frac{1}{m}$$

$$d = \frac{1+m}{m} - \frac{1}{m} = 1$$

$$a_n = \frac{1}{m} + (n-1)1$$

Hence , $$a_n = \frac{1}{m} + n - 1$$

10. What is the common difference of an A.P. which $a_{21} - a_7 = 84$.

Ans : [Board Outside Delhi Set I, II, III, 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have
$$\begin{aligned} a_{21} - a_7 &= 84 \\ a + 20d - a - 6d &= 84 \\ 14d &= 84 \\ d &= \frac{84}{14} = 6 \end{aligned}$$

Hence common difference is 6.

11. Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4} ....$ is the first negative.

Ans : [Board Outside Delhi Set I, II, III 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 20$ and $d = -\frac{3}{4}$

Let the n^{th} term be first negative term, then

$$\begin{aligned} a + (n-1)d &< 0 \\ 20 + (n-1)\left(-\frac{3}{4}\right) &< 0 \\ 20 - \frac{3}{4}n + \frac{3}{4} &< 0 \\ 3n &> 83 \\ n &> \frac{83}{3} = 27\frac{2}{3} \end{aligned}$$

Hence 28^{th} term is first negative.

SHORT ANSWER TYPE QUESTIONS - I

1. How many terms of the A.P. $65, 60, 55, ...$ be taken so that their sum is zero?

Ans : [Delhi Set III, 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a = 65, d = -5, S_n = 0$

Now $$S_n = \frac{n}{2}[2a + (n-1)d]$$

Let sum of n term be zero, then we have

$$\begin{aligned} \frac{n}{2}[130 + (n-1)(-5)] &= 0 \\ \frac{n}{2}[130 + 5n + 5] &= 0 \\ 135n - 5n^2 &= 0 \\ n(135 - 5n) &= 0 \\ 5n &= 135 \\ n &= 27 \end{aligned}$$

2. How many terms of the A.P. $18, 16, 14 .....$ be taken so that their sum is zero?

Ans : [Delhi Set I, 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 18, d = -2, S_n = 0$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Let sum of n term be zero, then we have

$$\begin{aligned} \frac{n}{2}[36 + (n-1)(-2)] &= 0 \\ n(38 - 2n) &= 0 \\ n &= 19 \end{aligned}$$

3. How many terms of the A.P. $27, 24, 21 ....$ should be taken so that their sum is zero?

Ans : [Delhi Set II, 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $$a = 27, d = -3, S_n = 0$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Let sum of n term be zero, then we have

$$\frac{n}{2}[54 + (n-1)(-3)] = 0$$

$$\begin{aligned} n(-3n+57) &= 0 \\ n &= 19 \end{aligned}$$

4. In an A.P., if $S_3+S_7=167$ and $S_{10}=235$, then find the A.P., where S_n donotes the sum of first n terms.

Ans : [Outside Delhi CBSE Board, Term-2, 2015, Set I, II, III]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\begin{aligned} S_5+S_7 &= 167 \\ \frac{5}{2}(2a+4d)+\frac{7}{2}(2a+6d) &= 167 \\ 24a+62d &= 334 \\ 12a+31d &= 167 \qquad ...(1) \\ S_{10} &= 235 \\ 5(2a+9d) &= 235 \\ 2a+9d &= 47 \qquad (2) \end{aligned}$$

Solving (1) and (2), we get

$$a = 1, d=5$$

Thus AP is 1, 6, 11....

5. Find the sum of sixteen terms of an A.P. $-1,-5,-9,.......$

Ans : [Board Term-2, 2012 Set (8)]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here, $a_1=-1, a_2=-5$ and $d=-4$

Now $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\begin{aligned} S_{16} &= \frac{16}{6}[2\times(-1)+(16-1)(-4)] \\ &= 8[-2-60]=8(-62) \\ &=-496 \end{aligned}$$

6. If the n^{th} term of an A.P. is $7-3n$, find the sum of twenty five terms.

Ans : [Board Term-2, 2012 Set (16)]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $n=25, a_n=7-3n$

Taking $n=1,2,3,....$ we have

$$\begin{aligned} a_1 &= 7-3\times 1 = 4 \\ a_2 &= 7-3\times 2 = 1 \\ a_3 &= 7-3\times 3 = -2 \end{aligned}$$

Thus required AP is $4,1,-2,.....$

Here, $a=4, d=1-4=-3$

Now, $$\begin{aligned} S_n &= \frac{n}{2}[2a+(n-1)d] \\ &= \frac{25}{2}[2\times 4+(25-1)(-3)] \\ &= \frac{25}{2}[8+24(-3)] \\ &= \frac{25}{2}(8-72) = -800 \end{aligned}$$

7. If the 1^{st} term of a series is 7 and 13^{th} term is 35. Find the sum of 13 terms of the swquence.

Ans : [Board Term-2, 2012, Set (36)]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a=7, a_{13}=35$

$$\begin{aligned} a_n &= a+(n-1)d \\ a_{13} &= a+12d \\ 35 &= 7+12d \Rightarrow d = \frac{7}{3} \end{aligned}$$

Now $$\begin{aligned} S_n &= \frac{n}{2}[2a+(n-1)d] \\ S_{13} &= \frac{13}{2}\left[2\times 7+12\times\left(\frac{7}{3}\right)\right] \\ &= \frac{13}{2}[14+28] \\ &= \frac{13}{2}\times 42 = 273 \end{aligned}$$

8. If the n^{th} term of a squence is $3-2n$. Find the sum of fifteen terms.

Ans : [Board Term-2, 2012 Set (38)]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here, $a_n=3-2n$

Taking $n=1$, $a_1 = 3-2=1$

15th term, $a_{15} = 3-2\times 15 = 3-30=-27$

Now $$\begin{aligned} S_n &= \frac{n}{2}(a+1) \\ S_{15} &= \frac{15}{2}[1+(-27)] \\ &= \frac{15}{2}[-26] \\ &= 15\times(-13)=-195 \end{aligned}$$

9. If S_n denotes the sum of n terms of an A.P. whose common difference is d and first term is a, find $S_n-2S_{n-1}+S_{n-2}$.

Ans : [Board Term-2, 2011 (A1)]

We have $$\begin{aligned} a_n &= S_n - S_{n-1} \\ a_{n-1} &= S_{n-1}-S_{n-2} \\ S_n-2S_{n-1}+S_{n-2} &= S_n-S_{n-1}-S_{n-1}+S_{n-2} \\ &= (S_n-S_{n-1})-(S_{n-1}-S_{n-2}) \\ &= a_n-a_{n-1}=d \end{aligned}$$

10. The sum of first n terms of an A.P. is $5n-n^2$. Find the n^{th} term of the A.P.

Ans : [Foreign Set I, II, III, 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have, $S_n = 5n-n^2$

Now, n^{th}term of A.P.

$$\begin{aligned} a_n &= S_n - S_{n-1} \\ &= (5n-n^2)-\left[5(n-1)-(n-1)^2\right] \end{aligned}$$

$$\begin{aligned} &= 5n - n^2 - [5n - 5 - (n^2 + 1 - 2n)] \\ &= 5n - n^2 - (5n - 5 - n^2 - 1 + 2n) \\ &= 5n - n^2 - n + 6 + n^2 \\ &= -2n + 6 \\ a_n &= -2(n - 3) \end{aligned}$$

Thus n^{th} term is $= -2(n - 3)$

11. The first and last term of an A.P. are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

Ans : [Board Term-2, 2012 Set (19)]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
We have $a = 5, a_n = 45$

$$\begin{aligned} \text{Now} \qquad 45 &= 5 + (n - 1)d \\ (n - 1)d &= 40 \qquad ...(1) \\ \text{Given,} \qquad S_n &= 400 \\ \text{Now} \qquad S_n &= \frac{n}{2}(a + l) \\ 400 &= \frac{n}{2}(5 + 45) \\ 800 &= 50n \\ n &= 16 \end{aligned}$$

Substituting this value of n in (1) we have

$$\begin{aligned} (n - 1)d &= 40 \\ 15d &= 40 \\ d &= \frac{40}{15} = \frac{8}{3} \end{aligned}$$

12. If the sum of the first 7 terms of an A.P. is 49 and that of the first 17 terms is 289, find the sum of its first n terms.

Ans : [Board Foreign Set-II, 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n - 1)d] \\ \text{Now} \qquad S_7 &= \frac{7}{2}(2a + 6d) = 49 \\ a + 3d &= 7 \qquad ...(1) \\ \text{and} \qquad S_{17} &= \frac{17}{2}(2a + 16d) = 289 \\ a + 8d &= 17 \end{aligned}$$

Subtracting (1) from (2), we get

$$5d = 10 \Rightarrow d = 2$$

Substituting this value of d in (1) we have

$$\begin{aligned} a &= 1 \\ \text{Now} \qquad S_n &= \frac{n}{2}[2 \times 1(n - 1)2] \\ &= \frac{n}{2}[2 + 2n - 2] = n^2 \end{aligned}$$

Hence, sum of n terms is n^2.

13. How many terms of the A.P. $-6, \frac{-11}{2}, -5, -\frac{9}{2}$.... are needed to give their sum zero.

Ans : [Board outside Delhi compt. Set-III, 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = -6, d = -\frac{11}{2} - (-6) = \frac{1}{2}$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Let sum of n term be zero, then we have

$$\begin{aligned} \frac{n}{2}\left[2 \times -6 + (n - 1)\frac{1}{2}\right] &= 0 \\ \frac{n}{2}\left[-12 + \frac{n}{2} - \frac{1}{2}\right] &= 0 \\ \frac{n}{2}\left[\frac{n}{2} - \frac{25}{2}\right] &= 0 \\ n^2 - 25n &= 0 \\ n(n - 25) &= 0 \\ n &= 25 \end{aligned}$$

Hence 25 terms are needed.

14. Which term of the A.P. $3, 12, 21, 30, ...$ will be 90 more than its 50^{th} term.

Ans : [Board Compt. Set-III 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$\begin{aligned} \text{We have} \qquad a &= 3, d = 9 \\ \text{Now} \qquad a_n &= a + (n - 1)d \\ a_{50} &= 3 + 49 \times 9 = 444 \\ \text{Now,} \qquad a_n - a_{50} &= 90 \\ 3 + (n - 1)9 - 444 &= 90 \\ (n - 1)9 &= 90 + 441 \\ (n - 1) &= \frac{531}{9} = 49 \\ n &= 49 + 1 = 60 \end{aligned}$$

15. The 10^{th} term of an A.P. is $-4$and its 22^{nd} term is (-16). Find its 38^{th} term.

Ans : [Board Delhi compt. Set-I, 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$\begin{aligned} a_{10} &= a + 9d = -4 \qquad (1) \\ \text{and} \qquad a_{22} &= a + 21d = -16 \qquad (2) \end{aligned}$$

Subtracting (2) from (1) we have

$$12d = -12 \Rightarrow d = -16$$

Substituting this value of d in (1) we get

$$\begin{aligned} a &= 5 \\ \text{Thus} \qquad a_{38} &= 5 + 37 \times -1 = -32 \\ \text{Hence,} \qquad a_{38} &= -32 \end{aligned}$$

16. Find how many integers between 200 and 500 are divisible by 8.

Ans : [Board Delhi compt. Set-I, II, III, 2017]

Number divisible by 8 are 208, 2016, 224, ... 496.
Which is an A.P.
Let the first term be a, common difference be d and

nth term be a_n.
We have $aa = 208, d = 8$ and $a_n = 496$

$$\begin{aligned} \text{Now} \qquad a+(n-1)d &= a_n \\ 208+(n-1)d &= 496 \\ (n-1)8 &= 496-208 \\ n-1 &= \frac{288}{8} = 36 \\ n &= 36+1 = 37 \end{aligned}$$

Hence, required numbers divisible by 8 is 37.

17. The fifth term of an A.P. is 26 and its 10^{th} term is 51. Find the A.P.

Ans : [Outside Delhi Compt. set-II, 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$\begin{aligned} a_5 &= a+4d = 26 & \text{...(1)} \\ a_{10} &= a+9d = 51 & \text{...(2)} \end{aligned}$$

Subtracting (1) from (2) we have

$$\begin{aligned} 5d &= 25 \\ d &= 5 \end{aligned}$$

Substituting this value of d in (1) we get

$$a = 6$$

Hence, the AP is 6, 11, 17, ...

18. Find the A.P. whose third term is 5 and seventh term is 9.

Ans : [Board Outside Delhi Compt. Set-I, 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$\begin{aligned} \text{Now} \qquad a_3 &= a+2d = 5 & \text{...(1)} \\ \text{and} \qquad a_7 &= a+6d = 9 & \text{...(2)} \end{aligned}$$

Subtracting (2) from (1) we have

$$4d = 4 \Rightarrow d = 1$$

Substituting this value of d in (1) we get

$$a = 3$$

Hence AP is 3, 4, 5, 6, ...

19. Find whether -150 is a term of the A.P. $11, 8, 5, 2, ...$

Ans : [Board Delhi Compt. Set-I, 2017]

Let the first term be a, common difference be d and nth term be a_n.
Let the n^{th} term of given A.P. $11, 8, 5, 2, ...$ be -150
Hence $a = 11,\ d = 8-11 = -3$ and $a_n = -150$

$$\begin{aligned} a+(n-1)d &= a_n \\ 11+(n-1)(-3) &= -150 \\ (n-1)(-3) &= -161 \\ (n-1) &= \frac{-161}{-3} = 53\frac{2}{3} \end{aligned}$$

which is not a whole number. Hence -150 is not a term of given A.P.

20. If seven times the 7^{th} term of an A.P. is equal to eleven times the 11^{th} term, then what will be its 18^{th} term.

Ans : [Board Foreign Set-I, II, III, 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$\begin{aligned} 7a_7 &= 11a_{11} \\ \text{Now} \qquad 7(a+6d) &= 11(a+10d) \\ 7a+42d &= 11a+110d \\ 11a-7a &= 42d-110d \\ ,\qquad 4a &= -68d \\ 4a+68d &= 0 \\ 4(a+17d) &= 0 \\ a+17d &= 0 \\ \text{Hence,} \qquad a_{18} &= 0 \end{aligned}$$

21. In an A.P. of 50 terms, the sum of the first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P.

Ans : [Board Foreign SET-III, 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$\begin{aligned} S_{10} &= 210 \\ S_n &= \frac{n}{2}[2a+(n-1)d] \\ \frac{10}{2}(2a+9d) &= 42 \\ 10a+45d &= 42 & (1) \\ S_{50} &= \frac{50}{2}[2a+(50-1)d] \\ S_{35} &= \frac{35}{2}[2a+(35-1)d] \\ a_{36} &= a+35d \\ a_{50} &= a+49d \end{aligned}$$

Sum of last 15 terms

$$\begin{aligned} &= \frac{n}{2}(a_{36}+a_{50}) \\ 2565 &= \frac{15}{2}(a+35d+a+49d) \\ 171 &= \frac{1}{2}(2a+84d) \\ a+42d &= 171 & (2) \end{aligned}$$

Solving (1) and (2) we get

$$a = 3 \text{ and } d = 4$$

Hence, AP is 3, 7, 11, ...

SHORT ANSWER TYPE QUESTIONS - II

1. In an A.P. the sum of first n terms is $\frac{3n^2}{2}+\frac{13n}{2}$. Find the 25^{th} term.

Ans : [Board Sample Paper, 2016]

$$\begin{aligned} \text{We have} \quad S_n &= \frac{3n^2+13n}{2} \\ a_n &= S_n - S_{n-1} \end{aligned}$$

$$\begin{aligned} a_{25} &= S_{25} - S_{24} \\ &= \frac{3(25)^2 + 13(25)}{2} - \frac{3(24)^2 + 13(24)}{2} \\ &= \frac{1}{2}\{3(25^2 - 24^2) + 13(25 - 24)\} \\ &= \frac{1}{3}(3 \times 49 + 13) = 80 \end{aligned}$$

2. The sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each A.P. is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

Ans : [O.D. Set III, 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $S_1 = 1 + 2 + 3 + ... n$

$S_2 = 1 + 3 + 5 + ...$ up to n terms

$S_3 = 1 + 4 + 7 + ...$ upto n terms

Now $S_1 = \dfrac{n(n+1)}{2}$

$$S_2 = \frac{n}{2}[2 \times 1 + (n-1)2] = \frac{n}{2}[2n] = n^2$$

and $S_3 = \dfrac{n}{2}[2 \times 1 + (n-1)3] = \dfrac{n(3n-1)}{2}$

$$\begin{aligned} \text{Now, } S_1 + S_3 &= \frac{n(n+1)}{2} + \frac{n(3n-1)}{2} \\ &= \frac{n[n + 1 + 3n - 1]}{2} \\ &= \frac{n[4n]}{2} \\ &= 2n^2 = 2s_2 \end{aligned}$$

Hence Proved

3. If S_n denotes, the sum of the first n terms of an A.P. prove that $S_{12} = 3(S_8 - S_4)$.

Ans : [Delhi CBSE Board, 2015, Set I]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\begin{aligned} S_{12} &= 6[2a + 11d] = 12a + 66d \\ S_8 &= 4[2a + 7d] = 8a + 28d \\ S_4 &= 2[2a + 3d] = 4a + 6d \\ 3(S_8 - S_4) &= 3[(8a + 28d) - (4a + 6d)] \\ &= 3[4a + 22d] = 12a + 66d \\ &= 6[2a + 11d] = S_{12} \end{aligned}$$

Hence Proved

4. The 14^{th} term of an A.P. is twice its 8^{th} term. If the 6^{th} term is -8, then find the sum of its first 20 terms.

Ans : [Outside Delhi CBSE Board, 2015, Set I]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here, $a_{14} = 2a_8$ and $a_6 = -8$

$$\begin{aligned} \text{Now} \quad a + 13d &= 2(a + 7d) \\ a + 13d &= 2a + 14d \\ a &= -d \qquad ...(1) \end{aligned}$$

$$\begin{aligned} \text{and} \quad a_6 &= -8 \\ a + 5d &= -8 \qquad ...(2) \end{aligned}$$

Solving (1) and (2), we get

$$a = 2, d = -2$$

$$\begin{aligned} \text{Now} \quad S_{20} &= \frac{20}{2}[2 \times 2 + (20-1)(-2)] \\ &= 10[4 + 19 \times (-2)] \\ &= 10(4 - 38) \\ &= 10 \times (-34) = -340 \end{aligned}$$

5. If the ratio of the sums of first n terms of two A.P.'s is $(7n + 1):(4n + 27)$, find the ratio of their m^{th} terms.

Ans : [O.D. Set I, 2016]

Let a, and A be the first term and d and D be the common difference of two AP's, then we have

$$\begin{aligned} \frac{S_n}{S'_n} &= \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2A + (n-1)D]} = \frac{7n+1}{4n+27} \\ &= \frac{2a + (n-1)d}{2A + (n-1)D} = \frac{7n+1}{4n+27} \end{aligned}$$

$$\frac{a + (\frac{n-1}{2})d}{A + (\frac{n-1}{2})D} = \frac{7n+1}{4n+27}$$

Putting $\dfrac{n-1}{2} = m - 1$ or $n = 2m - 1$ we get

$$\frac{a + (m-1)d}{A + (m-1)D} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$$

Hence, $\dfrac{a_m}{A_m} = \dfrac{14m-6}{8m+23}$

6. If the sum of the first n terms of an A.P. is $\frac{1}{2}[3n^2 + 7n]$, then find its n^{th} term. Hence write its 20^{th} term.

Ans : [Delhi CBSE Board Term-2, 2015, set II]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Sum of n term $S_n = \dfrac{1}{2}[3n^2 + 7n]$

$$\begin{aligned} \text{Sum of 1 term} \quad S_1 &= \frac{1}{2}[3 \times (1)^2 + 7(1)] \\ &= \frac{1}{2}[3 + 7] = \frac{1}{2} \times 10 = 5 \end{aligned}$$

$$\begin{aligned} \text{Sum of 2 term} \quad S_2 &= \frac{1}{2}[3(2)^2 + 7 \times 2] \\ &= \frac{1}{2}[12 + 14] = \frac{1}{2} \times 26 = 13 \end{aligned}$$

$$\begin{aligned} \text{Now} \quad a_1 &= S_1 = 5 \\ a_2 &= S_2 - S_1 = 13 - 5 = 8 \\ d &= a_2 - a_1 = 8 - 5 = 3 \end{aligned}$$

Now, A.P. is $5, 8, 11, ...$

$$\begin{aligned} n^{th} \text{ term,} \quad a_n &= a + (n-1)d \\ &= 5 + (n-1)3 \\ &= 5 + (20-1)(3) \end{aligned}$$

$$\begin{aligned} &= 5+57 \\ &= 62 \end{aligned}$$

Hence, $a_2 = 62$

7. In an A.P., if the 12^{th} term is -13 and the sum of its first four terms is 24, find the sum of its first ten terms.

Ans : [Foreign Set I, II, 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$\begin{aligned} a_{12} &= a+11d = -13 && ...(1) \\ S_n &= \frac{n}{2}[2a+(n-1)d] \end{aligned}$$

Now $S_4 = 2[2a+3d] = 24$

$$2a+3d = 12 \qquad ...(2)$$

Multiplying (1) by 2 and subtracting (2) from it we get

$$\begin{aligned} (2a+22d)-(2a+3d) &= -26-12 \\ 19d &= -38 \\ d &= -2 \end{aligned}$$

Substituting the value of d in (1) we get

$$\begin{aligned} a+11\times -2 &= -13 \\ a &= -13+22 \\ a &= 9 \end{aligned}$$

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$

$$\begin{aligned} S_{10} &= \frac{10}{2}(2\times 9+9\times -2) \\ &= 5\times(18-18) = 0 \end{aligned}$$

Hence, $S_{10} = 0$

8. The tenth term of an A.P., is -37 and the sum of its first six terms is -27. Find the sum of its first eight terms.

Ans : [Foreign Set III, 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$\begin{aligned} a_n &= a+(n-1)d \\ S_n &= \frac{n}{2}[2a+(n-1)d] \end{aligned}$$

$$\begin{aligned} a+9d &= -37 && ...(1) \\ 3(2a+5d) &= -27 \\ 2a+5d &= -9 && ...(2) \end{aligned}$$

Multiplying (1) by 2 and subtracting (2) from it, we get

$$\begin{aligned} (2a+18d)-(2a+5d) &= -74+9 \\ 13d &= -65 \\ d &= -5 \end{aligned}$$

Substituting the value of d in (1) we get

$$\begin{aligned} a+9\times -5 &= -37 \\ a &= -37+45 \\ a &= 8 \end{aligned}$$

Now $S_n = \frac{n}{2}[2a+(n-1)d]$

$$\begin{aligned} &= \frac{8}{2}[2\times 8+(8-1)(-5)] \\ &= 4[16-35] \\ &= 4\times -19 = -76 \end{aligned}$$

Hence, $S_n = -76$

9. Find the sum of first seventeen terms of A.P. whose 4^{th} and 9^{th} terms are -15 and -30 respectively.

Ans : [Board Term-2, 2014]

Let the first term be a, common difference be d and nth term be a_n.

Now $a_4 = a+3d = -15 \qquad ...(1)$

$$a_9 = a+8d = -30 \qquad ...(2)$$

Subtracting eqn (1) from eqn (2), we obtain

$$\begin{aligned} (a+8d)-(a+3d) &= -30-(-15) \\ 5d &= -15 \Rightarrow d = \frac{-15}{5} = -3 \end{aligned}$$

Substituting the value of d in (1) we get

$$\begin{aligned} a+3d &= -15 \\ a+3(-3) &= -15 \\ a &= -15+9 = -6 \end{aligned}$$

Now $S_{17} = \frac{17}{2}[2\times(-6)+(17-1)(-3)]$

$$\begin{aligned} &= \frac{17}{2}[--12+16\times(-3)] \\ &= \frac{17}{2}[-12-48] \\ &= \frac{17}{2}[-60] = 17\times(-30) \\ &= -510 \end{aligned}$$

Thus $S_{17} = -510$

10. The common difference of an A.P. is -2. Find its sum, if first term is 100 and last term is -10.

Ans : [Board Term-2, 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 100, d = -2, t_n = -10$

Now $a_n = a+(n-1)d$

$$\begin{aligned} -10 &= 100+(n-1)(-2) \\ -10 &= 100-2n+2 \\ 2n &= 112 \\ n &= 56 \end{aligned}$$

Thus 56^{th} term is -10 and number of terms in A.P. are 56.

Now $S_n = \frac{n}{2}(a+1)$

$$\begin{aligned} S_{56} &= \frac{56}{2}(100-10) \\ &= \frac{56}{2}(90) = 56\times 45 = 2520 \end{aligned}$$

Thus $S_n = 2520$

11. The 16^{th} term of an A.P. is finve times its third term. If its 10^{th} term is 41, then find the sum of its first fifteen terms.

Ans : [Outside Delhi CBSE, 2015, Set II]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have, $a_{16} = 5a_3$

$$a + 15d = 5(a + 2d)$$

$$4a = 5d \quad ...(1)$$

and $a_{10} = 41$

$$a + 9d = 41 \quad ...(2)$$

Solving (1) and (2), we get

$$a = 5, d = 4$$

Now $S_{15} = \frac{15}{2}[2 \times 5 + (15 - 1) \times 4]$

$$= \frac{15}{2}[10 + 56]$$

$$= \frac{15}{2} \times 66 = 15 \times 33 = 495$$

Thus $S_{15} = 495$

12. The 13^{th} term of an A.P. is four times its 3^{rd} term. If the fifth term is 16, then find the sum of its first ten terms.

Ans : [Outside Delhi, 2015 Set III]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a_{13} = 4a_3$

$$a + 12d = 4(a + 2d)$$

$$3a = 4d \quad ...(1)$$

and $a_5 = 16$

$$a + 4d = 16 \quad ...(2)$$

Substituting the value of $a = \frac{4}{3}d$ in (2)

$$\frac{4}{3}d + 4d = 16$$

$$16d = 48 \Rightarrow d = 3$$

Thus $a = 4$ and $d = 3$

Now $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{10} = \frac{10}{2}[2 \times 4 + (10 - 1)3]$$

$$= 5[8 + 27] = 5 \times 35 = 175$$

Thus $S_{10} = 175$

13. The n^{th} term of an A.P. is given by $(-4n + 15)$. Find the sum of first 20 terms of this A.P.

Ans : [Board Term-2, 2013]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a_n = -4n + 15$

$$a_1 = -4 \times 1 + 15 = 11$$

$$a_2 = -4 \times 2 + 15 = 7$$

$$a_3 = -4 \times 3 + 15 = 3$$

$$d = a_2 - a_1 = 7 - 11 = -4$$

Now, we have $a = 11$, $d = -4$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{20} = \frac{20}{2}[2 \times 11 + (20 - 1) \times (-4)]$$

$$= 10[22 - 76]$$

$$= 10 \times (-54) = -540$$

Thus $S_{20} = -540$

14. The sum of first 7 terms of an A.P. is 63 and sum of its next 7 terms is 161. Find 28^{th} term of A.P.

Ans : [Foreign Set I, II, III, 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Now, $S_7 = 63$

$$\frac{7}{2}[2a + 6d] = 63$$

$$2a + 6d = 18 \quad ...(1)$$

Also, sum of next 7 terms,

$$S_{14} = S_{first7} + S_{next7} = 63 + 161$$

$$\frac{14}{2}[2a + 13d] = 224$$

$$2a + 13d = 32 \quad (2)$$

Subtracting (1) form (2)

$$7d = 14 \Rightarrow d = 2$$

Substituting the value of d in (1) we get

$$a = 3$$

Now $a_n = a + (n - 1)d$

$$a_{28} = 3 + 2 \times (27)$$

$$= 57$$

Thus 28^{th} term is 57.

15. The sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$. Determine the A.P. and the 12^{th} term.

Ans : [Delhi CBSE Term-2, 2014] [Board Term-2, 2012 set (13)]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = 3n^2 - 4n$$

$$S_1 = 3(1)^2 - 4(1) = -1$$

$$S_2 = 3(2)^2 - 4(2) = 4$$

$$a_1 = S_1 = -1$$

$$a_2 = S_2 - S_1 = 4 - (-1) = 5$$

$$d = a_2 - a_1 = 5 - (-1) = 6$$

Thus AP is $-1, 5, 11, ...$

Now $a_{12} = a + 11d$

$$= -1 + 11 \times 6 = 65$$

16. Find the sum of all two digit natural numbers which are divisible by 4.

Ans : [Delhi Compt. Set-III, 2017]

First two digit multiple of 4 is 12 and last is 96
So, $a = 12, d = 4$. Let n^{th} term be last term $a_n = 96$

$$\begin{aligned} \text{Now} \quad a + (n-1)d &= a_n \\ 12 + (n-1)4 &= 96 \\ (n-1)4 &= 96 - 12 = 84 \\ n - 1 &= 21 \\ n &= 21 + 1 = 22 \\ \text{Now,} \quad S_{22} &= \frac{22}{2}[12 + 96] \\ &= 11 \times 108 \\ &= 1188 \end{aligned}$$

17. Find the sum of the following series.
$5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + ... + (-5) + 81 + (-3)$

Ans : [Board foreign set-I, 2017]

The series can be written as
$(5 + 9 + 13 + ... + 81) + (-41) + (-39) + (-37) + (-35) ...(-5) + (-3)$
For the series $(5 + 9 + 13 .....81)$

$$\begin{aligned} a &= 5 \\ d &= 4 \\ \text{and} \quad a_n &= 81 \\ \text{Now} \quad a_n &= 5 + (n-1)4 = 81 \\ 81 &= 5 + (n-1)4 \\ (n-1)4 &= 76 \\ n &= 20 \\ S_n &= \frac{20}{2}(5 + 81) = 860 \end{aligned}$$

For series $(-41) + (-39) + (-37) + ... + (-5) + (-3)$

$$\begin{aligned} a_n &= -3 \\ a &= -41 \\ d &= 2 \\ a_n &= -41 + (n-1)(2) \\ -3 &= -41 + 2n - 2 \Rightarrow n = 20 \\ \text{Now} \quad S_n &= \frac{20}{2}[-41 + -3] = -440 \end{aligned}$$

$$\text{Sum of the series} = 860 - 440 = 420$$

18. Find the sum of n terms of the series
$\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + ...$

Ans : [CBSE Board Delhi Set-I, II, III, 2017]

Let sum of n term be S_n

$$s_n = \left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + ... \text{ up to } n\text{term}$$

$$\begin{aligned} &= (4 + 4 + 4 + ... \text{ up to } n \text{ terms}) + \\ &\quad + \left(-\frac{1}{n} - \frac{2}{n} - \frac{3}{n} - ...... \text{ up to } n \text{ terms}\right) \\ &= (4 + 4 + 4 + ... \text{ up to } n \text{ terms}) + \\ &\quad -\frac{1}{n}(1 + 2 + 3 + ... \text{ up to } n \text{ terms}) \\ &= 4n - \frac{1}{n} \times \frac{n(n+1)}{2} \\ &= 4n - \frac{n+1}{2} = \frac{7n-1}{2} \end{aligned}$$

Hence, sum of n terms $= \frac{7n-1}{2}$

19. Find the number of multiple of 9 lying between 300 and 700.

Ans : [Outside Delhi Compt. Set-II, 2017]

The numbers, multiple of 9 between 300 and 700 are 306, 315, 324, ... 693.
Let the first term be a, common difference be d and nth term be $a_n = 693$

$$\begin{aligned} a_n &= 306 + (n-1)9 \\ 693 &= 306 + (n-1)9 \\ (n-1)9 &= 693 - 306 = 387 \\ n - 1 & \frac{387}{9} = 43 \\ n &= 43 + 4 = 44 \end{aligned}$$

Hence there are 44 terms.

20. If the sum of the first 14 terms of an A.P. is 1050 and its first term is 10 find it 20^{th} term.

Ans : [Board Outside Delhi Compt. Set-III, 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
We have $a = 10$, and $S_{14} = 1050$

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ S_{14} &= \frac{14}{2}[2 \times 10 + (14-1)d] \\ 1050 &= 7[20 + 13d] \\ 20 + 13d &= \frac{1050}{7} = 150 \\ 13d &= 130 \Rightarrow d = 10 \\ a_{20} &= a + (n-1)d \\ &= 10 + 19 \times 10 = 200 \end{aligned}$$

Hence $a_{20} = 200$

21. If the tenth term of an A.P. is 52 and the 17^{th} term is 20 more than the 13^{th} term, find A.P.

Ans : [Board Outside Delhi Set-I, 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$\begin{aligned} \text{Now} \quad a_{10} &= 52 \\ a + 9d &= 52 \qquad ...(1) \\ \text{Also} \quad a_{17} - a_{13} &= 20 \\ a + 16d - (a + 12d) &= 20 \\ 4d &= 20 \\ d &= 5 \end{aligned}$$

Substituting this valued d in (1), we get

$$a = 7$$

Hence AP is 7, 12, 17, 22, ...

22. Find the sum of all odd number between 0 and 50.

Ans : [Delhi Compt. Set-III, 2017]

Let the first term be a, common difference be d, nth

term be a_n and sum of n term be S_n.
Given AP is $1+3+5+7+.....+49$
Let total number of terms be n.

$$\begin{aligned} a_n &= 1+(n-1)\times 2 \\ 49 &= 1+2n-2 \\ 50 &= 2n \Rightarrow n = 25 \end{aligned}$$

Now
$$\begin{aligned} S_{25} &= \frac{n}{2}(a+a_n) \\ &= \frac{25}{2}(1+49) \\ &= 25\times 25 = 625 \end{aligned}$$

Hence, Sum of odd number is 625

23. Find the sum of first 15 multiples of 8.

Ans : [Delhi Compt. Set-I, 2017]

Let the first term be $a = 8$, common difference be $d = 8$, nth term be a_n and sum of n term be S_n.
First term of given A.P. Be 8 and common difference be 8. Than

$$\begin{aligned} S_n &= \frac{n}{2}[2a+(n-1)d] \\ S_{15} &= \frac{15}{2}[2\times 8+(15-1)8] \\ &= \frac{15}{2}[16+112] \\ &= \frac{15}{2}\times 128 = 996 \end{aligned}$$

Hence, the sum of 15 terms is 960.

24. If m^{th} term of an AP is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$ find the sum of first mn terms.

Ans : [CBSE Board Set-I, 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Now
$$a_m = a+(m-1)d = \frac{1}{n} \qquad ...(1)$$

$$a_n = a+(n-1)d = \frac{1}{m} \qquad ...(2)$$

Subtracting (2) from (1) we get

$$\begin{aligned} (m-n)d &= \frac{1}{n}-\frac{1}{m} = \frac{m-n}{mn} \\ d &= \frac{1}{mn} \end{aligned}$$

Substituting this valued d in (1), we get

$$a = \frac{1}{mn}$$

Now,
$$\begin{aligned} S_{mn} &= \frac{mn}{2}\left(\frac{2}{mn}+(mn-1)\frac{1}{mn}\right) \\ &= 1+\frac{mn}{2}-\frac{1}{2} = \frac{1}{2}+\frac{mn}{2} \\ &= \frac{1}{2}[mn+1] \end{aligned}$$

Hence, the sum on mn term is $\frac{1}{2}[mn+1]$.

25. How many terms of an A.P. $9, 17, 25, ...$ must be taken to give a sum of 636?

Ans :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
We have $a = 9, d = 8, S_n = 636$

Now
$$\begin{aligned} S_n &= \frac{n}{2}[2a+(n-1)d] \\ 636 &= \frac{n}{2}[18+(n-1)8] \\ 636 &= n[9+(n-1)4] \\ 636 &= n(9+4n-4) \\ 636 &= n(5+4n) \\ 636 &= 5n+4n^2 \\ 4n^2+5n-636 &= 0 \\ 4n^2-48n+53n-636 &= 0 \\ 4n(n-12)+53(n-12) &= 0 \\ (4n+53)(n-12) &= 0 \end{aligned}$$

Thus
$$n = \frac{-53}{4} \text{ or } 12$$

As n is a natural number $n = 12$. Hence 12 terms are required to give sum 636.

LONG ANSWER TYPE QUESTIONS

1. The minimum age of children to be eligible to participate in a painting competition is 8 years. It is observed that the age of youngest boy was 8 years and the ages of rest of participants are having a common difference of 4 months. If the sum of ages of all the participants is 168 years, find the age of eldest participant in the paining competition.

Ans : [Board Sample Paper, 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
We have $a = 8, d = 4$ month $= \frac{1}{3}$ years, $S_n = 168$

$$\begin{aligned} S_n &= \frac{n}{2}[2a+(n-1)d] \\ 168 &= \frac{n}{2}\left[2(8)+(n-1)\frac{1}{3}\right] \\ n^2+47n-1008 &= 0 \\ n^2+63n-16n-1008 &= 0 \\ (n-16)(n+63) &= 0 \\ n &= 16 \text{ or } n = -63 \end{aligned}$$

As n cannot be negative, we take $n = 16$

Age of the eldest participant $= a+15d = 13$ years

2. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after, the thief to catch him. He goes with a speed of 100/minute in the first minute and increased his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief.

Ans : [Delhi Set I, II, 2016]

Let total time to catch the thief be n minutes
Total distance covered by thief $= (100n)$

Total distance covered by policeman

$$\begin{aligned} &= 100 + 110 + 120 + ... + (n-1) \text{ terms} \\ 100n &= \frac{n-1}{2}[200 + (n-2)10] \\ n^2 - 3n - 18 &= 0 \\ (n-6)(n+3) &= 0 \\ n &= 6 \end{aligned}$$

Policeman takes 5 minutes to catch the thief.

3. If S_n denotes the sum of first n terms of an A.P., Prove that, $S_{30} = 3(S_{20} - S_{10})$

Ans : [Delhi 2015 Set III, Foreign Set I, II, III, 2014]

Let the first term be a, and common difference be d.

Now $$S_{30} = \frac{30}{2}(2a + 29d) \qquad ...(1)$$

$$\begin{aligned} &= 15(2a + 29d) \\ 3(S_{20} - S_{10}) &= 3\left[10(2a + 19d) - 5(2a + 9d)\right] \\ &= 3\left[20a + 190d - 10a - 45d\right] \\ &= 3\left[10a + 145d\right] \\ &= 15\left[2a + 29d\right] \qquad ...(2) \end{aligned}$$

Hence $$S_{30} = 3(S_{20} - S_{10})$$

4. The sum of first 20 terms of an A.P. is 400 and sum of first 40 terms is 1600. Find the sum of its first 10 terms.

Ans : [Board Term-2, 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We know $$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Now $$\begin{aligned} S_{20} &= \frac{20}{2}(2a + 19d) \\ 400 &= \frac{20}{2}(2a + 19d) \\ 400 &= 10[2a + 19d] \\ 2a + 19d &= 40 \qquad (1) \end{aligned}$$

Also, $$S_{40} = \frac{40}{2}(2a + 39d)$$

or, $$1600 = 20[2a + 39d]$$

or, $$2a + 39d = 80 \qquad (2)$$

Solving (1) and (2), we get $a = 1$ and $d = 2$.

Now $$\begin{aligned} S_{10} &= \frac{10}{2}[2 \times 1 + (10-1)(2)] \\ &= 5[2 + 9 \times 2] \\ &= 5[2 + 18] \\ &= 5 \times 20 = 100 \end{aligned}$$

5. Find $\left(4 - \frac{1}{n}\right) + \left(7 - \frac{2}{n}\right) + \left(10 - \frac{3}{n}\right) + ...$ upto n terms.

Ans : [Board Term-2, 2015]

Let sum of n term be S_n, then we have

$$\begin{aligned} s_n &= \left(4 - \frac{1}{n}\right) + \left(7 - \frac{2}{n}\right) + \left(40 - \frac{3}{n}\right) + ... \text{ upto } n \text{ terms.} \\ &= (4 + 7 + 10 + ... + n\, terms) - \left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n} ..... + 1\right) \\ &= (4 + 7 + 10 + ... + n\, terms) - \frac{1}{n}(1 + 2 + 3 + ...n) \\ &= \frac{n}{2}[2 \times 4 + (n-1)(3)] - \frac{1}{n} \times \frac{n}{2}[2 \times 1 + (n-1)(1)] \\ &= \frac{n}{2}[8 + 3n - 3] - \frac{1}{2}[2 + n - 1] \\ &= \frac{n}{2}(3n + 5) - \frac{1}{2}(n + 1) \\ &= \frac{3n^2 + 5n - n - 1}{2} \\ &= \frac{3n^2 + 4n - 1}{2} \end{aligned}$$

6. Find the 60^{th} term of the A.P. $8, 10, 12, ...,$ if it has a total of 60 terms and hence find the sum of its last 10 terms.

Ans : [Outside Delhi, CBSE Board, 2015 Set I, II]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a = 8, d = 10 - 8 = 2$

$$a_n = a + (n-1)d$$

Now $$a_{60} = 8 + (60-1)2 = 8 + 59 \times 2 = 126$$

and $$a_{51} = 8 + 50 \times 2 = 8 + 100 = 108$$

Sum of last 10 terms,

$$\begin{aligned} S_{51-60} &= \frac{n}{2}(a_{51} + a_{60}) \\ &= \frac{10}{2}(108 + 126) \\ &= 5 \times 234 = 1170 \end{aligned}$$

Hence sum of last 10 terms is 1170.

7. An arithmetic progression $5, 12, 19, ...$ has 50 terms. Find its last term. Hence find the sum of its last 15 terms.

Ans : [Outside, Delhi CBSE Board, 2015, Set III]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a = 5$, $d = 12 - 5 = 7$ and $n = 50$

$$\begin{aligned} a_{50} &= 5 + (50-1)7 \\ &= 5 + 49 \times 7 = 348 \end{aligned}$$

Also the first term of the A.P. of last 15 terms be a_{36}

$$\begin{aligned} a_{36} &= 5 + 35 \times 7 \\ &= 5 + 245 = 250 \end{aligned}$$

Now, sum of last 15 terms

$$\begin{aligned} S_{36-50} &= \frac{15}{2}[S_{36} + S_{50}] \\ &= \frac{15}{2}[250 + 348] \\ &= \frac{15}{2} \times 598 = 4485 \end{aligned}$$

Hence, sum of last 15 terms is 4485.

8. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 3, when divided by 4. Also find the sum of all numbers on both sides of the middle terms separately.

Ans : [Foreignset I, 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

The three digit numbers which leaves 3 as remainder when divided by 4 are: $103, 107, 111, ... 999$

Now, the first number $a = 103$, last number $a_n = 999$ and common differenced $d = 4$

Let the number of terms in this sequence be n.

$$\begin{aligned} a_n &= a + (n-1)d \\ 999 &= 103 + (n-1)4 \\ 896 &= (n-1)4 \\ (n-1) &= \frac{896}{4} = 224 \\ n &= 224 + 1 = 225 \\ \text{Middle term} &= \frac{225+1}{2} \\ &= 113^{th} \text{ term} \\ a_{113} &= 103 + 112 \times 4 = 551 \end{aligned}$$

and $a_{112} = 551 - 4 = 547$

Sum of Ist 112 terms

$$\begin{aligned} S_{112} &= \frac{112}{2}(a + a_{112}) \\ &= 56(103 + 547) \\ &= 56 \times 650 = 36400 \end{aligned}$$

and $a_{114} = 551 + 4 = 555$

The sum of last 112 terms

$$\begin{aligned} &= \frac{112}{2}(s_{114} + a_{225}) \\ &= 56(555 + 999) \\ &= 56 \times 1554 = 87024 \end{aligned}$$

9. Find the middle term of the sequence formed all numbers between 9 and 95, which leave a remainder 1 when divided by 3. Also find the sum of the numbers on both sides of the middle term separately.

Ans : [Foreign Set II, 2015]

The sequence is 10, 13, ... 94

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

$$\begin{aligned} 94 &= 10 + (n-1)3 \\ 84 &= (n-1)3 \\ n &= \frac{84}{3} + 1 = 29 \end{aligned}$$

Therefore $\frac{29+1}{2} = 15^{th}$ term is the middle term.

Middle term $a_{15} = a + (15-1)d$

$$\begin{aligned} &= 10 + 14 \times 3 = 52 \\ a_{16} &= 52 + 3 = 55 \end{aligned}$$

Sum of first 14 terms,

$$\begin{aligned} s_{14} & \\ &= \frac{14}{2}[2 \times 10 + (14-1) \times 3] \\ &= 7[20 + 13 \times 3] = 413 \\ S_n &= \frac{n}{2}[2a + (n-1)d] \end{aligned}$$

Sum of the last 14 terms,

$$\begin{aligned} &= \frac{14}{2}[2s_{16} + (14-1)d] \\ &= \frac{14}{2}[2 \times 55 + (14-1) \times 3] \\ &= 7[110 + 13 \times 3] \\ &= 1043 \end{aligned}$$

10. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 5 when divided by 7. Also find the sum of all numbers on both sides of the middle term separately.

Ans : [Foreign Set III, 2014, 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

The sequence is $103, 110, ..., 999$

Here $a = 103$, $d = 7$, $a_n = 999$

$$\begin{aligned} a_n &= a + (n-1)d \\ 999 &= 103 + (n-1) \times 7 \\ n &= \frac{999 - 103}{7} + 1 = 129 \end{aligned}$$

Therefore $\frac{129+1}{2} = 65^{th}$ term is the middle term.

Middle term $a_{65} = 103 + (64 \times 7) = 551$

$$a_{66} = 551 + 7 = 558$$

Sum of first 64 terms,

$$\begin{aligned} S_{64} &= \frac{64}{2}[2a + (64-1)d] \\ &= 32[2 \times 103 + 63 \times 7] \\ &= 32[206 + 441] = 20704 \end{aligned}$$

Sum of last 64 terms

$$\begin{aligned} S_{66-129} &= \frac{64}{2}(558 + 999) \\ &= 32 \times 1557 \\ &= 49824 \end{aligned}$$

11. If the sum of first n term of an an A.P. is given by $S_n = 3n^2 + 4n$. Determine the A.P. and the n^{th} term.

Ans : [Board Term-2, 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $S_n = 3n^2 + 4n.$

$$\begin{aligned} a_1 &= S_1 3(1)^2 + 4(1) = 7 \\ a_1 + a_2 &= S_2 = 3(2)^2 + 4(2) \\ &= 12 + 8 = 20 \\ a_2 &= S_2 - S_1 = 20 - 7 = 13 \\ a + d &= 13 \end{aligned}$$

or, $7 + d = 13$

Thus $d = 13 - 7 = 6$

Hence AP is $7, 13, 19, ...$

Now,
$$\begin{aligned} a_n &= a+(n-1)d \\ &= 7+(n-1)(6) \\ &= 7+6n-6 \\ &= 6n+1 \\ a_n &= 6n+1 \end{aligned}$$

12. The sum of the 3^{rd} and 7^{th} terms of an A.P. is 6 and their product is 8. Find the sum of first 20 terms of the A.P.

Ans : [Board Term-2, 2012 Set (21)]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have
$$\begin{aligned} a_3+a_7 &= 6 \\ a+2d+a+6d &= 6 \\ a+4d &= 3 \qquad (1) \end{aligned}$$

and
$$\begin{aligned} a_3\times a_7 &= 8 \\ (a+2d)(a+6d) &= 8 \qquad (2) \end{aligned}$$

Substituting the value $a=(3-4d)$ in (2) we get
$$(3-4d+2d)(3-4d+6d) = 8$$
or,
$$(3+2d)(3-2d) = 8$$
or,
$$9-4d^2 = 8$$
$$4d^2 = 1 \Rightarrow d^2=\frac{1}{4} \Rightarrow d = \pm\frac{1}{2}$$

CASE 1 : Substituting $d=\frac{1}{2}$ in equation (1), $a=1$.

$$\begin{aligned} S_{20} &= \frac{n}{2}[2a+(n-1)d]^2 \\ &= \frac{20}{2}\left[2+\frac{19}{2}\right] = 115 \end{aligned}$$

Thus $d=\frac{1}{2}, a=1$ and $S_{20}=115$

CASE 2 : Substituting $d=-\frac{1}{2}$ in equation (1) $a=5$

$$\begin{aligned} S_{20} &= \frac{20}{2}\left[2\times5+19\times\left(-\frac{1}{2}\right)\right] \\ &= 10\left[10-\frac{19}{2}\right] = 15 \end{aligned}$$

Thus $d=-\frac{1}{2}, a=5$ and $S_{20}=15$

13. A sum of Rs. 280 is to be used towards four prizes. If each prize after the first is Rs. 20 less than its preceding prize, find the value of each of the prizes.

Ans : [Board Term-2, 2012(44)]

Let I^{st} prize be Rs. x, then series of prize is
$x, x-20, x-40, x-60, ...$
Here series is AP and $a=x, d=-20, S_n=280, n=4$

$$\begin{aligned} S_n &= \frac{n}{2}[2a+(n-1)d] \\ 280 &= \frac{4}{2}[2x+3(-20)] \\ 280 &= 2[2x-60] \\ 140 &= 2x-60 \\ x &= \frac{140+40}{2} = 100 \end{aligned}$$

Thus prizes are Rs. 100, Rs. 80, Rs. 60, Rs. 40.

14. In a garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row. How many rows are there of rose plants> also find the total number of roes plants in the garden.

Ans : [Board Term-2, 2012(1)]

The number of rose plants in the $1^{st}, 2^{nd}, ...$ are $23, 21, 19, ..., 5$.
Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n
Here $a=23, d=-2, a_n=5$

$$\begin{aligned} a_n &= a+(n-1)d \\ 5 &= 23+(n-1)(-2) \\ n &= 10 \end{aligned}$$

Total number of roes plants in the flower bed,

$$\begin{aligned} S_n &= \frac{n}{2}[2a+(n-1)d] \\ S_{10} &= 5(46-18) = 140 \end{aligned}$$

15. A sum of Rs. 1890 is to be used to given seven cash prizes to students of a school for their overall academic performance. If each prize is Rs. 50 less than its preceding prize, find the value of each of the prizes.

Ans : [Board Term-2, 2012(5)]

Let I^{st} prize be Rs. x, then series of prize is
$x, x-50, x-100, x-150, ...$
Here series is AP and
$a=s, d=-50, S_n=1890, n=7$

$$\begin{aligned} S_n &= \frac{n}{2}[2a+(n-1)d] \\ 1890 &= \frac{7}{2}[2x+(-50)\times 6] \\ 270 &= x+(-50)\times 3 = x-150 \\ x &= 270+150 = 420 \end{aligned}$$

The prizes are Rs. 420, Rs. 370, Rs. 320, Rs. 270, Rs. 220, Rs. 170, Rs. 120.

16. If the sum of first m terms of an A.P. is same as the sum of its first n terms $(m\neq n)$, show that the sum of its first $(m+n)$ terms is zero.

Ans : [Board Term-2, 2012 (12)]

Let the first term be a, common difference be d, nth term be a_n, and sum of n term be S_n

Now
$$\begin{aligned} S_m &= S_n \\ \frac{m}{2}[2a+(m-1)d] &= \frac{n}{2}[2a+(n-1)d] \\ 2a(m-n)+\{(m^2-n^2)-m-nd\} &= 0 \\ 2a(m-n)+\{(m-n)(m+n)-(m-n)d\} &= 0 \\ (m-n)[2a+(m+n-1)d] &= 0 \\ 2a+(m+n-1)d &= 0 \qquad [m-n\neq 0] \\ S_{m+n} &= \frac{m+n}{2}[2a+(m+n-1)d] \end{aligned}$$

$$= \frac{m+n}{2} \times 0 = 0$$

17. A man repays a loan of Rs. 3250 by paying Rs. 20 in the first month and then increases the payment by Rs. 15 every month. How long will it take him to clear the loan?

Ans : [Board Term-2, 2012 Set (34)]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 20, d = 15$

Now
$$\begin{aligned} S_n &= 3250 \\ S_n &= \frac{n}{2}[2a + (n-1)d] \\ 3250 &= \frac{n}{2}[2a + (n-1) \times 15] \\ 3250 \times 2 &= n[40 + 15n - 15] \\ 6500 &= n[25 + 15n] \\ 1300 &= n[5 + 3n] \\ 3n^2 + 65n - 60n - 1300 &= 0 \\ n(3n+65) - 20(3n+65) &= 0 \\ (n-20)(3n+65) &= 0 \end{aligned}$$

Since $n = -65/3$, is not possible, $n = 20$

Man will repay loan in 20 months.

18. If $1 + 4 + 7 + 10 ..... + x = 287$, Find the value of x.

Ans : [Board Foreign Set-I, 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a = 1, d = 3$

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ \frac{n}{2}[2 \times 1 + (n-1)3] &= 287 \\ \frac{n}{2}[2 + (3n-3)] &= 287 \\ 3n^2 - n &= 574 \\ 3n^2 - n - 574 &= 0 \\ 3n^2 - 42n + 41n - 574 &= 0 \\ 3n(n-14) + 41(n-14) &= 0 \\ (n-14)(3n+41) &= 0 \end{aligned}$$

Since negative value is not possible, $n = 14$

$$\begin{aligned} a_{14} &= a + (n-1)d \\ &= 1 + 13 \times 3 = 40 \end{aligned}$$

19. Find the sum of first 24 terms of an A.P. whose n^{th} term given by $a_n = 3 + 2n$.

Ans : [Board Outside Delhi Comptt. Set I, II, III, 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have
$$\begin{aligned} a_n &= 3 + 2n \\ a_1 &= 3 + 2 \times 1 = 5 \\ a_2 &= 2 + 2 \times 2 = 7 \\ a_3 &= 3 + 2 \times 3 = 9 \end{aligned}$$

Thus the series is 5, 7, 9, ... in which

$$a = 5 \text{ and } d = 2$$

Now
$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ S_{24} &= \frac{24}{2}(2 \times 5 + 23 \times 2) \\ &= 12 \times 56 \end{aligned}$$

Hence, $S_{24} = 672$

HOTS QUESTIONS

1. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Ans : [CBSE O.D. 2014]

The sequence goes like 110, 120, 130, ... 990
Since they have a common difference of 10, they form an A.P.
Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n
Here $a = 110, a_n = 990, d = 10$

$$\begin{aligned} a_n &= a + (n-1)d \\ 990 &= 110 + (n-1) \times 10 \\ 990 - 110 &= 10(n-1) \\ 880 &= 10(n-1) \\ 88 &= n - 1 \\ n &= 88 + 1 = 89 \end{aligned}$$

Hence, there are 89 terms between 101 and 999 divisible by both 2 and 5.

2. How many thee digit natural numbers are divisible by 7?

Ans : [Board Term-2, 2013]

Let A.P. is 105, 112, 119, ..., 994 which is divisible by 7.
Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n
Here, $a = 105, d = 112 - 105 = 7, t_n = 994$, then

$$\begin{aligned} a_n &= a + (n-1)d \\ 994 &= 105 + (n-1) \times 7 \\ 889 &= (n-1) \times 7 \\ n - 1 &= \frac{889}{7} = 127 \\ n &= 127 + 1 = 128 \end{aligned}$$

Hence, there 128 terms divisible by 7 in A.P.

3. How many two digit numbers are divisible by 7?

Ans : [Board Sample paper, 2016]

Two digit numbers which are divisible by 7 are 14, 21, 28, ... 98. It forms an A.P.
Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 14,\ d = 7, a_n = 98$

Now
$$\begin{aligned} a_n &= a + (n-1)d \\ 98 &= 14 + (n-1)7 \\ 98 - 14 &= 7n - 7 \end{aligned}$$

$$84+7 = 7n$$

$$7n = 91 \Rightarrow n = 13$$

4. How many three digit numbers are such that when divided by 7, leave a remainder 3 in each case?

Ans : [Board Term-2, 2012 Set (1)]

When a three digit number divided by 7 and leave 3 as remainder are 101, 108, 115, ... 997

These are in A.P.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 101, d = 7, a_n = 997$

Now
$$a_n = a+(n-1)d$$

$$997 = 101+(n-1)7$$

$$997-101 = 896 = (n-1)7$$

$$\frac{896}{7} = n-1$$

$$n = 128+1 = 129$$

Hence, 129 numbers are divided by 7 which leaves remainder is 3.

5. How many multiples of 4 lie between 11 and 266?

Ans : [Board Term-2, 2012, Set (21)]

First multiple of 4 is 12 and last multiple of 4 is 264. It forms a AP. Let multiples of 4 be n.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here, $a = 12, a_n = 264, d = 4$

$$a_n = a+(n-1)d$$

$$264 = 12+(n-1)4$$

$$n = \frac{264-12}{4}+1$$

Hence, there are 64 multiples of 4 that lie between 11 and 266.

6. Prove that the n^{th} term of an A.P. can not be n^2+1. Justify your answer.

Ans : [Board Term-2, 2015]

Let n^{th} term of A.P.

$$a_n = n^2+1$$

Substituting the value of $n = 1, 2, 3, ...$ we get

$$a_1 = 1^2+1 = 2$$

$$a_2 = 2^2+1 = 5$$

$$a_3 = 3^2+1 = 10$$

The obtained sequence is 2, 5, 10, 17,......

Its common difference

$$a_2-a_1 = a_3-a_2 = a_4-a_3$$

$$5-2 \neq 10-5 \neq 17-10$$

$$3 \neq 5 \neq 7$$

Since the sequence has no. common difference, n^2+1 is not a form of n^{th} term of an A.P.

7. Find the sum of all two digits odd positive numbers.

Ans : [KVS 2014]

The list of 2 digits odd positive numbers are 11, 13 ...... 99. It forms an AP.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 11, d = 2, l = 991$

Now
$$a_n = a+(n-1)d$$

$$99 = 11+(n-1)2$$

$$88 = (n-1)2$$

$$n = 44+1 = 45$$

$$S_n = \frac{n}{2}[a+a_n]$$

$$= \frac{45}{2}[11+99]$$

$$S_n = \frac{15\times 108}{2} = 2475$$

Hence the sum of given A.P. is $S_n = 2475$

8. Find the sum of the two digits numbers divisible by 6.

Ans : [Board Term-2, 2013]

Series of two digits numbers divisible by 6 is: 12, 18, 24, ...96. It forms and AP.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 12, d = 18-12 = 6, a_n = 96$

$$a_n = a+(n-1)d$$

$$96 = 12+(n-1)\times 6$$

$$84 = 6(n-1)$$

$$n = 14+1 = 15$$

$$S_n = \frac{n}{2}[a+a_n]$$

$$= \frac{15}{2}[12+96]$$

$$= \frac{15\times 2}{2}[8]$$

$$= 15\times 54 = 810$$

Hence the sum of given AP is 810.

9. Find the sum of the integers between 100 and 200 that are divisible by 6.

Ans : [Board Term-2, 2012 Set (5)]

The series as per question is 102, 108, 114, ... 198. which is an A.P.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 102, d = 6$ and $l = 198$

Now
$$198 = 102+(n-1)6$$

$$96 = (n-1)6$$

$$\frac{96}{6} = n-1$$

$$n = 17$$

Now
$$S_{17} = \frac{n}{2}(a+l)$$

$$= \frac{17}{2}[102+198]$$

$$= \frac{17}{2} \times 300 = 17 \times 150 = 2550$$

Hence the sum of given AP is 2550.

10. Find the number of terms of the A.P. $-12, -9, -6, ..., 21$. If 1 is added to each term of this A.P., then find the sum of all the terms of the A.P. thus obtained.

Ans : [Board Term-2, 2013]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a = -12, d = 9-(-12) = 3$

$$\begin{aligned} a_n &= a+(n-1)d \\ 21 &= -12+(n-1)\times 3 \\ 21+12 &= (n-1)\times 3 \\ 33 &= (n-1)\times 3 \\ n-1 &= 11 \\ n &= 11+1 = 12 \end{aligned}$$

Now, if 1 is added to each term we have a New A.P. with

$-12+1, -a+1, -6+1......21+1$

Now we have $a = -11, d = 3$ and $a_n = 22$ and $n = 12$

Sum of this obtained A.P.

$$\begin{aligned} S_{12} &= \frac{12}{2}[-11+22] \\ &= 6 \times 11 = 66 \end{aligned}$$

Hence the sum of new AP is 66.

11. How many terms of the A.P. $-6, \frac{11}{2}, -5, ...$ are needed to given the sum -25? Explain the double answer.

Ans : [Board Term-2, 2012 Set (13)]

A.P. is $-6, -\frac{11}{2}, -5 ....$

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here we have $a = -6$

$$\begin{aligned} d &= -\frac{11}{2}+\frac{6}{1} = \frac{1}{2} \\ S_n &= -25 \\ S_n &= \frac{n}{2}[2a+(n-1)d] \\ -25 &= \frac{n}{2}\left[-12+(n-1)\times\frac{1}{2}\right] \\ -50 &= n\left[\frac{-24+(n-1)}{2}\right] \\ -100 &= n[n-25] \\ n^2-25n+100 &= 0 \\ (n-20)(n-5) &= 0 \\ n &= 20, 5 \end{aligned}$$

or, $S_{20} = S_3$

Here we have got two answers because two value of $n$some of AP is same. Since a is negative and d is positive; the sum of the terms from 6^{th} to 20^{th} is zero.

12. If S_1, S_2, S_3 be the sum of $n, 2n, 3n$ terms respectively of an A.P. Prove that $S_3 = 3(S_2 - S_1)$.

Ans : [Board Term-2, 2012 Set (59)]

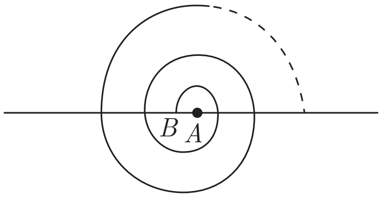
Let the first term be a, and common difference be d.

Now
$$\begin{aligned} S_1 &= \frac{n}{2}[2a(n-1)d] \\ S_2 &= \frac{2n}{2}[2a+(2n-1)d] \\ S_3 &= \frac{3n}{2}[2a+(3n-1)d] \end{aligned}$$

$3(S_2 - S_1)$

$$\begin{aligned} &= 3\left[\frac{2n}{2}[2a+(2n-1)d] - \frac{n}{2}[2a+(n-1)d]\right] \\ &= 3\left[\frac{n}{2}[4a+2(2n-1)d] - [2a+(n-1)d]\right] \\ &= 3\left[\frac{n}{2}(4a+4nd-2d-2a-nd+d)\right] \\ &= 3\left[\frac{n}{2}(2a+3nd-d)\right] \\ &= \frac{3n}{2}[2a+(3n-1)d] = S_3 \end{aligned}$$

13. A spiral is made up of successive semi-circles with centres alternately a A and B starting with A, of radii 1 cm, 2 cm, 3 cm, ... as shown in the figure. What is the total length of spiral made up of eleven consecutive semi-circles? (Use $\pi = 3.14$)



Ans : [Board Term-2, 2012 Set (50); [NCERT]]

Let r_1, r_2............. be the radii of semi-circles and l_1, l_2,............ be the lengths of circumferences of semi-circles, than

$$\begin{aligned} l_1 &= \pi r_1 = \pi(1) = \pi\, cm \\ l_2 &= \pi r_2 = \pi(2) = 2\pi\, cm \\ l_3 &= 3\pi\, cm \\ &............... \\ &............... \\ l_{11} &= 11\pi\, cm \end{aligned}$$

Total length of spiral

$$\begin{aligned} L &= l_1 + l_2 + ... + l_{11} \\ &= \pi + 2\pi + 3\pi + ... + 11\pi \\ &= \pi(1+2+3+.......+11) \\ &= \pi \times \frac{11 \times 12}{2} \\ &= 66 \times 3.14 \\ &= 207.24 \text{ cm} \end{aligned}$$

14. The ratio of the sums of first m and first n terms of an A.P. is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is $(2m-1):(2n-1)$.

Ans : [CBSE Board Delhi Set I, 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

$$\begin{aligned}
\frac{S_m}{n_n} &= \frac{m^2}{n^2} \\
\frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]} &= \frac{m^2}{n^2} \\
\frac{2a+(m-1)d}{2a+(n-1)d} &= \frac{m^2}{n^2} \times \frac{n}{m} = \frac{m}{n} \\
m(2a+(n-1)d) &= n\left[2a+(m-1)d\right] \\
2ma + mnd - md &= 2na + nmd - nd \\
2ma - 2na &= md - nd \\
d &= 2a
\end{aligned}$$

Now,

$$\begin{aligned}
\frac{a_m}{a_n} &= \frac{a+(m-1)d}{a+(n-1)d} \\
&= \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a} \\
&= \frac{a+2ma-2a}{a+2na-2a} \\
&= \frac{2ma-a}{2na-a} = \frac{a(2m-1)}{a(2n-1)} \\
&= 2m-1 : 2n-1
\end{aligned}$$

15. If the p^{th} term of an A.p. is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$. Prove that the sum of first pq term of the A.P. is $\left[\frac{pq+1}{2}\right]$

Ans : [CBSE Board Delhi Set III, 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

$$a_p = a+(p-1)d = \frac{1}{q} \qquad \text{...(1)}$$

and

$$a_q = a+(q-1)d = \frac{1}{p} \qquad \text{...(2)}$$

Solving (1) and (2) we get

$$a = \frac{1}{pq} \text{ and } d = \frac{1}{p}$$

$$\begin{aligned}
S_{pq} &= \frac{pq}{2}\left[2 \times \frac{1}{8q} + (pq-1)\frac{1}{pq}\right] \\
&= \frac{pq+1}{2}
\end{aligned}$$

16. If the ratio of the 11^{th} term of an A.P. to its 18^{th} term is 2 : 3, find the ratio of the sum of the first five term of the sum of its first 10 terms.

Ans : [Delhi Compt. Set I, II, III 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now

$$\frac{a_{11}}{a_{18}} = \frac{a+10d}{a+17d} = \frac{2}{3}$$

$$\begin{aligned}
2(a+17d) &= 3(a+10d) \\
a &= 4d \qquad \text{...(1)}
\end{aligned}$$

Now,

$$\frac{S_5}{S_{10}} = \frac{\frac{5}{2}(2a+4d)}{\frac{10}{2}[2a+9d]} = \frac{(a+2d)}{[2a+9d]}$$

Substituting the value $a = 4d$ we have

or,

$$\frac{S_5}{S_{10}} = \frac{4d+2d}{8d+9d} = \frac{6}{17}$$

Hence $S_5 : S_{10} = 6:17$

17. An A.P. Consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the past three terms is 429. Find the A.P.

Ans : [Sample Paper 2017]

Let the middle most terms of the A.P. be $(x-d), x, (x+d)$

We have

$$\begin{aligned}
x - d + x + x + d &= 225 \\
3x &= 225
\end{aligned}$$

or,

$$x = 75$$

and the middle term $= \frac{37+1}{2} = 19^{th}$ term

Thus AP is

$(x-18d), ...(x-2d), (x-d), x, (x+d), (x+2d), ...$

$(x-18d)$

Sum of last three terms,

$$\begin{aligned}
(x+18d)+(x+17d)+(x+16d) &= 429 \\
3x+51d &= 429
\end{aligned}$$

,

$$225+51d = 429 \text{ or, } d = 4$$

First term $a_1 = x - 18d = 75 - 18 \times 4 = 3$

$$a_2 = 3 + 4 = 7$$

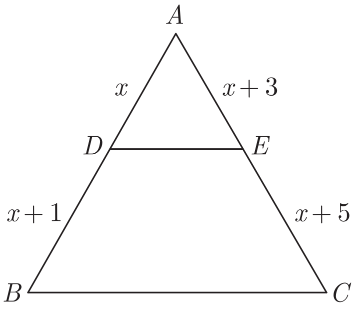
Hence A.P. = 3, 7, 11, ..., 147.

CHAPTER 6

Triangles

VERY SHORT ANSWER TYPE QUESTIONS

1. In $\Delta ABC, DE || BC$, find the value of x.



Ans : [Board Term-1, 2016, Set-O4YP6G7]

In the given figure $DE || BC$, thus

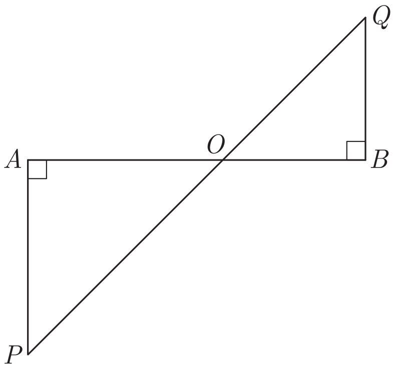
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{x+1} = \frac{x+3}{x+5}$$

$$x^2 + 5x = x^2 + 4x + 3$$

$$x = 3$$

2. In the given figure, if $\angle A = 90^0, \angle B = 90^0$, $OB = 4.5\,cm$, $OA = 6\,cm$ and $AP = 4cm$, then find QB.



Ans : [Board Term-1, 2015, DDEE]

In ΔPAO and ΔQBO we have

$$\angle A = \angle B = 90^0$$

Vertically opposite angle,

$$\angle POA = \angle QOB$$

Thus $\Delta PAO \sim \Delta QBO$

$$\frac{OA}{OB} = \frac{PA}{QB}$$

$$\frac{6}{4.5} = \frac{4}{QB}$$

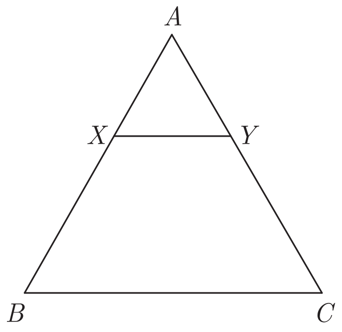
$$QB = \frac{4 \times 4.5}{6} = 3 \text{ cm}$$

Thus $QB = 3$ cm

3. In ΔABC, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, $AY = 5$ and $YC = 9$, then state whether XY and BC parallel or not.

Ans : [Term-1, 2016, MV98HN3], [Term-1, 2015, CJTOQ]

As per question we have drawn figure given below.



In this figure we have

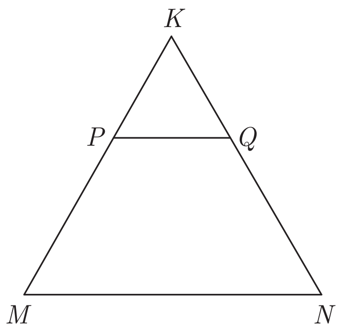
$$\frac{AX}{XB} = \frac{3}{4}, AY = 5, YC = 9$$

$$\frac{AX}{XB} = \frac{3}{4} \text{ and } \frac{AY}{YC} = \frac{5}{9}$$

Since $$\frac{AX}{XB} \neq \frac{AY}{YC}$$

Hence XY is not parallel to BC.

4. In the figure, PQ is parallel to MN. If $\frac{KP}{PM} = \frac{4}{13}$ and $KN = 20.4cm$, then find KQ.



Ans :

In the given figure $PQ || MN$, thus

$$\frac{KP}{PM} = \frac{KQ}{QN} \qquad \text{(By BPT)}$$

$$\frac{KP}{PM} = \frac{KQ}{KN - KQ}$$

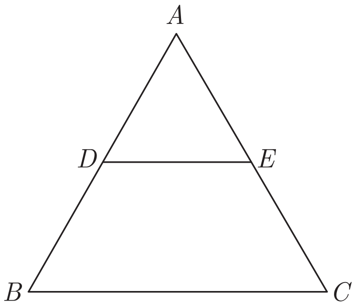
$$\frac{4}{13} = \frac{KQ}{20.4 - KQ}$$

$$4 \times 20.4 - 4KQ = 13KQ$$

$$17KQ = 4 \times 20.4$$

$$KQ = \frac{20.4 \times 4}{17} = 4.8 \text{ cm}$$

5. In given figure $DE || BC$. If $AD = 3c, DB = 4cm$ and $AE = 6cm$, then find EC.



Ans : [Board Term-1, 2016, Set-ORDAWEZ]

In the given figure $DE \parallel BC$, thus

$$\frac{AD}{AB} = \frac{AE}{EC}$$

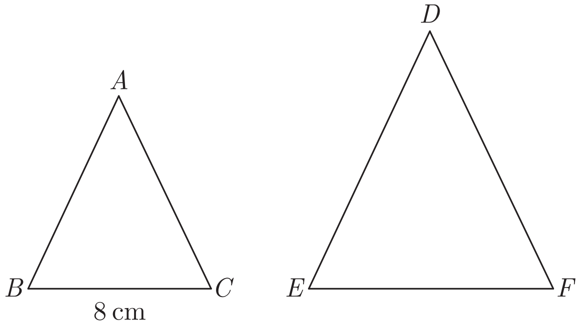
$$\frac{3}{4} = \frac{6}{EC}$$

$$EC = 8 \text{ cm}$$

6. If triangle ABC is similar to triangle DEF such that $2AB = DE$ and $BC = 8cm$, then find EF.

Ans :

As per given condition we have drawn the figure below.



Here we have $2AB = DE$ and $BC = 8$ cm

Since $\Delta ABC \sim \Delta DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{AB}{8} = \frac{2AB}{EF}$$

$$EF = 2 \times 8 = 16 \text{ cm}$$

7. Are two triangles with equal corresponding sides always similar?

Ans : [Board Term-1, 2015, Set-FHN8MGD]

Yes, Two triangles having equal corresponding sides are are congruent and all congruent Δs have equal angles, hence they are similar too.

8. If ratio of corresponding sides of two similar triangles is $5:6$, then find ratio of their areas.

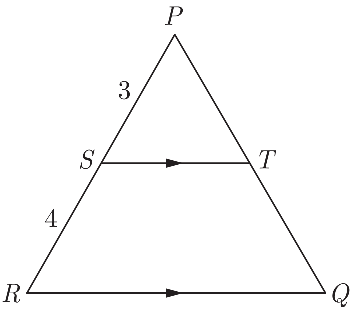
Ans : [Board Term-1, 2015, Set-WJQZQBN]

Let the triangles be ΔABC and ΔDEF

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

$$25:36$$

9. In the given figure, $ST || RQ, PS = 3$ cm and $SR = 4cm$. Find the ratio of the area of ΔPST to the area of ΔPRQ.



Ans : [Sample Question paper 2017]

We have $PS = 3$ cm, $SR = 4$ cm, and $ST || RQ$.

Now $PR = PS + SR$

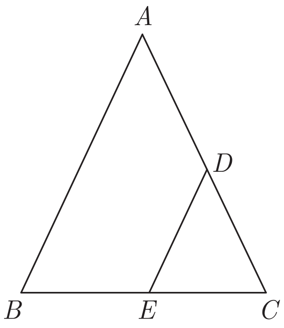
$$= 3 + 4 = 7cm$$

$$\frac{ar\Delta PST}{ar\Delta PQR} = \frac{PS^2}{PR^2} = \frac{3^2}{7^2} = \frac{9}{49}$$

Hence required ratio is $9:49$

SHORT ANSWER TYPE QUESTIONS - I

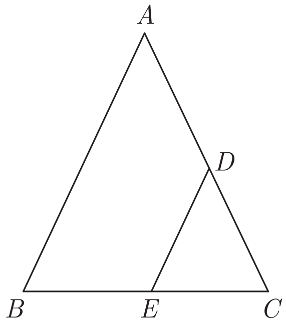
1. In the figure of ΔABC, the points D and E are on the sides CA, CB respectively such that $DE || AB$, $AD = 2x, DC = x + 3, BE = 2x - 1$ and $CE = x$. Then, find x.



OR

In the figure of $\Delta ABC, DE || AB$. $DE || AB$. If

$AD = 2x$, $DC = x+3$, $BE = 2x-1$ and $CE = x$, then find the value of x.



Ans : , [Term-1, 2016, GRKEGO], [Term-1, 2015, DDE-M]

We have
$$\begin{aligned} \frac{CD}{AD} &= \frac{CE}{BE} \\ \frac{x+3}{2x} &= \frac{x}{2x-1} \\ 5x &= 3 \text{ or, } x = \frac{3}{5} \end{aligned}$$

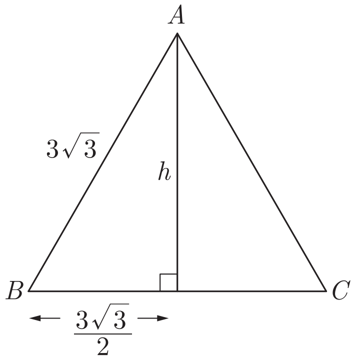
Alternative Method :

In ABC, $DE \parallel AB$, thus
$$\begin{aligned} \frac{CD}{CA} &= \frac{CE}{CB} \\ \frac{CD}{CD+AD} &= \frac{CE}{CE+BE} \\ \frac{x+3}{x+3+2x} &= \frac{x}{2+2x-1} \\ \frac{x+3}{3x+3} &= \frac{x}{3x-1} \\ (x+3)(3x-1) &= x(3x+3) \\ 3x^2 - x + 9x - 3 &= 3x^2 + 3x \\ 8x - 3 &= 3x \\ 8x - 3x &= 3 \\ 5x &= 3 \\ x &= \frac{3}{5} \end{aligned}$$

2. In an equilateral triangle of side $3\sqrt{3}$ cm find the length of the altitude.

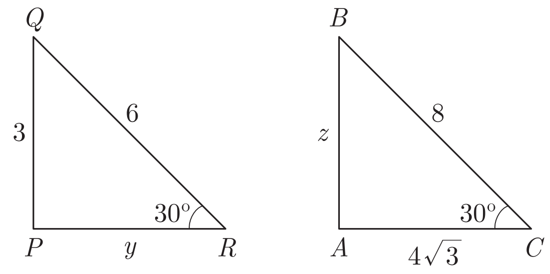
Ans : [Board Term-1, 2016, Set-MV98HN3]

Let ΔABC be an equilateral triangle of side $3\sqrt{3}$ cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



Now
$$\begin{aligned} (3\sqrt{3})^2 &= h^2 + \left(\frac{3\sqrt{3}}{2}\right)^2 \\ 27 &= h^2 + \frac{27}{4} \\ h^2 &= 27 - \frac{27}{4} = \frac{81}{4} \\ h &= \frac{9}{2} = 4.5 \text{ cm} \end{aligned}$$

3. In the given figure, $\Delta ABC \sim \Delta PQR$. Find the value of $y+z$.



Ans : [Board Term-1, Set-CJTOQ]

In the given figure $\Delta ABC \sim \Delta PQR$

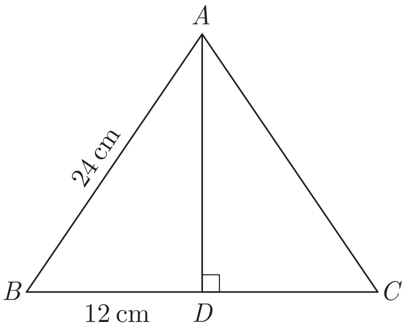
Thus
$$\begin{aligned} \frac{AB}{PQ} &= \frac{BC}{QR} = \frac{AC}{PR} \\ \frac{z}{3} &= \frac{8}{6} = \frac{4\sqrt{3}}{y} \\ \frac{z}{3} &= \frac{8}{6} \text{ and } \frac{8}{6} = \frac{4\sqrt{3}}{y} \\ z &= \frac{8\times 3}{6} \text{ and } y = \frac{4\sqrt{3}\times 6}{8} \\ z &= 4 \text{ and } y = 3\sqrt{3} \end{aligned}$$

Thus $y + z = 3\sqrt{3} + 4$

4. In an equilateral triangle of side 24 cm, find the length of the altitude.

Ans : [Board Term-1, 2015, Set-DDE-E]

Let ΔABC be an equilateral triangle of side 24 cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



Now
$$\begin{aligned} BD &= \frac{BC}{2} = \frac{24}{2} = 12 \text{ cm} \\ AB &= 24 \text{ cm} \end{aligned}$$

$$\begin{aligned} AD &= \sqrt{AB^2 - BD^2} \\ &= \sqrt{(24)^2 - (12)^2} \\ &= \sqrt{576 - 144} \\ &= \sqrt{432} = 12\sqrt{3} \end{aligned}$$

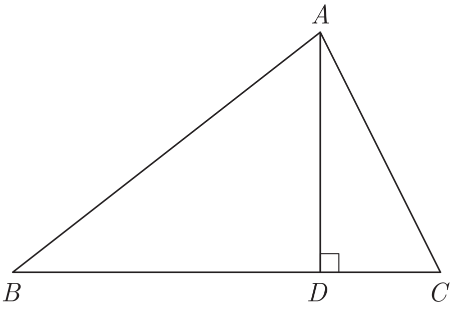
Thus $AD = 12\sqrt{3}$ cm

$\therefore$ The length of the altitude is $12\sqrt{3}$ cm.

5. In $\Delta ABC, AD \perp BC$, such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled at A.

Ans : [Board Term-1, 2015, Set-FHN8MGD]

As per given condition we have drawn the figure below.



We have $AD^2 = BD \times CD$

$$\frac{AD}{CD} = \frac{BD}{AD}$$

Since $\angle D = 90°$, by SAS we have

$$\Delta ADC \sim \Delta BDA$$

and $\angle BAD = \angle ACD;$

Since corresponding angles of similar triangles are equal

$$\angle DAC = \angle DBA$$

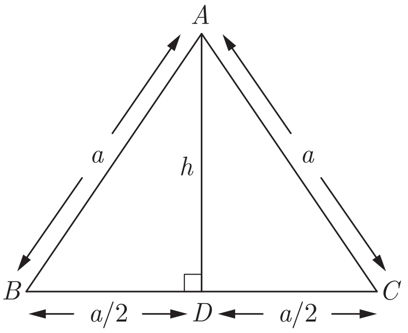
$$\begin{aligned} \angle BAD + \angle ACD + \angle DAC + \angle DBA &= 180° \\ 2\angle BAD + 2\angle DAC &= 180° \\ \angle BAD + \angle DAC &= 90° \\ \angle A &= 90° \end{aligned}$$

Thus ΔABC is right angled at A.

6. Find the altitude of an equilateral triangle when each of its side is 'a' cm.

Ans : [Board Term-1, 2016, Set-O4YP6G7]

Let ΔABC be an equilateral triangle of side a and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



In ΔABD, $\quad a^2 = \left(\frac{a}{2}\right)^2 + h^2$

$$h^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

Thus $\quad h = \frac{\sqrt{3}a}{2}$

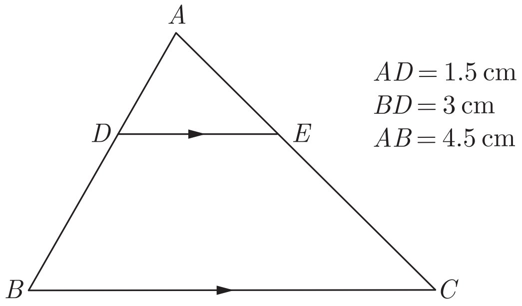
7. Let $\Delta ABC \sim \Delta DEF$. if ar $(\Delta ABC) = 100$ cm^2, $ar(DEF) = 196$ cm^2 and $DE = 7$, then find AB.

Ans : [Board Term-1, 2015, Set-DDE-M]

We have $\Delta ABC \sim \Delta DEF$, thus

$$\begin{aligned} \frac{ar(\Delta ABC)}{are(\Delta DEF)} &= \frac{AB^2}{DE^2} \\ \frac{100}{196} &= \frac{AB^2}{(7)^2} \\ \frac{100}{196} &= \frac{AB^2}{49} \\ AB^2 &= \frac{49 \times 100}{196} \\ AB^2 &= 25 \\ AB &= 5 \text{ cm} \end{aligned}$$

8. In the given figure, $DE || BC$. If $AD = 1.5$ cm $BD = 2AD$, then find $\frac{ar(\Delta ADE)}{ar(trapezium\ BCED)}$



Ans : [Board Term-1, 2013, set FFC]

We have $\quad AD = 1.5$ cm, $BD = 3$

and $\quad AB = AD + BD = 1.5 + 3.0 = 4.5cm$ cm

In triangle ADE and ABC, $\angle A$ is common and $DE || BC$

Thus $\quad \angle ADE = \angle ABC$

$$\angle AED = \angle ACB$$

(corresponding angles)

By AA similarity we have

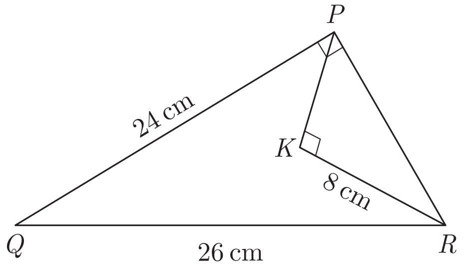
$$\Delta ADE \sim \Delta ABC$$

Now $\quad \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{AD^2}{AB^2} = \frac{(1.5)^2}{(4.5)^2} = \frac{1}{9}$

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC) - ar(\Delta ADE)} = \frac{1}{9-1}$$

$$\frac{ar(\Delta ADE)}{ar(trapezium\ BCED)} = \frac{1}{8}$$

9. In the given triangle $PQR, \angle QPR = 90°, PQ = 24$ cm and $QR = 26$ cm and in $\Delta PKR, \angle PKR = 90^o$ and $KR = 8$ cm, find PK.



Ans : [Board Term-1, 2012, Set-21]

In the given triangle we have

$$\angle QPR = 90^0$$

Thus $$QR^2 = QP^2 + PR^2$$

$$PR = \sqrt{26^2 - 24^2}$$

$$= \sqrt{100} = 10 \text{ cm}$$

Now $$\angle PKR = 90^0$$

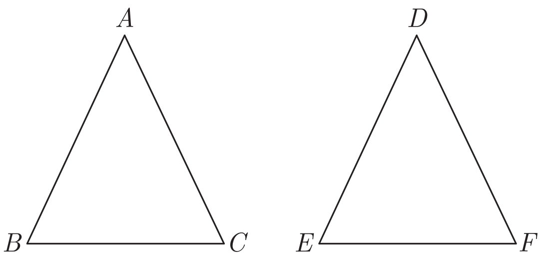
Thus $$PK = \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$$

$$= \sqrt{36} = 6 \text{ cm}$$

10. The sides AB and AC and the perimeter P_1 of ΔABC are respectively three times the corresponding sides DE and DF and the parameter P_2 of ΔDEF. Are the two triangles similar? If yes, find $\frac{ar(\Delta ABC)}{ar(\Delta DEF)}$

Ans : [Board Term-1, 2012, SEt-39]

As per given condition we have drawn the figure below.



In ΔABC and ΔDEF,

$$AB = 3DE$$

and $$AC = 3DF$$

$$\frac{AB}{DE} = 3; \ \frac{AC}{DF} = 3;$$

Since $P_1 = 3P_2$, $BC = 3EF$

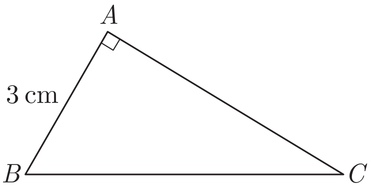
Thus $$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = 3$$

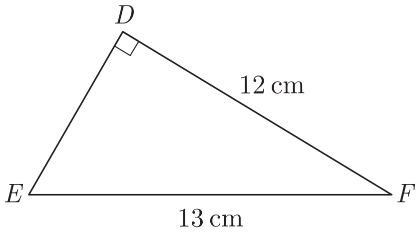
By SSS criterion we have

$$\Delta ABC \sim \Delta DEF$$

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = (3)^2 = 9$$

11. Given $\Delta ABC \sim \Delta DEF$, find $\frac{\Delta ABC}{\Delta DEF}$





Ans : [Board Term-1, 2016, Set-ORDAWEZ]

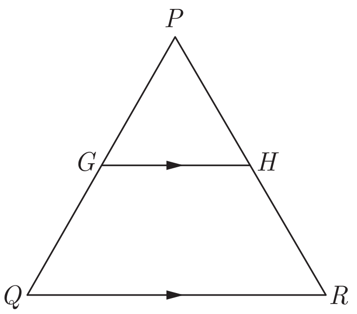
In ΔDEF, we have

$$DE = \sqrt{(13)^2 - (12)^2}$$

$$= \sqrt{169 - 144} = \sqrt{25} = 5$$

Thus $$\frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

12. In the given figure, G is the mid-point of the side PQ of ΔPQR and $GH || QR$. Prove that H is the mid-point of the side PR or the triangle PQR.



Ans : [Board Term-1, 2012, Set-43]

Since G is the mid-point of PQ we have

$$PG = GQ$$

$$\frac{PG}{GQ} = 1$$

According to the question, $GH || QR$, thus

$$\frac{PG}{GQ} = \frac{PH}{HR} \qquad \text{(By BPT)}$$

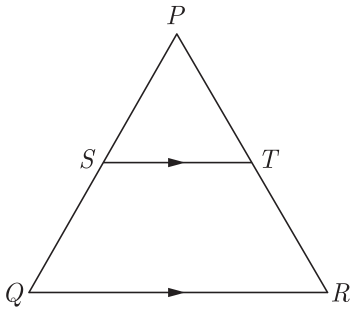
$$1 = \frac{PH}{HR}$$

$$PH = HR. \qquad \text{Hence proved.}$$

Hence, H is the mid-point of PR.

13. In the given figure, in a triangle $PQR, ST || QR$ and

$\frac{PS}{SQ} = \frac{3}{5}$ and $PR = 28$ cm, find PT.

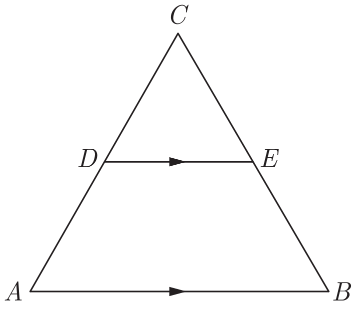


Ans : [Board Term-1, 2012, Set-64; Set-I 2011]

We have
$$\frac{PS}{SQ} = \frac{3}{5}$$
$$\frac{PS}{PS+SQ} = \frac{3}{3+5}$$
$$\frac{PS}{PQ} = \frac{3}{8}$$

According to the question, $ST \,||\, QR$, thus
$$\frac{PS}{PQ} = \frac{PT}{PR} \qquad \text{(By BPT)}$$
$$PT = \frac{PS}{PQ} \times PR$$
$$= \frac{3 \times 28}{8} = 10.5 \text{ cm}$$

14. In the given figure, $\angle A = \angle B$ and $AD = BE$. Show that $DE \,||\, AB$.



Ans : [Board Term-1, 2012, set-63]

In $\Delta\, CAB$, we have
$$\angle A \ \angle B \qquad (1)$$

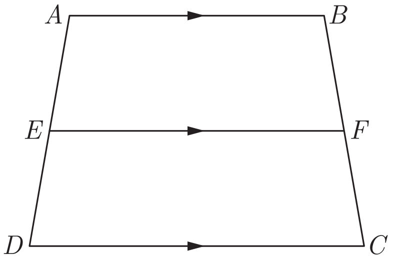
By isoscales triangle property we have
$$AC = CB$$

But, we have been given
$$AD = BE \qquad (2)$$

Dividing equation (2) by (1) we get,
$$\frac{CD}{AD} = \frac{CE}{BE}$$

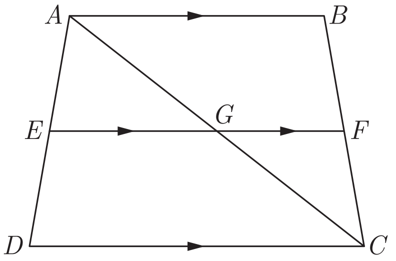
By converse of BPT,
$$DE \,||\, AB. \qquad \text{Hence Proved}$$

15. In the given figure, if $ABCD$ is a trapezium in which $AB \,||\, CD \,||\, EF$, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$



Ans : [Board Term-1, 2012, Set-25]

We draw, AC intersecting EF at G as shown below.



In $\Delta\, CAB, GF \,||\, AB$, thus by BPT we have
$$\frac{AG}{CG} = \frac{BF}{FC} \qquad ...(1)$$

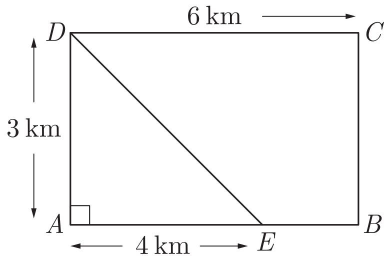
In $\Delta\, ADC$, $EG \,||\, DC$, thus by BPT we have
$$\frac{AE}{ED} = \frac{AG}{CG} \qquad ...(2)$$

From equations (1) and (2),
$$\frac{AE}{ED} = \frac{BF}{FC}. \qquad \text{Hence Proved.}$$

16. In a rectangle $ABCD, E$ is a point on AB such that $AE = \frac{2}{3}AB$. If $AB = 6$ km and $AD = 3$ km, then find DE.

Ans : [Board Term-1, 2016, Set-LGRKEGO]

As per given condition we have drawn the figure below.



We have
$$AE = \frac{2}{3}AB = \frac{2}{3} \times 6 = 4 \text{ km}$$

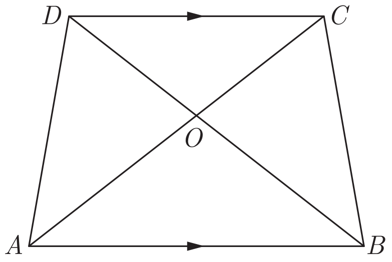
In right triangle ADE,
$$DE^2 = (3)^2 + (4)^2 = 25$$

Thus
$$DE = 5 \text{ km}$$

17. $ABCD$ is a trapezium in which $AB \,||\, CD$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Ans : [Board Term-1, 2012, Set-66]

As per given condition we have drawn the figure below.



In ΔAOB and ΔCOD, $AB \parallel CD$,

Thus $\angle OAB = \angle DCO$

and $\angle OBA = \angle ODC$ (Alternate angles)

By AA similarity we have

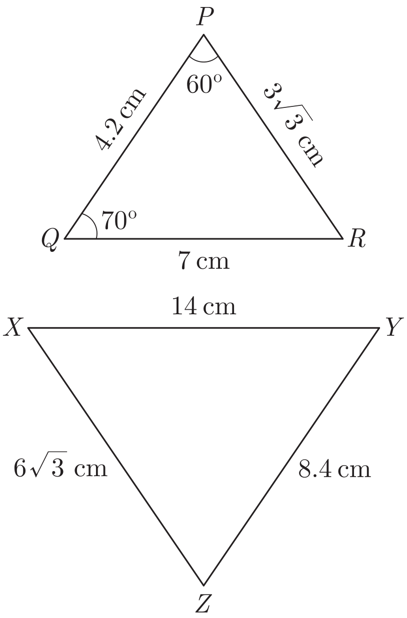
$$\Delta AOB \sim \Delta COD$$

For corresponding sides of similar triangles we have

$$\frac{AO}{CO} = \frac{BO}{DO}$$

$$\frac{AO}{BO} = \frac{CO}{DO}.$$ Hence Proved

18. In the given figures, find the measure of $\angle X$.



Ans : [Board Term-1, 2012, Set-38]

From given figures,

$$\frac{PQ}{ZY} = \frac{4.2}{8.4} = \frac{1}{2},$$

$$\frac{PR}{ZX} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

and $$\frac{QR}{YX} = \frac{7}{14} = \frac{1}{2}$$

Thus $$\frac{QP}{ZY} = \frac{PR}{ZX} = \frac{QR}{YX}$$

By SSS criterion we have

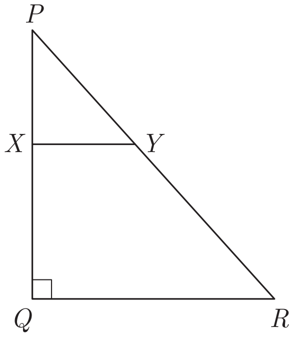
$$\Delta PQR \sim \Delta ZYX$$

Thus $\angle X = \angle R$

$$= 180^{\circ} - (60^{\circ} + 70^{\circ}) = 50^{\circ}$$

Thus $\angle X = 50^{\circ}$

19. In the given figure, PQR is a triangle right angled at Q and $XY \parallel QR$. If $PQ = 6$ cm, $PY = 4$ cm and $PX : XQ = 1:2$. Calculate the length of PR and QR.



Ans : [Board Term-1, 2012, Set-44]

Since $XY \parallel OR$, by BPT we have

$$\frac{PX}{XQ} = \frac{PY}{YR}$$

$$\frac{1}{2} = \frac{PY}{PR - PY}$$

$$= \frac{4}{PR - 4}$$

$$PR - 4 = 8$$

$$PR = 12 \text{ cm}$$

In right ΔPQR we have

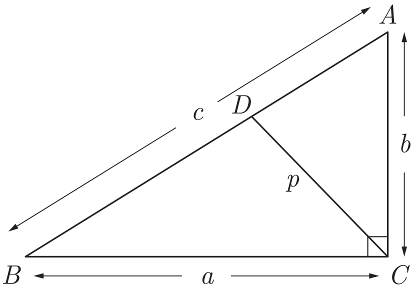
$$QR^2 = PR^2 - PQ^2$$

$$= 12^2 - 6^2$$

$$= 144 - 36 = 108$$

Thus $QR = 6\sqrt{3}$ cm

20. ABC is a right triangle right angled at C. Let $BC = a$, $CA = b, AB = c$ and p be the length of perpendicular from C to AB. Prove that $cp = ab$.



Ans : [Board Term-1, 2012, Set-65]

In the given figure $CD \perp AB$, and

$$CD = p$$

Area $$\Delta ABC = \frac{1}{2} \times base \times height$$

$$= \frac{1}{2} \times AB \times CD = \frac{1}{2}cp$$

Also,Area of $\Delta ABC = \frac{1}{2} \times BC \times AC = \frac{1}{2} ab$

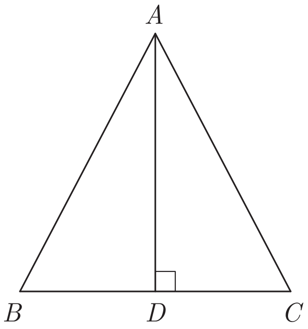
Thus $\frac{1}{2} cp = \frac{1}{2} ab$

$cp = ab$ Proved

21. In an equilateral triangle ABC, AD is drawn perpendicular to BC meeting BC in D. Prove that $AD^2 = 3BD^2$.

Ans : [Board Terom-1, 2012, Set-40]

In ΔABD, from Pythagoras theorem,



$$AB^2 = AD^2 + BD^2$$

Since $AB = BC = CA$, we get

$$BC^2 = AD^2 + BD^2,$$

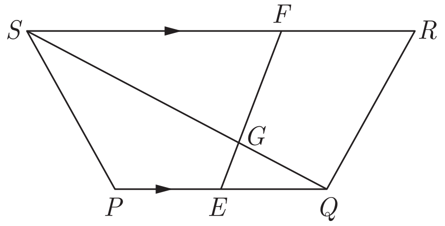
Since $\perp$ is the median in an equilateral Δ, $BC = 2BD$

$$(2BD)^2 = AD^2 + BD^2$$

$$4BD^2 - BD^2 = AD^2$$

$$3BD^2 = AD^2$$

22. In the figure, $PQRS$ is a trapezium in which $PQ || RS$. On PQ and RS, there are points E and F respectively such that EF intersects SQ at G. Prove that $EQ \times GS = GQ \times FS$.



Ans : [Board Term-1, 2016, Set-O4YP6G7]

In ΔGEQ and ΔGFS

$\angle EGQ = \angle FGS$ (vert. opp. angles)

$\angle EQG = \angle FSG$ (alt. angles)

Thus by AA similarity we have

$$\Delta GEQ \sim GFS$$

$$\frac{EQ}{FS} = \frac{GQ}{GS}$$

$$EQ \times GS = GQ \times FS$$

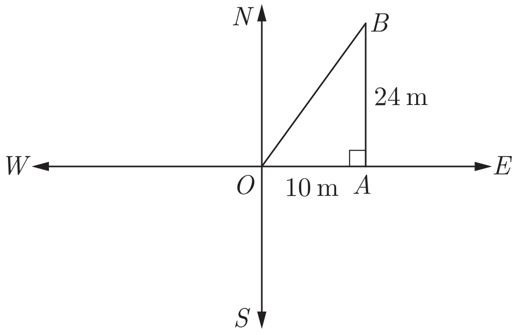
23. A man steadily goes 10 m due east and then 24 m due north.

(1) Find the distance from the starting point.

(2) Which mathematical concept is used in this problem?

Ans :

(1) Let the initial position of the man be at O and his final position be B. Since the man goes to 10 m due east and then 24 m due north. Therefore, ΔAOB is a right triangle right angled at A such that $OA = 10$ m and $AB = 24$ m. We have shown this condition in figure below.



By Pythagoras theorem,

$$OB^2 = OA^2 + AB^2$$

$$= (10)^2 + (24)^2$$

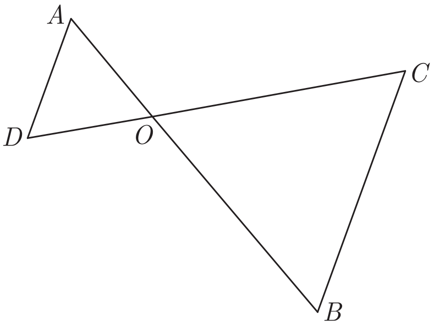
$$= 100 + 576 = 676$$

or, $OB = \sqrt{676} = 26$ m

Hence, the man is at a distance of 26 m from the starting point.

(2) Pythagoras Theorem

24. In the given figure, $OA \times OB = OC \times OD$, show that $\angle A = \angle C$ and $\angle B = \angle D$.



Ans : [Board Term-1, 2012, Set-71]

We have $OA \times OB = OC \times OD$

$$\frac{OA}{OD} = \frac{OB}{OC}$$

$$\angle AOD = \angle COB$$

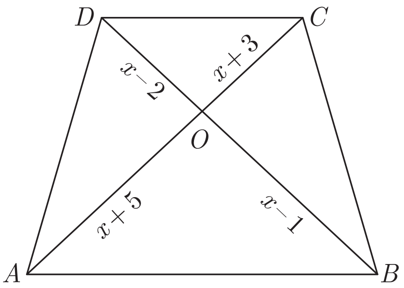
(Vertically opposite angles)

Thus by SAS similarity we have

$$\Delta AOD \sim \Delta COB$$

Thus $\angle A = \angle C$ and $\angle B = \angle D$. because of corresponding angles of similar triangles.

25. In the given figure, if $AB || DC$, find the value of x.



Ans : [Board Term-1, 2012, Set-35]

We know that diagonals of a trapezium divide each other proportionally. Therefore

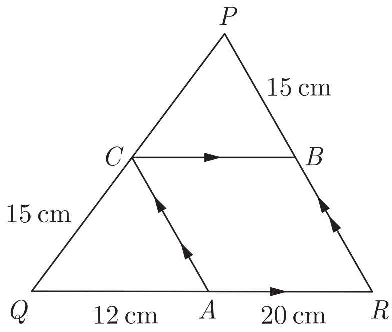
$$\frac{OA}{OC} = \frac{BO}{OD}$$

$$\frac{x+5}{x+3} = \frac{x-1}{x-2}$$

$$\begin{aligned}(x+5)(x-2) &= (x-1)(x+3)\\ x^2+2x+85x-10 &= x^2+3x-x-3\\ 3x-2x &= 10-3\\ x &= 7\end{aligned}$$

Thus $x = 7$.

26. In the given figure, $CB || QR$ and $CA || PR$. If $AQ = 12$ cm, $AR = 20$ cm, $PB = CQ = 15$ cm, calculate PC and BR.



Ans : [Board Term-1, 2012, Set-55]

In ΔPQR, $CA || PR$

By BPT similarity we have

$$\frac{PC}{CQ} = \frac{RA}{AQ}$$

$$\frac{PC}{15} = \frac{20}{12}$$

$$PC = \frac{15 \times 20}{12} = 25 \text{ cm}$$

In ΔPQR, $CB || QR$

Thus $$\frac{PC}{CQ} = \frac{PR}{BR}$$

$$\frac{25}{15} = \frac{15}{BR}$$

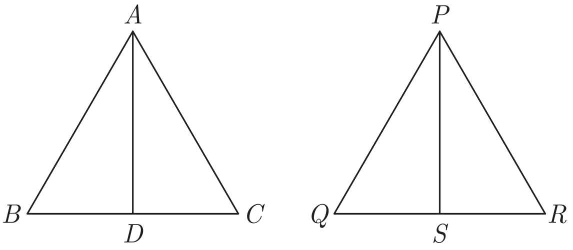
$$BR = \frac{15 \times 15}{25} = 9 \text{ cm}$$

SHORT ANSWER TYPE QUESTIONS - II

1. If $\Delta ABC \sim \Delta PQR$ and AD and PS are bisectors of corresponding angles A and P, then prove that $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PS^2}$.

Ans : [Board Term-1, 2016, Set-MV98HN3]

As per given condition we have drawn the figure below.



Since $\Delta ABC \sim \Delta PQR$ we have

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$$

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} \quad \text{...(1)}$$

Now $$\angle A = \angle P$$

$$\frac{1}{2}\angle A = \frac{1}{2}\angle P$$

$$\angle BAD = \angle QPS$$

By AA similarity we have

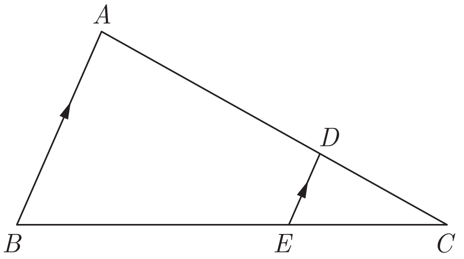
$$\Delta BAD \sim \Delta QPS$$

$$\frac{BA}{QP} = \frac{AD}{PS} \quad \text{...(2)}$$

By equation (1) and (2), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PS^2}$$ Hence proved

2. In given figure, D is a point on AC such that $AD = 2CD$, also $DE || AB$.



Find : $\frac{ar\Delta ACB}{ar\Delta DCE}$

Ans : [Board Term-1, 2015, Set-FHN8MGD]

In given figure we have

$$AD = 2CD$$

In ΔCDE and ΔCAB

$$\angle C = \angle C \quad \text{(Common)}$$

$$\angle CDE = \angle CAB$$

(Corresponding angles)

By AA similarity rule we get

$$\Delta CDE \sim \Delta CAB$$

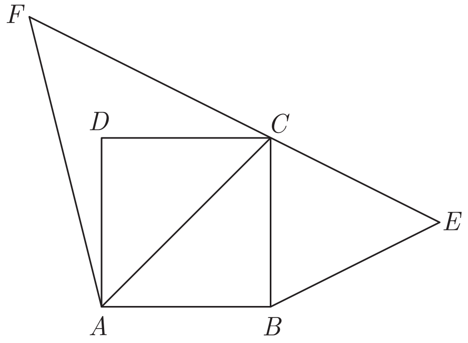
Now, $$\frac{ar(\Delta DCE)}{ar(\Delta ACB)} = \frac{CD^2}{CA^2} = \frac{CD^2}{(AD+DC)^2}$$

$$= \frac{CD^2}{(2DC+DC)^2} = \frac{CD^2}{(3CD)^2} = \frac{1}{9}$$

3. Prove that area of the equilateral triangle described on the side of a square is half of this area of the equilateral triangle described on its diagonal.

Ans : [Board Term-1, 2015, WJQZQBN]

As per given condition we have drawn the figure below.



Equilateral triangle are equiangular also and they are similar by AAA similarity criterion.

Thus $$\Delta BCE \sim \Delta ACF$$

Here ΔABC is a right triangle.

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

Here $AB = BC$ because of sides of a square,

$$AC^2 = 2BC^2$$

$$AC = \sqrt{2}\,BC$$

Now, $$\frac{ar\Delta ACF}{ar\Delta BCE} = \frac{AC^2}{DC^2} = \frac{(\sqrt{2}\,BC)^2}{BC^2} = 2$$

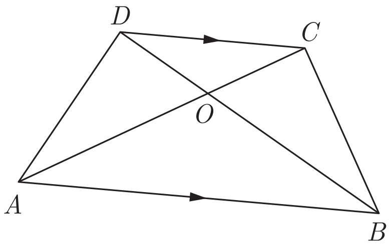
$$ar(\Delta ACF) = 2ar(\Delta BEC)$$

$$ar(\Delta BEC) = \frac{1}{2}ar(\Delta ACF)$$ Hence Proved.

4. In a trapezium $ABCD$, diagonals AC and BD intersect at O. I $AB = 3DC$, then find ratio of areas of triangles COD and AOB.

Ans : [Board Term-1, 2015, Set-FHN8MGD]

As per given condition we have drawn the figure below.



because of AA similarity we have

$$\Delta AOB \sim \Delta COD$$

$$\frac{ar(\Delta COD)}{ar(\Delta AOB)} = \frac{CD^2}{AB^2}$$

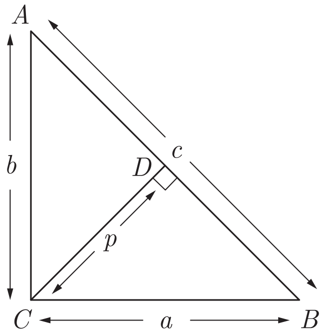
$$= \frac{CD^2}{(3CD)^2} = \frac{CD^2}{9CD^2} = \frac{1}{9}$$

ratio $= 1:9$

5. ΔABC is right angled at C. If p is the length of the perpendicular from C to AB and a, b, c are the lengths of the sides opposite $\angle A, \angle B$ and $\angle C$ respectively, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Ans : [Board Term-1, 2016, Set-O4YP6G7]

As per given condition we have drawn the figure below.



In ΔACB and ΔCDB

$$\angle ABC = \angle CDB = 90^0$$

$$\angle B = \angle B \quad \text{(common)}$$

Because of AA similarity we have

$$\Delta ABC \sim \Delta CDB$$

Now $$\frac{b}{p} = \frac{c}{a}$$

$$\frac{1}{p} = \frac{c}{ab}$$

$$\frac{1}{p^2} = \frac{c^2}{a^2b^2}$$

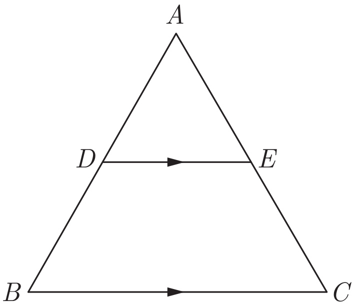
$$\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2}$$

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$ Hence Proved

6. In $\Delta ABC, DE || BC$. If $AD = x+2, DB = 3x+16, AE = x$ and $EC = 3x+5$, them find x.

Ans : [Board Term-1, 2015, Set-DDE-E]

As per given condition we have drawn the figure below.



In the give figure

$$DE \parallel BC$$

By BPT we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

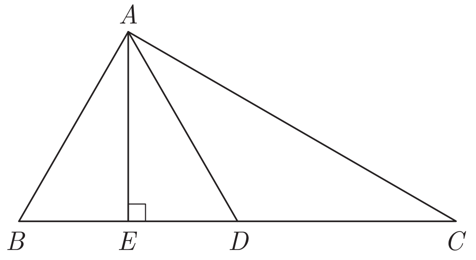
$$\frac{x+2}{3x+16} = \frac{x}{x3+5}$$

$$\begin{aligned}(x+2)(3x+5) &= x(3x+16)\\ 3x^2+5x+6x+10 &= 3x^2+16x\\ 11x+10 &= 16x\\ 11x+10 &= 10\\ 5x &= 10\\ x &= 2\end{aligned}$$

7. If in $\Delta\,ABC$, AD is median and $AE \perp BC$, then prove that $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$.

Ans : [Board Term-1, 2015, Set-DEE-E]

As per given condition we have drawn the figure below.



In $\Delta\,ABE$, using Pythagoras theorem we have

$$\begin{aligned}AB^2 &= AE^2 + BE^2\\ &= AD^2 - DE^2 + (BD - DE)^2\\ &= AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE\\ &= AD^2 + BD^2 - 2BD \times DE \qquad ...(1)\end{aligned}$$

In $\Delta\,AEC$,

$$\begin{aligned}AC^2 &= AE^2 + EC^2\\ &= (AD^2 - ED^2) + (ED + DC)^2\\ &= AD^2 - ED^2 + ED^2 + DC^2 + 2ED \times DC\\ &= AD^2 + CD^2 + 2ED \times CD\\ &= AD^2 + DC^2 + 2DC \times DE \qquad ...(2)\end{aligned}$$

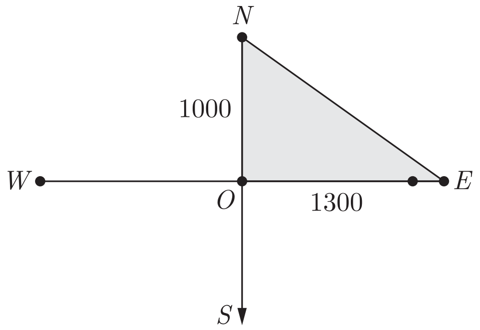
Adding equation (1) and (2) we have

$$\begin{aligned}AB^2 + AC^2 &= 2(AD^2 + BD^2) \qquad (BD = DC)\\ &= \left[2AD^2 + 2\left(\tfrac{1}{2}BC\right)^2\right] \qquad (\; BD = \tfrac{1}{2}BC)\\ &= 2AD^2 + \frac{1}{2}BC^2 \qquad \text{Hence Proves}\end{aligned}$$

8. From an airport, two aeroplanes start at the same time. If speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h then find the distance between the two aeroplanes after 2 hours.

Ans : [Board Term-1, 2015, Set-DDE-E]

As per given condition we have drawn the figure below.



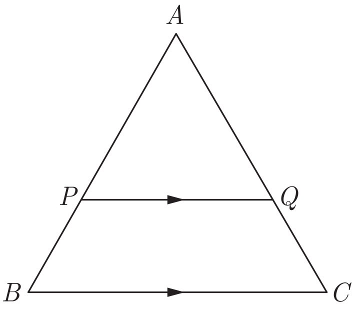
Distance covered by first aeroplane due North after two hours = 500 × 2 = 1,000 km.
Distance covered by second aeroplane due East after two hours = 650 × 2 = 1,300 km.
Distance between two aeroplane after 2 hours

$$\begin{aligned}NE &= \sqrt{ON^2 + OE^2}\\ &= \sqrt{(1000)^2 + (1300)^2}\\ &= \sqrt{1000000 + 1690000}\\ &= \sqrt{2690000}\\ &= 1640.12 \text{ km}\end{aligned}$$

9. ABC is a triangle, PQ is the line segment intersecting AB in P and AC in Q such that $PQ \,||\, BC$ and divides $\Delta\,ABC$ into two parts, equal in area, find $BP{:}\,AB$,

Ans : [Board Term-1, 2012, LK-59]

As per given condition we have drawn the figure below.



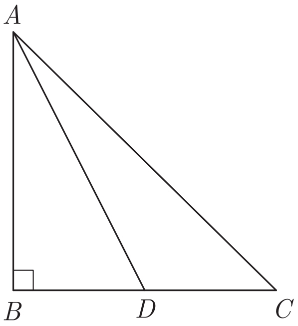
Here, Since $PQ \,||\, BC$ and PQ divides $\Delta\,ABC$ into two equal parts, thus $\Delta\,APQ \sim \Delta\,ABC$

Now $$\frac{ar(\Delta\,APQ)}{ar(\Delta\,ABC)} = \frac{AP^2}{AB^2}$$

$$\begin{aligned}\frac{1}{2} &= \frac{AP^2}{AB^2}\\ \frac{1}{\sqrt{2}} &= \frac{AP}{AB}\\ \frac{1}{\sqrt{2}} &= \frac{AB - BP}{AB} \qquad (AB = AP + BP)\\ \frac{1}{\sqrt{2}} &= 1 - \frac{BP}{AB}\\ \frac{BP}{AB} &= 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}\\ BP : AB &= (\sqrt{2} - 1) : \sqrt{2}\end{aligned}$$

10. In the given figure, ABC is a right angled triangle, $\angle B = 90^\circ$. D is the mid-point of BC. Show that

$AC^2 = AD^2 + 3CD^2$.



Ans : [Board Term-1, 2016, Set-ORDAWEZ 2011, Set-60]

We have $$BD = CD = \frac{BC}{2}$$

$$BC = 2BD$$

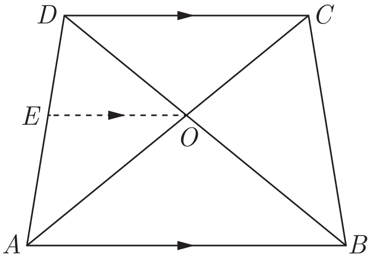
Using Pythagoras theorem in the right ΔABC, we have

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= AB^2 + 4BD^2 \\ &= (AB^2 + BD^2) + 3BD^2 \\ AC^2 &= AD^2 + 3CD^2 \end{aligned}$$

11. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

Ans : [Board Term-1, 2011, Set-39]

As per given condition we have drawn the figure below.



We have drawn $EO \| AB$ on DA.

In quadrilateral $ABCD$,

$$\frac{AO}{BO} = \frac{CO}{DO} \qquad \text{(Given)}$$

or, $$\frac{AO}{CO} = \frac{BO}{DO} \qquad ...(1)$$

In ΔABD, $EO \| AB$

By BPT we have

$$\frac{AE}{ED} = \frac{BO}{DO} \qquad ...(2)$$

From equation (1) and (2), we get

$$\frac{AE}{ED} = \frac{AO}{CO}$$

In ΔADC, $$\frac{AE}{ED} = \frac{AO}{CO}$$

$$EO \| DC \qquad \text{(Converse of BPT)}$$

$$EO \| AB \qquad \text{(Construction)}$$

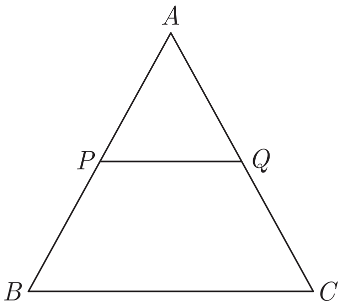
$$AB \| DC$$

Thus in quadrilateral $ABCD$ we have

$$AB = DC$$

Thus $ABCD$ is a trapezium. Hence Proved

12. In the given figure, P and Q are the points on the sides AB and AC respectively of ΔABC, such that $AP = 3.5$cm, $PB = 7$ cm, $AQ = 3$ cm and $QC = 6$ cm. If $PQ = 4.5$ cm, find BC.



Ans : [Board Term-1, 2011, Set-40]

We have $$\frac{AP}{AB} = \frac{3.5}{10.5} = \frac{1}{3}$$

$$\frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$$

In ΔABC, $\frac{AP}{AB} = \frac{AQ}{AC}$ and $\angle A$ is common.

Thus due to SAS we have

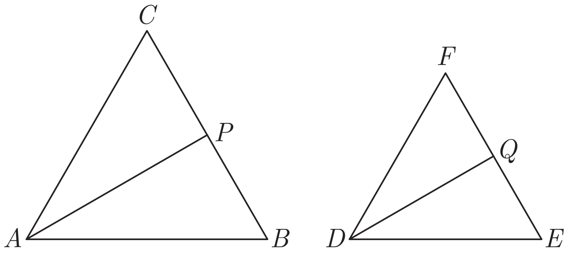
$$\Delta APQ \sim \Delta ABC$$

$$\frac{AP}{AB} = \frac{PQ}{BC}$$

$$\frac{1}{3} = \frac{4.5}{BC}$$

$$BC = 13.5 \text{ cm.}$$

13. In given figure $\Delta ABC \sim \Delta DEF$. AP bisects $\angle CAB$ and DQ bisects $\angle FDE$.



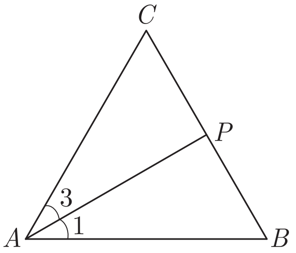
Prove that :

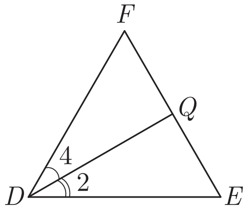
(1) $\frac{AP}{DQ} = \frac{AB}{DE}$

(2) $\Delta CAP \sim \Delta FDQ$.

Ans : [Board Term-1, 2016, Set-LGRKEGO]

As per given condition we have redrawn the figure below.





(1) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D \quad \text{(Corresponding angles)}$$

$$2\angle 1 = 2\angle 2$$

Also $\angle B = \angle E$ (Corresponding angles)

$$\frac{AP}{DQ} = \frac{AB}{DE} \qquad \text{Hence Proved}$$

(2) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D$$

and $\angle C = \angle F$

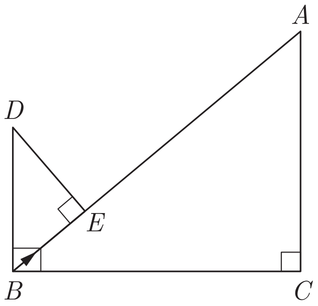
$$2\angle 3 = 2\angle 4$$

$$\angle 3 = \angle 4$$

By AA similarity we have

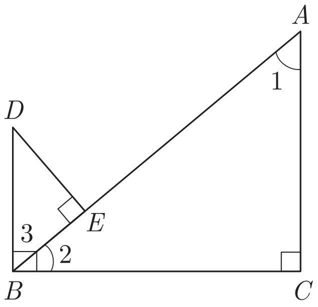
$$\Delta CAP \sim \Delta FDQ$$

14. In the given figure, $DB \perp BC, DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.



Ans : [Board Term-1, 2011, Set-40]

As per given condition we have redrawn the figure below.



We have $DB \perp BC, DE \perp AB$ and $AC \perp BC$.

In ΔABC,

$$\angle 1 + \angle 2 = 90^{\circ} \qquad [\angle C = 90^{\circ}]$$

But we have been given

$$\angle 2 + \angle 3 = 90^{\circ}$$

Hence $\angle 1 = \angle 3$

In ΔABC and ΔBDE,

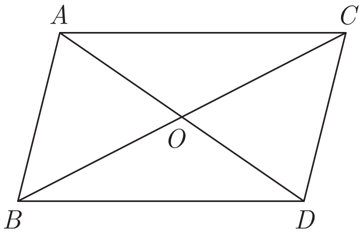
$$\angle 1 = \angle 3 \qquad \text{(Proved)}$$

$$\angle ACB = \angle DEB = 90^{\circ} \qquad \text{(Given)}$$

$$\Delta ABC \sim \Delta BDE \qquad \text{(By } AA \text{ Similarity)}$$

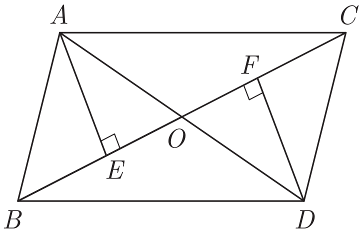
Thus $\frac{AC}{BC} = \frac{BE}{DE}$. Hence Proved

15. In the given figure, ΔABC and ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.



Ans : [Term-1, 2011, Set-40], [Term-1, 2016, ORDAWEZ]

As per given condition we have redrawn the figure below. Here we have drawn $AE \perp BC$ and $DF \perp BC$.



In ΔAOE and ΔDOF,

$$\angle AOE = \angle DOF$$

(Vertically opposite angles)

$$\angle AEO = \angle DFO = 90^{\circ} \quad \text{(Construction)}$$

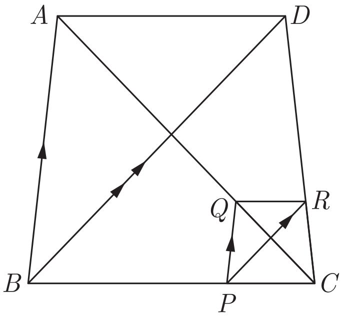
or, $\Delta AOE \sim \Delta DOF$ (By AA similarity)

Thus $\frac{AO}{DO} = \frac{AE}{DF}$ ...(1)

Now, $$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF}$$

$$= \frac{AE}{DF} = \frac{AO}{DO} \text{From equation (1)}$$

16. In the given figure, two triangles ABC and DBC lie on the same side of BC such that $PQ || BA$ and $PR || BD$. Prove that $QR || AD$.



Ans : [Boar term-1, 2011, Set-21]

In ΔABC, we have $PQ || AB$ and $PR || BD$

$$\frac{BP}{PC} = \frac{AQ}{QC} \qquad \text{(by BPT) ...(1)}$$

Again in ΔBCD, we have

$$PR \; || \; BD$$

$$\frac{BP}{PC} = \frac{DR}{RC} \qquad \text{(by BPT) ...(2)}$$

$$\frac{AQ}{QC} = \frac{DR}{RC}$$

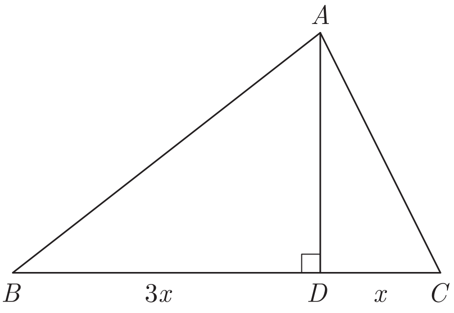
$$PR \; || \; AD \qquad \text{(By converse of BPT)}$$

Hence proved

17. The perpendicular AD on the base BC of a ΔABC intersects BC at D so that $DB = 3CD$. Prove that $2(AB)^2 = 2(AC)^2 + BC^2$.

Ans : [Board Term-1, 2011, Set-44, 60, 2012, 2016 Set-39, Set-NH3]

As per given condition we have drawn the figure below.



In ΔADB, we have

$$AB^2 = AD^2 + BD^2 \qquad \text{...(1)}$$

(Pythagoras Theorem)

In ΔADC, $\quad AC^2 = AD^2 + CD^2 \qquad$ ...(2)

(Pythagoras theorem)

Subtracting eqn. (2) from eqn. (1), we get

$$AB^2 - AC^2 = BD^2 - CD^2$$

$$= \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$$

$$= \frac{9}{16}BC^2 - \frac{1}{16}BC^2 = \frac{BC^2}{2}$$

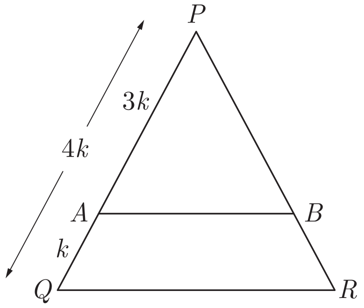
$$2(AB^2 - AC^2) = BC^2$$

$$2(AB)^2 = 2AC^2 + BC^2. \qquad \text{Hence Proved}$$

18. In the given figure, $\frac{PA}{AQ} = \frac{BR}{BR} = 3$. If the area of ΔPQR is $32\,cm^2$, then find the area of the quadrilateral $AQRB$.

Ans : [Board Term-1, 2011, Set-44]

As per given condition we have drawn the figure below.



Since $\angle P$ common and $\frac{PA}{AQ} = \frac{PB}{BR}$, therefore

We have, $\Delta PQR \sim \Delta PAB$

$$\frac{area(\Delta PQR)}{area(\Delta PAB)} = \frac{PQ^2}{PA^2}$$

$$\frac{32}{are(\Delta PAB)} = \frac{(4k)^2}{(3k^2)} = \frac{16k^2}{9k^2}$$

$$\text{area}\Delta PAB = 18\,cm^2$$

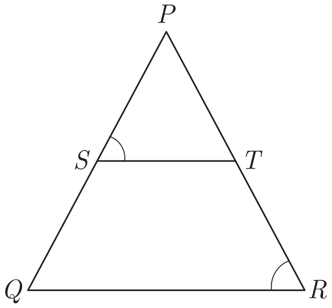
Thus area of quadrilateral $AQRB$,

$$= area\ of\ \Delta PQR - area\ of\ \Delta PAB$$

$$= 32 - 18$$

$$= 14\,cm^2$$

19. In the given figure, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.



Ans : [Board Term-1, 2011, Set-74]

We have $\qquad \frac{PS}{SQ} = \frac{PT}{TR}$

and $\qquad \angle PST = \angle PRQ$

By converse of BPT,

$$ST \; || \; QR$$

$$\angle PST = \angle PQR$$

(Corresponding angles)

and $\qquad \angle PST = \angle PRQ$

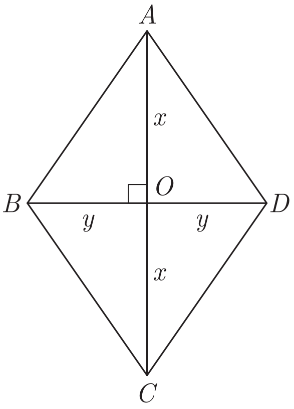
Thus $\qquad \angle PQR = \angle PRQ$

So, ΔPQR is an isosceles triangle. Hence Proved

20. Prove that the sum of squares on the sides of a rhombus is equal to sum of squares of its diagonals.

Ans : [Board Term-1, 2011, Set-21]

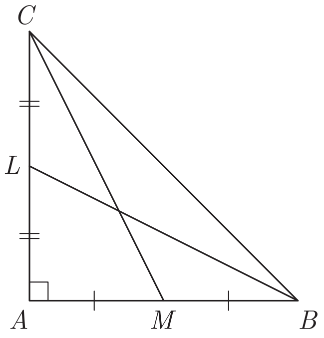
Let, $ABCD$ is a rhombus and since diagonals of a rhombus bisect each other at 90°.



$$\begin{aligned} \text{Now} \quad AO &= OC \Rightarrow AO^2 = OC^2 \\ BO &= OD \Rightarrow BO^2 = OD^2 \\ \text{and} \quad \angle AOB &= 90^0 \\ AB^2 &= OA^2 + BO^2 \\ \text{Similarly,} \quad AD^2 &= x^2 + y^2 = BC^2 = CD^2 \\ AB^2 + BC^2 + CD^2 + DA^2 &= 4AO^2 + 4DO^2 \\ &= (2AO)^2 + (2DO)^2 \\ &= (2x)^2 + (2y)^2 \\ AB^2 + BC^2 + CD^2 + AD^2 &= AC^2 + BD^2 \end{aligned}$$

Hence Proved

21. In the given figure, BL and CM are medians of ΔABC, fight angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$.



Ans : [Board Term-1, 2011, Set-74]

We have a right angled triangle ΔABC at A where BL and CM are medians.

$$\begin{aligned} \text{In } \Delta ABL, \quad BL^2 &= AB^2 + AL^2 \\ &= AB^2 + \left(\frac{AC}{2}\right)^2 \quad (BL \text{ is median}) \\ \text{In } \Delta ACM, \quad CM^2 &= AC^2 + AM^2 \\ &= AC^2 + \left(\frac{AB}{2}\right)^2 \quad (CM \text{ is median}) \end{aligned}$$

Now

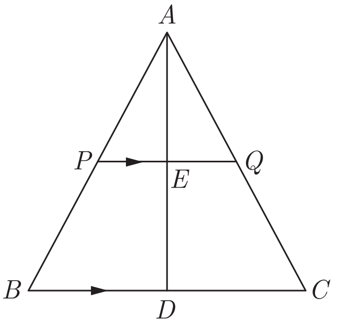
$$\begin{aligned} BL^2 + CM^2 &= AB^2 + AC^2 + \frac{AC^2}{4} + \frac{AB^2}{4} \\ 4(BL^2 + CM^2) &= 5AB^2 + 5AC^2 \\ &= 5(AB^2 + AC^2) \\ &= 5BC^2. \end{aligned}$$

Hence Proved

22. In a ΔABC, let P and Q be points on AB and AC respectively such that $PQ || BC$. Prove that the median AD bisects PQ.

Ans : [Board Term-1, 2011, Set-70]

As per given condition we have drawn the figure below.



The median AD intersects PQ at E.

$$\begin{aligned} \text{We have,} \quad PQ &\parallel \text{BE} \\ \text{or,} \quad \angle ApE &= \angle B \text{ and } \angle AQE = \angle C \end{aligned}$$

(Corresponding angles)

Thus in ΔAPE and ΔABD we have

$$\begin{aligned} \angle APE &= \angle ABD \\ \angle PAE &= \angle BAD \\ \text{Thus} \quad \Delta APE &\sim \Delta ABD \\ \frac{PE}{BD} &= \frac{AE}{AD} \qquad \ldots(1) \\ \text{Similarly,} \quad \Delta AQE &\sim \Delta ACD \\ \text{or,} \quad \frac{QE}{CD} &= \frac{AE}{AD} \qquad \ldots(2) \end{aligned}$$

From eqns. (1) and (2),

$$\frac{PE}{BD} = \frac{QE}{CD}$$

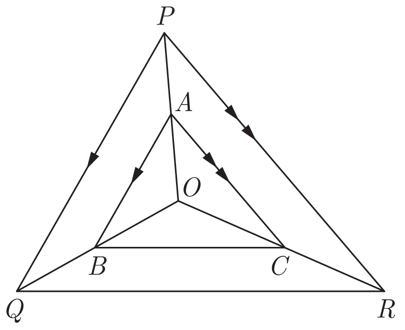
As $CD = BD$, we get

$$\frac{PE}{BD} = \frac{QE}{BD}$$

$$\text{or,} \quad PE = QE$$

Hence, AD bisects PQ.

23. In the given figure A, B and C are points on OP, OQ and OR respectively such that $AB || PQ$ and $AC || PR$. Prove that $BC || QR$.



Ans : [Board Term-1, 2012, Set-66]

In ΔPOQ, $AB \parallel PQ$

By BPT $\dfrac{AO}{AP} = \dfrac{OB}{BQ}$ ...(1)

In ΔOPR, $AC \parallel PR$,

By BPT $\dfrac{OA}{AP} = \dfrac{OC}{CR}$ (2)

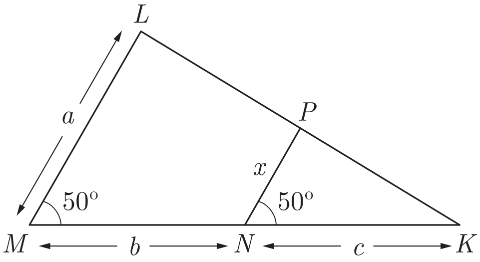
From equations (1) and (2), we have

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

(By converse of BPT)

$BC \parallel QR$ Hence Proved

24. In the given figure, find the value of x in terms of a, b and c.



Ans : [Board Term-1, 2012, Set-52]

In triangles LMK and PNK,

$\angle M = \angle N = 50°$ (Given)

$\angle K = \angle K$ (Common)

Due to AA similarity,

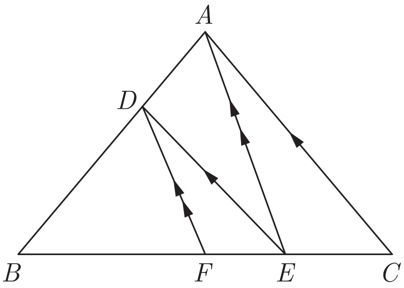
$$\Delta LMK \sim \Delta PNK$$

$$\frac{LM}{PN} = \frac{KM}{KN}$$

$$\frac{a}{x} = \frac{b+c}{c}$$

$$x = \frac{ac}{b+c}$$

25. In the given figure, $DE \,||\, AC$ and $DF \,||\, AE$. Prove that $\dfrac{BE}{FE} = \dfrac{BE}{EC}$.



Ans : [Board Term-1, 2012 Set-66]

In ΔABC, $DE \parallel AC$, (Given)

By BPT $\dfrac{BD}{DA} = \dfrac{BE}{EC}$ ...(1)

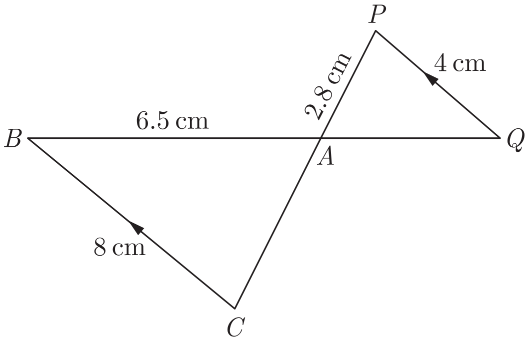
In ΔABE, $DF \parallel AE$, (Given)

By BPT $\dfrac{BD}{DA} = \dfrac{BF}{FE}$ ...(2)

From (1) and (2), we have

$\dfrac{BF}{FE} = \dfrac{BE}{EC}$. Hence proved

26. In the given figure, $BC \,||\, PQ$ and $BC = 8cm$, $PQ = 4cm$, $BA = 6.5\,cm$, $AP = 2.8cm$. Find CA and AQ.



Ans : [Board Term-1, 2012, Set-66]

In ΔABC and ΔAPQ, $AB = 6.5$ cm, $BC = 8$ cm, $PQ = 4$ cm and $AP = 2.8$ cm

$BC \parallel PQ$ (Given)

Due to alternate angles

$$\angle CBA = \angle AQP$$

Due to vertically opposite angles,

$$\angle BAC = \angle PAQ$$

Due to AA similarity

$$\Delta ABC \sim \Delta AQP$$

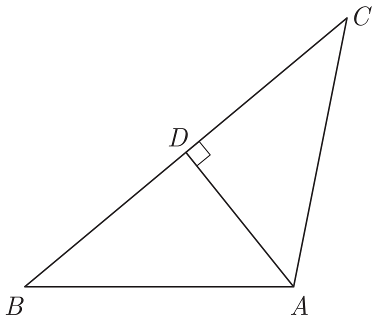
$$\frac{AB}{AQ} = \frac{BC}{QP} = \frac{AC}{AP}$$

$$\frac{6.5}{AQ} = \frac{8}{4} = \frac{AC}{AP}$$

$$AQ = \frac{6.5}{2} = 3.25 \text{ cm}$$

$$AC = 2 \times 2.5 = 5.6 \text{ cm}$$

27. In the given figure, if $AD \perp BC$, prove that $AB^2 + CD^2 = BD^2 + AC^2$.



In right angled ΔADC,

$AC^2 = AD^2 + CD^2$ ...(1)

In right $\Delta\, ADB$,

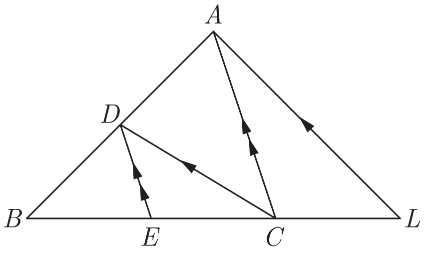
$$AB^2 = AD^2 + BD^2 \qquad ...(2)$$

Subtracting eqn. (1) from eqn. (2)

$$AB^2 - AC^2 = BD^2 - CD^2$$

$$AB^2 + CD^2 = AC^2 + BD^2. \qquad \text{Hence proved.}$$

28. In the given figure, $CD \,||\, LA$ and $DE \,||\, AC$. Find the length of CL, if $BE = 4$ cm and $EC = 2$ cm.



Ans : [Board Term-1, 2012, Set-39]

In $\Delta\, ABC$,, $DE \,||\, AC$, $BE = 4$ cm and $EC = 2$cm

By BPT $\dfrac{BD}{DA} = \dfrac{BE}{EC}$ ...(1)

In $\Delta\, ABL$, $DC \,||\, AL$

By BPT $\dfrac{BD}{DA} = \dfrac{BC}{CL}$ ...(2)

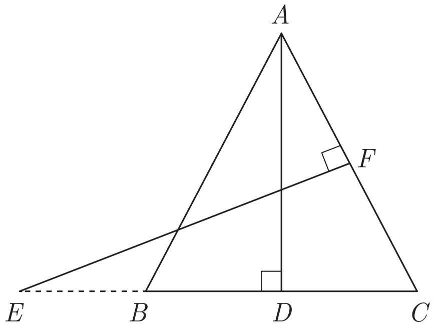
From equations (1) and (2),

$$\frac{BE}{EC} = \frac{BC}{CL}$$

$$\frac{4}{2} = \frac{6}{CL}$$

$$CL = 3 \text{ cm}$$

29. In the given figure, $AB = AC$. E is a point on CB produced. If AD is perpendicular to BC and EF perpendicular to AC. Prove that $\Delta\, ABD$ is similar to $\Delta\, CEF$.



Ans : [Board Term-1, 2012, Set-60]

In $\Delta\, ABD$ and $\Delta\, CEF$, we have

$$AB = AC$$

Thus $\angle ABC = \angle ACB$

$$\angle ABD = \angle ECF$$

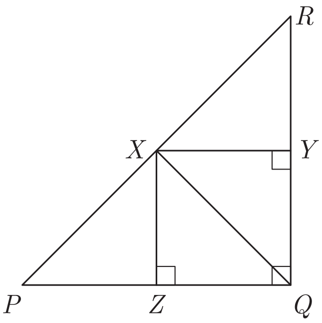
$$\angle ADB = \angle EFC \qquad \text{(each } 90^\circ\text{)}$$

Due to AA similarity

$$\Delta\, ABD \sim \Delta\, ECF \qquad \text{Hence proved}$$

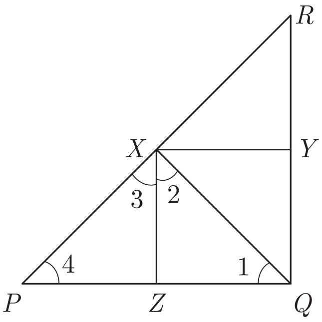
LONG ANSWER TYPE QUESTIONS

1. $\Delta\, PQR$ is right angled at Q. $Q.QX \perp PR, XY \perp RQ$ and $XZ \perp PQ$ are drawn. Prove that $XZ^2 = PZ \times ZQ$.



Ans : [Board Term-1, 2015, Set-MV98HN3]

We have redrawn the given figure as below.



It may be easily seen that $RQ \perp PQ$

and $XZ \perp PQ$ or $XZ \,||\, YQ$

Similarly $XY \,||\, ZQ$

Thus $XYQZ$ is a rectangle.

In $\Delta\, XZQ$, $\angle 1 + \angle 2 = 90^\circ$ ...(1)

and in $\Delta\, PZX$, $\angle 3 + \angle 4 = 90^\circ$ ...(2)

$XQ \perp PR$ or, $\angle 2 + \angle 3 = 90^\circ$ ...(3)

From eq. (1) and (3), $\angle 1 = \angle 3$

From eq. (2) and (3), $\angle 2 = \angle 4$

Due to AA similarity

$$\Delta\, PZX \sim \Delta\, XZQ$$

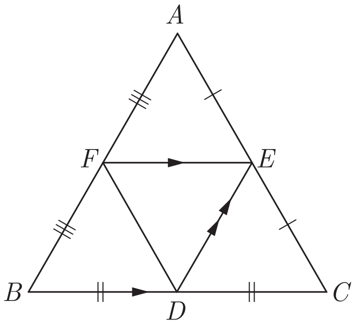
$$\frac{PZ}{XZ} = \frac{XZ}{ZQ}$$

$$XZ^2 = PZ \times ZQ \qquad \text{Hence proved}$$

2. In $\Delta\, ABC$, the mid-points of sides BC, CA and AB are D, E and F respectively. Find ratio of $ar(\Delta\, DEF)$ to $ar(\Delta\, ABC.)$

Ans : [Board Term-1, 2015, Set-DDE-M]

As per given condition we have drawn the figure below. Here F, E and D are the mid-points of AB, AC and BC respectively.



Hence, $FE || BC, DE || AB$ and $DF || AC$
By mid-point theorem,

If $\quad DE \parallel BA$ then $DE \parallel BF$

and if $\quad FE \parallel BC$ then $FE \parallel BD$

Therefore $FEDB$ is a parallelogram in which DF is diagonal and a diagonal of Parallelogram divides it into two equal Areas.

$$\text{Hence} \quad ar(\Delta BDF) = ar(\Delta DEF) \qquad ...(1)$$

$$\text{Similarly } ar(\Delta CDE) = ar(\Delta DEF) \qquad ...(2)$$

$$(\Delta AFE) = ar(\Delta DEF) \qquad ...(3)$$

$$(\Delta DEF) = ar(\Delta DEF) \qquad ...(4)$$

Adding equation (1), (2), (3) and (4), we have

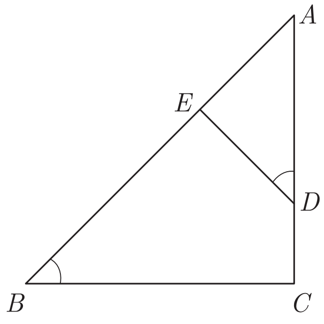
$$ar(\Delta BDF) + ar(\Delta CDE) + ar(\Delta AFE) + ar(\Delta DEF)$$

$$= 4ar(\Delta DEF)$$

$$ar(\Delta ABC) = 4ar(\Delta DEF)$$

$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

3. In ΔABC, if $\angle ADE = \angle B$, then prove that $\Delta ADE \sim \Delta ABC$.
Also, if $AD = 7.6$ cm, $AE = 7.2$ cm, $BE = 4.2$ cm and $BC = 8.4$ cm, then find DE.



Ans : [Board Term-1, 2015, WJQZQBN]

In ΔADE and ΔABC, $\angle A$ is common

$$\text{and} \quad \angle ADE = \angle ABC \qquad \text{(Given)}$$

Due to AA similarity

$$\Delta ADE \sim \Delta ABC$$

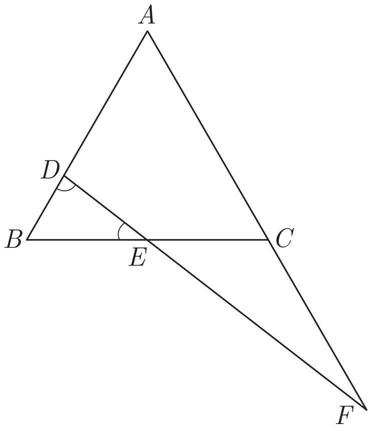
$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{AD}{AE + BE} = \frac{DE}{BC}$$

$$\frac{7.6}{4.2 + 4.2} = \frac{DE}{8.4}$$

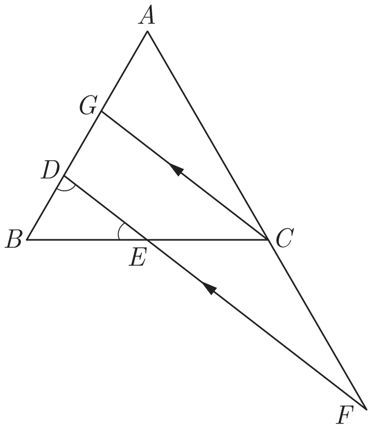
$$DE = \frac{7.6 \times 8.4}{11.4} = 5.6 \text{ cm}$$

4. In the figure, $\angle BED = \angle BDE$ and E is the mid-point of BC. Prove that $\frac{AF}{CF} = \frac{AD}{BE}$.



Ans :

We have redrawn the given figure as below. Here $CG || FD$.



$$\text{We have} \quad \angle BED = \angle BDE$$

Since E is mid-point of BC,

$$\text{or,} \quad BE = BD = EC \qquad ...(1)$$

$$\text{In } \Delta BCG, \quad DE \parallel FG$$

$$\frac{BD}{DG} = \frac{BE}{EC} = 1 \qquad \text{(from (1))}$$

$$BD = DG = EC = BE \qquad \text{[using (1)]}$$

$$\text{In } \Delta ADF, \quad CG \parallel FD$$

$$\frac{AG}{GD} = \frac{AC}{CF} \qquad \text{(By BPT)}$$

$$\frac{AG + GD}{GD} = \frac{AF + CF}{CF}$$

$$, \quad \frac{AD}{GD} = \frac{AF}{CF}$$

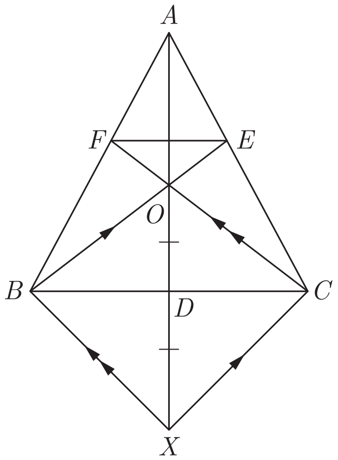
$$\text{Thus} \quad \frac{AF}{CF} = \frac{AD}{BE} \qquad \text{(using (1))}$$

5. In $\Delta ABC, AD$ is a median and O is any point on AD. BO and CO on producing meet AC and AB at E and F respectively. Now AD is produced to X such that $OD = DX$ as shown in figure.

Prove that :

(1) $EF || BC$

(2) $AO : AX = AF : AB$



Ans : [board Term-1, 2015, Set-O4YP6G7]

Since BC and OX bisect each other, $BXCO$ is a parallelogram. Therefore $BE || XC$ and $BX || CF$.

In ΔABX, by BPT we get,

$$\frac{AF}{FB} = \frac{AO}{OX} \quad ..(1)$$

In ΔAXC,
$$\frac{AE}{EC} = \frac{AO}{OX} \quad ...(2)$$

From (1) and (2) we get

$$\frac{AF}{FB} = \frac{AE}{EC}$$

By converse of BPT we have

$$EF \ || \ BC$$

From (1) we get
$$\frac{OX}{OA} = \frac{FB}{AF}$$

$$\frac{OX + OA}{OA} = \frac{FB + AF}{AF}$$

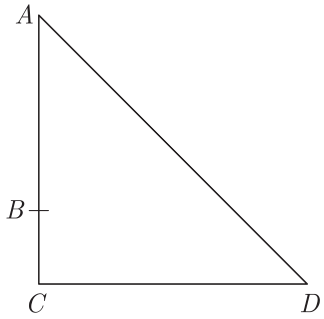
$$\frac{AX}{OA} = \frac{AB}{AF}$$

$$\frac{AO}{AX} = \frac{AF}{AB}$$

Thus $AO : AX = AF : AB$ Hence Proved

6. In the right triangle, B is a point on AC such that $AB + AD = BC + CD$. If $AB = x, BC = h$ and $CD = d$, then find x (in term of h and d).

Ans : [Board Term-1, 2015, Set-FHN8MGD]



We have
$$AB + AD = BC + CD$$

$$AD = BC + CD - AB$$

$$AD = h + d - x$$

In right angled triangle ΔACD,

$$AD^2 = AC^2 + DC^2$$

$$(h + d - x)^2 = (x + h)^2 + d^2$$

$$(h + d - x)^2 - (x + h)^2 = d^2$$

$$(h + d - x - x - h)(h + d - x + x + h) = d^2$$

$$(d - 2x)(2h + d) = d^2$$

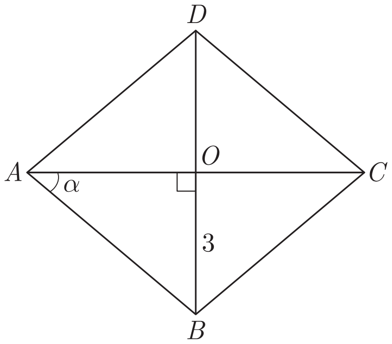
$$2hd + d^2 - 4hx - 2xd = d^2$$

$$2hd = 4hx + 2xd$$

$$= 2(2h + d)x$$

or,
$$x = \frac{hd}{2h + d}$$

7. $ABCD$ is a rhombus whose diagonal AC makes an angle α with AB. If $\cos\alpha = \frac{2}{3}$ and $OB = 3$ cm, find the length of its diagonals AC and BD.



Ans : [Board Term-1, 2013, Set FFC]

We have
$$\cos\alpha = \frac{2}{3} \text{ and } OB = 3 \text{ cm}$$

In ΔAOB,
$$\cos\alpha = \frac{2}{3} = \frac{AO}{AB}$$

Let $OA = 2x$ then $AB = 3x$

Now in right angled triangle ΔAOB we have

$$AB^2 = AO^2 + OB^2$$

$$(3x)^2 = (2x)^2 + (3)^2$$

$$9x^2 = 4x^2 + 9$$

$$5x^2 = 9$$

$$x = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}}$$

Hence,
$$OA = 2x = 2\left(\frac{3}{\sqrt{5}}\right) = \frac{6}{\sqrt{5}} \text{ cm}$$

and
$$AB = 3x = 3\left(\frac{3}{\sqrt{5}}\right) = \frac{9}{\sqrt{5}} \text{ cm}$$

Diagonal
$$BD = 2 \times OB = 2 \times 3 = 6 \text{ cm}$$

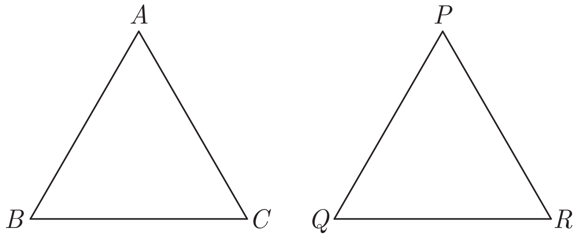
and
$$AC = 2AO$$

$$= 2 \times \frac{6}{\sqrt{5}} = \frac{12}{\sqrt{5}} \text{ cm}$$

8. If the area of two similar triangles are equal, prove that they are congruent.

Ans : [Board Term-1, 2012, Set-35]

As per given condition we have drawn the figure below.



We have $\Delta ABC \sim \Delta PQR$,

and $ar\Delta ABC = ar\Delta PQR$

Since $\Delta ABC \sim \Delta PQR$, we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} \quad ...(1)$$

Since $ar(\Delta ABC) = ar(\Delta PQR)$ we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = 1$$

From equation (1), we get

$$\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} = 1$$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = 1$$

$$AB = PQ,$$

$$BC = QR$$

and $CA = RA$

By SSS similarity we have

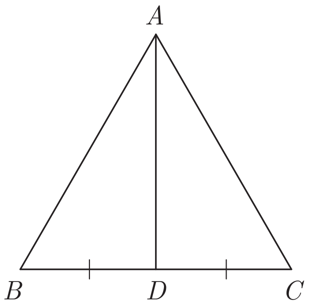
$$\Delta ABC \cong \Delta PQR$$

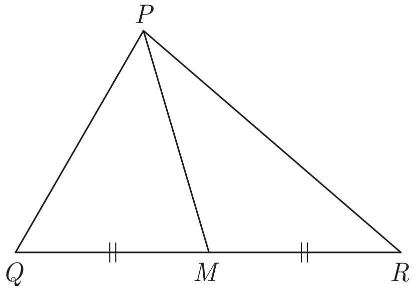
9. In ΔABC, AD is the median to BC and in ΔPQR, PM is the median to QR. If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$. Prove that $\Delta ABC \sim \Delta PQR$.

Prove that $\Delta ABC \sim \Delta PQR$.

Ans : [Board Term-1, 2013 FFC; 2012, SEt-48]

As per given condition we have drawn the figure below.





In ΔABC AD is the median, therefore

$$BC = 2BD$$

and in ΔPQR, PM is the median,

$$QR = 2QM$$

Given, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$$

or, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2BD}{2QM}$$

In triangles ABD and PQM,

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$$

By SSS similarity we have

$$\Delta ABD \sim \Delta PQM$$

By CPST we have

$$\angle B = \angle Q,$$

In ΔABC and ΔPQR,

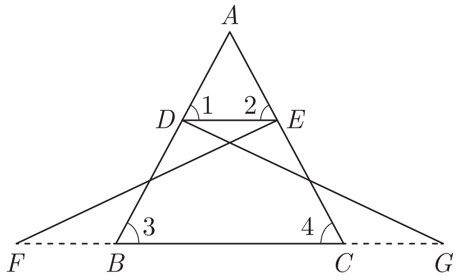
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

By SAS similarity we have

$$\angle B = \angle Q,$$

Thus $\Delta ABC \sim \Delta PQR$. Hence Proved.

10. In the following figure, $\Delta FEC \cong \Delta GBD$ and $\angle 1 = \angle 2$. Prove that $\Delta ADE \cong \Delta ABC$.



Ans : [Board Term-1, 2012, Set-21]

Since $\Delta FEC \cong \Delta GBD$

$$EC = BD \quad ...(1)$$

Since $\angle 1 = \angle 2$, using isosceles triangle property

$$AE = AD \quad ...(2)$$

From equation (1) and (2), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$$DE \parallel BC, \quad \text{(Converse of BPT)}$$

Due to corresponding angles we have

$$\angle 1 = \angle 3 \text{ and } \angle 2 = \angle 4$$

Thus in ΔADE and ΔABC,

$$\angle A = \angle A$$

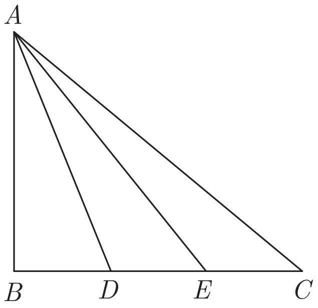
$$\angle 1 = \angle 3$$

$$\angle 2 = \angle 4$$

Sy by AAA criterion of similarity,

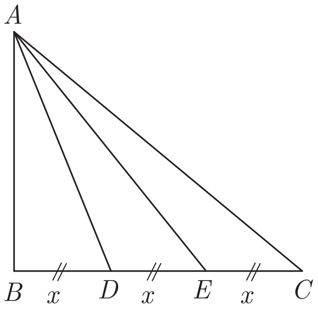
$\Delta ADE \sim \Delta ABC$ Hence proved

11. In the given figure, D and E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.



Ans : [Board Term-1, 2013, LK-59]

As per given condition we have drawn the figure below.



Since D and E trisect BC, let $BD = DE = EC$ be x.

Then $BE = 2x$ and $BC = 3x$

In ΔABE, $AE^2 = AB^2 + BE^2 = AB^2 + 4x^2$ ...(1)

In ΔABC, $AC^2 = AB^2 + BC^2 = AB^2 + 9x^2$ ...(2)

In ΔADB, $AD^2 = AB^2 + BD^2 = AB^2 + x^2$ ...(3)

Multiplying (2) by 3 and (3) by 5 and adding we have

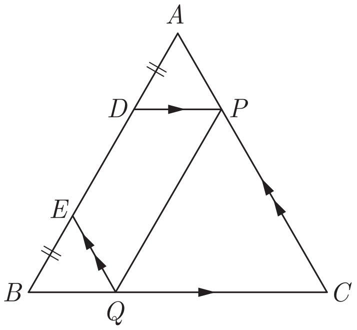
$$\begin{aligned} 3AC^2 + 5AD^2 &= 3(AB^2 + 9x^2) + (AB^2 + x^2) \\ &= 3AB^2 + 27x^2 + 5AB^2 + 5x^2 \\ &= 8AB^2 + 32x^2 \\ &= 8(AB^2 + 4x^2) = 8AE^2 \end{aligned}$$

Thus $3AC^2 + 5AD^2 = 8AE^2$ Hence Proved

12. Let ABC be a triangle D and E be two points on side AB such that $AD = BE$. If $DP \| BC$ and $EQ \| AC$, then prove that $PQ \| AB$.

Ans : [Board Term-1, 2012, Set-44]

As per given condition we have drawn the figure below.



In ΔABC, $DP \| BC$

By BPT we have $\dfrac{AD}{DB} = \dfrac{AP}{PC}$, ...(1)

Similarly, in ΔABC, $EQ \| AC$

$$\frac{BQ}{QC} = \frac{BE}{EA} \quad ...(2)$$

From figure,

$$\begin{aligned} EA &= AD + DE \\ &= BE + ED \qquad (\ BE = AD) \\ &= BD \end{aligned}$$

Therefore equation (2) becomes,

$$\frac{BQ}{QC} = \frac{AD}{BD} \quad ...(3)$$

From (1) and (3), we get

$$\frac{AP}{PC} = \frac{BQ}{QC}$$

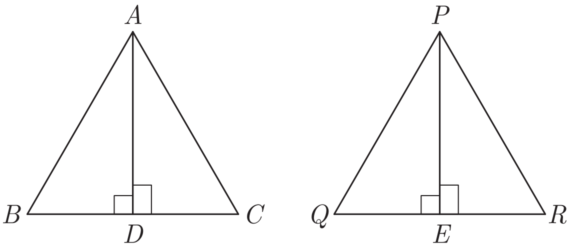
By converse of BPT,

$PQ \| AB$ Hence Proved

13. Prove that ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Ans : [Board Term-1, 2015, FHN8MGD, Sample Paper 2017]

As per given condition we have drawn the figure below. Here $\Delta ABC \sim \Delta PQR$



We have drawn $AD \perp BC$ and $PE \perp QR$

Since $\Delta ABC \sim \Delta PQR$, due to corresponding sides of similar triangles

$$\frac{AB}{PQ} \ \frac{AC}{QR} = \frac{AC}{PR} \quad ...(1)$$

$$\angle B = \angle Q$$

In ΔADB and ΔPEQ,

$\angle B = \angle Q$ (Proved)

$\angle ADB = \angle PEQ$ [each 90°]

$\Delta ADB \sim \Delta PEQ$ (AA Similarity)

Corresponding sides of similar triangle,

$$\frac{AD}{PE} = \frac{AB}{PQ} \quad ...(2)$$

From eq. (1) and eq. (2),

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PE} \quad ...(3)$$

Now, $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PE}$$

$$= \left(\frac{BC}{QR}\right) \times \left(\frac{AD}{PE}\right)$$

$$= \frac{BC}{QR} \times \frac{BC}{QR}$$

From equation (3) we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2} \qquad ...(4)$$

From equation (3) and equation (4) we have

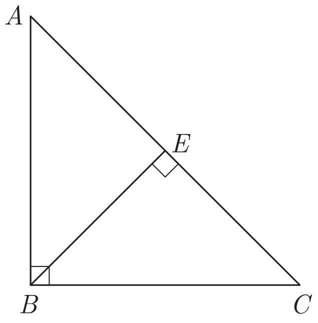
$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

14. Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. Using the above result, prove that, in rhombus $ABCD$, $4AB^2 = AC^2 + BD^2$.

Ans : [Board Term-1, 2015 CJTOQ, [Sample Paper 2017]

(1) As per given condition we have drawn the figure below. Here $AB \perp BC$.

We have drawn $BE \perp AC$



In ΔAEB and ΔABC $\angle A$ common and

$$\angle E = \angle B \qquad \text{(each } 90^\circ)$$

By AA similarity we have

$$\Delta AEB \sim \Delta ABC$$

$$\frac{AE}{AB} = \frac{AB}{AC}$$

$$AB^2 = AE \times AC$$

Now, In ΔCEB and ΔCBA, $\angle C$ common and

$$\angle E = \angle B \qquad \text{(each } 90^\circ)$$

By AA similarity we have

$$\Delta AEB \sim \Delta CBA$$

$$\frac{CE}{BC} = \frac{BC}{AC}$$

$$BC^2 = CE \times AC \qquad ...(2)$$

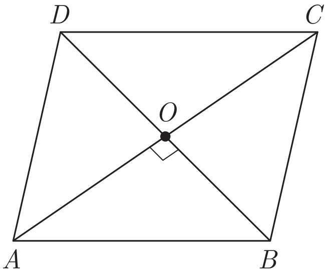
Adding eqns. (1) and (2) we have

$$AB^2 + BC^2 = AE \times AC + CE \times AC$$

$$AB^2 + BC^2 = AC \times AC$$

$$AB^2 + BC^2 = AC^2 \qquad \text{Hence proved}$$

(2) As per given condition we have drawn the figure below. Here $ABCD$ is a rhombus.



We have drawn diagonal AC and BD.

$$AO = OC = \frac{1}{2}AC$$

and

$$BO = OD = \frac{1}{2}BD$$

$$AC \perp BD$$

Since diagonal of rhombus bisect each other at right angle,

$$\angle AOB = 90^\circ$$

$$AB^2 = OA^2 + OB^2$$

$$= \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$$= \frac{AC^2}{4} + \frac{BD^2}{4}$$

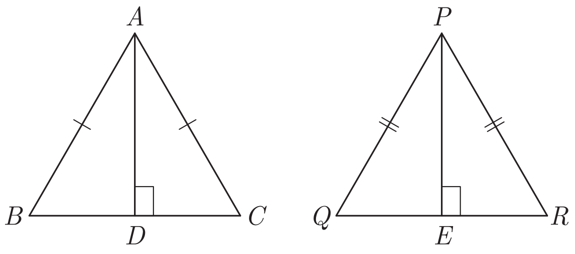
or

$$4AB^2 = AC^2 + BD^2 \qquad \text{Hence proved}$$

15. Vertical angles of two isosceles triangles are equal. If their areas are in the ratio $16:25$, then find the ratio of their altitudes drawn from vertex to the opposite side.

Ans : [Board Term-1, 2015, Set CJTOQ]

As per given condition we have drawn the figure below.



Here

$$\angle A = \angle P$$

$$\angle B = \angle C, \angle Q = \angle R$$

Let $\angle A = \angle P$ be x.

In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$

$$x^2 + \angle B + \angle B = 180^\circ \qquad (\angle B = \angle C)$$

$$2\angle B = 180^\circ - x$$

$$\angle B = \frac{180^\circ - x}{2} \qquad ...(1)$$

Now, in ΔPQR

$$\angle P + \angle Q + \angle R = 180^\circ \qquad (\angle Q = \angle R)$$

$$x^2 + \angle Q + \angle Q = 180^\circ$$

$$2\angle Q = 180^\circ - x$$

$$\angle Q = \frac{180^\circ - x}{2} \qquad ...(2)$$

In ΔABC and ΔPQR,

$$\angle A = \angle P \qquad \text{[Given]}$$
$$\angle B = \angle Q \qquad \text{[From eq. (1) and (2)]}$$

Due to AA similarity,

$$\Delta ABC \sim \Delta PQR$$

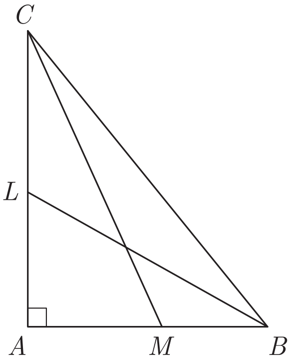
Now $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PE^2}$$

$$\frac{16}{25} = \frac{AD^2}{PE^2}$$

$$\frac{4}{5} = \frac{AD}{PE}$$

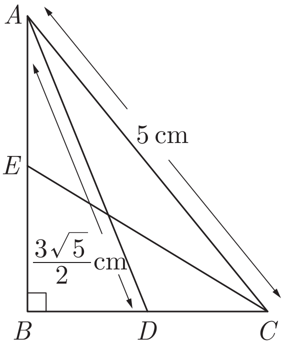
$$\frac{AD}{PE} = \frac{4}{5}$$

16. In the figure, ABC is a right triangle, right angled at B. AD and CE are two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE.



Ans : [Board Term-1, 2013 FFC]

We have redrawn the given figure as below.



Here in ΔABC, $\angle B = 90°$, AD and CE are two medians.

By Pythagoras theorem we get

$$AC^2 = AB^2 + BC^2 = (5)^2 = 25 \qquad \text{...(1)}$$

In ΔABD, $$AD^2 = AB^2 + BD^2$$

$$\left(\frac{3\sqrt{5}}{2}\right)^2 = AB^2 + \frac{BC^2}{4}$$

$$\frac{45}{4} = AB^2 + \frac{BC^2}{4} \qquad \text{...(2)}$$

In ΔEBC, $$CE^2 = BC^2 + \frac{AB^2}{4} \qquad \text{..(3)}$$

Subtracting equation (2) from equation (1),

$$\frac{3BC^2}{4} = 25 - \frac{45}{4} = \frac{55}{4}$$

$$BC^2 = \frac{55}{3} \qquad \text{...(4)}$$

From equation (2) we have

$$AB^2 + \frac{55}{12} = \frac{45}{4}$$

$$AB^2 = \frac{45}{4} - \frac{55}{12} = \frac{20}{3}$$

From equation (3) we get

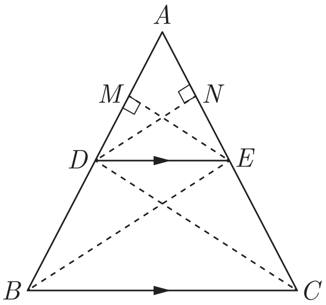
$$CE^2 = \frac{55}{3} + \frac{20}{3 \times 4} = \frac{240}{12} = 20$$

Thus $$CE = \sqrt{20} = 2\sqrt{5} \text{ cm.}$$

17. If a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove it.

Ans : [Board Term-1, 2012 FFC, 2012 Set 15]

A triangle ABC is given in which $DE || BC$. We have drawn $DN \perp AE$ and $EM \perp AD$ as shown below. We have joined BE and CD.



In ΔADE,

$$\text{area } (\Delta ADE) = \frac{1}{2} \times AE \times DN \qquad \text{...(1)}$$

In ΔDEC,

$$\text{area } (\Delta DCE) = \frac{1}{2} \times CE \times DN \qquad \text{...(2)}$$

Dividing eqn. (1) by eqn. (2),

$$\frac{area(\Delta ADE)}{area(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$$

or, $$\frac{area(\Delta ADE)}{area(\Delta DEC)} = \frac{AE}{CE} \qquad \text{...(3)}$$

Now in ΔADE,

$$area(\Delta ADE) = \frac{1}{2} \times AD \times EM \qquad \text{...(4)}$$

and in ΔDEB,

$$area(\Delta DEB) = \frac{1}{2} \times EM \times BD \qquad \text{...(5)}$$

Dividing eqn. (4) by eqn. (5),

$$\frac{area(\Delta ADE)}{area(\Delta DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$$

or, $$\frac{area(\Delta ADE)}{area(\Delta DEB)} \frac{AD}{BD} \qquad \text{...(6)}$$

Since ΔDEB and ΔDEC lie on the same base DE and between two parallel lines DE and BC.

$$\text{area }(\Delta DEB) = area(\Delta DEC)$$

From equation (3) we have

$$\frac{area(\Delta ADE)}{area(\Delta DEB)} = \frac{AE}{CE} \qquad ...(7)$$

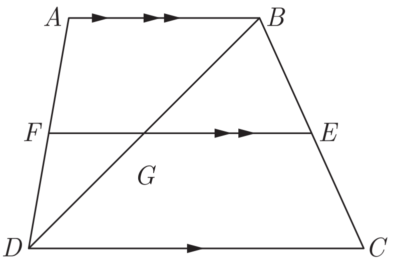
From equations (6) and (7) we get

$$\frac{AE}{CE} = \frac{AD}{BD}. \qquad \text{Hence proved.}$$

18. In a trapezium $ABCD, AB || DC$ and $DC = 2AB$. $EF = AB$, where E and F lies on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$ diagonal DB intersects EF at G. Prove that, $7EF = 11AB$.

Ans : [Board Term-1, 2012, Set-65]

As per given condition we have drawn the figure below.



In trapezium $ABCD$,

$$AB || DC \text{ and } DC = 2AB.$$

Also, $$\frac{BE}{EC} = \frac{4}{3}$$

In trapezium $ABCD$,

$$EF || AB || CD$$

$$\frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$$

In ΔBGE and ΔBDC, $\angle B$ is common and

Due to corresponding angles,

$$\angle BEG = \angle BCD$$

Due to AA similarity we get

$$\Delta BGE \sim \Delta BDC$$

, $$\frac{EG}{CD} = \frac{BE}{BC} \qquad ...(1)$$

As, $$\frac{BE}{EC} = \frac{4}{3}$$

$$\frac{BE}{BE+EC} = \frac{4}{4+3} = \frac{4}{7}$$

$$\frac{BE}{BC} = \frac{4}{7} \qquad ...(2)$$

From (1) and (2) we have

$$\frac{EG}{CD} = \frac{4}{7}$$

, $$EG = \frac{4}{7}CD \qquad ...(3)$$

Similarly, $$\Delta DGF \sim \Delta DBA$$

$$\frac{DF}{DA} = \frac{FG}{AB}$$

$$\frac{FG}{AB} = \frac{3}{7}$$

$$FG = \frac{3}{7}AB \qquad ...(4)$$

$$\left[\frac{AF}{AD} = \frac{4}{7} = \frac{BE}{BC} \Rightarrow \frac{EC}{BC} = \frac{3}{7} = \frac{DE}{DA}\right]$$

Adding equation (3) and (4) we have

$$EG + FG = \frac{4}{7}DC + \frac{3}{7}AB$$

oe $$EF = \frac{4}{7} \times (2AB) + \frac{3}{7}AB$$

$$= \frac{8}{7}AB + \frac{3}{7}AB = \frac{77}{7}AB$$

$$7EF = 11AB. \qquad \text{Hence proved.}$$

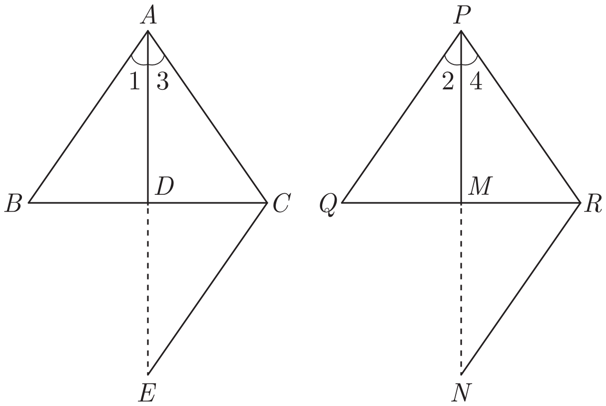
19. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Ans : [Board Term-1, 2012, Set-62]

It is given that in ΔABC and ΔPQR, AD and PM are their medians,

such that $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$$

We have produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$. Join CE As per given condition we have drawn the figure below. and RN.



In ΔABD and ΔEDC,

$$AD = DE \qquad \text{(By construction)}$$

$$\angle ADB = \angle EDC \qquad \text{(VOA)}$$

$$BD = DC \qquad (AD \text{ is a median})$$

By SAS congruency

$$\Delta ABD \cong \Delta EDC$$

$$AB = CE \qquad \text{(By CPCT)}$$

Similarly, $$PQ = RN \text{ and } \angle A = \angle 2$$

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PE} \qquad \text{(Given)}$$

, $$\frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$$

$$\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$$

By SSS similarity, we have

$$\Delta AEC \sim \Delta PNR$$

$$\angle 3 = \angle 4$$

$$\angle 1 = \angle 2$$

$$\angle 1 + \angle 3 = \angle 2 + \angle 4$$

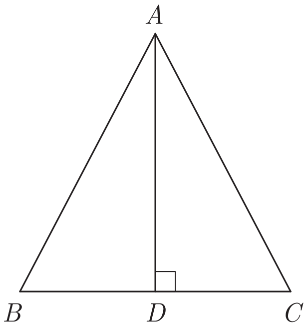
By SSS similarity, we have

$$\Delta ABC \sim \Delta PQR \qquad \text{Hence Proved}$$

20. In $\Delta ABC, AD \perp BC$ and point D lies on BC such that $2DB = 3CD$. Prove that $5AB^2 = 5AC^2 + BC^2$.

Ans : [Board Term-1, 2015 Set DDE-E]

It is given in a triangle $\Delta ABC, AD \perp BC$ and point D lies on BC such that $2DB = 3CD$.
As per given condition we have drawn the figure below.



Since $\qquad 2DB = 3CD$

$$\frac{DB}{CD} = \frac{3}{2}$$

Let DB be $3x$, then CD will be $2x$ so $BC = 5x$

Since $CD = 90°$ in ΔADB, we have

$$AB^2 = AD^2 + DB^2 = AD^2 + (3x)^2$$

$$= AD^2 + 9x^2$$

or, $\qquad 5AB^2 = 5AD^2 + 45x^2$

$$5AD^2 = 5AB^2 - 45x^2 \qquad ...(1)$$

and $\qquad AC^2 = AD^2 + CD^2 = AD^2 + (2x)^2$

$$= AD^2 + 4x^2$$

or, $\qquad 5AC^2 = 5AD^2 + 20x^2$

$$5AD^2 = 5AC^2 - 20x^2 \qquad ...(2)$$

Comparing eq. (1) and eq. (2) we have

$$5AB^2 - 45x^2 = 5AC^2 - 20x^2$$

$$5AB^2 = 5AC^2 - 20x^2 + 45x^2$$

$$= 5AC^2 + 25x^2$$

$$= 5AC^2 + (5x)^2$$

$$= 5AC^2 + BC^2 \qquad [\ BC = 5x]$$

Therefore $\qquad 5AB^2 = 5AC^2 + BC^2 \qquad$ Hence proved

HOTS QUESTIONS

1. In a right triangle ABC, right angled at C. P and Q are points of the sides CA and CB respectively, which divide these sides in the ratio $2:1$.

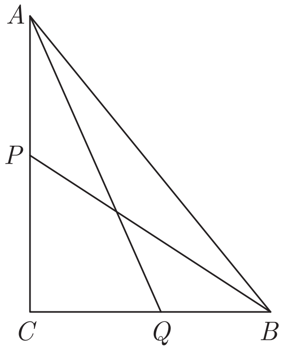
Prove that : $\qquad 9AQ^2 = 9AC^2 + 4BC^2$

$$9BP^2 = 9BC^2 + 4AC^2$$

$$9(AQ^2 + BP^2) = 13AB^2$$

Ans :

As per given condition we have drawn the figure below.



Since P divides AC in the ratio $2:1$

$$CP = \frac{2}{3}AC$$

$$QC = \frac{2}{3}BC$$

$$AQ^2 = QC^2 + AC^2$$

$$= \frac{4}{9}BC^2 + AC^2$$

or, $\qquad 9AQ^2 = 4BC^2 + 9AC^2 \qquad ...(1)$

Similarly, we get

$$9BP^2 = BC^2 + 4A^2 \qquad ...(2)$$

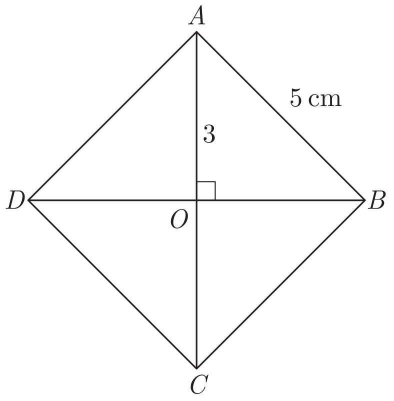
Adding equation (1) and (2), we get

$$9(AQ^2 + BP^2) = 13AB^2$$

2. Find the length of the second diagonal of a rhombus, whose side is 5 cm and one of the diagonals is 6 cm.

Ans :

As per given condition we have drawn the figure below.



We have $AB = BC = CD = AD = 5$ cm and $AC = 6$ cm

Since $AO = OC, \qquad AO = 3$ cm

Here ΔAOB is right angled triangle as diagonals of rhombus intersect at right angle.

By Pythagoras theorem,

$$OB = 4 \text{ cm.}$$

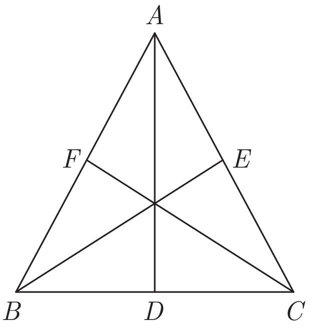
Since $DO = OB, BD = 8$ cm, length of the other diagonal $= 2(BO)$ where $BO = 4$ cm

Hence $$BD = 2 \times BO = 2 \times 4 = 8 \text{ cm}$$

3. Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

Ans :

As per given condition we have drawn the figure below.



In triangle sum of squares of any two sides is equal to twice the square of half of the third side, together with twice the square of median bisecting it.

If AD is the median,

$$AB^2 + AC^2 = 2\left\{AD^2 + \frac{BC^2}{4}\right\}$$

$$2(AB^2 + AC^2) = 4AD^2 + BC^2 \quad ...(1)$$

Similarly by taking BE and CF as medians,

$$2(AB^2 + AC^2) = 4BE^2 + AC^2 \quad ...(2)$$

and $$2(AC^2 + BC^2) = 4CF^2 + AB^2 \quad ...(3)$$

Adding, (1), (2) and (iii), we get

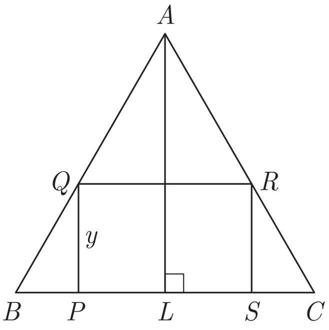
$$3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$$

Hence proved

4. ABC is an isosceles triangle in which $AB = AC = 10cm$. $BC = 12cm$. $PQRS$ is a rectangle inside the isosceles triangle. Given $PQ = SR = ycm, PS = PR = 2x$. Prove that $x = 6 - \frac{3y}{4}$.

Ans :

As per given condition we have drawn the figure below.



Here we have drawn $AL \perp BC$.

Since it is isosceles triangle, AL is median of BC,

$$BL = LC = 6 \text{ cm.}$$

In right ΔALB, by Pythagoras theorem,

$$AL^2 = AB^2 - BL^2$$

$$= 10^2 - 6^2 = 64 = 8^2$$

Thus $AL = 8$ cm.

In ΔBPQ and ΔBLA,

$$\angle B = \angle C \quad \text{(Isosceles triangle)}$$

$$\angle BPQ = \angle BLA = 90^\circ$$

Thus by AA similarity we get

$$\Delta BPQ \sim \angle BLA$$

$$\frac{PB}{PQ} = \frac{BL}{AL}$$

$$\frac{6-x}{y} = \frac{6}{8}$$

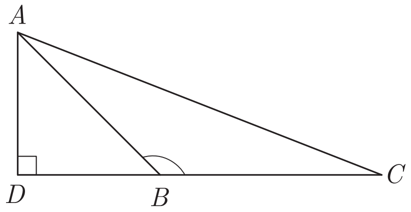
$$x = 6 - \frac{3y}{4}$$ Hence proved.

5. If ΔABC is an obtuse angled triangle, obtuse angled at B and if $AD \perp CB$. Prove that :

$AC^2 \; AB^2 + BC^2 + 2BC \times BD$

Ans :

As per given condition we have drawn the figure below.



In ΔADB, By Pythagoras theorem

$$AB^2 = AD^2 + BD^2 \quad ...(1)$$

In ΔADC, By Pythagoras theorem,

$$AC^2 = AD^2 + CD^2$$

$$= AD^2 + (BC + BD)^2$$

$$= AD^2 + BC^2 + 2BC \times BD + BD^2$$

$$= (AD^2 + BD^2) + 2BC \times BD$$

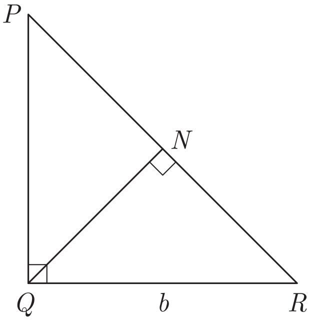
Substituting $(AD^2 + BD^2) = AB^2$ we have

$$AC^2 = AB^2 + BC^2 + 2BC \times BD$$

6. If A be the area of a right triangle and b be one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4 + 4A^2}}$.

Ans :

As per given condition we have drawn the figure below.



Let $QR = b$, then we have

$$\begin{aligned} A &= ar(\Delta PQR) \\ &= \frac{1}{2} \times b \times PQ \end{aligned}$$

or,
$$PQ = \frac{2 \cdot A}{b} \qquad \text{...(1)}$$

Due to AA similarity we have

$$\begin{aligned} \Delta PNQ &\sim \Delta PQR \\ \frac{PQ}{PR} &= \frac{NQ}{QR} \qquad \text{...(2)} \end{aligned}$$

From ΔPQR

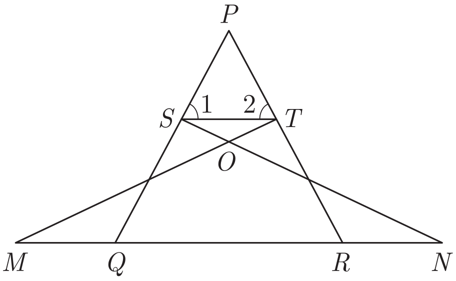
$$\begin{aligned} PQ^2 + QR^2 &= PR^2 \\ \frac{4A^2}{b^2} + b^2 &= PR^2 \\ PR &= \sqrt{\frac{4A^2 + b^4}{b^2}} = \frac{\sqrt{4A^2 + b^4}}{b} \end{aligned}$$

Equation (2) becomes

$$\begin{aligned} \frac{2A}{b \times PR} &= \frac{NQ}{b} \\ NQ &= \frac{2A}{PR} \end{aligned}$$

Altitude
$$NQ = \frac{2Ab}{\sqrt{4A^2 + b^4}}$$
Hence Proved.

7. In given figure $\angle 1 = \angle 2$ and $\Delta NSQ \sim \Delta MTR$, then prove that $\Delta PTS \sim \Delta PRO$.



Ans : [Sample Question Paper 2017]

Given, $\Delta NSQ \cong \Delta MTR$

By CPCT we have

$$\angle SQN = \angle TRM$$

From angle sum property we have

$$\begin{aligned} \angle P + \angle 1 + \angle 2 &= \angle P + \angle PQR + \angle PRQ \\ \angle 1 + \angle 2 &= \angle PQR + \angle PRQ \end{aligned}$$

Since $\because \angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$ we get

$$\begin{aligned} 2\angle 1 &= 2\angle PQR \\ \angle 1 &= \angle PQR \end{aligned}$$

Also
$$\angle 2 = \angle QPR$$
common

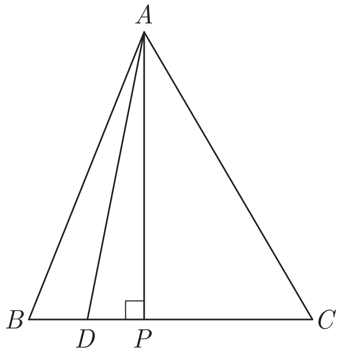
Thus by AAA Similarity

$$\Delta PTS \sim \Delta PRQ$$

8. In an equilateral triangle ABC, D is a point on the side BC such the $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

Ans : [Sample Question Paper 2017]

As per given condition we have drawn the figure below. Here we have drawn $AP \perp BC$



Here $AB = BC = CA$ and $BD = \frac{1}{3}BC$.

In ΔADP,

$$\begin{aligned} AD^2 &= AP^2 + DP^2 \\ &= AP^2 + (BP - BD)^2 \\ &= AP^2 + BP^2 + BD^2 + 2BP \cdot BD \end{aligned}$$

From ΔAPB using $AP^2 + BP^2 = AB^2$ we have

$$\begin{aligned} AD^2 &= AB^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right) \\ &= AB^2 + \frac{AB^2}{9} - \frac{AB^2}{3} = \frac{7}{9}AB^2 \\ 9AD^2 &= 7AB^2 \end{aligned}$$

Hence Proved

CHAPTER 7

Coordinate Geometry

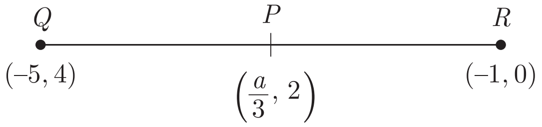
TOPIC 1 : DISTANCE BETWEEN TWO POINTS AND SECTION FORMULA

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the value of a, for which point $P(\frac{a}{3},2)$ is the midpoint of the line segment joining the Points $Q(-5,4)$ and $R(-1,0)$.

Ans : [Board Sample Paper, 2016]

As per question, line diagram is shown below.



Since P is mid-point of QR, we have

$$\frac{a}{3} = \frac{-5+(-1)}{2} = \frac{-6}{2} = -3$$

or, $$a = -9$$

2. The ordinate of a point A on y-axis is 5 and B has co-ordinates $(-3,1)$. Find the length of AB.

Ans : [Delhi CBSE, Term-2, 2014]

We have $A(0,5)$ and $B(-3,1)$.

Distance between A and B,

$$\begin{aligned} AB &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-3-0)^2+(1-5)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} = 5 \end{aligned}$$

3. Find the perpendicular distance of $A(5,12)$ from the y-axis.

Ans : [Board Terms-2, 2011 Set (A1)]

As per question, line diagram is shown below.
Perpendicular from point $A(5,12)$ on y-axis touch it at $(0,12)$.
Distance between $(5,12)$ and $(0,12)$ is,

$$\begin{aligned} d &= \sqrt{(0-5)^2+(12-12)^2} \\ &= \sqrt{25} \\ &= 5 \text{ units.} \end{aligned}$$

4. If the centre and radius of circle is $(3,4)$ and 7 units respectively,, then what it the position of the point $A(5,8)$ with respect to circle?

Ans : [Board Term-2, 2013]

Distance of the point, from the centre

$$\begin{aligned} a &= \sqrt{(5-3)^2+(8-4)^2} \\ &= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \end{aligned}$$

Since $2\sqrt{5}$ is less than 7, the point lies inside the circle.

5. Find the perimeter of a triangle with vertices $(0,4)$,$(0,0)$ and $(3,0)$.

Ans : [Board Term-2, 2011 Set (B1)]

We have $A(0,4)$, $B(0,0)$, and $C(3,0)$.

$$\begin{aligned} AB &= \sqrt{(0-2)^2+(0-4)^2} = \sqrt{16} = 4 \\ BC &= \sqrt{(3-0)^2+(0-0)^2} = \sqrt{9} = 3 \\ CA &= \sqrt{(0-3)^2+(4-0)^2} \\ &= \sqrt{9+16} = \sqrt{25} = 5 \end{aligned}$$

Thus Perimeter of triangle $= 4+3+5 = 12$

6. To locate a point Q on line segment AB such that $BQ = \frac{5}{7}\times AB$. What is the ratio of line segment in which AB is divided?

Ans : [Board Term-2, 2013]

We have $$BQ = \frac{5}{7}AB$$

$$\frac{BQ}{AB} = \frac{5}{7} \Rightarrow \frac{AB}{BQ} = \frac{7}{5}$$

$$\frac{AB-BQ}{BQ} = \frac{AQ}{BQ} = \frac{7-5}{5} = \frac{2}{5}$$

$$AQ{:}BQ = 2{:}5$$

7. Find the distance of the point $(-4,-7)$ from the y-axis.

Ans : [Board Term-2, 2013]

As per question, line diagram is shown below.
Perpendicular from point $A(-4,-7)$ on y-axis touch it at $(0,-7)$.
Distance between $(-4,-7)$ and $(0,-7)$ is

$$\begin{aligned} d &= \sqrt{(0+4)^2+(-7+7)^2} \\ &= \sqrt{4^2+0} = \sqrt{16} = 4 \text{ units} \end{aligned}$$

8. If the distance between the points $(4,k)$ and $(1,0)$ is 5, then what can be the possible values of k.

Ans : [Delhi Set I, II, III 2017]

Using distance formula

$$\begin{aligned} \sqrt{(4-1)^2+(k-0)^2} &= 5 \\ 3^2+k^2 &= 25 \\ k &\pm 4 \end{aligned}$$

9. Find the coordinates of the point on y-axis which is nearest to the point $(-2,5)$.

Ans : [Sample Question Poper, 2017]

The point on y-axis that is nearest to the point $(-2,5)$ is $(0,5)$.

10. In what ratio does the x-axis divide the line segment joining the points $(-4,-6)$ and $(-1,7)$? Find the coordinates of the point of division.

Ans : [Board Sample Paper, 2017]

Let x-axis be divides the line-segment joining $(-4,-6)$ and $(-1,7)$ at the point $P(x,y)$ in the ratio $1:k$.

Now, the coordinates of point of division P,

$$\begin{aligned}(x,y) &= \frac{1(-1)+k(-4)}{k+1}, \frac{1(7)+k(-6)}{k+1}\\ &= \frac{-1-4k}{k+1}, \frac{7-6k}{k+1}\end{aligned}$$

Since P lies on x axis, therefore $y=0$, which gives

$$\begin{aligned}\frac{7-6k}{k+1} &= 0\\ 7-6k &= 0\\ k &= \frac{7}{6}\end{aligned}$$

Hence, the ratio is $1:\frac{7}{6}$ or, $6:7$ and the coordinates of P are $\left(-\frac{34}{13}, 0\right)$

SHORT ANSWER TYPE QUESTIONS - I

1. Find a relation between x and y such that the point $P(x,y)$ is equidistant from the points $A(-5,3)$ and $B(7,2)$.

Ans : [Board Sample Paper, 2016]

Let $P(x,y)$ is equidistant from $A(-5,3)$ and $B(7,2)$, then we have

$$\begin{aligned}AP &= BP\\ \sqrt{(x+5)^2+(y-3)^2} &= \sqrt{(x-7)^2+(y-2)^2}\\ (x+5)^2+(y-3)^2 &= (x-7)^2+(y-2)^2\\ 10x+25-6y+9 &= -14x+49-4y+4\\ 24x+34 &= 2y+53\\ 24x-2y &= 19\end{aligned}$$

Thus $24x-2y-19 = 0$ is the required relation.

2. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from $Q(2,-5)$ and $R(-3,6)$, find the co-ordinates of P.

Ans : [Delhi Set I, II, III, 2016]

Let the point P $(2y,y)$,

Since $PQ = PR$, we have

$$\begin{aligned}\sqrt{(2y-2)^2+(y+5)^2} &= \sqrt{(2y+3)^2+(y-6)^2}\\ (2y-2)^2+(y+5)^2 &= (2y+3)^2+(y-6)^2\\ -8y+4+10y+25 &= 12y+9-12y+36\\ 2y+29 &= 45\\ y &= 8\end{aligned}$$

Hence, coordinates of point P are $(16,8)$

3. Find the ratio in which y-axis divides the line segment joining the points $A(5,-6)$ and $B(-1,-4)$. Also find the co-ordinates of the point of division.

Ans : [Delhi Set I, II, III, 2016]

Let y-axis be divides the line-segment joining $A(5,-6)$ and $B(-1,-4)$ at the point $P(x,y)$ in the ratio $AP:PB = k:1$

Now, the coordinates of point of division P,

$$\begin{aligned}(x,y) &= \frac{k(-1)+1(5)}{k+1}, \frac{k(-4)+1(-6)}{k+1}\\ &= \frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\end{aligned}$$

Since P lies on y axis, therefore $x=0$, which gives

$$\begin{aligned}\frac{5-k}{k+1} &= 0\\ k &= 5\end{aligned}$$

Hence required ratio is $5:1$

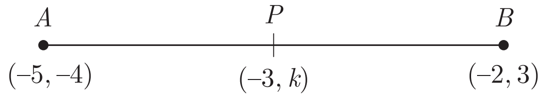
Now $$y = \frac{-4(5)-6}{6} = \frac{-13}{3}$$

Hence point on y-axis is $(0,-\frac{13}{3})$.

4. Find the ratio in which the point $(-3,k)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$. Also find the value of k.

Ans : [Foreign Set I, II, III, 2016]

As per question, line diagram is shown below.



Let AB be divides by P in ratio $n:1$.
x co-ordinate for section formula

$$\begin{aligned}-3 &= \frac{(-2)n+1(-5)}{n+1}\\ -3(n+1) &= -2n-5\\ -3n-3 &= -2n-5\\ 5-3 &= 3n-2n\\ 2 &= n\end{aligned}$$

Ratio $$\frac{n}{1} = \frac{2}{1} \text{ or } 2:1$$

Now, y co-ordinate,

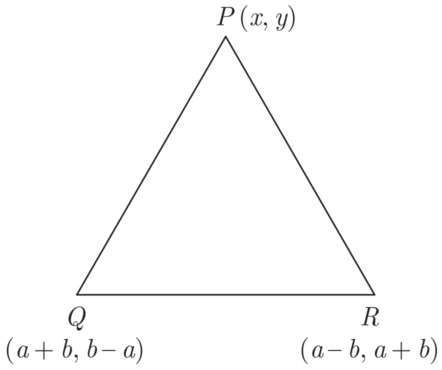
$$k = \frac{2(3)+1(-4)}{2+1} = \frac{6-4}{3} = \frac{2}{3}$$

5. If the point $P(x,y)$ is equidistant from the points $Q(a+b,b-a)$ and $R(a-b,a+b)$, then prove that $bx=ay$.

Ans : [O.D. Set I, II, III, 2016]
[Board Term-2, 2012 Set (12)]

We have $|PQ| = |PR|$

$$\sqrt{\left[x-(a+b)\right]^2+\left[y-(b-a)\right]^2} = \sqrt{\left[x-(a-b)\right]^2+\left[y-(b+a)\right]^2}$$



$$\left[x-(a+b)\right]^2+\left[y-(b-a)\right]^2 = \left[x-(a-b)\right]^2+\left[y-(a+b)\right]^2$$

$$-2x(a+b)-2y(b-a) = -2x(a-b)-2y(a+b)$$

$$2x(a+b)+2y(b-a) = 2x(a-b)+2y(a+b)$$

$$2x(a+b-a+b)+2y(b-a-a-b) = 0$$

$$2x(2b)+2y(-2a) = 0$$

$$xb-ay = 0$$

$$bx = ay$$

Hence Proved

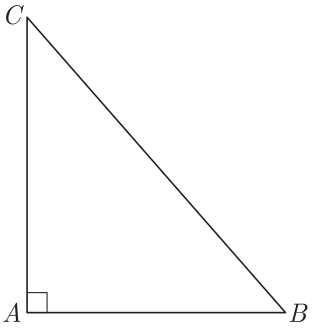
6. Prove that the point $(3,0)$, $(6,4)$ and $(-1,3)$ are the vertices of a right angled isosceles triange.

Ans : [O.D. Set I, II, III, 2016]

We have $A(3,0)$, $B(6,4)$ and $C(-1,3)$

Now
$$AB^2 = (3-6)^2+(0-4)^2$$
$$= 9+16 = 25$$
$$BC^2 = (6+1)^2+(4-3)^2$$
$$= 49+1 = 50$$
$$CA^2 = (-1-3)^2+(3-0)^2$$
$$= 16+9 = 25$$
$$AB^2 = CA^2 \text{ or, } AB = CA$$

Hence triangle is isosceles.



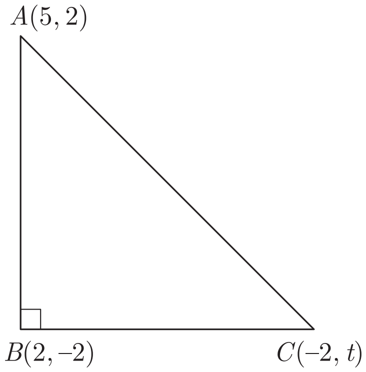
Also,
$$25+25 = 50$$
or,
$$AB^2+CA^2 = BC^2$$

Since pythagoras theorem is verified, therefore triangle is a right angled triangle.

7. If $A(5,2)$, $B(2,-2)$ and $C(-2,t)$ are the vertices of a right angled triangle with $\angle B = 90°$, then find the value of t.

Ans : [Delhi CBSE Board, 2015][Set I, II, III]

As per question, triangle is shown below.



Now
$$AB^2 = (2-5)^2+(-2-2)^2 = 9+16 = 25$$
$$BC^2 = (-2-2)^2+(t+2)^2 = 16+(t+2)^2$$
$$AC^2 = (5+2)^2+(2-t)^2 = 49+(2+t^2)$$

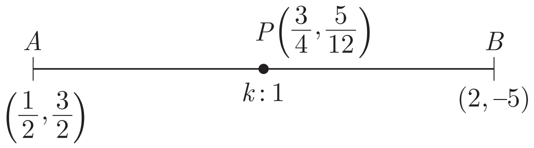
Since $\Delta\,ABC$ is a right angled triangle

$$AC^2 = AB^2+BC^2$$
$$49+(2-t)^2 = 25+16+(t+2)^2$$
$$49+4-4t+t^2 = 41+t^2+4t+4$$
$$53-4t = 45+4t$$
$$8t = 8$$
$$t = 1$$

8. Find the ratio in which the pont $P(\frac{3}{4},\frac{5}{12})$ divides the line segment joining the point $A\,(\frac{1}{2},\frac{3}{2})$ and $(2,-5)$.

Ans : [Delhi CBSE Term-2, 2015, Set I, II, III]

Let P divides AB in the ration $k:1$. Line diagram is shown below.



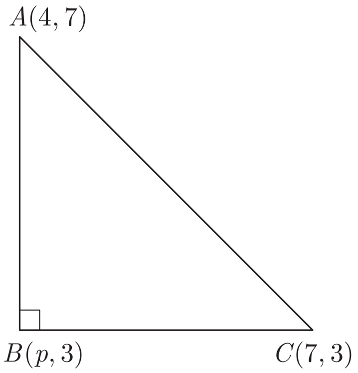
Now
$$\frac{k(2)+1(\frac{1}{2})}{k+1} = \frac{3}{4}$$
$$8k+2 = 3k+3$$
$$k = \frac{1}{5}$$

Thus required ratio is $\frac{1}{5}$:1 or 1:5.

9. The points $(4,7)$, $B(p,3)$ and $C(7,3)$ are the vertices of a right triangle, right-angled at B. Find the value of p.

Ans : [Outside Delhi CBSE, 2015, Set I, II]

As per question, triangle is shown below. Here ΔABC is a right angle triangle,

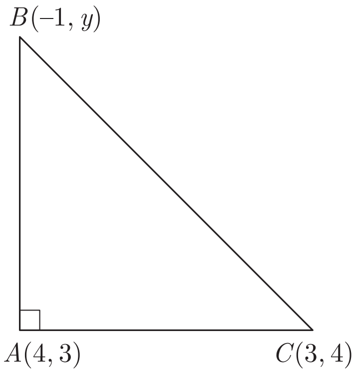


$$\begin{aligned} AB^2+BC^2 &= AC^2 \\ (p-4)^2+(3-7)^2+(7-p)^2+(3-3)^2 & \\ &= (7-4)^2+(3-4)^2 \\ (p-4)^2+(-4)^2+(7-p)^2+0 &= (3)^2+(-4)^2 \\ p^2-8p+16+16+49+p^2-14p &= 9+16 \\ 2p^2-22p+81 &= 25 \\ 2p^2-22p+56 &= 0 \\ p^2-11p+28 &= 0 \\ (p-4)(p-7) &= 0 \\ p &= 7 \text{ or } 4 \end{aligned}$$

10. If $A(4,3)$, $B(-1,y)$, and $C(3,4)$ are the vertices of a right triangle ABC, right angled at A, then find the value of y.

Ans : [Outside Delhi Board, 2015, Set II]

As per question, triangle is shown below.



We have
$$\begin{aligned} AB^2+AC^2 &= BC^2 \\ (4+1)^2+(3-y)^2+(4-3)^2 &= (3+1)^2+(4-y)^2 \\ (5)^2+(3-y)^2+(-1)^2+(1)^2 &= (4)^2+(4-y)^2 \\ 25+9-6y+y^2+1+1 &= 16+16-8y+y^2 \\ 36+2y-32 &= 0 \\ 2y+4 &= 0 \\ y &= -2 \end{aligned}$$

11. Show that the points (a,a), $(-a,-a)$ and $(-\sqrt{3}\,a, \sqrt{3}\,a)$ are the vertices of an equilateral triangle.

Ans : [Foreign Set I, II, III, 2015]

Let $A(a,a), B(-a,-a)$ and $C(-\sqrt{3}\,a, \sqrt{3}\,a)$

$$\begin{aligned} AB &= \sqrt{(a+a)^2+(a+a)^2} \\ &= \sqrt{4a^2+4a^2} \\ &= 2\sqrt{2}\,a \\ BC &= \sqrt{(-a+\sqrt{3}\,a)^2+(-a-\sqrt{3}\,a)^2} \\ &= \sqrt{a^2-2\sqrt{3}\,a^2+3a^2+a^2+2\sqrt{3}\,a^2+3a^2} \\ &= 2\sqrt{2}\,a \\ AC &= \sqrt{(a+\sqrt{3}\,a)^2+(a-\sqrt{3}\,a)^2} \end{aligned}$$

$$= \sqrt{a^2+2\sqrt{3}\,a^2+3a^2+a^2-2\sqrt{3}\,a^2+3a^2}$$
$$= 2\sqrt{2}\,a$$

Since $AB = BC = AC$, therefore ABC is an equilateral triangle.

12. If the mid-point of the line segment joining $A\left[\frac{x}{2}, \frac{y+1}{2}\right]$ and $B(x+1, y-3)$ is $C(5,-2)$, find x, y.

Ans : [Delhi CBSE, Term II, 2014][Board Term-2, 2012 Set (1)]

If the mid-point of the line segment joining $A\left[\frac{x}{2}, \frac{y+1}{2}\right]$ and $B(x+1, y-3)$ is $C(5,-2)$, then at mid point,

$$\begin{aligned} \frac{\frac{x}{2}+x+1}{2} &= 5 \\ \frac{3x}{2}+1 &= 10 \\ 3x &= 18 \end{aligned}$$

or, $$x = 6$$

also $$\begin{aligned} \frac{\frac{y+1}{2}+y-3}{2} &= -2 \\ \frac{y+1}{2}+y-3 &= -4 \\ y+1+2y-6 &= -8 \\ y &= -1 \end{aligned}$$

13. Find the point on the x-axis which is equidistant from the points $(2,-5)$ and $(-2,9)$.

Ans : [Board Term-2, 2012 Set (22)]

Let the point $P(x,0)$ on the x-axis is equidistant from points $A(2,-5)$ and $B(-2,9)$.

$$\begin{aligned} PA^2 &= PB^2 \\ (2-x)^2+(-5-0)^2 &= (-2-x)^2+(9-0)^2 \\ 4-4x+x^2+25 &= 4+4x+x^2+81 \\ -8x &= 56 \\ x &= -7 \end{aligned}$$

Thus point is $(-7,0)$.

14. Show that $A(6,4)$, $B(5,-2)$ and $C(7,-2)$ are the vertices of an isosceles triangle.

Ans : [Board Term-2, 2012 Set (44)]

We have $A(6,4), B(5,-2), C(7,-2)$.

$$\begin{aligned} \text{Now} \quad AB &= \sqrt{(6-5)^2+(4+2)^2} \\ &= \sqrt{1^2+6^2} = \sqrt{37} \\ BC &= \sqrt{(5-7)^2+(-2+2)^2} \\ &= \sqrt{(-2)^2+0^2} = 2 \\ CA &= \sqrt{(7-6)^2+(-2-4)^2} \\ &= \sqrt{1^2+6^2} = \sqrt{37} \\ AB &= BC = \sqrt{37} \end{aligned}$$

Since two sides of a triangle are equal in length, triangle is an isosceles triangle.

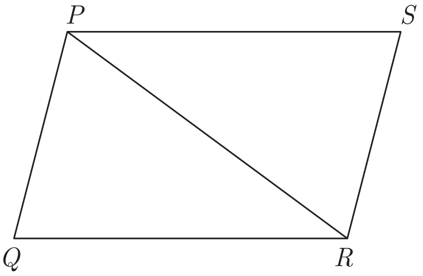
15. If $P(2,-1), Q(3,4), R(-2,3)$ and $S(-3,-2)$ be four points in a plane, show that $PQRS$ is a rhombus but not a square.

Ans : [Board Term-2, 2012 (28)]

We have $P(2,-1), Q(3,4), R(-2,3), S(-3,-2)$

$$\begin{aligned} PQ &= \sqrt{1^2+5^2} = \sqrt{26} \\ QR &= \sqrt{5^2+1^2} \ \sqrt{26} \\ RS &= \sqrt{1^2+5^2} = \sqrt{26} \\ PS &= \sqrt{5^2+1^2} = \sqrt{26} \end{aligned}$$

Since all the four sides are equal, $PQRS$ is a rhombus.



$$\begin{aligned} \text{Now} \quad PR &= \sqrt{1^2+5^2} = \sqrt{26} \\ &= \sqrt{4^2+4^2} = \sqrt{32} \\ PQ^2+QR^2 &= 2 \times 26 = 52 \neq (\sqrt{32})^2 \end{aligned}$$

Since ΔPQR is not a right triangle, $PQRS$ is a rhombus but not a square.

16. Show that $A(-1,0), B(3,1), C(2,2)$ and $D(-2,1)$ are the vertices of a parallelogram $ABCD$.

Ans : [Board Term-2, 2012 Set (1)]

Mid-point of AC

$$\left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right)$$

Mid-point BD

$$\left(\frac{3-2}{2}, \frac{1+1}{2}\right) = \left(\frac{1}{2}, 1\right)$$

Here Mid-point of AC = Mid-point of BD

Since diagonals of a quadrilateral bisect each other, $ABCD$ is a parallelogram.

17. If $(3,2)$ and $(-3,2)$ are two vertices of an equilateral triangle which contains the origin, find the third vertex.

Ans : [Board Term-2, 2012 Set (12)]

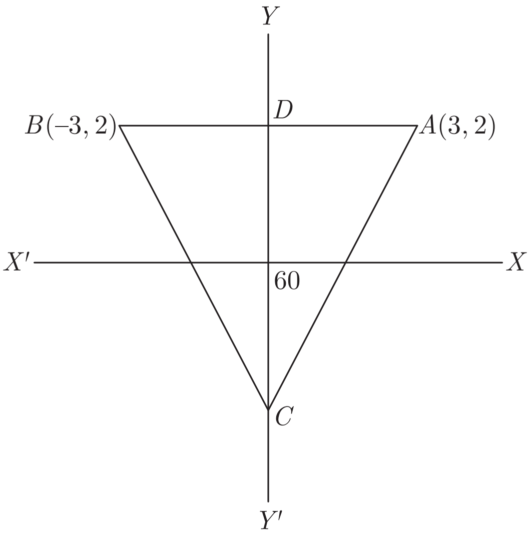
We have $A(3,2)$ and $B(-3,2)$.

It can be easily seen that mid-point of AB is lying on y-axis. Thus AB is equal distance from x-axis everywhere.

Also $OD \perp AB$

Hence 3^{rd} vertex of ΔABC is also lying on y-axis.

The digram of triangle should be as given below.



Let $C(x,y)$ be the coordinate of 3^{rd} vertex of ΔABC.

$$\begin{aligned} \text{Now} \qquad AB^2 &= (3+3)^2+(2-2)^2 = 36 \\ BC^2 &= (x+3)^2+(y-2)^2 \\ AC^2 &= (x-3)^2+(y-2)^2 \\ \text{Since} \qquad AB^2 &= AC^2 = BC^2 \end{aligned}$$

$$(x+3)^2+(y-2)^2 = 36 \qquad (1)$$

$$(x-3)^2+(y-2)^2 = 36 \qquad (2)$$

Since $P(x,y)$ lie on y-axis, substituting $x=0$ in(1) we have

$$\begin{aligned} 3^2+(y-2)^2 &= 36-9 = 27 \\ (y-2)^2 &= 36-9 = 27 \end{aligned}$$

Taking square root both side

$$\begin{aligned} y-2 &= \pm 3\sqrt{3} \\ y &= 2 \pm 3\sqrt{3} \end{aligned}$$

Since origin is inside the given triangle, coordinate of C below the origin,

$$y = 2-3\sqrt{3}$$

Hence Coordinate of C is $(0, 2-3\sqrt{3})$

18. Find a so that $(3,a)$ lies on the line represented by $2x-3y-5=0$. Also, find the co-ordinates of the point where the line cuts the x-axis.

Ans : [Board Term-2 Set (34)]

Since $(3,a)$ lies on $2x-3y-5=0$, it must satisfy this equation. Therefore

$$\begin{aligned} 2 \times 3 - 3a - 5 &= 0 \\ 6-3a-5 &= 0 \\ 1 &= 3a \\ a &= \frac{1}{3} \end{aligned}$$

Line $2x-3y-5=0$ will cut the x-axis at $(x,0)$. and it must satisfy the equation of line.

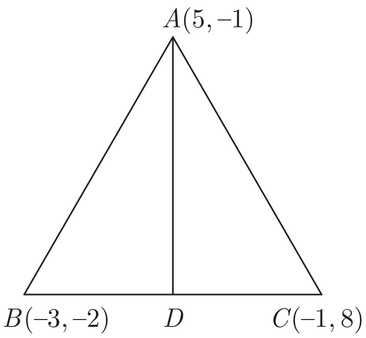
$$2x-5=0 \Rightarrow x=\frac{5}{2}$$

Hence point is $\left(\frac{5}{2},0\right)$

19. If the vertices of ΔABC are $A(5,-1), B(-3,-2)$, $C(-1,8)$, Find the length of median through A.

Ans : [Board Term-2, 2012 Set (17)]

Let AD be the median. As per question, triangle is shown below.



Since D is mid-point of BC, co-ordinates of D,

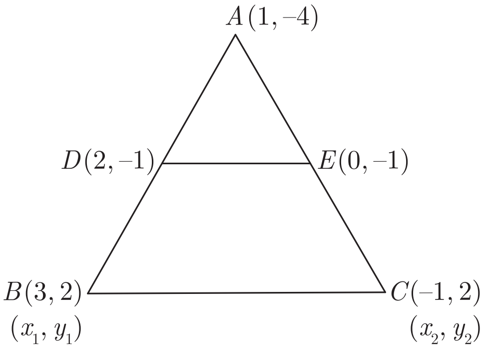
$$\begin{aligned}(x_1,\ y_2) &= \left(\frac{-3-1}{2},\frac{-2+8}{2}\right)\\ &= (-2,\ 3)\\ AD &= \sqrt{(5+2)^2+(-1-3)^2}\\ &= \sqrt{(7)^2+(4)^2}\\ &= \sqrt{49+16}\\ &= \sqrt{65} \text{ units}\end{aligned}$$

Thus length of median is $\sqrt{65}$

20. Find the mid-point of side BC of ΔABC, with $A(1,-4)$ and the mid-points of the sides through A being $(2,-1)$ and $(0,-1)$.

Ans : [Board Term-2, 2012 Set (21)]

Assume co-ordinates of B and C are (x_1, y_1) and (x_2, y_2) respectively. As per question, triangle is shown below.



Now $$2 = \frac{1+x_1}{2} \Rightarrow x_1 = 3$$

and $$-1 = \frac{-4+y_1}{2} \Rightarrow y_1 = 2$$

$$0 = \frac{1+x_2}{2} \Rightarrow x = -1$$

$$-1 = \frac{-4+y_2}{2} \Rightarrow y_2 = 2$$

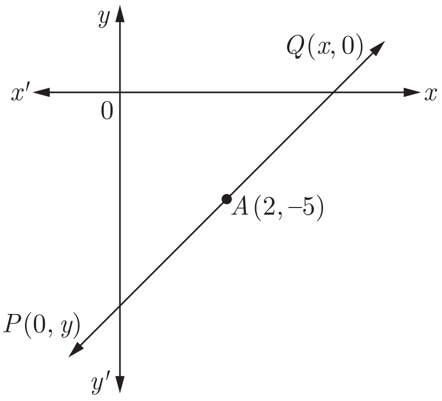
Thus $$\begin{aligned}B(x_1, y_1) &= (3,2),\\ C(x_2, y_2) &= (-1,2)\end{aligned}$$

So, mid-point of BC is $\left(\frac{3-1}{2},\frac{2+2}{2}\right) = (1,2)$

21. A line intersects the y-axis and x-axis at the points P and Q respectively. If $(2,-5)$ is the mid-point of PQ, then find the coordinates of P and Q.

Ans : [Outside Delhi, Set-III, 2017]

Let coordinates of P be $(0,y)$ and of Q be $(x,0)$.
$A(2,-5)$ is mid point of PQ.
As per question, line diagram is shown below.



Using section formula,

$$(2,-5) = \left(\frac{0+x}{2}+\frac{y+0}{2}\right)$$

$$2 = \frac{x}{2} \Rightarrow x = 4$$

and $$-5 = \frac{y}{2} \Rightarrow y = -10$$

Thus P is $(0,-10)$ and Q is $(4,0)$

22. If $(1,\frac{p}{3})$ is the mid point of the line segment joining the points $(2,0)$ and $(0,\frac{2}{9})$, then show that the line $5x+3y+2=0$ passes through the point $(-1,3p)$.

Ans :

Since $(1,\frac{p}{3})$ is the mid point of the line segment joining the points $(2,0)$ and $(0,\frac{2}{9})$, we have

$$\frac{p}{3} = \frac{0+\frac{2}{9}}{2} = \frac{1}{9}$$

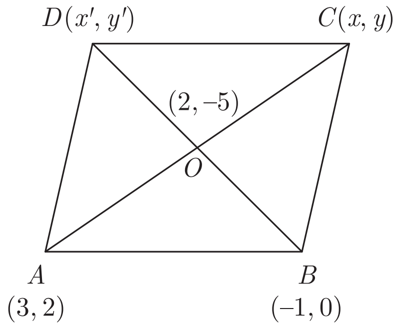
$$p = \frac{1}{3}$$

Now the point $(-1,3p)$ is $(-1,1)$.
The line $5x+3y+2=0$, passes through the point $(-1,1)$ as $5(-5)+3(1)+2=0$

23. If two adjacent vertices of a parallelogram are $(3,2)$ and $(-1,0)$ and the diagonals intersect at $(2,-5)$ then find the co-ordinates of the other two vertices.

Ans : [Board Foreign Set I, II, III, 2017]

Let two other co-ordinates be (x,y) and (x',y') respectively using mid-point formula.
As per question parallelogram is shown below.



Now $\quad 2 = \frac{x+3}{2} \Rightarrow x = 1$

and $\quad -5 = \frac{2+y}{2} \Rightarrow y = -12$

Again, $\quad \frac{-1+x'}{2} = 2 \Rightarrow x' = 5$

and $\quad \frac{0+y'}{2} = -5 \Rightarrow y' = -10$

Hence, coordinates of $C(1,-12)$ and $D(5,-10)$

24. In what ratio does the point $P(-4,6)$ divides the line segment joining the points $A(-6,10)$ and $B(3,-8)$?

Ans : [Delhi Compt. Set-I, II, III 2017]

Let $\quad AP:PB = k:1$

Now $\quad \frac{3k-6}{k+1} = -4$

$$3k-6 = -4k-4$$

$$7k = 2$$

$$k = \frac{7}{2}$$

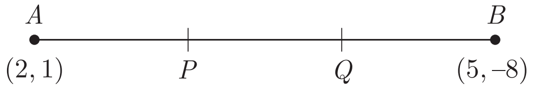
]

Hence, $\quad AP:PB = 7:2$

25. If the line segment joining the points $A(2,1)$ and $B(5,-8)$ is trisected at the points P and Q, find the coordinates P.

Ans : [Outside Delhi Compt. Set-I, III, 2017]

As per question, line diagram is shown below.



Let $P(x,y)$ divides AB in the ratio $1:2$
Using section formula we get

$$x = \frac{1 \times 5 + 2 \times 2}{1+2} = 3$$

$$y = \frac{1 \times -8 + 2 \times 1}{1+2} = 2$$

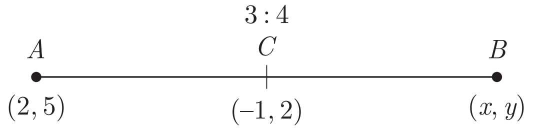
Hence coordinates of P are $(3,-2)$.

SHORT ANSWER TYPE QUESTIONS - II

1. If the point $C(-1,2)$ divides internally the line segment joining the points $A(2,5)$ and $B(x,y)$ in the ratio $3:4$, find the value of x^2+y^2.

Ans : [Foreign Set I, II, III, 2016]

As per question, line diagram is shown below.



We have $\quad \frac{AC}{BC} = \frac{3}{4}$

Applying section formula for x co-ordinate,

$$-1 = \frac{3x+4(2)}{3+4}$$

$$-7 = 3x+8$$

$$x = -5$$

Similarly applying section formula for y co-ordinate,

$$2 = \frac{3y+4(5)}{3+4}$$

$$14 = 3y+20$$

$$y = 2$$

Thus (x,y) is $(-5,-2)$.

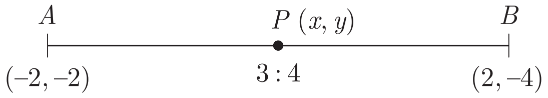
Now $\quad x^2+y^2 = (-5)^2+(-2)^2$
$\quad = 25+4 = 29$

2. If the co-ordinates of points A and B are $(-2,-2)$ and $(2,-4)$ respectively, find the co-ordinates of P such that $AP = \frac{3}{7}AB$, where P lies on the line segment AB.

Ans : [Outside Delhi, 2015, Set I, II]

We have $\quad AP = \frac{3}{7}AB \Rightarrow AP:PB = 3:4$

As per question, line diagram is shown below.



Section formula :

$$x = \frac{mx_2+nx_1}{m+n} \text{ and } y = \frac{my_2+nx_1}{m+n}$$

Applying section formula we get

$$x = \frac{3 \times 2 + 4 \times -2}{3+4} = -\frac{2}{7}$$

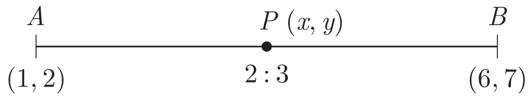
$$y = \frac{3 \times -4 + 4 \times -2}{3+4} = -\frac{20}{7}$$

Hence P is $\left(-\frac{2}{7}, -\frac{20}{7}\right)$

3. Find the co-ordinate of a point P on the line segment joining $A(1,2)$ and $B(6,7)$ such that $AP = \frac{2}{5}AB$

Ans : [Outside Delhi, 2015, Set III]

As per question, line diagram is shown below.



We have $AP = \frac{2}{5}AB \Rightarrow AP:PB = 2:3$

Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + nx_1}{m+n}$$

Applying section formula we get

$$x = \frac{2 \times 6 + 3 \times 1}{2+3} = \frac{12+3}{5} = 3$$

and $$y = \frac{2 \times 7 + 3 \times 2}{2+3} = \frac{14+6}{5} = 4$$

Thus $P(x,y) = (3,4)$

4. If the distance of $P(x,y)$ from $A(6,2)$ and $B(-2,6)$ are equal, prove that $y = 2x$.

Ans : [CBSE Board Term-2, 2015]

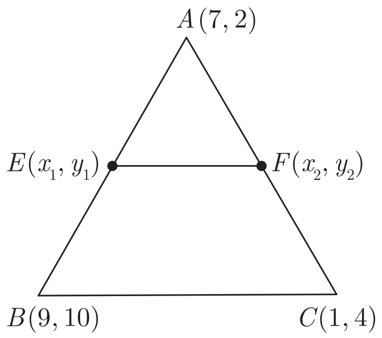
We have $P(x,y), A(6,2), B(-2,6)$

Now
$$\begin{aligned} PA &= PB \\ PA^2 &= PB^2 \\ (x-6)^2 + (y-2)^2 &= (x+2)^2 + (y-6)^2 \\ -12x + 36 - 4y + 4 &= 4x + 4 - 12y + 36 \\ -12x - 4y &= 4x - 12y \\ 12y - 4y &= 4x + 12x \\ 8y &= 16x \\ y &= 2x \end{aligned}$$
Hence Proved

5. The co-ordinates of the vertices of ΔABC are $A(7,2)$, $B(9,10)$ and $C(1,4)$. If E and F are the mid-points of AB and AC respectively, prove that $EF = \frac{1}{2}BC$.

Ans : [Board Term-2 2015]

Let the mid-points of AB and AC be $E(x_1, y_1)$ and $F(x_2, y_2)$. As per question, triangle is shown below.



Co-ordinates of point E

$$(x_1, y_1) = \left(\frac{9+7}{2}, \frac{10+2}{2}\right) = (8,6)$$

Co-ordinates of point F

$$(x_2, y_2) = \left(\frac{7+1}{2}, \frac{2+4}{2}\right) = (4,3)$$

Length, $$\begin{aligned} EF &= \sqrt{(x-4)^2 + (6-3)^2} \\ &= \sqrt{(4)^2 + (3)^2} \\ &= 5 \text{ units} \quad ...(1) \end{aligned}$$

Length $$\begin{aligned} BC &= \sqrt{(9-1)^2 + (10-4)^2} \\ &= \sqrt{(8)^2 + (6)^2} \\ &= 10 \text{ units} \quad ...(2) \end{aligned}$$

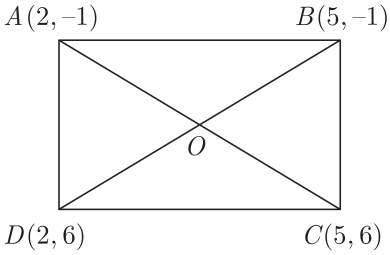
From equation (1) and (2) we get

$EF = \frac{1}{2}BC$ Hence proved.

6. Prove that the diagonals of a rectangle $ABCD$, with vertices $A(2,-1), B(5,-1), C(5,6)$ and $D(2,6)$ are equal and bisect each other.

Ans : [CBSE O.D. 2014]

As per question, rectangle $ABCD$, is shown below.



Now $$\begin{aligned} AC &= \sqrt{(5-2)^2 + (6+1)^2} = \sqrt{3^2 + 7^2} \\ &= \sqrt{9+49} = \sqrt{58} \\ BD &= \sqrt{(5-2)^2 + (-1-6)^2} = \sqrt{3^2 + 7^2} \\ &= \sqrt{9+49} = \sqrt{58} \end{aligned}$$

Since $AC = BD = \sqrt{58}$ the diagonals of rectangle $ABCD$ are equal

Mid-point of AC

$$= \left(\frac{2+5}{2}, \frac{-1+6}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$$

Mid-point of BD

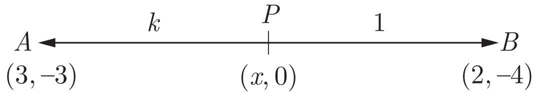
$$= \left(\frac{2+5}{2}, \frac{6+-1}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$$

Since the mid-point of diagonal AC and mid-point of diagonal BD is same and equal to $\left(\frac{7}{5}, \frac{5}{2}\right)$. Hence they bisect each other.

7. Find the ratio in which the line segment joining the points $A(3,-3)$ and $B(-2,7)$ is divided by x-axis. Also find the co-ordinates of point of division.

Ans : [Delhi, Term-2, 2014]

y co-ordinate of any point on the x will be zero. Let $(x,0)$ be point on x axis which cut the line. As per question, line diagram is shown below.



Let the ratio be $k:1$.
Using section formula for y co-ordinate we have

$$0 = \frac{1(-3) + k(7)}{1+k}$$

$$k = \frac{3}{7}$$

Using section formula for x co-ordinate we have

$$x = \frac{1(3)+k(-2)}{1+k} = \frac{3-2\times\frac{3}{7}}{1+\frac{3}{7}} = \frac{3}{2}$$

Thus co-ordinates of point are $\left(\frac{3}{2},0\right)$.

8. Find the ratio in which $(11,15)$ divides the line segment joining the points $(15,5)$ and $(9,20)$

Ans : [board Term-2, 2014]

Let the two points $(15,5)$ and $(9,20)$ are divided in the ratio $k:1$ by point $P(11,15)$
Using Section formula, we get

$$x = \frac{m_2x_1+m_1x_2}{m_2+m_1}$$

$$11 = \frac{1(15)+k(9)}{1+k}$$

$$11+11k = 15+9k$$

$$k = 2$$

Thus ratio is 2 : 1.

9. Find the point on y-axis which is equidistant from the points $(5,-2)$ and $(-3,2)$.

Ans : [Delhi Set, 2014]
[Board Term-2, 2012 Set (13)]

Let point be $(0,y)$

$$5^2+(y+2)^2 = (3)^2+(y-2)^2$$

or, $y^2+25+4y+4 = 9-4y+4$

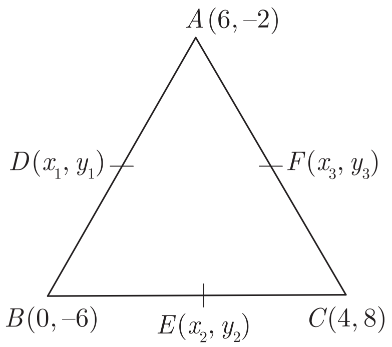
$$8y = -16 \text{ or, } y = -2$$

or, Point $(0,-2)$

10. The vertices of ΔABC are $A(6,-2)$, $B(0,-6)$ and $C(4,8)$. Find the co-ordinates of mid-points of AB, BC and AC.

Ans : [Board Term-2, 2014]

Let mid-point of AB, BC and AC be $D(x_1,y_1)$, $E(x_2,y_2)$ and $F(x_2,y_3)$. As per question, triangle is shown below.



Using section formula, the co-ordinates of the points D,E,F are

For D, $$x_1 = \frac{6+0}{2} = 3$$

$$y_1 = \frac{-2-6}{2} = -4$$

For E, $$x_2 = \frac{0+4}{2} = 2$$

$$y_2 = \frac{-6+8}{2} = 1$$

For F, $$x_3 = \frac{4+6}{2} = 5$$

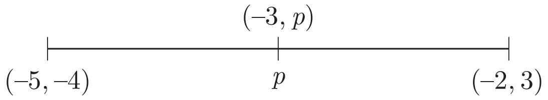
$$y_3 = \frac{-2+8}{2} = 3$$

The co-ordinates of the mid-points of AB, BC and AC are $D(3,-4)$, $E(2,1)$ and $F(5,3)$ respectively.

11. Find the ratio in which the point $(-3,p)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$. Hence find the value of p.

Ans : [Board Term-2, 2012]

As per question, line diagram is shown below.



Let $X(-3,p)$ divides the line joining of $A(-5,-4)$ and $B(-2,3)$ in the ratio $k:1$.
The co-ordinates of p are $\left[\frac{-2k-5}{k+1},\frac{3k-4}{k+1}\right]$
But co-ordinates of P are $(-3,p)$. Therefore we get

$$\frac{-2k-5}{k+1} = -3 \Rightarrow k = 2$$

and $$\frac{3k-4}{k+1} = p$$

Substituting $k=2$ gives

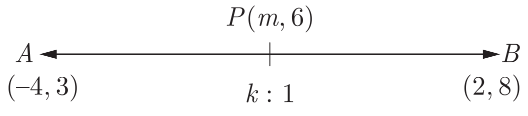
$$p = \frac{2}{3}$$

Hence ratio of division is 2:1 and $p = \frac{2}{3}$

12. Find the ratio in which the point $p(m,6)$ divides the line segment joining the points $A(-4,3)$ and $B(2,8)$. Also find the vlaue of m.

Ans : [Board Term-2, 2012 set (31)]

As per question, line diagram is shown below.



Let the ratio be $k:1$
Using section formula, we have

$$m = \frac{2k+(-4)}{k+1} \quad (1)$$

$$6 = \frac{8k+3}{k+1} \quad (2)$$

$$8k+3 = 6k+6$$

$$2k = 3$$

$$k = \frac{3}{2}$$

Thus ratio is $\frac{3}{2}:1$ or $3:2$.

Substituting value of k in (1) we have

$$m = \frac{2\left(\frac{3}{2}\right)+(-4)}{\frac{3}{2}+1} = \frac{3-4}{\frac{5}{2}} = \frac{-1}{\frac{5}{2}} = \frac{-2}{5}$$

13. If $A(4,-1), B(5,3), C(2,y)$ and $D(1,1)$ are the vertices of a parallelogram $ABCD$, find y.

Ans : [board Term-2, 2012 Set (5)]

Diagonals of a parallelogram bisect each other.
Mid-points of AC and BD are same.

Thus $$\left(3,\frac{-1+y}{2}\right) = (3,2)$$

$$\frac{-1+y}{2} = 2 \Rightarrow y = 5$$

14. Find the co-ordinates of the points of trisection of the line segment joining the points $A(1,-2)$ and $B(-3,4)$.

Ans : [Board Term-2, 2012 Set(34)]

Let $P(x_1,y_1), Q(x_2,y_2)$ divides AB into 3 equal parts.
Thus P divides AB in the ratio of 1:2.
As per question, line diagram is shown below.

Now $$x_1 = \frac{1(-3)+2(1)}{1+3} = \frac{-3+2}{3} = \frac{-1}{3}$$

$$y_1 = \frac{1(4)+2(-2)}{1+2} = \frac{4-4}{3} = 0$$

Co-ordinates of P is $\left(-\frac{1}{3},0\right)$.

Here Q is mid-point of PB.

Thus $$x_2 = \frac{-\frac{1}{3}+(-3)}{2} = \frac{-10}{6} = \frac{-5}{3}$$

$$y_2 = \frac{0+4}{2} = 2$$

Thus co-ordinates of Q is $\left(-\frac{5}{2},2\right)$.

15. If (a,b) is the mid-point of the segment joining the points $A(10,-6)$ and $B(k,4)$ and $a-2b=18$, find the value of k and the distance AB.

Ans : [Board Term-2, 2012 Set(21)]

We have $A(10,-6)$ and $B(k,4)$.
If $P(a,b)$ is mid-point of AB, then we have

$$(a,b) = \left(\frac{k+10}{2},\frac{-6+4}{2}\right)$$

$$a = \frac{k+10}{2} \text{ and } b = -1$$

From given condition we have

$$a-2b = 18$$

Substituting value $b=-1$ we obtain

$$a+2 = 18 \Rightarrow a = 16$$

$$a = \frac{k+10}{2} = 16 \Rightarrow k = 22$$

$$P(a,b) = (16,1)$$

$$AB = \sqrt{(22-10)^2+(4+6)^2}$$

$$= 2\sqrt{61} \text{ units}$$

16. Find the ratio in which the line $2x+3y-5=0$ divides the line segment joining the points $(8,-9)$ and $(2,1)$. Also find the co-ordinates of the point of division.

Ans : [Board Term-2, 2012 Set(21)]

Let a point $P(x,y)$ on line $2x+3y-5=0$ divides AB in the ratio $k:1$.

Now $$x = \frac{2k+8}{k+1}$$

and $$y = \frac{k-9}{k+1}$$

Substituting above value in line $2x+3y-5=0$ we have

$$2\left(\frac{2k+8}{k+1}\right)+3\left(\frac{k-9}{k+1}\right)-5 = 0$$

$$4k+16+3k-27-5k-5 = 0$$

$$2k-16 = 0$$

$$k = 8$$

Thus ratio is 8 : 1.

Substituting the value $k = 8$ we get

$$x = \left(\frac{2\times 8+8}{8+1}\right) = \frac{8}{3}$$

$$y = \left(\frac{8-9}{8+1}\right) = -\frac{1}{9}$$

Thus $$P(x,y) = \left(\frac{8}{3},-\frac{1}{9}\right)$$

17. Find the area of the rhombus of vertices $(3,0),(4,5),(-1,4)$ and $(-2,-1)$ taken in order.

Ans : [Board Term-2, 2012 Set (40)]

We have $A(3,0), B(4,5), C(-1,4), D(-2,-1)$

Diagonal AC, $$d_1 = \sqrt{(3+1)^2+(0-4)^2}$$
$$= \sqrt{16+16} = \sqrt{32}$$
$$= \sqrt{16\times 2} = 4\sqrt{2}$$

Diagonal BD, $$d_2 = \sqrt{(4+2)^2+(5+1)^2}$$
$$= \sqrt{36+36} = \sqrt{72}$$
$$= \sqrt{36\times 2} = 6\sqrt{2}$$

Area of rhombus $$= \frac{1}{2}\times d_1\times d_2$$
$$= \frac{1}{2}4\sqrt{2}\times 6\sqrt{2}$$
$$= 24 \text{ sq. unit.}$$

18. Find the ratio in which the line joining points $(a+b,b+a)$ and $(a-b,b-a)$ is divided by the point (a,b).

Ans : [Board Term-2, 2013]

Let $A(a+b,b+a), B(a-b,b-a)$ and $P(a,b)$ and P divides AB in $k:1$, then we have

$$a = \frac{k(a-b)+1(a+b)}{k+1}$$

$$a(k+1) = k(a-b)+a+b$$

$$ak+a = ak-bk+a+b$$

$$bk = b$$

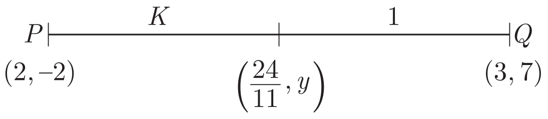
$$k = 1$$

Thus (a,b) divides $A(a+b,b+a)$ and $B(a-b,b-a)$ in 1:1 internally.

19. In what ratio does the point $\left(\frac{24}{11},\ y\right)$ divides the line segment joining the points $P(2,-2)$ and $Q(3,7)$? Also find the value of$y$.

Ans : [CBSE Marking Scheme, 2017]

As per question, line diagram is shown below.



Let $P\left(\frac{24}{11},\ y\right)$ divides the segment joining the points $P(2,-2)$ and $Q(3,7)$ in ratio $k:1$.

Using intersection formula $x=\frac{mx_2+nx_1}{m+1}$ we have

$$\frac{3k+2}{k+1} = \frac{24}{11}$$

$$33k+22 = 24k+24$$

$$9k = 2$$

$$k = \frac{2}{9}$$

Hence, $$y = \frac{-18+14}{11} = -\frac{4}{11}$$

20. Find the co-ordinates of the points which divide the line segment joining the points $(5,7)$ and $(8,10)$ in 3 equal parts.

Ans : [Outside Delhi Compt. Set-II, 2017]

Let $P(x_1,y_2)$ and $Q(x_2,y_2)$ trisect AB. Thus P divides AB in the ratio 1:2

As per question, line diagram is shown below.

Now $$x = \frac{1(8)+2(7)}{3} = 6$$

$$y = \frac{1(10)+2(7)}{3} = 8$$

Thus $P(x_1,y_1)$ is $P(6,8)$. Since Q is the mid point of PB, we have

$$x_1 = \frac{6+8}{2} = 7$$

$$y_1 = \frac{8+10}{2} = 9$$

Thus $Q(x_2,y_2)$ is $Q(7,9)$

21. Find the co-ordinates of a point on the axis which is equidistant from the points $A(2,-5)$ and $B(-2,9)$.

Ans : [Delhi Compt. Set-I, 2017]

Let the point P on the x axis be $(x,0)$. Since it is equidistant from the given points $A(2,-5)$ and $B(-2,9)$

$$PA = PB$$

$$PA^2 = PB^2$$

$$(x-2)^2+[0-(-5)]^2 = (x-(-2))^2+(0-9)^2$$

$$x^2-4x+4+25 = x^2+4x+4+81$$

$$-4x+29 = 4x+85$$

$$x = -\frac{56}{8} = -7$$

Hence the point on x axis is $(-7,0)$

22. The line segment joining the points $A(3,-4)$ and $B(1,2)$ is trisected at the points P and Q. Find the coordinate of the PQ.

Ans : [Delhi Compt. Set-II, 2017]

Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ trisect AB. Thus P divides AB in the ratio 1:2

As per question, line diagram is shown below.

Using intersection formula

$$x = \frac{1\times 1+2\times 3}{1+2} = \frac{7}{3}$$

$$y = \frac{1\times 2+2\times -4}{1+2} = -2$$

Hence point P is $\left(\frac{7}{3},-2\right)$

23. Show that ΔABC with vertices $A(-2,0)$, $B(0,2)$ and $C(2,0)$ is similar to ΔDEF with vertices $D(-4,0)$, $F(4,0)$ and $E(0,4)$.

Ans : [Board Foreign Set-I, II 2017], [Delhi Board Set-I, II, II, II 2017]

Using distance formula

$$AB = \sqrt{(0+2)^2+(2-0)^2} = \sqrt{4+4}$$
$$= 2\sqrt{2} \text{ units}$$

$$BC = \sqrt{(2-0)^2+(0-2)^2} = \sqrt{4+4}$$
$$= 2\sqrt{2} \text{ units}$$

$$CA = \sqrt{(-2,-2)^2+(0-0)^2} = \sqrt{16}$$
$$= 4 \text{ units}$$

and $$DE = \sqrt{(0+4)^2+(4-0)^2} = \sqrt{32}$$
$$= 4\sqrt{2} \text{ units}$$

$$EF = \sqrt{(4-0)^2+(0-4)^2} = \sqrt{32}$$
$$= 4\sqrt{2} \text{ units}$$

$$FD = \sqrt{(-4-4)^2+(0-0)^2} = \sqrt{64}$$
$$= 8 \text{ units}$$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{2\sqrt{2}}{4\sqrt{2}} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{4}{8} = \frac{1}{2}$$

Since Ratio of the corresponding sides of two similar Δ s is equal, we have

$$\Delta ABC \sim \Delta DEF$$ Hence Proved.

24. Find the co-ordinates of the point on the y-axis which is equidistant from the points $A(5,3)$ and $B(1,-5)$

Ans : [Delhi Compt. Set-III, 2017]

Let the points on y-axis be $P(0,y)$

Now $$PA = PB$$

$$PA^2 = PB^2$$
$$(0-5)^2+(y-3)^2 = (0-1)^2+(y+5)^2$$
$$5^2+y^2-6y+9 = 1+y^2+10y+25$$
$$16y = 8$$
$$y = \frac{1}{2}$$

Hence point on y-axis is $(0,\frac{1}{2})$.

25. In the given figure ΔABC is an equilateral triangle of side 3 units. Find the co-ordinates of the other two vertices.

Ans : [Board Foreign Set-I, II, 2017]

The co-ordinates of B will be $(2+3,0)$ or $.(5,0)$
Let co-ordinates of C be (x,y)

Since triangle is equilateral, we have

$$AC^2 = BC^2$$
$$(x-2)^2(y-0)^2 = (x-5)^2+(y-0)^2$$
$$x^2+4-4x+y^2 = x^2+25-10x+y^2$$
$$6x = 21$$
$$x = \frac{7}{2}$$

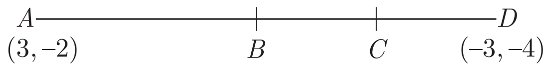
And $$(x-2)^2+(y-0)^2 = 9$$
$$\left(\frac{7}{2}-2\right)^2+y^2 = 9$$
$$\frac{9}{4}+y^2 = 9 \text{ or, } y^2 = 9-\frac{9}{4}$$
$$y^2 = \frac{27}{4} = \frac{3\sqrt{3}}{2}$$

Hence C is $\left(\frac{4}{3},\frac{3\sqrt{3}}{2}\right)$.

26. Find the co-ordinates of the points of trisection of the line segment joining the points $(3,-2)$ and $(-3,-4)$.

Ans : [Board Foreign Set-I, II, III 2017]

Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ trisect the line joining $A(3,-2)$ and $B(-3,-4)$.
As per question, line diagram is shown below.



Thus P divides AB in the ratio 1:2

Using intersection formula $x = \frac{mx_2+nx_1}{m+n}$ and $y = \frac{my_2+my_1}{m+n}$

$$x_1 = \frac{1(-3)+2(3)}{1+2} = 1$$

and $$y_1 = \frac{1(-4)+2(-2)}{1+2} = -\frac{8}{3}$$

Thus we have $x = 1$ and $y = -\frac{8}{3}$

Since Q is at the mid-point of PB, using mid-point formula

$$x_2 = \frac{1-3}{2} = -1$$

and $$y_2 = \frac{-\frac{8}{3}+(-4)}{2} = -\frac{10}{3}$$

Hence the co-ordinates of P and Q are $(1,-\frac{8}{3})$ and $(-1,-\frac{10}{3})$

27. If the distances of $P(x,y)$ from $A(5,1)$ and $B(-1,5)$ are equal, then prove that $3x = 2y$.

Ans : [Outside Delhi, Set-II, 2016]

Since $P(x,y)$ is equidistant from the given points $A(5,1)$ and $B(-1,5)$,

$$PA = PB$$
$$PA^2 = PB^2$$

Using distance formula,

$$(5-x)^2+(1-y)^2 = (-1-x)^2+(5-y)^2$$
$$(5-x)^2+(1-y)^2 = (1+x)^2+(5-y)^2$$
$$25-10x+1-2y = 1+2x+25-10y$$
$$-10x-2y = 2x-10y$$
$$8y = 12x$$
$$3x = 2y$$

Hence proved.

LONG ANSWER TYPE QUESTIONS

1. If $P(9a-2,-b)$ divides the line segment joining $A(3a+1,-3)$ and $B(8x,5)$ in the ratio 3:1. Find the values of a and b.

Ans : [Board Sample Paper, 2016]

Using section formula we have

$$9a-2 = \frac{3(8a)+1+(3a+1)}{3+1} \quad ...(1)$$
$$-b = \frac{3(5)+1(-3)}{3+1} \quad ...(2)$$

Form (2) $$-b = \frac{15-3}{4} = 3 \Rightarrow b = -3$$

From (1), $$9a-2 = \frac{24a+3a+1}{4}$$
$$4(9a-2) = 27a+1$$
$$36a-8 = 27a+1$$
$$9a = 9$$
$$a = 1$$

2. Find the coordinates of the point which divide the line segment joining $A(2,-3)$ and $B(-4,-6)$ into three equal parts.

Ans : [Board Sample paper, 2016]

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ trisect the line joining $A(3, -2)$ and $B(-3, -4)$.
As per question, line diagram is shown below.
P divides AB in the ratio of 1:2 and Q divides AB in the ratio 2:1.
By section formula

$$x_1 = \frac{mx_2 + nx_1}{1+2} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

$$P(x_1, y_1) = \left(\frac{1(-4) + 2(2)}{2+1}, \frac{2(-6) + 1(-3)}{2+1}\right)$$

$$= \left(\frac{-4+4}{3}, \frac{-6-(-6)}{3}\right)$$

$$= (0, -4)$$

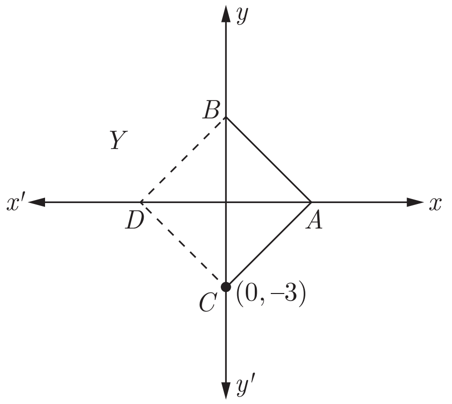
$$Q(x_2, y_2) = \left(\frac{2(-4) + 1(2)}{2+1}, \frac{2(-6) + 1(-3)}{2+1}\right)$$

$$= \left(\frac{-8+2}{3}, -\frac{12+(-3)}{3}\right) = (-2, -5)$$

3. The base BC of an equilateral triangle ABC lies on y-axis. The co-ordinates of point C are $(0,3)$. The origin is the mid-point of the base. Find the co-ordinates of the point A and B. Also find the co-ordinates of another point D such that $BACD$ is a rhombus.

Ans : [Foreign Set I, II, 2015]

As per question, diagram of rhombus is shown below.



Co-ordinates of point B are $(0,3)$

Thus $BC = 6$ unit

Let the co-ordinates of point A be $(x,0)$

Now $AB = \sqrt{x^2 + 9}$

Since $AB = BC$, thus

$$x^2 + 9 = 36$$

$$x^2 = 27 \Rightarrow x = \pm 3\sqrt{3}$$

Co-ordinates of point A is $\left(3\sqrt{3}, 0\right)$

Since $ABCD$ is a rhombus

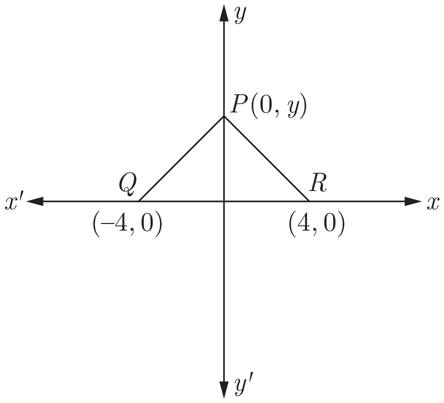
$$AB = AC = CD = DB$$

Thus co-ordinate of point D is $\left(-3\sqrt{3}, 0\right)$

4. The base QR of an equilateral triangle PQR lies on x-axis. The co-ordinates of point Q are $(-4,0)$ and the origin is the mid-point of the base. find the co-ordinates of the point P and R.

Ans : [Foreign set III, 2015]

As per question, line diagram is shown below.



Co-ordinates of point R is $(4,0)$

Thus $QR = 8$ units

Let the co-ordinates of point P be $(0, y)$

Since $PQ = QR$

$$(-4-0)^2 + (0-y)^2 = 64$$

$$16 + y^2 = 64$$

$$y = \pm 4\sqrt{3}$$

Coordinates of P are $\left(0, 4\sqrt{3}\right)$ or $\left(0, -4\sqrt{3}\right)$

TOPIC 2 : AREA OF TRIANGLE

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the area of the triangle with vertices $(0,0)(6,0)$ and $(0,5)$

Ans : [Board Term-2, 2015]

Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[0(0-5) + 6(5-0) + 0(0-0)]$$

$$= \frac{1}{2}[6 \times 5] = 15 \text{ sq. units}$$

2. If the points $A(x, 2), B(-3, -4), C(7, -5)$ are collinear, then find the value of x.
Since the points are collinear, then

$$\text{Area of triangle} = 0$$

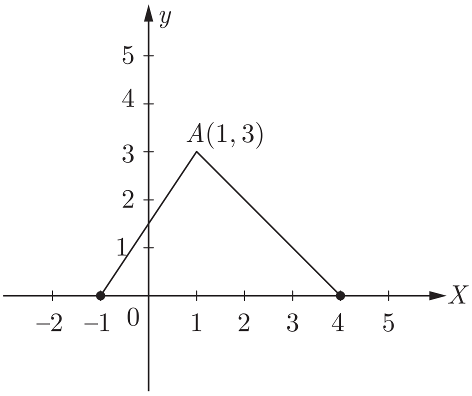
$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$[x(-4+5) + (-3)(-5-2) + 7(2+4)] = 0$$

$$x + 21 + 42 = 0$$

$$x = -63$$

3. In Fig., find the area of triangle ABC (in sq. units)?



Ans : [Board Term-2, 2013]

Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

$$= \frac{1}{2}[1(0-0)+(-1)(0-3)+4(3-0)]$$

$$= \frac{1}{2}[2+12] = \frac{15}{2} = 7.5 \text{ s, units}$$

4. If the point $(0,0),(1,2)$ and (x,y) are collinear, then find x.

Ans : [Board Term-2, 2011, Set A1]

The points are collinear, then area of triangle must be zero.

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$[0(2-y)+1(y-0)+x(0-2)] = 0$$

$$[y-2x] = 0$$

$$x = \frac{y}{2}$$

SHORT ANSWER TYPE QUESTIONS - I

1. Show that the points $A(0,1), B(2,3)$ and $C(3,4)$ are collinear.

Ans : [CBSE Term-2, 2016 Set-HODM4OL]

If the area of the triangle formed by the points is zero, then points are collinear.
We have $A(0,1), B(2,3)$ and $C(3,4)$

$$\Delta = \frac{1}{2}|0(3-4)+2(4-1)+3(1-3)|$$

$$= \frac{1}{2}|0+(2)(3)+(3)(-2)|$$

$$= \frac{1}{2}|6-6| = 0$$

Thus given points are collinear.

2. Prove that the points $(2,-2),(-2,1)$ and $(5,2)$ are the vertices of a right angled triangle. Also find the area of this triangle.

Ans : [Foreign Set I, II, III, 2016]

We have $A(2,-2), B(-2,1)$ and $(5,2)$

Applying distance formula we get

$$AB^2 = (2+2)^2+(-2-1)^2$$

$$= 16+9 = 25$$

Thus $AB = 5$

Similarly $AC^2 = (-2-5)^2+(1-2)^2$

$$= 49+1 = 50$$

$$BC^2 = 50 \Rightarrow BC = 5\sqrt{2}$$

$$AC^2 = (2-5)^2+(-2-2)^2$$

$$= 9+16$$

$$= 25$$

$$AC^2 = 25 \Rightarrow AC = 5$$

Clearly $AB^2 + AC^2 = BC^2$

$$25+25 = 50$$

Hence the triangle is right angled,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times Base \times Height$$

$$= \frac{1}{2} \times 5 \times 5 = \frac{25}{2} sq. unit.$$

3. Find the relation between x and y, if the point $A(x,y), B(-5,7)$ and $C(-4,5)$ are collinear.

Ans : [Outside Delhi CBSE Board, 2015, Set I, II, III]

If the area of the triangle formed by the points is zero, then points are collinear.

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$[x(7-5)-5(5-y)-4(y-7)] = 0$$

$$2x-25+5y-4y+28 = 0$$

$$2x+y+3 = 0$$

4. For what values of k are the points $(8,1),(3,-2k)$ and $(k,-5)$ collinear?

Ans : [Foreign Set I, II, III 2015]

Since points $(8,1),(3,-2k)$ and $(k,-5)$ are collinear, area of triangle formed must be zero.

$$\frac{1}{2}[8(-2k+5)+3(-5,-1)+k(1+2k)] = 0$$

$$2k^2-15k+22 = 0$$

$$k = 2, \frac{11}{2}$$

SHORT ANSWER TYPE QUESTIONS - II

1. Find the value of p, if the points $A(2,3), B(4,p)$, $C(6,-3)$ are collinear.

Ans : [Baord Term-2, 2012 sEt (17)]

Since points $A(2,3), B(4,p)$ and $C(6,-3)$ are collinear, area of triangle formed must be zero.

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$[2(p+3)+4(-3-3)+6(3-p)] = 0$$

$$\begin{aligned}[2p+6-24+18-6p] &= 0\\ [-4p] &= 0\\ 4p &= 0\\ p &= 0\end{aligned}$$

2. If $(5,2),(-3,4)$ and (x,y) are collinear, show that $x+4y-13=0$

Ans : [CBSE Board Term-2, 2015]

Since points $(5,2),(-3,4)$ and (x,y) are collinear, area of triangle formed must be zero.

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

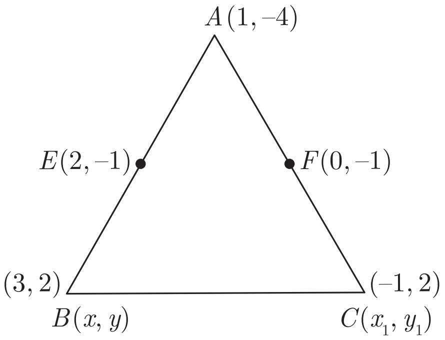
$$\begin{aligned}[5(4-y)+(-3)(y-2)+x(2-4)] &= 0\\ [20-5y-3y+6+(-2x)] &= 0\\ [-2x-8y+26] &= 0\\ x+4y-13 &= 0\end{aligned}$$

Hence proved

3. Find the area of a triangle ABC with $A(1,-4)$ and mid-points of sides through A being $(2,-1)$ and $(0,-1)$.

Ans : [Delhi CBSE Board, 2015, Set I, III]

Let $B(x_1,y_1)$ and $C(x_2,y_2)$ be other vertices of triangle. As per question, triangle is shown below.



Let $E(2,-1)$ be the mid point of AB and $F(0,-1)$ be the mid point of AC.

Now $\dfrac{x_1+1}{2} = 2 \Rightarrow x_1 = 3$

and $\dfrac{y_1+(-4)}{2} = -1 \Rightarrow y_2 = 2$

Thus point B is $(3,2)$.

Again $\dfrac{x_2-1}{2} = 0 \Rightarrow x_1 = -1$

$\dfrac{y_2+(-4)}{2} = -1 \Rightarrow y_1 = 2$

Thus point C is $(-1,2)$

Now the co-ordinates are $A(1,-4), B(3,2), C(-1,2)$

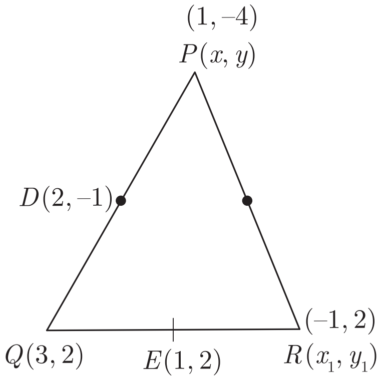
Area of triangle

$$\begin{aligned}\Delta &= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\\ &= \frac{1}{2}[1(2-2)+3(2+4)-1(-4-2)]\\ &= \frac{1}{2}[0+18+6] = 12 \text{ sq. units}\end{aligned}$$

4. Find the area of the triangle PQR with $Q(3,2)$ and mid-points of the sides through Q being $(2,-1)$ and $(1,2)$.

Ans : [Delhi CBSE Board, 2015 Set III]

Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ be other vertices of triangle. As per question, triangle is shown below.



Let $D(2,-1)$ be the mid point of PQ and $E(1,2)$ be the mid point of AC.

Let the co-ordinate of p be (x,y) and $R(x_1,y_1)$

Now $\dfrac{x_1+3}{2} = 2 \Rightarrow x_1 = 1$

$\dfrac{y_1+2}{2} = -1 \Rightarrow y_2 = -4$

Thus point is $P(1,-4)$

Again $\dfrac{x_2+3}{2} = 1 \Rightarrow x_1 = -1$

$\dfrac{y_2+2}{2} = 2 \Rightarrow y_1 = 2$

Thus point is $R(-1,2)$

Now we have $P(1,-4), Q(3,2), R(-1,2)$

Area of triangle

$$\begin{aligned}\Delta &= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\\ &= \frac{1}{2}[1(2-2)+3(2+4)+(-1)(-4-2)]\\ &= \frac{1}{2}[0+18+6] = \frac{1}{2}\times 24 = 12 \text{ sq. units}\end{aligned}$$

5. If the points $A(-2,1), B(a,b)$ and $C(4,1)$ are collinear and $a-b=1$, find a and b.

Ans : [Delhi CBSE Term-2, 2014]

If three points are collinear, then area covered by given points must be zero.

Thus area

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$\begin{aligned}\frac{1}{2}[-2(b-1)+a(1-1)+4(1-b)] &= 0\\ [-2b+2+0+4(1-b)] &= 0\\ -6b+6 &= 0 \Rightarrow b = 1\end{aligned}$$

Substituting $b=1$ in given condition $a-b=1$ we have

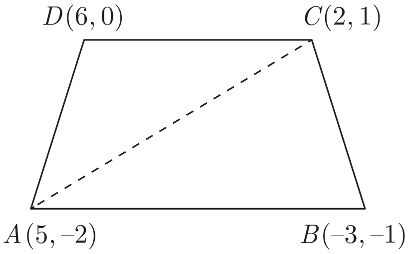
$$a - 1 = 1$$
$$a = 2$$

This $a = 2$ and $b = 1$.

6. Find the area of the quadrilateral $ABCD$, the co-ordinates of whose vertices are $A(5, -2), B(-3, -1)$, $C(2,1)$ and $D(6,0)$.

Ans : [Delhi Set, 2014], [Board Term-2, 2012 set (13)]

As per question the quadrilateral $ABCD$ is shown below.



Area of quadrilateral

$$= \Delta_{ABC} + \Delta_{ADC}$$

$$ABCD = ar(\Delta\, ABC) + ar(ADC)$$

$$\text{Area}_{ABCD} = \frac{1}{2}[(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2)]$$

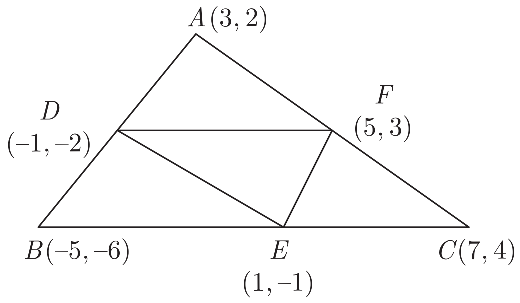
$$+(x_3 y_4 - x_4 y_3) + (x_4 y_1 - x_1 y_4)$$

$$= \frac{1}{2}[5(-1) - (-2)(-3) + (-3)(1)$$

$$-(-1)(2) + (2 \times 0 - 1 \times 6) + 6(-2) - (0 \times 5)$$

$$= \frac{1}{2}[-30] = |-15| = 15 \text{ sq. units}$$

7. In the given triangle ABC as shown in the diagram D, E and F are the mid-points of AB, BC and AC respectively. Find the area of $\Delta\, DEF$.



Ans : [Board Term-2, 2012 Set (5)]

Mid-point BA $\quad x_D = \frac{3 + -5}{2} = -1$

and $\quad y_D = \frac{2-6}{2} = -2$

Thus point D is $(-1, -2)$

Mid-point BC, $\quad x_E = \frac{-5+7}{2} = 1$

and $\quad y_E \; \frac{-6+4}{2} = -1$

Thus point is E is $(1, -1)$.

Mid- Point CA, $\quad x_F = \frac{7+3}{2} = 5$

$$y_F = \frac{4+2}{2} = 3$$

Thus point F is $(5,3)$

Now, area $\Delta\, DEF$

$$\Delta = \frac{1}{2}[-(-1-3) + 1(3+2) + 5(-2+1)]$$

$$= \frac{1}{2}[4 + 5 - 5]$$

$$= 2 \text{ Unit}$$

8. Find the area of the triangle formed by joining the mid-points of the sides of a triangle, whose co-ordinates of vertices are $(0, -1), (2,1)$ and $(0,3)$.

Ans : [Outside Delhi Compt. Set I, III 2017]

Let the vertices of given triangle be $A(0, -1), B(2,1)$ and $C(0,3)$. As per question the triangle is shown below.

Let the coordinates of mid-points

$$P = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1, 0)$$

$$Q = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

$$Q = \left(\frac{0+0}{2}, \frac{-1+3}{2}\right) = (0, 1)$$

Area of $\Delta\, PQR$

$$\Delta = \frac{1}{2}[x_1(y_2 - y_1) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[(2-1) + 1(1-0) + 0(0-2)]$$

$$= \frac{1}{2}(1 + 1 + 0) = 1 \text{ sq. units}$$

9. The area of a triangle is 5 sq. units. Two of its vertices are $(2,1)$ and $(3, -2)$. If the third vertex is $(\frac{7}{2}, y)$, Find the value of y.

Ans : [Delhi Set II 2017]

We have $\Delta\, ABC = 5$ sq. units

$$\frac{1}{2}\left[2(-2-y) + (y-1) + \frac{7}{2}(1+2)\right] = 5$$

$$\frac{1}{2}\left[-4 - 2y + 3y - 3 + \frac{21}{2}\right] = 5$$

$$y + \frac{7}{2} = 10$$

$$y = 10 - \frac{7}{2} = \frac{13}{2}$$

If we consider possibility of negative area then, we have

$$y + \frac{7}{2} = -10$$

$$y = -10 - \frac{7}{2} = -\frac{27}{2}$$

Hence the value of y is $\frac{13}{2}$ or $-\frac{27}{2}$

LONG ANSWER TYPE QUESTIONS

1. Prove that the area of a triangle with vertices $(t, t-2), (t+2, t+2)$ and $(t+3)$ is independent of t.

Ans : [Delhi Set I, II, III, 2016]

Area of the triangle

$$\Delta = \frac{1}{2}|\, t(t+2-t)+(t+2)(t-t+2)+ +(t+3)(t-2-t-2)|$$

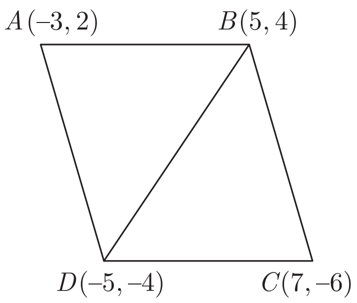
$$= \frac{1}{2}[2t+2t+4-4t-12]$$

$$= 4 \text{ sq. units. which is independent of } t.$$

2. Find the area of a quadrilateral $ABCD$, the co-ordinates of whose vertices are $A(-3,2), B(5,4), C(7,-6)$ and $D(-5,-4)$.

Ans : [Foreign Set III, 2016]

As per question the quadrilateral is shown below.



Area of triangle ABD

$$\Delta_{ABD} = \frac{1}{2}|-3(8)+5(-6)+-5(2-4)|$$

$$= 22 \text{ sq. units}$$

Area of triangle BCD

$$\Delta_{BCD} = \frac{1}{2}|5(-2)+7(-8)-5(10)|$$

$$= 58 \text{ sq. units}$$

$$\text{Area}_{ABCD} = \Delta_{ABD}+\Delta_{BCD}$$

$$= 22+58 = 80 \text{ sq. units}$$

3. If $A(-4,8), B(-3,-4), C(0,-5)$ and $D(5,6)$ are the vertices of a quadrilateral $ABCD$, find its area.

Ans : [Delhi CBSE Board, 2015 Set I, III]

We have $A(-4,8)$, $B(-3,-4)$, $C(0,5)$ and $D(5,6)$

Area of quadrilateral

$$= \frac{1}{2}[(x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+(x_3y_4-x_4y_3) +(x_4y_1-x_1y_4)]$$

$$\text{Area} = \frac{1}{2}[\{-4\times(-4)-(-3)(8)\}+\{(-3)(-5)-0 \times(-4)\}+\{0\times 6-5(-5)\}+[\{5\times 8-(-4)(6)\}]$$

$$= \frac{1}{2}[16+24+15-0+0+25+40+24]$$

$$= \frac{1}{2}[40+15+25+40+24] = \frac{1}{2}\times 144 = 72 \text{ sq. units}$$

4. If $P(-5,-3), Q(-4,-6), R(2,-3)$ and $S(1,2)$ are the vertices of a quadrilateral $PQRS$, find its area.

Ans : [Delhi CBSE Board, 2015 Set II]

We have $P(-5,-3)$, $Q(-4,-6)$, $R(2,-3)$ and $S(1,2)$

Area of quadrilateral

$$= \frac{1}{2}[(x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+(x_3y_4-x_4y_3) +(x_4y_1-x_1y_4)]$$

Area

$$= \frac{1}{2}[-5(-6)-(-4)(-3)+(-4)(-3)-2(-6) +(2)(2)-1\times(-3)+1\times(-3)-(-5)(2)]$$

$$= \frac{1}{2}[30-12+12+12+4+3-3+10]$$

$$= \frac{1}{2}[30+12+4+10] = \frac{1}{2}[56] = 28 \text{ sq. units}$$

5. Find the values of k so that the area of the triangle with vertices $(1,-1),(-4,2k)$ and $(-k,-5)$ is 24 sq. units.

Ans : [Outside Delhi CBSE Board, 2015, Set I]

We have $(1,-1),(-4,2k)$ and $(-k,-5)$

Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

$$24 = \frac{1}{2}[1(2k+5)-4(-5+1)-k(-1-2k)]$$

$$48 = 2k+5+16+k+2k^2$$

$$2k^2+3k-27 = 0$$

$$(k-3)(2k+9) = 0$$

$$k = 3, -\frac{9}{2}$$

6. Find the values of k so that the area of the triangle with vertices $(k+1,1),(4,-3)$ and $(7,-k)$ is 6 sq. units.

Ans : [Outside Delhi CBSE Board, 2015, Set I]

We have $(k+1,1),(4,-3)$ and $(7,-k)$

Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

$$6 = \frac{1}{2}[(k+1)(-3+k)+4(-k-1)+7(1+3)]$$

$$12 = [k^2-2k-3-4k-4+28]$$

$$12 = k^2-6k+21$$

$$k^2-6k+9 = 0$$

$$(k-3)(k-3) = 0$$

$$k = 3,3$$

7. Find the values of k for which the points $A(k+1,2k)$, $B(3k,2k+3)$ and $C(5k-1,5k)$ are collinear.

Ans : [Outside Delhi CBSE Board, 2015, Set III]

If three points are collinear, then area covered by given points must be zero.

$$=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$[(k+1)(2k+3-5k)+3k(5k-2k)+$$
$$+(5k-1)(2k-2k-3) = 0$$
$$-3k^2+3k-3k+3+9k^2-15k+3=0 = 0$$
$$6k^2-15k+6 = 0$$
$$2k^2-5k+2 = 0$$
$$(k-2)(2k-1) = 0$$

Thus $k = 2$ or $k = \frac{1}{2}$

8. The vertices of quadrilateral $ABCD$ are $A(5,-1)$, $B(8,3)$, $C(4,0)$ and $D(1,-4)$. Prove that $ABCD$ is a rhombus.

Ans : [Board Term-2, 2015]

The vertices of the quadrilateral $ABCD$ are $A(5,-1)$, $B(8,3)$, $C(4,0)\,D(1,-4)$.

Now $AB = \sqrt{(8-5)^2+(3+1)^2}$

$= \sqrt{3^2+4^2} = 5$ units

$BC = \sqrt{(8-4)^2+(3-0)^2}$

$= \sqrt{4^2+3^2} = 5$ units

$CD = \sqrt{(4-1)^2+(0+4)^2}$

$= \sqrt{(3)^2+(4)^2} = 5$ units

$AD = \sqrt{(5-1)^2+(-1+4)^2}$

$= \sqrt{(4)^2+(3)^2} = 5$ units

Diagonal, $AC = \sqrt{(5-4)^2+(-1-0)^2}$

$= \sqrt{1^2+1^2} = \sqrt{2}$ units

Diagonal $BD = \sqrt{(8-1)^2+(3+4)^2}$

$= \sqrt{(7)^2+(7)^2} = 7\sqrt{2}$ units

As the length of all the sides are equal but the length of the diagonals are not equal. Thus $ABCD$ is not square but a rhombus.

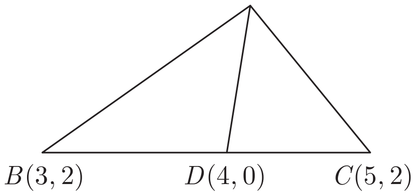
9. $A(4,-6), B(3,-2)$ and $C(5,2)$ are the vertices of a $\Delta\,ABC$ and AD is its median. Prove that the median AD divides $\Delta\,ABC$ into two triangles or equal areas.

Ans : [CBSE O.D. 2014]

Since AD is the median of $\Delta\,ABC$ from vertex A, we have

$$D(x,y) = \left(\frac{3+5}{2}+\frac{-2+2}{2}\right) = (4,0)$$

As per question statement triangle is shown below.



Area of $\Delta\,ADB$,

$$\Delta_{ADB} = \frac{1}{2}\times(4(0+2)+(-2+6)+3(-6-0))$$

$$=\frac{1}{2}\times(8+16+-18)$$

$$=\frac{1}{2}\times 3 = 3 \text{ square units} \quad (1)$$

Area of $\Delta\,ACB$

$$\Delta_{ACB} = \frac{1}{2}\times(4(0-2)+4(2+6)+5(-6-0))$$

$$=\frac{1}{2}\times(-8+32-30)$$

$$=\frac{1}{2}\times -6 = -3$$

Since area can not be negative, we take positive value.

Thus $\Delta_{ACB} = 3$ square units (2)

From (1) and (2) we seen that $\Delta_{ADB} = \Delta_{ACB}$. It is verified that median of $\Delta\,ABC$ divides it into two triangles of equal areas.

10. The co-ordinates of vertices of $\Delta\,ABC$ are $A(0,0), B(0,2)$ and $C(2,0)$. Prove that $\Delta\,ABC$ is an isosceles triangle. Also find its area.

Ans : [Board Term-2, 2014]

Using distance formula $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ we have

$$AB = \sqrt{(0-0)^2+(0-2)^2} = \sqrt{4} = 2$$
$$AC = \sqrt{(0-2)^2+(0-0)^2} = \sqrt{4} = 2$$
$$BC = \sqrt{(0-2)^2+(2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$$

Clearly, $AB = AC \neq BC$

Thus $\Delta\,ABC$ is an isosceles Triangle

Now, $AB^2+AC^2 = 2^2+2^2 = 4+4 = 8$

also, $BC^2 = (2\sqrt{2})^2 = 8$

$AB^2+AC^2 = BC^2$

Thus $\Delta\,ABC$ is an isosceles right angled triangle.

Now, area of $\Delta\,ABC$

$$\Delta_{ABC} = \frac{1}{2} base \times height$$

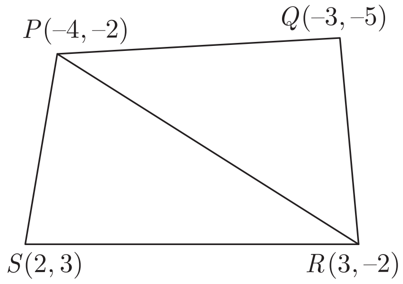
$$=\frac{1}{2}\times 2\times 2$$

$$= 2 \text{ sq. units.}$$

11. Find the area of the quadrilateral $PQRS$. The co-ordinates of whose vertices are $P(-4,-2), Q(-3,-5)$, $R(3,-2)$ and $5(2,3)$.

Ans : [Outside Delhi Set-II, 2017]

As per question quadrilateral $PQRS$ is shown below.



Area $\square_{PQRS} = \Delta_{PQR} + \Delta_{PRS}$

Area ΔPQR

$$\Delta_{PQR} = \frac{1}{2}[x_1(y_2-y_1)+x_2(y_3-y_1)+x_2(y_1-y_2)]$$

$$=\frac{1}{2}[-4(-2-(-5))+3(-5-(-2))+-3(-2-(-2))]$$

$$=\frac{1}{2}[-4\times 3+3\times -3+3\times 0]$$

$=\frac{1}{2}\times(12+9)=\frac{21}{2}$ sq. units

Area ΔPRS

$$\Delta_{PRS}=\frac{1}{2}[-4(-2-3)+3(3+2)+2(-2+2)]$$

$$=\frac{1}{2}[-4\times -5+3\times 5+0]$$

$=\frac{1}{2}\times(20+15)=\frac{35}{2}$ sq. units

Area $\square_{PQRS} = \frac{21}{2}+\frac{35}{2} = 28$ sq. units

12. If the co-ordinates of two points are $A(3,4), B(5,-2)$ and a point $P(x,5)$ is such that $PA = PB$ then find the area of ΔPAB.

Ans : [Outside Delhi Compt. Set-I, 2017]

Since $PA = PB$

$PA^2 = PB^2$

Using distance formula we have

$$(x-3)^2+(5-4)^2 = (x-5)^2+(5+2)^2$$

$$x^2-6x+9+1 = x^2-10x+25+49$$

$$10x-6x = 74-10$$

$$x = 16$$

Thus area ΔPAB

$$\Delta_{PAB} = \frac{1}{2}[16(4+2)+3(-2-5)+5(5-4)]$$

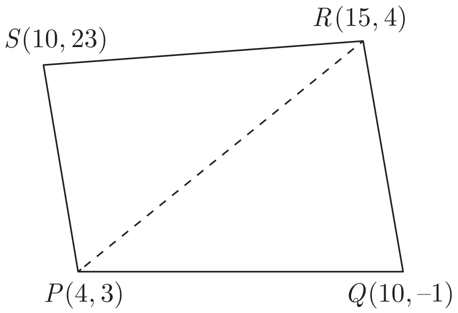
$$=\frac{1}{2}[96-21+5]=40$$

Hence, area of triangle is 40 sq. units

13. Find the area of a quadrilateral $PQRS$ whose vertices are $P(4,3), Q(10,-1), R(15,4)$ and $S(10,23)$.

Ans : [Delhi Compt. Set III 2017]

As per question quadrilateral $PQRS$ is shown below.



Area $\square_{PQRS} = \Delta_{PQR} + \Delta_{PRS}$

Area ΔPQR

$$\Delta_{PQR} = \frac{1}{2}[x_1(y_2-y_1)+x_2(y_3-y_1)+x_2(y_1-y_2)]$$

$$=\frac{1}{2}[4(-5)+10(1)+15(4)]$$

$=\frac{1}{2}\times 50 = 25$ sq. units

Area ΔPRS

$$\Delta_{PRS} = \frac{1}{2}[4(-19)+15(20)+10(-1)]$$

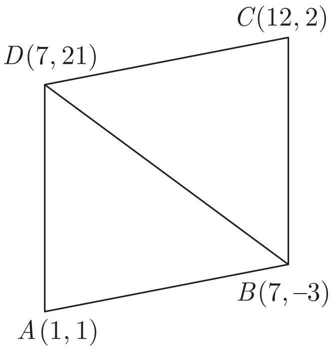
$=\frac{1}{2}\times 214 = 107$ sq. units

Area $\square_{PQRS} = 25+107 = 132$ sq. unit

14. Find the area of a quadrilateral $ABCD$, whose vertices are $A(1,1), B(7,-3), C(12,2)$ and $D(7,21)$.

Ans : [Delhi Compt. Set I 2017]

As per question quadrilateral $ABCD$ is shown below.



Area of quadrilateral $ABCD$

$$\square_{ABCD} = \Delta_{ABD} + \Delta_{BCD}$$

Area ΔABD,

$$\Delta_{ABD} = \frac{1}{2}[1(-3-21)+7(21-1)+7(1+3)]$$

$$=\frac{1}{2}[-24+7\times 20+7\times 4]$$

$$=\frac{1}{2}[-24+140+28]$$

$=\frac{1}{2}\times 144 = 72$ sq. units

Area ΔBCD,

$$\Delta_{BCD} = \frac{1}{2}[7(2-21)+12(21+3)+7(-3-2)]$$

$$=\frac{1}{2}[7\times -19+12\times 24+7\times -5]$$

$$=\frac{1}{2}[-133+288-35]$$

$$=\frac{1}{2}[288-168]$$

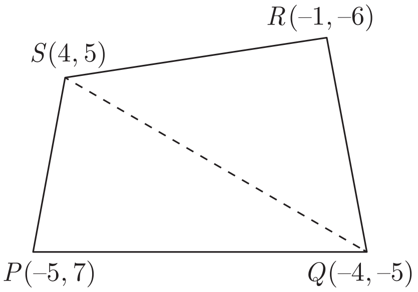
$=\frac{1}{2}\times 120 = 60$ sq. units

Area $\square_{ABCD} = 72+60 = 132$ sq. units.

15. Find the area of a quadrilateral $PQRS$ whose vertices area $P(-5,7), R(-1,-6)$ and $S(4,5)$

Ans : [Delhi Compt. Set II, 2017]

As per question quadrilateral $PQRS$ is shown below.



Area $\square_{PQRS} = \Delta_{PQR} + \Delta_{QRS}$

Area $\Delta\, PQR$

$$\Delta_{PQR} = \frac{1}{2}[x_1(y_2-y_1)+x_2(y_3-y_1)+x_2(y_1-y_2)]$$

$$= \frac{1}{2}[-5(-5-5)+-4(5-7)+4(7+5)]$$

$$= \frac{1}{2}[50+8+48]$$

$$= \frac{1}{2} \times 106 = 53 \text{ sq. units.}$$

Area $\Delta\, QRS$

$$\Delta_{QRS} = \frac{1}{2}[-4(-6-5)+-1(5+5)+4(-5+6)]$$

$$= \frac{1}{2}[44+(-10)+4]$$

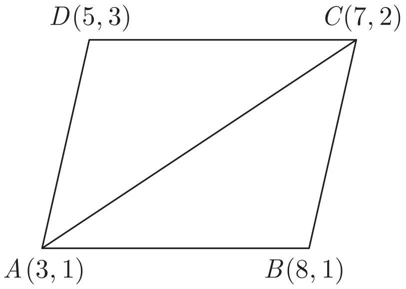
$$= \frac{1}{2} \times 38 = 19 \text{ sq. units}$$

Area $\square_{PQRS} = 53+19 = 72$ sq. units

16. Find the area of the quadrilateral whose vertices are $A(3,1), B(8,1), C(7,2)$ and $D(5,3)$

Ans : [Delhi Compt. Set II 2017]

As per question quadrilateral $ABCD$ is shown below.



Area of quadrilateral $ABCD$

$$\square_{ABCD} = \Delta_{ABC} + \Delta_{ACD}$$

Area of triangle

$$\Delta = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_2(y_1-y_2)]$$

Area $\Delta\, ABC$

$$\Delta_{ABC} = \frac{1}{2}[3(1-2)+8(2-1)+7(1-1)]$$

$$= \frac{1}{2}(3 \times -1 + 8 \times 1 + 7 \times 0)$$

$$= \frac{1}{2}[-3+8] = \frac{5}{2} \text{ sq. units.}$$

Area $\Delta\, ACD$

$$\Delta_{ACD} = \frac{1}{2}[3(2-3)+7(3-1)+5(1-2)]$$

$$= \frac{1}{2}[3 \times -1 + 7 \times 2 + 5 \times -1]$$

$$= \frac{1}{2}[-3+14-5]$$

$$= 3 \text{ units}$$

Area $\square_{ABCD} = \frac{5}{2} + 3 = \frac{11}{2}$ sq. units.

HOTS QUESTIONS

1. Find the ratio is which the line segment joining the points $A(3,-3)$ and $B(-2,7)$ is divided by x-axis. Also find the co-ordinates of the point of division.

Ans : [CBSE O.D. 2014]

We have $A(3,-3)$ and $B(-2,7)$

At any point on x-axis y-coordinate is always zero.

So, let the point be $(x,0)$ that divides line segment AB in ratio $k:1$.

Now $$(x,0) = \left(\frac{-2k+3}{k+1}, \frac{7k-3}{k+1}\right)$$

$$\frac{7k-3}{k+1} = 0$$

$$7k-3 = 0 \Rightarrow k = \frac{3}{7}$$

The line is divided in the ratio of 3 : 7

Now $$\frac{-2k+3}{k+1} = x$$

$$\frac{-2 \times \frac{3}{7}+3}{\frac{3}{7}+1} = x$$

$$\frac{-6+21}{3+7} = x$$

$$\frac{15}{10} = x$$

$$x = \frac{3}{2}$$

The coordinates of the point is $\left(\frac{3}{2},0\right)$.

2. Determine the ratio in which the straight line $x-y-2=0$ divides the line segment joining $(3,-1)$ and $(8,9)$.

Ans : [Board Term-2, 2012 Set (44)]

Let co-ordinates of P be (x_1,y_1) and it divides line AB in the ratio $k:1$.

$$\text{Now} \qquad x_1 = \frac{8k+3}{k+1}$$

$$y_1 = \frac{9k-1}{k+1}$$

Since point $P(x_1,y_1)$ lies on line $x-y-2=0$, so co-ordinates of P must satisfy the equation of line.

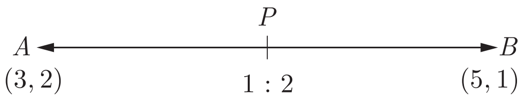
$$\begin{aligned} \text{Thus} \qquad \frac{8k+3}{k+1}-\frac{9k-1}{k+1}-2 &= 0 \\ 8k+3-9k+1-2k-2 &= 0 \\ -3k+2 &= 0 \\ k &= \frac{2}{3} \end{aligned}$$

So, line $x-y-2=0$ divides AB in the ratio $2:3$

3. The line segment joining the points $A(3,2)$ and $B(5,1)$ is divided at the point P in the ratio $1:2$ and P lies on the line $3x-18y+k=0$. Find the value of k.

Ans : [Board Term-2, 2012 Set (I)]

Let co-ordinates of P be (x_1,y_1) and it divides line AB in the ratio $1:2$.



$$x_1 = \frac{mx_2+nx_1}{m+n} = \frac{1\times5+2\times3}{1+2} = \frac{11}{3}$$

$$y_2 = \frac{my_2+ny_1}{m+n} = \frac{1\times2+2\times2}{1+2} = \frac{5}{3}$$

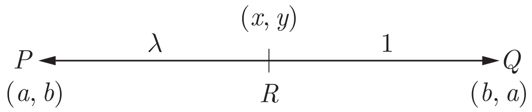
Since point $P(x_1,y_1)$ lies on line ,$3x-18y+k=0$, so co-ordinates of P must satisfy the equation of line.

$$\begin{aligned} 3\times\frac{11}{3}-18\times\frac{5}{3}+k &= 0 \\ k &= 19 \end{aligned}$$

4. If $R(x,y)$ is a point on the line segment joining the points $P(a,b)$ and $Q(b,a)$, then prove that $x+y=a+b$.

Ans : [Board Term-2, 2012 Set (28)]

As per question line is shown below.



Let point $R(x,y)$ divides the line joining P and Q in the ratio $k:1$, then we have

$$x = \frac{kb+a}{k+1}$$

$$\text{and} \qquad y = \frac{ka+b}{k+1}$$

$$\begin{aligned} \text{Adding,} \qquad x+y &= \frac{kb+a+ka+b}{k+1} \\ &= \frac{k(a+b)+(a+b)}{k+1} \\ &= \frac{(k+1)(a+b)}{k+1} = a+b \\ x+y &= a+b \end{aligned}$$

Hence Proved

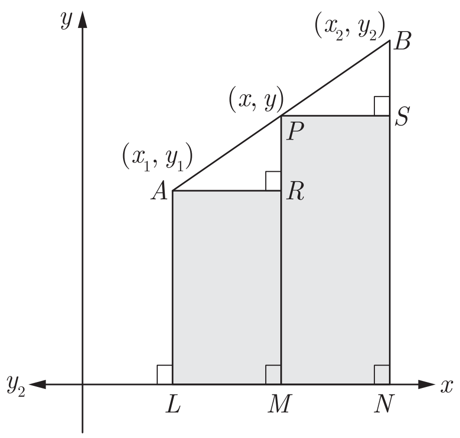
5. (i) Derive section formula.

(ii) In what ratio does $(-4,6)$ divides the line segment joining the point $A(-6,4)$ and $B(3,-8)$

Ans : [KVS 2014]

(i) Section Formula : Let $A(x_1,y_1)$ and $B(x_2,y_2)$ are two points. Let $P(x,y)$ be a point on line, joining A and B, such that P divides it in the ratio $m_1:m_2$.

$$\text{Now} \qquad (x,y) = \left(\frac{m_2x_1+m_1x_2}{m_1+m_2},\frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$$



Proof : Let AB be a line segment joining the points. $A(x_1,y_1), B(x_2,y_2)$.

Let P divides AB in the ratio $m_1:m_2$. Let P have co-ordinates (x,y).

Draw $AL, PM, PN, \perp$ to x-axis

It is clear form figure, that

$$\begin{aligned} AR &= LM = OM-OL = x-x_1 \\ PR &= PM-RM = y-y_1. \\ \text{also,} \qquad PS &= ON-OM = x_2-x \\ BS &= BN-SN = y_2-y \end{aligned}$$

$$\text{Now} \qquad \Delta APR \sim \Delta PBS \qquad [AAA]$$

$$\text{Thus} \qquad \frac{AR}{PS} = \frac{PR}{BS} = \frac{AP}{PB}$$

$$\text{and} \qquad \frac{AR}{PS} = \frac{AP}{PB}$$

$$\begin{aligned} \frac{x-x_1}{x_2-x} &= \frac{m_1}{m_2} \\ m_2x-m_2x_1 &= m_1x_2-m_1x \\ x &= \frac{m_1x_2+m_2x_1}{m_1+m_2} \end{aligned}$$

$$\text{Now} \qquad \frac{PR}{BS} = \frac{AP}{PB}$$

$$\frac{y-y_2}{y_2-y} = \frac{m_1}{m_2}$$

$$y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

Thus co-ordinates of P are $\left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$

(ii) Assume that $(-4,6)$ divides the line segment joining the point $A(-6,4)$ and $B(3,-8)$ in ratio $k:1$

Using section formula for x co-ordinate we have

$$-4 = \frac{k(3)-6}{k+1}$$

$$-4k-4 = 3k-6 \Rightarrow k = \tfrac{2}{7}$$

6. If the points $A(0,1), B(6,3)$ and $C(x,5)$ are the vertices of a triangle, find the value of x such that area of $\Delta ABC = 10$

Ans : [CBSE S.A.2 2016 HODM4OL]

We have $A(0,1), B(6,3)$ and $C(x,5)$
Since area of the triangle ABC is 10, we have

$$\tfrac{1}{2}[0(3-5)+6(5-1)+x(1-3)] = 10$$

$$\tfrac{1}{2}[0+24-2x] = 10$$

Here area may be negative also. So we have to consider the negative area also.
For positive area

$$24-2x = 20 \Rightarrow x = 2$$

For negative area,

$$24-2x = -20 \Rightarrow x = 22$$

7. The co-ordinates of the points A, B and C are $(6,3), (-3,5)$ and $(4,-2)$ respectively. $P(x,y)$ is any points in the plane. Show that $\frac{ar(\Delta PBC)}{ar(\Delta ABC)} = \left|\frac{x+y-2}{7}\right|$

Ans : [Foreign Set I, 2016]

We have $A(6,3), B(-3,5), C(4,-2)$ and $P(x,y)$

Area of ΔPBC,

$$\text{ar}(\Delta PBC) = \tfrac{1}{2}|x(7)+3(2+y)+4(y-5)|$$

$$= \tfrac{1}{2}|7x+7y-14|$$

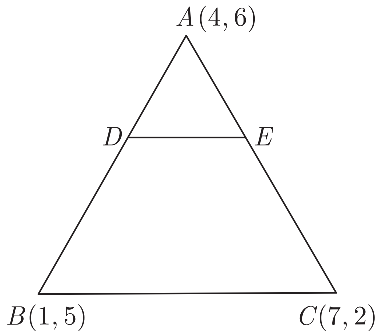
Area of ΔABC,

$$\text{ar}(\Delta ABC) = \tfrac{1}{2}|6\times 7-3(-5)+4(3-5)| = \frac{49}{2}$$

Thus $$\frac{\text{ar}(\Delta PBC)}{\text{ar}(\Delta ABC)} = \frac{\frac{1}{2}(7x+7y-14)}{\frac{49}{2}}$$

$$= \frac{7(x+y-2)}{49} = \left|\frac{x+y-2}{7}\right|$$

8. In the given figure, the vertices of ΔABC are $A(4,6)$, $B(1,5)$ and $C(7,2)$. A line-segment DE is drawn to intersect sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of ΔADE and compare it with area of ΔABC.



Ans : [O.D. Set I, II, III, 2016]

Area of a triangle having vertices $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3) is given by

$$\Delta = \tfrac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

Thus area of triangle ABC is,

$$\Delta_{ABC} = \tfrac{1}{2}[4(5-2)+1(2-6)+7(6-5)]$$

$$= \tfrac{1}{2}[12+(-4)+7] = \frac{15}{2} \text{ sq units}$$

In ΔADE and ΔABC, we have

$$\frac{AD}{AB} = \frac{AE}{EC} = \frac{1}{3}$$

and $\angle DAE = \angle BAC$

Hence $\Delta DAE \sim \Delta ABC$

Now $$\frac{\Delta_{ADE}}{\Delta_{ABC}} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$\frac{\Delta_{ADE}}{\frac{15}{2}} = \frac{1}{9}$$

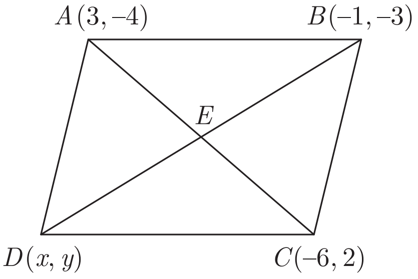
Area $$\Delta_{ADE} = \frac{1.5}{2\times 9} = \frac{5}{6} \text{ Sq. units}$$

Area $$\Delta_{ADE} : \Delta_{ABC} = \frac{5}{6} : \frac{15}{2} = 1:9$$

9. The three vertices of a parallelogram $ABCD$ are $A(3,-4), B(-1,-3)$ and $C(-6,2)$. Find the co-ordinates of vertex D and find the area of $ABCD$.

Ans : [Board Term-2, 2013]

Let 4th vertices of parallelogram be $D(x,y)$. As per question the parallelogram is shown below.



Diagonals of a parallelogram bisect each other. Here E is mid-point of AC and BD.
From bisection of AC we have

$$E = \left(\frac{3-6}{2}, \frac{-4+2}{2}\right) = \left(\frac{-3}{2}, 1\right) \qquad (1)$$

From bisection of BD we have

$$E = \left(\frac{x-1}{2}, \frac{y-3}{2}\right) \qquad (2)$$

From (1) and (2) we have

$$\frac{x-1}{2} = -\frac{3}{2} \Rightarrow x = -3+1 \Rightarrow x = -2$$

and $$\frac{y-3}{2} = -1 \Rightarrow y-3 = -2 \Rightarrow y = 1$$

Thus fourth vertex D is $(-2,1)$

Area of ΔABC

$$\Delta_{ABC} = \frac{1}{2}[x_1(y_2-y_1)+x_2(y_3-y_1)+x_2(y_1-y_2)]$$

$$= \frac{1}{2}[3(-3-2)-1(2+4)-6(-4+3)]$$

$$= \frac{1}{2}[-15-6+6]$$

$$= \frac{1}{2}\times(-15) = -\frac{15}{2} = \frac{15}{2} \text{ sq. units}$$

Since diagonal divides parallelogram into two equal parts, So Area of parallelogram $ABCD$

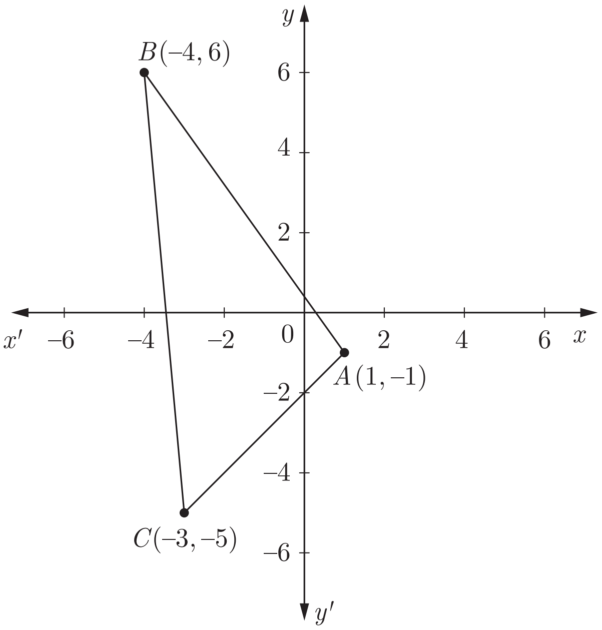
$$\square_{ABCD} = 2 \times \Delta_{ABC}$$

$$= 2 \times \frac{15}{2} = 15 \text{ sq. units}$$

10. The co-ordinates of vertices of ΔABC are $A(1,-1)$, $B(-4,6)$ and $C(-3,-5)$. Draw the figure and prove that ΔABC a scalene triangle. Find its area also.

Ans : [Board Term-2, 2014]

As per question diagram is shown below.



The co-ordinates of the vertices of ΔABC are $A(1,-1)$, $B(-4,6)$ and $C(-3,-5)$ respectively

Now $$AB = \sqrt{(1+4)^2+(-1-6)^2}$$

$$= \sqrt{25+49} = \sqrt{74} = \sqrt{74}$$

$$BC = \sqrt{(-4+3)^2+(6+5)^2}$$

$$= \sqrt{1+121} = \sqrt{122} = \sqrt{122}$$

$$AC = \sqrt{(1+3)^2-(-1+5)^2}$$

$$= \sqrt{16+16} = 4\sqrt{2}$$

Since $AB \neq BC \neq AC$ triangle ΔABC is scalene.

Now, area of ΔABC,

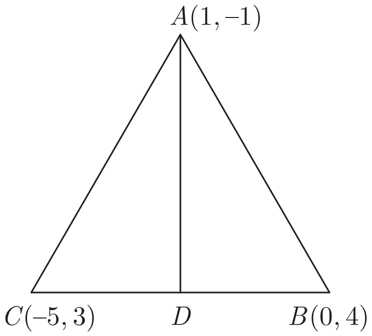
$$= \frac{1}{2}[1(6+5)+(-4)(-5+1)+(-3)(-1-6)]$$

$$= \frac{1}{2}[11+16+21] = 24 \text{ sq. units}$$

11. $(1,-1),(0,4)$ and $(-5,3)$ are vertices of a triangle. Check whether it is a scalene triangle, isosceles triangle or an equilateral triangle. Also, find the length of its median joining the vertex $(1,-1)$ the mid-point of the opposite side.

Ans : [Board Term-2, 2015]

Let the vertices of ΔABC be $A(1,-1)$, $B(0,4)$ and $C(-5,3)$. Let $D(x,y)$ be mid point of BC. Now the triangle is shown below.



Using distance formula, we get

$$AB = \sqrt{(1-0)^2+(-1-4)^2} = \sqrt{1+5^2} = \sqrt{26}$$

$$BC = \sqrt{(-5,0)^2+(3-4)^2} = \sqrt{25+1} = \sqrt{26}$$

$$AC = \sqrt{(-5-1)^2+(3+1)^2} = \sqrt{36+16} = 2\sqrt{13}$$

Since $AB = BC \neq AC$, triangle ΔABC is isosceles.

Now, using mid-section formula, the co-ordinates of mid-point of BC are

$$x = \frac{-5+0}{2} = -\frac{5}{2}$$

$$y = \frac{3+4}{2} = \frac{7}{2}$$

$$D(x,y) = \left(-\frac{5}{2}, \frac{7}{2}\right)$$

Length of median AD

$$AD = \sqrt{\left(\frac{-5}{2}-1\right)^2+\left(\frac{7}{2}+1\right)^2}$$

$$= \sqrt{\left(\frac{-7}{2}\right)+\left(\frac{9}{2}\right)^2}$$

$$= \sqrt{\frac{130}{4}} = \frac{\sqrt{130}}{2}\ unit^2$$

Thus length of median AD is $\frac{\sqrt{130}}{2}$ units.

12. If $a \neq b \neq 0$, prove that the points $(a,a^2),(b,b^2),(0,0)$ will not be collinear.

Ans : [Delhi Set I, II, III 2017]

If three points are collinear, then area covered by given points must be zero.

$$\text{area} = \frac{1}{2}\left[a(b^2-0)+b(0-a^2)+0(a^2-b2)\right]$$

$$= \frac{1}{2}\left[ab^2-a^2b+0\right]$$

$$= \frac{1}{2}\left[ab(b-a)\right] \neq 0 \text{ as } a \neq b \neq 0$$

Hence, the given points are not collinear.

13. If the points $A(k+1,2k), B(3k,2k+3)$ and $C(5k-1,5k)$ are collinear, then find the value of k.

Ans : [Delhi Set I, II, III, 2017]

14. If the points $A(k+1,2k), B(3k,2k+2)$ and $C(5k-1,5k)$ are collinear, then find the value of k.

Ans : [Outside Delhi, Set-II, 2017]

If three points are collinear, then area covered by given points must be zero.

Thus area

$$\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right] = 0$$

$$x_1(y_2-y_3)+x_2(y_3-y_1)+x_2(y_1-y_2) = 0$$

Here $x_1 = k+1, x_2 = 3k, x_3 = 5k-1$

$y_1 = 2k, y_2 = 2k+3, y_3 = 5k.$

$$(k+1)(2k+3-5k)+3k(5k-2k)+$$
$$+(5k-1)(2k-2k-3) = 0$$
$$(k+1)(3-3k)+3k(3k)+(5k-1)(-3) = 0$$
$$3(1+k)(1-k)+3(k)(3k)-3(5k-1) = 0$$
$$3[1-k^2+3k^2-5k+1] = 0$$
$$2k^2-5k+2 = 0$$
$$2k^2-4k-k+2 = 0$$
$$2k(k-2)-1(k-2) = 0$$
$$(2k-1)(k-2) = 0$$

Thus $k = 2$ and $\frac{1}{2}$.

15. Thus $k = 2$ and $\frac{1}{2}$.The points $A(4,-2), B(7,2), C(0,9)$ and $D(-3,5)$ from a parallelogram. Find the length of altitude of the parallelogram on the base AB.

Ans : [Sample Question Paper 2017]

Let the height of parallelogram taking AB as based be h.

Now
$$AB = \sqrt{(7-4)^2+(2+2)^2}$$
$$= \sqrt{3^2+4^2} = \sqrt{9+16}$$
$$= 5 \text{ units}$$

Area of $\Delta\, ABC$

$$\Delta_{ABC} = \frac{1}{2}\left[x_1(y_2-y_1)+x_2(y_3-y_1)+x_2(y_1-y_2)\right]$$

$$= \frac{1}{2}\left[4(2-9)+7(9+2)+0(2-2)\right]$$

$$= \frac{1}{2}\times 49 = \frac{49}{2} \text{ sq. units}$$

Now,
$$\frac{1}{2}\times AB \times h = \frac{49}{2}$$

$$\frac{1}{2}\times 5 \times h = 49$$

$$h = \frac{49}{5} = 9.8 \text{ units.}$$

16. Point $(-1,y)$ and $B(5,7)$ lie on a circle with centre $O(2,-3y)$. Find the values of y. Hence find the radius of the circle.

Ans : [Delhi CBSE, Term-2, 2014]

Since, $A(-1,y)$ and $B(5,7)$ lie on a circle with centre $O(2,-3y)$, OA and OB are the radius of circle and are equal. Thus

$$OA = OB$$
$$\sqrt{(-1-2)^2+(y+3y)^2} = \sqrt{(5-2)^2+(7+3y)^2}$$
$$9+16y^2 = 9y^2+42y+58$$
$$y^2-6y-7 = 0$$
$$(y+1)(y-7) = 0$$
$$y = -1, 7$$

When $y = -1$, centre is $O(2,-3y) = (2,3)$ and radius

$$OB = \left|\sqrt{(5-2)^2+(7-3)^2}\right|$$
$$= \sqrt{9+16} = 5 \text{ unit}$$

When $y = 7$, centre is $O(2,-3y) = (2,-21)$ and radius

$$OB = \left|\sqrt{(2-5)^2+(-21-7)^2}\right|$$
$$= \left|\sqrt{9+784}\right| = \sqrt{793} \text{ unit}$$

CHAPTER 8

Introduction of Trignometry

VERY SHORT ANSWER TYPE QUESTIONS

1. In a triangle ABC, write $\cos\left(\frac{B+C}{2}\right)$ in terms of angle A.

Ans : [Board Term-1, 2016, Set-OYPG7]

$$\begin{aligned} \text{In a triangle} \quad A+B+C &= 180^o \\ \text{or,} \quad B+C &= 180^o - A \\ \text{Thus} \quad \cos\left(\frac{B+C}{2}\right) &= \cos\left[\frac{180^o - A}{2}\right] \\ &= \cos\left[90 - \frac{A}{2}\right] \\ &= \sin\frac{A}{2} \end{aligned}$$

2. If $\sec\theta.\sin\theta = 0$, then find the value of θ.

Ans : [Board Term-1, 2016, Set-O4YP6G7]

$$\begin{aligned} \text{We have} \quad \sec\theta.\sin\theta &= 0 \\ \frac{\sin\theta}{\cos\theta} &= 0 \\ \tan\theta &= 0 = \tan 0^o \end{aligned}$$

Thus $\theta = 0^o$

3. If $A+B = 90^o$ and $\sec A = \frac{2}{3}$, then find the value of $\operatorname{cosec} B$.

Ans : [Board Term-1, 2016, Set-ORDAWEZ]

$$\begin{aligned} \text{We have} \quad A+B &= 90^o \\ \text{and} \quad \sec A &= \frac{5}{3} \\ \text{or,} \quad \sec(90^o - B) &= \frac{5}{3}. \\ \text{Thus} \quad \operatorname{cosec} B &= \frac{5}{3} \end{aligned}$$

4. If $\tan 2A = \cot(A+60^o)$, find the value of A where $2A$ is an acute angle.

Ans : [Board Term-1, 2016, Set-LGRKRO]

$$\begin{aligned} \text{We have} \quad \tan 2A &= \cot(A+60^o) \\ \cot(90 - 2A) &= \cot(A+60^o) \\ 90 - 2A &= A + 60^o \\ 3A &= 30^o \\ \text{Thus} \quad A &= 10^o \end{aligned}$$

5. Find the value of $\frac{\sin 25^o}{\cos 65^o} + \frac{\tan 23^o}{\cot 67^o}$

Ans : [Board Term-1, 2015, Set-FHN8MGD]

$$\begin{aligned} \frac{\sin 25^o}{\cos 65^o} + \frac{\tan 23^o}{\cot 67^o} &= \frac{\sin 25^o}{\sin(90^o - 65^o)} + \frac{\tan 23^o}{\tan(90^o - 67^o)} \\ &= \frac{\sin 25^o}{\sin 25^o} + \frac{\tan 23^o}{\tan 23^o} \\ &= 1 + 1 = 2 \end{aligned}$$

6. If $\cos 2A = \sin(A - 15)$, find A.

Ans : [Board Term1, 2015, Set-FHN8MGD]

$$\begin{aligned} \text{We have} \quad \cos 2A &= \sin(A - 15) \\ \sin(90^o - 2A) &= \sin(A - 15^o) \\ 90^o - 2A &= A - 15 \\ 3A &= 105^o \\ A &= 35^o \end{aligned}$$

7. If $\tan(3x + 30^o) = 1$ them find the value of x.

Ans : [Board Term-1, 2015, Set-WjQZQBN]

$$\begin{aligned} \text{We have} \quad \tan(3x + 30^o) &= 1 = \tan 45^o \\ 3x + 30^o &= 45^o \\ x &= 5^o \end{aligned}$$

8. What happens to value of $\cos\theta$ when θ increases from 0^o to 90^o.

Ans : [Board Term-1, 2015, Set-WJQZQBN]

$\cos\theta$ decreases from 1 to θ.

9. Find the value of $\tan^2 10^o - \cot^2 80^o$.

Ans : [DDE-M, 2015]

We have

$$\begin{aligned} \tan^2 10^o - \cot^2 80^o &= \tan^2(90^o - 80^o) - \cot^2 80^o \\ & \qquad [\therefore \tan(90^o - \theta) = \cot\theta] \\ &= \cot^2 80^o - \cot^2 80^o \\ &= 0 \end{aligned}$$

10. If A and B are acute angles and $\sin A = \cos B$, then find the value of $A+B$.

Ans : [Board Term-1, 2016, Set-MV98HN3]

$$\begin{aligned} \text{We have} \quad \sin A &= \cos B \\ \sin A &= \sin(90^o - B) \\ A &= 90^o - B \\ A + B &= 90^o \end{aligned}$$

11. If A and B are acute angles and $\operatorname{cosec} A = \sec B$, then find the value of $A+B$.

Ans : [DDE-E, 2015]

$$\begin{aligned} \text{We have} \quad \operatorname{cosec} A &= \sec B \\ \operatorname{cosec} A &= \operatorname{cosec}(90^o - B) \\ A &= 90^o - B \end{aligned}$$

$$A + B = 90^o$$

12. Find the value of $\cot 10^o . \cot 30^o . \cot 80^o$

Ans : [CTOQ, 2015]

$$\begin{aligned}\cot 10^o \cot 30^o \cot 80^o &= \cot(90^o - 80^o)\cot 30^o \cot 80^o \\ &= \tan 80^o \cot 30^o \frac{1}{\tan 80^o} \\ &= \cot 30^o = \sqrt{3}\end{aligned}$$

SHORT ANSWER TYPE QUESTIONS - I

1. Evaluate :

$$\frac{3\tan^2 30^o + \tan^2 60^o + \operatorname{cosec} 30^o - \tan 45^o}{\cot^2 45^o}$$

Ans : [Board term-1, 2016, Set-MV98HN3]

$$\begin{aligned}\frac{3\tan^2 30^o + \tan^2 60^o + \operatorname{cosec} 30^o - \tan 45^o}{\cot^2 45^o} &= \frac{3\times\left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2} \\ &= \frac{3\times\frac{1}{3} + 3 + 2 - 1}{1} \\ &= 1 + 3 + 2 - 1 = 5\end{aligned}$$

2. If $\sin(A+B) = 1$ and $\sin(A-B) = \frac{1}{2}$, $0 \le A + B = 90^o$ and $A > B$, then find A and B.

Ans : [Board Term-1, 2016, Set-O4YP6G7]

We have $\sin(A+B) = 1 = \sin 90^o$

$$A + B = 90^o \qquad ...(1)$$

and $\sin(A-B) = \frac{1}{2} = \sin 30^o$

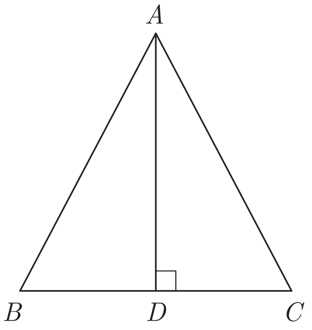
$$A - B = 30^o \qquad ...(2)$$

Solving eq. (1) and (2), we obtain

$$A = 60^o \text{ and } B = 30^o$$

3. Find $\operatorname{cosec} 30^o$ and $\cos 60^o$ geometrically.

Ans : [Board Term-1, 2015, Set-FHN8MGD]



Let a triangle ABC with each side equal to $2a$.

$$\angle A = \angle B = \angle C = 60^o$$

Draw AD perpendicular to BC

$$\Delta BDA \cong \Delta CDA \text{ by } RHS$$

$$BD = CD$$

$$\angle BAD = CAD = 30^o \text{ by } CPCT$$

$$AD = \sqrt{3a}$$

In ΔBDA, $\operatorname{cosec} 30^o = \frac{AB}{BD} = \frac{2a}{a} = 2$

and $\cos 60^o = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$

4. Evaluate : $\frac{\operatorname{cosec} 13^o}{\sec 77^o} - \frac{\cot 20^o}{\tan 70^o}$

Ans : [JTOQ, 2015]

$$\begin{aligned}\frac{\operatorname{cosec} 13^o}{\sec 77^o} - \frac{\cot 20^o}{\tan 70^o} &= \frac{\operatorname{cosec}(90^o - 77^o)}{\sec 77^o} - \frac{\cot(90^o - 70^o)}{\tan 70^o} \\ &= \frac{\sec 77^o}{\sec 77^o} - \frac{\tan 70^o}{\tan 70^o} \\ &= 1 - 1 = 0\end{aligned}$$

5. Evaluate : $\frac{\sin 90^o}{\cos 45^o} + \frac{1}{\operatorname{cosec} 30^o}$

Ans : [Board Term-1, 2013, Set-FFC]

We have $\frac{\sin 90^o}{\cos 45^o} + \frac{1}{\operatorname{cosec} 30^o} = \frac{1}{\frac{1}{\sqrt{2}}} + \frac{1}{2}$

$$= \sqrt{2} + \frac{1}{2} = \frac{2\sqrt{2}+1}{2}$$

6. If $\sin(36+\theta)^o = \cos(16+\theta)^o$, then find θ, where $(36+\theta)^o$ and $(16+\theta)^o$ are both acute angles.

Ans : [Board Term-1, 2012, Set-68]

We have

$$\begin{aligned}\sin(36+\theta)^o &= \cos(16+\theta)^o \\ \cos\left[90^o - (36+\theta)^o\right] &= \cos(16+\theta)^o \\ 90^o - 36^o - \theta &= 16^o + \theta\end{aligned}$$

$$2\theta \;\; 90^o - 36^o - 16^o = 38^o$$

$$\theta = \frac{38^o}{2} = 19^o.$$

7. If $\sqrt{2}\sin\theta = 1$, find the value of $\sec^2\theta - \operatorname{cosec}^2\theta$.

Ans : [Board Term-1, 2012, Set-67]

We have $\sqrt{2}\sin\theta = 1$

$$\sin\theta = \frac{1}{\sqrt{2}} = \sin 45^o$$

Thus $\theta = 45^o$

Now $\sec^2\theta - \operatorname{cosec}^2\theta = \sec^2 45^o - \operatorname{cosec}^2 45^o$

$$= (\sqrt{2})^2 - (\sqrt{2})^2$$

$$= 0$$

8. If $4\cos\theta = 11\sin\theta$, find the value of $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$.

Ans : [Board Term-1, 2012, Set-50]

We have $4\cos\theta = 11\sin\theta$

or, $\cos\theta = \frac{11}{4}\sin\theta$

Now $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta} = \frac{11\times\frac{11}{4}\sin\theta - 7\sin\theta}{11\times\frac{11}{4}\sin\theta + 7\sin\theta}$

$$= \frac{\sin\theta\left(\frac{121}{4} - 7\right)}{\sin\theta\left(\frac{121}{4} + 7\right)}$$

$$= \frac{121 - 28}{121 + 28} = \frac{93}{149}$$

9. If $\tan(A+B) = \sqrt{3}$, $\tan(A-B) = \frac{1}{\sqrt{3},}$ $0^o < A + B$

$\leq 90^{\circ}, A > B$, then find A and B.

Ans : [Board Term-1, 2012, Set-69]

We have $\tan(A+B) = \sqrt{3} = \tan 60^{\circ}$

$$A+B = 60^{\circ} \qquad \text{...(1)}$$

Again $\tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30^{\circ}$

or, $$A-B = 30^{\circ} \qquad \text{...(2)}$$

Adding equations (1) and (2), we obtain,

$$2A = 90^{\circ}$$

$$A = \frac{90^{\circ}}{2} = 45^{\circ}$$

Putting this value of A in equation (1), we get

$$B = 60^{\circ} - A = 60^{\circ} - 45^{\circ} = 15^{\circ}$$

Hence, $A = 45^{\circ}$ and $B = 15^{\circ}$

10. If $\cos(A-B) = \frac{\sqrt{3}}{2}$ and $\sin(A+B) = \frac{\sqrt{3}}{2}$, find $\sin A$ and B, where $(A+B)$ and $(A-B)$ are acute angles.

Ans : [Board Term-1, 2012, Set-70]

We have $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30^{\circ}$

$$A-B = 30^{\circ} \qquad \text{...(1)}$$

Also $\sin(A+B) = \frac{\sqrt{3}}{2} = \sin 60^{\circ}$

$$A+B = 60^{\circ} \qquad \text{...(2)}$$

Adding equations (1) and (2), we obtain,

$$2A = 90^{\circ}$$

$$A = 45^{\circ}$$

Putting this value of A in equation (1), we get $B = 15^{\circ}$

11. Express $\cos 68^{\circ} + \tan 76^{\circ}$ in terms of the angles between 0° and 45°.

Ans : [Board Term-1, 2012, Set-64]

Here we will use $\cos(90^{\circ} - \theta) = \sin\theta$ and $\tan(90^{\circ} - \theta) = \cot\theta$.

$$\cos 68^{\circ} + \tan 76^{\circ} = \cos(90^{\circ} - 22^{\circ}) + \tan(90^{\circ} - 14^{\circ})$$

$$= \sin 22^{\circ} + \cot 14^{\circ}$$

12. Find the value of $\cos 2\theta$, if $2\sin 2\theta = \sqrt{3}$.

Ans : [Board Term-1, 2012, Set-25]

We have $2\sin 2\theta = \sqrt{3}$

$$\sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^{\circ}$$

$$2\theta = 60^{\circ}$$

Hence, $\cos 2\theta = \cos 60^{\circ} = \frac{1}{2}$.

13. Find the value of $\sin 30^{\circ}.\cos 60^{\circ} + \cos 30^{\circ}.\sin 60^{\circ}$ is it equal to $\sin 90^{\circ}$ or $\cos 90^{\circ}$?

Ans : [Board Term-1, 2016, Set-ORDAWEZ]

$$\sin 30^{\circ}\cos 60^{\circ} + \cos 30^{\circ}\sin 60^{\circ} = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

It is equal to $\sin 90^{\circ} = 1$ but not equal to $\cos 90^{\circ}$ as $\cos 90^{\circ} = 0$.

14. Evaluate : $\frac{6\sin 23^{\circ} + \sec 79^{\circ} + 3\tan 48^{\circ}}{\text{cosec}\, 11^{\circ} + 3\cot 42^{\circ} + 6\cos 67^{\circ}}$

Ans : [Board Term-1, 2012, Set-55]

$$\frac{6\sin 23^{\circ} + \sec 79^{\circ} + 3\tan 48^{\circ}}{\text{cosec}\, 11^{\circ} + 3\cot 42^{\circ} + 6\cos 67^{\circ}}$$

$$= \frac{6\sin(90^{\circ} - 23)^{\circ} + \text{cosec}(90^{\circ} - 79^{\circ}) + 3\cot(90^{\circ} - 48^{\circ})}{\text{cosec}\, 11^{\circ} + 3\cot 42^{\circ} + 6\cos 67^{\circ}}$$

$$= \frac{6\cos 67^{\circ} + \text{cosec}\, 11^{\circ} + 3\cot 42^{\circ}}{\text{cosec}\, 11^{\circ} + 3\cot 42^{\circ} + 6\cos 67^{\circ}}$$

$$= 1$$

15. If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0^{\circ} < \theta < 90^{\circ}$, find the value of θ.

Ans : [Boar Term-1, 2012, Set-35]

We have $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0^{\circ} < \theta < 90^{\circ}$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \tan 30^{\circ} \qquad \left[\tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\theta = 30^{\circ}$$

16. Evaluate : $\frac{\cos 45^{\circ}}{\sec 30^{\circ}} + \frac{1}{\sec 60^{\circ}}$

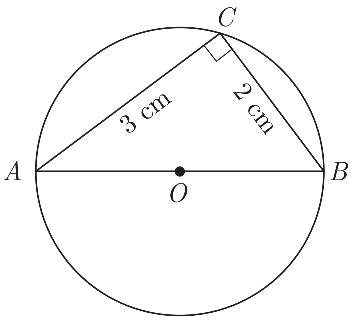
Ans : [Board Term-1, 2012, Set-63]

We have $\frac{\cos 45^{\circ}}{\sec 30^{\circ}} + \frac{1}{\sec 60^{\circ}} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} + \frac{1}{2}$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{1}{2}$$

$$= \frac{\sqrt{6}+2}{4}$$

17. In the given figure, AOB is a diameter of a circle with center O. fing $\tan A \tan B$.



Ans : [Board Term-1, 2012, Set-52]

In ΔABC, $\angle C$ in a semi-circle angle, thus

$$\angle C = 90^{\circ}$$

$$\tan A = \frac{BC}{AC} = \frac{2}{3}$$

and $$\tan B = \frac{AC}{BC} = \frac{3}{2}$$

$$\tan A.\tan B = \frac{2}{3} \times \frac{3}{2} = 1$$

18. If $\sin\phi = \frac{1}{2}$, show that $3\cos\phi - 4\cos^3\phi = 0$.

Ans :

We have $\sin\theta = \frac{1}{2}$

$$\phi = 30^o$$

Now substituting this value of θ in LHS we have

$$3\cos\phi - 4\cos^3\phi = 3\cos 30^o - 4\cos^3 30^o$$

$$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{2}}{2}$$

$$= 0$$

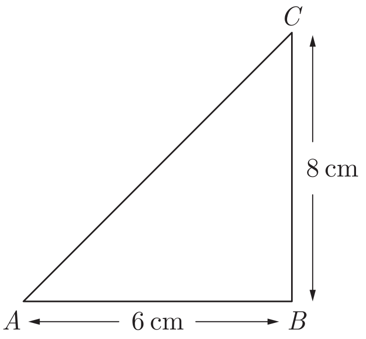
Hence Prove

SHORT ANSWER TYPE QUESTIONS - II

1. If in a triangle ABC right angled at $B, AB = 6$ units and $BC = 8$ units, then find the value of $\sin A.\cos C + \cos A.\sin C$.

Ans : [Board Term-1, 2016, set-O4YP6G7]

As per question statement figure is shown below.



We have $AC^2 = 8^2 + 6^2 = 100$

$$AC = 10$$

Now $\sin A = \frac{8}{10}, \cos A = \frac{6}{10}$

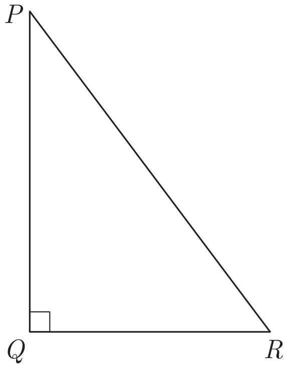
and $\sin C = \frac{6}{10}, \cos C = \frac{8}{10}$

Thus $\sin A\cos C + \cos A\sin C = \frac{8}{10}\times\frac{8}{10} + \frac{6}{10}\times\frac{6}{10}$

$$= \frac{64}{100} + \frac{36}{100}$$

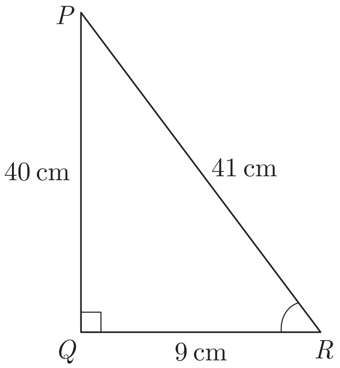
$$= \frac{100}{100} = 1$$

2. In the given $\angle PQR$, right-angled at $Q, QR = 9$ cm and $PR - PQ = 1$ cm. Determine the value of $\sin R + \cos R$.



Ans : [Board Term-1, 2015, Set-FHN8MGD]

We redraw the figure as shown below.



Using Pythagoras theorem we have

$$PQ^2 + QR^2 = PR^2$$

$$PQ^2 + 9^2 = (PQ+1)^2$$

$$PQ^2 + 81 = (PQ+1)^2$$

$$PQ^2 + 81 = PQ^2 + 1 + 2PQ$$

$$PQ = 40$$

Since $PR - PQ = 1$, thus

$$PR = 1 + 40 = 41$$

$$\sin R + \cos R = \frac{40}{41} + \frac{9}{41} = \frac{49}{41}$$

3. Express $\cos 71^o - \sin 57^o + \tan 63^o$ in terms of trigonometric ratios of angles between 0^o and 45^o.

Ans : [JTOQ, 2015]

$\cos 71^o - \sin 57^o + \tan 63^o$

$$= \cos(90^o - 19^o) - \sin(90^o - 33^o) + \tan(90^o - 27^o)$$

$$= \sin 19^o - \cos 33^o + \cot 27^o$$

4. If $\cos(40^o + x) = \sin 30^o$, find the value of x.

Ans : [DDE-E, 2015]

We have $\cos(40^o - x) = \sin 30^o$

$$\cos(40^o + x) = \sin(90^o - 60^o)$$

$$\cos(40^o + x) = \cos 60^o$$

$$40^o + x = 60^o$$

$$x = 60^o - 40^o = 20^o$$

Thus $x = 20^o$

5. Evaluate : $\dfrac{5\cos^2 60^o + 4\cos^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 60^o}$

Ans : [Board Term-1, 2013, Set-Lk-59]

$$\frac{5\cos^2 60^o + 4\cos^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 60^o}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{\sqrt{3}}{2}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{5}{4} + 3 - 1}{\frac{1}{4} + \frac{1}{4}}$$

$$= \frac{\frac{5}{4} + 2}{\frac{1}{2}} = \frac{\frac{13}{4}}{\frac{1}{2}} = \frac{13}{2}$$

6. If $\sin 30^o = \cos(\theta - 6^o)$, where 3θ and $\theta - 6^o$ are both acute angles, find the value of θ.

Ans : [board Term-1, 2011, Set-21]

According to the question,

$$\begin{aligned} \sin 3\theta &= \cos(\theta - 6^o) \\ \cos(90^o - 3\theta) &= \cos(\theta - 6^o) \\ 90^o - 3\theta &= \theta - 6^o \\ 4\theta &= 90^o + 6^o = 96^o \end{aligned}$$

Thus $\theta = \dfrac{96^o}{4} = 24^o$

7. Simplify :

$$\frac{\sin\theta\sec(90^o - \theta)\tan\theta}{\operatorname{cosec}(90^o - \theta)\cos\theta\cot(90^o - \theta)} - \frac{\tan(90^o - \theta)}{\cot\theta}$$

Ans : [Board Term-1, 2011, Set-66]

$$\begin{aligned} \sec(90^o - \theta) &= \operatorname{cosec}\theta, \\ \tan(90^o - \theta) &= \cot\theta, \\ \cot(90^o - \theta) &= \tan\theta, \\ \operatorname{cosec}(90^o - \theta) &= \sec\theta \end{aligned}$$

Hence,

$$\frac{\sin\theta\sec(90^o - \theta)\tan\theta}{\operatorname{cosec}(90^o - \theta)\cos\theta\cot(90^o - \theta)} - \frac{\tan(90^o - \theta)}{\cot\theta}.$$

$$= \frac{\sin\theta\operatorname{cosec}\theta\tan\theta}{\sec\theta\cos\theta\tan\theta} - \frac{\cot\theta}{\cot\theta}$$

$$= \frac{\sin\theta \times \frac{1}{\sin\theta} \times \tan\theta}{\frac{1}{\cos\theta} \times \cos\theta\tan\theta} - 1$$

$$= 1 - 1 = 0$$

8. Verify : $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{\sin\theta}{1+\cos\theta}$, for $\theta = 60^o$

Ans :

$$\text{LHS} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\cos 60^o}{1+\cos 60^o}}$$

$$= \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}} = \sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}} = \frac{1}{\sqrt{3}} \quad \left(\cos 60^o = \frac{1}{2}\right)$$

$$\text{RHS} = \frac{\sin\theta}{1+\cos\theta} = \frac{\sin 60^o}{1+\cos 60^o}$$

$$= \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}}$$

$$\text{RHS} = \text{LHS}$$

Hence, relation is verified for $\theta = 60^o$.

9. If $\tan A + \cot A = 2$, then find the value of $\tan^2 A + \cot^2 A$.

Ans : [DDE-M, 2015]

We have $\tan A + \cot A = 2$

Squaring both sides, we have

$$\begin{aligned} (\tan A + \cot A)^2 &= (2)^2 \\ \tan^2 A + \cot^2 A + 2\tan A.\cot A &= 4 \\ \tan^2 A + \cot^2 A + 2\tan A \times \frac{1}{\tan A} &= 4 \\ \tan^2 A + \cot^2 A + 2 &= 4 \\ \tan^2 A + \cot^2 A &= 4 - 2 \\ \tan^2 A + \cot^2 A &= 2 \end{aligned}$$

10. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\cos\theta$.

Ans : [Board Term-1, 2011, Set-74]

We have $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$$\begin{aligned} \sin\theta &= \cos\theta(\sqrt{2} - 1) \\ &= \frac{\cos\theta(\sqrt{2} - 1)(\sqrt{2} + 1)}{(\sqrt{2} + 1)} \end{aligned}$$

or, $\sin\theta = \dfrac{\cos\theta(2 - 1)}{\sqrt{2} + 1}$

$$\begin{aligned} (\sqrt{2} + 1)\sin\theta &= \cos\theta \\ \sqrt{2}\sin\theta + \sin\theta &= \cos\theta \\ coa\theta - \sin\theta &\ \sqrt{2}\sin\theta \end{aligned}$$

Hence proved.

11. Prove that : $\dfrac{\cos A}{1 - \tan A} + \dfrac{\sin A}{1 - \cot A} = \sin A + \cos A.$

Ans : [Board Term-1, 2013, FFC: 2011, Set-74]

$$\text{LHS} = \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$$

$$= \frac{\cos A}{1 - \left(\frac{\sin A}{\cos A}\right)} + \frac{\sin A}{1 - \left(\frac{\cos A}{\sin A}\right)}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)}$$

$$= \cos A + \sin A$$

$$= \sin A + \cos A$$

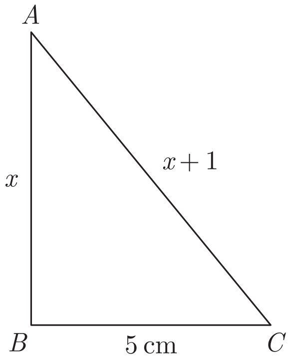
$$= \text{RHS}$$

Hence proved.

12. In ΔABC, $\angle B = 90^o$, $BC = 5$ cm, $AC - AB = 1$, Evaluate : $\frac{1+\sin C}{1+\cos C}$.

Ans : [Board Term-1, 2011, Set-52]

As per question we have drawn the figure given below.



$$
\begin{aligned}
\text{We have} \quad AC - AB &= 1 \\
\text{Let } AB &= x\text{, then we have} \\
AC \;\; & x + 1 \\
\text{Now} \quad AC^2 &= AB^2 + BC^2 \\
(x+1)^2 &= x^2 + 5^2 \\
x^2 + 2x + 1 &= x^2 + 25 \\
2x &= 24 \\
x &= \frac{24}{2} = 12 \text{ cm}
\end{aligned}
$$

Hence, $AB = 12$ cm and $AC = 13$ cm

$$
\begin{aligned}
\text{Now} \quad \sin C &= \frac{AB}{AC} = \frac{12}{13} \\
\cos C &= \frac{BC}{AC} = \frac{5}{13} \\
\text{Now} \quad \frac{1+\sin C}{1+\cos C} &= \frac{1+\frac{12}{13}}{1+\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}
\end{aligned}
$$

LONG ANSWER TYPE QUESTIONS

1. Evaluate :

$\tan^2 30^o \sin 30^o + \cos 60^o \sin^2 90^o \tan^2 60^o - 2\tan 45^o \cos^2 0^o \sin 90^o$

Ans : [Board Term-1, 2015, WJQZQBN]

$\tan^2 30^o \sin 30^o + \cos 60^o \sin^2 90^o \tan^2 60^o - 2\tan 45^o \cos^2 0^o \sin 90^o$

$$
\begin{aligned}
&= \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{2} + \frac{1}{2} \times (1)^2 \times (\sqrt{3})^2 - 2 \times 1 \times 1^2 \times 1 \\
&= \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times 1 \times 3 \times - 2 \times 1 \times 1 \times 1 \\
&= \frac{1}{6} + \frac{3}{2} - 2 = \frac{1+9-12}{6} = -\frac{2}{6} = -\frac{1}{3}
\end{aligned}
$$

2. Given that $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, find the values of $\tan 75^o$ and $\tan 90^o$ by taking suitable values of A and B.

Ans : [NCERT]

$$
\begin{aligned}
\text{We have } \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\
\text{(i)} \qquad \tan 75^o &= \tan(45^o + 30^o) \\
&= \frac{\tan 45^o + \tan 30^o}{1 \cdot \tan 45^o \cdot \tan 30^o} \\
&= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \\
&= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} \\
&= \frac{3+2\sqrt{3}+1}{(\sqrt{3})^2 - (1)^2} = \frac{4+2\sqrt{3}}{2}
\end{aligned}
$$

Hence $\tan 75^o = 2 + \sqrt{3}$

$$
\begin{aligned}
\text{(ii)} \qquad \tan 90^o &= \tan(60^o + 30^o) \\
&= \frac{\tan 60^o + \tan 30^o}{1 - \tan 60^o \tan 30^o} \\
&= \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{1 - \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{\frac{3+1}{\sqrt{3}}}{0}
\end{aligned}
$$

Hence, $\tan 90^o = \infty$

3. Evaluate :

$\sin^2 30^o \cos^2 45^o + 4\tan^2 30^o + \frac{1}{2}\sin 90^o - 2\cos^2 90^o + \frac{1}{24}$

Ans : [Board Term-1, 2013, LK-59]

$\sin^2 30^o \cos^2 45^o + 4\tan^2 30^o + \frac{1}{2}\sin 90^o - 2\cos^2 90^o + \frac{1}{24}$

$$
\begin{aligned}
&= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2(0) + \frac{1}{24} \\
&= \frac{1}{4}\left(\frac{1}{2}\right) + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} = \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} \\
&= \frac{3+32+12+1}{24} = \frac{48}{24} = 2
\end{aligned}
$$

4. Evaluate : $4(\sin^4 30^o + \cos^4 60^o) - 3(\cos^2 45 - \sin^2 90^o)$

Ans : [Board Term-1, 2013, Set-FFC]

$4(\sin^4 30^o + \cos^4 60^o) - 3(\cos^2 45 - \sin^2 90^o)$

$$
\begin{aligned}
&= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right] \\
&= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right] \\
&= 4 \times \frac{2}{16} - 3 \times -\frac{1}{2} \\
&= \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2
\end{aligned}
$$

5. If $15\tan^2\theta + 4\sec^2\theta = 23$, then find the value of $(\sec\theta + \text{cosec}\,\theta)^2 - \sin^2\theta$.

Ans : [Board Term-1, 2012, Set-38]

$$
\begin{aligned}
\text{We have} \quad 15\tan^2\theta + 4\sec^2\theta &= 23 \\
15\tan^2\theta + 4(\tan^2\theta + 1) &= 23 \quad (\sec^2\theta = 1 + \tan^2\theta) \\
15\tan^2\theta + 4\tan^2\theta + 4 &= 23 \\
19\tan^2\theta &= 19 \\
\tan\theta &= 1 = \tan 45^o \\
\text{Thus} \quad \theta &= 45^o
\end{aligned}
$$

Now, $(\sec\theta + \text{cosec}\,\theta)^2 - \sin^2\theta$

$$
\begin{aligned}
&= (\sec 45^o + \text{cosec}\,45^o)^2 - \sin^2 45^o \\
&= (\sqrt{2} + \sqrt{2})^2 - \left(\frac{1}{\sqrt{2}}\right)^2
\end{aligned}
$$

$$= (2\sqrt{2})^2 - \frac{1}{2} = 8 - \frac{1}{2} = \frac{15}{2}$$

6. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$.

Ans : [board Term-1, 2012, Set-48]

We have $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$

Let $\cot\theta = x$, then we have

$$\sqrt{3}x^2 - 4x + \sqrt{3} = 0$$

$$\sqrt{3}x^2 - 3x - x + \sqrt{3} = 0$$

$$(x - \sqrt{3})(\sqrt{3x} - 1) = 0$$

Thus $x = \sqrt{3}$ or $\frac{1}{\sqrt{3}}$

or $\cot\theta = \sqrt{3}$ or $\cot\theta = \frac{1}{\sqrt{3}}$

Therefore $\theta = 30^\circ$ or $\theta = 60^\circ$

If $\theta = 30^\circ$, then

$$\cot^2 30^\circ + \tan^2 30^\circ = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \frac{1}{3} = \frac{10}{3}$$

If $\theta = 60^\circ$, then

$$\cot^2 60^\circ + \tan^2 60^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$$

$$= \frac{1}{3} + 3 = \frac{10}{3}.$$

7. Evaluate the following :

$$\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$$

Ans : [Board Term-1, 2012, Set-43]

$$\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$$

$$= \frac{2\left(\frac{1}{2}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2(1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \frac{\frac{2}{4} + 4 - 2}{\frac{1}{4} + \frac{1}{2}} = \frac{10}{3}$$

8. Evaluate : $\frac{\cos 65^\circ}{\sin 25^\circ} - \frac{\tan 20^\circ}{\cot 70^\circ} - \sin 90^\circ + \tan 5^\circ \tan 35^\circ \tan 60^\circ \tan 55^\circ \tan 85^\circ$.

Ans : [Board Term-1, 2012, Set-50]

We have $\frac{\cos 65^\circ}{\sin 25^\circ} = \frac{\cos 65^\circ}{\sin(90^\circ - 65^\circ)} = \frac{\cos 65^\circ}{\cos 65^\circ} = 1,$

$$\frac{\tan 20^\circ}{\cot 70^\circ} = \frac{\tan(90^\circ - 70^\circ)}{\cot 70^\circ} = \frac{\cot 70^\circ}{\cot 70^\circ} = 1$$

and $\sin 90^\circ = 1$

$\tan 5^\circ \tan 35^\circ \tan 60^\circ \tan 55^\circ \tan 85^\circ$

$$= \tan(90^\circ - 85^\circ)\tan(90^\circ - 55^\circ)$$
$$\tan 55^\circ \tan 60^\circ \tan 85^\circ.$$

$$= \cot 85^\circ \tan 85^\circ \cot 55^\circ \tan 55^\circ \cdot \sqrt{3}$$

$$= 1 \times 1 \times \sqrt{3} = \sqrt{3}$$

Now given expression $= 1 - 1 - 1 + \sqrt{3} = \sqrt{3} - 1$

9. Prove that : $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta.$

Ans : [Board Term-1, 2012, Set-48]

$$\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = \frac{\tan\theta}{1 - \frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1 - \tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{(1 - \tan\theta)\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{(\tan\theta - 1)\tan\theta}$$

$$= \frac{\tan^3\theta - 1}{(\tan\theta - 1)\tan\theta}$$

$$[\therefore a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{(\tan\theta - 1)(\tan\theta)}$$

$$= \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$$

$$= \tan\theta + 1 + \cot\theta$$

Hence Proved.

10. In an acute angled triangle ABC, *if* $\sin(A + B - C) = \frac{1}{2}$ and $\cos(B + C - A) = \frac{1}{\sqrt{2}}$, find $\angle A, \angle B$ and $\angle C$.

Ans : [Board Term-1, 2012, Set-39]

We have $\sin(A + B - C) = \frac{1}{2} = \sin 30^\circ$

or, $A + B - C = 30^\circ$ ...(1)

and $\cos(B + C - A) = \frac{1}{\sqrt{2}} = \cos 45^\circ$

or, $B + C - A = 45^\circ$ ...(2)

Adding equation (1) and (2), we get

$$2B = 75^\circ$$

or, $B = 37.5^\circ$

Now subtracting equation (2) from equation (1) we get,

$$2(A - C) = -15^\circ$$

or, $A - C = 7.5^\circ$ ...(3)

Now $A + B + C = 180^\circ$

$$A + B + C = 180^\circ$$

$A + C = 180^\circ - 37.5^\circ = 142.5^\circ$ ...(4)

Adding equation (3) and (4), we have

$$2A = 135^\circ$$

or, $A = 67.5^\circ$

and, $C = 75^\circ$

Hence, $\angle A = 67.5^\circ, \angle B = 37.5^\circ, \angle C = 75^\circ$

VERY SHORT ANSWER TYPE QUESTIONS

1. If $k + 1 = \sec^2\theta(1 + \sin\theta)(1 - \sin\theta)$, then find the value of k.

Ans : [Board Term-1, 2015, Set-JJOQ]

We have $k + 1 = \sec^2\theta(1 + \sin\theta)(1 - \sin\theta)$

$$\begin{aligned} &= \sec^2\theta(1-\sin^2\theta) \\ &= \sec^2\theta.\cos^2\theta \\ &= \sec^2\theta \times \frac{1}{\sec^2\theta} \end{aligned}$$

or, $k+1 = 1$

or, $k = 1-1 = 0$

Thus $k = 0$

2. Find the value of $\sin^2 41^o + \sin^2 49^o$

Ans : [DDE-M, 2015][NCERT]

We have

$$\begin{aligned} \sin^2 41 + \sin^2 49 &= \sin^2(90^o - 49^o) + \sin^2 49^o \\ &= \cos^2 49 + \sin^2 49^o \\ &= 1 \qquad [\cos^2\theta + \sin^2\theta = 1] \end{aligned}$$

SHORT ANSWER TYPE QUESTIONS - I

1. Express the trigonometric ratio of $\sec A$ and $\tan A$ in terms of $\sin A$.

Ans : [Board Term-1, 2015, Set-FHN8MGD]

We have $\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1-\sin^2 A}}$

and $\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1-\sin^2 A}}$

2. Prove that : $\frac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} = 1$

Ans : [Board Term-1, 2015, Set-WJQZQBN]

$$\begin{aligned} \frac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} &= \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{1 - 2\sin^2\theta\cos^2\theta} \\ &= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\theta\cos^2\theta} \\ &= \frac{1 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2 + \cos^2\theta} \\ &= 1 \end{aligned}$$

Hence Prove

3. Prove that : $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

Ans : [DDE-M, 2015]

To prove $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

We have

$$\begin{aligned} \sec^4\theta - \sec^2\theta &= \sec^2\theta(\sec^2\theta - 1) \\ & \qquad [[1 + \tan^2\theta = \sec^2\theta]] \\ &= \sec^2\theta(\tan^2\theta) \\ &= (1 + \tan^2\theta)\tan^2\theta \\ &= \tan^2\theta + \tan^4\theta \end{aligned}$$

Hence Proved.

4. Find the value of θ, if, $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4; \theta \leq 90^o$

Ans : [DDE-E, 2015]

We have

$$\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$$

$$\frac{\cos\theta(1+\sin\theta) + \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 4$$

$$\frac{\cos\theta[1+\sin\theta+1-\sin\theta]}{1-\sin\theta} = 4$$

$$\frac{\cos\theta(2)}{\cos^2\theta} = 4$$

$$\frac{1}{\cos\theta} = 2$$

$$\cos\theta = \frac{1}{2}$$

$$\cos\theta = \cos 60^o$$

Thus $\theta = 60^o$

5. Prove that : $-1 + \frac{\sin A\sin(90^o - A)}{\cot(90^o - A)} = -\sin^2 A$

Ans : [Board Term-1, 2012, Set-62]

$$\begin{aligned} -1 + \frac{\sin A\sin(90^o - A)}{\cot(90^o - A)} &= -1 + \frac{\sin A\cos A}{\tan A} \\ &= -1 + \sin A\cos A \times \cot A \\ &= -1 + \sin A\cos A \times \frac{\cos A}{\sin A} \\ &= -1 + \cos^2 A = -(1 - \cos^2\theta) \\ &= -\sin^2 A \end{aligned}$$

Hence Proved.

6. Prove that : $\sqrt{\frac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$

Ans : [Board Term-1,2012, Set-74]

$$\begin{aligned} \sqrt{\frac{1-\cos A}{1+\cos A}} &= \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}} \\ &= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}} \\ &= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}} \qquad [\because \sin^2\theta + \cos^2\theta = 1] \\ &= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A} \\ &= \operatorname{cosec} A - \cot A \end{aligned}$$

Hence Proved.

7. If $\sin\theta - \cos\theta = \frac{1}{2}$, then find the value of $\sin\theta + \cos\theta$.

Ans : [board Term-1, 2013, Set-FFC]

We have $\sin\theta - \cos\theta = \frac{1}{2}$

Squaring both sides, we get

$$\begin{aligned} (\sin\theta - \cos\theta)^2 &= \left(\frac{1}{2}\right)^2 \\ \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta &= \frac{1}{4} \\ 1 - 2\sin\theta\cos\theta &= \frac{1}{4} \\ 2\sin\theta\cos\theta &= 1 - \frac{1}{4} = \frac{3}{4} \end{aligned}$$

Again, $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$$\begin{aligned} &= 1 + 2\sin\theta\cos\theta \\ &= 1 + \frac{3}{4} = \frac{7}{4} \end{aligned}$$

Thus $\sin\theta + \cos\theta = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$

8. If θ be an acute angle and $5\operatorname{cosec}\theta = 7$, Then evaluate $\sin\theta + \cos^2\theta - 1$.

Ans : [Board Term-1, 2012, Set-43]

We have
$$5\operatorname{cosec}\theta = 7$$
$$\operatorname{cosec}\theta = \frac{7}{5}$$
$$\sin\theta = \frac{5}{7} \qquad [\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}]$$
$$\sin\theta + \cos^2\theta - 1 = \sin\theta - (1-\cos^2\theta)$$
$$= \sin\theta - \sin^2\theta \quad [\sin^2\theta + \cos^2\theta = 1]$$
$$= \frac{5}{7} - \left(\frac{5}{7}\right)^2 = \frac{35-25}{49} = \frac{10}{49}$$

9. If $\sin A = \frac{\sqrt{3}}{2}$, find the value of $2\cot^2 A - 1$.

Ans : [Board Term-1, 2012, Set-21]

Using $\because \cot^2\theta = -1 + \operatorname{cosec}^2\theta$ we have
$$2\cot^2 A - 1 = 2(\operatorname{cosec}^2 A - 1) - 1$$
$$= \frac{2}{\sin^2 A} - 3$$
$$= \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} - 3 = \frac{8}{3} - 3 = \frac{-1}{3}$$

Thus
$$2\cot^2 A - 1 = \frac{-1}{3}$$

SHORT ANSWER TYPE QUESTIONS - II

1. Prove that : $\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A} = \cos A - \sin A$

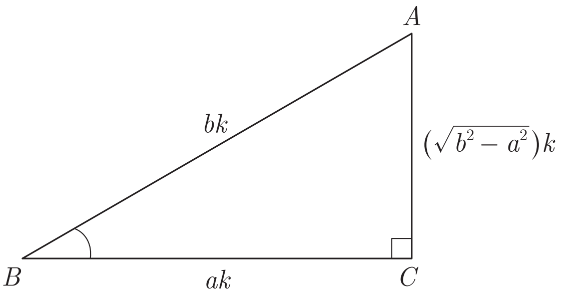
Ans : [Board Term-1, 2016, Set-MV98HN3]

$$\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A} = \frac{\cos A}{1+\frac{\sin A}{\cos A}} - \frac{\sin A}{1+\frac{\cos A}{\sin A}}$$
$$= \frac{\cos^2 A}{\cos A + \sin A} - \frac{\sin^2 A}{\sin A + \cos A}$$
$$= \frac{\cos^2 A - \sin^2 A}{(\sin A + \cos A)}$$
$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A + \cos A}$$
$$= \cos A - \sin A \qquad \text{Hence Proved.}$$

2. If $b\cos\theta = a$, then prove that $\operatorname{cosec}\theta + \cot\theta = \sqrt{\frac{b+a}{b-a}}$.

Ans : [Board Term-1, 2015, Set-WJQZQBN]

Consider the triangle shown below.



$$b\cos\theta = a$$
$$AC^2 = AB^2 - BC^2$$

or,
$$\cos\theta = \frac{a}{b}$$
$$AC = \sqrt{b^2 - a^2 k}$$
$$\operatorname{cosec}\theta = \frac{b}{\sqrt{b^2-a^2}}, \cot\theta = \frac{1}{\sqrt{b^2-a^2}}$$
$$\operatorname{cosec}\theta + \cot\theta = \frac{b+a}{\sqrt{b^2-a^2}} = \sqrt{\frac{b+a}{b-a}}$$

3. Prove that : $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \tan\theta$

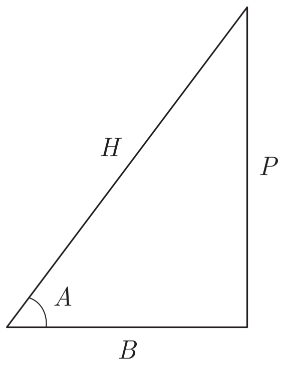
Ans : [Bard Term-1, 2015, Set-WJQZQBN, FHN8MGD]

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$
$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$
$$= \frac{\tan\theta(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta - \sin^2\theta)}$$
$$= \tan\theta \qquad \text{Hence Proved}$$

4. When is an equation called 'an identity'. Prove the trigonometric identity $1 + \tan^2 A = \sec^2 A$.

Ans : [DDE-E, 2015][NCERT]

Consider the triangle shown below.



Let $\tan A = \frac{P}{B}$ and $\sec A = \frac{H}{B}$
$$H^2 = P^2 + B^2$$

Now
$$1 + \tan^2 A = 1 + \left(\frac{P}{B}\right)^2 = 1 + \frac{P^2}{B^2}$$
$$= \frac{B^2 + P^2}{B^2} = \frac{H^2}{B^2}$$
$$= \left(\frac{H}{B}\right)^2$$
$$= \sec^2 A \qquad \text{Hence Proved.}$$

Equations that are true no matter what value is plugged in for the variable. On simplifying an identity equation, one always get a true statement.

5. Prove that : $(\cot\theta - \operatorname{cosec}\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Ans : [Board Term-1, 2015, Set JTOQ, 2015]

To prove $(\cot\theta-\operatorname{cosec}\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

$$(\cot\theta-\operatorname{cosec}\theta)^2=\left(\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)^2$$

$$=\left(\frac{\cos\theta-1}{\sin\theta}\right)^2$$

$$=\frac{(1-\cos\theta)^2}{\sin^2\theta}\quad[[\sin^2\theta+\cos^2\theta=1]]$$

$$=\frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$

$$=\frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$=\frac{1-\cos\theta}{1+\cos\theta}$$

Hence Proved.

6. Prove that :

$(\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)=1$

Ans : [DDE-M, 2015]

$$\text{LHS}=(\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)$$

$$=\left(\frac{1}{\sin\theta}-\sin\theta\right)\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$$

$$=\left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta.\cos\theta}\right)$$

$$=\frac{\cos^2\theta}{\sin\theta}\times\frac{\sin^2\theta}{\cos\theta}\times\left(\frac{1}{\sin\theta.\cos\theta}\right)\quad[\sin^2\theta+\cos^2\theta=1]$$

$$=\cos\theta\sin\theta\times\frac{1}{\sin\theta\cos\theta}=1$$

Hence Proved.

7. Show that :

$\operatorname{cosec}^2\theta-\tan^2(90^\circ-\theta)=\sin^2\theta+\sin(90^\circ-\theta)$

Ans : [Board Term-1, 2013, LK-59]

$$\operatorname{cosec}^2\theta-\tan^2(90^\circ-\theta)=\frac{1}{\sin^2}-\frac{\sin^2(90^\circ-\theta)}{\cos^2(90-\theta)}$$

$$=\frac{1}{\sin^2\theta}-\frac{\sin^2(90^\circ-\theta)}{\sin^2\theta}$$

$$=\frac{1}{\sin^2\theta}-\frac{\cos^2\theta}{\sin^2\theta}$$

$$=\frac{1-\cos^2\theta}{\sin^2\theta}=\frac{\sin^2\theta}{\sin^2\theta}$$

$$=1$$

$$=\sin^2\theta+\cos^2\theta$$

$$=\sin^2\theta+\sin^2(90^\circ-\theta)$$

Hence Proved

8. Prove that : $\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta-1}-\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta+1}=2\sec^2\theta$

Ans : [Board Term-1, 2013, FFC]

We have

$$\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta-1}-\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta+1}=\operatorname{cosec}^2\theta\left[\frac{1}{\frac{1}{\sin\theta}-1}-\frac{1}{\frac{1}{\sin\theta}+1}\right]$$

$$=\operatorname{cosec}^2\theta\left[\frac{\sin\theta}{1-\sin\theta}-\frac{\sin\theta}{1+\sin\theta}\right]$$

$$=\frac{1\times\sin\theta}{\sin^2\theta}\left[\frac{(1+\sin\theta)-(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}\right]$$

$$=\frac{1}{\sin\theta}\left[\frac{2\sin\theta}{1-\sin^2\theta}\right]$$

$$=\frac{2}{\cos^2\theta}=2\sec^2\theta$$

Hence Proved

9. Prove that :

$\frac{1}{\operatorname{cosec}A-\cot A}-\frac{1}{\sin A}=\frac{1}{\sin A}-\frac{1}{\operatorname{cosec}A+\cot A}.$

Ans : [Board Term-1, 2011, Set-66]

$$\frac{1}{\operatorname{cosec}A-\cot A}-\frac{1}{\sin A}=\frac{1}{\sin A}-\frac{1}{\operatorname{cosec}A+\cot A}$$

$$\frac{1}{\operatorname{cosec}A-\cot A}+\frac{1}{\operatorname{cosec}A+\cot A}=\frac{1}{\sin A}+\frac{1}{\sin A}$$

$$\frac{1}{\operatorname{cosec}A-\cot A}+\frac{1}{\operatorname{cosec}A+\cot A}=\frac{2}{\sin A}$$

$$\frac{1}{\operatorname{cosec}A-\cot A}+\frac{1}{\operatorname{cosec}A+\cot A}=\frac{2}{\sin A}$$

$$\frac{\operatorname{cosec}A+\cot A+\operatorname{cosec}A-\cot A}{(\operatorname{cosec}A-\cot A)(\operatorname{cosec}A+\cot a)}=\frac{2}{\sin A}$$

$$\frac{2\operatorname{cosec}A}{\operatorname{cosec}^2A-\cot^2A}=\frac{2}{\sin A}$$

$$\frac{2}{\frac{\sin A}{1}}=\frac{2}{\sin A}$$

$$\frac{\sin A}{2}=\frac{2}{\sin A}$$

Hence Proved.

10. If $\sec\theta=x+\frac{1}{4x}$, prove that $\sec\theta+\tan\theta=2x$ or, $\frac{1}{2x}$.

Ans : [Board Term-1, 2011, Set-55]

We have $\sec\theta=x+\frac{1}{4x}$

Squaring both side we have

$$\sec^2\theta=x^2+\frac{1}{16x^2}+2.x\frac{1}{4x}$$

$$1+\tan^2\theta=x^2+\frac{1}{16x^2}+\frac{1}{2}$$

$$\tan^2\theta=x^2+\frac{1}{16x^2}+2-1$$

$$=x^2+\frac{1}{16x^2}-1$$

$$=x^2+\frac{1}{16x^2}-2.x.\frac{1}{4x}$$

$$\tan^2\theta=\left(x-\frac{1}{4x}\right)^2$$

Taking square root both sides we obtain

$$\tan\theta=\pm\left(x-\frac{1}{4x}\right)$$

Now take $\tan\theta=x-\frac{1}{4x}$

Now $\sec\theta=x+\frac{1}{4x}$ Given

$$\tan\theta+\sec\theta=2x$$

Now take $\tan\theta=-\left(x-\frac{1}{4x}\right)=-x+\frac{1}{4x}$

$$\sec\theta = x + \frac{1}{4x}$$

$$\sec\theta + \tan\theta = \frac{1}{4x} + \frac{1}{4x} = \frac{1}{2x} \quad \text{Hence proved.}$$

11. Prove that : $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \dfrac{2}{2\sin^2\theta - 1}$

Ans : [Board Term-1, 2011, Set-39]

$$\text{LHS} = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) - 2\sin\theta\cos\theta + (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta}{\sin^2\theta - (1 - \sin^2\theta)}$$

$$= \frac{1+1}{\sin^2\theta - 1 + \sin^2\theta}$$

$$= \frac{2}{2\sin\theta - 1} = \text{RHS}$$

Hence Proved.

12. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, Prove that $x^2 + y^2 = 1$.

Ans : [Board Term-1, 2011, Set-44]

We have $\quad x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta \qquad (1)$

and $\quad x\sin\theta = y\cos\theta$

or, $\quad x = \dfrac{y\cos\theta}{\sin\theta} \qquad (2)$

Eliminating x from eqn. (1) and eqn. (2) we obtain,

$$\frac{y\cos\theta}{\sin\theta}\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$$

$$y\cos\theta \times 1 = \sin\theta\cos\theta$$

$$y = \sin\theta \qquad ...(3)$$

Substituting this value of y in eqn. (2) we have,

$$x = \cos\theta \qquad (4)$$

Squaring and adding eqn. (3) and eqn. (4), we get

$$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1 \qquad \text{Hence Proved.}$$

13. Prove that $\dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta + \sin\theta} + \dfrac{\cos^2\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2$

Ans : [Board Term-1, 2011, Set-40]

$$\text{LHS} = \frac{\cos^2\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)}$$

$$+ \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$$

$$= (1 - \sin\theta\cos\theta) + (1 + \sin\theta\cos\theta)$$

$$= 2 - \sin\theta\cos\theta + \sin\theta\cos\theta$$

$$= 2 = \text{RHS} \qquad \text{Hence Proved.}$$

14. Evaluate the following :

$$\frac{\sec^2(90^{\circ} - \theta) - \cot^2\theta}{2(\sin^2 25^{\circ} + \sin^2 65^{\circ})} - \frac{2\cos^2 60^{\circ}\tan^2 28^{\circ}\tan^2 62^{\circ}}{3(\sec^2 43^{\circ} - \cot^2 47^{\circ})}$$

Ans : [Board Term-1, 2011, Set-60]

$$\frac{\sec^2(90^{\circ} - \theta) - \cot^2\theta}{2(\sin^2 25^{\circ} + \sin^2 65^{\circ})} - \frac{2\cos^2 60^{\circ}\tan^2 28^{\circ}\tan^2 62^{\circ}}{3(\sec^2 43^{\circ} - \cot^2 47^{\circ})}$$

$$= \frac{(\operatorname{cosec}^2\theta - \cot^2\theta)}{2(\sin^2 25^{\circ} + \cos^2 25^{\circ})} - \frac{2 \times \frac{1}{2} \times \frac{1}{2}\tan^2 28^{\circ} \times \cot^2 28^{\circ}}{3(\sin^2 43^{\circ} - \tan^2 43^{\circ})}$$

$$= \frac{1}{2 \times 1} - \frac{\frac{1}{2} \times \tan^2 28^{\circ} \times \frac{1}{\tan^2 28^{\circ}}}{3}$$

$$= \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$$

15. Evaluate :

$$\frac{\sec 41^{\circ}\sin 49^{\circ} + \cos 29^{\circ}\operatorname{cosec} 61^{\circ} - \frac{2}{\sqrt{3}}(\tan 20^{\circ}\tan 60^{\circ}\tan 70^{\circ})}{3(\sin^2 31^{\circ} + \sin^2 59^{\circ})}$$

Ans : [Board Term-1, 2011, Set-25]

$$\frac{\sec 41^{\circ}\sin 49^{\circ} + \cos 29^{\circ}\operatorname{cosec} 61^{\circ} - \frac{2}{\sqrt{3}}(\tan 20^{\circ}\tan 60^{\circ}\tan 70^{\circ})}{3(\sin^2 31^{\circ} + \sin^2 59^{\circ})}$$

$$= \frac{\sec(90^{\circ} - 41^{\circ})\sin 49^{\circ} + \cos 29^{\circ}\operatorname{cosec}(90^{\circ} - 29^{\circ}) - \frac{2}{\sqrt{3}}[\tan 20^{\circ}\sqrt{3}\tan(90^{\circ} - 20^{\circ})]}{3[\sin^2 31^{\circ}\sin^2(90^{\circ} - 31^{\circ})]}$$

$$= \frac{\operatorname{cosec} 49^{\circ}\sin 49^{\circ} + \cos 29^{\circ}\sec 29^{\circ} - \frac{2}{\sqrt{3}}[\tan 20^{\circ}\sqrt{3}\tan(90^{\circ} - 20^{\circ})]}{3[\sin^2 31^{\circ} + \sin^2(90^{\circ} - 31^{\circ})]}$$

$$= \frac{1 + 1 - 2[\tan 20^{\circ}\cot 20^{\circ}]}{3[\sin^2 31^{\circ} + \cos^2 31^{\circ}]} = \frac{1+1-2}{3} = \frac{2-2}{3} = 0$$

16. Evaluate :

$$\frac{\cos^2(45^{\circ} + \theta) + \cos^2(45^{\circ} - \theta)}{\tan(60^{\circ} + \theta)\tan(30^{\circ} - \theta)} + \operatorname{cosec}(75^{\circ} + \theta) - \sec(15^{\circ} - \theta)$$

Ans : [Board Term-1, 2011, Set-40]

$$\frac{\cos^2(15^{\circ} + \theta) + \cos^2(45^{\circ} - \theta)}{\tan(60^{\circ} + \theta)\tan(30^{\circ} - \theta)}$$

$$+ \operatorname{cosec}(75^{\circ} + \theta) - \sec(15^{\circ} - \theta)$$

$$= \frac{\cos^2(45^{\circ} + \theta) + \sin^2(45^{\circ} + \theta)}{\tan(60^{\circ} + \theta)\cot(60^{\circ} + \theta)}$$

$$+ \operatorname{cosec}(75^{\circ} + \theta) - \operatorname{cosec}(90^{\circ} - 15^{\circ} + \theta)$$

$$= \frac{\cos^2(45^{\circ} + \theta) + \sin^2(45^{\circ} + \theta)}{\tan(60^{\circ} + \theta)\cot(60^{\circ} + \theta)} +$$

$$+ \operatorname{cosec}(75^{\circ} + \theta) - \operatorname{cosec}(75^{\circ} + \theta)$$

$$= \frac{1}{1} = 1$$

17. Express : $\sin A, \tan A$ and $\operatorname{cosec} A$ in terms of $\sec A$.

Ans : [Board Term-1, 2011, Set-25]

(1) $\quad \sin^2 A + \cos^2 A = 1$

$$\sin A = \sqrt{1 - \cos^2 A}$$

$$= \sqrt{1 - \frac{1}{\sec^2 A}}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

(2)
$$\tan A = \frac{\sin A}{\cos A} = \sin A \sec A$$
$$= \frac{\sqrt{\sec^2 A - 1}}{\sec A} \times \sec A$$
$$= \sqrt{\sec^2 A - 1}$$

(iii)
$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

18. Find the value of the following without using trigonometric tables :
$$\frac{\cos 50^\circ}{2\sin 40^\circ} + \frac{4(\operatorname{cosec}^2 59^\circ - \tan^2 31^\circ)}{3\tan^2 45^\circ} - \frac{2}{3}\tan 12^\circ \tan 78^\circ . \sin 90^\circ$$

Ans : [Board Term-1, 2011, Set-21]

We have
$$\cos 50^\circ = \cos(90^\circ - 40^\circ) = \sin 40^\circ$$
$$\operatorname{cosec}^2 59^\circ = \operatorname{cosec}^2(90^\circ - 31^\circ) = \sec^2 31^\circ$$
and
$$\tan 78^\circ = \tan(90^\circ - 12^\circ) = \cot 12^\circ$$

Hence,
$$\frac{\cos 50^\circ}{2\sin 40^\circ} + \frac{4(\operatorname{cosec}^2 59^\circ - \tan^2 31^\circ)}{3\tan^2 45^\circ} - \frac{2}{3}\tan 12^\circ \tan 78^\circ . \sin 90^\circ$$
$$= \frac{\sin 40^\circ}{2\sin 40^\circ} + \frac{4(\sec^2 31^\circ - \tan^2 31^\circ)}{3\tan^2 45^\circ} - \frac{2}{3}\tan 12^\circ \cot 12^\circ \times 1$$
$$= \frac{1}{2} + \frac{4}{3} - \frac{2}{3} = \frac{7}{6}$$

19. Evaluate :
$$\frac{\operatorname{cosec}^2 63^\circ + \tan 24^\circ}{\cos^2 66^\circ + \sec^2 27^\circ} + \frac{\sin^2 63^\circ + \cos 63^\circ . \sin 27^\circ + \sin 27^\circ \sec 63^\circ}{2(\operatorname{cosec}^2 65^\circ - \tan^2 25^\circ)}$$

Ans : [Sample Question Paper 2017-18]

$$\frac{\operatorname{cosec}^2 63^\circ + \tan^2 24^\circ}{\cot^2(90^\circ - 24^\circ) + \sec^2(90^\circ - 63^\circ)} +$$
$$\frac{\sin^2 63^\circ + \cos 63^\circ . \sin(90^\circ - 63^\circ) + \sin 27^\circ . \sec(90 - 27)}{2\left(\operatorname{cosec}^2 65^\circ - \tan^2(90^\circ - 65^\circ)\right)}$$
$$= \frac{\operatorname{cosec}^2 63^\circ + \tan^2 24^\circ}{\tan^2 24^\circ + \operatorname{cosec}^2 63^\circ} +$$
$$+ \frac{\sin^2 63^\circ + \cos 63^\circ . \cos 63^\circ + \sin 27^\circ . \operatorname{cosec} 27^\circ}{2(\operatorname{cosec}^2 65^\circ - \cot^2 65^\circ)}$$
$$= 1 + \frac{\sin^2 63^\circ + \cos^2 63^\circ + \sin 27^\circ \times \frac{1}{\sin 27^\circ}}{2 \times 1}$$
$$= 1 + \frac{1+1}{2} = 1 + 1 = 2$$

20. If $\sin\theta + \cos\theta = \sqrt{2}$, then evaluate $\tan\theta + \cot\theta$.

Ans : [Sample Question Paper 2017-18]

We have
$$\sin\theta + \cos\theta = \sqrt{2}$$

Squaring both sides, we get
$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$
$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$
$$1 + 2\sin\theta\cos\theta = 2$$
$$2\sin\theta\cos\theta \;\; 2 - 1 = 1$$
$$\frac{1}{\sin\theta\cos\theta} = 2$$

Now,
$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$
$$= \frac{1}{\cos\theta\sin\theta} = 2$$

LONG ANSWER TYPE QUESTIONS

1. Prove that $b^2x^2 - a^2y^2 = a^2b^2$, if :
(1) $x = a\sec\theta, y = b\tan\theta$, or
(2) $x = a\operatorname{cosec}\theta, y = b\cot\theta$

Ans : [Board Term-1, 2015, WJQZQBN]

(1) We have $x = a\sec\theta, y = b\tan\theta$,
$$\frac{x^2}{a^2} = \sec^2\theta, \; \frac{y^2}{b^2} = \tan^2\theta$$
or,
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta = 1$$

Thus $b^2x^2 - a^2y^2 = a^2b^2$ Hence Proved

(ii) We have $x = a\operatorname{cosec}\theta, y = b\cot\theta$
$$\frac{x^2}{a^2} = \operatorname{cosec}^2\theta, \; \frac{y^2}{b^2} = \cot^2\theta$$
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \operatorname{cosec}^2\theta - \cot\theta = 1$$

Thus $b^2x^2 - a^2y^2 = a^2b^2$ Hence Proved

2. If $\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta$, then prove that $\operatorname{cosec}\theta + \cot\theta = \sqrt{2}\operatorname{cosec}\theta$.

Ans : [Board Term-1, 2015, WJQZQBN]

We have
$$\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta$$

Squaring both sides we have
$$\operatorname{cosec}^2\theta + \cot^2\theta - 2\operatorname{cosec}\theta\cot\theta = 2\cot^2\theta$$
$$\operatorname{cosec}^2\theta - \cot^2\theta = 2\operatorname{cosec}\theta\cot\theta$$
$$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 2\operatorname{cosec}\theta\cot\theta$$
$$(\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta)$$
$$(\operatorname{cosec}\theta + \cot\theta)\sqrt{2}\cot\theta = 2\operatorname{cosec}\theta\cot\theta$$
$$\operatorname{cosec}\theta + \cot\theta = \sqrt{2}\operatorname{cosec}\theta$$

Hence Proved.

3. Prove that :
$$\frac{\cot^3\theta . \sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\tan^3\theta . \cos^2\theta}{(\cos\theta + \sin\theta)} = \frac{\sec\theta\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + \sec\theta}$$

Ans : [Set-FHN8MGD, 2015]

$$\frac{\cot^3\theta . \sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\tan^3\theta . \cos^2\theta}{(\cos\theta + \sin\theta)}$$
$$= \frac{\frac{\cos^3\theta}{\sin^3\theta} \times \sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\frac{\sin^3\theta}{\cos^3\theta} \times \cos^3\theta}{(\cos\theta + \sin\theta)^2}$$
$$= \frac{\cos^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\sin^3\theta}{(\cos\theta + \sin\theta)^2}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)^2}$$

$$= \frac{1 - \sin\theta\cos\theta}{\cos\theta + \sin\theta} = \frac{\frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}}{\frac{\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta}{\cos\theta\sin\theta}}$$

$$= \frac{\operatorname{cosec}\theta\sec\theta - 1}{\operatorname{cosec}\theta + \sec\theta} \qquad \text{Hence Proved}$$

4. Prove that : $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\operatorname{cosec}\theta.$

Ans : [Board Terim-1, 2012, Set-9]

$$\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = \frac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{(\sec\theta + 1)(\sec\theta - 1)}}$$

$$= \frac{2\sec\theta}{\sqrt{\sec^2\theta.1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} = \frac{2\sec\theta}{\tan\theta}$$

$$(\tan^2\theta = \sec^2\theta - 1)$$

$$= 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= 2 \times \frac{1}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta \qquad \text{Hence Prove}$$

5. Prove that : $\frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\sec\theta + 1}{\sec\theta - 1}.$

Ans : [Board Term-1, 2012, Set-21]

We have
$$\frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \sin\theta}{\frac{\sin\theta}{\cos\theta} - \sin\theta}$$

$$= \frac{\sin\theta(\frac{1}{\cos\theta} + 1)}{\sin\theta(\frac{1}{\cos\theta} - 1)}$$

$$= \frac{\sec\theta + 1}{\sec\theta - 1}$$

Hence Proved.

6. Prove that : $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

Ans : [Board Term-1, 2012, Set-62]

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1}$$

$$= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)}$$

$$= \frac{2\operatorname{cosec}^2 A}{\operatorname{cosec}^2 A - 1} = \frac{2\operatorname{cosec}^2 A}{\cot^2 A}$$

$$= \frac{\frac{2}{\sin^2 A}}{\frac{\cos^2 A}{\sin^2 A}} = \frac{2}{\sin^2 A} \times \frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{2}{\cos^2 A} = 2\sec^2 A \qquad \text{Hence Proved.}$$

7. If $\operatorname{cosec}\theta + \cot\theta = p$, then prove that $\cos\theta = \frac{p^2 - 1}{p^2 + 1}$.

Ans : [Board Term-1, 2016, Set-MV98HN3]

$$\frac{p^2 - 1}{p^2 + 1} = \frac{(\operatorname{cosec}\theta + \cot\theta) - 1}{(\operatorname{cosec}\theta + \cot\theta)^2 + 1}$$

$$= \frac{\operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta - 1}{\operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta + 1}$$

$$= \frac{1 + \cot^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta - 1}{\operatorname{cosec}^2\theta + \operatorname{cosec}^2\theta - 1 + 2\operatorname{cosec}\theta\cot\theta + 1}$$

$$= \frac{2\cot\theta(\cot\theta + \operatorname{cosec}\theta)}{2\operatorname{cosec}\theta(\operatorname{cosec}\theta + \cot\theta)}$$

$$= \frac{\cos\theta}{\sin\theta} \times \sin\theta = \operatorname{cosec}\theta \qquad \text{Hence proved}$$

8. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $m^2 + n^2 = a^2 + b^2$

Ans : [Board Term-1, 2012, Set-58]

We have

$$m^2 = a^2\cos^2\theta + 2ab\sin\theta\cos\theta + b^2\sin^2\theta \quad ...(1)$$

and,
$$n^2 = a^2\sin^2\theta - 2ab\sin\theta\cos\theta + b^2\cos^2\theta \quad ...(2)$$

Adding equations (1) and (2) we get

$$m^2 + n^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$$

$$= a^2(1) + b^2(1)$$

$$= a^2 + b^2 \qquad \text{Hence Proved.}$$

9. Prove that : $\frac{\cos^2\theta}{1 - \tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} = 1 + \sin\theta\cos\theta.$

Ans : [Board Term-1, 2012, Set-50]

$$\frac{\cos^2\theta}{1 - \tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} = \frac{\cos^2\theta}{1 - \frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\cos^3\theta}{\cos\theta - \sin\theta} - \frac{\sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$$

$$= 1 + \sin\theta\cos\theta \qquad \text{Hence Proved}$$

10. If $\cos\theta + \sin\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, prove that $q(p^2 - 1) = 2p$

Ans : [Board Term-1, 2012, Set-38]

We have $\cos\theta + \sin\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$

$$q(p^2 - 1) = (\sec\theta + \operatorname{cosec}\theta)[(\cos\theta + \sin\theta)^2 - 1]$$

$$= (\sec\theta + \operatorname{cosec}\theta)(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1)$$

$$= (\sec\theta + \operatorname{cosec}\theta)[1 + 2\sin\theta\cos\theta - 1]$$

$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$

$$= \frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta} \times 2\sin\theta\cos\theta$$

$$= 2(\sin\theta + \cos\theta) = 2p \qquad \text{Hence Proved.}$$

11. If $x = r\sin A\cos C, y = r\sin A\sin C$ and $z = r\cos A$, then prove that $x^2 + y^2 + z^2 = r^2$

Ans : [Board Term-1, 2012, Set-50]

Since,
$$x^2 = r^2\sin^2 A\cos^2 C$$

$$y^2 = r^2\sin^2 A\sin^2 C$$

and
$$z^2 = r^2\cos^2 A$$

$$x^2 + y^2 + z^2 = r^2\sin^2 A\cos^2 C + r^2\sin^2 A\sin^2 C + r^2\cos^2 A$$

$$= r^2\sin^2 A(\cos^2 C + \sin^2 C) + r^2\cos^2 A$$

$$= r^2\sin^2 A + r^2\cos^2 A$$

$$= r^2(\sin^2 A + \cos^2 A)$$

$$= r^2 \qquad \text{Hence Proved.}$$

12. Prove that: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=2\sec\theta$.

Ans : [Board Term-1, 2012, Set-40]

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)}\times\frac{(1+\sin\theta)}{(1+\sin\theta)}}+\sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)}\times\frac{(1-\sin\theta)}{(1-\sin\theta)}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}}+\sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$=\frac{1+\sin\theta}{\cos\theta}+\frac{1-\sin\theta}{\cos\theta}=\frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$$

$$=\frac{2}{\cos\theta}=2\sec\theta \qquad \text{Hence Prove}$$

13. Prove that
$(1-\sin\theta+\cos\theta)^2=2(1+\cos\theta)(1-\sin\theta)$.

Ans : [board Term-1, 2012, Set-62]

$$(1-\sin\theta+\cos\theta)^2$$

$$=1+\sin^2\theta+\cos^2\theta-2\sin\theta-2\sin\theta\cos\theta+2\cos\theta$$

$$=1+1-2\sin\theta-2\sin\theta\cos\theta+2\cos\theta$$

$$=2+2\cos\theta-2\sin\theta-2\sin\theta\cos\theta$$

$$=2(1+\cos\theta)-2\sin\theta(1+\cos\theta)$$

$$=(1+\cos\theta)(2-2\sin\theta)$$

$$=2(1+\cos\theta)(1-\sin\theta) \qquad \text{Hence Proved}$$

14. Prove that : $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta-1}=\sec\theta+\tan\theta$

Ans : [Board Term-1, 2012, Set-43]

$$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$$

$$=\frac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$$

$$=\frac{(\tan\theta+\sec\theta)-(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}$$

$$=\frac{(\tan\theta+\sec\theta)-[1-\sec\theta+\tan\theta]}{\tan\theta-\sec\theta+1}$$

$$=\tan\theta+\sec\theta \qquad \text{Hence Proved}$$

15. Prove that :
$(\sin\theta+\mathrm{cosec}\,\theta)^2+(\cos\theta+\sec\theta)^2=7+\tan^2\theta+\cot^2\theta\cot^2\theta$

Ans : [Board Term-1,2012, Set-52]

$$(\sin\theta+\mathrm{cosec}\,\theta)^2+(\cos\theta+\sec\theta)^2$$

$$=\sin^2\theta+\mathrm{cosec}^2\theta+2\sin\theta\,\mathrm{cosec}\,\theta+\cos^2\theta$$
$$+\sec^2\theta+2\cos\theta\sec\theta$$

$$=(\sin^2\theta+\cos^2\theta)+\mathrm{cosec}^2\theta+2\sin\theta\times\frac{1}{\sin\theta}$$
$$+\sec^2\theta+2\cos\theta\times\frac{1}{\sin\theta}$$

$$=1+(1+\cot^2\theta)+2+(1+\tan^2\theta)+2$$

$$=7+\tan^2\theta+\cot^2\theta \qquad \text{Hence Proved}$$

16. If $\sin\theta=\frac{c}{\sqrt{c^2+d^2}}$ and $d>0$, find the value of $\cos\theta$ and $\tan\theta$.

Ans : [Board Term - 1, 2013 LK-59]

We have $\qquad \sin\theta=\frac{c}{\sqrt{c^2+d^2}}$

Now $\qquad \cos^2\theta=1-\sin^2\theta$

$$=1-\left(\frac{c}{\sqrt{c^2+d^2}}\right)^2$$

$$=1-\frac{c^2}{c^2+d^2}$$

$$=\frac{c^2+d^2-c^2}{c^2+d^2}=\frac{d^2}{c^2+d^2}$$

Thus $\qquad \cos\theta=\frac{d}{\sqrt{c^2+d^2}}$

Again, $\qquad \tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\frac{c}{\sqrt{c^2+d^2}}}{\frac{d}{\sqrt{c^2+d^2}}}$

Thus $\qquad \tan\theta=\frac{c}{d}$

17. If $\tan\theta=\frac{1}{\sqrt5}$,

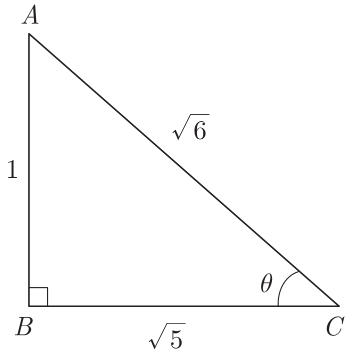
(1) Evaluate : $\frac{\mathrm{cosec}^2\theta-\sec^2\theta}{\mathrm{cosec}^2\theta+\sec^2\theta}$

(2) Verify the identity : $\sin^2\theta+\cos^2\theta=1$

Ans : [Board Term-1, 2012, Set-60]

We have $\qquad \tan\theta=\frac{1}{\sqrt5}$

We draw the triangle as shown below and write all dimensions.



(i) $\frac{\mathrm{cosec}^2\theta-\sec^2\theta}{\mathrm{cosec}^2\theta+\sec^2\theta}=\frac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$

$$=\frac{\cot^2\theta-\tan^2\theta}{2+\cot^2\theta+\tan^2\theta}$$

$$=\frac{(\sqrt5)^2-(\frac15)^2}{2(\sqrt5)^2+(\frac{1}{\sqrt5})^2}$$

$$=\frac{5-\frac15}{2+5+\frac15}=\frac{25-1}{35+1}=\frac{24}{36}=\frac23$$

(2)
$$\begin{aligned}\sin^2\theta+\cos^2\theta &= \left(\frac{1}{\sqrt{6}}\right)^2+\left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2\\ &= \frac{1}{6}+\frac{5}{6}=\frac{6}{6}\\ &= 1 \qquad \text{Hence proved.}\end{aligned}$$

18. Evaluate :
$$\frac{\cot(90^\circ-\theta)\sin(90^\circ-\theta)}{\sin\theta}+\frac{\cot 40^\circ}{\tan 50^\circ}-(\cos^2 20^\circ+\cos^2 70^\circ)$$

Ans : [Board Term-1, 2012, Set-35]

Given expression :
$$\frac{\cot(90^\circ-\theta)\sin(90^\circ-\theta)}{\sin\theta}+\frac{\cot 40^\circ}{\tan 50^\circ}-(\cos^2 20^\circ+\cos^2 70^\circ)$$

$$=\frac{\tan\theta\cos\theta}{\sin\theta}+\frac{\cot(90^\circ-50^\circ)}{\tan 50^\circ}-\left[\cos^2 20^\circ+\cos^2(90^\circ-20^\circ)\right]$$

$$=\frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}+\frac{\tan 50^\circ}{\tan 50^\circ}-\left[\cos^2 20^\circ+\sin^2 20^\circ\right]$$

$$=1+1-1=1$$

19. Evaluate :
$$\frac{\operatorname{cosec}^2(90^\circ-\theta)-\tan^2\theta}{4(\cos^2 40^\circ+\cos^2 50^\circ)}-\frac{2\tan^2 30^\circ\sec^2 52^\circ\sin^2 38^\circ}{3(\operatorname{cosec}^2 70^\circ-\tan^2 20^\circ)}$$

Ans : [Board Terim-1, 2012, Set-52]

$$\begin{aligned}\operatorname{cosec}^2(90^\circ-\theta) &= \sec^2\theta\\ \sec^2\theta-\tan^2\theta &= 1\\ \cos^2 40^\circ+\cos^2 50^\circ &= \cos^2(90^\circ-50^\circ)+\cos^2 50^\circ\\ \sin^2 50^\circ+\cos^2 50^\circ &= 1\\ \tan^2 30^\circ &= \left(\frac{1}{\sqrt{3}}\right)^2=\frac{1}{3}\end{aligned}$$

$$\begin{aligned}\sec^2 52^\circ\sin^2 38^\circ &= \sec^2 52^\circ\sin^2(90^\circ-52^\circ)\\ &= \sec^2 52\cos^2 52^\circ=1\end{aligned}$$

and
$$\begin{aligned}\operatorname{cosec}^2 70^\circ-\tan^2 20^\circ &= \operatorname{cosec}^2(90^\circ-20^\circ)-\tan^2 20^\circ\\ &= \sec^2 20^\circ-\tan^2 20^\circ=1\end{aligned}$$

Thus given expression becomes
$$=\frac{1}{4}-\frac{2\times\frac{1}{3}\times 1}{3(1)}$$
$$=\frac{1}{4}-\frac{2}{9}=\frac{9-8}{36}=\frac{1}{36}$$

20. If $\sec\theta+\tan\theta=p$, show that $\sec\theta-\tan\theta=\frac{1}{p}$, Hence, find the values of $\cos\theta$ and $\sin\theta$.

Ans : [Board Term-1, 2015]

We have $\sec\theta+\tan\theta = p$

Now
$$\begin{aligned}\frac{1}{p} &= \frac{1}{\sec\theta+\tan\theta}\times\frac{(\sec\theta-\tan\theta)}{\sec\theta-\tan\theta}\\ &= \frac{\sec\theta-\tan\theta}{\sec^2\theta-\tan^2\theta}=\sec\theta-\tan\theta\\ &= \sec\theta-\tan\theta\end{aligned}$$

Solving $\sec\theta+\tan\theta = p$ and $\sec\theta-\tan\theta=\frac{1}{p}$,

$$\sec\theta = \frac{1}{2}\left(p+\frac{1}{p}\right)=\frac{p^2+1}{2p}$$

and
$$\tan\theta = \frac{1}{2}\left(p-\frac{1}{p}\right)=\frac{p^2-1}{2p}$$

Thus
$$\cos\theta = \frac{2p}{p^2+1}$$

and
$$\sin\theta = \tan\theta\cos\theta = \frac{p^2-1}{p^2+1}$$

21. Prove that : $(\operatorname{cosec}\theta+\cot\theta)^2=\frac{\sec\theta+1}{\sec\theta-1}$

Ans :

$$\begin{aligned}(\operatorname{cosec}\theta+\cot\theta)^2 &= \operatorname{cosec}^2\theta+\cot^2\theta+2\operatorname{cosec}\theta.\cot\theta\\ &= \left(\frac{1}{\sin\theta}\right)^2+\left(\frac{\cos\theta}{\sin\theta}\right)+\frac{2\times 1}{\sin\theta}\times\frac{\cos\theta}{\sin\theta}\\ &= \frac{1}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}+\frac{2\cos\theta}{\sin^2\theta}\\ &= \frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta}=\frac{(1+\cos\theta)^2}{\sin^2\theta}\\ &= \frac{(1+\cos\theta)(1+\cos\theta)}{1-\cos^2\theta}\\ &= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}\\ &= \frac{1+\cos\theta}{1-\cos\theta}=\frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}}\\ &= \frac{\sec\theta+1}{\sec\theta-1} \qquad \text{Hence Prove.}\end{aligned}$$

22. Prove that :
$(\sin A+\sec A)^2+(\cos A+\operatorname{cosec} A)^2=(1+\sec A\operatorname{cosec} A)^2$

Ans : [Board Term-I, 2012, Set 25]

$$\begin{aligned}\text{LHS} &= (\sin A+\sec A)^2+(\cos A+\operatorname{cosec} A)^2\\ &= \left(\sin A+\frac{1}{\cos A}\right)^2+\left(\cos A+\frac{1}{\sin A}\right)^2\\ &= \sin^2 A+\frac{1}{\cos^2 A}+2\frac{\sin A}{\cos A}+\cos^2 A\\ &\qquad +\frac{1}{\sin^2 A}+2\frac{\cos A}{\sin A}\\ &= \sin^2 A+\cos^2 A+\frac{1}{\sin^2 A}+\frac{1}{\cos^2 A}\\ &\qquad +2\left(\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right)\\ &= 1+\frac{\sin^2 A+\cos^2 A}{\sin^2 A\cos^2 A}+2\left(\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}\right)\\ &= 1+\frac{1}{\sin^2 A\cos^2 A}+\frac{2}{\sin A\cos A}\\ &= \left(1+\frac{1}{\sin A\cos A}\right)^2\\ &= (1+\sec A.\operatorname{cosec} A)^2 \qquad \text{Hence Proved}\end{aligned}$$

23. If $(\sec A+\tan A)(\sec B+\tan B)(\sec C+\tan C)$
$=(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)$
Prove that each of the side is equal to ± 1.

Ans : [Board Term-I, 2012, Set-12]

We have

$$(\sec A+\tan A)(\sec B+\tan B)(\sec C+\tan C)$$
$$=(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)$$

Multiply both sides by

$$(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)$$

or, $(\sec A+\tan A)(\sec B+\tan B)(\sec C+\tan C)\times$

$$(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)$$
$$=(\sec A-\tan A)^2(\sec B-\tan B)^2(\sec C-\tan C)^2$$

or, $(\sec^2 A-\tan^2 A)(\sec^2 B-\tan^2 B)(\sec^2 C-\tan^2 C)$

$$=(\sec A-\tan A)^2(\sec A-\tan B)^2(\sec C-\tan C)^2$$

or, $1=\left[(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)\right]^2$

or, $(\sec A-\tan A)(\sec B-\tan B)(\sec C+\tan C)=\pm 1$

24. If $4\sin\theta=3$, find the value of x if

$$\sqrt{\frac{\operatorname{cosec}^2\theta\cot^2\theta}{\sec^2\theta-1}}+2\cot\theta=\frac{\sqrt{7}}{x}+\cos\theta$$

Ans : [Board Term-1, 2012, Set-40]

We have $\sin\theta=\frac{3}{4}$

or, $\sin^2\theta=\frac{9}{16}$

Since $\sin^2\theta+\cos^2=1$, we have

$$\cos^2\theta=1-\sin^2\theta=1-\frac{9}{16}=\frac{7}{16}$$

$$\cos\theta=\frac{\sqrt{7}}{4}$$

and $\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\frac{3}{4}}{\frac{\sqrt{7}}{4}}=\frac{3}{\sqrt{7}}$

Thus $\sqrt{\frac{\operatorname{cosec}^2\theta-\cot^2\theta}{\sec^2\theta-1}}+2\cot\theta=\frac{\sqrt{7}}{x}+\cos\theta$

$$\sqrt{\frac{1}{\tan^2\theta}}+2\times\frac{\sqrt{7}}{3}=\frac{\sqrt{7}}{x}+\frac{\sqrt{7}}{4}$$

$$\frac{1}{\tan\theta}+\frac{2\sqrt{7}}{3}=\frac{\sqrt{7}}{x}+\frac{\sqrt{7}}{4}$$

$$\frac{\sqrt{7}}{3}+\frac{2\sqrt{7}}{3}-\frac{\sqrt{7}}{4}=\frac{\sqrt{7}}{x}$$

$$\frac{4\sqrt{7}-\sqrt{7}}{4}=\frac{\sqrt{7}}{x}$$

$$\frac{3\sqrt{7}}{4}=\frac{\sqrt{7}}{x}$$

Thus $x=\frac{4}{3}$

HOTS QUESTIONS

1. Prove that $\sec^2\theta+\operatorname{cosec}^2\theta$ can never be less than 2.

Ans :

Let $\sec^2\theta+\operatorname{cosec}^2\theta=x$

$$1+\tan^2\theta+1+\cot^2\theta=x$$
$$2+\tan^2\theta+\cot^2\theta=x$$
$$2+\tan^2\theta+\cot^2\theta=x$$

$\tan^2\theta\geq 0$ and $\cot^2\theta\geq x$

Thus $x>2$

Thus $\sec^2\theta+\operatorname{cosec}^2\theta>2$

Hence $\sec^2\theta+\operatorname{cosec}^2\theta$ can never be less than 2.

2. (a) Solve for ϕ, if $\tan 5\phi=1$

(b) Solve for ϕ, if $\frac{\sin\phi}{1+\cos\phi}+\frac{1+\cos\phi}{\sin\phi}=4$

Ans :

(a) $\tan 5\phi=1$

$$\tan 5\phi=\tan 45^\circ$$
$$5\phi=45^\circ$$

Thus $\phi=9^\circ$

(b) $\frac{\sin\phi}{1+\cos\phi}+\frac{1+\cos\phi}{\sin\phi}=4$

$$\frac{\sin^2\phi+(1+\cos\theta)^2}{\sin\phi(1+\cos\phi)}=4$$

$$\frac{\sin^2\phi+1+\cos^2\phi+2\cos\phi}{\sin\phi+\sin\phi\cos\phi}=4$$

$$\frac{2+2\cos\phi}{\sin\phi(1+\cos\phi)}=4$$

$$\frac{2(1+\cos\phi)}{\sin\phi(1+\cos\phi)}=4$$

$$\frac{2}{\sin\phi}=4$$

$$\sin\phi=\frac{1}{2}$$

$$\sin\phi=\sin 30$$

Thus $\phi=30^\circ$

3. If $\tan A+\sin A=m$ and $\tan A-\sin A=n$, show that $m^2-n^2=4\sqrt{mn}$.

Ans :

We have $\tan A+\sin A=m$

and $\tan A-\sin A=n$

$$m^2-n^2=(\tan A+\sin A)^2-(\tan A-\sin A)^2$$
$$=(\tan^2 A+\sin^2 A+2\sin A.\tan A)$$
$$-(\tan^2 A+\sin^2 A-2\sin A.\tan A)$$
$$=\tan^2 A+\sin^2 A+2\sin A\tan A$$
$$-\tan^2 A-\sin^2 A+2\sin A\tan A$$
$$=4\sin A\tan A$$

$$4\sqrt{mn}=4\sqrt{(\tan A+\sin A)(\tan A-\sin A)}$$
$$=4\sqrt{\tan^2 A-\sin^2 A}$$
$$=4\sqrt{\frac{\sin^2 A-\sin^2 A\cos^2 A}{\cos^2 A}}$$
$$=4\sqrt{\frac{\sin^2 A(1-\cos^2 A)}{\cos^2 A}}$$
$$=4\sqrt{\frac{\sin^2 A\times\sin^2 A}{\cos^2 A}}$$

$$= 4\sin A\tan A$$

Thus $m^2 - n^2 = 4\sqrt{mn}$ Hence Proved

4. If $\dfrac{\cos\alpha}{\cos\beta} = m$ and $\dfrac{\cos\alpha}{\sin\beta} = n$, show that $(m^2+n^2)\cos^2\beta = n^2$.

Ans :

We have $\dfrac{\cos\alpha}{\cos\beta} = m$ and $\dfrac{\cos\alpha}{\sin\beta} = n$

$$m^2 = \frac{\cos^2\alpha}{\cos^2\beta} \text{ and } n^2 = \frac{\cos^2\alpha}{\sin^2\beta}$$

$$\begin{aligned}(m^2+n^2)\cos^2\beta &= \left[\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right]\cos^2\beta \\ &= \cos^2\alpha\left[\frac{1}{\cos^2\beta} + \frac{1}{\sin^2\beta}\right]\cos^2\beta \\ &= \cos^2\alpha\frac{\sin^2\beta+\cos^2\beta}{\cos^2\beta\sin^2\beta}\cos^2\beta \\ &= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta \\ &= \frac{\cos^2\alpha}{\sin^2\beta} \\ &= n^2\end{aligned}$$

Hence Proved.

5. If $7\operatorname{cosec}\phi - 3\cot\phi = 7$, prove that $7\cot\phi - 3\operatorname{cosec}\phi = 3$.

Ans :

We have

$$\begin{aligned}7\operatorname{cosec}\phi - 3\cot\phi &= 7 \\ 7\operatorname{cosec}\phi - 7 &= 3\cot\phi \\ 7(\operatorname{cosec}\phi - 1) &= 3\cot\phi \\ 7(\operatorname{cosec}\phi-1)(\operatorname{cosec}\phi+1) &= 3\cot\phi(\operatorname{cosec}\phi+1) \\ 7(\operatorname{cosec}^2\phi - 1) &= 3\cot\phi(\operatorname{cosec}\phi+1) \\ 7\cot^2\phi &= 3\cot\phi(\operatorname{cosec}\phi+1) \\ 7\cot\phi &= 3(\operatorname{cosec}\phi+1) \\ 7\cot\phi - 3\operatorname{cosec}\phi &= 3\end{aligned}$$

Hence Proved

6. Prove that : $\dfrac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta$

Ans : [Sample Question Paper 2017-18]

$$\begin{aligned}\frac{\cos\theta-\sin\theta+1}{\cos\theta+\sin\theta-1} &= \frac{\sin\theta(\cos\theta-\sin\theta+1)}{\sin\theta(\cos\theta+\sin\theta-1)} \\ &= \frac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta}{\sin\theta(\cos\theta+\sin\theta-1)} \\ &= \frac{\sin\theta\cos\theta + \sin\theta - (1-\cos^2\theta)}{\sin\theta(\cos\theta+\sin\theta-1)} \\ &= \frac{\sin\theta(\cos\theta+1) - [(1-\cos\theta)(1+\cos\theta)]}{\sin\theta(\cos\theta+\sin\theta-1)} \\ &= \frac{(1+\cos\theta)(\sin\theta-1+\cos\theta)}{\sin\theta(\cos\theta+\sin\theta-1)} \\ &= \frac{(1+\cos\theta)(\cos\theta+\sin\theta-1)}{\sin\theta(\cos\theta+\sin\theta-1)} \\ &= \frac{1+\cos\theta}{\sin\theta} \\ &= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} \\ &= \operatorname{cosec}\theta + \cot\theta\end{aligned}$$

Hence Proved

CHAPTER 9

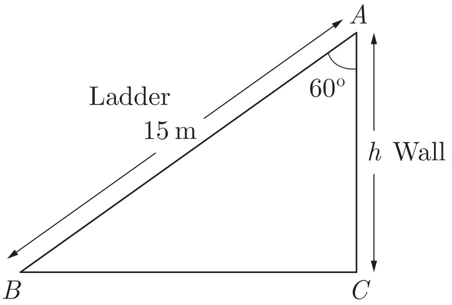
Some Applications of Trigonometry

VERY SHORT ANSWER TYPE QUESTIONS

1. A ladder 15 m long leans against a wall making an angle of 60^o with the wall. Find the height of the point where the ladder touches the wall.

Ans : [KVS 2014]

Let the height of wall be h. As per given in question we have drawn figure below.

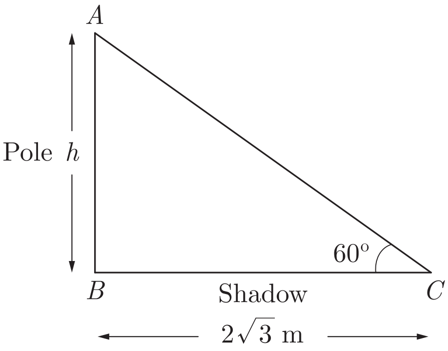


$$\begin{aligned} \frac{h}{15} &= \cos 60^o \\ h &= 15 \times \cos 60^o \\ &= 15 \times \frac{1}{2} = 7.5 \text{ m} \end{aligned}$$

2. A pole casts a shadow of length $2\sqrt{3}$ m on the ground, when the Sun's elevation is 60^o. Find the height of the pole.

Ans : [CBSE Foreign 2015]

Let the height of pole be h. As per given in question we have drawn figure below.

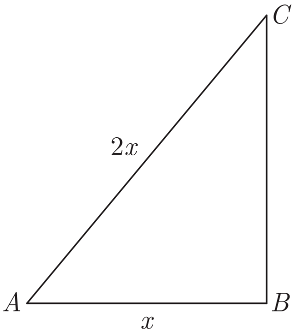


$$\begin{aligned} \frac{h}{2\sqrt{3}} &= \tan 60^o \\ h &= 2\sqrt{3} \tan 60^o \\ &= 2\sqrt{3} \times \sqrt{3} = 6 \text{ m} \end{aligned}$$

3. If the length of the ladder placed against a wall is twice the distance between the foot of the ladder and the wall. Find the angle made by the ladder with the horizontal.

Ans : [CBSE 2015, Set-HODM40L]

Let the distance between the foot of the ladder and the wall is x, then length of the ladder will be $2x$. As per given in question we have drawn figure below.

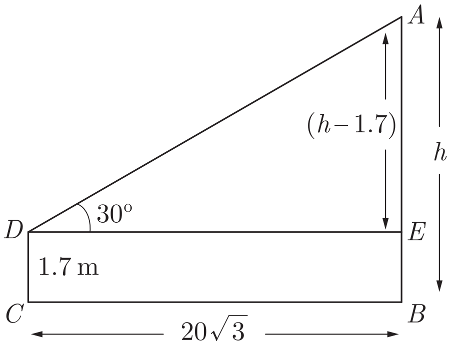


$$\begin{aligned} \text{In } \Delta ABC, \qquad \angle B &= 90^o \\ \cos A &= \frac{x}{2x} = \frac{1}{2} = \cos 60^o \\ A &= 60^o \end{aligned}$$

4. An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30^o. Find the height of tower.

Ans : [CBSE Foreign 2016]

Let height of the tower AB be h. As per given in question we have drawn figure below.



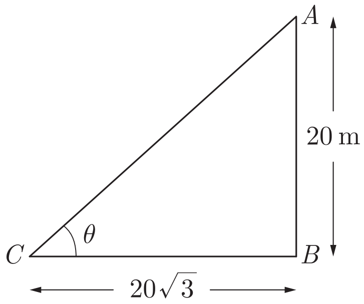
$$\begin{aligned} \text{Here} \qquad AE &= h - 1.7 \\ \text{and} \qquad BC &= DE = 20\sqrt{3} \\ \text{In } \Delta ADE, \qquad \angle E &= 90^o \\ \tan 30^o &= \frac{h - 1.7}{20\sqrt{3}} \end{aligned}$$

$$\frac{1}{\sqrt{3}} = \frac{h-1.7}{20\sqrt{3}}$$

$$h - 1.7 = 20$$

or $$h = 20 + 1.7 = 21.7 \text{ m}$$

5. In figure, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. find the Sun's altitude.



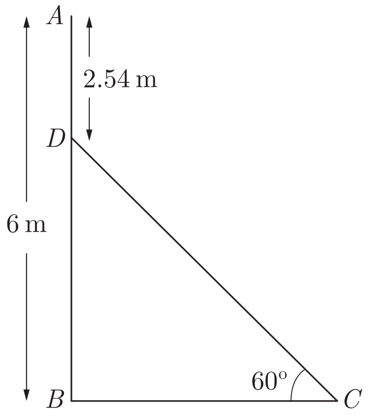
Ans : [CBSE Outside Delhi 2015]

Let the $\angle ACB$ be θ.

$$\tan\theta = \frac{AB}{BC} = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

Thus $$\theta = 30^\circ$$

6. In the given figure, AB is a 6 m high pole and DC is a ladder inclined at an angle of 60° to the horizontal and reaches up to point D of pole. If $AD = 2.54$ m, find the length of ladder. (use $\sqrt{3} = 1.73$)



Ans : [CBSE Delhi 2016]

We have $$AD = 2.54 \text{ m}$$

$$DB = 6 - 2.54 = 3.46 \text{ m}$$

In ΔBCD, $$\angle B = 90^\circ$$

$$\sin 60^\circ = \frac{BD}{DC}$$

$$\frac{\sqrt{3}}{2} = \frac{3.46}{DC}$$

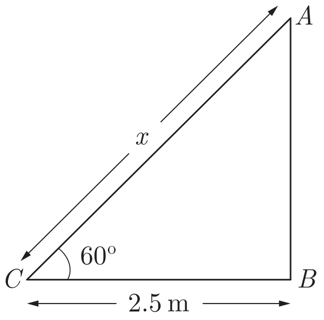
$$DC = \frac{3.46 \times 2}{\sqrt{3}} = \frac{3.46}{1.73} = 4$$

Thus length of ladder is 4 m.

7. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

Ans : [CBSE Board Term-2, 2011]

Let the length of ladder be x. As per given in question we have drawn figure below.



In ΔACB with $\angle C = 60^\circ$

$$\cos 60^\circ = \frac{2.5}{AC}$$

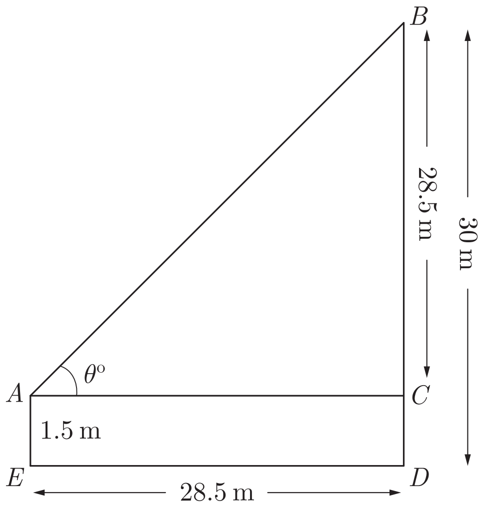
$$\frac{1}{2} = \frac{2.5}{AC}$$

$$AC = 2 \times 2.5 = 5 \text{ m}$$

8. An observer 1.5 m tall is 28.5 m away from a tower 30 m high. Find the angle of elevation of the top of the tower from his eye.

Ans : [CBSE Board Term-2, 2012]

As per given in question we have drawn figure below.



Here $AE = 1.5$ m is height of observer and $BD = 30$ m is tower.

Now $$BC = 30 - 1.5 = 28.5 \text{ m}$$

In ΔBAC, $$\tan\theta = \frac{BC}{AC}$$

$$\tan\theta = \frac{28.5}{28.5} = 1 = \tan 45^\circ$$

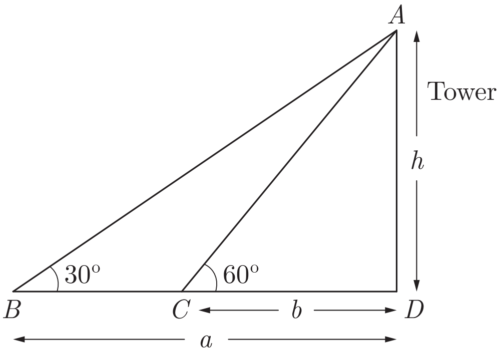
$$\theta = 45^\circ$$

Hence angle of elevation is 45°

9. If the angles of elevation of the top of a tower from two points distant a and $b(a > b)$ from its foot and in the same straight line from it are respectively 30° and 60°, then find the height of the tower.

Ans : [CBSE 2014]

Let the height of tower be h. As per given in question we have drawn figure below.



From ΔABD, $\frac{h}{a} = \tan 30°$

$$h = a \times \frac{1}{\sqrt{3}} = \frac{a}{\sqrt{3}} \quad ...(1)$$

From ΔACD, $\frac{h}{b} = \tan 60°$

$$h = b \times \sqrt{3} = b\sqrt{3} \quad ...(2)$$

From (1) $a = \sqrt{3}\,h$

From (2) $b = \frac{h}{\sqrt{3}}$

Thus $a \times b = \sqrt{3}\,h \times \frac{h}{\sqrt{3}}$

$$ab = h^2$$

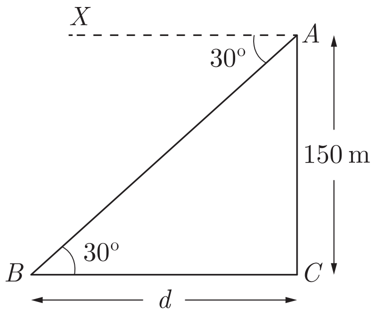
$$h = \sqrt{ab}$$

Hence, the height of the tower is $\sqrt{ab}$.

10. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. Find the distance of the car from the tower (in m).

Ans : [CBSE Outside Delhi, 2014]

Let the distance of the car from the tower be d. As per given in question we have drawn figure below.



Due to alternate angles we have

$$\angle BAX = \angle ABC = 30°$$

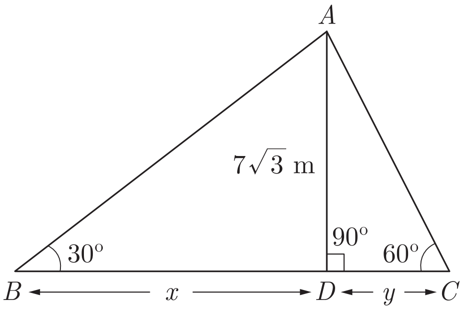
In ΔACB, $\angle C = 90°$

$$\tan 30° = \frac{150}{d}$$

$$\frac{1}{\sqrt{3}} = \frac{150}{d}$$

Thus $d = 150\sqrt{3}$ m.

11. In the given figure, if $AD = 7\sqrt{3}$ m, then find the value of BC.



Ans : [CBSE 2012]

Let $BD = x$ and $DC = y$

From ΔABD we get

$$\tan 30° = \frac{7\sqrt{3}}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{7\sqrt{3}}{x}$$

$$x = 7\sqrt{3} \times \sqrt{3} = 21 \text{ m}$$

From ΔADC,

$$\tan 60° = \frac{7\sqrt{3}}{y}$$

$$\sqrt{3} = \frac{7\sqrt{3}}{y}$$

$$y = 7 \text{ m.}$$

Now $BC = BD + DC$

$$= 21 + 7 = 28 \text{ m.}$$

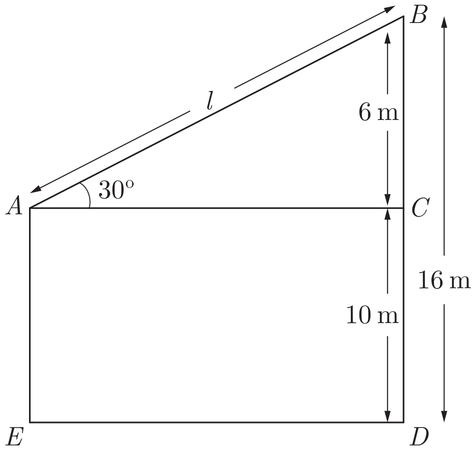
Hence, the value of BC is 28 m.

12. The top of two poles of height 16 m and 10 m are connected by a length l meter. If wire makes an angle of 30° with the horizontal, then find l.

Ans : [CBSE Board Term-2, 2012]

Let BD and AE be two poles, where $BD = 16$ m, $AE = 10$m.

As per given in question we have drawn figure below.



Length $BC = BD - CD = BD - AE$

$$= 16 - 10 = 6 \text{ m.}$$

From $\Delta ABC, \sin 30^\circ = \dfrac{BC}{l}$

$$\frac{1}{2} = \frac{BC}{l}$$

$$l = 2BC$$

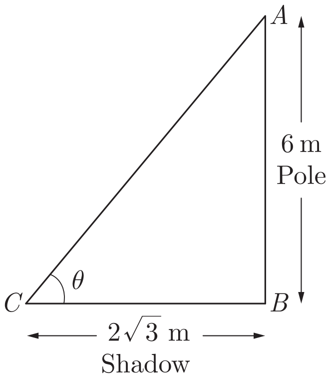
$$= 6 \times 2 = 12 \text{ m.}$$

Hence, the value of l is 12 m.

13. *A* pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then find the Sun's elevation.

Ans : [CBSE Board Term-2, 2012]

Let the Sun's elevation be θ. As per given in question we have drawn figure below.



Length of pole is 6 m and length of shadow is $2\sqrt{3}$ m.

From ΔABC, we have

$$\tan\theta = \frac{AB}{BC}$$

$$\tan\theta = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60^\circ$$

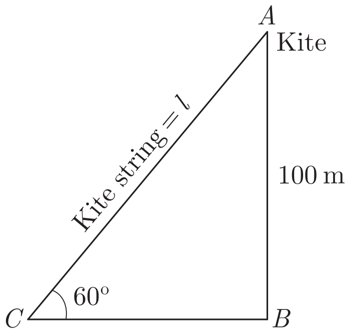
$$\theta = 60^\circ$$

Hence sun's elevation is 60°.

14. Find the length of kite string flying at 100 m above the ground with the elevation of 60°.

Ans : [CBSE Board Term-2, 2012]

Let the length of kite string $AC = l$ m. As per given in question we have drawn figure below.



Here $\angle ACB = 60^\circ$, height of kite $AB = 100$ m.

From ΔABC, we have

$$\sin 60^\circ = \frac{AB}{BC}$$

$$\frac{\sqrt{3}}{2} = \frac{100}{l}$$

$$l = \frac{2 \times 100}{\sqrt{3}} = \frac{200}{\sqrt{3}} \text{ m}$$

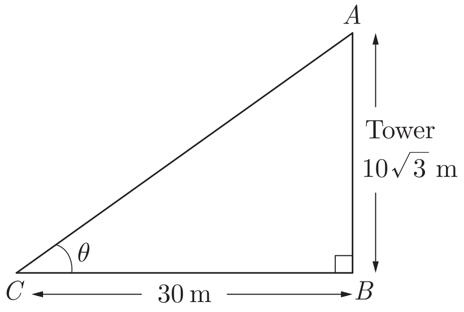
$$= \frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{200\sqrt{3}}{3} \text{ m}$$

Hence length the kids string is $\dfrac{200\sqrt{3}}{3}$

15. Find the angle of elevation of the top of the tower from the point on the ground which is 30 m away from the foot of the tower of height $10\sqrt{3}$ m.

Ans : [CBSE Board Term-2, 2012]

Let the angle of elevation of top of the tower be θ. As per given in question we have drawn figure below.



From ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{10\sqrt{3}}{30} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

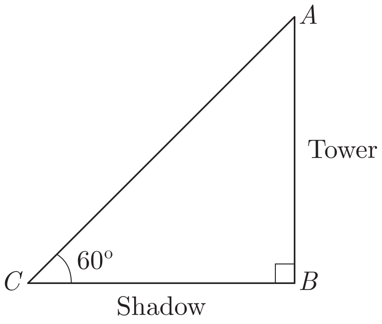
Thus $\theta = 30^\circ$

Hence angle of elevation is 30°.

16. If the altitude of the sun is 60°, what is the height of a tower which casts a shadow of length 30 m ?

Ans : [CBSE Board Term-2, 2011]

Let AB be the tower whose height be h m. As per given in question we have drawn figure below.



Here shadow is $BC = 30$ m.

From ΔABC, we get

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{h}{30} = \sqrt{3}$$

$$h = 30\sqrt{3} \text{ m}$$

Hence, height of tower is $30\sqrt{3}$ m.

17. If $\cos A = \frac{2}{5}$, find the value of $4 + 4\tan^2 A$.

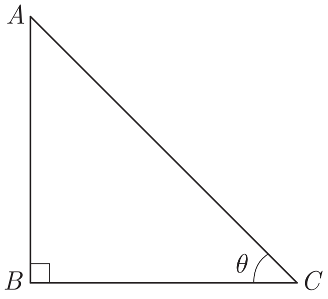
Ans : [CBSE Sample Question Paper 2017-18]

$$4 +| 4\tan^2 A = 4(1+\tan^2 A)$$
$$4\sec^2 A = \frac{4}{\cos^2 A} = \frac{4}{\left(\frac{2}{5}\right)^2} = 4 \times \frac{25}{4} = 25$$

18. The ratio of the height of a tower and the length of its shadow on the grow is $\sqrt{3} : 1$. What is the angle of elevation of the sun ?

Ans : [CBSE Board Term-2, 2016]

Let height of tower be AB and its shadow be BC. As per given in question we have drawn figure below.



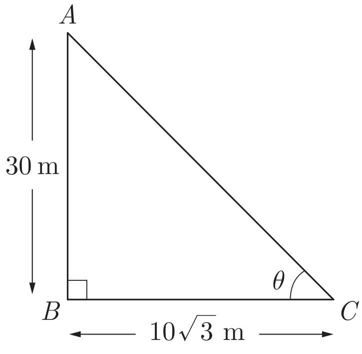
$$\frac{AB}{BC} = \tan\theta = \frac{\sqrt{3}}{1} = \tan 60^\circ$$

Hence, angle of elevation of sun is 60°.

19. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun ?

Ans : [CBSE Outside Delhi 2017]

Tower AB is 30 m and shadow BC is $10\sqrt{3}$. As per given in question we have drawn figure below.



In ΔABC which is right triangle,

$$\tan\theta = \frac{AB}{BC} = \frac{30}{10\sqrt{3}} = \sqrt{3} = \tan 60^\circ$$

Thus $\theta = 60^\circ$

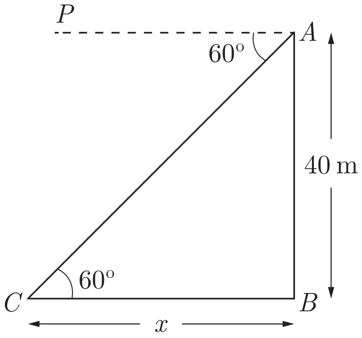
so, angle of elevation of sun is 60°.

SHORT ANSWER TYPE QUESTIONS - I

1. From the top of light house, 40 m above the water, the angle of depression of a small boat is 60°. Find how far the boat from the base of the light house.

Ans : [CBSE Board Term-2, 2015]

Let AB be the light house and C be the position of the boat. As per given in question we have drawn figure below.



Since $\angle PAC = 60^\circ \Rightarrow \angle ACB = 60^\circ$

Let $CB = x$

In ΔABC, $\tan 60^\circ = \frac{AB}{BC}$

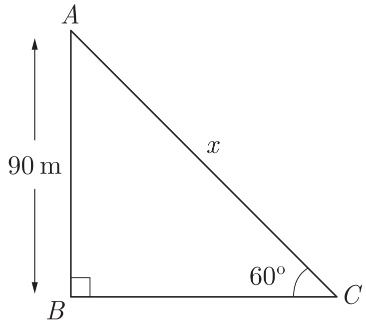
$$\sqrt{3} = \frac{40}{x}$$
$$x = \frac{40}{\sqrt{3}} = \frac{40 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{40\sqrt{3}}{3} \text{ m}$$

Hence, The boat is $\frac{40\sqrt{3}}{3}$ m away from the foot of light house.

2. A kite is flying at a height of 90 m above the ground. The string attached to the kite is temporarily tried to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is no slack in the string.

Ans : [CBSE Delhi Term-2, 2014, 2011]

As per given in question we have drawn figure below.



In right ΔABC, we have

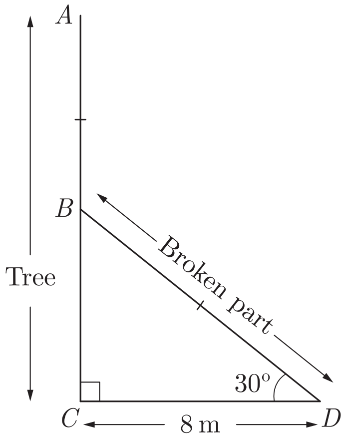
$$\sin 60^\circ = \frac{AB}{AC}$$
$$\frac{\sqrt{3}}{2} = \frac{90}{x}$$
$$x = \frac{90 \times 2}{\sqrt{3}} = \frac{180}{\sqrt{3}} = \frac{3 \times 60}{\sqrt{3}}$$
$$= 60\sqrt{3} = 60 \times 1.732$$

Hence length of string is 103.92 m.

3. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Ans : [CBSE Board Term-2, 2011, Set A1]

Let the tree be AC and is broken at B. The broken part touches at the point D on the ground. As per given in question we have drawn figure below.



In right ΔBCD,
$$\begin{aligned} \cos 30^\circ &= \frac{CD}{BD} \\ \frac{\sqrt{3}}{2} &= \frac{8}{BD} \\ BD &= \frac{16}{\sqrt{3}} \end{aligned}$$

and
$$\begin{aligned} \tan 30^\circ &= \frac{BC}{CD} \\ \frac{1}{\sqrt{3}} &= \frac{BC}{8} \\ BC &= \frac{8}{\sqrt{3}} \end{aligned}$$

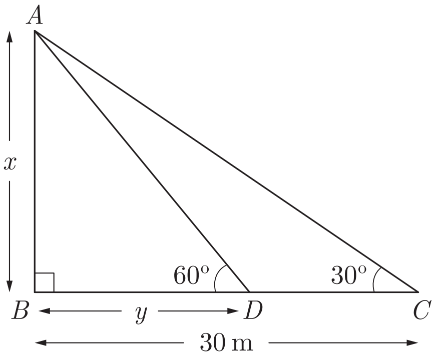
Height of tree,
$$BC + BD = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = 8\sqrt{3}$$

Hence the height of the tree is $8\sqrt{3}$ m.

4. If the shadow of a tower is 30 m long, when the Sun's elevation is 30°. What is the length of the shadow, when Sun's elevation is 60° ?

Ans : [CBSE Board Term-2, 2011, Set C1]

As per given in question we have drawn figure below.



In ΔABC,
$$\begin{aligned} \frac{AB}{BC} &= \tan 30^\circ \\ \frac{AB}{30} &= \tan 30^\circ = \frac{1}{\sqrt{3}} \\ AB &= \frac{30}{\sqrt{3}} = 10\sqrt{3} \end{aligned}$$

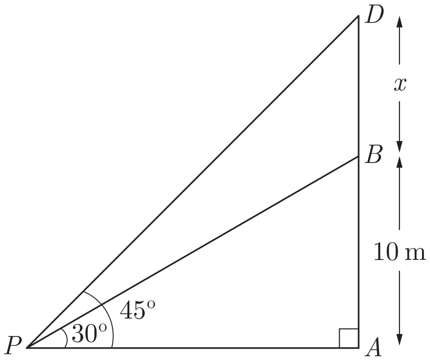
In ΔABD,
$$\begin{aligned} \frac{AB}{BD} &= \tan 60^\circ \\ \frac{10\sqrt{3}}{BD} &= \tan 60^\circ = \sqrt{3} \\ BD &= 10 \text{ m} \end{aligned}$$

Hence the length of shadow is 10 m.

5. From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top the of the building and the angle of elevation of the length of the flagstaff from P is 45°. Find the length of the flagstaff and distance of building from point P. [Take $\sqrt{3} = 1.732$]

Ans : [CBSE Board Term-2, 2012] [Delhi 2013] [Term-2, 2011]

Let height of flagstaff be $BD = x$ m. As per given in question we have drawn figure below.



$$\begin{aligned} \tan 30^\circ &= \frac{AB}{AP} \\ \frac{1}{\sqrt{3}} &= \frac{10}{AP} \\ AP &= 10\sqrt{3} \end{aligned}$$

Distance of the building from P,
$$= 10 \times 1.732 = 17.32 \text{ m}$$

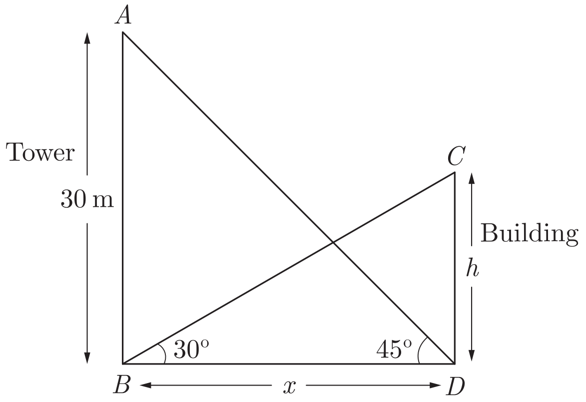
Now
$$\begin{aligned} \tan 45^\circ &= \frac{AD}{AP} \\ 1 &= \frac{10 + x}{17.32} \\ x &= 17.32 - 10.00 = 7.32 \text{ m} \end{aligned}$$

Hence, length of flagstaff is 7.32 m.

6. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building.

Ans : [Delhi CBSE 2015 Set I, II, III]

Let the height of the building be $AB = h$ m. and distant between tower and building be, $BD = x$ m. As per given in question we have drawn figure below.



In ΔABD $\quad \tan 45^{\circ} = \frac{AB}{BD}$

$$1 = \frac{30}{x}$$

$$x = 30 \qquad ...(1)$$

Now in ΔBDC,

$$\tan 30^{\circ} = \frac{CD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\sqrt{3}\,h = x \Rightarrow h = \frac{x}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2), we get

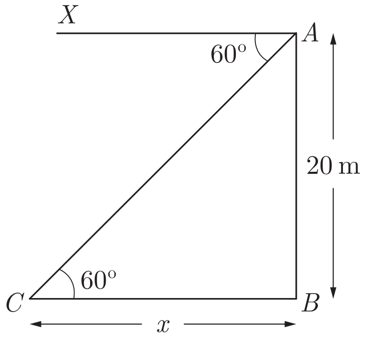
$$h = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m.}$$

Therefore height of the building is $10\sqrt{3}$ m

7. A player sitting the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. Take $\sqrt{3} = 1.732$

Ans : [CBSE Board Term-2, 2011, B1]

Let C be the point where the ball is lying. As per given in question we have drawn figure below.



Due to alternate angles we obtain

$$\angle XAC = \angle ACB = 60^{\circ}$$

In ΔABC, $\quad \tan 60^{\circ} = \frac{AB}{BC}$

$$\sqrt{3} = \frac{20}{x}$$

$$x = \frac{20}{\sqrt{3}} = 20\left(\frac{\sqrt{3}}{3}\right)$$

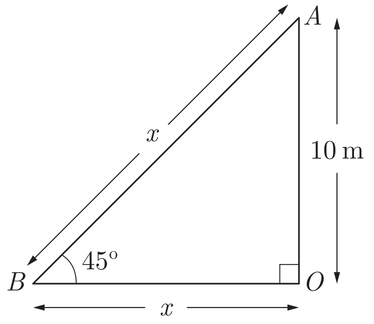
Hence, distance between ball and foot of tower is 11.53 m.

SHORT ANSWER TYPE QUESTIONS - II

1. An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire. [Use $\sqrt{2} = 1.414$]

Ans : [CBSE Term 2, 2016]

Let OA be the electric pole and B be the point on the ground to fix the pole. Let BA be x.
As per given in question we have drawn figure below.



In ΔABO, we have

$$\sin 45^{\circ} = \frac{AO}{AB}$$

$$\frac{1}{\sqrt{2}} = \frac{10}{x}$$

$$x = 10\sqrt{2} = 10 \times 1.414$$

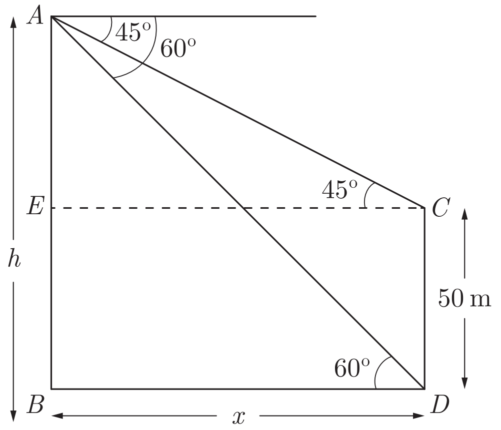
$$= 14.14 \text{ m}$$

Hence, the length of wire is 14.14 m

2. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (Use $\sqrt{3} = 1.73$)

Ans : [Delhi Set I, II, III, 2016]

As per given in question we have drawn figure below.



We have $\quad \tan 45^{\circ} = \frac{h-50}{x}$

$$x = h - 50 \qquad ...(1)$$

and $\quad \tan 60^{\circ} = \frac{h}{x}$

$$\sqrt{3} = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2) we hav

$$h - 50 = \frac{h}{\sqrt{3}}$$

$$\sqrt{3}\,h - 50\sqrt{3} = h$$

$$\sqrt{3}\,h - h = 50\sqrt{3}$$

$$h(\sqrt{3} - 1) = 50\sqrt{3}$$

$$h = \frac{50\sqrt{3}}{\sqrt{3} - 1} = \frac{50(3 + \sqrt{3})}{2}$$

$$h = 25(3 + \sqrt{3})$$

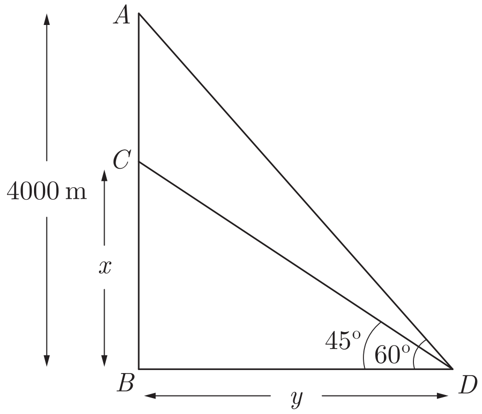
$$= 75 + 25\sqrt{3}$$

$$= 118.25 \text{ m}$$

3. An aeroplane, when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. (Use $\sqrt{3} = 1.73$)

Ans : [Foreign Set III, 2016]

Let the height first plane be $AB = 4000$ m and the height of second plane be $BC = x$ m. As per given in question we have drawn figure below.



Here $\angle BDC = \angle 45°$ and $\angle ADB = 60°$

In $\Delta\, CBD$, $\dfrac{x}{y} = \tan 45° = 1 \Rightarrow x = y$

and in $\Delta\, ABD$, $\dfrac{4000}{y} = \tan 60° = \sqrt{3}$

$$y = \frac{4000\sqrt{3}}{3}$$

$$= 2306.67 \text{ m}$$

Thus vertical distance between two,

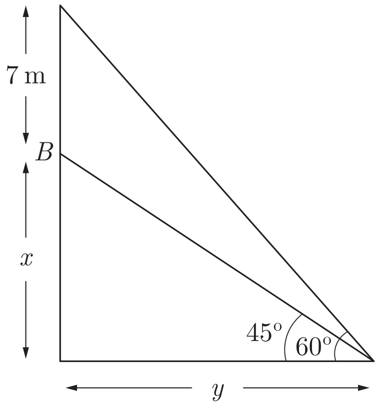
$$4000 - y = 4000 - 2306.67$$

$$= 1693.33 \text{ m}$$

4. A 7 m long flagstaff is fixed on the top of a tower standing on the horizontal plane. From point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60° and 45° respectively. Find the height of the tower correct to one place of decimal. (Use $\sqrt{3} = 1.73$)

Ans : [CBSE Foreign Set II, 2016]

As per given in question we have drawn figure below.



$$\frac{x}{y} = \tan 45° = 1 \Rightarrow x = y$$

$$\frac{x + 7}{x} = \tan 60° = \sqrt{3}$$

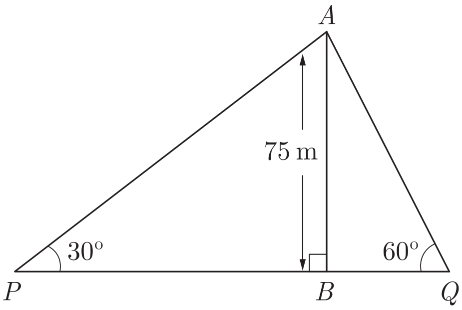
$$7 = (\sqrt{3} - 1)x$$

$$x = \frac{7(\sqrt{3} + 1)}{2} = \frac{7(2.73)}{2} = 9.6 \text{ m}$$

5. Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. find the distance between the two men. (Use $\sqrt{3} = 1.73$)

Ans : [CBSE Foreign Set I, 2016]

Let AB be the building and the two men are at P and Q. As per given in question we have drawn figure below.



In $\Delta\, ABP$, $\tan 30° = \dfrac{AB}{BP}$

$$\frac{1}{\sqrt{3}} = \frac{75}{BP}$$

$$BP = 75\sqrt{3} \text{ m}$$

In $\Delta\, ABQ$, $\tan 60° = \dfrac{AB}{BQ}$

$$\sqrt{3} = \frac{75}{BQ}$$

$$BQ = \frac{75}{\sqrt{3}} = 25\sqrt{3}$$

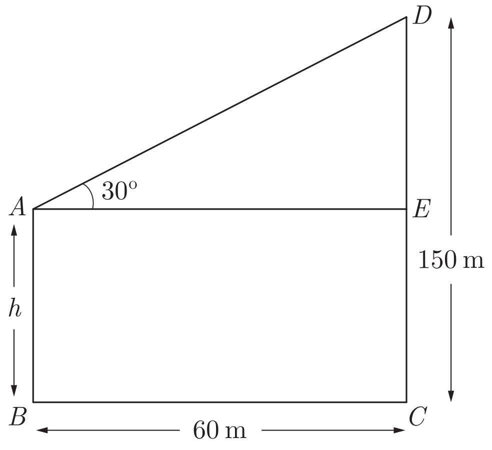
Distance between the two men,

$$PQ = BP + BQ = 75\sqrt{3} + 25\sqrt{3}$$
$$= 100\sqrt{3} = 100 \times 1.73 = 173$$

6. The horizontal distance between two towers is 60 m. The angle of elevation of the top of the taller tower as seen from the top of the shorter one is 30°. If the height of the taller tower is 150 m, then find the height of the shorter tower.

Ans : [CBSE Board Term-2, 2015]

Let AB and CD be two towers. Let the height of the shorter tower $AB = h$ m. As per given in question we have drawn figure below.



Here $BC = AE = 60$ m, $DE = DC - EC = (150 - h)$

In $\Delta\,AED$, $$\frac{DE}{AE} = \tan 30^{\circ}$$

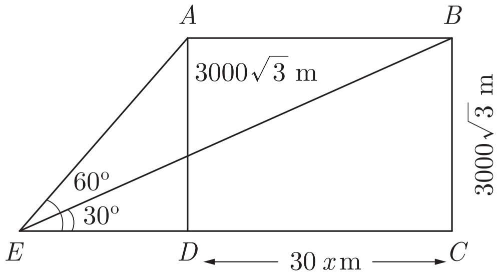
$$\frac{150-h}{60} = \tan 30^{\circ} = \frac{1}{\sqrt{3}}$$
$$150\sqrt{3} - h\sqrt{3} = 60$$
$$\sqrt{3}\,h = 150\sqrt{3} - 60$$
$$\sqrt{3}\,h = 150\sqrt{3} - 20\sqrt{3} \times \sqrt{3}$$

or $$h = (150 - 20\sqrt{3})\text{ m}$$

7. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.

Ans : [CBSE O.D. 2014]

As per given in question we have drawn figure below.



$$\angle AED = 60^{\circ}, \angle BED = 30^{\circ}$$

$$AD = BC = 3000\sqrt{3}\text{ m}$$

Let the speed of the aeroplane $= x$ m/s

$$AB = DC \times 30 \times x = 30x\text{ m} \quad ...(1)$$

In right $\Delta\,AED$, we have

$$\tan 60^{\circ} = \frac{AD}{DE}$$
$$\sqrt{3} = \frac{3000\sqrt{3}}{DE}$$
$$DE = 3000\text{ m} \quad ...(2)$$

In right $\Delta\,BEC$,

$$\tan 30^{\circ} = \frac{BC}{EC}$$
$$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{DE + CD}$$
$$DE + CD = 3000 \times 3$$
$$3000 + 30x = 9000$$
$$30x = 6000$$
$$x = 200\text{ m/s}$$

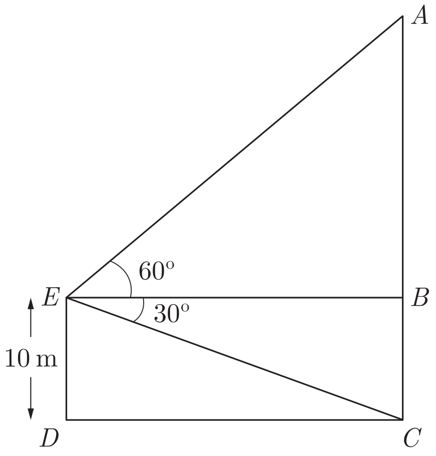
Hence, Speed of plane is 200 m/s

$$= 200 \times \frac{18}{5} = 720\text{ km/hr}$$

8. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill.

Ans : [Outside Delhi, Set-II, 2016]

As per given in question we have drawn figure below. Here AC is height of hill and man is at E. $ED = 10$ is height of ship from water level. As per given in question we have drawn figure below.



In $\Delta\,BCE$, $BC = 10$ m and

$$\angle BEC = 30^{\circ}$$

Now $$\tan 30^{\circ} = \frac{BC}{BE}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{BE}$$
$$BE = 10\sqrt{3}$$

Since $BE = CD$, distance of hill from ship

$$CD = 10\sqrt{3}\text{ m} = 10 \times 1.732\text{ m}$$

$$= 17.32 \text{ m}$$

Now in ΔABE, $\angle AEB = 60^o$

where $AB = hm$, $BE = 10\sqrt{3}$ m

and $\angle AEB = 60^o$

Thus $\tan 60^o = \frac{AB}{BE}$

$$\sqrt{3} = \frac{AB}{10\sqrt{3}}$$

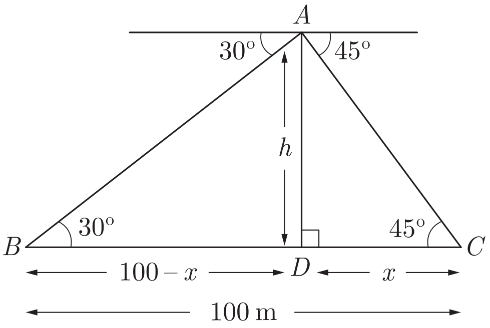
$$AB = 10\sqrt{3} \times \sqrt{3} = 30\,m$$

Thus height of hill $AB + 10 = 40$ m

9. Two ships are approaching a light house from opposite directions. The angle of depression of two ships from top of the light house are 30° and 45°. If the distance between two ships is 100 m, Find the height of light-house.

Ans : [CBSE Foreign 2014]

As per given in question we have drawn figure below. Here AD is light house of height h and BC is the distance between two ships.



We have $BC = 100$ m

In ΔADC $\tan 45^o = \frac{h}{x} \Rightarrow h = x$

In ΔABD, $\tan 30^o = \frac{h}{100 - x}$

$$\frac{1}{\sqrt{3}} = \frac{h}{100 - x}$$

$$100 - x = h\sqrt{3}$$

$$100 - h = h\sqrt{3} \qquad h = x$$

$$100 = h + h\sqrt{3}$$

$$= h(1 + \sqrt{3})$$

$$h = \frac{100}{1 + \sqrt{3}}$$

$$= \frac{100}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)}$$

$$= \frac{100(\sqrt{3} - 1)}{3 - 1}$$

$$= 50(\sqrt{3} - 1)$$

$$= 50(1.732 - 1)$$

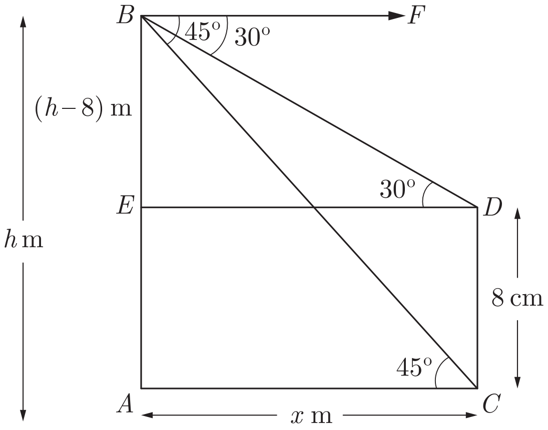
$$= 50 \times 0.732$$

Thus height of light house is 36.60 m.

10. The angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30° and 45°, respectively. Find the height of multi-storeyed building and distance between two buildings. [KVS 2014]

Ans :

As per given in question we have drawn figure below.



Here $AE = CD = 8$ m

$BE = AB - AE = (h - 8)$ m

and $AC = DE = x$ m

Also, $\angle FBD = \angle BDE = 30^\circ$

$\angle FBC = \angle BCA = 45^\circ$

In right angled ΔCAB we have

$$\tan 45^\circ = \frac{AB}{AC}$$

$$1 = \frac{h}{x} \Rightarrow x = h \qquad ...(1)$$

In right angled ΔEDB

$$\tan 30^\circ = \frac{BE}{ED}$$

$$\frac{1}{\sqrt{3}} = \frac{h - 8}{x}$$

$$x = \sqrt{3}(h - 8) \qquad ...(2)$$

From (1) and (2), we get

$$h = \sqrt{3}h - 8\sqrt{3}$$

$$8\sqrt{3} = \sqrt{3}h - h$$

$$h = \frac{8\sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= 4\sqrt{3}(\sqrt{3} + 1)$$

$$= (12 + 4\sqrt{3}) \text{ m}$$

Since, $x = h$, $x = (12 + 4\sqrt{3})$

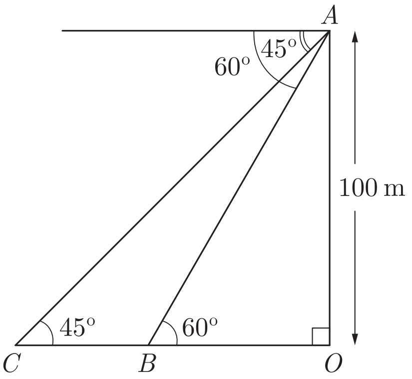
Distance $= (12 + 4\sqrt{3})$ m

Hence the height of multi storey building = distance $= (4\sqrt{3} + 12)$ m.

11. From a top of a building 100 m high the angle of depression of two objects are on the same side observed to be 45° and 60°. Find the distance between the objects.

Ans : [CBSE Board Term-2, 2014]

Let A be a point on top of building and B, C be two objects. As per given in question we have drawn figure below.



Here $\angle ACO = \angle CAX = 45^\circ$

and $\angle ABO = \angle XAB = 60^\circ$

In right ΔAOC, $\frac{AO}{CO} = \tan 45^\circ$

$$\frac{100}{CO} = 1$$

$$CO = 100 \text{ m}$$

Also in right ΔAOB,

$$\frac{AO}{OB} = \tan 60^\circ$$

$$\frac{100}{OB} = \sqrt{3}$$

$$OB = \frac{100}{\sqrt{3}}$$

Thus $BC = CO - OB = 100 - \frac{100}{\sqrt{3}}$

$$= 100\left(1 - \frac{1}{\sqrt{3}}\right) = 100\frac{(\sqrt{3}-1)}{\sqrt{3}}$$

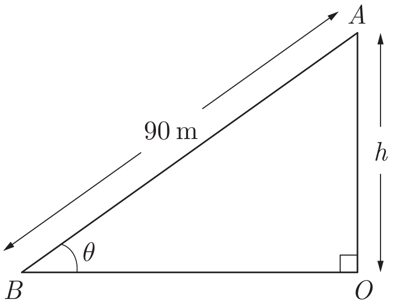
$$= 100\frac{(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{100(3-\sqrt{3})}{3} \text{ m}$$

12. A Boy, flying a kite with a string of 90 m long, which is making an angle θ with the ground. Find the height of the kite. (Given $\tan\theta = \frac{45}{8}$)

Ans : [CBSE Board Term-2, 2014]

Let A be the position of kite and AB be the string. As per given in question we have drawn figure below.



Since $\tan\theta = \frac{15}{8} = \frac{AO}{BO}$

Let AO be $15k$ and BO be $8k$

Now using Pythagoras Theorem

$$AB = \sqrt{BO^2 + AO^2}$$

$$= \sqrt{(15k)^2 + (8k)^2} = 17k$$

In ΔABO, $\frac{AO}{AB} = \sin\theta$

$$\frac{h}{90} = \frac{15k}{17k} = \frac{15}{17}$$

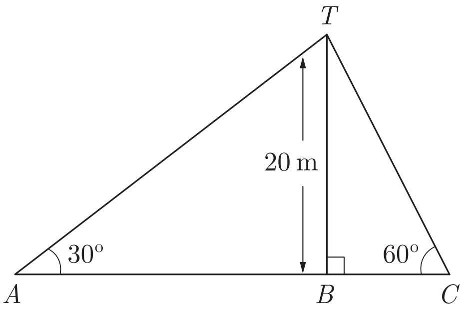
$$h = \frac{15 \times 90}{17} = 79.41 \text{ m}$$

Hence, height of kite is 79.41 m.

13. Two men standing on opposite sides of a tower measure the angles of elevation of he top of the tower as 30° and 60° respectively. If the height of the tower in 20 m, then find the distance between the two men.

Ans : [CBSE Board Term-2, 2013]

Let two men are standing at A and C and BT is the tower. As per given in question we have drawn figure below.



In right angle triangle ΔBTC,

$$\tan 60^\circ = \frac{BT}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{20}{BC}$$

$$BC = \frac{20}{\sqrt{3}}$$

In right angle triangle ΔBTC,

$$\tan 60^\circ = \frac{BT}{BC}$$

$$\sqrt{3} = \frac{20}{BC}$$

$$BC = \frac{20}{\sqrt{3}}$$

Thus distance between two men

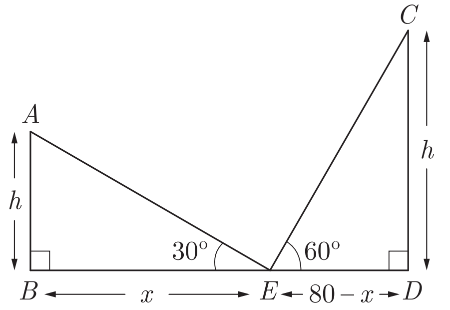
$$AB + BC = 20\sqrt{3} + \frac{20}{\sqrt{3}} = \frac{60+20}{\sqrt{3}} = \frac{80\sqrt{3}}{3} \text{ m.}$$

Hence, distance between the men is $\frac{80\sqrt{3}}{3}$ m.

14. Two poles of equal heights are standing opposite to each other on either side of a road, which is 80 m wide. From a point between them on the road, angles of elevation of their top are 30° and 60°. Find the height of the poles and distance of point from poles.

Ans : [CBSE Board Term-2, 2011 Set (B1), Delhi 2013]

Let the distance between pole AB and man E be x. As per given in question we have drawn figure below.



Here distance between pole CD and man is $80 - x$

In right angle triangle ΔABE,

$$\tan 30^\circ = \frac{h}{x}$$

$$h = \frac{x}{\sqrt{3}} \qquad ...(1)$$

In angle triangle ΔCDE,

$$\tan 60^\circ = \frac{h}{80 - x}$$

$$\sqrt{3} = \frac{h}{80 - x}$$

$$h = 80\sqrt{3} - x\sqrt{3} \qquad ...(2)$$

Comparing (1) and (2) we have

$$\frac{x}{\sqrt{3}} = 80\sqrt{3} - x\sqrt{3}$$

$$x = 80 \times 3 - x \times 3$$

$$4x = 240$$

$$x = \frac{240}{4} = 60 \text{ m}$$

Substituting this value of x in (1) we have

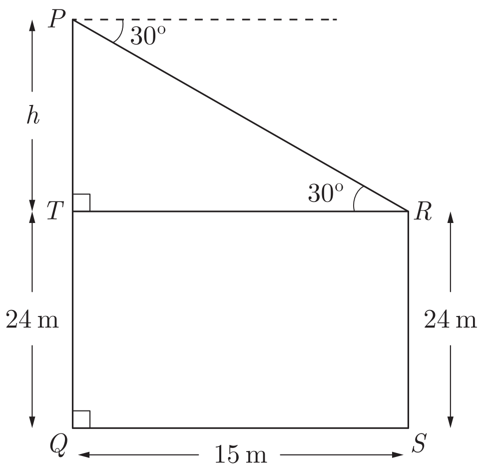
$$h = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

Hence, height of the pole is 34.64 m

15. The horizontal distance between two poles is 15 m. The angle of depression of the top of first pole as seen from the top of second pole is 30°. If the height of the first of the pole is 24 m, find the height of the second pole. [Use $\sqrt{3} = 1.732$]

Ans : [CBSE Board Term-2, 2013]

Let RS be first pole and PQ be second pole. As per given in question we have drawn figure below.



In right ΔPTR,

$$\tan 30^\circ = \frac{PT}{TR}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{15}$$

$$h = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$

$$= 5 \times 1.732 = 8.66$$

$$PQ = PT + TQ$$

$$= 8.66 + 24$$

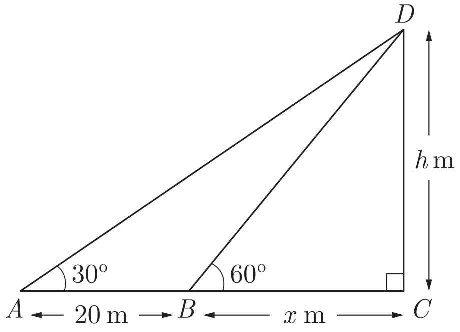
$$= 32.66 \text{ m}$$

Thus height of the second pole is 32.66 m.

16. The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metre towards the foot of the tower to a point B the angle of elevation increase to 60°. Find the height of the tower and the distance of the tower from the point A.

Ans : [CBSE Board Term-2, 2012]

Let height of tower be h and distance BC be x.
As per given in question we have drawn figure below.



In right ΔDBC, $\frac{h}{x} = \tan 60^\circ$

$$h = \sqrt{3}\,x \qquad ...(1)$$

In right ΔADC,

$$\frac{h}{x + 20} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = x + 20 \qquad ...(2)$$

Substituting the value of h from eq. (1) in eq. (2), we get

$$3x = x + 20$$

$$x = 10 \text{ m} \qquad ...(3)$$

Thus $AC = 20 + x = = 30$ m.

and $h = \sqrt{3} \times 10 = 10\sqrt{3}$

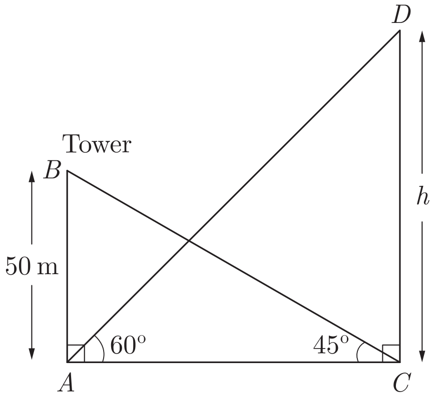
$$= 10 \times 1.732 = 17.32 \text{ m}$$

Hence, height of tower is 17.32 m and distance of tower from point A is 30 m.

17. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the holl is 30°. If the tower is 50 m high, find the height of the hill.

Ans : [CBSE Board Term-2, 2012]

Let AB be tower of height 50 m and DC be hill of height h. As per given in question we have drawn figure below.



In right $\Delta\, BAC$

$$\cos 30^\circ = \frac{AC}{50}$$

$$\sqrt{3} = \frac{AC}{50}$$

$$AC = 50\sqrt{3}$$

In right $\Delta\, ACD$,

$$\tan 60^\circ = \frac{CD}{50\sqrt{3}}$$

$$\sqrt{3} = \frac{CD}{50\sqrt{3}}$$

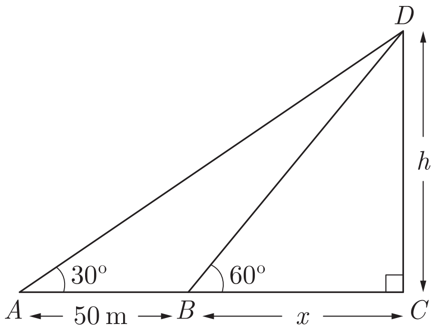
$$CD = 50\sqrt{3} \times \sqrt{3} = 150 \text{ m}$$

Thus height of the hill $CD = 150$ m

18. A person observed the angle of elevation of the top of a tower as 30°. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60°. Find the height of the tower.

Ans : [CBSE Board Term-2, 2012 Set(31)]

Let DC be tower of height h. As per given in question we have drawn figure below.



Here A is the point at elevation 30° and B is the point of elevation at 60°

Let BC be x.

Now $AC = (50 + x)$ m

In right $\Delta\, DCB$, $\frac{h}{x} = \tan 60^\circ = \sqrt{3}$

$$h = \sqrt{3}\,x \qquad \text{...(1)}$$

In right $\Delta\, DCA$,

$$\frac{h}{x+50} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = x + 50 \qquad (1)$$

Substituting the value of h from (1) in (2), we have

$$3x = x + 50$$

$$2x = 50 \Rightarrow x = 25 \text{ m}$$

$$h = 25\sqrt{3}$$

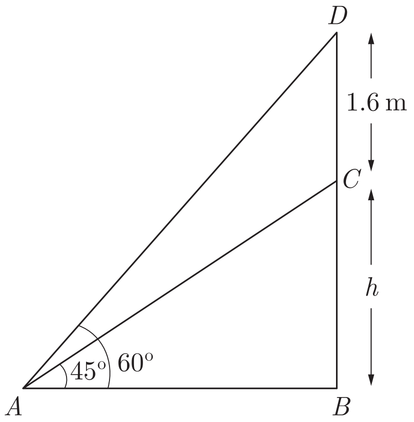
$$= 25 \times 1.732 = 43.3 \text{ m}$$

Hence height of tower is 43.3 m.

19. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal.

Ans : [CBSE Board Term-2, 2012 (50)]

Let CD be statue of 1.6 m and pedestal BC of height h. Let A be point on ground. As per given in question we have drawn figure below.



In right $\Delta\, ABD$,

$$\cos 60^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{h+1.6}$$

$$AB = \frac{h+1.6}{\sqrt{3}} \qquad \text{...(1)}$$

In right $\Delta\, ABC$,

$$\frac{AB}{BC} = \cot 45^\circ$$

$$1 = \frac{AB}{h}$$

$$AB = h \qquad \text{...(2)}$$

From (1) and (2), we get

$$h = \frac{h+1.6}{\sqrt{3}}$$

$$h\sqrt{3} = h + 1.6$$

$$h\sqrt{3} - h = 1.6$$

$$h(\sqrt{3} - 1) = 1.6$$

$$h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{1.732 - 1}$$

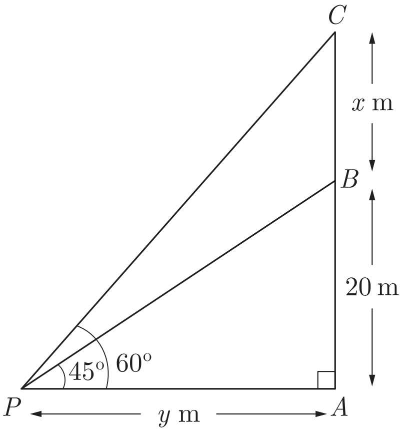
$$= \frac{1.6}{0.732} = 2.185 \text{ m}$$

Height of pedestal h is 2.2 m.

20. From a point on a ground, the angle of elevation of bottom and top a transmission tower fixed on the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Ans : [Outside Delhi Compt. 2017]

Let P be the point on ground, AB be the building of height 20 m and BC be the tower of height x. As per given in question we have drawn figure below.



In right $\Delta\, BAP$ we have

$$\frac{BA}{PA} = \tan 45^\circ$$

$$\frac{20}{y} = 1$$

$$y = 20$$

In right $\Delta\, CAP$

$$\frac{CA}{PA} = \tan 60^\circ$$

$$\frac{20+x}{y} = \sqrt{3}$$

$$20 + x = y\sqrt{3}$$

$$20 + x = 20\sqrt{3}$$

$$x = 20\sqrt{3} - 20$$

$$= 20(\sqrt{3} - 1)$$

$$= 20 \times (1.732 - 1)$$

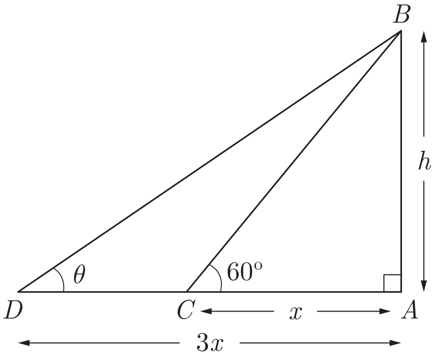
$$= 20 \times 0.73 = 14.64$$

Hence, height of the tower is 14.64 m.

21. The shadow of a tower at a time is three times as long as its shadow when the angle of elevation of the sun is 60°. Find the angle of elevation of the sun at the of the longer shadow.

Ans : [CBSE Foreign 2017]

Let AB be tower of height h, AC be the shadow at elevation of sun of 60°. As per given in question we have drawn figure below.



In right $\Delta\, BAC$,

$$\frac{AB}{AC} = \tan 60^\circ$$

$$\frac{h}{x} = \sqrt{3}$$

$$h = x\sqrt{3}$$

In right $\Delta\, BAD$,

$$\frac{AB}{AD} = \tan\theta$$

$$\frac{h}{3x} = \tan\theta$$

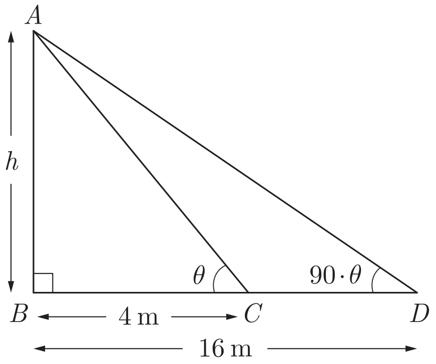
$$\frac{x\sqrt{3}}{3x} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

Thus $\theta = 30^\circ$.

22. On a straight line passing through the foot of a tower, two C and D are at distance of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.

Ans : [CBSE Outside Delhi 2017]

Let AB be tower of height h, C and D be the two point. As per given in question we have drawn figure below.



Since $\angle ACB$ and $\angle ADB$ are complementary,

$$\angle ACB = \theta \text{ and } \angle ADB = 90^\circ - \theta$$

Now, in right $\Delta\, ABC$,

$$\tan\theta = \frac{AB}{BC} = \frac{h}{4} \quad \text{...(1)}$$

In right $\Delta\, ABD$,

$$\tan(90 - \theta) = \frac{AB}{BD} = \frac{h}{16}$$

$$\cot\theta = \frac{h}{16} \qquad \tan(90-\theta) = \cot\theta$$

$$\tan\theta = \frac{16}{h} \qquad ...(2)$$

From (1) and (2) we have

$$\frac{h}{4} = \frac{16}{h}$$

$$h^2 = 4 \times 16 = 64 = 8^2$$

$$h = 8 \text{ m}$$

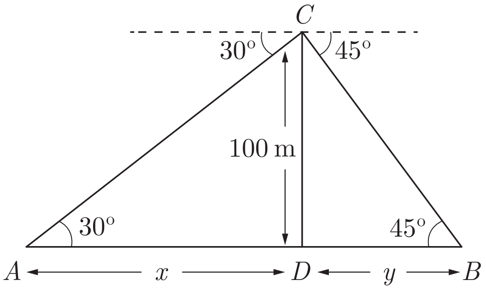
Thus height of tower is 18.8 m.

LONG ANSWER TYPE QUESTIONS

1. From the top of tower, 100 m high, a man observes two cars on the opposite sides of the tower with the angles of depression 30° & 45° respectively. Find the distance between the cars. (Use $\sqrt{3} = 1.73$)

Ans : [CBSE Board Sample Paper, 2016]

Let DC be tower of height 100 m. A and B be two car on the opposite side of tower. As per given in question we have drawn figure below.



In right ΔADC,

$$\tan 30° = \frac{CD}{AD}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{x}$$

$$x = 100\sqrt{3} \qquad ...(1)$$

In right ΔBDC,

$$\tan 45° = \frac{CD}{DB}$$

$$1 = \frac{100}{y}$$

$$\Rightarrow \qquad y = 100 \text{ m}$$

Distance between two cars

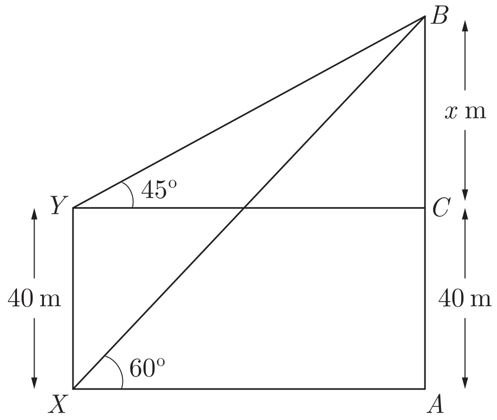
$$AB = AD + DB$$
$$= (100\sqrt{3} + 100)$$
$$= (100 \times 1.73 + 100) \text{ m}$$
$$= (173 + 100) \text{ m}$$
$$= 273 \text{ m}$$

Hence, distance between two cars is 273 m.

2. The angle of elevation of the top B of a tower AB from a point X on the ground is 60°. At point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Find the height of the tower AB and the distance XB.

Ans : [CBSE SA-2 2016]

As per given in question we have drawn figure below.



In right ΔYCB, we have

$$\tan 45° = \frac{BC}{YC}$$

$$1 = \frac{x}{YC}$$

$$YC = x$$

$$XA = x$$

In right ΔXAB we have

$$\tan 60° = \frac{AB}{XA}$$

$$\sqrt{3} = \frac{x+40}{x}$$

$$\sqrt{3}x = x + 40$$

$$x\sqrt{3} - x = 40$$

$$x = \frac{40}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$
$$= 20(\sqrt{3}+1)$$
$$= (20\sqrt{3}+1)$$

Thus height of the tower,

$$AB = x + 40$$
$$= 20\sqrt{3} + 20 + 40$$
$$= 20\sqrt{3} + 60$$
$$= 20(\sqrt{3}+3)$$

In right ΔXAB we have,

$$\sin 60° = \frac{AB}{BX}$$

$$\frac{\sqrt{3}}{2} = \frac{AB}{BX}$$

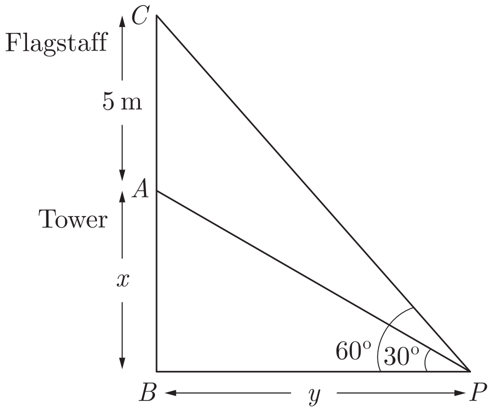
$$BX = \frac{2AB}{\sqrt{3}} = \frac{20 \times 2(\sqrt{3}+3)}{\sqrt{3}}$$
$$= 40(1+\sqrt{3})$$
$$= 40 \times 2.73 = 109.20$$

3. A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 5 m. From a point on the ground the angles of elevation of top and bottom of the flagstaff are 60° and 30° respectively.

Find the height of the tower and the distance of the point from the tower. (take $\sqrt{3} = 1.732$)

Ans : [CBSE Foreign Set I, 2016]

Let AB be tower of height x and AC be flag staff of height 5 m. As per given in question we have drawn figure below.



In right $\Delta\, ABP$,

$$\frac{AB}{BP} = \tan 30^\circ$$

$$\frac{x}{y} = \frac{1}{\sqrt{3}}$$

$$y = \sqrt{3}\, x \qquad ...(1)$$

In right $\Delta\, CBP$

$$\frac{x+5}{y} = \tan 60^\circ = \sqrt{3} \qquad ...(2)$$

Substituting the value of y from (1) we have

$$\frac{x+5}{\sqrt{3}\, x} = \sqrt{3}$$

$$x+5 = 3x \Rightarrow x = 2.5 \text{ m}$$

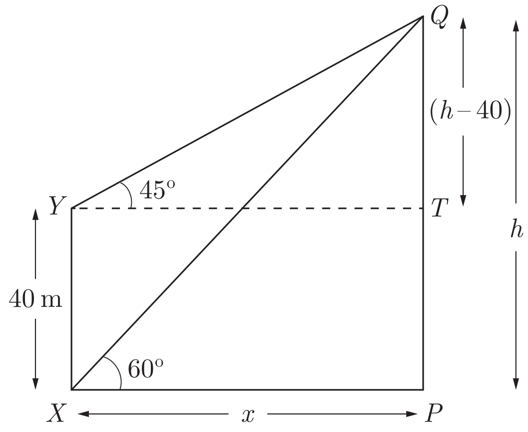
Height of tower is $= 2.5$ m

Distance of P from tower $= (2.5 \times 1.732)$ or 4.33 m.

4. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the PQ and the distance PX. (Use $\sqrt{3} = 1.73$)

Ans : [CBSE Outside Delhi 2016]

Let PX be x and PQ be h. As per given in question we have drawn figure below.



Now $\qquad QT = (h-40)$ m

In right $\Delta\, PQX$ we have,

$$\tan 60^\circ = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = \sqrt{3}\, x \qquad ...(1)$$

In right $\Delta\, QTY$ we have

$$\tan 45^\circ = \frac{h-40}{x}$$

$$1 = \frac{h-40}{x}$$

$$x = h-40 \qquad ...(2)$$

Solving (1) and (2), we get

$$\begin{aligned} x &= \sqrt{3}\, x - 40 \\ \sqrt{3}\, x - x &= 40 \\ (\sqrt{3}-1)x &= 40 \\ x &= \frac{40}{\sqrt{3}-1} = 20(\sqrt{3}+1) \text{ m} \end{aligned}$$

Thus

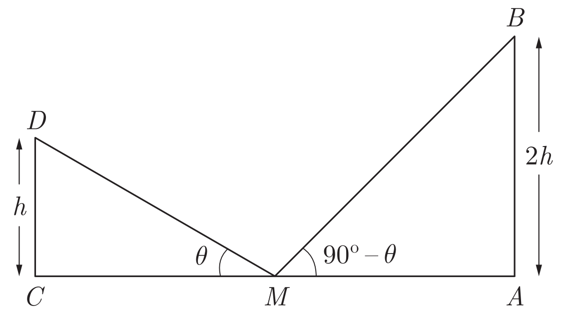
$$\begin{aligned} x &= \sqrt{3} \times 20(\sqrt{3}+1) \\ &= 20(3+\sqrt{3}) \text{ m} \\ &= 20(3+1.73) \\ &= 20 \times 4.73 \end{aligned}$$

Hence, height of tower is 94.6 m.

5. Two post are k metre apart and the height of one is double that of the other. If from the mid-point of the line segment joining their feet, an observer finds the angles of elevation of their tops to be complementary, then find the height of the shorted post.

Ans : [CBSE Foreign 2015]

Let AB and CD be the two posts such that $AB = 2$ CD. Let M be the mid-point of CA. As per given in question we have drawn figure below.



Here $\angle CMD = \theta$ and $\angle AMB = 90^\circ - \theta$

Clearly, $\qquad CM = MA = \frac{1}{2}k$

Let $CD = h$. then $AB = 2h$

Now,

$$\frac{AB}{AM} = \tan(90^\circ - \theta)$$

$$\frac{2h}{\frac{k}{2}} = \cot\theta$$

$$\frac{4h}{k} = \cot\theta \qquad ...(1)$$

Also in right $\Delta\, CMD$,

$$\frac{CD}{CM} = \tan\theta$$

$$\frac{h}{\frac{k}{2}} = \tan\theta$$

$$\frac{2h}{k} = \tan\theta \qquad ...(2)$$

Multiplying (1) and (2), we have

$$\frac{4h}{k} \times \frac{2h}{k} = \tan\theta \times \cot\theta = 1$$

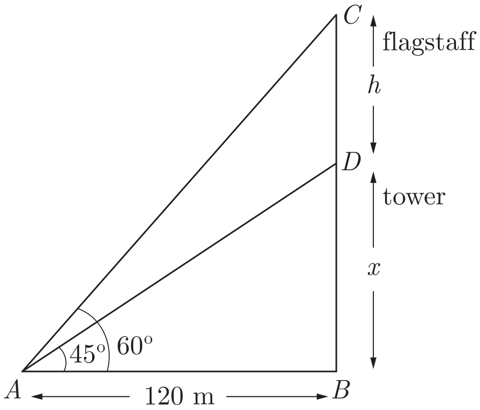
$$h^2 = \frac{k^2}{8}$$

$$h = \frac{k}{2\sqrt{2}} = \frac{k\sqrt{2}}{4}$$

6. The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground flagstaff fixed at the top of the tower, at A is 60°, then find the height of the flagstaff. [Use $\sqrt{3} = 1.73$]

Ans : [CBSE OD 2014]

Let BD be the tower of height x and CD be flagstaff of height h. As per given in question we have drawn figure below.



Here $\angle DAB = 45^\circ, \angle CAB = 60^\circ$

and $AB = 120$ m

In right angled $\Delta\, ABD$ we have

$$\frac{x}{AB} = \tan 45^\circ = 1$$

$$x = AB = 120 \text{ m}$$

In right angled $\Delta\, ACB$ we have

$$\frac{h+x}{120} = \tan 60^\circ = \sqrt{3}$$

$$h + 120 = 120\sqrt{3}$$

$$h = 120\sqrt{3} - 120$$

$$= 120(\sqrt{3} - 1)$$

$$= 120(1.73 - 1)$$

$$= 120 \times 0.73$$

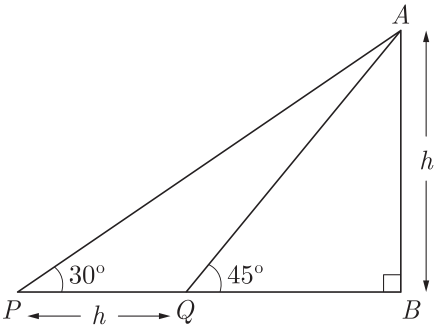
$$h = 87.6 \text{ m}$$

Hence, height of the flagstaff is 87.6 m.

7. A man on the top of a vertical tower observes a car moving at a uniform speed towards him. If it takes 12 min. for the angle of depression to change from 30° to 45°, how soon after this, the car will reach the tower ?

Ans : [KVS 2014]

Let AB be the tower of height h. As per given in question we have drawn figure below.



Car is at P at 30° and is at Q at 45° elevation.

Here $\angle AQB = 45^\circ$

Now, in right $\Delta\, ABQ$ we have,

$$\tan 45^\circ = \frac{AB}{BQ}$$

$$1 = \frac{h}{BQ}$$

$$BQ = h$$

In right $\Delta\, APB$ we have,

$$\tan 30^\circ = \frac{AB}{PB}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+h}$$

$$x + h = h\sqrt{3}$$

$$x = h(\sqrt{3} - 1)$$

Thus, $$\text{Speed} = \frac{h(\sqrt{3}-1)}{12} \text{ m/min}$$

Time for remaining distance,

$$t = \frac{h}{\frac{h(\sqrt{3}-1)}{12}} = \frac{12}{(\sqrt{3}-1)}$$

$$= \frac{12(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{12(\sqrt{3}+1)}{3-1}$$

$$= \frac{12}{2}(\sqrt{3}+1)$$

$$= 6(\sqrt{3}+1)$$

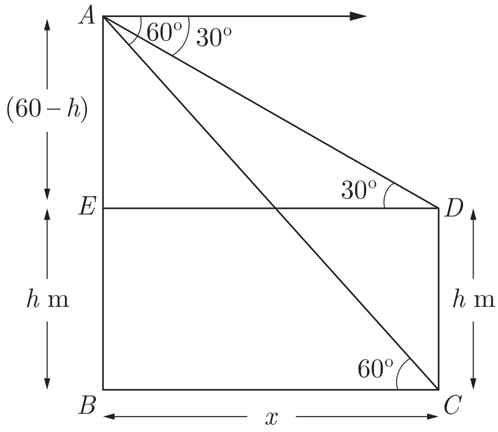
$$t = 6 \times 2.73 = 16.38$$

Hence, time taken by car is 16.38 minutes.

8. From the top of a building 60 m high the angles of depression of the top and the bottom of a tower are observed to be 30° and 60°. Find the height of the tower.

Ans : [Delhi, Term-2 2014], [CBSE Board Term-2 2012 Set 3, 2011 Set B1]

Let AB be the building of height 60 m and CD be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here $DC = EB = h$ m and let $BC = x$

$$AE = (60-h) \text{ m}$$

In right angled ΔAED we have

$$\frac{60-h}{ED} = \tan 30^\circ$$

$$\frac{60-h}{x} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}(60-h) = x \qquad ...(1)$$

In right ΔABC we have

$$\frac{60}{x} = \tan 60^\circ$$

$$60 = \sqrt{3}\,x \qquad ...(2)$$

Substituting the value of x from equation (1) in equation (2), we have

$$60 = \sqrt{3} \times \sqrt{3}(60-h)$$

$$60 = 3 \times (60-h)$$

$$20 = 60-h$$

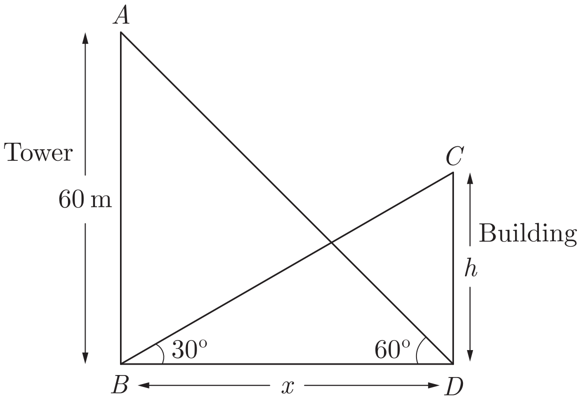
$$h = 40 \text{ m}$$

Hence, Height of tower is 40 m.

9. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

Ans : [CBSE Delhi 2013]

Let AB be the tower of 60 m height and CD be the building of h height. As per given in question we have drawn figure below.



In right ΔABD we have

$$\tan 60^\circ = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{60}{x}$$

$$x = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

Now, in right ΔBCD we have

$$\tan 30^\circ = \frac{CD}{BD} = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{20\sqrt{3}}$$

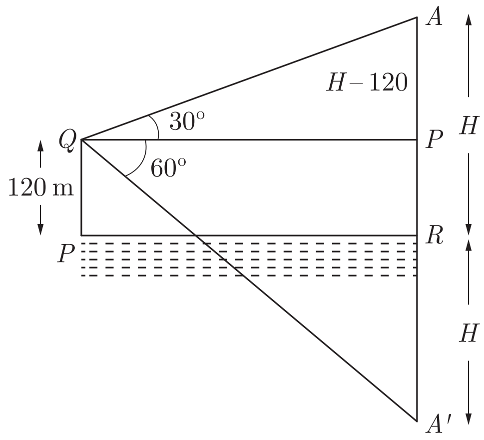
$$h = \frac{20\sqrt{3}}{\sqrt{3}} = 20$$

Hence height of the building is 20 m.

10. The angle of elevation of a cloud from a point 120 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud.

Ans : [CBSE Board Term-2, 2012]

As per given in question we have drawn figure below.



Here A is cloud and A' is refection of cloud.

In right ΔAOP we have

$$\tan 30^\circ = \frac{H-120}{OP}$$

$$\frac{1}{\sqrt{3}} = \frac{H-120}{OP}$$

$$OP = (H-120)\sqrt{3} \qquad ...(1)$$

In right $\Delta OPA'$ we have

$$\tan 60^\circ = \frac{H+120}{OP}$$

$$OP = \frac{H+120}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2), we get

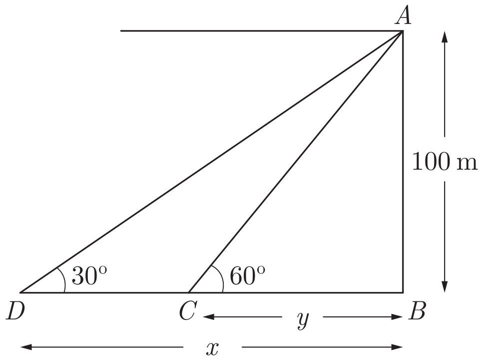
$$\frac{H+120}{\sqrt{3}} = \sqrt{3}(H-120)$$

Thus height of cloud is 240 m.

11. As observed from the top of a light house, 100 m high above sea level, the angles of depression of a ship, sailing directly towards it, changes from 30° to 60° . Find the distance travelled by the ship during the period of observation. (Use $\sqrt{3} = 1.73$)

Ans : [CBSE Outside DelhiI 2016]

Let AB be the light house of height 100 m. Let C and D be the position of ship at elevation 60° and 30°. As per given in question we have drawn figure below.



In right $\Delta\,ABC$ we have

$$\frac{AB}{BC} = \tan 60^{\text{o}}$$

$$\frac{100}{y} = \sqrt{3}$$

$$y = \frac{100}{\sqrt{3}}$$

In right $\Delta\,ABD$, we have

$$\frac{AB}{BD} = \tan 30^{\text{o}}$$

$$\frac{100}{x} = \frac{1}{\sqrt{3}}$$

$$x = 100\sqrt{3}$$

Required distance travelled by ship,

$$x - y = 100\sqrt{3} - \frac{100}{\sqrt{3}} \text{ m}$$

$$= 100\left[\frac{3-1}{\sqrt{3}}\right]$$

$$= \frac{100 \times 2}{\sqrt{3}}$$

$$= \frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3}$$

$$CD = x - y$$

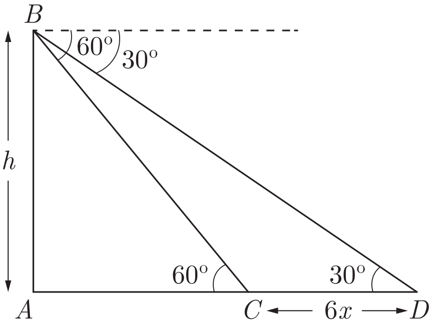
$$= \frac{200 \times 1.73}{3} = \frac{3.46}{3} \text{ m}$$

115.33 m

12. A straight highway leads to the foot of a tower. A man standing on its top observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. 6 seconds later, the angle of depression of the car becomes 60°. Find the time taken by the car to reach the foot of tower from this point.

Ans : [Delhi Compt Set-I, III, 2017]

Let AB be the tower of fight h. Let point C and D be location of car. As per given in question we have drawn figure below.



Let the speed of car be x m/sec.
Thus distance covered in 6 sec $= 6x$.

Hence $DC = 6x$ m

Let distance (remaining) CA covered in t sec.

$$CA = tx$$

Now in right $\Delta\,ADB$,

$$AD = AC + CD = 6x + 9x$$

$$\tan 30^\circ = \frac{h}{6x + tx}$$

$$\frac{h}{x} = \frac{6+t}{\sqrt{3}} \qquad \text{...(1)}$$

In right $\Delta\,ACB$ we have,

$$\tan 60^\circ = \frac{h}{tx}$$

$$\sqrt{3}\,t = \frac{h}{tx} \qquad \text{...(2)}$$

From eqn. (1) and (2) we get

$$\sqrt{3}\,t = \frac{6+t}{\sqrt{3}}$$

$$3t = 6 + t$$

$$2t = 6$$

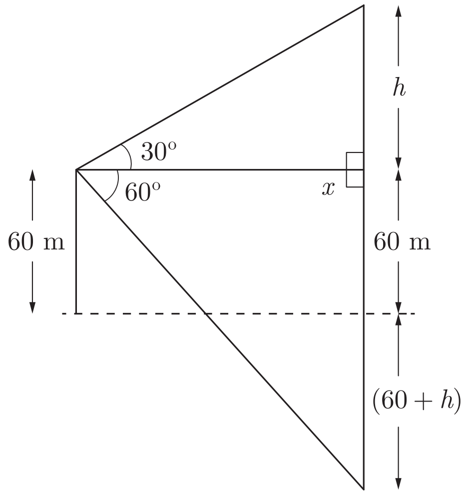
$$t = 3$$

Hence, car takes 3 seconds.

13. An angle of elevation of a cloud from a point 60m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water is 60°. Find the height of the cloud from the surface of water.

Ans : [CBSE Delhi Set-I]

As per given in question we have drawn figure below.



Here $$\frac{h}{x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$x = h\sqrt{3} \qquad ...(1)$$

and $$\frac{h+60+60}{x} = \tan 60^\circ$$

$$\frac{h+120}{x} = \sqrt{3}$$

$$h+120 = x\sqrt{3} \qquad ...(2)$$

From (1) and (2) we get

$$h+120 = \sqrt{3}\,h \times \sqrt{3}$$

$$h+120 = 3h$$

$$h = \frac{120}{2} = 60 \text{ m}$$

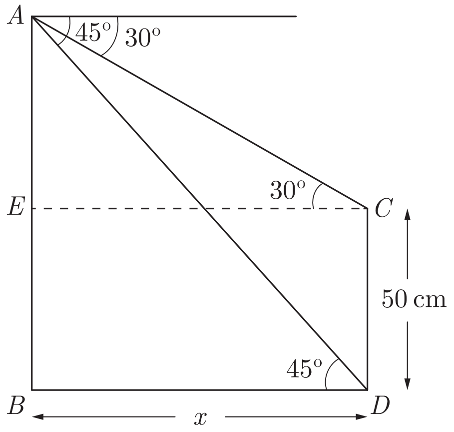
Hence height of cloud from surface of water

$$= 60 + 60 = 120 \text{ m}$$

14. The angle of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 45° respectively. Find the height of the tower and also the horizontal distance between the building and the tower.

Ans : [Sample Question Paper 2017-18]

Let CD be the building of height 50 m and AB be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.



Let distance between BO be x.
Now, in right ΔABD

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{h}{x} = 10$$

$$h = x \qquad ...(1)$$

In right ΔAEC we have

, $$\frac{AE}{EC} = \tan 30^\circ$$

$$\frac{h-50}{x} = \frac{1}{\sqrt{3}}$$

$$x = h\sqrt{3} - 50\sqrt{3} \qquad ...(2)$$

From (1) and (2) we get

$$h = h\sqrt{3} - 50\sqrt{3}$$

$$h\sqrt{3} - h = 50\sqrt{3}$$

$$h(\sqrt{3}-1) = 50\sqrt{3}$$

$$h = \frac{50\sqrt{3}}{\sqrt{3}-1} = \frac{50\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{50(3-\sqrt{3})}{3-1}$$

$$h = 25(3+\sqrt{3})$$

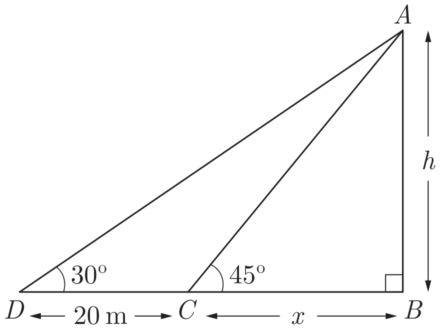
$$= 25 \times 4.732 = 118.3 \text{ m}$$

Hence, the height of tower = distance between building and tower = 118.3 m

15. An observer finals the angle of elevation of the top of the tower from a certain point on the ground as 30°. If the observer moves 20 m. Towards the base of the tower, the angle of elevation of the top increase by 15°, find the height of the tower.

Ans : [CBSE Delhi Set-III 2017]

Let AB be the tower of height h. Angle of elevation from point D and C are given 30° and 45° respectively. As per given in question we have drawn figure below.



Here $CB = x$ and $DC = 20$ m

Now in right ΔABC,

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{h}{x} = 1$$

$$h = x$$

In right ΔABC we have

$$\frac{AB}{DB} = \tan 30^\circ$$

$$\frac{h}{(20+x)} = \frac{1}{\sqrt{3}}$$

$$h\sqrt{3} = 20 + x$$

Substituting the value of x from (1) in (2)

$$h\sqrt{3} = 20 + h$$

$$h\sqrt{3} - h = 20$$

$$h(\sqrt{3}-1) = 20$$

$$h = \frac{20}{\sqrt{3}-1} = \frac{20(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{20(\sqrt{3}+1)}{3-1}$$

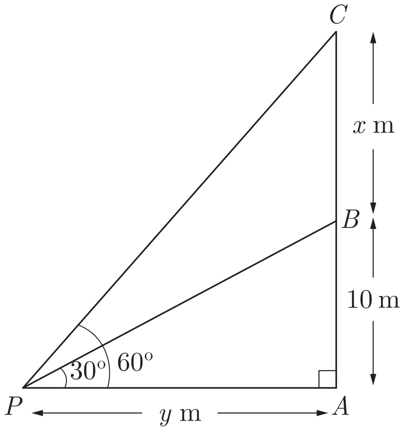
$$= 10(\sqrt{3}+1)$$

Hence, the height of tower $= 10(\sqrt{3}+1)$ m

16. From a point P on the ground, the angles of elevation of the top of a 10 m tall building and a helicopter, hovering at some height vertically over the top of the building are 30° and 60° respectively. Find the height of the helicopter above the ground.

Ans : [CBSE Outside Delhi Compt. 2017]

Let AB be the building of height 10 m and the height of the helicopter from top the building be x. As per given in question we have drawn figure below.



Let the distance between point and building be y.
Height of the helicopter from ground

$$= (10+x) \text{ m}$$

In right ΔBAP we have

$$\frac{AB}{BP} = \tan 30^\circ$$

$$\frac{10}{y} = \frac{1}{\sqrt{3}} \qquad ...(1)$$

$$y = 10\sqrt{3}$$

In right ΔCAP,

$$\frac{AC}{PA} = \tan 60^\circ$$

$$\frac{10+x}{y} = \sqrt{3}$$

$$10+x = y\sqrt{3} \qquad ...(2)$$

From (1) and (2)

$$10+x = 10\sqrt{3}\times\sqrt{3} = 40$$

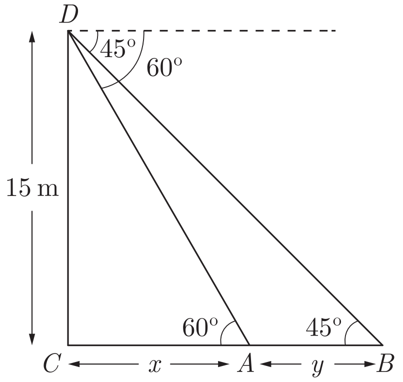
$$x = 30$$

Hence height of the helicopter is 30 m.

17. Two points A and B are on the same side of a tower and in the same straight line with its base. The angle of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points.

Ans : [CBSE Delhi 2017]

Let CD be the tower of height 15 m. Let A and B point on same side of tower As per given in question we have drawn figure below.



In right ΔDCA we have

$$\frac{DC}{CA} = \tan 60^\circ$$

$$\frac{15}{x} = \sqrt{3}$$

$$x = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$

In right ΔDCB we have

$$\frac{DC}{CB} = \tan 45^\circ$$

$$\frac{15}{x+y}z = 1$$

$$x+y = 15$$

$$5\sqrt{3}+y = 15$$

$$y = 15-5\sqrt{3}$$

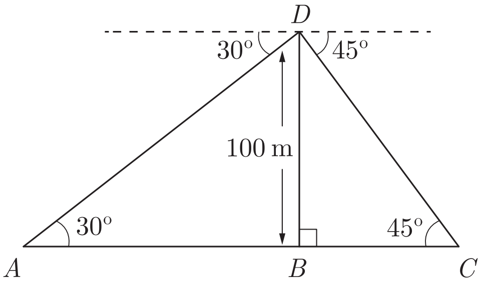
$$= 5(3-\sqrt{3}) \text{ m}$$

Hence, the distance between points $= 5(3-\sqrt{3})$ m

18. From the top of a tower, 100 m high, a man observes two cars on the opposite sides of the tower and in same straight line with its base, with angles of depression 30° and 45°. Find the distance between the cars. [Take $\sqrt{3} = 1.732$]

Ans : [CBSE Outside Delhi Compt. Set-III 2017]

Let BD be the tower of height 100 m. Let A and C be location of car on opposite side of tower As per given in question we have drawn figure below.



In right ΔABD,

$\angle DAB = 30^\circ$

In ΔBDC, $\angle BCD = 45^\circ$

also, $BD = 100$ m

In right ΔABD we have,

$$\tan 30^\circ = \frac{DB}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{AB}$$

$$AB = 100\sqrt{3} \text{ m}$$

In right ΔDBC we have,

$$\tan 45^\circ = \frac{DB}{BC}$$

$$1 = \frac{100}{BC}$$

$$BC = 100 \text{ m}$$

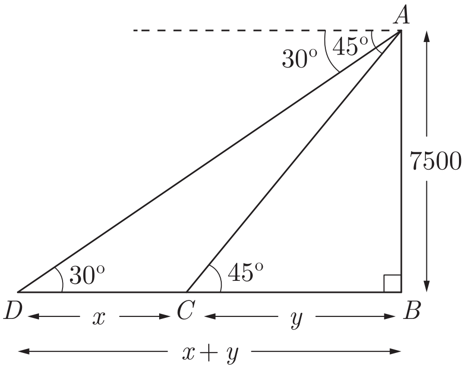
Now, $AB + BC = 100 + 100\sqrt{3} = 100(\sqrt{3}+1)$

$$= 100 + 173.2 = 273.2 \text{ m}$$

19. The angle of depression of two ships from an aeroplane flying at the height of 7500 m are 30° and 45°. if both the ships are in the same that one ship is exactly behind the other, find the distance between the ships.

Ans : [CBSE Foreign 2017]

Let A, C and D be the position of aeroplane and two ship respectively. Aeroplane is flying at 7500 m height from point B. As per given in question we have drawn figure below.



In right ΔABC we have

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{7500}{y} = y$$

$$y = 7500 \quad \text{...(1)}$$

In right ΔABD we have

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{7500}{x+y} = \frac{1}{\sqrt{3}}$$

$$x + y = 7500\sqrt{3} \quad \text{...(2)}$$

Substituting the value of y from (1) in (2) we have

$$x + 7500 = 7500\sqrt{3}$$

$$x = 7500\sqrt{3} - 7500$$

$$= 7500(\sqrt{3} - 1)$$

$$= 7500(1.73 - 1)$$

$$= 7500 \times 0.73$$

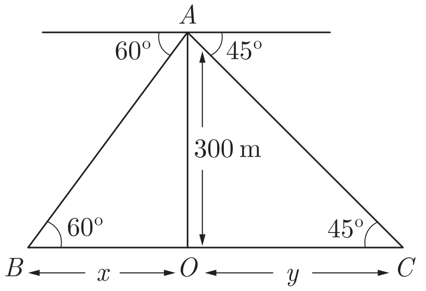
$$= 5475 \text{ m}$$

Hence, the distance between two ships is 5475 m.

20. An aeroplane is flying at a height of 300 m above the ground. Flying at this height the angle of depression from the aeroplane of two points on both banks of a respectively. Find the width of the river. River in opposite direction are 45° and 60°.

Ans : [CBSE Outside Delhi Set-I 2017]

Let A be helicopter flying at a height of 300 m above the point O on ground. Let B and C be the bank of river. As per given in question we have drawn figure below.



Let BO be x and OC be y.

In right ΔAOC we have

$$\frac{AO}{OC} = \tan 45^\circ$$

$$\frac{300}{y} = 1$$

$$y = 300$$

In right ΔAOB we have

$$\frac{AO}{BO} = \tan 60^\circ$$

$$\frac{300}{x} = \sqrt{3}$$

$$x\sqrt{3} = 300$$

$$x = \frac{300}{\sqrt{3}} = 100\sqrt{3}$$

$$BC = y + x = 300 + 100\sqrt{3}$$

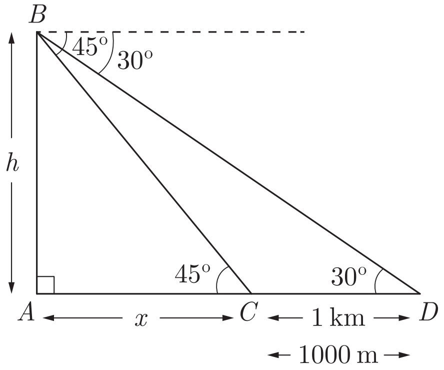
$$= 300 + 100 \times 1.732 = 473.2 \text{ m}$$

Hence, the width of river is 473.2 m

21. From the top of a hill, the angle of depression of two consecutive kilometre stones due east are found to be 45° and 30° respectively. Find the height of the hill. [Use $\sqrt{3} = 1.73$]

Ans : [CBSE Outside Delhi 2016]

Let AB be the hill of height h. Angle of depression from point D and C are given 30° and 45° respectively. As per given in question we have drawn figure below.



In right ΔABC we have

$$\begin{aligned} \frac{AB}{AC} &= \tan 45^\circ \\ \frac{h}{x} &= 1 \\ h &= x \end{aligned}$$

In right ΔABD we have

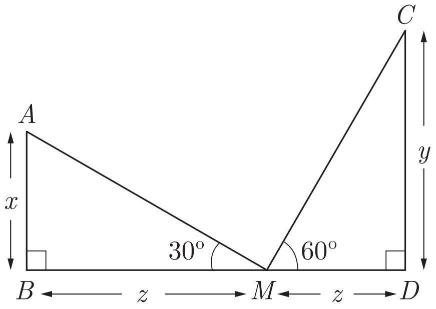
$$\begin{aligned} \frac{AB}{AC+CD} &= \tan 30^\circ \\ \frac{h}{x+1000} &= \frac{1}{\sqrt{3}} \\ h\sqrt{3} &= h+1000 \\ h(\sqrt{3}-1) &= 1000 \\ h &= \frac{1000}{\sqrt{3}-1} = \frac{1000(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} \\ &= \frac{1000(\sqrt{3}+1)}{3-1} \\ &= 500(\sqrt{3}+1) \quad = 500(1.73+1) \\ &= 500 \times 2.73 = 1365 \end{aligned}$$

Hence height of the hill is 1365 m.

22. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x:y$.

Ans : [Delhi CBSE Term-2, 2015]

Let AB be the tower of height x and CD be the tower of height y. Angle of depressions of both tower at centre point M are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here M is the centre of the line joining their feet.
Let $BM = MD = z$

In right ΔABM we have,

$$\begin{aligned} \frac{x}{z} &= \tan 30^\circ \\ x &= z \times \frac{1}{\sqrt{3}} \end{aligned}$$

In right ΔCDM we have

$$\begin{aligned} \frac{y}{z} &= \tan 60^\circ \\ y &= z \times \sqrt{3} \end{aligned}$$

From (1) and (2), we get

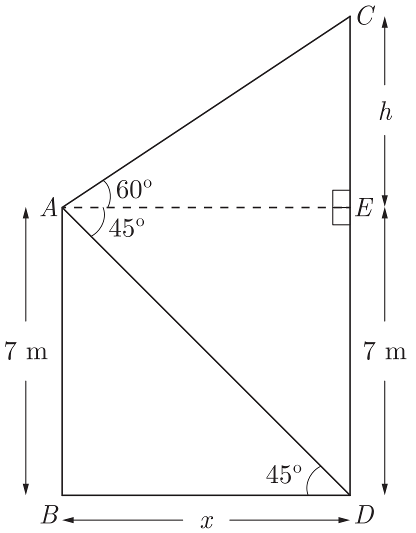
$$\begin{aligned} \frac{x}{y} &= \frac{z \times \frac{1}{\sqrt{3}}}{z \times \sqrt{3}} \\ \frac{x}{y} &= \frac{1}{3} \end{aligned}$$

Thus $\qquad x:y = 1:3$

23. From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower. (Use $\sqrt{3} = 1.732$)

Ans : [Foreign Set-I, II]

Let AB be the building of height 7 m and CD be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here $\angle CBD = \angle ECB = 45^\circ$ due to alternate angles.

In right ΔABC we have

$$\begin{aligned} \frac{CD}{BD} &= \tan 45^\circ \\ \frac{7}{x} &= 1 \\ x &= 7 \end{aligned}$$

In right ΔAEC we have

$$\begin{aligned} \frac{CE}{AE} &= \tan 60^\circ \\ \frac{h-7}{x} &= \sqrt{3} \\ h-7 &= x\sqrt{3} \\ h-7 &= 7\sqrt{3} \\ h &= 7\sqrt{3}+7 \\ &= 7(\sqrt{3}+1) \\ &= 7(1.732+1) \end{aligned}$$

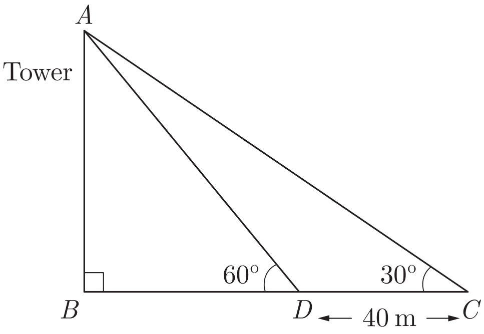
Hence, height of tower = 19.124 m

HOTS QUESTIONS

1. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30°, than when it is 60°. Find the height of the tower.

Ans : [CBSE Board Term-2, 2011]

Let AB be the tower of height h. Let BC be the shadow at 60° and BD be shadow at 30°.
As per given in question we have drawn figure below.



In right $\Delta\, ABC$ we get,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = \sqrt{3}\, x$$

In right $\Delta\, ABD$ we have,

$$\tan 30^\circ = \frac{AB}{BC+40}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+40}$$

$$x+40 = \sqrt{3}\, h = \sqrt{3} \times \sqrt{3}\, x = 3x$$

$$40 = 2x \Rightarrow x = 20 \text{ m}$$

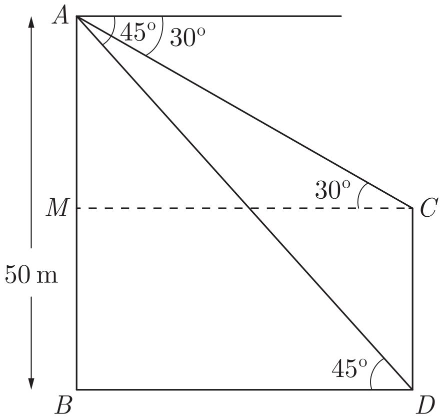
$$h = \sqrt{3} \times 20 = 20\sqrt{3} \text{ m}$$

Thus height of tower is $20\sqrt{3}$ m

2. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30° and 45° respectively. Find :
(1) How far the pole is from the bottom of the tower,
(2) The height of the pole. (Use $\sqrt{3} = 1.732$)

Ans : [CBSE Foreign 2015]

Let AB be the tower of height 50 m and CD be the pole of height h. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30° and 45° respectively. As per given in question we have drawn figure below.



In right $\Delta\, ABD$ we have,

$$\tan 45^\circ = \frac{AB}{BD} = 1$$

$$1 = \frac{50}{x}$$

$$x = 50 \text{ m}$$

(1) Thus distance of pole from bottom of tower is 50 m.

Now in $\Delta\, AMC$ we have

$$\tan 30^\circ = \frac{AM}{MC} = \frac{AM}{x}$$

$$AM = \frac{50}{\sqrt{3}} \text{ or } 28.87 \text{ m.}$$

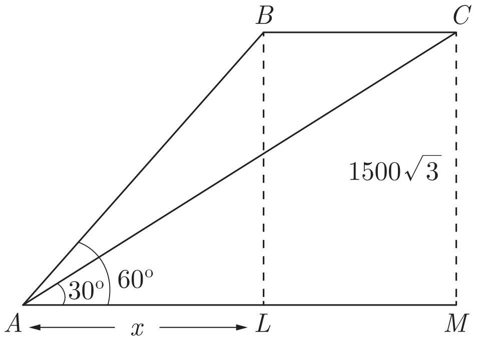
(2) Height pole $h = CD = BM$

$$= 50 - 28.87 = 21.13 \text{ m.}$$

3. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changed to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr.

Ans : [CBSE Outside Delhi 2015]

Let A be the point on ground, B and C be the point of location of aeroplane at height of $1500\sqrt{3}$ m. As per given in question we have drawn figure below.



In right $\Delta\, BAL$

$$\frac{BL}{AL} = \tan 60^\circ$$

$$\frac{1500\sqrt{3}}{x} = \sqrt{3} \qquad BL = CM$$

$$x = 1500 \text{ m.}$$

In right $\Delta\, CAM$ we have

$$\frac{CM}{AL+LM} = \tan 30^\circ$$

$$\frac{1500\sqrt{3}}{x+y} = \frac{1}{\sqrt{3}}$$

$$x+y = 1500 \times 3$$

$$1500+y = 4500$$

$$y = 3000 \text{ m.}$$

$$\text{Speed} = \frac{Distan\,ce}{Time} = \frac{y}{t}$$

$$= \frac{3000}{15} = 200 \text{ m/s}$$

$$= \frac{200}{1000} \times 60 \times 60$$

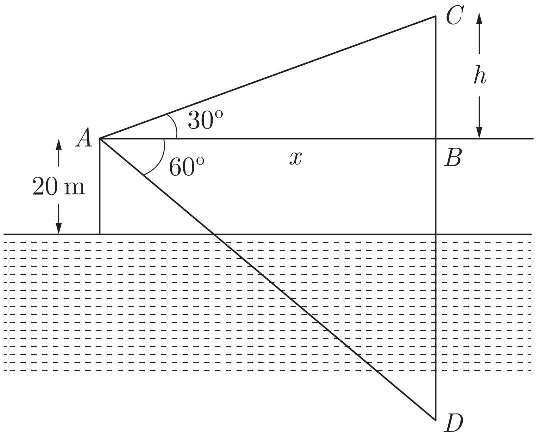
$$= 720 \text{ km/hr.}$$

Hence, the speed of the aeroplane is 720 km/hr.

4. At a point A, 20 metre above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A ?

Ans : [CBSE Outside Delhi, 2015]

As per given in question we have drawn figure below. Here cloud is at C, D is reflection of cloud in water.



In right ΔABC we have

$$\frac{h}{x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$x = \sqrt{3}\,h \qquad \text{...(1)}$$

Here $DE = EC$ because D is reflection of cloud and E is at water level.

In right ΔABD we have

$$\frac{BD}{BA} = \tan 60^\circ$$

$$\frac{DC+EB}{x} = \sqrt{3}$$

$$\frac{EC+EB}{x} = \sqrt{3}$$

$$\frac{h+20+20}{x} = \sqrt{3}$$

$$h+40 = \sqrt{3}\,x \qquad \text{...(2)}$$

From (1) and (2),

$$h+40 = \sqrt{3}\times\sqrt{3}\,h = 3h$$

$$h = 20 \text{ m}$$

$$x = 20\sqrt{3}$$

Now
$$AC = \sqrt{(BC)^2+(AB)^2}$$

$$= \sqrt{(20)^2+(20\sqrt{3})^2}$$

$$= \sqrt{400+1200}$$

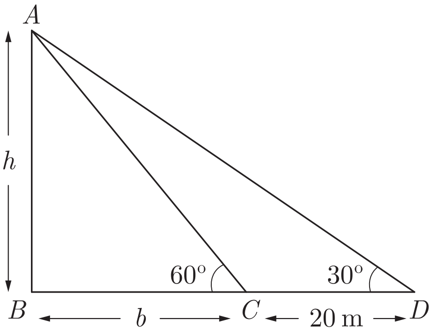
$$= 40 \text{ m.}$$

Hence distance of the cloud is 40 m.

5. A person standing on the bank of a river, observes that the angle of elevation of the top of the tree standing on the opposite bank is 60°. When he retreats 20 m from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the breadth of the river.

Ans : [CBSE Board Term-2, 2012]

Let AB be the tree of height h and breadth of river be b. As per given in question we have drawn figure below. Here point C and D are the location of person .



In right ΔABC we have,

$$\frac{h}{b} = \tan 60^\circ = \sqrt{3}$$

$$h = \sqrt{3}\,b \qquad \text{...(1)}$$

In right ΔABD we have

$$\frac{h}{b+20} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$h = \frac{b+20}{\sqrt{3}} \qquad \text{...(2)}$$

From (1) and (2) we have

$$b\sqrt{3} = \frac{b+20}{\sqrt{3}}$$

$$3b = b+20 \Rightarrow b = 10 \text{ m}$$

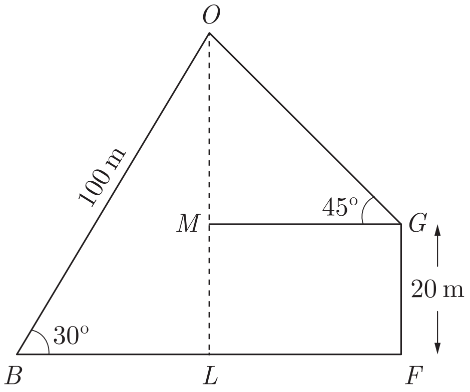
$$h = b\sqrt{3} = 10\times 1.73 = 17.3 \text{ m}$$

Thus height of tree is 17.3 m and breadth of river is 10 m.

6. A boy observes that the angle of elevation of a bird flying at a distance of 100 m is 30°. At the same distance from the boy, a girl finds the angle of elevation of the same bird from a building 20 m high is 45°. Find the distance of the bird from the girl.

Ans : [CBSE Borad Term-2, 2014]

Let O be the position of the bird and B be the position of the boy. Let FG be the building and G be the position of the girl. As per given in question we have drawn figure below.



In right $\Delta\, OLB$ we have

$$\frac{OL}{BO} = \sin 30^\circ$$

$$\frac{OL}{100} = \frac{1}{2}$$

$$OL = 50 \text{ m}$$

$$OM = OL - ML$$

$$= OL - FG = 50 - 20 = 30 \text{ m}$$

In right $\Delta\, OMG$ we have

$$\frac{OM}{OG} = \sin 45^\circ$$

$$\frac{OM}{OG} = \frac{1}{\sqrt{2}}$$

$$\frac{30}{OG} = \frac{1}{\sqrt{2}}$$

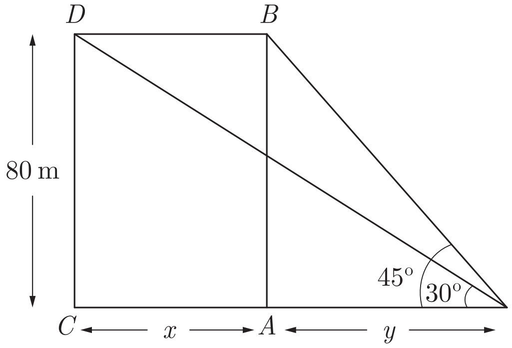
$$OG = 30\sqrt{2} \text{ m}$$

Hence, distance of the bird from the girl is $30\sqrt{2}$ m.

7. A bird sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. (Take $\sqrt{3} = 1.732$)

Ans : [CBSE Delhi Set I, III, 2016]

Let CD be the tree of height 80 m and bird is sitting at D. Point O on ground is reference point from where we observe bird. As per given in question we have drawn figure below.



In right AOB we have

$$\tan 45^\circ = \frac{80}{y}$$

$$y = 80$$

In right DOC we have

$$\tan 30^\circ = \frac{80}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{x+y}$$

$$x + y = 80\sqrt{3}$$

$$x = 80\sqrt{3} - y = 80\sqrt{3} - 80$$

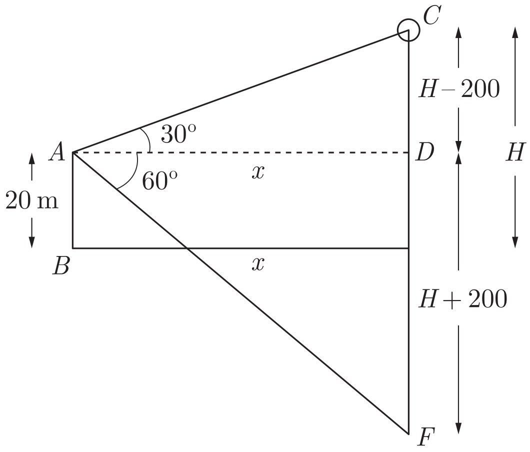
$$x = 80(\sqrt{3} - 1) = 58.4 \text{ m.}$$

Hence, speed of bird $= \frac{58.4}{2} = 29.2$ m

8. The angle of elevation of a cloud from a point 200 m above the lake is 30° and the angle of depression of its reflection in the lake is 60°, find the height of the cloud above the lake.

Ans : [CBSE Board Term-2 2012 Set (59), 2011, Set B1]

Let h be the height of cloud at C from lake. Let x be the horizontal distance of cloud from point of A. As per given in question we have drawn figure below.



Here BE is water level of lake and F is the reflection of cloud seen from A.

In right $\Delta\, ADC$ we have

$$\tan 30^\circ = \frac{h-200}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h-200}{x}$$

$$x = \sqrt{3}(h-200) \qquad ...(1)$$

In right $\Delta\, ADF$ we have

$$\tan 60^\circ = \frac{h+200}{x}$$

$$\sqrt{3} = \frac{h+200}{x}$$

$$\sqrt{3}\,x = h + 200 \qquad ...(2)$$

From (1) and (2) we have

$$3(h-200) = h + 200$$

$$3h - h = 200 + 600$$

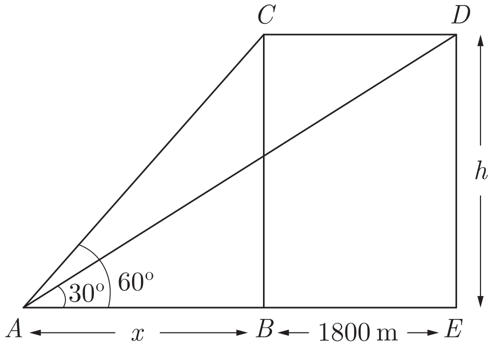
$$2h = 800$$

So, height of cloud $H = 400$ m.

9. The angle of elevation of a jet fighter point A on ground is 60°. After flying 19 seconds, the angle changes to 30°. If the jet is flying at a speed of 648 km/hour, find the constant height at which the jet is flying.

Ans : [CBSE Board Term-2, 2012]

Let C and D are the point of location of jet at height h. Point B and E are foot print on ground of get at thee location. As per given in question we have drawn figure below.



In 3600 sec distance travelled by plane = 648000 m

In 10 sec distance travelled by plane - $\dfrac{648000}{3600} \times 10$

$$= 1800 \text{ m}$$

In right ΔABC, we have

$$\frac{h}{x} = \tan 60^{\text{o}} = \sqrt{3}$$

$$h = x\sqrt{3} \qquad \text{...(1)}$$

In right ΔADE we have

$$\frac{h}{x+1800} = \tan 30^{\text{o}} = \frac{1}{\sqrt{3}}$$

$$h = \frac{x+1800}{\sqrt{3}} \qquad \text{...(2)}$$

From equations (1) and (2), we get

$$x\sqrt{3} = \frac{x+1800}{\sqrt{3}}$$

$$3x = x+1800$$

$$2x = 1800$$

$$x = 900 \text{ m}$$

$$h = x\sqrt{3}$$

$$= 900 \times 1.732$$

$$= 1558.5 \text{ m}$$

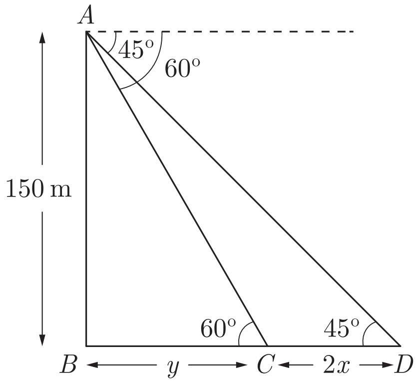
Thus height of jet is 1558.8 m.

10. A moving boat observed from the top of a 150 m high cliff. moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat.

Ans : [Delhi Set-I 2017]

Let AB be the cliff of height 150 m. Let C and D be the point of boat at 60° and 45°. Let the speed of the boat be x m/min. Let BC be y

As per given in question we have drawn figure below.



Here distance covered in 2 minutes is $2x$.

Thus $\quad CD = 2x$

In right ΔABD we have

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{150}{y} = \sqrt{3}$$

$$y = \frac{150}{\sqrt{3}} = 50\sqrt{3} \qquad \text{...(1)}$$

In right ΔABD we have

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{150}{y+2x} = 1$$

$$y+2x = 150 \qquad \text{...(2)}$$

From equations (1) and (2), we get

$$50\sqrt{3}+2x = 150$$

$$2x = 150 - 50\sqrt{3}$$

$$2x = 50(3-\sqrt{3})$$

$$x = 25(3-\sqrt{3})$$

$$\text{Speed of the boat} = 25(3-\sqrt{3}) \text{ m/min.}$$

$$= \frac{25(3-\sqrt{3}) \times 60}{1000}$$

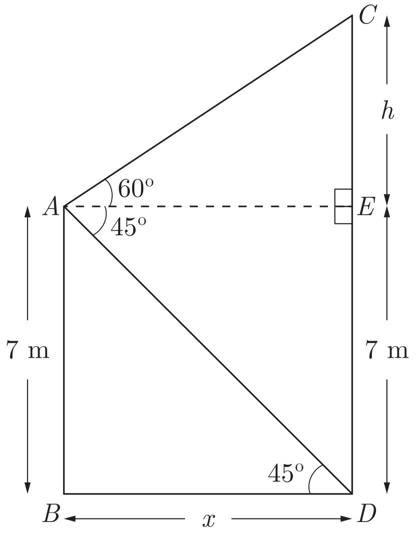
$$= \frac{3}{2}(3-\sqrt{3}) \text{ km/hr.}$$

11. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower.

Ans : [CBSE Delhi Set-I, II, III, 2017]

Let AB be the building of height 7 m and CD be the tower of height h. Let distance between two be x. Angle of depressions of top and bottom of tower are given 60° and 45° respectively. As per given in question we have drawn figure below.

Let AB be building = 7 m



CD be the height of tower $= (7+h)$ m

$$BD = AE = x \text{ m}$$

In right $\Delta\, ABD$ we have

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{7}{x} = 1$$

$$x = 7 \text{ m} \qquad ...(1)$$

In right $\Delta\, CEA$ we have

$$\frac{CE}{AE} = \tan 60^\circ$$

$$\frac{h-7}{x} = \sqrt{3}$$

$$h - 7 = x\sqrt{3} \qquad ...(2)$$

From equations (1) and (2), we get

$$h - 7 = 7\sqrt{3}$$

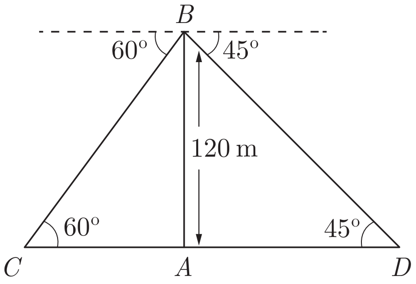
$$h = 7 + 7\sqrt{3} = 7(1+\sqrt{3}) \text{ m}$$

Hence, the height of tower is $7(1+\sqrt{3})$ m

12. From the top of a 120 m high tower, a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of repression as 60° and 45°. Find the distance between two cars.

Ans : [Delhi Compt. Set-III, II, 2017]

Let AB be the tower of height 120 m. Let C and D be location of car on opposite side of tower. As per given in question we have drawn figure below.



In right $\Delta\, BAD$ we have

$$\frac{AB}{AD} = \tan 45^\circ$$

$$\frac{120}{AB} = 1$$

$$AB = 120$$

In right $\Delta\, BAC$ we have

$$\frac{AB}{CA} = \tan 60^\circ$$

$$\frac{120}{CA} = \sqrt{3}$$

$$CA = \frac{120}{\sqrt{3}} = 40\sqrt{3}$$

$$\begin{aligned} CD &= CA + AD \\ &= 120 + 40\sqrt{3} \\ &= 120 + 40 \times 1.732 \\ &= 189.28 \text{ m} \end{aligned}$$

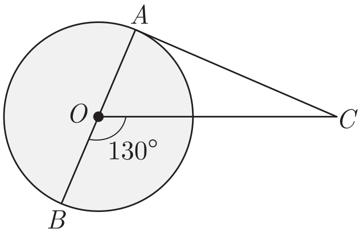
Hence the distance between two men is 189.28 m.

CHAPTER 10

Circle

VERY SHORT ANSWER TYPE QUESTIONS

1. In the given figure, AOB is a diameter of the circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$, the find $\angle ACO$.



Ans : [Foreign Set I, II, III, 2016]

Here OA is radius and AC is tangent at A, since radius is always perpendicular to tangent, we have

$$\angle OAC = 90^o$$

From exterior angle property,

$$\angle BOC = OAC + \angle ACO$$

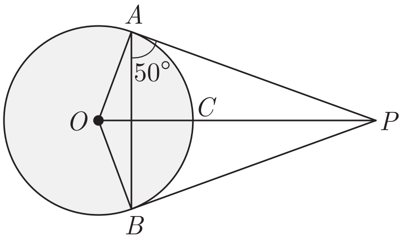
$$130^o = 90° + \angle ACO$$

$$\angle ACO \; 130^o - 90^o = 40^o$$

2. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50^o$, then find $\angle AOB$.

Ans : [Delhi Set I, II, III, 2016]

As per the given question we draw the figure as below.



Since $PA \perp OA$, $\angle OAP = 90°$

$$\angle OAB = \angle OAP - \angle BAP$$

$$= 90^o - 50^o = 40^o$$

Since OA and OB are radii, we have

$$\angle OAB = \angle OBA = 40^o$$

Now

$$\angle AOB + \angle OAB + \angle OBA = 180^o$$

$$\angle AOB + 40^o + 40^o = 180^o$$

$$\angle AOB = 180^o - 80^o = 100^o$$

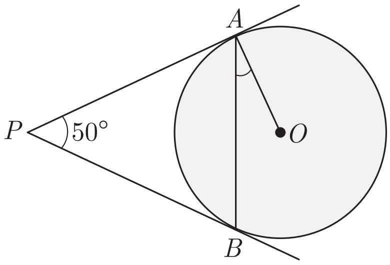
Hence $\angle AOB = 100^o$

3. What is the maximum number of parallel tangents a circle can have on a diameter?

Ans : [Board Term-2, 2012 Set (34)]

Tangent touches a circle on a distinct point. Thus on the diameter of a circle only two parallel tangents can be drawn. It has been shown in figure given below.

4. In figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50^o$. Write the measure of $\angle OAB$.



Ans : [Delhi CBSE Board, 2015, Set I, II, III]

We have $\angle APB = 50^o$

$$\angle PAB = \angle PBA = \frac{180^o - 50^o}{2} = 65^o$$

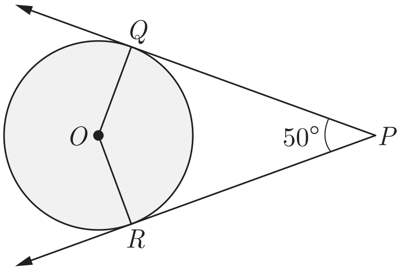
Here OA is radius and AP is tangent at A, since radius is always perpendicular to tangent at point of contact, we have

$$\angle OAP = 90^o$$

Now $\angle OAB = \angle OAP - \angle PAB$

$$= 90^o - 65^o = 25^o$$

5. In the given figure, PQ and PR are tangents to the circle with centre O such that $\angle QPR = 50^o$, Then find $\angle OQR$.



Ans : [Board Term-2, 2012] ,Delhi CBSE Term II, 2015 Set I, II, III]

We have $\angle QPR = \angle 50^o$ (Given)

Since $\angle QOR$ and $\angle QPR$ are supplementary angles

$$\angle QOR + \angle QPR = 180^o$$

$$\angle QOR = 180^o - \angle QPR$$

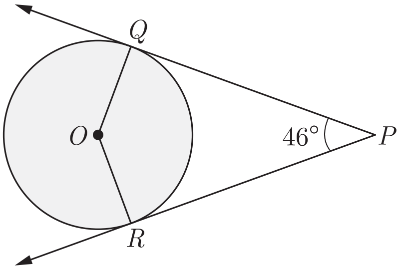
$$= 180^o - 50^o = 130^o$$

From $\Delta\, OQR$ we have

$$\angle OQR = \angle ORQ = \frac{180^o - 130^o}{2}$$

$$= \frac{50^o}{2} = 25^o$$

6. If PQ and PR are two tangents to a circle with center O. If $\angle QPR = 46^o$ then find $\angle QOR$.



Ans : [Delhi, CBSE, Term-2, 2014]

We have $\angle QPR = 46^o$

Since $\angle QOR$ and $\angle QPR$ are supplementary angles

$$\angle QOR + \angle QPR = 180^o$$

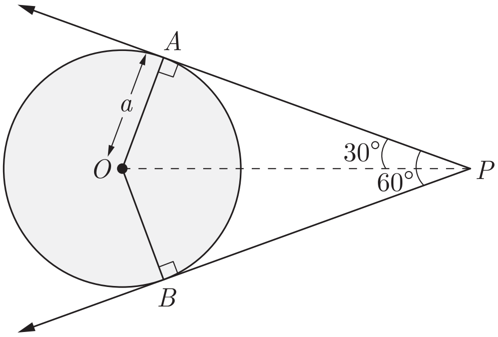
$$\angle QOR + 46^o = 180^o$$

$$\angle QOR = 180^o - 46^o = 134^o$$

7. If the angle between two tangents drawn from an external point P to a circle of radius a and center O, is 60^o, then find the length of OP.

Ans : [Outside Delhi Set II, 2018]

As per the given question we draw the figure as below.



Tangents are always equally inclined to line joining the external point P to center O.

$$\angle APO = \angle BPO = \frac{60}{2} = 30^o$$

Also radius is also perpendicular to tangent at point of contact.

In right $\Delta\, OAP$ we have

$$\angle APO = 30^o$$

Now, $$\sin 30^o = \frac{OA}{OP}$$

Here OA is radius whose length is a, thus

$$\frac{1}{2} = \frac{a}{OP}$$

or $$OP = 2a$$

8. If a circle can be inscribed in a parallelogram how will the parallelogram change?

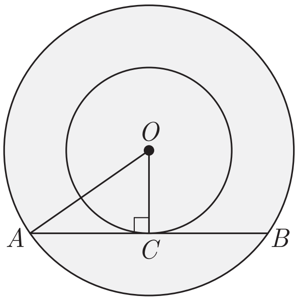
Ans : [Board Term II, 2014]

It changes into a rectangle or a square.

9. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of larger circle (In cm) which touches the smaller circle.

Ans : [Foreign Set I, II, III, 2014]

As per the given question we draw the figure as below.



Here AB is the chord of large circle which touch the smaller circle at point C. We can see easliy that $\Delta\, AOC$ is right angled triangle.

Here, $AO = 5$ cm, $OC = 3$ cm

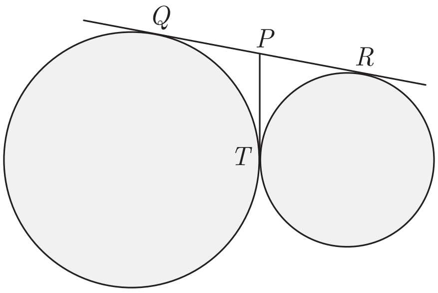
$$AC = \sqrt{AO^2 - OC^2}$$

$$= \sqrt{5^2 - 3^2}$$

$$= \sqrt{25 - 9} = \sqrt{16} = 4 \text{ cm}$$

Length of chord, $AB = 8$ cm.

10. In the figure, QR is a common tangent to given circle which meet at T. Tangent at T meets QR at P. If $QP = 3.8$ cm, then find length of QR.



Ans : [Delhi, Set 2014] [Board, Term-2, 2012 Set (44)]

Let us first consider large circle. Since length of tangents from external points are equal, we can write

$$QP = PT$$

Thus $$QP = PT = 3.8 \qquad ....(1)$$

Now consider the small circle. For this circle we can also write using same logic,

$$PR = PT$$

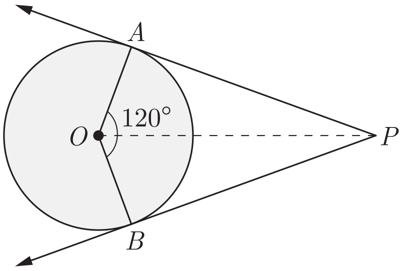
But we have $PT = 3.8$ cm

Thus $PR = PT = 3.8$ cm

Now $QR = QP + PR$

$= 3\cdot 8 + 3\cdot 8 = 7.6$ cm.

11. In the figure, PA and PB are tangents to a circle with centre O. If $\angle AOB = 120^\circ$, then find $\angle OPA$.



Ans : [Delhi, Set 2014], [Board Term-2, 2012 Set (44)]

Here OA is radius and AP is tangent at A, since radius is always perpendicular to tangent at point of contact, we have

$$\angle OAP = 90^\circ$$

Due to symmetry we have

$$\angle AOP = \frac{\angle AOB}{2} = \frac{120^\circ}{2} = 60^\circ$$

Now in right ΔAOP we have

$$\angle APO + \angle OAP + \angle AOP = 180^\circ$$

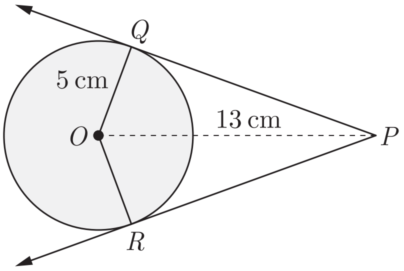
$$\angle APO + 90^\circ + 60^\circ = 180^\circ$$

$$\angle APO = 180^\circ - 150^\circ = 30^\circ.$$

12. From a point P, which is at a distant of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ are drawn to the circle, then the area of the quadrilateral $PQOR$ (in cm^2).

Ans : [Board Term-2, 2012 Set (31)]

As per the given question we draw the figure as below.



Here OQ is radius and QP is tangent at Q, since radius is always perpendicular to tangent at point of contact, $\Delta\, OQP$ is right angle triangle.

Now $PQ = \sqrt{OP^2 - OR^2}$

$= \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25}$

$= \sqrt{144} = 12$ cm

Area of triangle $\Delta\, OQP$

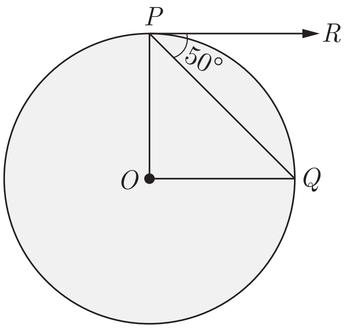
$$\Delta = \tfrac{1}{2}(OQ)(QP)$$

$$= \tfrac{1}{2} \times 12 \times 5 = 30$$

Area of quadrilateral $PQOR$,

$$2 \times \Delta\, POQ = 2 \times 30 = 60 \text{ cm}^2$$

13. If O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, find $\angle POQ$.



Ans : [Board Term-2, 2012 Set (26)]

We have $\angle RPQ = 50^\circ$

Since $\angle OPQ + \angle QPQ$ is right angle triangle,

$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

Since, $OP = OQ$ because of radii of circle, we have

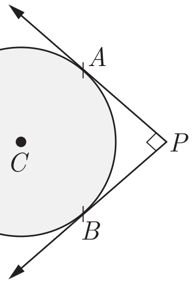
$$\angle OPQ = \angle OQR = 40^\circ$$

In ΔPOQ we have

$$\angle POQ = 180^\circ - (40^\circ + 40^\circ)$$

$$= 100^\circ$$

14. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If $PA \perp PB$, then find the length of each tangent.



Ans : [Board Term-2, 2013]

Here tangent drawn on circle from external point P are at aright angle, $CAPB$ will be a square

Thus $CA = AP = PB = BC = 4$ cm

Thus length of tangent is 4 cm.

15. What is the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm ?

Ans : [Board Term-2, 2012, Set (22)]

As per the given question we draw the figure as below.

$$\begin{aligned}\text{Length of the tangent } &= \sqrt{d^2 - r^2}\\ &= \sqrt{8^2 - 6^2}\\ &= \sqrt{64 - 36}\\ &= \sqrt{28} = 2\sqrt{3} \text{ cm.}\end{aligned}$$

16. If the angel between two radii of a circle is 130°, then what is the angle between the tangents at the end points of radii at their point of intersection ?

Ans : [Board Term II, 2012 Set (22)]

Sum of the angles between radii and between intersection point of tangent is always 180°.
Thus angle at the point of intersection of tangents

$$= 180° - 130° = 50°$$

17. To draw a pair of tangents to a circle which are inclined to each other at an angle of 30°, it is required to draw tangents at end points of two radii of the circle, what will be the angle between them ?

Ans : [Board Term II, 2012 Set (31)]

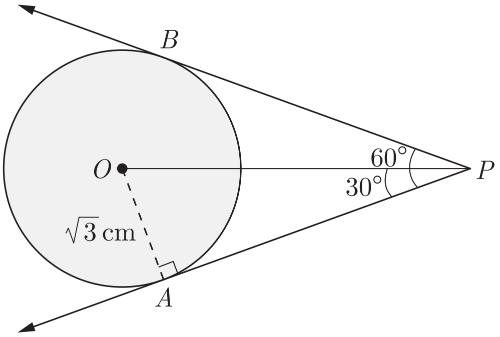
Sum of the angles between radii and between intersection point of tangent is always 180°.

Angle between the radii $= 180° - 30° = 150°$

18. Two tangents making an angle af 60° between them are drawn to a circle of radius $\sqrt{3}$ cm, then find the length of each tangent.

Ans : [Board, Term-2, 2013]

As per the given question we draw the figure as below.



Since, $\tan\theta = \dfrac{Altitude}{Base}$

So. $\tan 30° = \dfrac{OA}{AP}$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{AP}$$

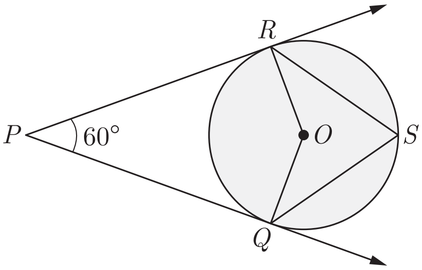
$$AP = \sqrt{3} \times \sqrt{3} = 3 \text{ cm.}$$

19. If a line intersects a circle in two distinct points, what is it called ?

Ans : [Board Term-2, 2012 Set (17)]

The line which intersects a circle in two distinct points is called secant.

20. In the given figure, find $\angle QSR$.



Ans : [Board Term-2, 2012 Set (5)]

Sum of the angles between radii and between intersection point of tangent is always 180°.

Thus $\angle ROQ + \angle RPQ = 180°$

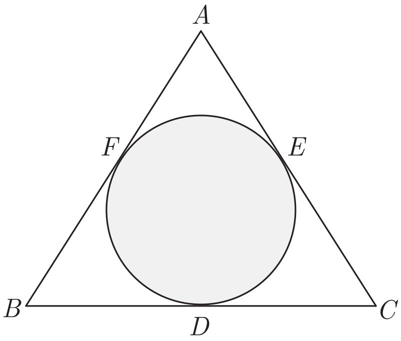
$$\angle ROQ = 180° - 60° = 120°$$

We know that angle subtended on the centre of a circle is twice of the angle subtended on circumference of circle

Thus $\angle QSR = \frac{1}{2}\angle ROQ = \frac{1}{2} \times 120°$

$$= 60°$$

21. A triangle ABC is drawn to circumscribe a circle. If $AB = 13$ cm, $BC = 14$ cm and $AE = 7$ cm, then find AC.



Ans : [Board Term-2, 2012 Set (26)]

Since AF and AE are tangent of the circle, $AF = AE$

Thus $AF = AE = 7$ cm

Now $BF = AB - AF = 13 - 7 = 6$ cm

Since BF and BD are tangent of the circle, $BF = BD$

Thus $BD = BF = 6$ cm

Now $CD = BC - BD = 14 - 6 = 8$ cm

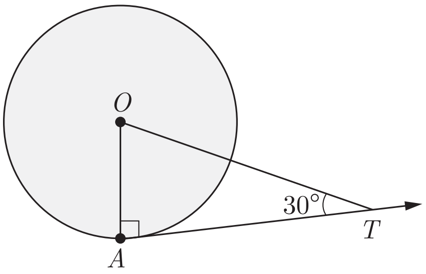
Since CD and CE are tangent of the circle, $CD = CE$

Thus $CE = CD = 8$ cm

Now $AC = AE + EC$

$$= 7 + 8 = 15 \text{ cm.}$$

22. In given figure, if AT is a tangent to the circle with centre O, such that $OT = 4$ cm and $\angle OTA = 30°$, then find the length of AT (in cm).



Ans : [Board Term-2, 2012 Set (13)]

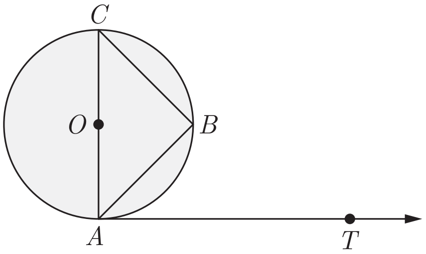
Since AT is a tangent to the circle, ΔOAT is right angle triangle

Now $\cos 30° = \frac{AT}{OT}$ $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$

$$AT = OT \cos 30°$$

or, $$AT = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3} \text{ cm.}$$

23. In the given figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50°$. If AT is the tangent to the circle at the point A, find $\angle BAT$.



Ans : [Board Term-2, 2012 Set (32)]

We have $\angle ACB = 50°$

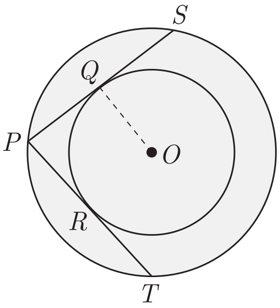
Since $\angle CBA$ is angle in semi-circle.

$$\angle CBA = 90°$$

Now $\angle OAB = 180° - 90° - 50° = 40°$

$$\angle BAT = 90° - \angle OAB$$
$$= 90° - 40° = 50°$$

24. In the figure there are two concentric circles with centre O. PRT and PQS are tangents to the inner circle from a point P lying on the outer circle. If $PR = 5$ cm find the length of PS.



Ans : [Delhi Compt. Set I, II, III 2017]

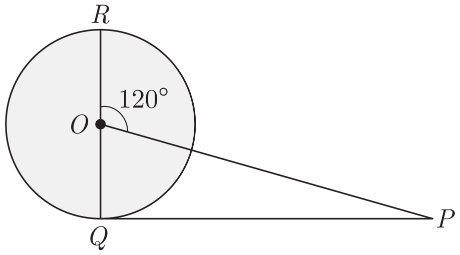
Since PQ and PR are tangent of the circle, $PQ = PR$

$$PQ = PR = 5 \text{ cm}$$

Since PS is chord of circle and point Q bisect it, thus

$$PQ = QS$$
$$PS = 2PQ$$
$$= 2 \times 5 = 10 \text{ cm}$$

25. PQ is a tangent drawn from an external point P to a circle with centre O, QOR is the diameter of the circle. If $\angle POR = 120°$, What is the measure of $\angle OPQ$?



Ans : [Foreign Set I, II, III, 2017]

Since PQ is a tangent to the circle, ΔOQP is right angle triangle

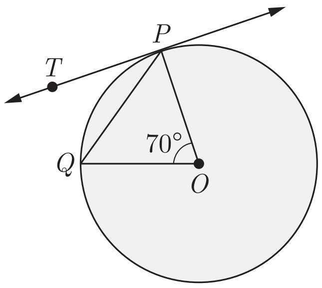
In ΔOQP $\angle POR = \angle OQP + \angle OPQ$

(Exterior angle)

Thus $\angle OPQ = \angle POR - \angle OQP$

$$= 120° - 90° = 30°$$

26. In figure, O is the centre of the circle, PQ is a chord and PT is tangent to the circle at P.



Ans : [Outside Delhi Set I, II, III 2017]

We have $\angle OPQ = \angle OQP$

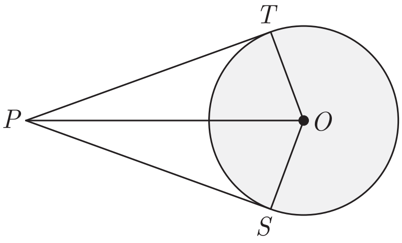
$$= \frac{180 - 70}{2} = 55°$$

Thus $TPQ = 90° - 55° = 35°$

SHORT ANSWER TYPE QUESTIONS - I

1. In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that

$\angle SPT = 120^\circ$, Prove that $OP = 2PS$.



Ans : [Foreign Set I, II, III, 2016]

Given that $\angle SPT = 120^\circ$

As OP bisects $\angle SPT$,

$$\angle OPS = \frac{120^\circ}{2} = 60^\circ$$

Since radius is always perpendicular to tangent,

$$\angle PTO = 90^\circ$$

Now in right triangle POS, we have

$$\cos 60^\circ = \frac{PS}{OP}$$

$$\frac{1}{2} = \frac{PS}{OP}$$

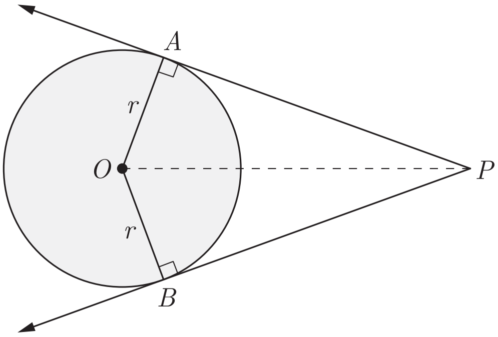
$$OP = 2PS$$ Hence proved.

2. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

Ans : [Outside Delhi Set, II, 2017]

Consider is circle of radius r and centre at O as shown in figure below. Here we have drwan two tangent from P at A and B. We have to prove that

$$AP = PB$$



We join OA, OB and OP.

Proof :

In ΔPAO and ΔPBO, OP is common and

$$OA = OB$$ radius of same circle

Since radius is always perpendicular to tangent, at point of contact,

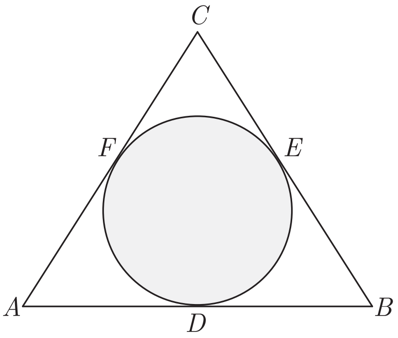
$$\angle OAP = \angle OBP = 90^\circ$$

Thus $\Delta PAO \cong \Delta PBO$.

and hence, $AP = BP$

Thus length of 2 tangents drawn from an external point to a circle are equal.

3. In the given figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sodes AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



Ans : [Delhi Set I, II, III, 2016]

Since AF and AD are tangent of the circle, $AF = AD$

Let $AF = AD = x$

Now $DB = AB - AD = 12 - x$

Since BD and BE are tangent of the circle, $BD = BE$

Thus $BE = BD = 12 - x$

Now $CE = CB - BE = 8 - (12 - x)$

Since CF and CE are tangent of the circle, $CF = CE$

Thus $CF = CE = 8 - (12 - x)$ cm

But $AC = CF + FA$

Substituting values we have

$$10 = 8 - (12 - x) + x$$

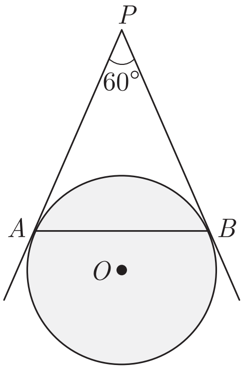
$$10 = 2x - 4$$

$$2x = 10 + 4 = 14$$

$$x = 7$$

Thus $AD = 7$ cm, $BE = 5$ cm, $CF = 3$ cm

4. In figure, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60^\circ$. Find the length of chord AB.



Ans : [Delhi Set I, II, III, 2016]

Since length of 2 tangents drawn from an external point to a circle are equal, we have

$$PA = PB$$

Thus $\angle PAB = \angle PBA = 60^\circ$

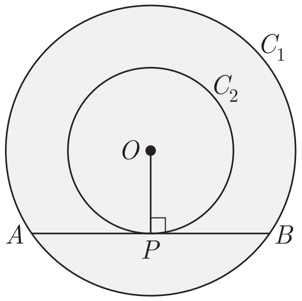
Hence ΔPAB is an equilateral triangle.

Therefore $AB = PA = 5$ cm.

5. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle is bisected at the point of contact.

Ans : [Board Term-2, 2012 Set (17, 40)]

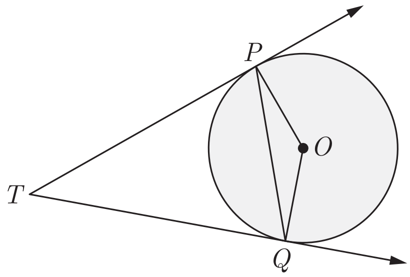
As per the given question we draw the figure as below.



Since OP is radius and APB is tangent, $OP \perp AB$. Now for bigger circle, O is centre and AB is chord such that $OP \perp AB$.

Thus OP bisects AB.

6. In the given figure PQ is chord of length 6 cm of the circle of radius 6 cm. TP and TQ are tangents to the circle at points P and Q respectively. Find $\angle PTQ$.



Ans : [CBSE S.A.2 2016 Set HODM4OL]

We have $PQ = 6$ cm, $OP = OQ = 6$ cm

Since $PQ = OP = OQ$, triangle ΔPQO is an equilateral triangle.

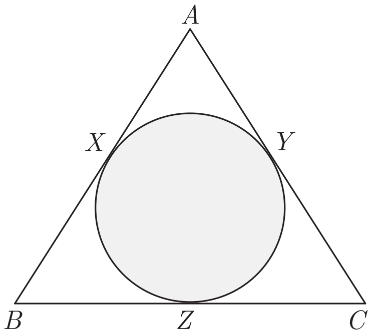
Thus $\angle POQ = 60°$

Now we know that $\angle POQ$ and $\angle PTQ$ are supplementary angle,

$$\angle POQ + \angle PTQ = 180°$$
$$\angle PTQ = 180° - \angle POQ$$
$$= 180° - 60° = 120°$$

Thus $\angle PTQ = 120°$

7. ABC is an isosceles triangle in which $AB = AC$ which is circumscribed about a circle as shown in the figure. Show that BC is bisected at the point of contact.



Ans : [Board Term-2, 2012 Set (22)]

We have $AB = AC$

Since tangents from an external point to a circle are equal,

$$AX = AY$$
$$BX = BZ$$
$$CZ = CY$$

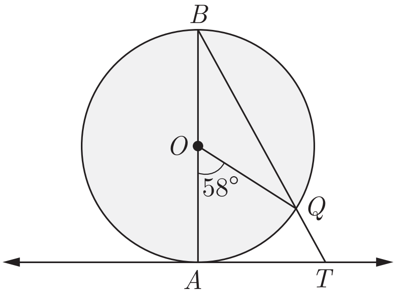
Now $AX + XB = AY + YC$

or $XB = YC$ $(AX = AY)$

or $BZ = CZ$

Thus Z is the mid-point of BC and Z is the point of contact. Hence, BC is bisected at he point of contact.

8. In given figure, AB is the diameter of a circle with center O and AT is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$.



Ans : [Board Term-2, 2015 Set I, II, III]

We have $\angle AOQ = 58°$

Since angle $\angle ABQ$ and $\angle AOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle ABQ = \frac{1}{2}\angle AOQ$$
$$= \frac{1}{2} \times 58° = 29°$$

Here OA is perpendicular to TA because OA is radius and TA is tangent at A.

Thus $\angle BAT = 90^\circ$

$\angle ABQ = \angle ABT$

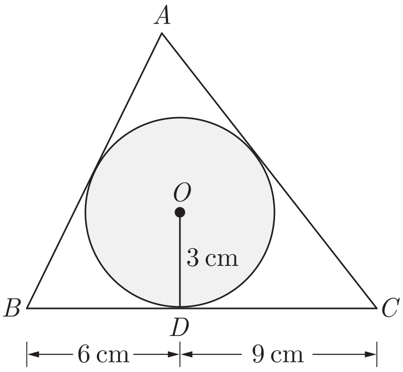
Now in ΔBAT,

$$\angle ATB = 90^\circ - \angle ABT$$

$$= 90^\circ - 29^\circ = 61^\circ$$

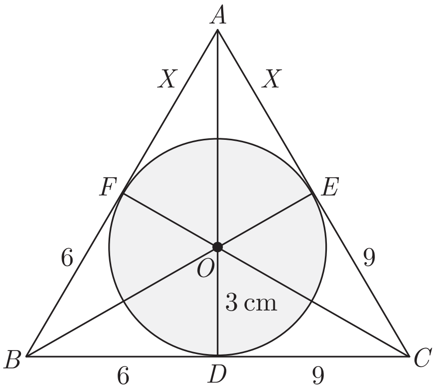
Thus $\angle ATQ = \angle ATB = 61^\circ$

9. In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm², then find the lengths of sides AB and AC.



Ans : [Outside Delhi CBSE, 2015, Set I, II, III]

We redraw the given circle as shown below.



Since tangents from an external point to a circle are equal,

$$AF = AE$$

$$BF = BD = 6 \text{ cm}$$

$$CE = CD = 9 \text{ cm}$$

Let $AF = AE = x$

Now $AB = AF + FA = 6 + x$

$$AC = AE + EC = x + 9$$

$$BC = 6 + 9 = 15 \text{ cm}$$

Perimeter of ΔABC,

$$p = 15 + 6 + x + 9 + x$$

$$= 30 + 2x$$

Now area $\Delta ABC = \frac{1}{2}rp$

Here $r = 3$ is the radius of circle. Substituting all values we have

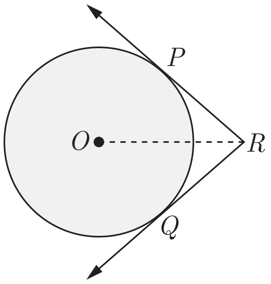
$$54 = \tfrac{1}{2} \times 3 \times (30 + 2x)$$

$$54 = 45 + 3x$$

or $x = 3$

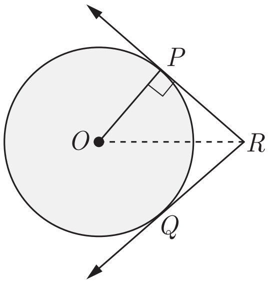
Thus $AB = 9$ cm, $AC = 12$ cm and $BC = 15$ cm.

10. In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120^\circ$, then prove that $OR = PR + RQ$.



Ans : [Outside Delhi CBSE Board, 2015, Set I, II, III]

We redraw the given figure by joining O to P as shown below.



$$\angle PRO = \tfrac{1}{2}\angle PRQ$$

$$= \frac{120^\circ}{2} = 60^\circ$$

Here ΔOPR is right angle triangle, thus

$$\angle POR = 90^\circ - \angle PRO$$

$$= 90^\circ - 60^\circ = 30^\circ$$

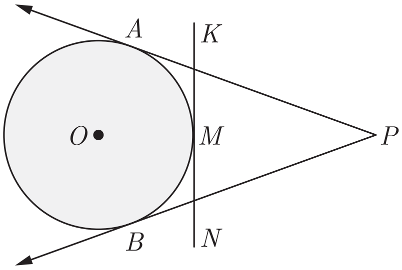
Now $$\frac{PR}{OR} = \sin 30^\circ = \frac{1}{2}$$

or, $OR = 2PR = PR + PR$

Since $PR = QR$,

$$OR = PR + QR \qquad \text{Hence Proved}$$

11. PA and PB are tangents from point P to the circle with centre O as shown in figure. At point M, a tangent is drawn cutting PA at K and PB at N. Prove that $KN = AK + BN$

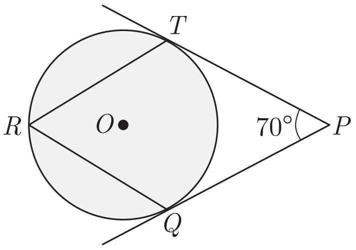


Ans : [Board Term-2, 2012 Set (28)]

Since length of tangents from an external point to a circle are equal,

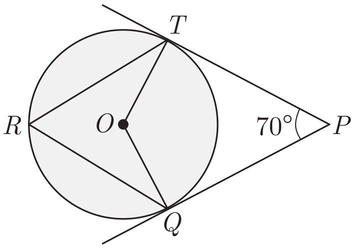
$$PA = PB, KA = KM, NB = NM,$$
$$KA + NB = KM + NM$$
$$AK + BN = KN. \qquad \text{Hence Proved}$$

12. In figure, O is the centre of a circle. PT are tangents to the circle from an external point P. If $\angle TPQ = 70^\circ$, find $\angle TRQ$.



Ans : [Foreign Set I, II, III, 2015]

We redraw the given figure by joining O to T and Q as shown below.



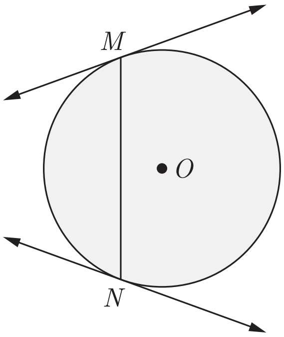
Here angle $\angle TOQ$ and $\angle TPQ$ are supplementary angle.

Thus $$\angle TOQ = 180^\circ - \angle TPQ$$
$$= 180^\circ - 70^\circ = 110^\circ$$

Since angle $\angle TRQ$ and $\angle TOQ$ are the angle on the circumference of the circle by the same arc,

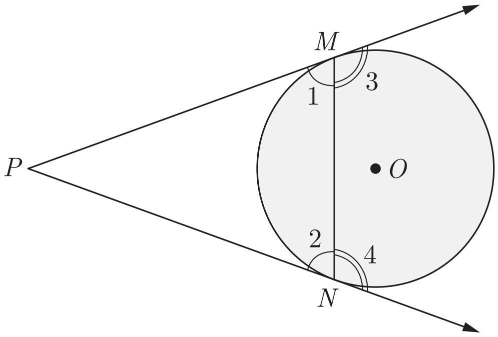
$$\angle TRQ = \frac{1}{2}\angle TOQ$$
$$= \frac{1}{2} \times 110^\circ = 55^\circ$$

13. Prove that tangents drawn at the ends of a chord of a circle make equal angles with the chord.



Ans : [Delhi Term-2, 2015]

We redraw the given figure by joining M and N to P as shown below.



Since length of tangents from an external point to a circle are equal,

$$PM = PN$$

Since angles opposite to equal sides are equal,

$$\angle 1 = \angle 2$$

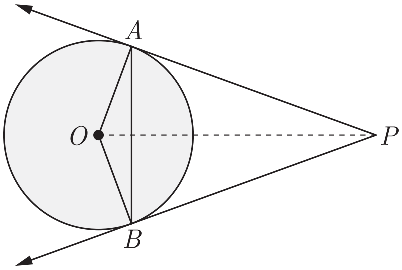
Now using property of linear pair we have

$$180^o - \angle 1 = 180^o - \angle 2$$
$$\angle 3 = \angle 4 \qquad \text{Hence Proved}$$

14. Two tangents PA and PB are drawn from an external point P to a circle inclined to each other at an angle of 70^o, then what is the value of $\angle PAB$?

Ans : [Board Term-2, 2012 Set (26, 34)]

As per question we draw the given circle as shown below.



Here angle $\angle AOB$ and $\angle APB$ are supplementary angle.

Thus $\angle AOB = 180^\circ - \angle APB$

$$= 180^\circ - 70^\circ = 110^\circ$$

OA and OB are radius of circle and equal in length, thus angle $\angle OAB$ and $\angle OBA$ are also equal. Thus in triangle ΔOAB we have

$$\angle OBA + \angle OAB + \angle AOB = 180^\circ$$

$$\angle OAB + \angle OAB = 180^\circ - \angle AOB$$

$$2\angle OAB = 180^\circ - 110^\circ$$

$$\angle OAB = 35^\circ$$

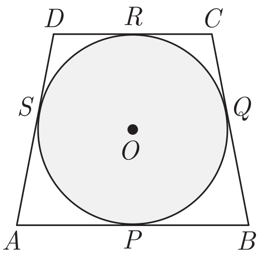
Since OA is radius and AP is tangent at A, $OA \perp AP$

$$\angle OAP = 90^o$$

Now $\angle PAB = \angle OAP - \angle OAB$

$$= 90^o - 35^o = 55^o$$

15. In Figure a quadrilateral $ABCD$ is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD, and DA touch the circle at the points P, Q, R and S respectively. Prove that. $AB + CD = BC + DA$.



Ans : [Outside Delhi Set, II, 2016]

Since length of tangents from an external point to a circle are equal,

At A, $AP = AS$ (1)

At B $BP = BQ$ (2)

At C $CR = CQ$ (3)

At D $DR = DS$ (4)

Adding eqn. (1), (2), (3), (4)

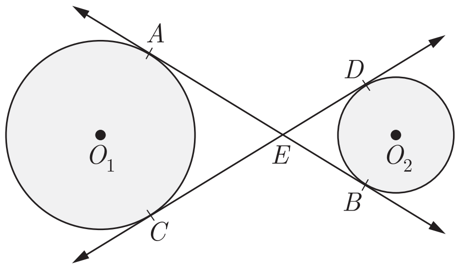
$$AP + BP + DR + CR = AS + DS + BQ + CQ$$

$$AP + BP + DR + RC = AS + SD + BQ + QC$$

$$AB + CD = AD + BC$$

Hence Proved

16. In Figure, common tangents AB and CD to the two circle with centres O_1 and O_2 intersect at E. Prove that $AB = CD$.



Ans : [CBSE O.D. 2014]

Since EA and EC are tangents from point E to the circle with centre Q_1

$$EA = EC \quad ...(1)$$

and EB and ED are tangents from point E to the circle with center O_2

$$EB = ED \quad (2)$$

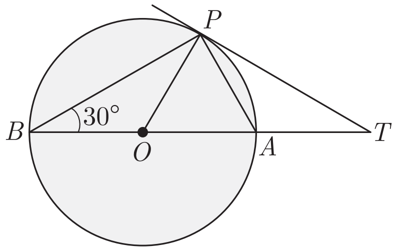
Adding eq (1) and (2) we have

$$EA + BE = CE + ED$$

$$AB = CD$$

Hence Proved

17. In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA when produced at T. If $\angle PBO = 30^0$, what is the measure of $\angle PTA$?



Ans : [Board Term-2, 2012 Set (21)]

Angle inscribed in a semicircle is always right angle.

$$\angle BPA = 90^0$$

Here OB and OP are radius of circle and equal in length, thus angle $\angle OBP$ and $\angle OPB$ are also equal.

Thus $\angle BPO = \angle PBO = 30^0$

Now $\angle POA = \angle OBP + \angle OPB$

$$= 30^\circ + 30^\circ = 60^\circ$$

Thus $\angle POT = \angle POA = 60^\circ$

Since OP is radius and PT is tangent at P, $OP \perp PT$

$$\angle OPT = 90^0$$

Now in right angle ΔOPT,

$$\angle PTO = 180^\circ - (\angle OPT + \angle POT)$$

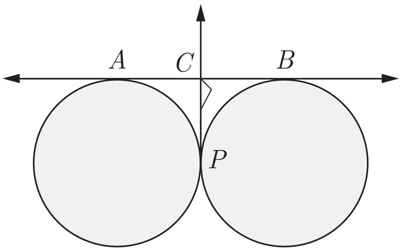
Substituting $\angle OPT = 90^0$ and $\angle POT = 60^\circ$ we have

$$\angle PTO = 180^\circ - (90^\circ + 60^\circ)$$

$$= 180^\circ - 150^\circ = 30^o$$

Thus $\angle PTA = \angle PTO = 30^o$

18. In the given figure, if $BC = 4.5$ cm, find the length of AB.



Ans : [Board Term-2, 2012 Set (59)]

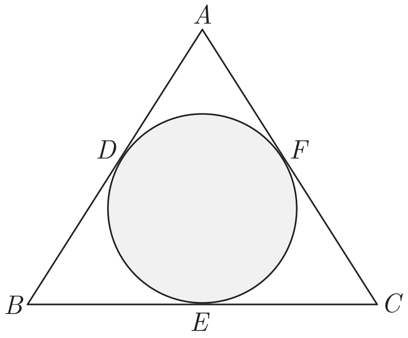
Since length of tangents from an external point to a circle are equal,

$$CB = CP = 4.5 \text{ cm}$$

and $CA = CP$

Now
$$\begin{aligned} AB &= AC + CB \\ &= CP + CP = 2CP \\ &= 2 \times 4.5 = 9 \text{ cm} \end{aligned}$$

19. In the given figure, if $AB = AC$, prove that $BE = CE$.



Ans : [Outside Delhi Set I, II, III 2017]

Since tangents from an external point to a circle are equal,

$$AD = AF \quad (1)$$
$$BD = BE \quad (2)$$
$$CE = CF \quad (3)$$

From $AB = AC$ we have

$$AD + DB = AF + FC$$

or $DB = FC$ $(AD = AF)$

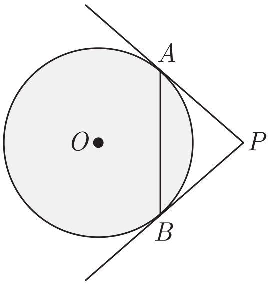
From eq (2) and (3) we have

$BE = EC$ Hence Proved

20. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

Ans : [Outside Delhi Set I, II, III 2017]

As per question we draw figure shown below.



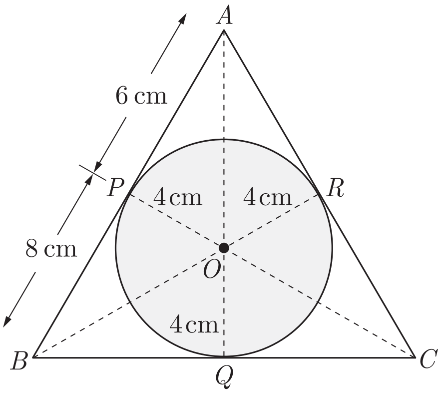
Since length of tangents from an external point to a circle are equal,

$$PA = PB$$

Since angles opposite to equal sides are equal,

$$\angle PAB = \angle PBA$$

21. In Figure the radius of incircle of ΔABC of area 84 cm^2 and the lengths of the segments AP and BP into which side AB is divided by the point of contact are 6 cm and 8 cm Find the lengths of the sides AC and BC.



Ans : [Outside Delhi Compt. Set I, II, III 2017]

Since length of tangents from an external point to a circle are equal,

At A, $AP = AR = 6$ cm (1)

At B $BP = BQ = 8$ cm (2)

At C $CR = CQ = x$ (3)

Perimeter of ΔABC,

$$\begin{aligned} p &= AP + PB + BQ + QC + CR + RA \\ &= 6 + 8 + 8 + x + x + 6 \\ &= 28 + 2x \end{aligned}$$

Now area $\Delta ABC = \frac{1}{2} rp$

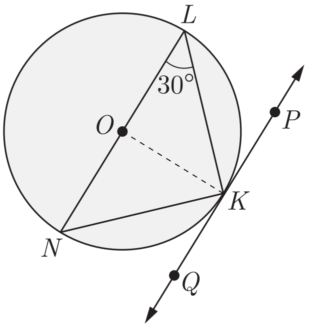
Here $r = 4$ is the radius of circle. Substituting all values we have

$$\begin{aligned} 84 &= \tfrac{1}{2} \times 4 \times (28 + 2x) \\ 84 &= 56 + 4x \\ 21 &= 14 + x \end{aligned}$$

or $x = 7$

Thus
$$\begin{aligned} AC &= AR + RC \\ &= 6 + 7 = 13 \\ BC &= BQ + QC \\ &= 8 + 7 = 15 \text{ cm} \end{aligned}$$

22. In figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at K and $\angle KLN = 30^\circ$, find $\angle PKL$.



Ans : [Outside Delhi Cmpt. Set I, II, III 2017]

Since OK and OL are radius of circle, thus

$$OK = OL$$

Angles opposite to equal sides are equal,

$$\angle OKL = \angle OLK = 30^o$$

Tangent is perpendicular to the end point of radius,

$$\angle OKP = 90^o \qquad \text{(Tangent)}$$

Now $\qquad \angle PKL = \angle OKP - \angle OKL$

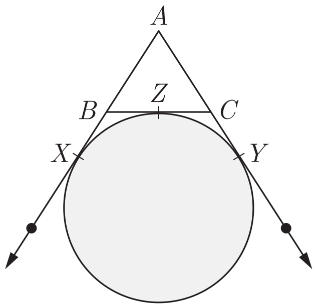
$$= 90^o - 30^o = 60^o$$

SHORT ANSWER TYPE QUESTIONS - II

1. ABC is a triangle. A circle touches sides AB and AC produced and side BC at BC at X, X, Y and Z respectively. Show that $AX = \frac{1}{2}$ perimeter of ΔABC.

Ans : [Board Term-2, 2016, Set-HODM4OL]

As per question we draw figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $\qquad AX = AY \qquad (1)$

At B $\qquad BX = BZ$ cm $\qquad (2)$

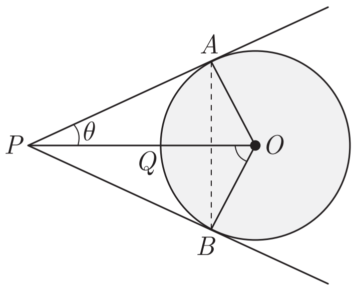
At C $\qquad CY = CZ \qquad (3)$

Perimeter of ΔABC,

$$p = AB + AC + BC$$
$$= (AX - BX) + (AY - CY) + BZ + ZC$$
$$= AX + AY - BX + BZ + ZC - CY$$
$$= AX + AY = 2AX$$

Thus $\qquad AX = \frac{1}{2}p \qquad$ Hence Proved

2. In the given figure, OP is equal to the diameter of a circle with center O and PA and PB are tangents. Prove that ABP is an equilateral triangle.



Ans : [Board Term-2, 2014]

We redraw the given figure by joining A to B as shown below.

Since OA is radius and PA is tangent at A, $OA \perp AP$. Now in right angle triangle ΔOAP, OP is equal to diameter of circle, thus

$$OP = 2OA$$

$$\frac{OA}{OP} = \frac{1}{2}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Since PO bisect the angle $\angle APB$,

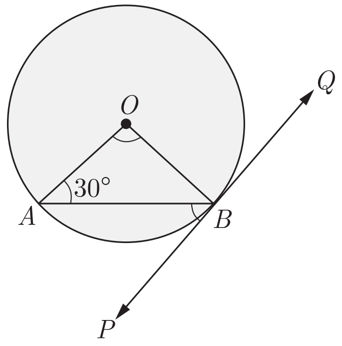
Hence, $\qquad \angle APB = 2 \times 30^\circ = 60^o$

Now, in ΔAPB,

$$AP = AB$$
$$\angle PAB = \angle PBA$$
$$= \frac{180^\circ - 60^\circ}{2} = 60^o$$

Thus ΔAPB is an equilateral triangle.

3. In the figure, PQ is a tangent to a circle with center O. If $\angle OAB = 30^o$, find $\angle ABP$ and $\angle AOB$.



Ans : [Board Term-2 2014]

Here OB is radius and QT is tangent at B, $OB \perp PQ$

$$\angle OBP = 90^o$$

Since the tangent is perpendicular to the end point of radius,

Here OA and OB are radius of circle and equal. Since angles opposite to equal sides are equal,

$$\angle OAB = \angle OBA = 30^o$$

Now $\qquad \angle AOB = 180^o - (30^o + 30^o)$

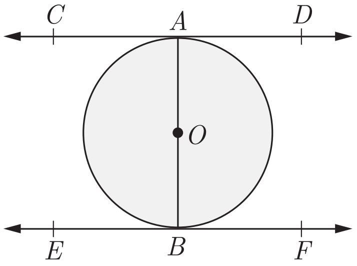
$$= 120^o$$

$$\angle ABP = \angle OBP - \angle OBA$$
$$= 90^o - 30^o = 60^o$$

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Ans : [Foreign Set I, II, III, Delhi CBSE, Term-2, 2014]
[Board Term-2, 2012 Set (12)]
[Delhi Set I, II, III 2017]

Let AB be a diameter of a given circle and let CD and RF be the tangents drawn to the circle at A and B respectively as shown in figure below.



Here $AB \perp CD$ and $AB \perp EF$

Thus $\angle CAB = 90^o$ and $\angle ABF = 90^o$

Hence $\angle CAB = \angle ABF$

and $\angle ABE = \angle BAD$

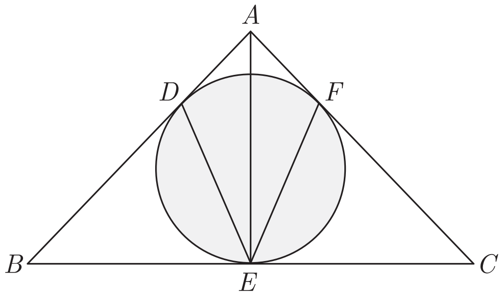
Hence $\angle CAB$ and $\angle ABF$ also $\angle ABE$ and $\angle BAD$ are alternate interior angles.

$CD || EF$ Hence Proved

5. In $\Delta ABD, AB = AC$. If the interior circle of ΔABC touches the sides AB, BC and CA at D, E and F respectively. Prove that E bisects BC.

Ans : [Board Term-2, 2014 Delhi Set, 2012 Set (40)]

As per question we draw figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $AF = AD$ (1)

At B $BE = BD$ (2)

At C $CE = CF$ (3)

Now we have $AB = AC$

$$AD + DB = AF + FC$$

$$BD = FC \qquad (AD = AF)$$

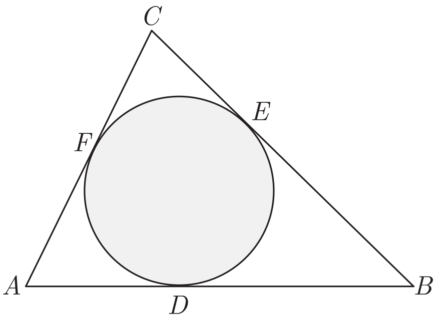
$$BE = EC \qquad (BD = BE, CE = CF)$$

Thus E bisects BC.

6. A circle is inscribed in a ΔABC, with sides AC, AB and BC as 8 cm, 10 cm and 12 cm respectively. Find the length of AD, BE and CF.

Ans : [Board Term-2, 2012 set (21); Delhi 2013]

As per question we draw figure shown below.



We have $AC = 8$ cm

$AB = 10$ cm

and $BC = 12$ cm

Let AF be x. Since length of tangents from an external point to a circle are equal,

At A, $AF = AD = x$ (1)

At B $BE = BD = AB - AD = 10 - x$ (2)

At C $CE = CF = AC - AF = 8 - x$ (3)

Now $BC = BE + EC$

$$12 = 10 - x + 8 - x$$

$$2x = 18 - 12 = 6$$

or $x = 3$

Now $AD = 3$ cm,

$BE = 10 - 3 = 7$ cm

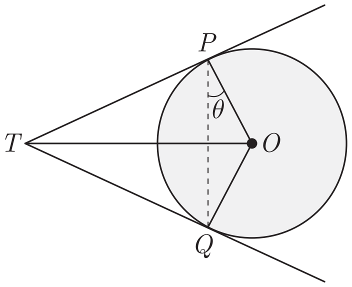
and $CF = 8 - 3 = 5$

7. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that

$$\angle PTO = \angle OPQ$$

Ans : [Delhi Set I, II, III, 2017]
[Delhi Compt Setm I, II, III, 2017]

As per question we draw figure shown below.



Let $\angle TPQ$ be θ. the tangent is perpendicular to the end point of radius,

$$\angle TPO = 90^o$$

Now $\angle TPQ = \angle TPO - \theta$

$$= (90^o - \theta)$$

Since, $TP = TQ$ and opposite angels of equal sides are always equal, we have

$$\angle TQP = (90^o - \theta)$$

Now in ΔTPQ we have

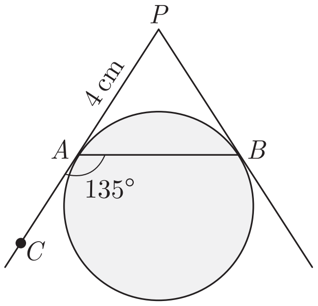
$$\angle TPQ + \angle TQP + \angle PTQ = 180^o$$

$$90^o - \theta + 90^o - \theta + \angle PTQ = 180^o$$

$$\angle PTQ = 180^o - 180^o + 2\theta = 2\theta$$

Hence $\angle PTQ = 2\angle OPQ$.

8. In the given figure, PA and PB are tangents to a circle from an external point P such that $PA = 4cm$ and $\angle BAC = 135^o$. Find the length of chord AB.



Ans : [Outside Delhi Set I, II, III, 2017]

Since length of tangents from an external point to a circle are equal,

$$PA = PB = 4 \text{ cm}$$

Here $\angle PAB$ and $\angle BAC$ are supplementary angles,

$$\angle PAB = 180^\circ - 135^\circ = 45^\circ$$

Angle $\angle ABP$ and $= \angle PAB = 45^\circ$ opposite angles of equal sides, thus

$$\angle ABP = \angle PAB = 45^\circ$$

Inb triangle ΔAPB we have

$$\angle APB = 180^\circ - \angle ABP - \angle BAP$$

$$= 180^\circ - 45^\circ - 45^\circ = 90^\circ$$

Thus ΔAPB is a isosceles right angled triangle

Now $$AB^2 = AP^2 + BP^2 = 2AP^2$$

$$= 2 \times 4^2 = 32$$

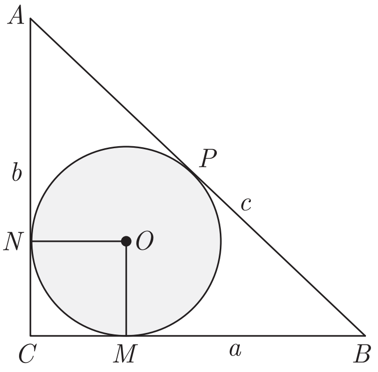
Hence $$AB = \sqrt{32} = 4\sqrt{2} \text{ cm}$$

LONG ANSWER TYPE QUESTIONS

1. a, b and c are the sides of a right triangle, where c is the hypotenuse. A circle, of radius r, touches the sides of the triangle. Prove that $r = \frac{a+b-c}{2}$.

Ans : [CBSE S.A.2 2016 Set HODM4OL]

As per question we draw figure shown below.



Let the circle touches CB at M, CA at N and AB at P.

Now $OM \perp CB$ and $ON \perp AC$ because radius is always perpendicular to tangent

OM and ON are radius of circle, thus

$$OM = ON$$

CM and CN are tangent from C, thus

$$CM = CN$$

Therefore $OMCN$ is a square. Let

Let $OM = r = CM = CN = ON$

Since length of tangents from an external point to a circle are equal,

$$AN = AP, CN = CM, BM = BP$$

$$AN = AP$$

$$AC - CN = AB - BP$$

$$b - r = c - BM$$

$$b - r = c - (a - r)$$

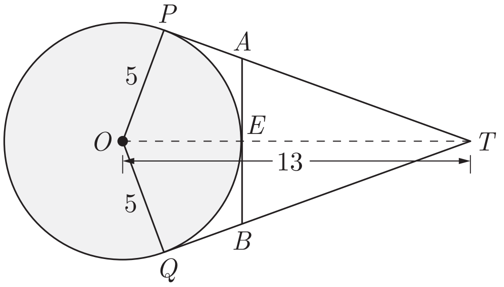
$$b - r = c - a + r$$

$$2r = a + b - c$$

$$r = \frac{a+b-c}{2}$$

Hence Proved.

2. In figure O is the centre of a circle of radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Ans : [Delhi Set I, II, III, 2016]

Here ΔOPT is right angled triangle because PT is tangent on radius OP.

Thus $$PT = \sqrt{13^2 - 5^2}$$
$$= \sqrt{169 - 25} = 12 \text{ cm}$$

and $$TE = OT - OE$$
$$= 13 - 5 = 8 \text{ cm}$$

Since length of tangents from an external point to a circle are equal,

Let $$PA = AE = x$$

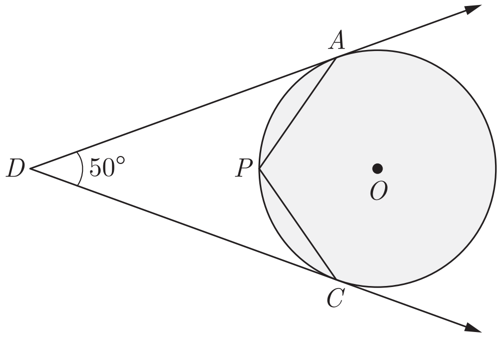
Here ΔAET is right angled triangle because AB is tangent on radius OE.

In ΔAET, $$TA^2 = TE^2 + EA^2$$
$$(TP - PA)^2 = 8^2 + x^2$$
$$(12 - x)^2 = 64 + x^2$$
$$144 - 24x + x^2 = 64 + x^2$$
$$24x = 144 - 64 = 80$$

or, $$x = 3.3 \text{ cm.}$$

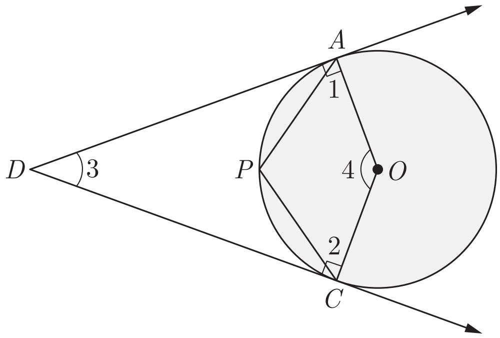
Thus $$AB = 2 \times x = 2 \times 3.3 = 6.6 \text{ cm.}$$

3. In the given figure, O is the centre of the circle. Determine $\angle APC$, if DA and DC are tangents and $\angle ADC = 50°$.



Ans : [Board Term-2, 2015]

We redraw the given figure by joining A and C to O as shown below.



Since DA and DC are tangents from point D to the circle with centre O, and radius is always perpendicular to tangent, thus

$$\angle DAO = \angle DCO = 90°$$

and

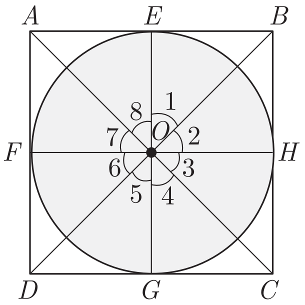
$$\angle ADC + \angle DAO + \angle DCO + \angle AOC = 360°$$
$$50° + 90° + 90° + \angle AOC = 360°$$
$$230° + \angle AOC = 360°$$
$$\angle AOC = 360° - 230° = 130°$$

Now $$\text{Reflex } \angle AOC = 360° - 130° = 230°$$
$$\angle APC = \frac{1}{2} \text{ reflex } \angle AOC$$
(angle subtended at centre...)
$$\angle APC = \frac{1}{2} \times 230° = 115°$$

4. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans : [Foreign Set I, II, III 2017][CBSE O. D. 2014]

A circle centre O is inscribed in a quadrilateral $ABCD$ as shown in figure given below.



Since OE and OF are radius of circle

$$OE = OF \quad \text{(radii of circle)}$$

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $$\angle OEA = \angle OFA = 90°$$

Now in ΔAEO and ΔAFO

$$OE = OF$$
$$\angle OEA = \angle OFA = 90°$$
$$OA = OA \quad \text{(Common side)}$$

Thus $$\Delta AEO \cong \Delta AFO \quad \text{(SAS congruency)}$$
$$\angle 7 = \angle 8$$

Similarly, $$\angle 1 = \angle 2$$
$$\angle 3 = \angle 4$$
$$\angle 5 = \angle 6$$

Since angle around a point is 360°,

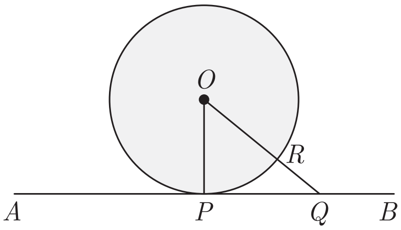
$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$$
$$2\angle 1 + 2\angle 8 + 2\angle 4 + 2\angle 5 = 360°$$
$$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180°$$
$$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$$
$$\angle AOB + \angle COD = 180°$$

Hence Proved.

5. Prove that tangent drawn at any point of a circle perpendicular to the radius through the point contact.

Ans : [Outside Delhi Set II, 2016]

Consider a circle with centre O with tangent AB at point of contact P as shown in figure below



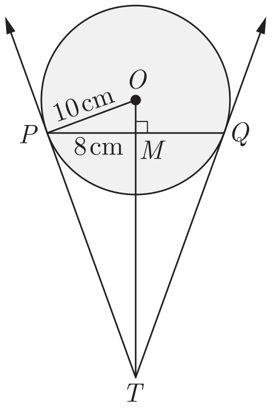
Let Q be point on AB and we join OQ. Suppose it touch the circle at R.

$$\begin{aligned} \text{We} \quad OP &= OR \quad \text{(Radius)} \\ \text{Clearly} \quad OQ &> OR \\ OQ &> OP \end{aligned}$$

Same will be the case with all other points on circle. Hence OP is the smallest line that connect AB and smallest line is perpendicular.

$$\begin{aligned} \text{Thus} \quad OP &\perp AB \\ \text{or,} \quad OP &\perp PQ \quad \text{Hence Proved} \end{aligned}$$

6. In figure, PQ, is a chord of length 16 cm, of a circle of radius 10 cm. the tangents at P and Q intersect at a point T. Find the length of TP.



Ans : [Delhi CBSE, Term-2, 2014]

Here PQ is chord of circle and OM will be perpendicular on it and it bisect PQ. Thus $\Delta\, OMP$ is a right angled triangle.

$$\begin{aligned} \text{We have} \quad OP &= 10 \text{ cm} \quad \text{(Radius)} \\ PM &= 8 \text{ cm} \quad (PQ = 16 \text{ cm}) \\ \text{Now in } \Delta\, OMP, OM &= \sqrt{10^2 - 8^2} \\ &= \sqrt{100 - 64} = \sqrt{36} \\ &= 6 \text{ cm} \end{aligned}$$

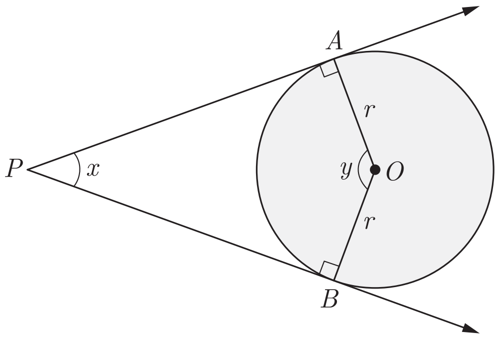
$$\begin{aligned} \text{Now} \quad \angle TPM + \angle MPO &= 90^\circ \\ \text{Also,} \quad \angle TPM + \angle PTM &= 90^\circ \\ \angle MPO &= \angle PTM \\ \angle TMP &= \angle OMP = 90^\circ \\ \Delta\, TMP &\sim \Delta\, PMO (AA) \\ \text{or,} \quad \frac{TP}{PO} &= \frac{MP}{MO} \\ \frac{TP}{10} &= \frac{8}{6} \\ TP &= \frac{80}{6} = \frac{40}{3} \end{aligned}$$

Hence length of TP is $\frac{40}{3}$ cm.

7. Two tangents PA and PB are drawn from an external point P to a circle with centre O, such that $\angle APB = \angle x$ and $\angle AOB = y$. Prove than opposite angles are supplementary.

Ans : [Board Term-2, 2011 (B1)]

As per question we draw figure shown below.



Now $OA \perp AP$ and $OB \perp BP$ because tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

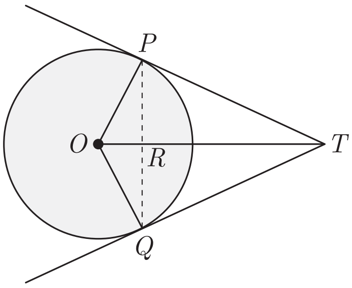
$$\text{Thus} \quad \angle A = \angle B = 90^\circ$$

Since, $AOBP$ is a quadrilateral,

$$\begin{aligned} \text{So,} \quad \angle A + \angle B + x + y &= 360^\circ \\ 90^\circ + 90^\circ + x + y &= 360^\circ \\ 180 + x + y &= 360^\circ \\ x + y &= 180^\circ \end{aligned}$$

Therefore opposite angle are supplementary.

8. In figure PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents drawn at P and Q intersect at T. Find the length of TP.



Ans : [Outside Delhi Compt. Set I, II, III 2017]

Since length of tangents from an external point to a circle are equal,

$$PT = QT$$

Thus ΔTPQ is an isosceles triangle and TO is the angle bisector of $\angle PTQ$.

Thus $OT \perp PQ$ and OT also bisects PQ.

$$\text{Thus} \quad PR = PQ = \tfrac{8}{2} = 4 \text{ cm}$$

Since $\Delta\, OPR$ is right angled isosceles triangle,

$$OR = \sqrt{OP^2 - PR^2}$$

$$= \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$$

$$= 3 \text{ cm}$$

Now, Let $TP = x$ and $TR = y$ then we have

$$x^2 = y^2 + 16 \qquad (1)$$

Also in $\Delta\, OPT$,

$$x^2 + (5)^2 = (y+3)^2 \qquad (2)$$

Solving (1) and (2) we get

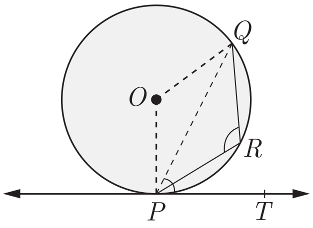
$$y = \frac{16}{3} \text{ and } x = \frac{20}{3}$$

Hence, $TP = \frac{20}{3}$

HOTS QUESTIONS

1. In figure, PQ is a chord of a circle O and PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$.

Ans : [Outside Delhi CBSE Board, 2015 Set I, II, III 2017]



We have $\angle QPT = 60^\circ$

Here $\angle OPT = 90^\circ$ because of tangent at radius.

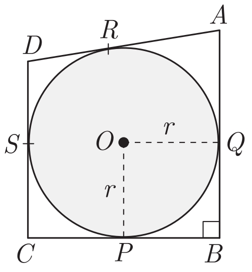
Now
$$\angle OPQ = \angle OQP$$
$$= \angle OPT - \angle QTP$$
$$= 90^\circ - 60^\circ = 30^\circ$$
$$\angle POQ = 180^\circ - (\angle OPQ + \angle OQP)$$
$$= 180^\circ - (30^\circ + 30^\circ)$$
$$= 180^\circ - 60^\circ = 120^\circ$$

Now Reflex $\angle POQ = 360^\circ - 120^\circ = 240^\circ$

$$\angle PRQ = \frac{1}{2} \text{ Reflex } \angle POQ$$

$$= \frac{1}{2} \times 240^\circ = 120^\circ$$

2. In figure, a circle with centre O is inscribed in a quadrilateral $ABCD$ such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29$ cm, $AD = 23$ cm, $\angle B = 90^\circ$ and $DS = 5$ cm, then find the radius of the circle (in cm).



Ans : [Board Term-2, 2013]

Since length of tangents from an external point to a circle are equal,

$$DR = DS = 5 \text{ cm}$$
$$AR = AQ$$
$$BQ = BP$$

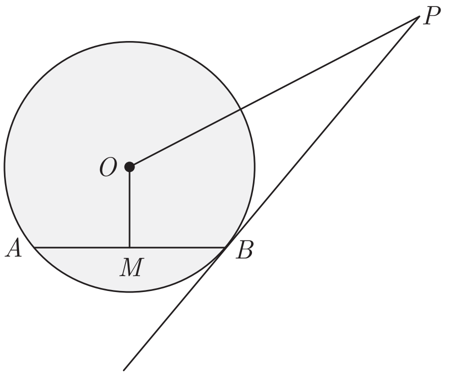
Now
$$AR = AD - DR$$
$$= 23 - 5 = 18 \text{ cm}$$
$$AQ = AQ = 18 \text{ cm}$$
$$QB = AB - AQ$$
$$= 29 - 18 = 11 \text{ cm}$$
$$PB = QB = 11$$

Now $\angle OQB = \angle OPB = 90^\circ$ because radius is always perpendicular to tangent.

Thus $OP = OQ = PB = BQ$

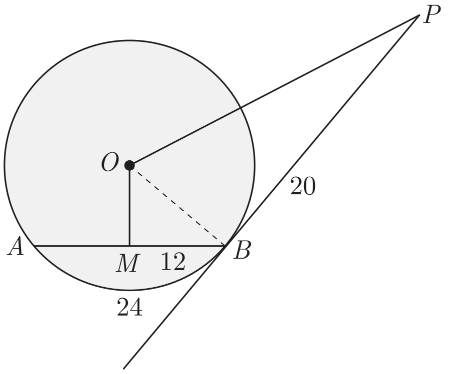
So, $POQB$ is a square. Hence, $r = OP = PB = 11$ cm

3. PB is a tangent to the circle with centre O to $B.AB$ is a chord of length 24 cm at a distance of 5 cm from the centre. It the tangent is length 20 cm, find the length of PO.



Ans : [Delhi Board Term-2, 2015]

We redraw the given figure by joining O to B as shown below.



Here $\Delta\, OMB$ right angled triangle because AB is chord and OM is perpendicular on it.

In right angled triangle $\Delta\, OMB$ we have,

$$OB^2 = OM^2 + MB^2$$
$$= 5^2 + 12^2 = 13^2$$

Thus $OB = 13$

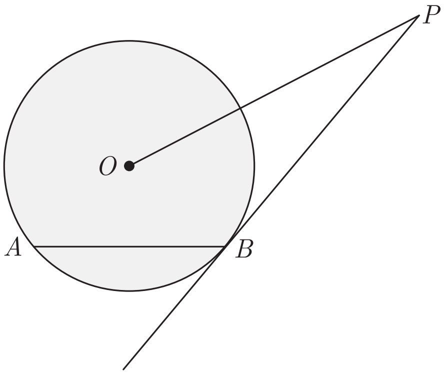
Here $\Delta\, OBP$ right angled triangle because PB is tangent on radius OB.

This in right angled triangle $\Delta\, OBP$ we have,

$$OP^2 = OB^2 + BP^2$$
$$= 13^2 + 20^2$$
$$= 569$$

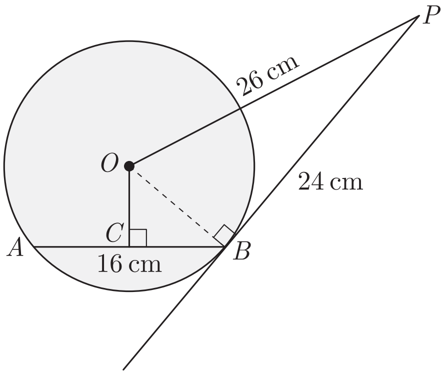
Thus $$OP = \sqrt{569} = 23.85 \text{ cm}$$

4. AB is a chord of circle with centre O. At B, a tangent PB is drawn such that its length is 24 cm. The distanace of P from the centre is 26 cm. If the chord AB is 16 cm, find its distance from the centre.



Ans : [Board Term-2, 2012 Set (40, 2014)]

We redraw the given figure by joining O to B as shown below.



Here we have drawn perpendicular OC on chord AB. Thus Triangle ΔOCB is also right angled triangle,
We have $PB = 24$ cm, $OP = 26$ cm.
Triangle ΔOPB is right angled triangle because PB is tangent at radius OB and $\angle OPB = 90°$.
In right angled ΔOPB, we have

$$OB = \sqrt{OP^2 - BP^2}$$
$$= \sqrt{26^2 - 24^2}$$
$$= \sqrt{676 - 576} = \sqrt{100}$$
$$= 10 \text{ cm}$$

Since perpendicular drawn from the centre to a chord bisect it, we have

$$BC = \frac{1}{2}AB = \frac{16}{2} = 8 \text{ cm}$$

Now in $\Delta\, OBC$, $OC^2 = OB^2 - BC^2$

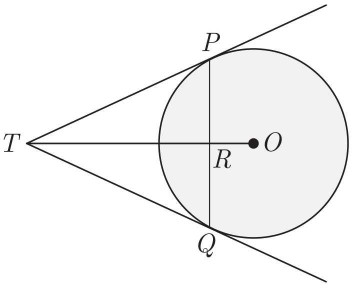
$$= 10^2 - 8^2 = 36$$
$$OC = 6 \text{ cm}$$

Thus distance of the chord from the centre is 6 cm.

5. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the rigth bisector of line segment PQ.

Ans : [Delhi CBSE Term-2, 2015 Set I, II, III]

A circle with centre O. Tangents TP and TQ are drawn from a point T outside a circle as shown in figure below.



Since length of tangents from an external point to a circle are equal,

$$TP = TQ$$

Angle $\angle TPR$ and $\angle TQR$ are opposite angle of equal sides, thus

$$\angle TPR = \angle TQR$$

Now in $\Delta\, PTR$ and $\Delta\, QTR$

$$TP = TQ$$
$$TR = TR \qquad \text{(Common)}$$
$$\angle TPR = \angle TQR$$

Thus $$\Delta\, PTR \cong \Delta\, QTR$$

and $$PR = QR$$

and $$\angle PRT = \angle QRT$$

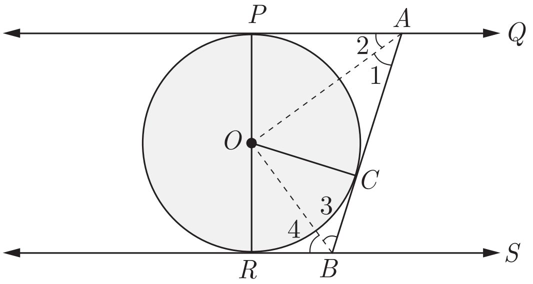
But $\angle PRT + QRT = 180°$ as PQ is line segment,

$$\angle PRT = \angle QRT = 90°$$

6. Therefore TR or OT is the right bisector of line segment PQ. Hence proved. Prove that the intercept of a tangent between a pair of parallel tangents to a circle subtend a right angle at the centre of the circle.

Ans : [Delhi CBSE, Term-2, 2014]
[Board Term-2, 2012 Set (22, 5)]

As per question we draw figure shown below.



Here PQ and RS are two parallel tangents to a circle with centre O.

AB is tangent to a circle at C, intersecting PQ and RS at A and B respectively.

Since $PA \,||\, RS$ and AB is transversal,

$$\angle 1+\angle 2+\angle 3+\angle 4 = 180^\circ$$

By congruency $\angle 1 = \angle 2, \angle 3 = \angle 4$, thus we have

$$\begin{aligned} 2\angle 1+2\angle 3 &= 180^\circ \\ \angle 1+\angle 3 &= 90^\circ \end{aligned}$$

In ΔAOB, by angle sum property of a triangle,

$$\begin{aligned} \angle AOB &= 180^\circ-(\angle 1+\angle 3) \\ &= 180^\circ-90^\circ \end{aligned}$$

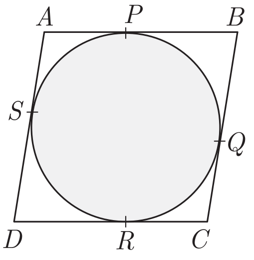
Thus $\angle AOB = 90^\circ$

7. Prove that the parallelogram circumscribing a circle is a rhombus.

Ans : [Delhi CBSE, Term-2, 2014]
[Board Term-2, 2012 Set (1); Delhi 2013]

Let $ABCD$ be the parallelogram.

$$AB = CD, AD = BC \qquad (1)$$



Since length of tangents from an external point to a circle are equal,

At A, $\quad AP = AS \qquad (2)$

At B $\quad BP = BQ \qquad (3)$

At C $\quad CR = CQ \qquad (4)$

At D $\quad DR = DS \qquad (5)$

Adding above 4 equation we have

$$AP+PB+CR+DR = AS+BQ+CQ+DS$$

or, $\quad AB+CD = AD+BC$

From (1) $\quad 2AB = 2AD$

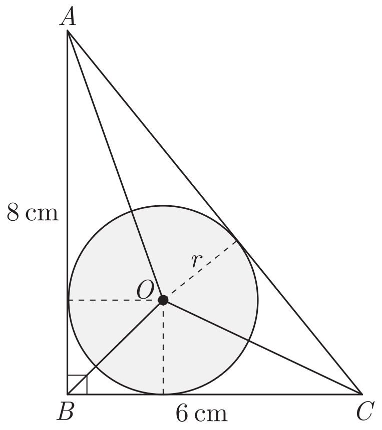
or $\quad AB = AD$

Thus $ABCD$ is a rhombus.

8. ABC is a right triangle in which $\angle B = 90^\circ$. A circle is inscribed in the triangle. It $AB = 8$ cm and $BC = 6$ cm, find the radius r of the circle.

Ans : [Board Term II, 2012 Set (44)]

As per question we draw figure shown below.



Area of triangle ΔABC,

$$\Delta ABC = \frac{1}{2}\times 8\times 6 = 24 \text{ cm}^2$$

and $\quad AC = \sqrt{8^2+6^2} = 10$ cm

Here we have joined AO, BO and CO

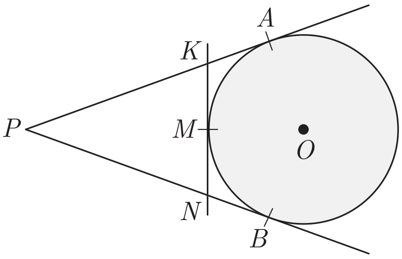
For area of triangle we have

$$\begin{aligned} \Delta ABC &= \Delta OBC+\Delta OCA+\Delta OAB \\ 24 &= \tfrac{1}{2}rBC+\tfrac{1}{2}rAC+\tfrac{1}{2}rAB \\ &= \tfrac{1}{2}r(BC+AC+AB) \\ &= \tfrac{1}{2}r(6+10+8) = 12r \end{aligned}$$

or $\quad 12r = 24$

Thus $r = 2$ cm.

9. In given figure, PA and PB are tangents from a point P to the circle wisth centre O. At the point M, nother tangent to the circle is drawn cutting PA and PB aat K and N. Prove that the perimeter of $\Delta PNK = 2PB$.



Ans : [Board Term-2, 2012 Set (1, 25)]

Since length of tangents from an external point to a circle are equal,

$$\begin{aligned} PA &= PB \\ KM &= KA \\ MN &= BN \end{aligned}$$

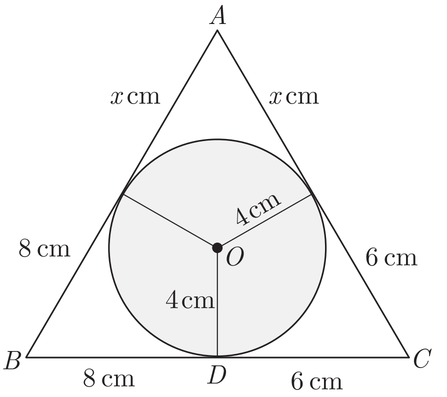
Now $\quad KN = KM+MN$

$$= KA+BN$$

Now perimeter of $\Delta\,PNK$

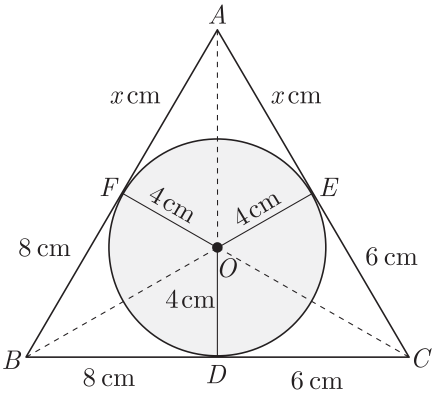
$$\begin{aligned} p &= PN + KN + PK \\ &= PN + BN + KA + PK \\ &= PB + PA \\ &= 2PB \qquad (PA = PB) \end{aligned}$$

10. In the figure, the $\Delta\,ABC$ is drawn to circumscribe a circle of radius 4 cm, such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find AB and AC.



Ans : [Board Term-2, 2012(34); Delhi CBSE Term II, 2014]

We redraw the given circle by joining AO, BO and CO shown in figure below. Let length of AF be x.



Since length of tangents from an external point to a circle are equal,

At A, $\quad AF = AE = x \qquad (2)$

At B $\quad BF = BD = 8$ cm $\qquad (3)$

At C $\quad CD = CE = 6$ cm $\qquad (4)$

Now $\quad AB = x + 8$

$\quad AC = x + 6$

$\quad BC = 8 + 6 = 14$ cm

Perimeter of circle

$$\begin{aligned} p &= AB + BC + CA \\ &= x + 8 + 14 + x + 6 \\ &= 2\,(x + 14) \end{aligned}$$

Semi-perimeter of circle

$$s = \tfrac{1}{2}p = x + 14$$

Area or triangle $\Delta\,ABC$

$$\begin{aligned} \Delta\,ABC &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{48x^2 + 672x} \qquad (1) \end{aligned}$$

Area or triangle $\Delta\,ABC$

$$\begin{aligned} \Delta\,ABC &= \tfrac{1}{2}rp \\ &= \tfrac{1}{2} \times 4 \times 2\,(x + 14) \\ &= 4\,(x + 14) \qquad (2) \end{aligned}$$

From equation (1) and (2) we have

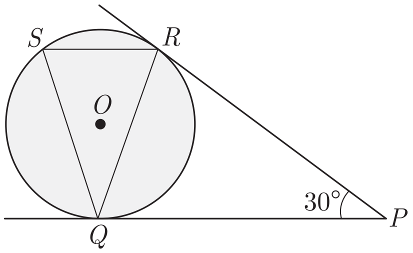
$$\begin{aligned} 48x^2 + 672x &= 16(x + 14)^2 \\ 48x(x + 14) &= 16(x + 14)^2 \\ 3x &= x + 14 \end{aligned}$$

or, $\quad x = 7$

Thus $\quad AC = 6 + 7 = 13$ cm

and $\quad AB\ 8 + 7 = 15$ cm.

11. In the figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.



Ans : [Delhi CBSE Term-2, 2015, (Set I, II, III)]

Since length of tangents from an external point to a circle are equal,

$$PR = PQ$$

Now $$\angle PRQ = \angle PQR = \frac{180^\circ - 30^\circ}{2}$$

$$= \frac{150^\circ}{2} = 75^\circ$$

Since $SR \parallel QP$, $\angle SRQ$ and $\angle RQP$ are alternate angle

$$\angle SRQ = \angle RQP = 75^\circ$$

Thus $\quad SQ = RQ$

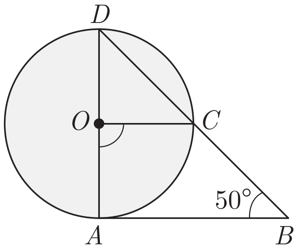
and $\quad \angle RSQ = \angle SRQ = 75^\circ$

In triangle $\Delta\,AQR$

$$\begin{aligned} \angle SQR + \angle QSR + \angle QRS &= 180^\circ \\ \angle SQR + 75^\circ + 75^\circ &= 180^\circ \\ \angle SQR &= 180^\circ - 150^\circ = 30^\circ \end{aligned}$$

Thus $\angle SQR = 30^\circ$.

12. In the given figure, AD is a diameter of a circle with centre O and AB is a tangent at A. C is a point on the circle such that DC produced intersects the tangent at B and $\angle ABC = 50^\circ$. Find $\angle AOC$.



Ans : [Board Term-2, 2015]

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Therefore $\angle A = 90^\circ$

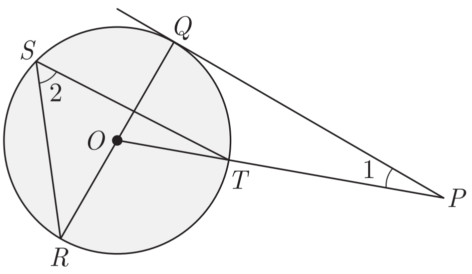
Now in ΔDAB we have

$$\begin{aligned} \angle D + \angle A + \angle B &= 180^\circ \\ \angle D + 90^\circ + 50^\circ &= 180^\circ \\ \angle D &= 40^\circ \end{aligned}$$

Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

$$\begin{aligned} \angle AOC &= 2\angle ADC = 2\angle D \\ &= 2 \times 40^\circ = 80^\circ \end{aligned}$$

13. In figure PQ is a tangent from an external point P to a circle with centre O and OP cuts the circle at T and $\angle QOR$ is a diameter. It $\angle POR = 130^\circ$ and S is a point on the circle, find $\angle 1 + \angle 2$.



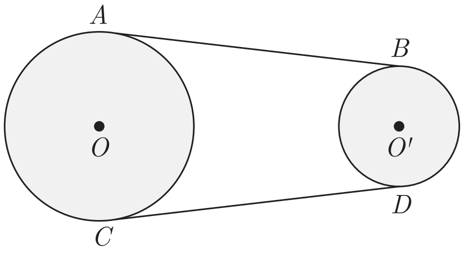
Ans : [Delhi Compt. Set I, II, III 2017]

Here $\angle OQP = 90^\circ$ because radius is always perpendicular to tangent at point of contact.
Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

$$\begin{aligned} \angle 2 &= \frac{1}{2}\angle TOR = \frac{1}{2}\angle POR \\ &= \frac{1}{2} \times 130^\circ = 65^\circ \end{aligned}$$

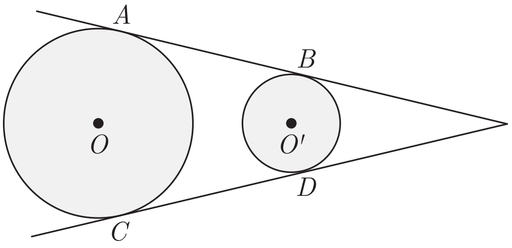
$$\begin{aligned} \text{Now} \qquad \angle POQ &= 180^\circ - 130^\circ = 50^\circ \\ \angle 1 &= 180^\circ - \angle OQP - \angle POQ \\ &= 180^\circ - 90^\circ - 50^\circ = 40^\circ \\ \text{Now} \qquad \angle 2 + \angle 1 &= 65^\circ + 40^\circ = 105^\circ \end{aligned}$$

14. In the figure AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$.



Ans : [Delhi Compt. Set III 2017]

We redraw the given figure by extending AB and BD which intersect at P as shown in figure below

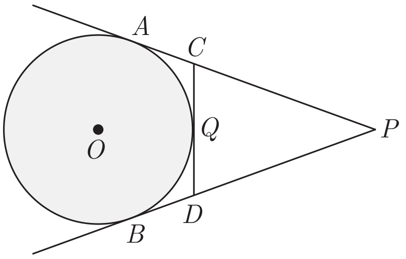


Since length of tangents from an external point to a circle are equal,

$$\begin{aligned} PA &= PC \\ \text{and} \qquad PB &= PD \\ \text{Now,} \qquad PA - PB &= PC - PD \\ AB &= CD \end{aligned}$$

Hence Proved

15. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at $PA = 12$ cm, $QC = QD = 3$ cm, then find $PC + PI$.



Ans : [Delhi Compt. Set I, II, III 2017]

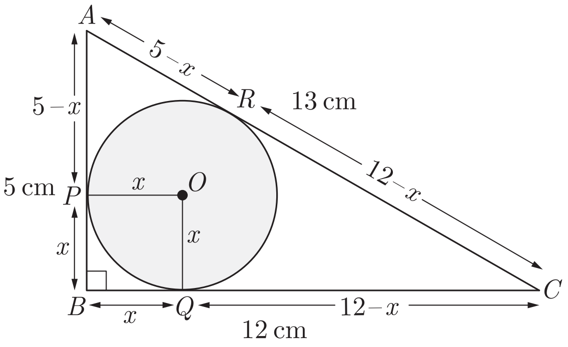
Since length of tangents from an external point to a circle are equal,

$$\begin{aligned} CA &= CQ = 3 \text{ cm} \\ DQ &= DB = 3 \text{ cm} \\ \text{and} \qquad PB &= PA = 12 \text{ cm} \\ PA + PB &= PC + CA + PD + DA \\ PC + PD &= PA - CA + PB - DB \\ &= 12 - 3 + 12 - 3 = 18 \text{ cm} \end{aligned}$$

16. In a right angle $\Delta\, ABC.$, $BC = 12\,cm$ and $AB = 5cm$. Find the radius of the circle inscribed in this triangle.

Ans : [Delhi CBSE Term-2, 2014]

Let the radius of circle be x. As per given in question we draw the figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $AP = AR = 5 - x$ (1)

At B $BP = BQ = x$ (2)

At C $CR = CQ = 12 - x$ (3)

Here, $AB = 5$ cm, $BC = 12$ cm and $\Delta B = 90^o$

Now
$$AC = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \text{ cm}$$

Now
$$AC = AR + RC$$
$$13 = 5 - x + 12 - x$$
$$2x = 17 - 13 = 4$$
$$x = \frac{4}{2} = 2 \text{ cm}$$

Hence, radius of the circle is 2 cm.

CHAPTER 12

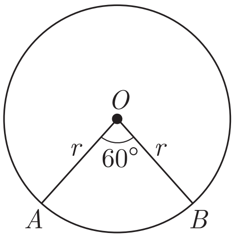
Areas Related to Circles

VERY SHORT ANSWER TYPE QUESTIONS

1. What is the perimeter of the sector with radius 10.5 cm and sector angle 60°.

Ans : [Board Term-2, 2012 Set (40)]

As per question the digram is shown below.



Perimeter of the sector,

$$p = 2r + \frac{2\pi r\theta}{360^\circ}$$

$$= 10.5 \times 2 + 2 \times \frac{22}{7} \times \frac{10.5 \times 60}{360}$$

$$= 21 + 11 = 32 \text{ cm}$$

2. If the circumferences of two concentric circles forming a ring are 88 cm and 66 cm respectively. Find the width of the ring.

Ans : [Delhi 2013]

Circumference of the outer circle $2\pi r_1 = 88$ cm

$$r_1 = \frac{88 \times 7}{22 \times 22} = 14 \text{ cm}$$

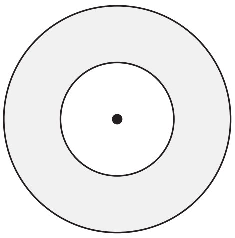
Circumference of the outer circle $2\pi r_2 = 66$ cm

$$r_2 = \frac{66 \times 7}{2 \times 22} = \frac{21}{2} \text{ cm} == 10.5 \text{ cm}$$

Width of the rirg,

$$r_1 - r_2 = 14 - 10.5 \text{ cm} = 35 \text{ cm}$$

3. Two coins of diameter 2 cm and 4 cm respectively are kept one over the other as shown in the figure, find the area of the shaded ring shaped region in square cm.



Ans : [CBSE Board Term-2, 2012]

$$\text{Area of circle} = \pi r^2$$

$$\text{Area of the shaded region} = \pi(2)^2 - \pi(1)^2$$

$$= 4\pi - \pi = 3\pi \text{ sq cm}$$

4. The diameter of two circle with centre A and B are 16 cm and 30 cm respectively. If area of another circle with centre C is equal to the sum of areas of these two circles, then find the circumference of the circle with centre C.

Ans : [Board Term-2, 2012 Set (22)]

Area of circle $= \pi r^2$, Let the radius of circle with centre $C = R$

According to question we have,

$$\pi(8)^2 + \pi(15)^2 = \pi R^2$$

$$64\pi + 225\pi = \pi R^2$$

$$289\pi = \pi R^2$$

$$R^2 = 289 \text{ or } R = 17 \text{ cm}$$

Circumference of circle

$$2\pi r = 2\pi \times 17$$

$$= 34\pi \text{ cm}$$

5. The diameter of a wheel is 1.26 m. What the distance covered in 500 revolutions.

Ans : [Board Term-2, 2012 Set (50)]

Distance covered in 1 revolution is equal to circumference of wheel and that is πd.

Distance covered in 500 revolutions

$$= 500 \times \pi \times 1.26$$

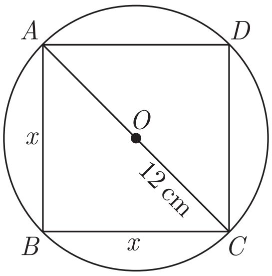
$$= 500 \times \frac{22}{7} \times 1.26$$

$$= 1980 \text{ m.} = 1.98 \text{ km}$$

6. What is the area of the largest square that can be inscribed in a circle of radius 12 cm.?

Ans : [Board Term-2, 2012 Set (31)]

As per question the digram is shown below.



Radius of the circle =12 cm

Diameter of circle = 24 cm

Diagonal of square = 24 cm

Let the side of square= x cm

From Pythagoras theorem we have

$$\begin{aligned} x^2 + x^2 &= (24)^2 \\ 2x^2 &= 24 \times 24 \\ x^2 &= \frac{24 \times 24}{2} = 288 \end{aligned}$$

Thus area of square,

$$x^2 = 288\,\text{cm}^2$$

7. What is the name of a line which intersects a circle at two distinct points?

Ans : [Board Term-2, 2012 (40)]

A line intersecting the circle at two distinct points is called a secant.

8. What is the perimeter of a sector of a circle whose central angle is 90° and radius is 7 cm?

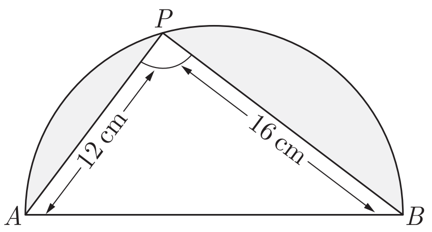
Ans : [Board Term-2, 2012 Set(59)]

As per question the digram is shown below.
Perimeter of the sector,

$$\begin{aligned} p &= 2r + \frac{2\pi r\theta}{360^\circ} \\ &= 2r + 7 + 2 \times \frac{22}{7} \times 7 \times \frac{90}{360} \end{aligned}$$

$$14 + 11 = 25 \text{ cm}$$

9. In the given figure, AB is the diameter where $AP = 12$ cm and $PB = 16$ cm. Taking the value of π as 3, find the perimeter of the shaded region.



Ans : [Board Term-2, 2012 Set (21)]

From Pythagoras theorem we have

$$\begin{aligned} AB &= \sqrt{(16)^2 + (12)^2} \\ &= \sqrt{256 + 144} \\ &= \sqrt{400} = 20cm \end{aligned}$$

Radius of circle $= 10$ cm.

Perimeter of shaded region

$$\begin{aligned} \pi r + AP + PB &= 3 \times 10 + 12 + 16 \\ &= 30 + 12 + 16 = 58 \text{ cm} \end{aligned}$$

10. Find the area of circle that can be inscribed in a square of side 10 cm.

Ans : [Board Term-2, 2012 Set (44)]

Radius of the circle $= \frac{10}{2} = 5$ cm

Area of the circle,

$$\pi r^2 = \pi \times (5)^2 = 25\pi\, cm^2$$

11. A thin wire is in the shape of a circle of radius 77 cm. It is bent into a square. Find the side of the square (Taking, $\pi = \frac{22}{7}$).

Ans : [Board Term-2, 2012 Set (5)]

Let side of square be x cm.

Perimeter of the circle = Perimeter of square

$$\begin{aligned} 2\pi r &= 4x \\ 2 \times \frac{22}{7} \times 77 &= 4x \\ x &= \frac{2 \times 22 \times 11}{4} = 121 \end{aligned}$$

Thus side of the square is 121 cm.

12. What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40 cm and 9 cm?

Ans : [Board Term-2, 2012 Set (34)]

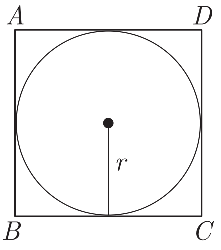
Area of the circle = sum of areas of two circles

$$\begin{aligned} \pi R^2 &= \pi \times (40)^2 + \pi(9)^2 \\ R^2 &= 1600 + 81 \\ R &= \sqrt{1681} = 41 \text{ cm} \end{aligned}$$

Thus diameter of given circle $= 41 \times 2 = 82$ cm

13. Find the area (in cm^2) of the circle that can be inscribed in a square of side 8 cm.

Ans : [board Term-2, 2012 Set (28, 32, 33)]



Side of square = diameter of circle = 8 cm

Radius of circle, $r = \frac{8}{2} = 4$ cm

Area of circle, $\pi r^2 = \pi \times 4 \times 4 = 16\pi$ cm^2

14. If the radius of a circle is doubled, what about its area?

Ans : [Board Term-2, 2012 Set (23)]

Let the radius of the circle be r. Then are will be πr^2

Now the radius is doubled

Area $= \pi(2r)^2 = 4\pi r^2 = 4 \times \pi r^2$

The area will be 4 times the area of the first circle.

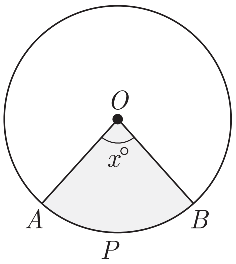
15. If the perimeter and the area of the circle are numerically equal, the find the radius of the circle.

Ans : [Board Term-2, 2012 Set(13)]

Perimeter of the circle = area of the circle.

$$\begin{aligned} 2\pi r &= \pi r^2 \\ r &= 2 \text{ units} \end{aligned}$$

16. In given fig., O is the centre of a circle. If the area of the sector $OAPB$ is $\frac{5}{36}$ times the area of the circle, then find the value of x.



Ans : [Board Term-2, 2012, Set (12)]

Area of sector $OAPB = \frac{5}{36}$ times the area of circle

Thus $$\pi r^2 \times \frac{x}{360} = \frac{5}{36}\pi r^2$$

$$\frac{x}{360} = \frac{5}{36}$$

$$x = 50^{o}$$

17. If circumference of a circle is 44 cm, then what will be the area of the circle?

Ans : [Board Term-2, 2012 (25)]

Circumference of a circle = 44 cm

$$\text{Radius of the circle} = \frac{22}{2 \times \frac{22}{7}} = 7 \text{ cm}$$

$$\text{Area of the circle} = \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\, cm^2$$

18. A steel wire when bent in the form of a square encloses an area or 121 cm2. If the same wire is bent in the form of a circle, then find the circumference of the circle.

Ans : [Board Term-2, 2012 (26)]

$$\text{Area of square} = (\text{side})^2 = 121 \text{ cm}^2$$

$$\text{Side of square} = = \sqrt{121} = 11 \text{ cm}$$

Parameter of square= $4 \times 11 = 44$ cm

Circumference of the circle = Perimeter of the square

= 44 cm

19. Find the radius of a circle whose circumference is equal to the sum of the circumference of two circles of diameter 36 cm and 20 cm

Ans : [Board term-2, 2012, A1]

Circumference of the circle = $2\pi r$

$$2\pi r = 2\pi \times 18 + 2\pi \times 10$$

$$r = 18 + 10$$

$$r = 28 \text{ cm}$$

Hence radius of given circle is 28 cm.

20. Find the diameter of a circle whose area is equal to the sum of areas of two circles os diameter 16 cm and 12 cm.

Ans : [Board Term-2, 2012, (22)]

Let r be the radius of the circle

Area of the circle = Sum of areas of two circles

$$\pi r^2 = \pi \times (8)^2 + \pi (6)^2$$

$$\pi r^2 = \pi (64 + 36)$$

$$r^2 = 100 \text{ or, } r = 10 \text{ cm}$$

Diameter of the circle = $2 \times 10 = 20$ cm.

21. If the circumference of a circle increases from 4π to 8π, then what about its area ?

Ans : [Delhi 2013]

Circumference of the circle = 4π cm or, $r = 2$ cm.

Increased circumference = 8π cm or, $r = 4$ cm.

Area of the 1^{st} circle = $\pi \times (2)^2 = 4\pi$ cm

Area of the new circle = $\pi(4)^2 = 16\pi = 4 \times 4\pi$

Area of the new circle= 4 times the area of first circle.

22. If the radius of the circle is 6 cm and the length of an arc 12 cm. Find the area of the sector.

Ans : [Board Term-2, 2014]

Area of the sector= $\frac{1}{2}$ ×(length of the corresponding arc)× radius

$$= \frac{1}{2} \times l \times r = \frac{1}{2} \times 12 \times 6$$

$$= 36 \text{ cm}^2$$

23. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find area of minor segment. ($Use\ \pi = 3.14$)

Ans : [Board Term-2, 2012 Set (5)]

Radius of circle $r = 10$ cm, central angle = $90°$
Area of minor segment

$$= \frac{1}{2} \times 10^2 \times \left[\frac{3.14 \times 90}{180} - \sin 90°\right]$$

$$= \frac{1}{2} \times 100 \times [1.57 - 1] = 28.5 \text{ cm}^2$$

24. If the perimeter of a semi-circular protractor is 36 cm, find its diameter. $\left(Use\ \pi = \frac{22}{7}\right)$

Ans : [Board Term-2, 2012 Set (59)]

Perimeter $\pi r + 2r = (\pi + 2) r = 36$

or, $\left(\frac{22}{7} + 2\right) r = 36$ or, $r = 7$

Diameter = 14 cm.

SHORT ANSWER TYPE QUESTIONS - I

1. Find the area of the square that can be inscribed in a circle of radius 8 cm.

Ans : [Board Term-2, 2015]

As per question the digram is shown below.

Radius of the circle =8 cm

Diameter of circle = 16 cm

Diagonal of square = 16 cm

Let the side of square = x cm

From Pythagoras theorem we have

$$\begin{aligned} x^2 + x^2 &= (16)^2 \\ 2x^2 &= 16 \times 16 \\ x^2 &= \frac{16 \times 16}{2} = 128 \end{aligned}$$

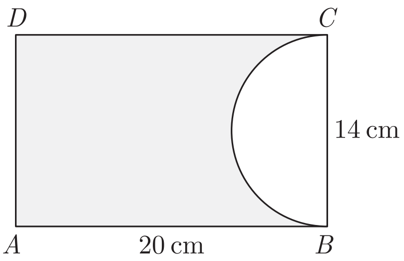
Thus area of square,

$$x^2 = 128 \text{ cm}^2$$

2. A paper is in the from of a rectangle $ABCD$ in which $AB = 20$ cm, $BC = 14$ cm. A semi-circular portion with BC as diameter is cut off. Find the area of the part. Use $\pi = \frac{22}{7}$.

Ans : [Foreign Set I, II, III, 2014] [Board Term-2 2012 Set (40)]

As per question the digram is shown below.



Area of remaining part

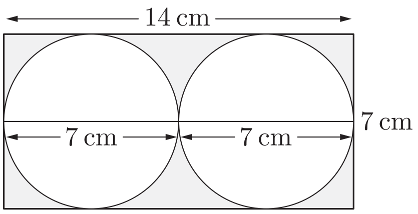
$$\begin{aligned} &= \text{Area of rectangle} - \text{Area of semi-circle} \\ &= 20 \times 14 - \frac{22 \times 7 \times 7}{7 \times 2} \\ &= 280 - 77] \end{aligned}$$

Hence, area of remaining part $= 203$ cm

3. Two circular pieces of equal radii and maximum areas, touching each other are cut out from a rectangular cardboard of dimensions 14 cm × 7 cm. find the area of the remaining cardboard. $\left(Use\ \pi = \frac{22}{7}\right)$

Ans : [Delhi 2013]

As per question the digram is shown below.



Area of the remaining cardboard

= Area of rectangular cardborad $-2 \times$ Area of circle

$$\begin{aligned} &= 14 \times 7 - 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \\ &= 98 - \frac{44}{7} \times \frac{49}{4} = 98 - 77 = 21 \end{aligned}$$

Hence, area of remaining card board $= 21$ cm^2

4. If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = \frac{22}{7}$, find the circumference (in cm) of the circle.

Ans : [Delhi 2012]

Let r be the radius of the circle

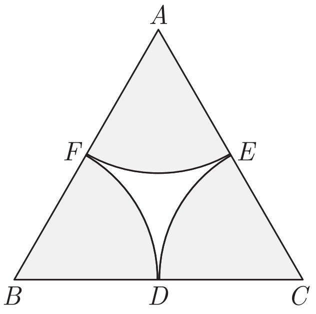
Now, circumference − radius $= 37$

$$\begin{aligned} 2\pi r - r &= 37 \\ 2 \times \frac{22}{7} r - r &= 37 \\ r\left(\frac{22-7}{7}\right) &= 37 \\ r \times \frac{37}{7} &= 37 \\ r &= \frac{37 \times 7}{37} = 7 \text{ cm} \end{aligned}$$

Circumference of the circle,

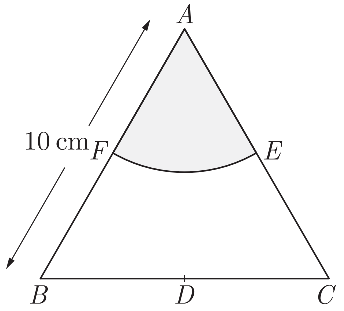
$$\begin{aligned} 2\pi r &= 2 \times \frac{22}{7} \times 7 \\ &= 44 \text{ cm.} \end{aligned}$$

5. In fig., arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm, to intersect the side BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region. (Use $\pi = 3.14$).



Ans : [Board Term-2, 2011 Set (34)]

We have redrawn figure as shown below.



Since ΔABC is an equilateral triangle

$$\angle A = \angle B = \angle C = 60^\circ$$

$$\begin{aligned} \text{Area of sector}, AFEA &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{60}{360} \times \pi(5)^2 = \frac{25}{6}\pi \text{ cm}^2 \end{aligned}$$

Here areas of all three sectors are equal.

Thus total area of shaded region

$$= 3\left(\frac{25}{6}\pi\right) = \frac{25 \times 3.14}{2}$$

$$= 39.25 \text{ cm}^2$$

6. If the perimeter of a protractor is 72 cm, calculate its are. Use $\pi = \frac{22}{7}$.

Ans : [Board Term-2, 2012 Set (22)]

Perimeter of semi-circle

$$\begin{aligned} \pi r + 2r &= 72 \text{ cm} \\ (\pi + 2)r &= 72 \text{ cm} \\ \left[\frac{22}{7} + 2\right] &= 72 \text{ cm} \\ r\left[\frac{22+14}{7}\right] &= 72 \text{ cm} \\ \frac{36}{7}r &= 72 \\ r &= 14 \text{ cm} \end{aligned}$$

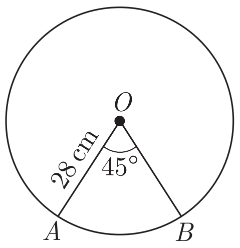
Area of protractor$= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$

$$= 308 \text{ cm}^2$$

7. Find the area of the corresponding major sector of a circle of radius 28 cm and the central angle 45°.

Ans : [Board Term-2, 2015]

As per question statement figure is shown below;



Area of major sector,

$$\begin{aligned} &= \text{area of circle} - \text{area of minor sector} \\ &= \pi r^2\left(1 - \frac{\theta}{360}\right) \\ &= \frac{22}{7} \times 28 \times 28\left(1 - \frac{45}{360}\right) \\ &= \frac{22}{7} \times 28 \times 28 \times \frac{7}{8} \\ &= 2156 \text{ cm}^2 \end{aligned}$$

8. The diameters of the front and rear wheels of a tractor are 80 cm and 200 cm respectively. Find the number of revolutions of rear wheel to cover the distance which the front wheel covers in 800 revolutions.

Ans : [Delhi 2013]

Circumference of front wheel

$$\pi d = \frac{22}{7} \times 80 = \frac{1760}{7} \text{ cm}$$

Distance covered by front wheel in 800 revolutions

$$= \frac{1760}{7} \times 800$$

Circumference of rear wheel

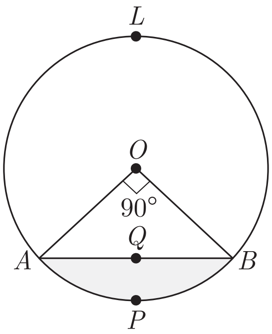
$$= \frac{22}{7} \times 200 = \frac{4400}{7} \text{ cm}$$

No. of revolutions made by rear wheel

$$= \frac{\frac{1760}{7} \times 800}{\frac{4400}{7}} = \frac{1760 \times 800}{4400} = 320$$

SHORT ANSWER TYPE QUESTIONS - II

1. In the given figure, a chord AB of the circle with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment $AQBP$. Hence find the area of major segment $A\angle LBQA$. (Use $\pi = 3.14$)



Ans : [Foreign Set I, II, III, 2016]

Area of sector $OAPB$,

$$= \frac{90}{360}\pi(10)^2 = 25\pi \text{ cm}^2$$

Area of ΔAOB,

$$= \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$$

Area of minor segment $AQBP$,

$$\begin{aligned} &= (25\pi - 50) \text{ cm}^2 \\ &= 25 \times 3.14 - 50 \\ &= 78.5 - 50 \\ &= 28.5 cm^2 \text{ cm}^2 \end{aligned}$$

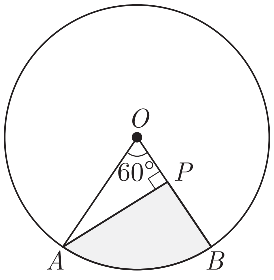
Also area of circle $= \pi(10)^2$

$$= 3.14 \times 100 = 314 \text{ cm}^2$$

Area of major segment $ALBQA = 314 - 28.5$

$$= 285.5 \text{ cm}^2$$

2. In the given figure, AOB is a sector of angle 60° of a circle with centre O and radius 17 cm. If $AP \perp OB$ and $AP = 15$ cm, find the area of the shaded region.



Ans : [CBSE S.A.2 2016 Set-HODM4OL]

Here $OA = 17$ cm $AP = 15$ cm and ΔOPA is right triangle

Using Pythagoras theorem, we have

$$OP = \sqrt{17^2 - 15^2} = 8 \text{ cm}$$

Area of the shaded region

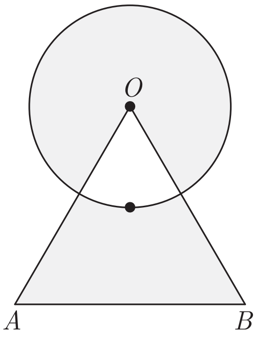
$= $ Area of the sector $\Delta\, OAB$ – Area of ΔOPA

$$= \frac{60}{360} \times \pi r^2 - \frac{1}{2} \times b \times h$$

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 17 \times 17 - \frac{1}{2} \times 8 \times 15$$

$$= 151.38 - 60 = 91.38\, cm^2$$

3. Find the area of shaded region shown in the given figure where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Ans : [Board Sample Paper 2016], [Foreign Set I, II, III, 2016]

Since OAB is an an equilateral triangle,we have

$$\angle AOB = 60^\circ$$

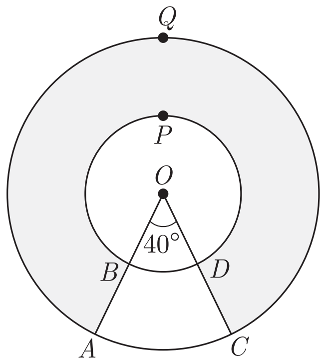
Area of shaded region $=$ Area of major sector $+$ (Area of $\Delta AOB -$ Area of minor sector)

$$= \frac{300}{360} \times \frac{22}{7} \times (6)^2 + \left(\frac{\sqrt{3}}{4}(12)^2 - \frac{60}{360} \times \frac{22}{7} \times 6^2\right)$$

$$= \frac{660}{7} + 36\sqrt{3} - \frac{132}{7}$$

$$= 36\sqrt{3} + \frac{528}{7} \text{ cm}^2$$

4. In the given figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40^\circ$. Use $\pi = \frac{22}{7}$.



Ans : [O.D. Set I, II, III, 2016]

Radii of two concentric circle $= 7$ cm and 14 cm

Angle $\angle AOC = 40^\circ$,

Angle $\angle AOC = 360^\circ - 40^\circ = 320^\circ$

Area of shaded region

$$= \frac{\theta}{360}\pi[R^2 - r^2]$$

$$= \frac{320}{360} \times \frac{22}{7}[14^2 - 7^2]$$

$$= \frac{8}{9} \times 22 \times (14 \times 2 - 7)$$

$$= \frac{8}{9} \times 22 \times 21 = \frac{8}{3} \times 22 \times 7$$

$$= \frac{8 \times 154}{3} \text{ cm}^2$$

Required area $\quad = \frac{1232}{3}$ cm^2

$$= 410.67 \text{ cm}^2$$

5. Find the area of minor segment of a circle of radius 14 cm, when its centre angle is 60°. Also find the area of corresponding major segment. Use $\pi = \frac{22}{7}$.

Ans : [Outside Delhi Set I, II, III, 2015]

Here, $r = 14$ cm, $\theta = 60^\circ$

Area of minor segment $= \pi r^2 \frac{\theta}{360} - \frac{1}{2} r^2 \sin\theta$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{60}{360^\circ} - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2}$$

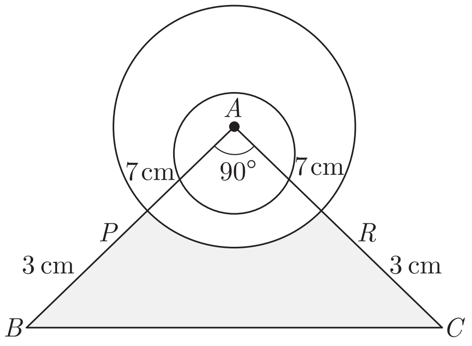
$$= \left(\frac{308}{3} - 49\sqrt{3}\right) = 17.9 \text{ cm}^2 \text{ approx.}$$

Area of major segment $= \pi r^2 - \left(\frac{308}{3} - 49\sqrt{3}\right)$

$$= \frac{1540}{3} + 49\sqrt{3} = 598.10$$

$$= 598 \text{ cm}^2 \text{ approx.}$$

6. A momento is made as shown in the figure. Its base $PBCR$ is silver plate from the Front side. Find the area which is silver plated. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2,2015]

From the given figure

Area of right-angled ΔABC

$$= \frac{1}{2} \times 10 \times 10 = 50$$

Area of quadrant APR of the circle of radii 7 cm

$$= \frac{1}{4} \times \pi \times (7)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 49 = 38.5 \text{ cm}^2$$

Area of base $PBCR$

$$= \text{Area of } \Delta ABC - \text{ Area of quadrant } APR$$
$$= 50 - 38.5 = 11.5 \text{ cm}^2$$

7. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle. Use $\pi = \frac{22}{7}$.

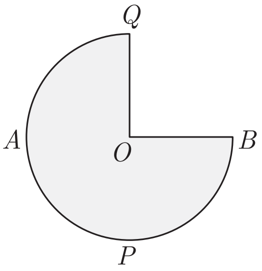
Ans : [Board Term-2, 2015]

Let radius of the circle be r cm

$$\text{Diameter} = 2r \text{ cm}$$
$$\text{Circumference} = 2\pi r$$
$$\text{Circumference} = \text{Diameter}+16.8$$
$$2\pi r = 2r + 16.8$$
$$2\left(\frac{22}{7}\right)r = 2r + 16.8$$
$$\frac{44}{7}r = 2r + 16.8$$
$$44r = 14r + 16.8 \times 7$$
$$30r = 177.6$$
$$r = \frac{117.6}{30} = 3.92$$

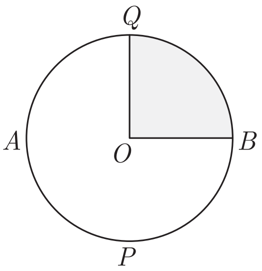
Thus $r = 3.92$ cm

8. In fig., APB and AQP are semi-circle, and $AO = OB$. If the perimeter of the figure is 47 cm, find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Delhi CBSE Board, 2015, Set I, II, III]

We have redrawn the given figure as shown below;



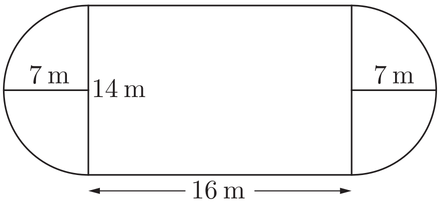
Let r be the radius of given circle
Perimeter of given figure $= 47$ cm
Perimeter of full circle$-$ perimeter of $\left(\frac{1}{4}\right)^{th}$ circle

$$= 47 + 2r$$
$$2\pi r - \frac{1}{4}(2\pi r) + 2r = 47$$
$$\frac{3\pi r}{2} + 2r = 47$$
$$r\left(\frac{3}{2} \times \frac{22}{7} + 2\right) = 47$$
$$r\left(\frac{33}{7} + 2\right) = 47$$
$$r = \frac{47 \times 7}{47} = 7 \text{ cm}$$

Now, area of shaded region

$$A = \text{area of circle } - \frac{1}{4} \text{ area of circle}$$
$$= \frac{3}{4} \text{ area of circle}$$
$$= \frac{3}{4} \times \pi r^2 = \frac{3}{4} \times \frac{22}{7} \times 7 \times 7$$
$$= \frac{3}{2} \times 77 = 115.5 \text{ cm}^2$$

9. Find the area of the adjoining diagram.



Ans : [Board Term-2, 2014]

Required area,

$= \text{area of two semi-circles of same radii} + \text{area of rectangle}$

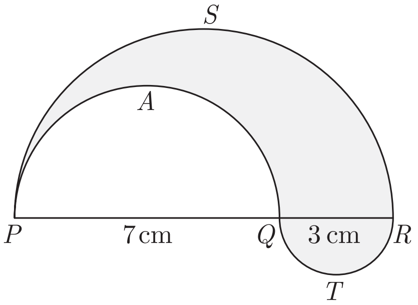
$= \text{area of one circle} + \text{area of rectangle}$

$= \pi r^2 + (l \times b)$

(where r is radius of circle and l and b are length and breadth of rectangle)

$$= \frac{22}{7} \times 7 \times 7 + (16 \times 14)$$
$$= 154 + 224 = 378 \, m^2$$

10. In the fig., PSR, RTQ and PAQ are three semi-circles of diameters 10 cm, 3 cm and 7 cm region. Use $\pi = \frac{22}{7}$.



Ans : [Delhi CBSE, Term II 2014]

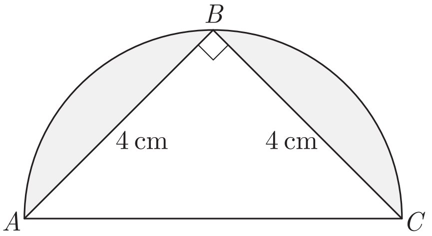
Perimeter of shaded region $=$ Perimeter of semi-circles $PSR + RTQ + PAQ$
Perimeter of shaded region

$$= \pi(5) + \pi(1.5) + \pi(3.5)$$
$$= \pi(10)$$

$$= \frac{22}{7} \times 10 = \frac{22}{7}$$

Perimeter of shaded region $= 31.4$ cm. (approx)

11. In the figure, ΔABC is in the semi-circle, find the area of the shaded region given that $AB = BC = 4$ cm.(Use $\pi = 3.14$)



Ans : [Board Term-2, 2014]

As ΔABC is a triangle in semi-circle

$$AC = \sqrt{4^2 + 4^2} = 4\sqrt{2} \text{ cm}$$

Radius of circle $\frac{4\sqrt{2}}{2} = 2\sqrt{2}$ cm

Area of shaded portion,

$$= \text{Area of the semi-circle} - (\text{Area of } \Delta ABC)$$

$$= \left\{\frac{1}{2}\pi \times (2\sqrt{2})^2\right\} - \left\{\frac{1}{2} \times 4 \times 4\right\}$$

$$= \left\{\frac{1}{2} \times 3.14 \times 8\right\} - 8$$

$$= 12.56 - 8 = 4.56 \text{ cm}^2$$

12. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of sector formed by the arc.

Ans : [Delhi Set Compt. Set-I, II, III 2017]

We have $r = 21$ cm and $\theta = 60^\circ$

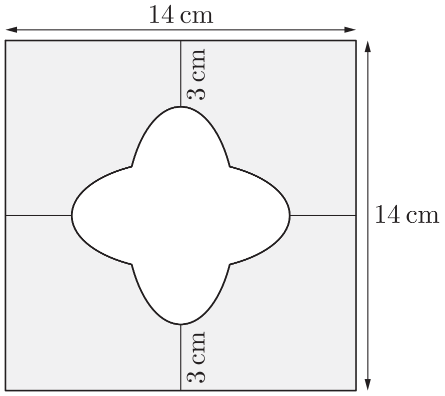
$$\text{Area formed the sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$$

$$= \frac{1}{6} \times 22 \times 3 \times 21$$

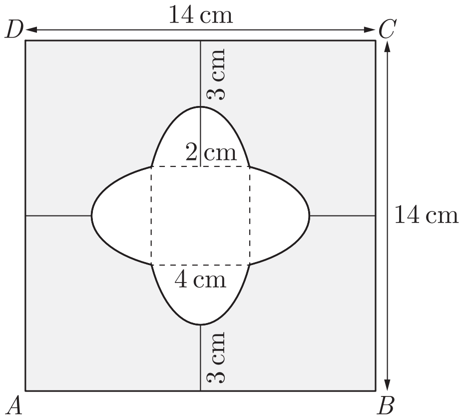
$$= 11 \times 21 = 231 \text{ cm}^2$$

13. In fig., find the area of the shaded region [$\pi = 3.14$]



Ans : [Delhi Set I, II, III 2015][Board Term-2, 2011 Set-B1]

We have redrawn the given figure as shown below;



Area of square $ABCD$

$$= 14 \times 14 = 196 \text{ cm}^2$$

Radius of the semi-circle formed inside $= 2$ cm

Area of 4 semi circle $= 4 \times \frac{1}{2}\pi r^2$

$$= 2 \times 3.14 \times 2 \times 2 = 25.12 \text{ cm}^2$$

Length of the side of square formed inside the semi-circle $= 4$ cm.

Area of the square $= 4 \times 4 = 16$ cm^2

Area of the shaded region,

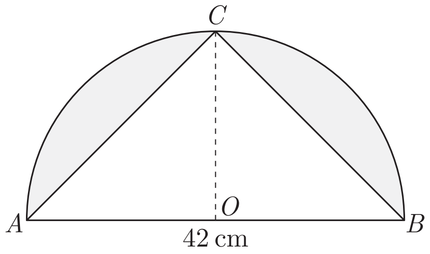
$$= \text{area of square } ABCD$$

$$- (\text{Area of 4 semi-circle} + \text{Area of square})$$

$$= 196 - (25.12 + 16)$$

$$= 196 - 41.12 = 154.88 \text{ cm}^2$$

14. In the figure, ΔACB is in the semi-circle. Find the area of shaded region given that $AB = 42$ cm.



Ans : [Board Term-2, 2014]

$$\text{Base of triangle} = \text{diameter of semicircle}$$

$$= 42 \text{ cm}$$

$$\text{and its heigh} = \text{radius of semicircle}$$

$$= \frac{42}{2} = 21 \text{ cm}$$

Area of shaded portion,

$$= \text{Area of semicircle} - \text{area of } \Delta ABC$$

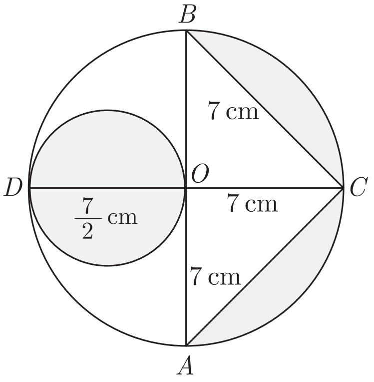
$$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times \frac{22}{7} \times 21 \times 21 - \frac{1}{2} \times 42 \times 21$$

$$= 693 - 441 = 252$$

Hence, the area of shaded portion $= 252 \text{ cm}^2$

15. AB and CD are two diameters of a circle perpendicular to each other and OD is the diameter of the smaller circle. If $OA = 7$ cm, find the area of the shaded region.



Ans : [Board Term-2, 2012 Set (13)]

Area of a circle with DO as diameter

$$\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{ sq.cm}$$

Area of semi-circle with AB as diameter

$$\frac{\pi r^2}{2} = \frac{22 \times 7 \times 7}{7 \times 2} = 77 \text{ sq.cm}$$

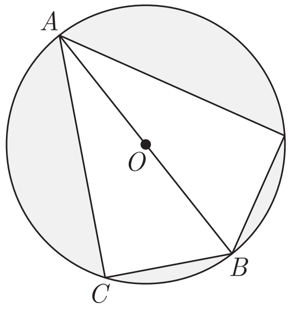
Area of ΔABC $= \frac{1}{2} \times 14 \times 7 = 49$ sq.cm

Area of shaded region

= Area of circle +Area of semi-circle −Area of ΔABC

$$= \frac{77}{2} + 77 - 49 = 66.5 \text{ cm}^2$$

16. Find the area of the shaded region in figure, if $BC = BD = 8$ cm, $AC = AD = 15$ cm and O is the centre of the circle. (Take $\pi = 3.14$)



Ans : [Board Term-2, 2012 Set (34)]

Since $\angle ADB$ and $\angle ACB$ angle in a semicircle,

$$\angle ADB = \angle ACB = 90^\circ$$

Since $\Delta ADB \cong \Delta ACB$

Thus $\text{ar}\Delta ADB = \text{ar}\Delta ACB$

$$= \frac{1}{2} \times 15 \times 8 = 60 \text{ cm}^2$$

and ar $\Delta ADB + ar\Delta ACB$

$$= 2 \times 60 = 120 \text{ cm}^2$$

Now in ΔABC $AB = \sqrt{AC^2 + BC^2}$

$$= \sqrt{15^2 + 8^2} = \sqrt{225 + 64}$$

$$= 17 \text{ cm}$$

Area of circle $\pi r^2 = \frac{22}{7} \times \frac{17}{2} \times \frac{17}{2}$

$$= 226.87 \text{ cm}^2$$

Area of shaded portion,

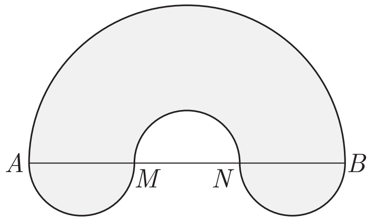
= area of circle−area of sum of ΔACB and ΔADB.

$= 226.87 - 120 = 106.87 \text{ cm}^2$

Hence, area of shaded region

$$= 106.87 \text{ cm}^2$$

17. In the given figure, AB is the diameter of the largest semi-circle. $AB = 21$ cm, $AM = MN = NB$. Semi-circle are drawn with AM, MN and NB as shown. Using $\pi = \frac{22}{7}$, calculate the area of the shaded region.



Ans : [Board Term-2, 2012 Set (21)]

Area of semi-circle with diameter 21 cm,

$$A = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{21}{7} \times \frac{21}{7} = \frac{693}{4} \text{ cm}^2$$

Here $AM = MN = NB = \frac{21}{3} = 7$ cm

Thus radii of smaller semi circle $= \frac{7}{2}$ cm

Area of semi-circle with diameter AM,

= Area of semi-circle with diameter MN

= Area of semi- circle with diameter NB

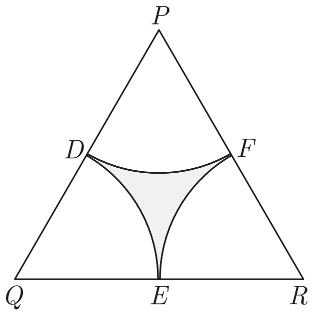
$$= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{4} \text{ cm}^2$$

Area of shaded region

= Area largest semicircle + smallest semicircle

$$= \frac{693}{4} + \frac{77}{4} = \frac{770}{4} = 192.5 \, cm^2$$

18. In the given figure, ΔPQR is an equilateral triangle of side 8 cm and D, E, F are centres of circular arcs, each of radius 4 cm. Find the area of shaded region. (Use $\pi = 3.14$) and $\sqrt{3} = 1.732$



Ans : [Board Term-2, 2012, Set (28)]

Area of shaded region

$$= \text{Area of } \Delta PQR - 3(\text{area of sector})$$

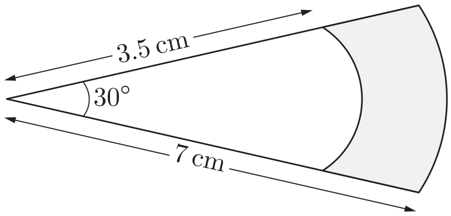
$$= \frac{\sqrt{3}}{4}(side)^2 - 3\left[\frac{\theta}{360^\circ} \times \pi r^2\right]$$

$$= \frac{\sqrt{3}}{4} \times 8 \times 8 - 3\left[\frac{60}{360^\circ} \times 3.14 \times 4 \times 4\right]$$

$$= 16\sqrt{3} - 3.14 \times 8 = 16 \times 1.732 - 25.12$$

$$= 27.712 - 25.12 = 2.59\, cm^2 \text{ cm}^2$$

19. In fig., sectors of two concentric circles of radii 7 cm and 3.5 cm are given. Find the area of shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2, 2012, Set B1]

Area of shaded region,

$$= \pi[R^2 - r^2]\frac{\theta}{360^\circ}$$

$$= \frac{22}{7}[7^2 - (3.5)^2]\frac{30^\circ}{360^\circ}$$

$$= \frac{22}{7}(7 + 3.5)(7 - 3.5) \times \frac{1}{12}$$

$$= \frac{22}{7} \times 10.5 \times 3.5 \times \frac{1}{12}$$

$$= 22 \times \frac{5}{10} \times \frac{35}{10} \times \frac{1}{4} = \frac{77}{8} = 9.62 \text{ cm}^2$$

20. A wire when bent in the form of an equilateral triangle encloses an area of $121\sqrt{3}$ cm2. If the wire is bent in the form of a circle, find the area enclosed by the circle. Use $\pi = \frac{22}{7}$.

Ans : [Outside Delhi Set-I, II, III 2017]

Let l be length of wire. If it is bent in the form of an equilateral triangle, side of triangle will be $\frac{l}{3}$

Area enclosed by the triangle,

$$\frac{\sqrt{3}}{4} \times \left(\frac{l}{3}\right)^2 = 121\sqrt{3}$$

$$\frac{1}{4} \times \left(\frac{l}{3}\right)^2 = 121$$

$$\frac{1}{2} \times \frac{l}{3} = 11$$

$$l = 66 \text{ cm}$$

Same wire is bent in the form of circle. Thus circumference of circle will be 66.

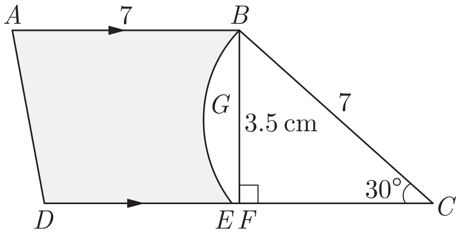
$$2\pi r = 66$$

$$r = \frac{66}{2\pi} = \frac{66}{2 \times \frac{22}{7}} = \frac{21}{2}$$

Area enclosed by the circle

$$\pi r^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{2} = 346.5 \text{ cm}^2$$

21. Adjoining fig, $ABCD$ is a trapezium with $AB || DC$ and $\angle BCD = 30^\circ$. Fig. $BGEC$ is a sector of a circle with centre C and $AB = BC = 7$ cm, $DE = 4$ cm and $BF = 3.5$ cm, then find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Outside Delhi Compt. Set-I, II, III 2017]

We have $AB = 7$ cm

$DE = 4$ cm, and

$BF = 3.5$ cm

Now $BC = DE + EC = 4 + 7 = 11$ cm

Area of Trapezium $ABCD$

$$= \frac{1}{2}(\text{Sum of } || \text{ lines}) \times \text{ distance between}$$

$$= \frac{1}{2}(11 + 7) \times 3.5 = \frac{1}{2} \times 18 \times 3.5$$

$$= 31.5 \text{ cm}^2$$

Area of shaded region

$$= \frac{30^o}{360^o} \times \frac{22}{7} \times 7 \times 7 = \frac{1}{12} \times 22 \times 7$$

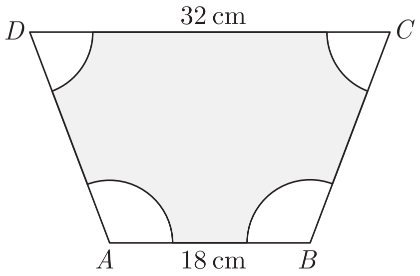
$$= 12.83 \text{ cm}^2$$

$$= \text{Area of trapezium} - \text{ar of sector}$$

$$= 31.5 - 12.83 = 18.67$$

$$= 18.67 \text{ cm}^2$$

22. In the given figure $ABCD$ is a trapezium with $AB || DC, AB = 18$ cm and $DC = 32$ cm and the distance between AB and AC is 14 cm. If arcs of equal radii 7 cm taking A, B, C and D have been drawn, then find the area of the shaded region.



Ans : [Foreign Set-I, II, III 2017]

In trapezium $ABCD, AB = 18$ we have $AB = 18$ cm, $CD = 32$ cm $AB || CD$ and distance between || lines = 14 cm and the radius of each sector = 7 cm.

Area of trapezium $ABCD$

$$= \frac{1}{2}(18 + 32) \times 14 = \frac{1}{2} \times 50 \times 14$$

$$= 350 \text{ cm}^2$$

Let, $\angle A = \theta, \angle B = \theta_2, \angle C = \theta_3$ and $\angle D = \theta_4$

Area of sector A,

$$\frac{\theta_1}{360}\pi r^2 = \frac{\theta_1}{360} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{\theta_1}{360} \times 154 \text{ cm}^2$$

$$\text{area of sector } B = \frac{\theta_2}{360} \times 154 \text{ cm}^2$$

$$\text{area of sector } C = \frac{\theta_3}{360} \times 154 \text{ cm}^2$$

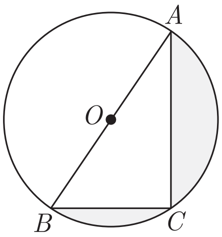
$$\text{area of sector } D = \frac{\theta_4}{360} \times 154 \text{ cm}^2$$

$$\text{area of 4 sectors} = \frac{\theta_1 + \theta_2 + \theta_3 + \theta_4}{360} \times 154$$

$$= \frac{360}{360} \times 154 = 154 \text{ cm}^2$$

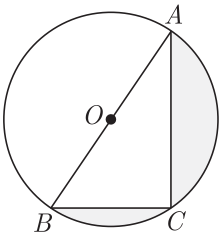
LONG ANSWER TYPE QUESTIONS

1. In the figure, O is the centre of circle such that diameter $AB = 13$ cm and $AC = 12$cm. BC is joined. Find the area of the shaded region. ($\pi = 3.14$)



Ans : [O.D. Set I, II, III, 2016]

We redraw the given figure as below.



Radius of semi circle $ACB = \frac{13}{2}$ cm

$$\text{Area of semicircle} = \frac{\pi}{2}r^2 = \frac{3.14}{2} \times \frac{13}{2} \times \frac{13}{2}$$

$$= \frac{3.14 \times 169}{8} = \frac{530.66}{8} \text{ cm}^2$$

Semicircle subtend 90° at circle, thus $\angle ACB = 90^\circ$

In ΔABC

$$AC^2 + BC^2 = AB^2$$

$$12^2 + BC^2 = 169$$

$$BC^2 = (160 - 144) = 25$$

$$BC = 5 \text{ cm}$$

Also area $\Delta = \frac{1}{2} \times \text{Base} \times \text{Hight}$

Area of ΔABC $\Delta = \frac{1}{2} \times AC \times BC$

$$= \frac{1}{2} \times 12 \times 5$$

$$= 30 \text{ cm}^2$$

$$\text{Area of shaded region} = \frac{530.66}{8} - 30$$

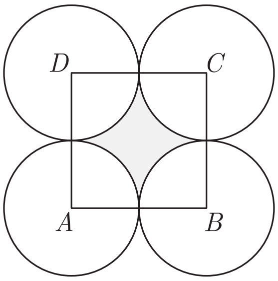
$$= (66.3325 - 30) \text{ cm}^2$$

$$= 36.3325 \text{ cm}^2$$

2. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circle is $\frac{24}{7}$ cm2. Find the radius of each circle.

Ans : [Board Sample paper, 2016]

As per question statement the figure is shown below.



Let r cm be the radius of each circle.

Area of square – Area of 4 sectors $= \frac{24}{7}$ cm^2

$$(2r)^2 - 4\left(\frac{90}{360^\circ} \times \pi r^2\right) = \frac{24}{7}$$

$$4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$$

$$\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$$

$$6r^2 = 24$$

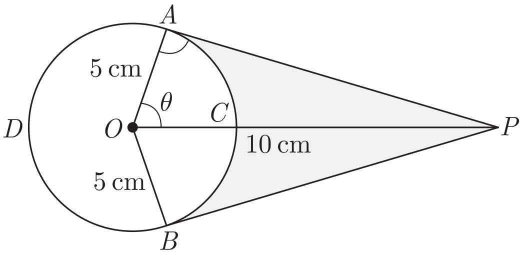
$$r^2 = 4$$

$$r = \pm 2$$

Thus radius of each circle is 2 cm.

3. An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)



Ans : [Delhi Set I, II, III, 2016]

Here AP is tangent at point A on circle.

Thus $\angle OAP = 90^\circ$

Now $\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$

or, $\theta = 60^o$

Reflex $\angle AOB = 360^\circ - 60^\circ - 60^\circ = 240^\circ$

Now arc $ADB \ \frac{2 \times 3.14 \times 5 \times 120}{360} = 20.93$ cm

Hence length of elastic in contact $= 20.93$ cm

Now, $AP = 5\sqrt{3}$ dm

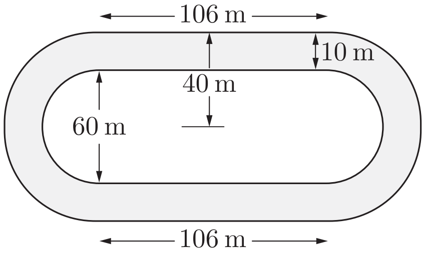
Area $(\Delta\, OAP + \Delta\, OBP) = 25\sqrt{3} = 43.25$ cm^2

Area of sector $OACB$

$$= \frac{25 \times 3.14 \times 120}{360} = 26.16 \text{ cm}^2.$$

Shaded Area $= 43.25 - 26.16 = 17.09$ cm^2

4. Fig. depicts a racing track whose left and right ends are semi-circular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide everywhere, find the area of the track.



Ans : [Board Term-2, 2011, Set B1]

Width of the inner parallel lines $= 60$ m
And the width of the outer lines $= 40 \times 2 = 80$ m

Radius of the inner semicircles $= \frac{60}{2} = 30$ m

Radius of the other semicircles $= \frac{80}{2} = 40$ m

Area of inner rectangle $= 106 \times 60 = 3180$ m^2

Area of outer rectangle $= 106 \times 80 = 4240$ m^2.

Area of the inner semicircle

$$= 2\frac{1}{2} \times \frac{22}{7} \times 30 \times 30 = \frac{19800}{7} \text{ m}^2$$

Area of outer semicircles

$$= 2 \times \frac{1}{2} \times \frac{22}{7} \times 40 \times 40 = \frac{35200}{7} \text{ m}^2$$

Area of racing track

= (area of outer rectangle + area of outer semicircles)

– (area of inner rectangle + area of inner semicircles)

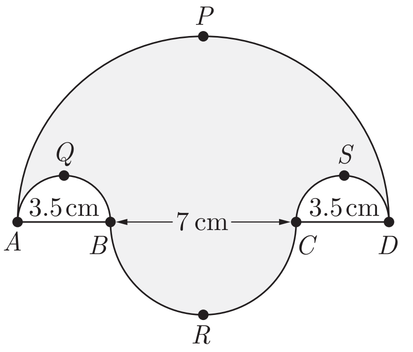
$$= 4240 + \frac{35200}{7} - \left(\frac{3180 + 19800}{7}\right)$$

$$= 1060 + \frac{15400}{7} = \frac{7420 + 15400}{7}$$

$$= \frac{22820}{7} = 3260 \text{ m}^2$$

Hence, area of track is 3260 m^2

5. Find the area of the shaded region in Figure, $\overset{\frown}{APD}, \overset{\frown}{AQB}, \overset{\frown}{BRC}$ and $\overset{\frown}{CSD}$, are semi-circles of diameter 14 cm, 3.5 cm, 7 cm and 3.5 cm respectively. Use $\pi = \frac{22}{7}$.



Ans : [Foreign Set I, II, III, 2016]

Diameter of the largest semi circle

$= 14$ cm

Radius $= \frac{14}{2} = 7$ cm

Diameter of two equal unshaded semicircle

$= 3.5$ cm

Radius of each circle $= \frac{3.5}{2}$ cm

Diameter of smaller shaded semi-circle $= 7$ cm

Radius $= 3.5$ cm

Area of shaded portion

= area of largest semi-circle+

+ area of smaller shades semicircle+

– area of two unshaded semicircles

$$= \frac{1}{2} \times \frac{88}{7} \times 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$- 2 \times \frac{22}{7} \times \frac{1}{2} \times \frac{3.5}{2} \times \frac{3.5}{2}$$

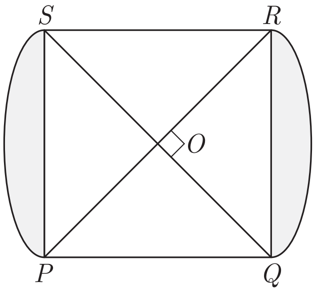
$$= \frac{1}{2} \times \frac{22}{7}\left[7^2 + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{4}\right)^2\right] \text{cm}^2$$

$$= \frac{1}{2} \times \frac{22}{7}\left[49 + \frac{49}{4} - \frac{49}{8}\right]$$

$$= \frac{1}{2} \times \frac{22}{7} \times 49\left[\frac{9}{8}\right]$$

$$= \frac{693}{8} \text{ sq. cm or } 86.625 \text{ cm}^2$$

6. In figure, $PQRS$ is square lawn with side $PQ = 42$ metre. Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).



Ans : [Outside Delhi Set I, II, III, 2015]

Radius of circle with centre O is OR.

Let $OR = x$ then using Pythagoras theorem we have

$$x^2 + x^2 = (42)^2 \text{ or } x = 21\sqrt{2} \text{ m}$$

Area of segment of circle with centre angle 90°

$$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (21\sqrt{2})^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 21 \times 21 \times 2$$

$$= 11 \times 3 \times 21 = 693$$

Area of triangle ΔROQ

$$= \frac{1}{2} \times (21\sqrt{2})^2 = 21 \times 21 = 441$$

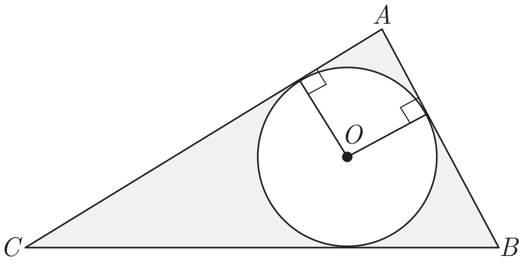
Area of the one side flower bed

$$= 693 - 441 = 252 \text{ m}^2$$

Area of flower bed of both

$$= 2 \times 252 = 504 \text{ m}^2$$

7. In the figure, ABC is a right angled triangle right angled at $\angle A$. Find the area of the shaded region, if $AB = 6$ cm, $BC = 10$ cm and O is the centre of the circle of the triangle ABC.



Ans : [Board Term-2, 2015]

Let r be the radius of in circle

Using the tangent properties we have

$$BC = 8 - r + 6 - r$$

$$10 = 14 - 2r$$

or, $\quad 2r = 4$or, $r = 2$ cm

Area of circle $\quad \pi r^2 = \frac{22}{7} \times 2 \times 2 = \frac{88}{7} = 12.57 \text{ cm}^2$

Now,area of ΔABC,

$$= \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

Area of shaded region

$$= \text{Area of } \Delta ABC - \text{ Area of the circle}$$

$$= 24. - 12.57 \text{ cm}^2 = 11.43 \text{ cm}^2$$

8. Two circular beads of different sizes are joined together such that the distance between their centres is 14cm. The sum of their areas is 130π cm2. Find the radius each bead.

Ans : [Board Term-2, 2015]

Let the radii of the circles are r_1 cm and r_2 cm

$$r_1 + r_2 = 14 \qquad ...(1)$$

Sum, of their areas,

$$130\pi = \pi(r_1^2 + r_2^2)$$

$$130\pi = \pi(r_1^2 + r_2^2)$$

$$r_1^2 + r_2^2 = 130 \qquad ...(2)$$

Now $\quad (r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$

$$(14)^2 = 130 + 2r_1r_2$$

$$2r_1r_2 = 196 - 130 = 66$$

$$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$$

$$= 130 - 66 = 64$$

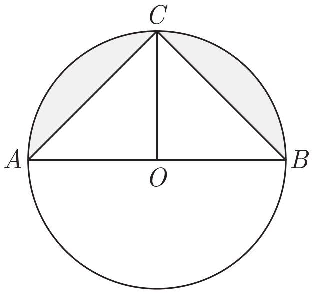
Thus $\quad r_1 - r_2 = 8 \qquad ...(3)$

From (1) and (3), $\quad 2r_1 = 22$

$$r_1 = 11 \text{ cm}$$

$$r_2 = 14 - 11 = 3 \text{ cm.}$$

9. A round thali has 2 inbuilt triangular for serving vegetables and a separate semi-circular area for keeping rice or chapati. If radius of thali is 21 cm, find the area of the thali that is shaded in the figure.



Ans : [Board Term-2, 2014]

Since AOB is the diameter of the circle. So Area of shaded region

$$= (\text{Area of semi-circle} - \text{Area of } \Delta ABC)$$

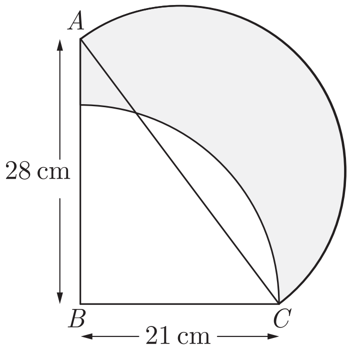
Area of semi-circle $= \frac{\pi r}{2} \times \frac{1}{2} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$

$$= \frac{1386}{2} = 693 \text{ cm}^2$$

Area of triangle $= \frac{1}{2} \times 42 \times 21 = 441 \text{ cm}^2$

Area of shaded region $= 693 - 441 = 252 \text{ cm}^2$

10. In the fig., ABC is a right-angle triangle, $\angle B = 90^\circ, AB = 28$ cm and $BC = 21$ cm. With AC as diameter, a semi-circle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region.



Ans : [CBSE Foreign 2014][CBSE Board Term-2 2011]

In right angled triangle ΔABC using Pythagoras theorem we have

$$AC^2 = AB^2 + BC^2$$

$$= 28^2 + 21^2 = 784 + 441$$

or $\quad AC^2 = 1225$

Thus $\quad AC = 35$ cm

Area of shaded region,

= area of $\Delta ABC +$

+ area of semi-circle with diameter $AC +$

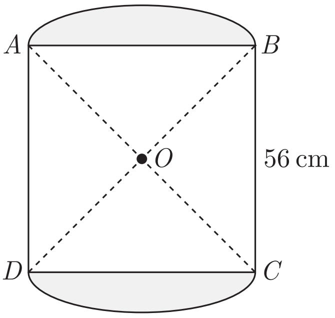
− area of quadrant with radius BC

$$= \frac{1}{2}(21 \times 28) + \frac{1}{2} \times \frac{22}{7} \times \left(\frac{35}{2}\right)^2 - \frac{1}{4} \times \frac{22}{7} \times (21)^2$$

$$= 294 + 481.25 - 346.5$$

$$= 775.25 - 346.5 = 428.75 \text{ cm}^2.$$

11. In fig., two circular flower beds have been shown on two sides of a square lawn $ABCD$ of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and flower beds.



Ans : [Board Term-2, 2011, Set A1]

Side of square = 56

Diagonal of square = $56\sqrt{2}$

Radius of circle = $\frac{1}{2} \times 56\sqrt{2} = 28\sqrt{2}$

Total area = Area of sector $OAB +$

+ Area of sector $ODC +$

+ Area of $\Delta OAD +$

+ Area of ΔOBC

$$= \frac{90°}{360°} \times \frac{22}{7} \times (28\sqrt{2})^2 + \frac{90°}{360°} \times \frac{22}{7} \times (28\sqrt{2})^2$$

$$+ \frac{1}{4} \times 56 \times 56 + \frac{1}{4} \times 56 \times 56$$

$$= \frac{1}{4} \times \frac{22}{7} \times (28\sqrt{2})^2 + \frac{1}{4} \times \frac{22}{7} \times (28\sqrt{2})^2$$

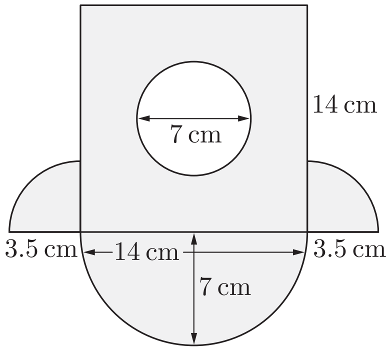
$$+ \frac{1}{4} \times 56 \times 56 + \frac{1}{4} \times 56 \times 56$$

$$= \frac{1}{4} \times 28 \times 56\left(\frac{22}{7} + \frac{22}{7} + 2 + 2\right) \text{ m}^2$$

$$= 7 \times 56\left(\frac{22 + 22 + 14 + 14}{7}\right) \text{ m}^2$$

$$= 56 \times 72 = 4032 \text{ m}^2.$$

12. In fig., find the area of the shaded region Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2, 2011, Set B1]

Area of square $= (14)^2 = 196 \text{ cm}^2$

Area of internal circle $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$

$$= \frac{77}{2} = 38.5 \text{ cm}^2$$

Area of semi-circle with 14 cm diameter

$$= \frac{1}{2} \times \frac{22}{7} \times 7^2 \text{ cm}^2$$

$$= 77 \text{ cm}^2$$

Area of two quarter circles of radius $\frac{7}{2}$ cm

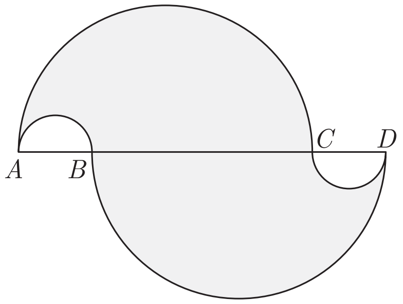
$$= 2 \times \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{77}{4} = 19.25 \text{ cm}^2$$

Shaded area $= 196 - 38.5 + 77 + 19.25$

$$= 292.25 - 38.5$$

$$= 253.75 \text{ cm}^2.$$

13. In fig., $AC = BD = 7$ cm and $AB = CD = 1.75$ cm. Semi-circles are drawn as shown in the figure. Find the area of the shaded region. $\left(Use\ \pi = \frac{22}{7}\right)$



Ans : [Board Term-2, 2011, Set B1]

Area of shaded region

= 2(Area of semi-circle of radius $\frac{7}{2}$ cm)

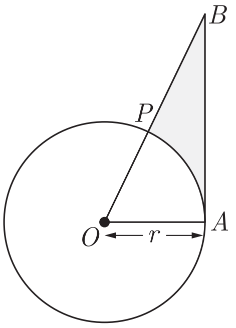
−2(Area of semi-circle of radius $\frac{7}{4}$ cm)

$$= 2\left[\frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right] - 2\left[\frac{1}{2} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}\right]$$

$$= \left(\frac{77}{2} - \frac{77}{8}\right) = \frac{77}{4}\left[1 - \frac{1}{4}\right] = \frac{77}{2} \times \frac{3}{4} = \frac{231}{8} \text{ cm}^2$$

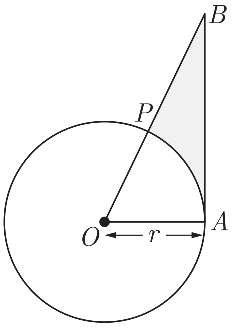
$$= 28.87 \text{ cm}^2$$

14. The given fig. is shown a sector OAP of a circle with centre O, containing $\angle\theta . AB$ is perpendicular to the radius OA and meets OP produced at B . Prove that the perimeter of shaded region is $r=\left[\tan\theta+\sec\theta+\frac{\pi\theta}{180}-1\right]$



Ans : [CBSE Outside 2015, 16]

As per question statement we have redrawn this figure as given below.



Here OAP is sectors of circle with centre O, $\angle POA=\theta$ and $OA\perp AB$

Perimeter of shaded region $= BP+AB+\widehat{AP}$ (1)

$$\text{Now}\qquad \tan\theta=\frac{AB}{r}\Rightarrow r\tan\theta=AB \qquad ...(2)$$

$$\sec\theta=\frac{OB}{r}\Rightarrow r\sec\theta=OB$$

$$OB-OP=BP\Rightarrow r\sec\theta-r=OP \qquad ...(3)$$

Length of arc AP

$$\begin{aligned}\widehat{AP}&=\frac{\theta}{360}\times2\pi r\\&=\frac{\theta}{360}\times2\pi r=\frac{\theta\pi r}{180} \qquad ...(4)\end{aligned}$$

Putting value from eq(2), (3), (4) in eq (1) we have

Perimeter of shaded region

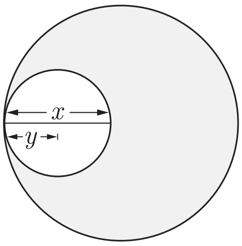
$$\begin{aligned}&=r\tan\theta+r\sec\theta-r+\frac{\theta\pi r}{180}\\&=r\left[\tan\theta+\sec\theta+\frac{\theta\pi}{180}-1\right]\end{aligned}$$

Hence, Proved.

15. Two circles touch internally. The sum of their areas is 1.16π and the difference between their centres is 6 cm. Find the radii of the circles.

Ans : [Foreign Set-I, II, III 2017]

Let the radius of larger circle be x and the radius of smaller circle be y. As per question statement we have shown diagram shown below.



$$\begin{aligned}\text{Now}\qquad x-y&=6 \qquad ...(1)\\ \text{and}\qquad \pi x^2+\pi y^2&=116\pi\\ \pi(x^2+y^2)&=116\pi\\ x^2+y^2&=116 \qquad ...(2)\end{aligned}$$

From (1) and (3)we have

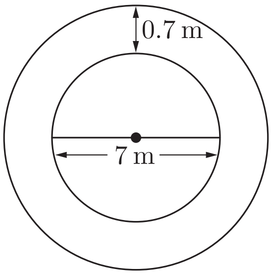
$$\begin{aligned}x^2+(x-6)^2&=116\\x^2+x^2-12x+36&=116\\x^2-6x-40&=0\\x^2-10x+4x-40&=0\\x(x-10)+4(x+10)&=0\\x&=10,\text{ and } y=10-6=4\end{aligned}$$

Hence, radii of the circles are 10 cm and 4 cm.

16. A park is of the shape of a circle of diameter 7 m. It is surrounded by a path of width of 0.7 m. Find the expenditure of cementing the path. If its cost is Rs.110 per sq. m.

Ans : [Foreign Set-I, II, III 2017]

As per question statement we have shown diagram shown below.



The diameter of park $= 7$ m

$$\text{radius}=\frac{7}{2}=3.5\text{ m}$$

$$\text{Width of path}=0.7\text{ m}$$

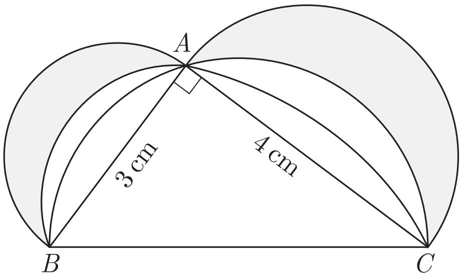
Radius of park with path

$$\begin{aligned}&=3.5+0.7=4.2\text{ m}\\ \text{Area of the path}&=\pi(4.2)^2-\pi(3.5)^2\\&=\frac{22}{7}(17.64-12.25)\\&=\frac{22}{7}\times5.39=22\times0.77\\&=16.94\text{ m}^2\end{aligned}$$

Cost of the cementing the path

$$= 16.94 \times 110$$
$$= Rs.1863.40$$

17. In the given figure, $\Delta\, ABC$ is a right angled triangle in which $\angle A = 90^\circ$. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.



Ans : [Outside Delhi Set-II 2017]

In $\Delta\, ABC$ we have

$\angle A = 90^\circ$, $AB = 3 = 3$ cm, and $AC = 4$ cm

Now $BC = \sqrt{AB^2 + AC^2} = \sqrt{3^2 + 4^2} = 5$ cm.

Area of shaded Area

$= $ Area of semicircle with radius $\frac{3}{2}$ cm

$+$ area of semi circle with radius $\frac{4}{2}$ cm

$+$ Area of triangle $\Delta\, ABC$)

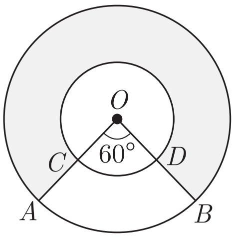
$-$ Area of semicircle with radius $\frac{5}{2}$ cm

$$= \frac{\pi}{2}\left(\frac{3}{2}\right)^2 + \frac{\pi}{2}(2)^2 + \frac{1}{2} \times 3 \times 4 - \frac{\pi}{2}\left(\frac{5}{2}\right)^2$$

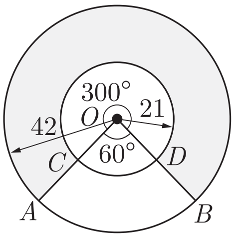
$$= \frac{9\pi}{8} + 2\pi + 6 - \frac{25\pi}{8} = \frac{9\pi + 16\pi - 25}{8} + 6$$

$$= 6 \text{ cm}^2$$

18. In the given figure, two concentric circle with centre O have radii 21 cm and 42 cm. If $\angle AOB = 60^\circ$, find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans :



Here $\angle AOB = 60^\circ$ and $\angle COD = 60^\circ$

$R = 42$ cm, $r = 21$ cm

Reflex of $\angle AOB$

$$\theta = (360^\circ - 60^\circ) = 300^\circ$$

Now, area of shaded region

$$= \frac{\theta}{360^\circ} \times \pi R^2 - \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{\theta}{360^\circ} \times \pi \times (R^2 - r^2)$$

$$= \frac{300^\circ}{360^\circ} \times \frac{22}{7} \times (42 - 21)(42 + 21)$$

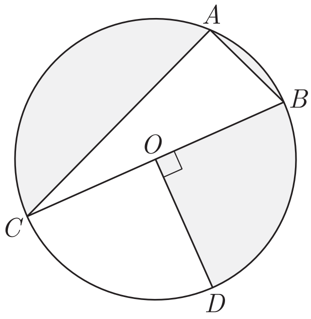
$$= \frac{5}{6} \times \frac{22}{7} \times 21 \times 63$$

$$= 5 \times 11 \times 63$$

$$= 3465 \text{ cm}^2$$

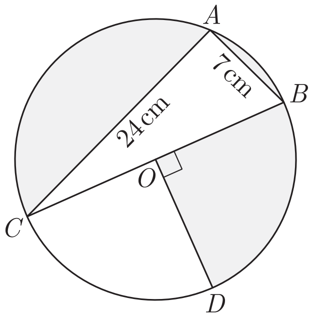
Thus area of shaded region is 3465 cm².

19. In the given figure, O is the centre of the circle with $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90^\circ$. Find the area of the shaded region.



Ans :

We have redrawn the given figure as shown below.



Here $\Delta\, CAB$ is right angle triangle with $\angle CAB = 90^\circ$
In right $\Delta\, CAB$, by Pythagoras theorem, we have

$$BC^2 = AC^2 + AB^2 = 24^2 + 7^2 = 576 + 49 = 625$$

Thus $BC = 25$ cm which is diameter.

Now radius is $\frac{25}{2}$ or 12.5 cm.

Area of shaded region,

$=$ area of semicircle $+$area of quadrant $-$ area of $\Delta\, ACB$

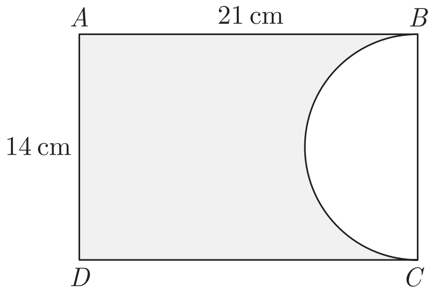
$$= \frac{1}{2}\pi r^2 + \frac{1}{2}\pi r^2 - \frac{1}{2} \times AB \times BC$$

$$= \frac{3}{4}\pi r^2 - \frac{1}{2} \times 7 \times 24 = \frac{3}{4} \times \frac{22}{7} \times \frac{625}{4} - 7 \times 12$$

$= 368.3035 - 84 \ = 284.3 \text{ cm}^2$

Thus area of shaded region $= 284.3035 \text{ cm}^2$

20. In the given figure, $ABCD$ is a rectangle of dimensions 21 cm $\times$ 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.



Ans : [Outside Delhi Set-I, 2017]

Area of shaded region

$= $ Area of rectangle $ABCD$ $-$ area of semicircle

$$= 21 \times 14 - \frac{1}{2} \times \pi \times 7 \times 7$$

$$= 294 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \quad = 294 - 77 \ = 217 \text{ cm}^2$$

Perimeter of shaded are

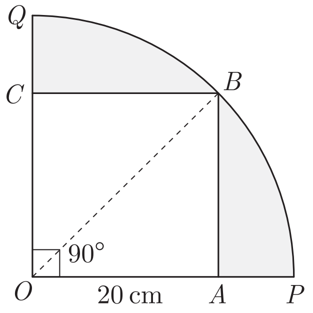
$$= AB + AD + CD + \overset{\frown}{CB}$$

$$= 21 + 14 + 21 + \frac{22}{7} \times 7$$

$$= 21 + 14 + 21 + 22 \ = 78 \text{ cm}$$

Hence, area of shaded region $= 217 \text{ cm}^2$ and perimeter $= 78$ cm.

21. A square $OABC$ is inscribed in a quadrant $OPBQ$ of a circle. If $OA = 20$ cm, find the area of the shaded region. [Use $\pi = 3.14$]



Ans : [Delhi CBSE, Term-2, 2014]

We have $\quad OB = \sqrt{OA^2 + AB^2}$

$$= \sqrt{20^2 + 20^2}$$

$$= \sqrt{800}$$

Thus $\quad OB = 20\sqrt{2}$ cm or,

radius $\quad r = 20\sqrt{2}$

Area of shaded region

$=$ Area of sector $OQBPO -$ Area of square $OABC$

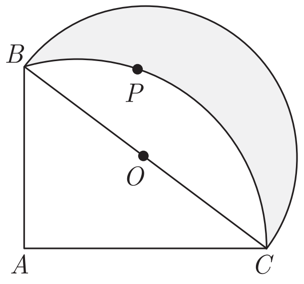
$$= \frac{90^\circ}{360^\circ} \times 3.14 \times 20\sqrt{2} \cdot 20\sqrt{2} - (20)^2$$

$$= \frac{1}{4} \times 3.14 \times 800 - 400$$

$$= 2(314) - 400 = 628 - 400$$

Required area is 228 cm^2.

22. In given figure $ABPC$ is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the are of the shaded region.



Ans : [Sample Question Paper 2017]

Radius of the quadrant $AB = AC = 14$ cm

$$BC = \sqrt{14^2 + 14^2} = 14\sqrt{2} \text{ cm}$$

Radius of semicircle $= \frac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

Area of semicircle $= \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2}$

$$= 154 \text{ cm}^2$$

Area of segment $BPCO$

$$\frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 = r^2\left(\frac{\pi\theta}{360} - \frac{1}{2}\right)$$

$$= 14 \times 14\left(\frac{22}{7} \times \frac{90}{360} - \frac{1}{2}\right)$$

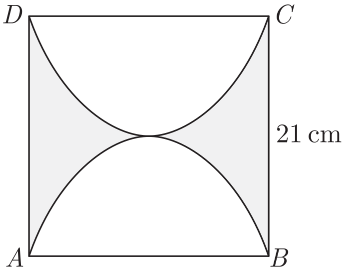
$$= 14 \times 14\left(\frac{11}{14} - \frac{1}{2}\right)$$

$$= 14 \times 14 \times \frac{2}{7} = 56 \text{ cm}^2$$

Hence, area of shaded region $= 56 \text{ cm}^2$

HOTS QUESTIONS

1. Find the perimeter of the shaded region if $ABCD$ is a square of side 21 cm and APB and CPD are semicircle. Use $\pi = \frac{22}{7}$.



Ans : [Board Sample paper 2016]

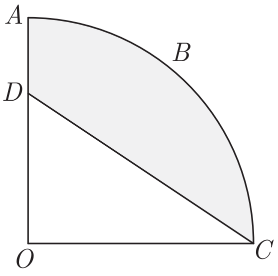
Perimeter of the shaded region

$= AD + BC +$

$+$lengths of the arcs of semi circles APB and CPD

$$= 21 + 21 + 2\left(\frac{22}{7} \times \frac{21}{2}\right) = 42 + 66 = 108 \text{ cm.}$$

2. In the figure $OABC$ is a quadrant of a circle of radius 7 cm. If $OD = 4$ cm, find the area of shaded region.



Ans : [Foreign Set I, II, III, 2014]

Area of shaded region

$=$ Area of sector $OCBAD -$ Area of $\Delta\, ODC$

$$= \frac{90^\circ}{360^\circ} \times \pi \times (7)^2 - \frac{1}{2} \times 7 \times 4$$

$$= \frac{49\pi}{4} - 14 = 24.5 \text{ cm}^2$$

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand from 9 a.m. to 9.35 a.m.

Ans : [Board Term-2, 2012 Set (13)]

Angle subtended in 1 minute

$\theta =$ angle subtended in 35 minutes

$= 35 \times 6 = 210^\circ$

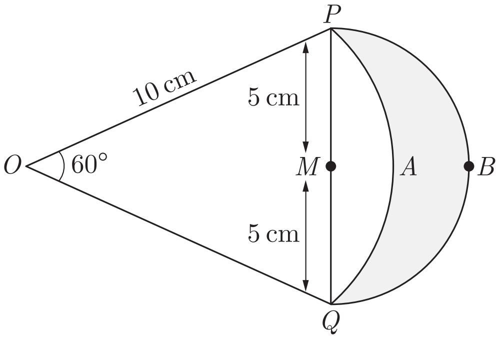
Area swept by the minute hand

$=$ Area of a sector

$$= \frac{\pi r^2 \theta}{360^\circ} = \frac{22}{7} \times \frac{14 \times 14 \times 210}{360}$$

$$= \frac{1078}{3} = 259.33 \text{ cm}^2$$

4. Figure shows two arcs PAQ and PQB. Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semi-circle drawn on PQ as diameter with centre M. If $OP = PQ = 10$ cm show that area of shaded region is $25\left(\sqrt{3} - \frac{\pi}{6}\right)$ cm2.



Ans : [Delhi Set I, II, III, 2016]

We have $OP = OQ = PQ = 10$

$\angle POQ = 60^\circ$

Area of segment $PAQM$

$$= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right) \text{ cm}^2$$

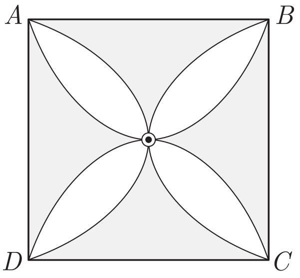
Area of semicircle $= \frac{25\pi}{2}$ cm^2

Area of shaded region

$$= \frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right)$$

$$= 25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{ cm}^2.$$

5. In fig. $ABCD$ is a square of side 14 cm. Semi-circle are drawn with each side of square as diameter. Find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [CBSE Delhi, 2016]

Area of square $= 196$ cm^2

Area of semicircle $= AOB + DOC$

$$= \frac{22}{7} \times 49 = 154 \text{ cm}^2$$

So, area of two shaded parts

$196 - 154 = 42$ cm^2

Hence, area of four shaded parts $= 84$ cm^2.

6. The long and short hands of a clock are 6 cm and 4 cm long respectively. Find the sum of distances travelled by their tips in 24 hours. (Use $\pi = 3.14$)

Ans : [Foreign Set I, II, III, 2015]

Long hand makes 24 rounds in 24 hours and short hand makes 2 round in 24 hours.

Radius of the circle formed by long hand = 6 cm. and radius of the circle formed by short hand = 4 cm.
Distance travelled by tips of hands in one round is equal to the circumference of circle.
Distance travelled by long hand in one round

$$= \text{circumference of the circle } 2 \times 6 \times \pi$$

Distance travelled by long hand in 24 rounds

$$= 24 \times 12\pi = 288\pi$$

Distance travelled by short hand in a round $= 2 \times 4\pi$

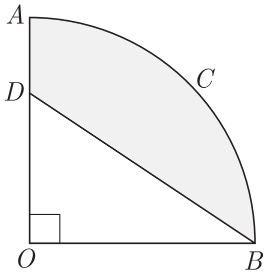
Distance travelled by short hand in 2 round

$$= 2 \times 8\pi = 16\pi$$

$$\text{Sum of the distance} = 288\pi + 16\pi = 304\pi$$

$$= 304 \times 3.14 = 954.56 \text{ cm}$$

7. In the given figure $DACB$ is a quadrant of a circle with centre O and radius 3.5 cm. If $OD = 2$ find the area of the region.



Ans : [Delhi Set-I, II, 2017]

Area of shaded region,

$$= \text{area of quadrant } OACB - \text{ar } \Delta DOB$$

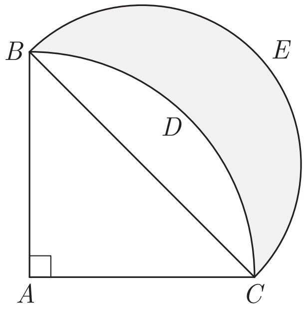
$$= \frac{1}{4}\pi r^2 - \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 - \frac{1}{2} \times 2 \times 3.5$$

$$= 19.625 - 3.5 = 6.125$$

Hence the area of shaded region is 6.125 cm.

8. As $ABDC$ is a quadrant of a circle of radius 28 cm and a semi-circle BEC is drawn with BC as diameter. Find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Sample Question Paper 2017]

As ABC is a quadrant of the circle, $\angle BAC$ will be $90°$.

In ΔABC, $BC^2 = AC^2 + AB^2$

$$= (28)^2 + (28)^2 = 2(28)^2$$

$$BC = 28\sqrt{2} \text{ cm}$$

Radius of semi-circle drawn on BC,

$$= \frac{28\sqrt{2}}{2} = 14\sqrt{2}$$

$$\text{Area of semi-circle} = \frac{1}{2} \times \frac{22}{7} \times (14\sqrt{2})^2$$

$$= 616 \text{ cm}^2$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 28 \times 28 = 392 \text{ cm}^2$$

$$\text{Area of quadrant} = \frac{1}{4} \times \frac{22}{7} \times 28 \times 28$$

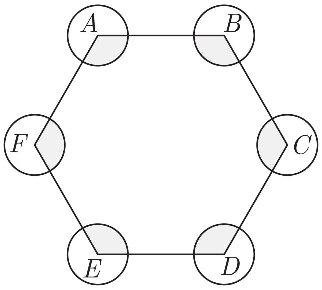
$$= 616 \text{ cm}^2$$

Area of the shaded region

= Area of semi-circle +area of Δ − Area of quadrant

$= 616 + 392 - 616 = 392 \text{ cm}^2$.

9. In fig., $ABCDEF$ is any regular hexagon with different vertices A, B, C, D, E, and F as the centres of circle with same radius 'r' are drawn. Find the area of the shaded portion.



Ans : [Board Term-2, 2011, B1]

Let number of sides is n.

$$n \times \text{each angle} = (n-2) \times 180°$$

$$6 \times \text{each angle} = 4 \times 180°$$

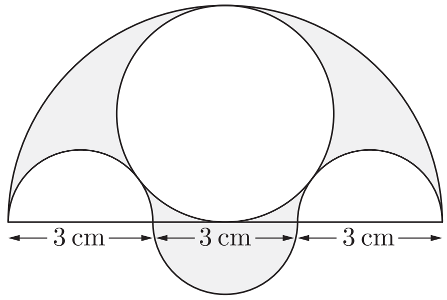
$$\text{each angle} = 120°$$

$$\text{area of a sector} = \frac{120°}{360°} \times \pi r^2$$

$$\text{Area of 6 shaded regions} = 6 \times \frac{120°}{360°} \times \pi r^2$$

$$= 2\pi r^2$$

10. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



Ans :

Area of shaded region

= Area of semicircle with $d = 9$ cm

+ Area of semicircle with $d = 3$ cm

$-2 \times$ area of semicircle with $d = 3$ cm

$-$ area of circle with $d = 4.5$ cm

$$= \frac{1}{2} \times \pi \times \left(\frac{9}{2}\right)^2 + \frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2$$

$$-2 \times \frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2 - \pi \times \left(\frac{4.5}{2}\right)^2$$

$$= \frac{\pi}{8}[(9)^2 + (3)^2 - 2(3)^2 - 2(4.5)^2]$$

$$= \frac{\pi}{8}[4(4.5)^2 + (3)^2 - 2(3)^2 - 2(4.5)^2]$$

$$= \frac{\pi}{8}[2(4.5)^2 - (3)^2] = \frac{\pi}{8}[2(3 \times 1.5)^2 - (3)^2]$$

$$= \frac{\pi(3)^2}{8}[2(1.5)^2 - 1] = \frac{9\pi}{8}[4.5 - 1]$$

$$= \frac{9 \times 22}{8 \times 7} \times 3.5 = \frac{99}{8} = 12.375 \text{ cm}^2$$

Thus area of shaded region is 12.375 cm^2

CHAPTER 13

Surface Areas and Volumes

VERY SHORT ANSWER TYPE QUESTIONS

1. The curved surface area of a cylinder is 264 m2 and its volume is 924 m3. Find the ratio of its height to its diameter.

Ans : [Board Term-2, 2014]

Curved Surface area of cylinder $= 2\pi rh$

Volume of cylinder $= \pi r^2 h$

$$\frac{\pi r^2 h}{2\pi rh} = \frac{924}{264} \Rightarrow \frac{r}{2} = \frac{7}{2}$$

Thus $r = 7$ m and substituting in $2\pi rh = 264$ we have

$$2 \times \frac{22}{7} \times 7 \times h = 264$$

$$h = 6 \text{ m}$$

Now $\quad \dfrac{h}{2r} = \dfrac{6}{14} = \dfrac{3}{6}$

Hence, $\quad h : d = 3 : 7$

2. A rectangular sheet paper 40 cm $\times$ 22 cm is rolled to form a hollow cylinder of height 40 cm. Find the radius of the cylinder.

Ans : [Foreign Set I, II, III, 2014]

Here, $h = 40$ cm, circumference $= 22$ cm

$$2\pi r = 22$$

$$r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} = 3.5 \text{ cm}$$

3. A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes.

Ans : [Delhi CBSE, 2014]

Volume of cylinder : Volume of cone : Volume of hemisphere

$$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3$$

$$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^2 \times h \qquad (h = r)$$

$$= 1 : \frac{1}{3} : \frac{2}{3}$$

or, $\quad 3 : 1 : 2$

4. What is the ratio of the total surface area of the solid hemisphere to the square of its radius.

Ans : [Board Term-2, 2012 Set (21,22)]

$$\frac{\text{Total surface area of hemisphere}}{\text{Square of its radius}} = \frac{3\pi r^2}{r^2} = \frac{3\pi}{1}$$

Total surface area of hemisphere : Square of radius $= 3\pi : 1$

5. Two cubes each of volume 8 cm3 are joined end to end, then what is the surface area of resulting cuboid.

Ans : [Board Term II, 2012 Set (23)]

Side of the cube, $a = \sqrt[3]{8} = \sqrt{2}$ cm

Now the length of cuboid

$$l = 4 \text{ cm}$$

Breadth, $\quad b = 2$ cm

Height, $\quad h = 2$ cm

Surface area of cuboid $= 2(l \times b + b \times h + h \times l)$

$$= 2(4 \times 2 + 2 \times 2 + 2 \times 4)$$

$$= 2 \times 20 = 40 \text{ cm}^2$$

6. The radius of sphere is r cm. It is divided into two equal parts. Find the whole surface of two parts.

Ans : [Board Term-2, 2012, Set (26)]

Whole surface of each part

$$= 2\pi r^2 + \pi r^2 = 3\pi r^2$$

Total surface of two parts

$$= 2 \times 3\pi r^2 = 6\pi r^2$$

7. What is the volume of a right circular cylinder of base radius 7 cm and height 10 cm ? Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 Set (59)]

Here $r = 7$ cm, $h = 10$ cm,

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times (7)^2 \times 10$$

$$= 1540 \text{ cm}^3$$

8. If the radius of the base of a right circular cylinder is halved, keeping the height same, find the ratio of the volume of the reduced cylinder to that of original cylinder.

Ans : [Board Term-2, 2012 Set (40)]

$$\frac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}} = \frac{\pi \times \left(\frac{r}{2}\right)^2 h}{\pi r^2 h}$$

$$= \frac{1}{4} = 1 : 4$$

9. If the area of three adjacent faces of a cuboid are X, Y, and Z respectively, then find the volume of cuboid.

Ans : [Board Term-2, 2012, Set (5)]

Let the length, breadth and height of the cuboid is l, b and h respectively.

$$X = l \times b$$

$$Y = b \times h$$

$$Z = l \times h$$

$$XYZ = l^2 \times b^2 \times h^2$$

$$\text{Volume of cuboid } = l \times b \times h$$

$$l^2 b^2 h^2 = XYZ$$

or, $$lbh = \sqrt{XYZ}$$

10. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3, find the ratio of their volumes.

Ans : [Board Term-2, 2012, Set (44)]

$$\frac{Volume\ of\ 1^{st}\ cylinder}{Volume\ of\ 2^{nd}\ cylinder} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$$

$$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$$

$$= \left(\frac{2}{3}\right)^2 \times \frac{5}{3}$$

$$= \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$$

$$= 20 : 27$$

11. Volume of two spheres are in the ratio 64 : 27, find the ratio of their surface areas.

Ans : [KVS 2014][Board Term-2, 2012, Set (22)]

$$\frac{Volume\ of\ I^{st}\ sphere}{Volume\ of\ II^{nd}} = \frac{64}{27}$$

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$$\frac{r_1^3}{r_2^3} = \frac{4^3}{3^3}$$

$$\frac{r_1}{r_2} = \frac{4}{3}$$

Ratio of their surface areas

$$\frac{2\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

12. A solid metallic object is shaped like a double cone as shown in figure. Radius of base of both cones is same but their heights are different. If this cone is immersed in water, find the quantity of water it will displace.

Ans : [Board Term-2, 2012]

Volume of the upper cone $= \frac{1}{3}\pi r^2 h$

Volume of the lower cone $= \frac{1}{3}\pi r^2 H$

Total volume of both the cones $= \frac{1}{3}\pi r^2 h + \frac{1}{3}\pi r^2 H$

$$= \frac{1}{3}\pi r^2 (h + H)$$

The quantity of water displaced will $\frac{1}{3}\pi r^2 (h + H)$ cube units.

13. Find the volume (in cm3) of the largest right circular cone that can be cut off from a cube of edge 4.2 cm.

Ans : [Board Term-2, 2012, Set (22)]

Edge of the cube = 4.2 cm.

Height of the cone = 4.2 cm.

Radius of the cone $= \frac{4.2}{2} = 2.1$ cm.

Volume of the cone$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 4.2$

$$= 19.4 \text{ cm}^3$$

14. The circumference of the edge of a hemisphere bowl is 132 cm. When π is taken as $\frac{22}{7}$, find the capacity of the bowl in cm3.

Ans : [Board Term-2, 2012]

Let r be the radius of bowl.

Circumference of bowl

$$2\pi r = 132$$

$$r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

Capacity i.e volume of the bowl

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= 19404 \text{ cm}^3$$

15. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere ?

Ans : [Delhi Set-I 2017]

Let radius of sphere be r.

Given, volume of sphere = S.A. of hemisphere

$$\frac{2}{3}\pi r^3 = 3\pi r^2$$

$$r = \frac{9}{2} \text{ units}$$

Diameter $$d = \frac{9}{2} \times 2 = 9 \text{ units}$$

16. Find the number of solid sphere of diameter 6 cm can be made by melting a solid metallic cylinder of height 45 cm and diameter 4 cm.

Ans : [Delhi CBSE Term-2, 2014]

Let the number of sphere = n

Radius of sphere = 3 cm, radius of cylinder = 2 cm

Volume of spheres = Volume of cylinder

$$n \times \frac{4}{3}\pi r^3 = \pi r_1^2 h$$

$$n \times \frac{4}{3} \times \frac{22}{7} \times (3)^3 = \frac{22}{7} \times (2)^2 \times 45$$

$$36n = 180$$

$$n = \frac{180}{36} = 5$$

Number of solid sphere = 5.

17. Three solid metallic spherical balls of radii 3 cm, 4 cm and 5 cm are melted into a single spherical ball, find its radius.

Ans : [Board Term-2, 2014]

Let the radius of spherical ball = R.

Volume of spherical ball = Volume of three balls

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left[(3)^2 + (4)^3 + (5)^3\right]$$

$$R^3 = 27 + 64 + 125 = 216$$

, $$R = 6 \text{ cm}$$

18. 12 solid spheres of the same size are made by melting a solid metallic cone of base radius 1 cm and height of 48 cm. Find the radius of each sphere.

Ans : [Board Term-2, 2014]

No. of spheres $= 12$

Radius of cone, $r = 1$ cm

Height of the cone $= 48$

Volume of 12 spheres = Volume of cone

Let the radius of sphere be R.

$$\begin{aligned} 12 \times \frac{4}{3}\pi R^3 &= \frac{1}{3}\pi r^2 h \\ 12 \times \frac{4}{3}\pi R^3 &= \frac{1}{3}\pi \times (1)^2 \times 48 \\ R^3 &= 1 \\ R &= 1 \text{ cm} \end{aligned}$$

19. Three cubes of iron whose edges are 3 cm, 4 cm and 5 cm respectively are melted and formed into a single cube, what will be the edge of the new cube formed ?

Ans : [Delhi CBSE Term-2, 2012 (13)]

Let the edge of single cube be x cm.

Volume of single cube= Volume of three cubes

$$\begin{aligned} x^3 &= (3)^3 + (4)^3 + (5)^3 \\ &= 27 + 64 + 125 = 216 \\ x &= 6 \text{ cm} \end{aligned}$$

20. A solid sphere of radius r melted and recast into the shape of a solid cone of height r. Find the radius of the base of a cone.

Ans : [Delhi Board Term-2, 2012, Set (22)]

Volume of sphere = Volume of cone

Let the radius of cone be R cm.

$$\begin{aligned} \frac{4}{3}\pi r^3 &= \frac{1}{3}\pi R^2 \times r \\ 4r^3 &= R^3 r \\ R^2 &= 4r^2 \\ R &= 2r \end{aligned}$$

21. If a cone is cut into two parts by a horizontal plane passing through the mid-points of its axis, find the ratio of the volume of the upper part and the cone.

Ans : [Board Term-2, 2011, Set A1]

As per question the figure is shown below.

$$\begin{aligned} \text{Volume of upper cone} &= \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 \times \frac{h}{2} \\ &= \frac{1}{3}\pi\frac{r^2}{4} \times \frac{h}{2} \\ &= \frac{1}{3}\pi\frac{r^2 h}{8} \\ \text{Volume of full cone} &= \frac{1}{3}\pi r^2 h \\ \frac{\text{Volume of upper of cone}}{\text{Volume of cone}} &= \frac{\frac{1}{3}\pi \times \frac{1}{8}r^2 h}{\frac{1}{3}\pi r^2 h} = \frac{1}{8} \\ &= 1 : 8 \end{aligned}$$

22. What is the frustum of a right circular cone of height 16 cm with radii of its circular ends as 8 cm and 29 cm has slant height equal to ?

Ans : [Board Term-2, 2014 A1]

As per question the figure is shown below.

Slant height of the frustum,

$$\begin{aligned} l &= \sqrt{h^2 + d^2} \\ &= \sqrt{(16)^2 + (12)^2} \\ &= \sqrt{256 + 144} \\ &= \sqrt{400} \\ &= 20 \text{ cm.} \end{aligned}$$

23. The slant height of a bucket is 26 cm. The diameter of upper and lower circular ends are 36 cm and 16 cm. Find the height of the bucket.

Ans : [Board Term-2, 2012 31]

Here, $l =$ 26 cm, upper radius = 18 cm,

lower radius = 8 cm

d =difference in radius = 18 – 8 = 10 cm.

Let h be the height of bucket

$$\begin{aligned} h &= \sqrt{l^2 - d^2} = \sqrt{(26)^2 - (10)^2} \\ &= \sqrt{676 - 100} = \sqrt{576} = 24 \text{ cm.} \end{aligned}$$

24. A cylinder and a cone have base radii 5 cm and 3 cm respectively and their respective heights are 4 cm and 8 cm. Find the ratio of their volumes.

Ans : [Board Term-2, 2012 Set (59)]

$$\begin{aligned} \text{Volume of cylinder} &= \pi(5)^2 \times 4 \text{ cm}^3 \\ &= 100\pi \text{ cm}^3 \\ \text{Volume of cone} &= \frac{1}{3}\pi \times 3^2 \times 8 \\ &= 24\pi \\ \text{Required ratio} &= 100\pi : 24\pi \\ &= 25 : 6. \end{aligned}$$

SHORT ANSWER TYPE QUESTIONS - I

1. A sphere of maximum volume is cut out from a solid hemisphere of radius 6 cm. Find the volume of the cut out sphere.

Ans : [Board Term-2, 2012 Set (5)]

$$\begin{aligned} \text{Diameter of sphere} &= \text{Radius of hemisphere} \\ &= 6 \text{ cm} \\ \text{Radius of sphere} &= 3 \text{ cm} \end{aligned}$$

$$\text{Volume} \qquad V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 3^3 \text{ cm}^3.$$

$$= 113.14 \text{ cm}^3.$$

2. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have ? Find the surface area of the solid.

Ans : [Board Term-2, 2012 Set (17)]

Diameter of hemisphere = Side of cubical block

$$2r = 7$$

or, $$r = \frac{7}{2}$$

Surface area of solid

$$= \text{Surface area of the cube}$$
$$- \text{ Area of base of hemisphere}$$
$$+ \text{ curved surface area of hemisphere}$$
$$= 6l^2 - \pi r^2 + 2\pi r^2$$
$$= 6 \times 49 - 11 \times \frac{7}{2} + 77 = 332.5 \text{ cm}^2$$

3. A glass cylinder with diameter 20 cm has water to a height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. Calculate the height by which water will rise in the cylinder. Use $\pi = \frac{22}{7}$

OR

A cylinder glass tube with radius 10 cm has water upto a height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. By how much the water will rise in the glass tube. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 Set(34)]

Let h be the height of water raised measured.

Volume of water displaced in cylinder $= \pi(10)^2 h$

Volume of cube, $\pi(10)^2 h = 8 \times 8 \times 8$

$$h = \frac{8 \times 8 \times 8 \times 7}{22 \times 10 \times 10}$$
$$= 1.629 \text{ cm.}$$

4. Two cubes of 5 cm each are kept together joining edge to edge to form a cuboid. Find the surface area of the cuboid so formed.

Ans : [Board Term-2, 2015]

Let l be the length of the cuboid so formed.

Thus $l = 5 + 5 = 10$ cm,$b = 5$ cm; $h = 5$ cm.

$$\text{Surface area} = 2(l \times b + b \times h + h \times l)$$
$$= 2(10 \times 5 + 5 \times 5 + 5 \times 10)$$
$$= 2(50 + 25 + 50)$$
$$= 2 \times 125$$
$$= 250 \text{ cm}^2.$$

5. If the total surface area of a solid hemisphere is 462 cm2, find its volume. Use $\pi = \frac{22}{7}$

Ans : [CBSE O.D. 2014]

Total surface area of hemisphere,

$$3\pi r^2 = 462 \text{ cm}^2$$
$$\frac{22}{7} \times r^2 = \frac{462}{3}$$
$$r^2 = \frac{462 \times 7}{22 \times 3} = 49$$
$$r = 7 \text{ cm.}$$

Volume of hemisphere,

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$
$$= \frac{2156}{3} = 718.67 \text{ cm}^3.$$

6. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of Rs.25 per meter.

Ans : [Delhi CBSE, Term-2, 2014], [Foreign Set I, II, III, 2014]

Given, radius $r = 7$ m and height $h = 24$ m

Slant height of tent,

$$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2}$$
$$= \sqrt{625} = 25 \text{ m.}$$

Curves surface area

$$\pi rl = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Curves surface area of tent will be required area of cloth. Let x meter of cloth is required

$$5x = 550 \text{ or, } x = \frac{550}{5} = 110 \text{ m.}$$

Thus 110 m of cloth is required.

$$\text{Cost of cloth} = 25 \times 110 = Rs.2750.$$

7. Find the number of plates, 1.5 cm in diameter and 0.2 cm thick, that can be fitted completely inside a right circular of height 10 cm and diameter 4.5 cm.

Ans : [Board Term-2, 2014]

Each one of he circular plate is also a cylinder.

Volume of plate $V_p = \pi r^r h = \pi \times (.75)^2 (.2)$

$$= \frac{9\pi}{80} \text{ cm}^3$$

Volume of right circular cylinder

$$V_c = \pi(2.25)^2(10) = 405\frac{\pi}{8} \text{ cm}^3$$

$$\text{Number of plates} = \frac{\frac{405\pi}{8}}{\frac{9\pi}{80}} = \frac{405\pi}{9\pi} \times \frac{80}{8}$$
$$= 450 \text{ plates.}$$

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the volume of the remaining solid to the nearest cm3. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 Set (44)]

As per question the figure is shown below.
Volume of remaining solid

$$= \text{Volume of cylinder} - \text{Volume of cone}$$
$$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$$
$$= \frac{2}{3} \times \frac{22}{7} \times 0.7 \times 0.7 \times 2.4$$
$$= 44 \times 0.1 \times 0.7 \times 0.8$$
$$= 4.4 \times .56 = 2.464 \text{ cm}^3.$$

9. A solid metallic of dimensions 9 m $\times$8m $\times$2 m is melted and recast into solid cubes of edge 2 m. Find the number of cubes so formed.

Ans : [Foreign Set-I, II 2017]

$$\text{Volume of cuboid} = 9 \times 8 \times 2 \text{ cm}^3$$
$$\text{Volume of cube} = 2 \times 2 \times 2 \text{ cm}^3$$

Let number of recast cubes be n.

$$\text{Volume of } n \text{ cubes} = \text{Volume of cuboid}$$

$$n \times 2 \times 2 \times 2 = 9 \times 8 \times 2$$

$$n = \frac{9 \times 8 \times 2}{2 \times 2 \times 2} = 18$$

Hence, number of cubes recast $= 18$

10. A solid metallic cylinder of radius 3.5 cm and height 14 cm melted and recast into a number of small solid metallic ball, each of radius $\frac{7}{12}$ cm. Find the number of balls so formed.

Ans : [CBSE S.A. 2 2016 Set-HODM4OL]

Let the number of recasted balls be N

Radius of cylinder $R = 3.5$ cm

Height of cylinder $h = 14$cm

Radius of recasted ball $r = \frac{7}{12}$

$$\text{Volume of balls} = \text{Volume of cylinder}$$

$$n\frac{4}{3}\pi r^2 = \pi R^2 h$$

$$n \times \frac{4}{3} \times \frac{7}{12} \times \frac{7}{12} \times \frac{7}{12} = 3.5 \times 3.5 \times 14$$

$$n = \frac{3.5 \times 3.5 \times 14 \times 3 \times 12 \times 12 \times 12}{4 \times 7 \times 7 \times 7}$$

$$= 0.5 \times 0.5 \times 2 \times 3 \times 3 \times 12 \times 12$$

$$= 648$$

Hence, number of recasted balls $= 648$

11. A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel ?

Ans : [Board Sample Paper, 2016]

Radius of sphere $\frac{6}{2} = 3$ cm

Radius of cylinder vessel $\frac{12}{2} = 6$ cm

Let the level of water rise in cylinder be h.

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi \times 3 \times 3 \times 3$$

$$= 36\pi \text{ cm}^3$$

$$\text{Volume of sphere} = \text{Increase volume in cylinder}$$

$$36\pi = \pi(6)^2 \times h$$

$$h = 1 \text{ cm}$$

Thus level of water rise in vessel is 1 cm.

12. Find the number of coins of 1.5 cm diameter and 0.2 cm thickness to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Ans : [Board Sample Paper 2016]

Volume of of any cylinder shape is $\pi r^2 h$.

$$\text{Volume of coin} = \pi(0.75)^2 \times 0.2 \text{ cm}^3$$

$$\text{Volume of cylinder} = \pi(2.25)^2 \times 10 \text{ cm}^3$$

$$\text{No. of coins} = \frac{\text{Volume of cylinder}}{\text{Volume of coin}}$$

$$= \frac{\pi(2.25)^2 \times 10}{\pi(0.75)^2 \times 0.2} = \frac{(3)^2 \times 10}{0.2}$$

$$= 450$$

13. A cone of height 24 cm and radius of base 6 cm is made up of clay. If we reshape it into a sphere, find the radius of sphere.

Ans : [KVS 2014]

$$\text{Volume of sphere} = \text{Volume of cone}$$

$$\frac{4}{3}\pi r_1^3 = \frac{1}{3}\pi r_2^2 h$$

$$\frac{4}{3} \times r_1^3 = (6)^2 \times \frac{24}{3}$$

$$4r_1^3 = 36 \times 24$$

$$r_1^3 = 6^3$$

$$r_1 = 6 \text{ cm}$$

Hence, radius of sphere is 6 cm.

14. A metallic sphere of total volume π is melted and recast into the shape of a right circular cylinder of radius 0.5 cm. What is the height of cylinder ?

Ans : [Board Term-2, 2012 (22)]

$$\text{Volume of cylinder} = \text{Volume of sphere,}$$

$$\pi r^2 h = \pi$$

where r and h are radius of base and height of cylinder

$$(0.5)^2 h = 1$$

$$\left(\frac{1}{2}\right)^2 h = 1$$

$$h = 4 \text{ cm.}$$

15. A metallic solid sphere of radius 4.2 cm is melted and recast into the shape of a solid cylinder of radius 6 cm. Find the height of the cylinder.

Ans : [Board Term-2, 2012 (1)]

$$\text{Volume of sphere} = \text{Volume of cylinder}$$

$$\frac{4}{3}\pi R^3 = \pi r^2 h$$

$$\frac{4}{3} \times (4.2)^3 = 6^2 \times h$$

$$h = \frac{4 \times 4.2 \times 4.2 \times 4.2}{3 \times 6 \times 6}$$

Hence, height of cylinder $h = 2.744$ cm.

SHORT ANSWER TYPE QUESTIONS - II

1. A right circular cone of radius 3 cm, has a curved surface area of 47.1 cm2. Find the volume of the cone. (Use $\pi = 3.14$)

Ans : [Delhi Set II, 2016]

We have $r = 3, \pi r l = 47.1$

Thus $$l = \frac{47.1}{3 \times 3.14} = 5$$

$$h = \sqrt{5^2 - 3^3} = 4 \text{ cm}$$

Volume of cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 3 \times 3 \times 4$$

$$= 37.68 \text{ cm}^3$$

2. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\pi = \frac{22}{7}$

Ans : [CBSE Delhi Set I, 2016]

Here, $r + h = 37$ (1)

and $2\pi r(r + h) = 1628$ (2)

Thus $2\pi r \times 37 = 1628$

$$2\pi r = \frac{1628}{37}$$

$$r = 7 \text{ cm}$$

Substituting $r = 7$ in (1) we have

$$h = 30 \text{ cm.}$$

Here volume of cylinder

$$\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ cm}^3$$

3. A tent is in the shape of cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs.500 per square meter. Use $\pi = \frac{22}{7}$

Ans : [O.D. Set I, II, III, 2016]

Height of cylinder $= 2.1$ m

Radius of cylinder $=$ radius of cone $= \frac{3}{2}$ m

Slant height of cone $= 2.8$ m

Surface area of tent

$$= C.S.A \text{ of cone } + C.S.A \text{ of cylinder.}$$

$$= \pi rl + 2\pi rh = \pi r(l + 2h)$$

Area of canvas required will be surface area of tent.

Thus $\pi r(l + 2h) = \frac{22}{7} \times \frac{3}{2}(2.8 + 2 \times 2.1)$

$$= \frac{33}{7} \times 7 = 33 \text{ m}^2$$

Total Cost $= 33 \times 500 Rs$

$$= 16,500 Rs$$

4. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have ? Find the cost of painting the total surface area of the solid so formed, at the rate of Rs.5 per 100 sq. cm.[Use $\pi = 3.14$]

Ans : [Outside Delhi CBSE Board 2015, Set I, II, III]

As per question the figure is shown below.

Side of given cube $a = 10$ cm

Area of cube(excluding base)

$$A_1 = \text{area of 4 walls + area of Top}$$

$$= 4a^2 + a^2 = 5a^2 = 5(10)^2 = 500 \text{ cm}^2$$

Let r be the largest radius of hemisphere. From fig. (ii) we have

$\square ABCD$, in the square of side 10 cm.

In ΔABC, $\angle B = 90$

From Pythagoras theorem we have

$$AC^2 = AB^2 + BC^2$$

$$(2r)^2 = (10)^2 + (10)^2$$

$$4r^2 = 200 \text{ cm}^2$$

$$r = \sqrt{\frac{200}{4}} = 5\sqrt{2} \text{ cm}$$

Hence, the required diameter of hemisphere

$$d = 2r = 2 \times 5\sqrt{2} = 10\sqrt{2} \text{ cm}$$

Now, area of unshaded part in fig (ii)

$$A_2 = \text{area of circle} - \text{area of square } ABCD$$

$$= \pi r^2 - (a)^2 = [\pi \times 50 - (10)^2]$$

$$= (157 - 100) = 57 \text{ cm}^2$$

Now, Total surface area of solid

$$A = A_1 + A_2 + 2\pi r^2$$

$$= [500 + 57 + 2 \times 3.14 \times 50]$$

$$= 871 \text{ cm}^2$$

The cost of painting of solid

$$= \left(871 \times \frac{5}{100}\right) = 43.55 \text{ Rs}$$

5. A hemispherical bowl of internal diameter 36 cm contains liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find. the height of the each bottle, if 10% liquid is wasted in this transfer.

Ans : [Outside Delhi CBSE Board, 2015, Set I, II, III]

$$\text{Volume of bowl} = \frac{2}{3}\pi r^3$$

$$\text{Volume of liquid in bowl} = \frac{2}{3}\pi \times (18)^3 \text{ cm}^3$$

$$\text{Volume of one after wastage} = \frac{2}{3}\pi(18)^3 \times \frac{90}{100} \text{ cm}^3$$

$$\text{Volume of one bottle} = \pi r^2 h$$

Volume of liquid in 72 bottles

$$= \pi \times (3)^2 \times h \times 72 \text{ cm}^2$$

$$\text{Volume of bottles} = \text{volume in liquid after wastage}$$

$$\pi \times (3)^2 \times h \times 72 = \frac{2}{3}\pi \times (18)^2 \times \frac{90}{100}$$

$$h = \frac{\frac{2}{3}\pi \times (18)^2 \times \frac{90}{100}}{\pi \times (3)^2 \times 72}$$

Hence, the height of bottle $= 5.4$cm

6. A metallic has radius 3 cm and height 5 cm. To reduce its weights, a conical hole is drilled in the cylinder. The conical hole has a radius of $\frac{3}{2}$ cm and its depth $\frac{8}{9}$ cm calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape.

Ans : [Foreign Set I, II, III, 2015]

$$\text{Volume of cylinder} = \pi r^2 h = \pi(3)^2 \times 5$$

$$= 45\pi \text{ cm}^3$$

$$\text{Volume of conical hole} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times \frac{8}{9}$$

$$= \frac{2}{3}\pi \text{ cm}^3$$

$$\text{Metal left in cylinder} = 45\pi - \frac{2}{3}\pi = \frac{133\pi}{3}$$

$$\frac{\text{Volume of metal left}}{\text{Volume of metal taken out}} = \frac{\frac{133}{3}\pi}{\frac{2}{3}\pi} = 133 : 2.$$

Hence, Volume of metal left : Volume of metal cut off $= 133 : 2$

7. A solid right-circular cone of height 60 cm and radius 30 cm is dropped in a right-circular cylinder full of water of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder in cubic metre. Use $\pi = \frac{22}{7}$

Ans : [Foreign Set I, II, III, 2015]

Volume of water in cylinder = Volume of cylinder

$$\pi r^2 h = \pi \times (60)^2 \times 180$$

$$= 648000\pi \text{ cm}^3$$

Water displaced on dropping cone is equal to the volume of solid cone, which is

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (30)^2 \times 60$$

$$= 18000\pi \text{ cm}^3$$

Volume of water left in cylinder

$$= \text{Volume of cylinder} - \text{Volume of cone}$$

$$= 648000\pi - 18000\pi = 630000\pi \text{ cm}^3$$

$$= \frac{630000 \times 22}{1000000 \times 7} \text{ m}^3 = 1.98 \text{ m}^3$$

8. The rain water from 22 m × 20 m roof drains into cylindrical vessel of diameter 2 m and height 3.5 m. If the rain water collect from the roof the root fills $\frac{4th}{5}$ of cylindrical vessel then find the rainfall in cm.

Ans : [Foreign Set I, II, III, 2015]

Volume of water collected in cylindrical vessel

$$= \frac{4}{5} \times \pi \times (1)^2 \times \left(\frac{7}{2}\right) \text{ m}^3 = \frac{44}{5} \text{ m}^3$$

Let the rainfall be h m.

Rain water from roof $= 22 \times 20 \times h$ m^3

$$22 \times 20 \times h = \frac{44}{5}$$

$$h = \frac{44}{5} \times \frac{1}{22 \times 20} = \frac{1}{50} \text{ m}^3$$

$$= \frac{1}{50} \times 100 = 2 \text{ cm}$$

9. A hollow cylindrical pipe is made up of copper. It is 21 dm long. The outer and inner diameters of the pipe are 10 cm and 6 cm respectively. Find the volume of copper used in making the pipe.

Ans : [Board Term-2, 2015]

Height of cylindrical pipe $h = 21$ dm

$$= 210 \text{ cm}$$

External Radius $R = \frac{10}{2} = 5$ cm

Internal Radius $r = \frac{6}{2} = 3$

Volume of copper used in making the pipe

= (Volume of External Cylinder)

− (Volume of Internal Cylinder)

$$= \pi R^2 h - \pi r^2 h$$

$$= \pi h(R^2 - h^2)$$

$$= \frac{22}{7} \times 210(5^2 - 3^2) = \frac{22}{7} \times 210 \times 8 \times 2$$

$$= 10560 \text{ cm}^3.$$

10. A glass is in the shape of a cylinder of radius 7 cm and height 10 cm. Find the volume of juice in litre required to fill 6 such glasses. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2015]

Radius of the glass = 7 cm

Height of the glass = 10 cm

Volume of 1 glass $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 10$$

$$= 1540 \text{ cm}^3$$

Volume of juice to fill 6 glasses

$$= 6 \times 1540 = 9240 \text{ cm}^3$$

$$\text{Volume in litre} = \frac{9240}{1000} = 9.240 \text{ litre.}$$

11. The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left. Use $\pi = \frac{22}{7}$

Ans : [CBSE O.D. 2014]

Side of cube $a = 7$ cm

The diameter of the largest possible sphere is the side of the cube.

Thus radius of sphere $= \frac{7}{2}$ cm.

Volume of the wood left

$$= \text{volume of cube} - \text{volume of sphere}$$

$$= a^3 - \frac{4}{3}\pi r^3$$

$$= 7 \times 7 \times 7 - \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 7 \times 7 \times 7\left(1 - \frac{11}{21}\right)$$

$$= 7 \times 7 \times 7 \times \frac{10}{21} = \frac{490}{3}$$

Hence, Volume of wood $= 163.3$ cm^3.

12. In the given figure a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs. 500/ sq. metre. Use $\pi = \frac{22}{7}$

Ans : [Outside Delhi, Set-II 2016]

Radius of cylinder as well as conical part $= \frac{3}{2}$cm

Height of cylinder $h = 2.1$ m

Slant height of cone $r = 2.8$ cm

Total canvas required

$$2\pi rh + \pi rl = \frac{22}{7} \times \frac{3}{2}[4.2 + 2.8] \text{ m}^2$$

$$= \frac{22}{7} \times \frac{3}{2} \times 7.0 = 33 \text{ m}^2$$

$$\text{Total cost} = 33 \times 500 = 16,500$$

13. A girl empties a cylindrical bucket, full of sand, of radius 18 cm and height 32 cm, on the floor to form a conical heap of sand. If the height of this conical heap is 24 cm, then find its slant height correct upto one place of decimal.

Ans : [Foreign Set I, II, III, 2014]

$$\text{Volume of cone} = \text{Volume of Cylinder}$$

$$\frac{1}{3}\pi r_2^2 h = \pi r_1 h^2$$

$$\frac{1}{3} \times r_2^2 \times 24 = 18 \times 18 \times 32$$

$$r_2^2 = 1296$$

Radius of cone $r_2 = 36$ cm

Now, slant height of cone

$$l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 36^2}$$

$$= \sqrt{576 + 1296} = 43.2 \text{ cm.}$$

14. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. Use $\pi = \frac{22}{7}$

Ans : [Delhi 2013]

As per question the figure is shown below.

Radius of toy = radius of cylinder = 3.5 cm

Vol. of toy = Vol. of cylinder − 2 × Vol. of hemisphere

$$= \pi r^2 h - 2 \times \frac{22}{7}\pi r^3$$

$$= \pi r^2\left[h - \frac{4}{3}r\right]$$

$$= \frac{22}{7} \times (3.5)^2\left[10 - \frac{4}{3} \times 3.5\right]$$

$$= 22 \times 0.5 \times 3.5 \times 5.3$$

$$= 205.205 \text{ cm}^3.$$

15. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the total surface area of the vessel. Use $\pi = \frac{22}{7}$

Ans : [Delhi 2013]

As per question the figure is shown below.

Radius of hemisphere $\frac{14}{2} = 7$ cm

Height of cylinder $= 13 - 7 = 6$ cm

Total slanted area of cylinder

= S.A of hemisphere + S.A. of hemisphere

$$= 2\pi r^2 + 2\pi rh$$

$$= 2\pi r(r + h)$$

$$= 2 \times \frac{22}{7} \times 7(7 + 6)$$

$$= 44 \times 13 = 572 \text{ cm}^2$$

16. The radii of two right circular cylinders are in the ratio of 2 : 3 and their height are in the ratio of 5 : 4. Calculate the ratio of their curved surface area and radio of their volumes.

Ans : [Board Term-2, 2012 Set (22)]

Let the radii of two cylinders be $2x$ and $3x$ and their heights be $5y$ and $4y$ respectively

Ratio of their curved surface areas

$$= \frac{2\pi \times 2x \times 5y}{2\pi \times 3x \times 4y} = \frac{5}{6}$$

Since their curved surface areas are in the ratio of 5 : 6.

Ratio of their volumes

$$= \frac{\pi \times (2x)^2 \times 5y}{2\pi \times 3x \times 4y} = \frac{5 \times 4}{4 \times 9} = \frac{5}{9}$$

Hence, their volumes are in the ratio of 5 : 9 and their *C.SA* are in the ratio of 5 : 6.

17. A toy is in the form of a cone surmounted on a hemisphere of common base of diameter 7 cm. If the height of the toy is 15.5 cm, find the total surface area of the toy. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 Set (21, 22)]

As per question the figure is shown below.

Radius $r = 3.5$ cm

and height $h = 12$ cm

Slant height of cone,

$$l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 3.5^2} = 12.5$$

Total surface area of the toy

= Surface area of hemisphere +

+ Curved surface area of cone

$$= 2\pi r^2 + \pi rl$$

$$= \pi r(2r_1 + l)$$

$$= \frac{22}{7} \times \frac{7}{2}\left(2 \times \frac{7}{2} + 12.5\right)$$

$$= 11 \times 19.5 = 214.5 \text{ cm}^2$$

18. Water is flowing at 7 m/s through a circular pipe of internal diameter of 4 cm into a cylindrical tank, the radius of whose base is 40 cm. Find the increase in water level in 30 minutes.

Ans : [Board Term-2, 2012 Set (40)]

Volume of water in 30 minutes

$$= \pi \times (2)^2 \times 700 \times 60 \times 30 \text{ cm}^3$$

Let height of water in tank $= h$ cm

and radius = 40 cm

Volume of water in the tank

$$\pi 40^2 \times h = 700 \times 60 \times 30 \times 4 \times \pi$$

$$h = \frac{700 \times 60 \times 30 \times 4}{40 \times 40}$$

$$= \frac{6300}{2} \text{ cm} = \frac{63}{2} \text{ m}$$

Hence, water level increased is 31.5 m.

19. The slant height of a frustum of a cone is 4 m and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Ans : [Outside Delhi Set-I 2017]

Let the radii of frustum be r_1 and r_2.

$$2\pi r_1 = 18 \text{ cm and } 2\pi r_2 = 6 \text{ cm}$$

$$\pi r_1 = \frac{18}{2} = 9 \text{ cm } \pi r_2 = \frac{6}{2} = 3 \text{ cm}$$

and slant height $= 4$ cm

Curved surface area of frustum

$$= \pi(r_1 + r_2) \times l$$
$$= (\pi r_1 + \pi r_2) \times l$$
$$= (9 + 3) \times 4$$
$$= 12 \times 4 = 48 \text{ cm}^2$$

Hence, curved surface area $= 48$ cm^2

20. A metallic solid sphere of radius 10.5 cm melted and recasted into smaller solid cones each of radius 3.5 cn and height 3 cm. How may cones will be made ?

Ans : [Delhi Set-II 2017]

Radius of given sphere $= 10.5$ cm

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi \times 10.5 \times 10.5 \times 10.5$$
$$= 4\pi \times 3.5 \times 10.5 \times 10.5 \text{ cm}^3$$

Radius of one recasted cone $= 3.5$cm

Height $= 3$ cm

$$\text{Volume} = \frac{1}{3}\pi \times 3.5 \times 3.5 \times 3$$
$$= \pi \times 3.5 \times 3.5 \text{ cm}^3$$

Let the number of recasted cones be n.

$$n \times \pi \times 3.5 = 4 \times \pi \times 3.5 \times 10.5 \times 10.5$$
$$n = \frac{4 \times 3.5 \times 10.5 \times 10.5}{3.5 \times 3.5} = 126$$

Hence, number of recasted cones $= 126$.

21. A solid metallic sphere of diameter 16 cm is melted and recasted into smaller solid cones, each of radius 4 cm and height 8 cm. Find the number of cones so formed.

Ans : [Delhi Set-III 2017]

Diameter of sphere $= 16$ cm

$$\text{radius} = \frac{16}{2} = 8 \text{ cm}$$
$$\text{Volume} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \pi \times 8 \times 8 \times 8\text{cm}^3$$

Radius and height of recasted cones $= 4$ cm and 8 cm respectively.

Volume of each cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{4} \times \pi \times 4 \times 4 \times 8 \text{ cm}^3$$

Let number of cones recasted be n

$$n = \frac{\text{Volume of Sphere}}{\text{Volume of One Cone}}$$
$$= \frac{\frac{4}{3} \times \pi \times 8 \times 8 \times 8}{\frac{1}{3} \times \pi \times 4 \times 4 \times 8} = 16$$

Hence number of recasted cones $= 16$ cm

22. A solid sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged into water, by how much will the level of water rise in the cylindrical vessel ?

Ans :

Let the rise in level of water be h cm.

Radius of sphere $= 3$ cm. radius of cylinder

$$= \frac{12}{2} = 6 \text{ cm}$$

Volume of water displaced in cylinder will be equal to the volume of sphere.

Thus $$\pi r^2 h = \frac{4}{3}\pi r^3$$

$$\pi \times 6 \times 6 \times h = \frac{4}{3} \times \pi \times 3 \times 3 \times 3$$
$$h = \frac{4 \times 3 \times 3 \times 3}{3 \times 6 \times 6} = 1 \text{ cm}$$

Hence the water level rises $= 1$ cm.

23. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Ans :

As per question the figure is shown below.

Height of hemisphere $r = 3.5$ cm

Height of cone $h = 15.5 - 3.5 = 12$

Slant height of cone

$$\sqrt{r^2 + h^2} = \sqrt{12.5 + 144}$$
$$= \sqrt{156.25} = 12.5 \text{ cm}$$

TSA of toy= CSA of cone + CSA of hemisphere

$$\pi r l + 2\pi r^2 = \frac{22}{7} \times 12.5 \times 3.5 + 2 \times \frac{22}{7} \times 3.5 \times 3.5$$
$$= 22 \times 12.5 \times 0.5 + 22 \times 3.5$$
$$= 22\left(12.5 \times \frac{5}{10} + 3.5\right)$$
$$= 22\left(12.5 \times \frac{1}{2} + 3.5\right)$$
$$= 22(6.25 + 3.5)$$
$$= 22(9.75) = 214.5 \text{ cm}^2$$

Thus total surface area of toy is 214.5 cm^2

24. A conical vessel, with base radius 5 cm height 24 cm, is full of water. This water emptied into a cylindrical vessel, of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. Use $\pi = \frac{22}{7}$

Ans : [Outside Delhi, Set-II 2016]

Here radius and height of conical vessel are 5 cm and 24 cm.

$$\text{Volume of cone } = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times 2.5 \times 24 \text{ cm}^3$$

When water is emptied into cylindrical vessel, water will rise in cylindrical vessel. Let rise in height be h.

Volume of water raised $= \pi r^2 h$

$$\frac{1}{3}\pi \times 25 \times 24 = \pi \times (10)^2 \times h$$

$$25 \times 8 = 100h$$

$$h = 2 \text{ cm}$$

25. Water is flowing at the rate of 0.7 m/sec through a circular pipe whose internal diameter is 2 cm into a cylindrical tank, the radius of whose base is 40 cm. Determine the increase in the level of water in half hour.

Ans : [Board Sample Paper, 2016]

Length of water that flows in 1 sec. $= 0.7$ m

Length of water that flows out in 30 minutes

$$= (0.7 \times 100 \times 60 \times 30) \text{ cm}$$

$$= 126000 \text{ cm}$$

Volume of water that flows out in 30 minutes

$$= \pi(1)^2 \times 126000 \text{ cm}^3$$

$$= 126000\pi \text{ cm}^3$$

Let the depth of water in the tank be x cm.

Volume of water tank $= \pi(40)^2 \times x$ cm^3

Volume of tank $=$ Volume of water flows

$$\pi(40)^2 \times x = 126000\pi$$

$$x = 78.75 \text{ cm}$$

26. A well of diameter 4 m dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment.

Ans : [Delhi Set I, II, III, 2016]

Diameter of earth dug out $= 4$m

radius of earth dug out $= 2$ m

Depth of the earth $= 21$,

Volume of earth $= \pi r^2 d$

$$= \frac{22}{7} \times 2 \times 2 \times 21$$

$$= 264 \text{ m}^2$$

Width of embankment $= 3$ m

Outer radius of ring $= 2 + 3 = 5$ m

Let the height of embankment be h.

Volume of embankment

$$\pi(R - r)^2 = 264$$

$$\frac{22}{7} \times (25 - 4) \times h = 264$$

$$h = \frac{264 \times 7}{22 \times 21} = 4$$

Height of embankment is 4 m.

27. A cylindrical tub, whose diameter is 12 cm and height 15 cm is full of ice-cream. The whole ice-cream is to be divided into 10 children in equal ice-cream cones, with conical base surmounted by hemispherical top. If the height of conical portion is twice the diameter of base, find the diameter of conical part of ice-cream cones.

Ans : [Foreign Set I, II, III, 2016]

For cylindrical tub,

Diameter $D = 12$ cm

Radius $R = 6$ cm

Height $H = 15$ cm.

Volume $\pi R^2 H = \pi(6)^2 \times 15$

$$= 540\pi \text{ cm}^3$$

Each child will get the ice-cream $\frac{540\pi}{10}$ cm^3

$$= 54\pi \text{ cm}^3$$

For cone, height $h = 2 \times d = 2(2r) = 4r$

Volume of cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \times 4r = \frac{4}{3}\pi r^3$$

Volume of hemisphere$= \frac{2}{3}\pi r^3$

Total volume of cone and hemisphere

$$= \frac{4}{3}\pi r^3 + \frac{2}{3}\pi r^3 = \frac{6}{3}\pi r^3 = 2\pi r^3$$

According to question,

$$2\pi r^3 = 54\pi$$

$$r^3 = 27$$

$$r = 3$$

Hence, Diameter $= 2r = 2 \times 3 = 6$ cm.

28. A hemispherical tank, of diameter 3 m, is full of water. It is being emptied by a pipe at the rate of $3\frac{4}{7}$ litre per second. How much time will it take to make the tank half empty ? Use $\pi = \frac{22}{7}$

Ans : [Foreign Set I, II, III, 2016]

Diameter of tank $= 3$m

Radius $r = \frac{3}{2}$ m

Volume of hemispherical tank,

$$V = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi\left(\frac{3}{2}\right)^3 \text{ m}^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{27}{8} \text{ m}^3$$

$$= \frac{11}{7} \times \frac{9}{2} = \frac{99}{14} \text{ m}^3$$

Since 1 m^3 = 1000 litre, we have

$$V = \frac{99}{14} \times 1000 \text{ litre}$$

Volume of hemisphere

$$\frac{V}{2} = \frac{1}{2} \times \frac{99}{14} \times 1000 \text{ Litres}$$

Let time taken for this volume to flow out be t sec. Then according to question,

$$t \times 3\frac{4}{7} = \frac{1}{2} \times \frac{99}{14} \times 1000$$

$$t \times \frac{25}{7} = \frac{1}{2} \times \frac{99}{14} \times 1000$$

$$t = \frac{7}{25} \times \frac{1}{2} \times \frac{99}{14} \times 1000$$

$$= 990 \text{ sec}$$

$$= 16 \text{ minutes } 30 \text{ sec.}$$

29. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. Use $\pi = \frac{22}{7}$

Ans : [Outside Delhi CBSE Board, 2015, Set I, II, III]

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

Volume of metal in 504 cones

$$= 504 \times \frac{\pi}{3} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3$$

$$\text{Volume of Sphere} = \frac{4}{3}\pi r^3 = \frac{4\pi}{3} \times r^3$$

$$= \text{Volume of 504 cones}$$

$$\frac{4\pi}{3} \times r^3 = 504 \times \frac{\pi}{3} \times \frac{35}{20} \times \frac{35}{20} \times 3$$

$$r^3 = 126 \times \frac{7}{4} \times \frac{7}{4} \times 3$$

$$= 7 \times 9 \times 2 \times \frac{7}{4} \times \frac{7}{4} \times 3$$

$$= 3 \times 3 \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \times 3$$

$$r = 3 \times \frac{7}{2} = 10.5 \text{ cm}$$

Thus diameter is 21 cm.

$$\text{Surface area} \quad 4\pi r^2 = 4 \times \frac{22}{7} \times 10.5 \times 10.5$$

$$= 1386 \text{ cm}^2$$

30. A solid metallic cone of radius 2 cm and height 8 cm is melted into a sphere. Find the radius of sphere.

Ans : [Board Term-2, 2014]

Let the radius of sphere be R.

$$\text{Volume of sphere} = \text{Volume of cone}$$

$$\frac{4}{3}\pi r^2 h = \frac{1}{3}\pi r^2 h$$

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times 2 \times 2 \times 8$$

$$R^3 = \frac{2 \times 2 \times 8}{4}$$

$$R^3 = 8$$

$$R = 2 \text{ cm}$$

31. A sphere of diameter 12 cm, is dropped in a right circular cylindrical verse, partly filled with water. If the sphere is completely submerged in water, the water level into the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

Ans : [Outside Delhi Set-II, 2016]

$$\text{Diameter of sphere} = 12 \text{ cm}$$

$$\text{Its radius} = 6 \text{ cm}$$

$$\text{Volume} = \frac{4}{3}\pi \times 6^3 \text{ cm}^3$$

It is submerged into water, in cylindrical vessel, then water turn rise by $3\frac{5}{9} = \frac{32}{9}$ cm

$$\text{Volume submerged} = \text{Volume rise}$$

Let radius of cylinder be r cm

$$\frac{4}{3}\pi \times 6^3 = \pi \times r^2 \times \frac{32}{9} \text{ cm}$$

$$\frac{216 \times 3 \times 4}{32} = r^2$$

$$\frac{4 \times 27 \times 3}{4} = r^2 \Rightarrow 4 \times \frac{81}{4} \text{ cm}^3 = r^2$$

$$r = 9 \text{ cm}$$

$$\text{Diameter} \qquad 2r = 2r = 2 \times 9 = 18 \text{ cm.}$$

32. The $\frac{3}{4}$th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel.

Ans : [Delhi Set-I 2017]

$$\text{Radius of conical vessel} = 5 \text{ cm}$$

$$\text{and its height} = 24 \text{ cm}$$

$$\text{Volume of this vessel} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 5 \times 5 \times 24$$

$$= 200\pi \text{ cm}^3$$

Internal radius of cylindrical vessel= 10

Let the height of emptied water be h.

Volume of water in cylinder,

$$\pi r^2 h = \frac{3}{4} \times \text{Volume of cone}$$

$$\pi \times 10 \times 10 \times h = \frac{3}{4} \times 200\pi$$

$$100\pi h = 150\pi$$

$$h = 1.5 \text{ cm}$$

Hence the height of water= 1.5 cm

33. Rampal decided to donate convas for 10 tents conical in shape with base diameter 14 m and height 24 m to a centre for handicapped person's welfare. If the cost of 2 m wide canvas is Rs. 40 per meter, find the amount by which Rampal helped the money.

Ans : [Outside Delhi Compt. Set-I, II, III 2017]

$$\text{Diameter of tent} = 14\text{m and height} = 24 \text{ m}$$

$$\text{radius of tent} = 7\,\text{m}$$

$$\text{Slant height} = \sqrt{h^2+r^2} = \sqrt{24^2+7^2}$$

$$= \sqrt{576+49} = 25 \text{ m}$$

Surface area of the tent

$$\pi rl = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Surface area of 10 tents

$$= 550 \times 10 = 5500$$

$$\text{Total cost} = 5500 \times \frac{40}{2} = 110000$$

Hence, Rampal helped the centre

$$= Rs. 110000$$

34. A cone of maximum size is curved out from a cube edge 14 cm. Find the surface area of remaining solid after the cone is curved out.

Ans : [Sample Question Paper 2017]

$$\text{Side of cube} = 14 \text{ cm.}$$

Cone of maximum size is curved out

$$\text{Diameter of cone} = 14 \text{ cm}$$

$$\text{Radius of cone} = 7 \text{ cm}$$

$$\text{Slant height } l = \sqrt{h^2+r^2} = \sqrt{14^2+7^2}$$

$$= \sqrt{196+49} = \sqrt{245}$$

$$= 15.65 \text{ cm.}$$

$$\text{Total surface area} = \text{Surface area cube + curved}$$

$$\text{Surface area of cone} - \text{Circular area of base of cone}$$

$$= 6a^2 + \pi rl - \pi r^2$$

$$= 6 \times 14 \times 14 + \frac{22}{7} \times 7 \times 15.65 - \frac{22}{7} \times 7 \times 7$$

$$= 1176 + [22(15.65-7)]$$

$$= 1176 \times 22 \times 8.65$$

$$= 223792.8 \text{ cm}^3$$

35. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation ?

Ans :

Water flow in 1 hr

$$= \text{Area of cross-section} \times \text{Speed of water}$$

$$= 5.4 \times 1.8 \times 25000 \text{ m}^3$$

$$= 54 \times 18 \times 250 \text{ m}^3$$

Water floe in 4 minutes

$$= 54 \times 18 \times 250 \times \frac{40}{60} \text{ m}^3$$

$$= 54 \times 6 \times 500 \text{ m}^3$$

$$\text{Irrigated area} \times \frac{10}{100} = 54 \times 6 \times 500$$

$$\text{Irrigated area} = 54 \times 6 \times 500 \times 10$$

$$= 1620000 \text{ m}^3$$

36. From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of same height and same base radius is hollowed out. Find the total surface area of the remaining solid. (Take $\pi = 3.14$)

Ans : [Outside Delhi Comp. Set-I, II III 2017]

$$\text{Height of cylinder} = \text{height of cone} = 8 \text{ cm}$$

$$\text{radius of cylinder} = \text{radius of cone} = 6 \text{ cm}$$

$$\text{Slant height of cone} = \sqrt{8^2+6^2} = \sqrt{64+36}$$

$$= 10 \text{ cm}$$

Total surface area of remaining solid

$$= \text{Surface area of cylinder+}$$

$$+ \text{ Surface area of cone + area of top}$$

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= \pi r(2h + l + r)$$

$$= \frac{22}{7} \times 6(2 \times 8 + 10 + 6)$$

$$= \frac{22}{7} \times 6 \times 32$$

$$= 603.43$$

Hence total surface area $= 603.43 \text{ cm}^2$

37. From a solid cylinder of height 24 cm and diameter 14 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid.

Ans : [Delhi Comp Set-I, II III 2017]

$$\text{Height of the cylinder} = \text{height of the cone}$$

$$= 24 \text{ cm.}$$

$$\text{and radius of cylinder} = \text{radius of cone}$$

$$= \frac{14}{2} = 7 \text{ cm}$$

$$\text{Slant height of cone} = \sqrt{h^2+r^2} = \sqrt{24^2+7^2}$$

$$= \sqrt{576+49} = 25 \text{ cm}$$

Total surface area of remaining part

$$= \text{Surface area of cylinder+}$$

$$+ \text{ Surface area of cone + area of top}$$

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= \pi r(2h + l + r)$$

$$= \frac{22}{7} \times 7(2 \times 24 + 25 + 7)$$

$$= 22 \times 80$$

$$= 1760 \text{ cm}^2$$

38. The perimeters of the ends of the frustum of a cone are 207.24 cm and 169.56 cm. If the height of the frustum be 8 cm, find the whole surface area of the frustum. (Use $\pi = 3.14$)

Ans : [Board Sample Paper, 2016]

Let R and r be the radii of the circular ends of the frustum where $R > r$

Now $\quad 2\pi R = 207.24$

$$R = \frac{207.24}{2 \times 3.14} = 33 = 33 \text{ cm}$$

and $\quad 2\pi r = 169.56$ cm

$$r = \frac{169.56}{2\times 3.14} = 27 = 27 \text{ cm}$$

Now $$l^2 = h^2 + (R-r)^2 = 8^2 + (33-27)^2 = 100$$

$$l = 10 \text{ cm}$$

Whole surface area of the frustum

$$\begin{aligned} &= \pi\left(R^2 + r^2 + (R+r)l\right) \\ &= 3.14\left[(33)^2 + (27)^2 + (33+27)10\right] \\ &= 3.14(1089 + 729 + 600) \\ &= 3.14 \times 2418 \text{ cm}^2 \\ &= 7592.52 \text{ cm}^2. \end{aligned}$$

39. A metal container, open from the top, is in the shape of a frustum of a cone of height 21 cm with radii of its lower and upper circular ends as 8 cm and 20 cm repetitively. Find the cost of milk which can completely fill the container at the rate of Rs. 35 per litre. Use $\pi = \frac{22}{7}$

Ans : [Foreign Set I, II, III, 2016]

As per question the figure is shown below.

If r_1 and r_2 be the radii of two circular ends and h be the height of frustum, then volume

$$= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$$

We have $$r_1 = 8 \text{ cm}$$

$$r_2 = 20 \text{ cm}$$

and $$h = 21 \text{ cm}$$

$$\begin{aligned} \text{Volume} &= \frac{1}{3} \times \frac{22}{7} \times 21\left[(8)^2 + (20)^2 + 8 \times 21\right] \\ &= 22[64 + 400 + 160] \\ &= 22 \times 624 \\ &= 13728 \text{ cm}^3 \\ &= \frac{13728}{1000} \text{ lit}(\because 1000 \text{ cm}^3 = 1 \text{ lit.}) \\ V &= 13.728 \text{ litres} \\ \text{Total Cost} &= Rs. 13.728 \times 35 \\ &= Rs.\, 480.48 \end{aligned}$$

40. A cone is cut by a plane parallel to the base and upper part is removed. If the curved surface area of upper cone is $\frac{1}{9}$ times the curved surface of original cone. Find the ratio of line segment to which the con's height is divided by the plane.

Ans : [Board Term-2, 2014]

As per question the figure is shown below.

$$\frac{\text{Curved surface of upper cone}}{\text{Curved surface of original cone}} = \frac{1}{9}$$

$$\frac{\pi rl}{\pi RL} = \frac{1}{9}$$

$$\frac{rl}{RL} = \frac{1}{9} \qquad ...(1)$$

Since by AA similarity $\Delta AOB - \Delta ACD$, thus

$$\frac{r}{R} = \frac{h}{H} = \frac{l}{L} \qquad (2)$$

Substituting (2) in (1) we have

$$\frac{h}{H} \times \frac{h}{H} = \frac{1}{9}$$

$$\frac{h^2}{H^2} = \frac{1}{9}$$

or, $$\frac{h}{H} = \frac{1}{3}$$

Hence $$\frac{\text{Height of upper cone}}{\text{Height of lower frustum}} = \frac{1}{3-1} = \frac{1}{2}$$

Ratio of the line segments $OA : OC = 1 : 2$

41. The slant height of a frustum of a cone is 4 cm and the perimeter (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 Set (12)]

As per question the figure is shown below.

We have $$l = 4 \text{ cm}$$

$$2\pi R = 18\text{cm}$$

$$R = \frac{18}{2\pi} = \frac{9}{\pi}$$

and $$2\pi r = 6$$

$$r = \frac{6}{2\pi} = \frac{3}{\pi} \text{ cm}$$

Curved surface area of frustum

$$\begin{aligned} \pi l(R+r) &= \pi \times 4\left(\frac{9}{\pi} + \frac{3}{\pi}\right) \\ &= 4\pi \times \frac{12}{\pi} = 48 \text{ cm}^2. \end{aligned}$$

LONG ANSWER TYPE QUESTIONS

1. A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 m high embankment. Find the width of the embankment.

Ans : [Delhi CBSE Board, 2012 Set I, II]

Depth of well = 14 m, radius = 12 m.

Volume of earth taken out

$$\pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 14$$

$$= 176 \text{ m}^3$$

Let r be the width of embankment. The radius of outer circle of embankment

$$= 2 + r$$

Area of upper surface of embankment

$$= \pi\left[(2+r)^2 - (2)^2\right]$$

Volume of embankment= Volume of earth taken out

$$\begin{aligned} \pi\left[(2+r)^2 - (2)^2\right] \times 0.4 &= 176 \\ \pi[4 + r^2 + 4r - 4] \times 0.4 &= 176 \\ r^2 + 4r &= \frac{176 \times 7}{0.4 \times 22} \\ r^2 + 4r &= 140 \\ r^2 + 4r - 140 &= 0 \\ (r+14)(r-10) &= 0 \\ r &= 10 \end{aligned}$$

Hence width of embankment $= 10$ m.

2. A hemispherical depression is cut from one face of a cubical block, such that diameter 'l' of hemisphere is equal to the edge of cube. find the surface area of the remaining solid.

Ans : [CBSE Set I, II, III, 2014]

Let the radius of hemisphere be r.

Therefore, $r = \frac{l}{2}$

Now, the required surface area

= Surface area of cubical block +

− Area of base of hemisphere +

+ Curved surface area of hemisphere.

$$= 6(l)^2 - \pi r^2 + 2\pi r^2$$

$$= 6l^2 - \pi\left(\frac{l}{2}\right)^2 + 2\pi\left(\frac{l}{2}\right)^2$$

$$= 6l^2 - \frac{\pi l^2}{4} + \frac{\pi}{2}l^2$$

$$= 6l^2 + \frac{\pi l^2}{4}$$

$$\text{Surface area} = \frac{1}{4}(24 + \pi)l^2 \text{ units.}$$

$$= \frac{1}{4}\left(24 + \frac{22}{7}\right)^2 l^2$$

$$= \frac{1}{4} \times \frac{190}{7} \times \frac{190}{7} l^2$$

$$= 184.18 l^2 \text{ unit}^2$$

3. Water in a canal 6 m wide and 1.5 m deep is flowing with a speed of 10 km/h. How much area in hectare will it irrigate in 30 minutes if 8 cm of standing water is needed ?

Ans : [KVS 2014][Delhi Set, 2014] [Board Term-2, 2012 (13)]

As per question the figure is shown below.

$$\text{Water flows in 1 hr} = 10 \text{ km}$$

$$\text{Water flows in } \frac{1}{2}\text{hr} = \frac{10}{2}$$

$$= 5 \text{ km}$$

$$= 5000 \text{ m}$$

Now volume of water flows in $\frac{1}{2}$ hr

$$lbh = 5000 \times 6 \times 1.5 \text{m}^3$$

$$= 45000 \text{ m}^3.$$

According to the question,

$$\text{Volume of water } \frac{1}{2} \text{ hr} = \text{area of irrigated field} \times \frac{8}{100}$$

$$45000 = \text{Area} \times \frac{8}{100}$$

$$\text{Area} = \frac{45000 \times 100}{8} = 562500 \text{ m}^2$$

$$= 56.25 \text{ hectare.}$$

4. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/hr, in how much time will the tank be filled ?

Ans : [Delhi Set, 2014]

[Board Term-2, 2012 (13)]

$$\text{Diameter of pipe} = 20\text{cm}.$$

$$\text{Radius of pipe} = \frac{20}{2} = 10 \text{ cm} = 0.10\text{m}$$

$$\text{Diameter of tank} = 10 \text{ cm}$$

$$\text{radius of the tank} = \frac{20}{2} = 5 \text{ cm}$$

$$\text{Depth of tank} = 2 \text{ m}$$

$$\text{Volume of tank} = \pi r^2 h = \pi \times 5 \times 5 \times 2 = 50\pi$$

Speed of the water 3 km/hr.

$$= \frac{300}{60} = 50 \text{ m/min}$$

Volume of water supplied in one minute

$$\pi r^2 h = \pi \times 0.10 \times 0.10 \times 50$$

$$\text{Time taken} \quad t = \frac{50\pi}{\pi \times 0.10 \times 0.10 \times 50} = 100$$

Hence time taken to fill the tank is 100 minutes.

5. The internal and external diameters of a hollow hemispherical vessel are 16 cm and 12 cm respectively. If the cost of painting 1 cm^2 of the surface area is Rs. 5.00, find the total cost of painting the vessel all over. (Use $\pi = 3.14$)

Ans :

$$\text{Here} \quad R = 8 \text{ cm}, r = 6 \text{ cm}$$

$$\text{Surface area} = 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$$

$$= \pi\left[2 \times 8^2 + 2 \times 6^2 + (8^2 - 6^2)\right]$$

$$= \pi[64 \times 2 + 36 \times 2 + (64 - 36)]$$

$$= \pi[128 + 72 + 28]$$

$$= 228 \times 3.14 = 715.92 \text{ cm}^2$$

$$\text{Total cost} = 715.92 \times 5 = 3579.60 \text{ Rs}$$

6. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.

Ans : [Delhi 2013]

Volume of water flowing through pipe in 1 sec.

$$\pi R^2 H = \pi \times (1)^2 \times 0.4 \times 100 \text{ cm}^3$$

Volume of water flowing in 30 min $(30 \times 60 \text{ sec})$

$$= \pi \times (1)^2 \times 0.4 \times 100 \times 30 \times 60$$

Volume of water in cylindrical talk in 30 min

$$\pi r^2 h = \pi \times (40)^2 \times h$$

Now

$$\pi \times (40)^2 \times h = \pi \times (1)^2 \times 0.4 \times 100 \times 30 \times 60$$

Rise in water level

$$h = \frac{\pi \times (1)^2 \times 0.4 \times 100 \times 30 \times 60}{\pi \times 40 \times 40}$$

$$= 45 \text{ cm.}$$

Thus level of water in the tank is 45 cm.

7. A toy is in the form of a cylinder of diameter $2\sqrt{2}$ m and height 3.5 m surmounted by a cone whose vertical angle is 90°. Find total surface area of the toy.

Ans : [Board Term-2, 2012 (44)]

As per question the figure is shown below.

Here $\angle C = 90°$ and $AC = BC = l$

Thus $AB^2 = AC^2 + BC^2 = l^2 + l^2 = 2l^2$

Now $(2\sqrt{2})^2 = 2l^2$

Thus $l = 2$ and $r = \sqrt{2}$ m

Slant height of conical portion

$$l = 2 \text{ m}$$

Total surface area of toy

$$2\pi rh + \pi r^2 + \pi rl = \pi r[7 + \sqrt{2} + 2] \text{ m}^2$$
$$= \pi\sqrt{2}[9 + \sqrt{2}] \text{ m}^2$$
$$= \pi[2 + 9\sqrt{2}] \text{ m}^2$$

8. Find the volume of the largest solid right circular cone that can be cut out off a solid cube of side 14 cm.

Ans : [Board Term-2, 2012 (1)]

The base of cone is the largest circle that can be inscribed in the face of the cube and the height will be equal to edge of the cube.

$$\text{Radius of cone} = \frac{14}{2} = 7 \text{ cm}$$

$$\text{Height of cone} = 14 \text{ cm}$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 14$$
$$= \frac{2156}{3} = 718.67.$$

9. Water is flowing at the rate of 15 km/hr through a cylindrical pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time the level of water in pond rise by 21 cm ?

Ans : [Board Term-2, 2012 Set (5)]

Speed of water flowing through the pipe

$$= 15 \text{ km/hr} = 15000 \text{ m/hr}$$

Volume of water flowing in 1 hr

$$\pi R^2 H = \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000 \text{ m}^3$$
$$= 231 \text{ m}^3$$

Volume of water in the tank when the depth is 21 cm

$$lbh = 50 \times 44 \times \frac{21}{100} \text{ m}^3 = 462 \text{ m}^3$$

Time taken to fill 462 m³

$$= \frac{462}{231} = 2 \text{ hrs.}$$

10. A medicine capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends, the length of the entire capsule is 15 mm and the diameter of the capsule is 5 mm. Find the Volume of the capsule.

Ans : [Board Term-2, 2012 Set (12)]

As per question the figure is shown below.

$$\text{Total height} = 14 \text{ mm}$$

$$\text{Height of cylinder} = 14 - 2 \times 2.5$$
$$= 14 - 5 = 9 \text{ mm}$$

$$\text{Radius of cylinder} = 2.5\text{mm}$$

$$\text{Radius of hemisphere} = 2.5 \text{ mm}$$

$$\text{Volume of capsule} = \text{Volume of two hemispheres} + \text{Volume of cylinder}$$
$$= 2 \times \frac{2}{3}\pi r^3 + \pi r^2 h$$
$$= \frac{4}{3}\pi\left(\frac{5}{2}\right)^3 + \pi\left(\frac{5}{2}\right)^2 \times 9$$
$$= \left(\frac{5}{2}\right)^2 \times \pi\left[\frac{4}{3} \times \frac{5}{2} + 9\right]$$
$$= \frac{25}{4}\pi\left[\frac{10}{3} + 9\right] = \frac{25}{4}\pi\left[\frac{10 + 27}{3}\right]$$
$$= \frac{25}{4}\pi\left[\frac{37}{3}\right] = \frac{25}{4} \times \frac{22}{7} \times \frac{37}{3}$$
$$= \frac{10175}{42} \text{ mm}^3$$
$$= 242.26 \text{ mm}^3.$$

11. A milk tanker cylindrical in shape having diameter 2 m and length 4.2 m supplies mild to the two booths in the ratio of 3 : 2. One of the milk booths has cuboidal vessel having base area 3.96 sq. m. and the other has a cylindrical vessel having radius 1 m. Find the level of milk in each of the vessels. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 (28)]

$$\text{Volume of milk} = \frac{22}{7} \times 1 \times 1 \times 4 \cdot 2 = 13 \cdot 2 \text{ m}^3$$

Supply of milk to booth I

$$= 13 \cdot 2 \times \frac{3}{5} = 2 \cdot 64 \times 3 = 7.92 \text{ m}^3$$

Supply of milk to booth II

$$= 13 \cdot 2 \times \frac{2}{5} = 2 \cdot 64 \times 2 = 5.28 \text{ m}^3$$

$$\text{Height in } 1^{sh} \text{ vessel} = \frac{7 \cdot 92}{3 \cdot 96} = 2 \text{ m}$$

$$\text{Height in } 2^{nd} \text{ vessel} = \frac{5 \cdot 28}{\frac{22}{7} \times 1} = \frac{5 \cdot 28 \times 7}{22} = 1.68 \text{ m}$$

12. In fig from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid. Use $\pi = \frac{22}{7}$ and $\sqrt{5} = 2.236$.

Ans : [Delhi CBSE Board, 2015 Set I, II, III]

Let r be the radius of the top after cutting

$$h = 12 - 4 = 8 \text{ cm}$$

Now $\frac{4}{r} = \frac{12}{6} \Rightarrow r = 2$ cm

Now slant length of frustum

$$l = \sqrt{h^2 + (R - r)^2}$$
$$= \sqrt{(8)^2 + (6 - 2)^2}$$
$$= \sqrt{64 + 16} = \sqrt{80}$$

$$= 4\sqrt{5} = 4 \times 2.236$$

$$= 8.944 \text{ cm}$$

Total surface area of frustum

$$= \pi\left[R^2 + r^2 + l(R + r)\right]$$

$$= \frac{22}{7}\left[(6)^2 + (2)^2 + 8.944(6 + 2)\right]$$

$$= \frac{22}{7}[36 + 4 + 71.552]$$

$$= \frac{22}{7} \times 111.552$$

$$= 350.59 \text{ cm}^2.$$

13. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire. Use $\pi = \frac{22}{7}$

Ans : [Outside Delhi Set I, II, III, 2015]

As per question the figure is shown below.

Volume of cylinder,

$$\pi r^2 h = \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times 10 \text{ cm}^3$$

$$= 554.40 \text{ cm}^3$$

Volume of metal scooped out

$$= 2 \times \text{Volume of hemisphere}$$

$$= 2 \times \frac{2}{3} \times \pi r^3 = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{42}{10}\right)^3$$

$$= 310.46 \text{ cm}^3$$

Volume of rest of cylinder

$$= 554.40 - 310.46 = 243.94 \text{ cm}^3$$

Now from rest volume a wire of thickness 1.4 cm i.e radius 0.7 cm is formed. Let length of wire be l. Thus volume of wire and rest cylinder will be equal.

Volume of wire, $\pi r^2 l = 243.94 \text{ cm}^3$

$$\frac{22}{7} \times \frac{7}{10} \times \frac{7}{10} \times l = 243.94 \text{ cm}^3$$

$$l = \frac{243.94 \times 10 \times 10}{22 \times 7}$$

$$h = 158.4 \text{ cm}$$

14. 150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.

Ans : [CBSE O.D. 2014]

Diameter of spherical marble = 14 cm

Radius $r_1 = \frac{1.4}{2} = 0.2 = \frac{7}{10}$ cm

Diameter of cylindrical vessel = 7 cm

Radius $R = \frac{7}{2} = 3.5$ cm

Let h be the rise in water level then,

Volume of 150 spherical marbles = volume of water rise

$$150 \times \frac{4}{3} \times \pi \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10} = \pi \times \frac{7}{2} \times \frac{7}{2} \times h$$

$$h = \frac{4 \times 7}{5}$$

$$\frac{28}{5} = h$$

$$h = 5.6 \text{ cm}$$

Thus 5.6 cm will be rise in the level of water.

15. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into toys in the shape of a cone of radius 3 cm and height 9 cm. Find the number of toys formed so.

Ans : [Outside Delhi Compt. 2017]

Height of cylinder = 15 cm

Diameter = 12 cm

Radius = 6 cm

Radius of cone = 3 cm

and height = 9 cm

Let the number of toys recast be n.

Volume of n conical toys = Volume of cylinder

$$n \times \frac{1}{3}\pi \times 3 \times 3 \times 9 = \pi \times 6 \times 6 \times 15$$

$$n = \frac{6 \times 6 \times 15}{3 \times 9}$$

$$= 20$$

Hence the number of toys = 20.

16. A well diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly around it to a width of 5 m. to form a embankment. Find the height of the embankment.

Ans : [CBSE Foreign 2017]

The volume of soil taken out from the well

$$\pi^2 rh = \frac{3}{2} \times \frac{3}{2} \times 14\pi \text{ m}^3$$

The radius of embankment with well

$$= \frac{3}{5} + 5 = \frac{13}{2} \text{ m}$$

Let the height of embankment be x. Then the volume of soil used in embankment,

$$\pi(R^2 - r^2)x = \pi r^2 h$$

$$\pi\left[\left(\frac{13}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right]x = \frac{3}{2} \times \frac{3}{2} \times 14\pi$$

$$\frac{160}{4}x = \frac{3}{2} \times \frac{3}{2} \times 14$$

$$x = \frac{3 \times 3 \times 14}{160} = 0.7875 \text{ m}$$

Hence the height of embankment = 78.75cm

17. Water is flowing at the rate of 5 km/hour through a pipe of diameter 14 cm into a rectangular tank of dimensions 50 m × 44 m. Find the time in which the level of water in the tank will rise by 7 cm.

Ans : [Delhi Compt 2017]

$$\text{Speed of water in pipe} = 5 \text{ km/hour}$$

$$\text{In an hour length of water} = 5000 \text{ m}$$

Let time taken to fill the tank be t.

$$\text{Total length of water} = t \times 5000 \text{ m}$$

Volume of water flown= Volume of water in tank

$$\pi r^2 h = l \times b \times h$$

$$\frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 500t = 50 \times 44 \times \frac{7}{100}$$

$$\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 5000t = 50 \times 44 \times \frac{7}{100}$$

$$t = \frac{50 \times 44}{22 \times 50} = 2$$

Hence, Time taken to fill the tank = 2 hours.

18. A bucket open at the top is in form of a frustum of a cone with a capacity of 12308.8 cm3. The radii of he top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. (Use $\pi = 3.14$)

Ans : [Delhi Set I, II, III, 2016]

Here $R = 20, r = 12, V = 12308.8$

$$V = \frac{1}{3}\pi(R^2 + r^2 + Rr)h$$

$$12308.8 = \frac{1}{3} \times 3.14(400 + 240 + 144)h$$

$$12308.8 = \frac{1}{3} \times 3.14 \times 784$$

$$h = 15 \text{ cm}$$

Now $l = \sqrt{(20-12)^2 + 15^2} = 17$ cm

Total area of metal sheet used,

$$= CSA + \text{Base area}$$

$$= \pi[(20+12) \times 17 + 12 \times 12]$$

$$= 2160.32 \text{ cm}^2$$

19. The radii of the circular ends of a frustum of cone of height 6 cm are 14 cm and 6 cm respectively. Find the lateral area and total surface area of the frustum.

Ans : [Board Term-2, 2012 Set (59)]

We have $r_1 = 14$ cm, $r_2 = 6$ cm, $h = 6$ cm

$$l = \sqrt{h^2 + (r_1 + r_2)^2}$$

$$= \sqrt{6^2 + (14-6)^2} = \sqrt{6^2 + 8^2}$$

$$= \sqrt{36 + 64} = 10\text{cm}$$

Lateral surface area,

$$\pi(r_1 + r_2)l = \frac{22}{7} \times (14+6) \times 10 \text{ cm}^2$$

$$= 628.57 \text{ cm}^2$$

Total surface area

$$\pi\left[r_1^2 + r_2^2 + l(r_1 + r_2)\right] = \frac{22}{7} \times [(196+36) + 20 \times 10]$$

$$= \frac{22}{7} \times 432 = 1357.71 \text{ cm}^2$$

20. A cone of radius 10 cm is divided into two parts by a plane parallel to its base through the mid-point of its height. Compose the Volume of the two parts.

Ans : [Delhi Set-III 2017]

As per question the figure is shown below.

Since $\Delta ABC \sim \Delta APQ$ we have

$$\frac{h}{2h} = \frac{r_1}{10} \Rightarrow r_1 = 5 \text{ cm}$$

Volume of smaller cone

$$= \frac{1}{3}\pi(5)^2 \times h$$

$$\text{Volume of frustum} = \frac{1}{2}\pi \times h(5^2 + 10 + 5 \times 10)$$

$$= \frac{1}{3}\pi \times h \times 175$$

$$\text{Required ratio} = \frac{\frac{1}{3} \times \pi \times 25 \times h}{\frac{1}{3} \times \pi \times h \times 175} = \frac{1}{7}$$

21. The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the two parts.

Ans : [Delhi Set-I, II, III 2017]

As per question the figure is shown below.
Let the radius of cone be r_2 and cut off cone be r_1

Height of the cone = 10 cm

And the height the cone cut off = 5cm

Since $\Delta AOC \sim \Delta AO'D$, we have

$$\frac{AO}{AO'} = \frac{r_2}{r_1} = \frac{10}{5}$$

$$r_2 = 2r_1$$

$$\text{Volume of cut off cone} = \frac{1}{3}\pi r_1^2 \times 5$$

$$= \frac{1}{3}\pi r_1^2 \text{ sq. units}$$

$$\text{Volume of original cone} = \frac{1}{3}\pi(2r_1)^2 \times 10$$

$$= \frac{40}{3}\pi r_1^2 \text{ sq. units}$$

Volume of frustum

= Volume of original cone − Volume of cut of cone

$$= \frac{40}{3}\pi r_1^2 - \frac{5}{3}\pi r_1^2 = \frac{35}{3}\pi r_1^2 \text{ sq.units}$$

$$\text{Ratio of two parts} = \frac{35\pi r_1^2}{5\pi r_1^2} = \frac{7}{1}$$

Hence the ratio of two parts = 7 : 1

22. A metallic right circular cone 20 cm high and whose vertical angel is 60° is cut into two parts at the middle of its height by a plane parallel to its base if the frustum so obtained be drawn into a wire of uniform diameter $\frac{1}{16}$ cm, find the length of the wire.

Ans : [Foreign Set-I 2017]

As per question the figure is shown below.

Total height of cone = 20 cm

and Vertex angle $= 30^\circ$

Let the radius of cone be r_2. Then we have

$$\frac{r_2}{20} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$r_2 = \frac{20}{\sqrt{3}} \text{ cm}$$

The height of the cone cut off $= 10$ cm

Let its radius be r_1. Then

$$\frac{r_1}{10} = \tan 30^\circ = \frac{1}{\sqrt{3}} \text{ cm}$$

$$r_1 = \frac{10}{\sqrt{30}} \text{ cm}$$

Let the length of wire be l. Its radius is $\frac{1}{32}$ cm.

Now Volume of frustum $=$ Volume of wire

$$\frac{1}{3}\pi \times h\left[(r_1)^2+(r_2)^2+(r_1 r_2)\right] = \pi r^2 l$$

$$\frac{1}{3} \times 10 \times \pi\left[\left(\frac{10}{3}\right)^2+\left(\frac{20}{3}\right)^2+\frac{10}{3}\times\frac{20}{3}\right] = \pi\left(\frac{1}{32}\right)^2 \times l$$

$$\frac{1}{3} \times 10\left[\frac{100}{9}+\frac{400}{9}+\frac{200}{9}\right] = \frac{1}{32 \times 32} \times l$$

$$\frac{1}{3} \times 10 \times \frac{700}{9} = \frac{1}{32} \times \frac{1}{32} \times l$$

$$l = \frac{32 \times 32 \times 700 \times 10}{3 \times 9}$$

$$= 796444.4 \text{ cm.}$$

Hence, the length of wire is 7964.44 m.

23. A right circular cone is divided into three parts trisecting its height by two planes drawn parallel to the base. Show that volumes of the three portions starting from the top are in the ratio 1 : 7 : 19.

Ans : [Foreign Set-III 2017]

As per question the figure is shown below.
Let the radii of three cones from top be r_1, r_2 and r_3 respectively.
Let the height of given cone be $3h$. So, the height of cone ADE is $2h$and height of cone ABC is h.

Since $\Delta ABC \sim ADE$,

$$\frac{r_1}{r_2} = \frac{h}{2h} \Rightarrow 2r_1 = r_2$$

Since $\Delta ADE \sim AFG$

$$\frac{r_1}{r_2} = \frac{h}{3h} \Rightarrow 3r_1 = r_3$$

Volume of cone ABC $= \frac{1}{3}\pi r_1^2 h$

Volume of cone ADE $= \frac{1}{3}\pi (r_2)^2 2h$

$$= \frac{1}{3}\pi (2r_1)^2 .2h$$

Volume of frustum $BCED$ $= \frac{1}{3}\pi 4r_1^2 2h - \frac{1}{3}\pi r_1^2 h$

$$= \frac{7}{3}\pi r_1^2 h$$

Volume of frustum $DEGF$

$$= \frac{1}{3}\pi r_3^2 .3h - \frac{1}{3}\pi r_1^2 .2h$$

$$= \frac{1}{3}\pi (3r_1)^2 3h - \frac{1}{3}(2r_1)^2 .2h$$

$$= \frac{1}{3}\pi r_1^2 h(27-8) = \frac{19}{3}\pi r_1^2 h$$

$$\text{Ratio} = \frac{1}{3}\pi r_1^2 h : \frac{7}{3}\pi r_1^2 h : \frac{19}{3}\pi r_1^2 h$$

Hence, required ratio $= 1 : 7 : 19$.

24. From a rectangular block of wood, having dimensions 15 cm × 10 cm × 3.5 cm, a pen stand is made by making four conical depressions. The radius of each one of the depression is 0.5 cm and the depth 2.1 cm. Find the volume of wood left in the pen stand.

Ans : [Delhi Compt. Set-I, II, III 2017]

Volume of cuboidal block

$$l \times b \times h = 15 \times 10 \times 3.5 = 525 \text{ cm}^3$$

Volume of one cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 2.1 \text{ cm}^3$$

$$= 0.55 \text{ cm}^3$$

Volume of 4 cones $= 0.55 \times 4 = 2.2 \text{ cm}^3$

Volume of wood remaining in pen stand

$$= 525 - 2.2 = 522.80 \text{ cm}^3$$

25. The height of a cone is 30 cm. From its topside a small cone is cut by a plane parallel to its base. If volume of smaller cone is $\frac{1}{27}$ of the cone then at what height it is cut from the base ?

Ans : [Delhi Set-II, 2017]

As per question the figure is shown below.
Let the radii of smaller cone and original cone be r_1 and r_2 respectively and the height of smaller cone be h.

Since $\Delta ABC \sim \Delta APQ$ we have

$$\frac{h}{30} = \frac{r_1}{r_2} \quad (1)$$

Volume smaller cone $= \frac{1}{27} \times$ Volume of original cone

$$\frac{1}{3}\pi r_1^2 \times h = \frac{1}{27} \times \frac{1}{3}\pi r_2^2 \times 30$$

$$\left(\frac{r_1}{r_2}\right)^2 \times \frac{h}{30} = \frac{1}{27}$$

From (1) using $\frac{h}{30} = \frac{r_1}{r_2}$ we have

$$\left(\frac{h}{30}\right)^2 \times \frac{h}{30} = \frac{1}{27}$$

$$\left(\frac{h}{30}\right)^3 = \frac{1}{27}$$

$$h^3 = \frac{30 \times 30 \times 30}{27}$$

$$h = 10 \text{ cm}$$

Hence, required height$= (30 - 10) = 20$ cm

HOTS QUESTIONS

1. The ratio of the volumes of two spheres is $8:27$. If r and R are the radii of sphere respectively, then find the $(R-r):r$.

Ans : [Board Term-2, 2012, Set (22)]

Ratio of volumes

$$\frac{\text{Volume of }1^{\text{st}}\text{sphere}}{\text{Volume of }2^{\text{nd}}\text{sphere}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{8}{27}$$

or, $$\frac{r}{R} = \frac{2}{3}$$

$$R = \frac{3}{2}r$$

$$(R-r):r = \left(\frac{3}{2}r - r\right):r$$

$$= \frac{r}{2}:r = 1:2$$

2. A decorative block, made up of two solids - a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. Use $\pi = \frac{22}{7}$.

Ans : [Delhi Set I, II, III, 2016]

Surface area of block

$$= 216 - \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} + 2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}$$

$$= 225.625 \text{ cm}^2.$$

3. In fig., from a cuboidal solid metallic block of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. Use $\pi = \frac{22}{7}$

Ans : [Delhi CBSE Board, 2015 Set-I, II, III]

$$\text{Total Surface area} = 2(lb + bh + hl) + 2\pi rh$$

Here, $l = 15$ cm, $b = 10$ cm, $h = 5$cm, $r = \frac{7}{2}$ cm

TSA of Cuboidal block

$$= 2(15 \times 10 + 10 \times 5 + 5 \times 15)$$

$$= 550 \text{ cm}^2.$$

Area of C.S. of Cylinder

$$2\pi rh = 2 \times \frac{22}{7} \times \frac{7}{2} \times 5$$

$$= 110 \text{ cm}^2$$

$$\text{Area of two Circular bases} = 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 77 \text{ cm}^2$$

$$\text{Required area} = 550 + 110 - 77 = 583 \text{ cm}^3.$$

4. A right triangle whose sides are 15 cm is made to revolve about its hypotenuse. Find the volume and the surface area of the double cone so formed. (Use $\pi = 3.14$)

Ans : [Board Term-2, 2012 Set (28)]

As per question the figure is shown below.

We have $$AC^2 = 20^2 + 15^2 = 625$$

$$AC = 25 \text{ cm}$$

$$ar(\Delta ABC) = ar(\Delta ABC)$$

$$\frac{1}{3} \times BC \times AB = \frac{1}{2} \times AC \times BD$$

$$15 \times 20 = 25 \times BD$$

$$BD = 12 \text{ cm}$$

Volume of double cone,

$$= \text{Volume of upper cone} + \text{Volume of lower cone}$$

$$= \frac{1}{3}\pi(BD)^2 \times AD + \frac{1}{3}\pi(BD)^2 \times CD$$

$$= \frac{1}{3}\pi(BD)^2\{AD + CD\} = \frac{1}{3}\pi(BD)^2(AC)$$

$$= \frac{1}{3} \times 3.14 \times 144 \times 25 = 3768 \text{ cm}^2$$

Surface area= C.S.A. of upper cone + C.S.A. of lower cone

$$= \pi(12)(20) + \pi(12)(15)$$

$$= 12\pi\{20 + 15\}$$

$$= 12 \times 3.14 \times 35$$

$$= 1318.8 \text{ cm}^2$$

5. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pipe, given that 1 cm3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)

Ans : [Board Term-2, 2012 Set (31)]

As per question the figure is shown below.

Radius of lower cylinder $R = 12$ cm

Radius of upper cylinder $r = 8$ cm

Height of upper cylinder $h = 60$ cm

Height of lower cylinder $H = 220$ cm

Volume of solid iron pole,

$$\pi R^2 H + \pi r^2 h = 3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$$

$$= 111532.8 \text{ cm}^3$$

$$\text{Mass of pole} = 111532.8 \times 8 \text{ g}$$

$$= 892.2624 \text{ kg.}$$

6. A heap of wheat is in the form of cone of diameter 6 m and height 3.5 m. Find its volume . How much canvas cloth is required to just cover the heap ? Use $\pi = \frac{22}{7}$

Ans :

Volume of wheat in the form of cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 3.5$$

$$= 11 \times 3 = 33 \text{ m}^3$$

$$l = \sqrt{3^2 + 3.5} = 4.609 \text{ m}$$

Canvas required to cover the heap

$$\pi rl = \frac{22}{7} \times 3 \times 4.609$$

$$= 43.45 \text{ m}^2.$$

7. A vessel full of water is in the form of an inverted cone of height 8 cm and the radius of its top, which is open, is 5 cm. 100 spherical lead balls are dropped into vessel. One-fourth of the water flows out of the vessel. Find the radius of a spherical ball.

Ans : [Foreign Set I, II, III, 2015]

Volume of water in cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (5)^3 \times 8 = \frac{200}{3}\pi \text{ cm}^3$$

Volume of water flows out

$$= \frac{1}{4} \times \frac{200}{3}\pi = \frac{50}{3}\pi \text{ cm}^3$$

Let the radius of one spherical ball be r cm

$$\frac{4}{3}\pi r^3 \times 100 = \frac{50}{3}\pi$$

$$r^3 = \frac{50}{4 \times 100} = \frac{1}{8}$$

or, $$r = \frac{1}{2} = 0.5 \text{ cm}$$

8. A cone is cut by a plane parallel to the base and upper part is removed. It the C.S.A. of the remainder is $\frac{15}{16}$ of the C.S.A. of whole cone, find the ratio of the line segments to which cone's height is divided by the plane.

Ans : [Board Term-2, 2014]

As per question the figure is shown below.

Let the height of larger cone be H and height of smaller cone be h. Let radius of larger and smaller cones be R and r

Since $\Delta\, ONC \sim \Delta\, OMA$, we have

$$\frac{h}{H} = \frac{r}{R} = \frac{l}{L}$$

C.S.A. of the frustum $= \frac{15}{16}$(C.S.A. of cone OAB)

C.S.A. of cone OCD

$$= 1 - \frac{15}{16} = \frac{1}{16} \text{ (C.S.A. of cone } OAB)$$

$$\frac{\text{C.S.A. of cone OCD}}{\text{C.S.A. of cone OAB}} = \frac{1}{16}$$

$$\frac{\pi rl}{\pi RL} = \frac{1}{16}$$

or, $$\left(\frac{r}{R}\right)\left(\frac{l}{L}\right) = \frac{1}{16}$$

$$\left(\frac{h}{H}\right)\left(\frac{h}{H}\right) = \frac{1}{16} \qquad \left(\frac{l}{L} = \frac{h}{H}\right)$$

$$\frac{h}{H} = \frac{1}{4}$$

$$h = \frac{1}{4}H$$

$$ON = \frac{1}{4}H$$

$$MN = \frac{3}{4}H$$

$$ON : MN = 1 : 3$$

9. A right angled triangle whose sides are 3 cm, 4 cm and 5 cm is revolved about the longest side. Find the surface area of figure obtained. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2012 (44)]

As per question the figure is shown below.

By revolving right triangle about longest side double cone is generated. Let radius of double cone $= x$ cm.

In $\Delta\, ADE$ and $\Delta\, ADC$,

$$\angle AED = \angle DAC = 90^\circ$$

$$\angle ADE = \angle ADC \text{ (common angle)}$$

Thus due to AA symmetry we have

or, $$\Delta\, ADC \sim \Delta\, ADC$$

$$\frac{x}{AC} = \frac{AD}{DC} = \frac{DE}{AD}$$

$$\frac{x}{4} = \frac{3}{5} = \frac{DE}{3}$$

$$x = \frac{12}{5} = 2.4 \text{ cm}$$

$$DE = \frac{9}{5} = 1.8 \text{ cm}$$

Surface area of double cone

$$\pi r l_1 + \pi r l_2 = \pi r(l_1 + l_2)$$

$$= \frac{22}{7} \times 2.4 \times (3 + 4)$$

$$= 22 \times 2.4 = 52.8 \text{ cm}^2.$$

10. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volume of the cylinder and toy. (Use $\pi = 3.14$)

Ans : [Board Term-2, 2012 Set (34)]

As per question the figure is shown below.

Let BPC is a hemisphere and ABC is a cone.

$$\text{Radius of hemisphere} = \text{Radius of cone}$$

$$= \frac{4}{2} = 2 \text{ cm}$$

$$h = \text{Height of cone} = 2 \text{ cm}$$

$$\text{Volume of toy} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$\frac{1}{3}\pi r^2(2r + h) = \frac{1}{3} \times 3.14 \times 2 \times 2(2 \times 2 + 2)$$

$$= \frac{1}{3} \times 3.14 \times 4 \times 6$$

$$= 25.12 \text{ cm}^3$$

Let right circular cylinder $EFGH$ circumscribe the given solid toy.

$$\text{Radius of cylinder} = 2 \text{ cm}$$

$$\text{Height of cylinder} = 4 \text{ cm}$$

Volume of right circular cylinder

$$\pi r^2 h = 3.14 \times (2)^2 \times 4 \text{ cm}^3 \qquad \text{...(ii)}$$

$$= 50.24 \text{ cm}^3$$

Difference of two volume

$$= \text{Volume of cylinder} - \text{Volume of toy}$$

$$= 50.24 - 25.12 = 25.12 \text{ cm}^3.$$

11. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm3. Find the height of the toy. Also find the cost of painting the hemisphere part of the toy at the rate of Rs. 10 per cm2. Use $\pi = \frac{22}{7}$

Ans : [Delhi CBSE Board 2015 set I, II, III]

As per question the figure is shown below.

Radius of cone = Radius of hemisphere

$$r = 3.5 \text{ cm}$$

Total volume, $V = 166\frac{5}{6} \text{ cm}^3 = \frac{1001}{6} \text{ cm}^3$

Let the height of cone be h.

Total volume

= Volume of cone + Volume of hemisphere

$$\frac{1001}{6} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$\frac{1001}{6} = \frac{1}{3}\pi(3.5)^2 h + \frac{2}{3}\pi(3.5)^2$$

$$\frac{1001}{6} = \frac{1}{3}\pi[12.25h + 2 \times 42.875]$$

$$\frac{1001 \times 3 \times 7}{6 \times 22} = 12.25h + 85.75$$

$$\frac{21021}{132} = 12.25h + 85.75$$

$$12.25h = 159.25 - 85.75$$

$$h = \frac{73.5}{12.25} = 6$$

Height of the toy $= 6 + 3.5 = 9.5$ cm.

Surface area of hemisphere

$$2\pi r^2 = 2 \times \frac{22}{7} \times 3.5 \times 3.5 = 77 \text{ cm}^2$$

Cost of painting $= 10 \times 77 = 770$ Rs

12. Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm, if the In crease in the level of the water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe.

Ans : [Delhi CBSE Board 2015 Set I, II, III]

Let the internal diameter of the pipe be r m.

Water flows in 1 hour = 2.52 km.

$$\text{Water flows in } \frac{1}{2} \text{ hour} = \frac{2.52}{2} = 1.26 \text{ km}$$

$$= 1260 \text{ m}$$

$$\text{Volume of water flows in } \frac{1}{2}\text{hour} = \pi r^2 h$$

$$= \pi r^2 \times 1260$$

Volume of the in cylindrical tank

$$= \pi \times \left(\frac{40}{100}\right)^2 \times 3.15$$

Volume of water flow = Volume of increase water

$$\pi r^2 \times 1260 = \pi\left(\frac{2}{5}\right)^2 \times 3.15$$

or, $$1260r^2 = \frac{2}{5} \times \frac{2}{5} \times 3.15$$

or, $$r^2 = \frac{4}{25} \times \frac{315}{100} \times \frac{1}{1260} = \frac{1}{2500}$$

or, $$r = \frac{1}{50} \text{ m} = 2 \text{ cm}$$

Internal diameter of pipe = 4 cm.

13. A solid is consisting of a right circular cone of height 120 cm and radius 60 cm standing on hemisphere of radius 60 cm. It is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm ant its height is 180 cm.

Ans : [Board Term-2, 2015]

As per question the figure is shown below.

Height of cone, $h = 120$ cm,

Radius of cone $r = 60$ cm

Radius of hemisphere $= 60$ cm.

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times 3.14 \times 60 \times 60 \times 120$$

$$= 3.14 \times 60 \times 60 \times 40$$

$$= 452160 \text{ cm}^3$$

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times 3.14 \times 60 \times 60 \times 60$$

$$= 452160 \text{ cm}^3$$

Total volume = Volume of cone + Volume of hemisphere

$$= 452160 + 452160$$

$$= 904320 \text{ cm}^3$$

Height of cylinder = 180 cm,

radius = 60 cm.

Volume of water in the cylinder

= Volume of cylinder

$$= \pi r^2 h$$

$$= 3.14 \times 60 \times 60 \times 180$$

$$= 2034720 \text{ cm}^3$$

Water left in the cylinder = Volume of water − Volume of (cone + sphere)

$$= 2034720 - 904320$$

$$= 1130400 \text{ cm}^3$$

14. A circus tent is in the shape of a cylinder surmounted by a conical top of same diameter. If there common diameter is 56 m, the height of cylindrical part is 6 m and the total height of the tent above the ground is 27 m, find the area of canvas used in the tent.

Ans : [Delhi Compt. Set-I, II, III 2017]

Total height of tent = 27 m

Height of cylindrical part = 6 m

Height of conical part $= 27 - 6 = 21$ m

$$\text{Slant height of cone} = \frac{56}{2} = 28 \text{ m}$$

$$\text{Slant height of cone} = \sqrt{r^2 + h^2}$$

$$= \sqrt{28^2 + 21^2}$$

$$= \sqrt{784 + 441} = \sqrt{1225}$$

$$= 35 \text{ m}$$

$$\text{Area of canvas used} = 2\pi rh + \pi rl$$

$$= \pi r(2h + l)$$

$$= \frac{22}{7} \times 28(2 \times 6 + 35)$$

$$= 22 \times 4 \times 47$$

$$= 4136 \text{ m}^2$$

15. From a right circular cylinder of height 2.4 cm and radius 0.7 cm, a right circular cone of same radius is cut-out. Find the total surface area of the remaining solid.

Ans : [Outside Delhi Set-II, III 2017]

Radius $r = 0.7$ cm

and height $h = 2.4$ cm

$$\text{Slant height} \quad l = \sqrt{h^2 + r^2} = \sqrt{(2.4)^2 + (0.7)^2}$$

$$= 2.5 \text{ m}$$

Total surface area of remaining solid

= C.S.A. of cylinder + C.S.A. of cone + area of top.

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= \frac{22}{7} \times 0.7(2 \times 2.4 + 2.5 + 0.7)$$

$$= \frac{22}{7} \times 0.7 \times 8 = \frac{176}{10}$$

Hence total surface area $= 17.6 \text{ cm}^2$