1 Real Numbers

EXERCISE 1.1

Choose the correct answer from the given four options in the following questions:

Q1. For some integer m, every even integer is of the form

(*a*) m (*b*) $m+1$ (*c*) $2m$ (*d*) $2m+1$

Sol. (*c*): Let p be any a positive integer. On dividing p by 2, we get m as quotient and r be the remainder. Then by Euclid's division algorithm, we have

$$p = 2m + r, \quad \text{where } 0 \le r < 2,$$

So, $r = 0, 1$

$\therefore$ $p = 2m$ and $p = 2m + 1$

$p = 2m$ for any integer m, then p is even.

Alternative Method: Even integers are 2, 4, 6, ...

So, these integers can be written in the form of

$$= 2\times1, 2\times2, 2\times3, 2\times4, \ldots$$
$$= 2m \quad \text{where } m = +1, +2, +3, \ldots$$

So, $2m$ becomes $\pm2, \pm4, \pm6, \pm8 \ldots$

Q2. For some integer q, every odd integer is of the form

(*a*) q (*b*) $q+1$ (*c*) $2q$ (*d*) $2q+1$

Sol. (*d*): Let p be any positive integer. On dividing p by 2, we obtain q as quotient and r is the remainder. Then by Euclid's division algorithm, we have

$$p = 2q + r \quad \text{where } 0 \le r < 2$$

So $r = 0$, and $r = 1$

$\therefore p = 2q$ and $p = 2q + 1$

Clearly, $p = 2q + 1$ is odd integer for any integer q.

Alternative Method: Odd integers are 1, 3, 5, 7... or $0\times1+1, 1\times2+1, 2\times3+1, \ldots$ or $2q+1$

where q is any integer so odd numbers are $q = 0, \pm1, \pm2, \pm3\ldots$ $\pm1, \pm3, \pm5, \pm7 \ldots$ are all odd integers or a number of the form $2q+1$ is odd.

Q3. $n^2 - 1$ is divisible by 8, if n is

(*a*) an integer (*b*) a natural number

(*c*) an odd integer (*d*) an even integer

Sol. (*c*): Let $p = n^2 - 1$, where n is any integer.

Case I: Let n is even, then $n = 2k$.

$\therefore$ $p = (2k)^2 - 1$

$$p = 4k^2 - 1$$

Let $k = 0$, then $p = 4(0)^2 - 1 = -1$, which is not divisible by 8

$k = 2$, then $p = 4(2)^2 - 1 = 15$, which is not divisible by 8

$k = 4$, then $p = 4(4)^2 = 64 - 1 = 63$, which is not divisible by 8

So, n can not be even integer.

Case II: Let n is odd then $n = 2k + 1$

$$p = (2k+1)^2 - 1$$
$$= 4k^2 + 1 + 4k - 1$$
$$p = 4k(k+1)$$

Let $k = 1$, $p = 4(1)[1+1] = 8$ is divisible by 8

$k = 3$ $p = 4 \times 3\,(3+1) = 48 = 8 \times 6$, is divisible by 8

$k = 5$ $p = 4(5)\,(5+1) = 120, = 8 \times 15$ is divisible by 8

So $n^2 - 1$ is divisible by 8 if n is odd number.

Q4. If the HCF of 65 and 117 is expressible in the form $65m - 117$, then the value of m is

(*a*) 4 (*b*) 2 (*c*) 1 (*d*) 3

Sol. (*b*): Find HCF of 65, 117 by any method let by factorisation

$$65 = 13 \times 5$$
$$117 = 13 \times 3 \times 3$$

So, HCF of 65 and 117 = 13

So, $65m - 117 = 13$

$\Rightarrow$ $65m = 130$

$\Rightarrow$ $m = 2$

Q5. The largest number which divides 70 and 125, leaving remainders 5 and 8 respectively is

(*a*) 13 (*b*) 65 (*c*) 875 (*d*) 1750

Sol. (*a*): **Main concept:** Required number is largest so problem is related to HCF.

Subtract 5 and 8 from 70 and 125 respectively.

So, $70 - 5 = 65$ and $125 - 8 = 117$

HCF of 65 and 117 is 13 (by any method). So, 13 is the largest number which leaves remainder 5 and 8 after dividing 70, and 125 by 13 respectively.

Q6. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers then HCF (a, b) is

(*a*) xy (*b*) xy^2 (*c*) x^3y^3 (*d*) x^2y^2

Sol. (*b*):

$$\left.\begin{array}{l} a = x^3y^2 \\ b = xy^3 \end{array}\right\} \text{prime factorisation}$$

So, HCF of a and $b = xy^2$ [common terms from a and b]

Q7. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$ where a and b being prime numbers, then LCM (p, q) is

(*a*) ab (*b*) a^2b^2 (*c*) a^3b^2 (*d*) a^3b^3

Sol. (*c*): $p = ab^2$

$q = a^3b$

LCM = Product of the highest powers of each factor.

So, LCM $= a^3b^2$.

Q8. The product of a non-zero rational and an irrational number is

(*a*) always irrational (*b*) always rational

(*c*) rational or irrational (*d*) one

Sol. (*a*): Product of a rational $\frac{5}{2}$, and an irrational $\frac{\sqrt{3}}{2} = \frac{5}{2} \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{4}$ is also irrational.

Q9. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is

(*a*) 10 (*b*) 100 (*c*) 504 (*d*) 2520

Sol. (*d*): As we require least number so problem is based on LCM.

Prime factor from 1 – 10

$1 = 1,\quad 2 = 2,\quad 3 = 3,$

$4 = 2 \times 2,\quad 5 = 5,\quad 6 = 2 \times 3,\quad 7 = 7,$

$8 = 2 \times 2 \times 2,\quad 9 = 3 \times 3,\quad 10 = 2 \times 5$

LCM of all numbers 1 to 10 $= 1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3$

LCM $= 2^3 \times 3^2 \times 5^1 \times 7^1 = 72 \times 35 = 2520$

Q10. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after:

(*a*) one decimal place (*b*) two decimal places

(*c*) three decimal places (*d*) four decimal places

Sol. (*d*): Number is $\frac{14587}{1250} = \frac{14587}{5^4 \times 2} = \frac{14587}{5^4 \times 2^4} \times 2^3$

$$= \frac{14587}{(10)^4} \times 8 = \frac{116696}{10000} = 11.6696$$

EXERCISE 1.2

Q1. Write whether every positive integer can be of the form $(4q + 2)$, where q is an integer. Justify your answer.

Sol. 'No'. By Euclid's division lemma, we have

dividend = divisor × quotient + remainder

$$a = bq + r$$

Let $b = 4$ then

$a = 4q + r$ where q, r are positive

Since $0 \le r < 4$ $\therefore$ $r = 0, 1, 2, 3$

So, a become of the form, $4q$, $4q+1$, $4q+2$ and $4q+3$

So, all integers can be represented by all $4q$, $4q+1$, $4q+2$, and $4q+3$ not only by $4q+2$.

Q2. "The product of two consecutive positive integers is divisible by 2". Is this statement true or false? Give reasons.

Sol. Yes, from any two consecutive numbers one will be even and other will be odd i.e. n, $(n+1)$. So, their product will be even which will be divisible by 2.

Hence, the product of two consecutive positive integers is divisible by 2.

Q3. "The product of three consecutive positive integers is divisible by 6". Is this statement true or false? Justify your answer.

Sol. Yes, the given statement is true.

Three consecutive positive integers are n, $(n+1)$, $(n+2)$. Out of 3 consecutive integers, one will be even and other will be divisible by 3. So, the product of all three becomes divisible by 6,

e.g., 13, 14, 15 here 14 is even, 15 is divisible by 3.

So, $13 \times 14 \times 15$ is divisible by 6.

Q4. Write whether the square of any positive integer can be of the form of $(3m+2)$, where m is a natural number. Justify your answer.

Sol. By Euclid's division lemma, $b = aq + r$

where a, b, q, r are +ve integers and here $a = 3$ then $b = 3q + r$ then $0 \le r < 3$ or $r = 0, 1, 2$, so b becomes $b = 3q, 3q+1, 3q+2$,

$$b = 3q$$

$$\Rightarrow \quad (b)^2 = (3q)^2$$

$$\Rightarrow \quad b^2 = 3 \cdot 3q^2 = 3m \quad \text{where, } 3q^2 = m$$

So, as b^2 is perfect square so $3m$ will also be perfect square.

When $r = 1$, $\quad b = 3q + 1$

$$\Rightarrow \quad (b)^2 = (3q+1)^2$$

$$\Rightarrow \quad b^2 = 9q^2 + 1 + 2 \times 3q$$

$$\Rightarrow \quad b^2 = 3[3q^2 + q] + 1$$

$$\Rightarrow \quad b^2 = 3m + 1 \text{ and } m = 3q^2 + 2q$$

So, b^2 is perfect square or a number of the form $3m+1$ is perfect square.

When $r = 2$, $\quad b = 3q + 2$

$$\Rightarrow \quad b^2 = 9q^2 + 4 + 2 \cdot 3q \cdot 2$$

$$= 9q^2 + 3 + 3 \times 4q + 1$$

$$= 3[3q^2 + 1 + 4q] + 1$$

$$\Rightarrow \quad b^2 = 3m + 1$$

Again, a number of the form $3m+1$ is perfect square.

Hence, a number of the form $(3m+2)$ can never be perfect square. But a number of the form $3m$, and $3m+1$ are perfect squares.

Q5. A positive integer is of the form $(3q + 1)$, q being a natural number. Can you write its square in any form other than $(3m + 1)$ i.e., $3m$ or $(3m + 2)$ for some integer m? Justify your answer.

Sol. By Euclid's division lemma,

$b = aq + r$ where b, q, r are natural numbers and $a = 3$

$\therefore$ $b = 3q + r$ where $0 \leq r < 3$ so $r = 0, 1, 2,$

At $r = 0$, $b = 3q$

$\Rightarrow$ $b^2 = (3q)^2 = 3 \cdot 3q^2$

$\Rightarrow$ $b^2 = 3m$, where $m = 3q^2$

So, a number of the form $3m$ is perfect square.

At $r = 1$, $b = 3q + 1$

$\Rightarrow$ $b^2 = (3q + 1)^2$

$\Rightarrow$ $b^2 = 9q^2 + 1 + 6q$

$\Rightarrow$ $b^2 = 3[3q^2 + 2q] + 1$

$\Rightarrow$ $b^2 = 3m + 1$, where $m = 3q^2 + 2q$

So, a number of the form $(3m + 1)$ is also perfect square.

At $r = 2$, $b = 3q + 2$

$\Rightarrow$ $b^2 = (3q)^2 + (2)^2 + 2(3q)(2)$

$= 9q^2 + 4 + 3 \times 4q$

$= 9q^2 + 3 + 3 \times 4q + 1 = 3[3q^2 + 1 + 4q] + 1$

$\Rightarrow$ $b^2 = 3m + 1$, where $m = 3q^2 + 4q + 1$

Hence, a perfect square will be of the form $3m$ and $(3m + 1)$ for m being a natural number.

Q6. The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75, what is HCF of (3000, 525)? Justify your answer.

Sol. The numbers 525 and 3000 both are divisible by 3, 5, 15, 25 and 75. So, highest common factor out of 3, 5, 15, 25 and 75 is 75 or HCF of (525, 3000) is 75.

Verification: $525 = 5 \times 5 \times 3 \times 7 = 3 \times 5^2 \times 7^1$

$3000 = 2^3 \times 5^3 \times 3^1 = 2^3 \times 3^1 \times 5^3$

$\text{HCF} = 3^1 \times 5^2 = 75$

Hence, verified.

Q7. Explain why $3 \times 5 \times 7 + 7$ is a composite number.

Sol. Main Concept: A number which is not prime is composite.

$3 \times 5 \times 7 + 7 = 7[3 \times 5 + 1] = 7[15 + 1]$

$= 7 \times 16$ have prime factors $= 7 \times 2 \times 2 \times 2 \times 2$

So, number $(3 \times 5 \times 7 + 7)$ is not prime hence, it is composite.

Q8. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.

Sol. As we know that

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = (a \times b)$$

18 must be factor of 380.

So, $\frac{380}{18}$ should be a natural number.

But $\frac{380}{18}$ is not a natural number or 380 is not divisible by 18.

So, 380 and 18 are not the LCM and HCF of any two numbers.

Q9. Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating (repeating) decimal expansion. Give reasons for your answer.

Sol. $$\frac{987}{10500} = \frac{3 \times 7 \times 47}{2^2 \times 3^1 \times 5^3 \times 7^1} = \frac{47}{2^2 \times 5^3}$$

3	987
7	329
	47

5	10500
3	2100
7	700
5	100
5	20
2	4
	2

As denominator has prime factors only in 2 and 5 so number $\frac{987}{10500}$ is terminating decimal.

$$\frac{47}{2^3 \times 5^3} \times 2 = \frac{94}{1000} = 0.094$$

Q10. A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form p/q? Give reasons.

Sol. 327.7081 is terminating decimal so in the form of

$$\frac{p}{q} = \frac{3277081}{10000}$$

$$q = 2^4 \times 5^4$$

So, q has only factors of 2 and 5 so it is terminating decimal.

EXERCISE 1.3

Q1. Show that the square of any positive integer is either of the form $4q$ or $4q + 1$ for some integer q.

Sol. Main concept: $\boxed{a = 4q + r}$ $0 \leq r < 4$.

By Euclid's division lemma,

$$a = 4m + r \quad \text{...}(i)$$

where a, m, r are integers and $0 \leq r < 4$

or $r = 0, 1, 2, 3$

When $r = 0$, $a = 4m$ [From (i)]

$\Rightarrow$ $a^2 = (4m)^2$ [Squaring both sides]

$\Rightarrow$ $a^2 = 4.4m^2$

$\Rightarrow \quad a^2 = 4q$, where $q = 4m^2 \Rightarrow 4q$ is a perfect square

When $r = 1$, $\quad a = 4m + 1$ [From (*i*)]

$\Rightarrow \quad a^2 = (4m + 1)^2$ [Squaring both sides]

$= (4m)^2 + (1)^2 + 2(4m)(1)$

$= 4[4m^2 + 2m] + 1$

$\Rightarrow \quad a^2 = 4q + 1, \quad$ where $q = 4m^2 + 2m$

$\because \quad a^2$ is perfect square so $4q + 1$ is also perfect square.

When $r = 2$, $\quad a = (4m + 2)$ [From (*i*)]

$\Rightarrow \quad a^2 = (4m)^2 + (2)^2 + 2(4m)(2)$ [Squaring both sides]

$\Rightarrow \quad a^2 = 4[4m^2 + 1 + 4m]$

$\Rightarrow \quad a^2 = 4q, \quad$ where $q = 4m^2 + 4m + 1$

$\because \quad a^2$ is perfect square. So, $4q$ will also be perfect square.

When $r = 3$, then $\quad a = 4m + 3$ [From (*i*)]

$\Rightarrow \quad (a)^2 = (4m + 3)^2$ [Squaring both sides]

$\Rightarrow \quad a^2 = (4m)^2 + (3)^2 + 2(4m)(3)$

$\Rightarrow \quad a^2 = 16m^2 + 9 + 24m$

$= 16m^2 + 8 + 24m + 1$

$= 4[4m^2 + 2 + 6m] + 1$

$\Rightarrow \quad a^2 = 4q + 1, \quad$ where $q = 4m^2 + 6m + 2$

As a^2 is perfect square so $4q + 1$ will also be perfect square.

Hence, number of the form $4q$ and $4q + 1$ is the perfect square.

Q2. Show that the cube of any positive integer is of the form $4m$, $4m + 1$ or $4m + 3$ for some integer m.

Sol. By Euclid's division algorithm, corresponding to the positive integer a and 4

$$a = 4q + r \quad \text{...}(i)$$

where a, q, r are non-negative integers and $0 \le r < 4$ i.e., $r = 0, 1, 2, 3$

Now, at $r = 0$, $\quad a = 4q + 0$ [From (*i*)]

$\Rightarrow \quad a^3 = (4q)^3$ [Cubing both sides]

$\Rightarrow \quad a^3 = 4 \cdot (16q^3)$

$\Rightarrow \quad a^3 = 4m, \quad$ where $m = 16q^3$

$\because a^3$ is perfect cube so $4m$ will also be perfect cube for some specified value of m.

Now, at $r = 1$, $\quad a = 4q + 1$ [From (*i*)]

$\Rightarrow \quad a^3 = (4q + 1)^3$ [Cubing both sides]

$\Rightarrow \quad a^3 = (4q)^3 + (1)^3 + 3(4q)^2(1) + 3(4q)(1)^2$

$= 4 \cdot 16q^3 + 1 + 4 \cdot 12q^2 + 4 \cdot 3q$

$= 4(16q^3 + 12q^2 + 3q) + 1$

$\Rightarrow \quad a^3 = 4m + 1, \quad$ where $m = 16q^3 + 12q^2 + 3q$

$\because a^3$ is perfect cube so $4m + 1$ will also be perfect cube for some specified value of m.

At $r = 2$, $a = 4q + 2$ [From (*i*)]

$\Rightarrow$ $a^3 = (4q + 2)^3$ [Cubing both sides]

$\Rightarrow$ $a^3 = (4q)^3 + (2)^3 + 3(4q)^2 (2) + 3(4q) (2)^2$

$= 4 \cdot 16q^3 + 8 + 4 \times 24q^2 + 4 \times 12q$

$= 4[16q^3 + 2 + 24q^2 + 12q]$

$\Rightarrow$ $a^3 = 4m$, where $m = 16q^3 + 24q^2 + 12q + 2$

As a^3 is perfect cube so, $4m$ is also perfect cube for some value of positive integer m.

At $r = 3$, $a = 4q + 3$ [From (*i*)]

$\Rightarrow$ $a^3 = (4q + 3)^3$ [Cubing both sides]

$\Rightarrow$ $a^3 = (4 \cdot q)^3 + (3)^3 + 3(4q)^2 (3) + 3(4q) (3)^2$

$\Rightarrow$ $a^3 = 4 \times 16q^3 + 27 + 4 \times 36q^2 + 4q \times 27$

$\Rightarrow$ $a^3 = 4 \times 16q^3 + 24 + 3 + 4 \times 36 \cdot q^2 + 4 \times 27q$

$= 4[16q^3 + 6 + 36q^2 + 27q] + 3$

$\Rightarrow$ $a^3 = 4m + 3$, where $m = 16q^3 + 36q^2 + 27q + 6$

Hence, a number of the form $4m$, $4m + 1$ and $4m + 3$ is perfect cube for specified natural value of m.

Q3. Show that the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q.

Sol. By Euclid's division algorithm, consider the positive integer a and 5

$$a = 5m + r \quad ...(i)$$

where, a, m, r are positive integers and $0 \le r < 5$ or $r = 0, 1, 2, 3, 4$

Squaring (*i*) both sides, we get

$a^2 = (5m)^2 + (r)^2 + 2(5m)(r) = 25m^2 + r^2 + 10mr$

$\Rightarrow$ $a^2 = 5(5m^2 + 2mr) + r^2$ ...(*ii*)

At $r = 0$, $a^2 = 5[5m^2 + 2m \cdot 0] + 0$ [From (*ii*)]

$\Rightarrow$ $a^2 = 5(5m^2)$

$\Rightarrow$ $a^2 = 5q$, where $q = 5m^2$

At $r = 1$, $a^2 = 5[5m^2 + 2m] + 1$ [From (*ii*)]

$\Rightarrow$ $a^2 = 5q + 1$, where $q = 5m^2 + 2m$

At $r = 2$, $a^2 = 5[5m^2 + 2.2m] + (2)^2$ [From (*ii*)]

$\Rightarrow$ $a^2 = 5q + 4$, where $q = 5m^2 + 4m$

At $r = 3$, $a^2 = 5[5m^2 + 2m \cdot 3] + 3^2$ [From (*ii*)]

$\Rightarrow$ $a^2 = 5[5m^2 + 6m] + 5 + 4$

$= 5[5m^2 + 6m + 1] + 4$

$\Rightarrow$ $a^2 = 5q + 4$, where $q = 5m^2 + 6m + 1$

At $r = 4$, $a^2 = 5[5m^2 + 2m \cdot 4] + 4^2$ [From (*ii*)]

$\Rightarrow$ $a^2 = 5(5m^2 + 8m) + 15 + 1$

$= 5[5m^2 + 8m + 3] + 1$

$\Rightarrow$ $a^2 = 5q + 1$, where $q = 5m^2 + 8m + 3$

Hence, the numbers of the form $5q$, $5q+1$, $5q+4$ are perfect squares and the numbers of the form $(5q+2)$, $(5q+3)$ are not perfect squares for some positive integers.

Q4. Show that the square of any positive integer cannot be of the form $(6m+2)$, or $(6m+5)$ for any integer m.

Sol. By Euclid's division algorithm, we have

$a = 6q + r$, where $0 \le r < 6$

or $r = 0, 1, 2, 3, 4, 5$

Consider $a = 6q + r$

$\Rightarrow a^2 = (6q)^2 + (r)^2 + 2(6q)(r)$ [Squaring both sides]

$\Rightarrow a^2 = 6[6q^2 + 2qr] + r^2$ ...(*i*)

At $r = 0$, $a^2 = 6[6q^2 + 2q \times 0] + 0^2$ [From (*i*)]

$\Rightarrow a^2 = 36q^2$

$\Rightarrow a^2 = 6m$, where $m = 6q^2$

At $r = 1$, $a^2 = 6[6q^2 + 2q \times 1] + 1^2$ [From (*i*)]

$= 6[6q^2 + 2q] + 1$

$\Rightarrow a^2 = 6m + 1$, where $m = 6q^2 + 2q$

At $r = 2$, $a^2 = 6[6q^2 + 2q \cdot 2] + 2^2$ [From (*i*)]

$a^2 = 6m + 4$, where $m = (6q^2 + 4q)$

At $r = 3$, $a^2 = 6[6q^2 + 2q \cdot 3] + 3^2$ [From (*i*)]

$= 6[6q^2 + 6q] + 6 + 3$

$= 6[6q^2 + 6q + 1] + 3$

$\Rightarrow a^2 = 6m + 3$, where $m = 6q^2 + 6q + 1$

At $r = 4$, $a^2 = 6[6q^2 + 2q \cdot 4] + 4^2$

$\Rightarrow a^2 = 6[6q^2 + 8q] + 12 + 4$

$= 6[6q^2 + 8q + 2] + 4$

$\Rightarrow a^2 = 6m + 4$ is perfect square, where $m = 6q^2 + 8q + 2$

At $r = 5$, $a^2 = 6[6q^2 + 2q \cdot 5] + 5^2$ [From (*i*)]

$\Rightarrow a^2 = 6[6q^2 + 10q] + 24 + 1$

$= 6[6q^2 + 10q + 4] + 1$

$\Rightarrow a^2 = 6m + 1$ is perfect square, where $m = 6q^2 + 10q + 4$

Hence, the numbers of the form $6m$, $(6m+1)$, $(6m+3)$ and $(6m+4)$ are perfect squares and $(6m+2)$, and $(6m+5)$ are not perfect squares for some value of m.

Q5. Show that the square of any odd integer is of the form $(4q+1)$ for some integer q.

Sol. By Euclid's division algorithm, $a = bq + r$ where a, b, q, r are non-negative integers and $0 \le r < 4$.

On putting $b = 4$ we get

$\boxed{a = 4q + r}$ ...(*i*)

When $r = 0$, $a = 4q$ which is even (as it is divisible by 2)
When $r = 1$, $a = 4q + 1$ which is odd ($\because$ it is not divisible by 2)
Squaring the odd number $(4q + 1)$, we get

$$= (4q+1)^2$$
$$= (4q)^2 + (1^2) + 2(4q)$$
$$= 4[4q^2 + 2q] + 1$$
$$= 4m + 1 \text{ is perfect square for } m = 4q^2 + 2q$$

When $r = 2$, $a = 4q + 2$ [From (i)]
$\Rightarrow$ $a = 2(2q + 1)$ is divisible by 2 so it is even.
When $r = 3$, $a = 4q + 3 = 4q + 2 + 1$
$= 2[2q + 1] + 1$ is not divisible by 2 so it is odd.
Squaring the odd number $(4q + 3)$, we get

$$(4q+3)^2 = (4q)^2 + (3)^2 + 2(4q)(3)$$
$$= 16q^2 + 9 + 24q$$
$$= 16q^2 + 24q + 8 + 1$$
$$= 4[4q^2 + 6q + 2] + 1$$
$$= 4m + 1 \text{ is perfect square for some value of } m.$$

Q6. If n is an odd integer, then show that $n^2 - 1$ is divisible by 8.
Sol. Let $a = n^2 - 1$ ...(i)
Where n is odd number, i.e., $n = 1, 3, 5, 7$
When $n = 1$, $a = 1^2 - 1 = 0$, which is divisible by 8. [From eq. (i)]
When $n = 3$, $a = 3^2 - 1 = 9 - 1 = 8$, which is also divisible by 8.
When $n = 5$, [From eq. (i)]
$a = 5^2 - 1 = 25 - 1 = 24 = 8 \times 3$, which is divisible by 8.
[From eq. (i)]
Hence, $n^2 - 1$ is divisible by 8 when n is odd.

Q7. Prove that, if x and y, both are odd positive integers, then $(x^2 + y^2)$ is even but not divisible by 4.
Sol. Let we have any two odd numbers $x = (2m + 1)$ and $y = (2m + 5)$.
Then, $x^2 + y^2 = (2m + 1)^2 + (2m + 5)^2$

$$= 4m^2 + 1 + 4m + 4m^2 + 25 + 20m$$
$$= 8m^2 + 24m + 26$$
$$= 2[4m^2 + 12m + 13]$$

So, $x^2 + y^2$ is even but it is not divisible by 4.

Q8. Use Euclid's division algorithm to find HCF of 441, 567 and 693.
Sol. Let $a = 693$ and $b = 567$
By Euclid's division algorithm, $a = bq + r$

$$\therefore \quad 693 = 567 \times 1 + 126$$
$$567 = 126 \times 4 + 63$$
$$126 = 63 \times 2 + 0$$

Hence, HCF (693 and 567) = 63.
Now, take 441 and HCF = 63

By Euclid's division algorithm, $c = dq + r$

$$c = 441 \text{ and } d = 63$$

$$\Rightarrow \quad 441 = 63 \times 7 + 0$$

Hence, HCF (693, 567, 441) = 63.

Q9. Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainders, 1, 2, and 3 respectively.

Sol. As 1, 2, and 3 are the remainders when required largest number (HCF) divides 1251, 9377 and 15628 respectively.

We have the numbers for HCF (1251 – 1), (9377 – 2) and (15628 – 3) i.e., 1250, 9375, 15625

For HCF of 1250, 9375, 15625 let $a = 15625$, $b = 9375$

By Euclid's division algorithm, $a = bq + r$

$$\therefore \quad 15625 = 9375 \times 1 + 6250$$

$$9375 = 6250 \times 1 + 3125$$

$$6250 = 3125 \times 2 + 0$$

$\therefore$ HCF (15625, 9375) = 3125

Now, let $d = 1250$ and $c = 3125$

By Euclid's division algorithm, $c = dq + r$

$$\therefore \quad 3125 = 1250 \times 2 + 625$$

$$1250 = 625 \times 2 + 0$$

Hence, required HCF (15625, 1250 and 9375) is 625.

Q10. Prove that $\sqrt{3} + \sqrt{5}$ is irrational.

Sol. Let us consider $\sqrt{3} + \sqrt{5}$ is a rational number that can be written as

$$\sqrt{3} + \sqrt{5} = a$$

$$\Rightarrow \quad \sqrt{5} = a - \sqrt{3}$$

Squaring both sides, we get

$$(\sqrt{5})^2 = (a - \sqrt{3})^2$$

$$\Rightarrow \quad 5 = (a)^2 + (\sqrt{3})^2 - 2(a)(\sqrt{3})$$

$$\Rightarrow \quad 2a\sqrt{3} = a^2 + 3 - 5$$

$$\Rightarrow \quad \sqrt{3} = \frac{a^2 - 2}{2a}$$

As $a^2 - 2, 2a$ are rational so $\frac{a^2 - 2}{2a}$ is also rational but $\sqrt{3}$ is not rational which contradicts our consideration. So, $\sqrt{3} + \sqrt{5}$ is irrational.

Q11. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

Sol. Number ending at 0 or 5 is divisible by 5.

Now, $(12)^n = (2 \times 2 \times 3)^n = 2^{2n} \times 3^n$

It has no any 5 in its prime factorisation. So, 12^n can never end with 5 and zero.

Q12. On a morning walk, three persons, step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk, so that each can cover the same distance in complete steps?

Sol. We have to find minimum distance (i.e., LCM) covered by steps.

$$40 = 2^3 \times 5$$
$$42 = 2 \times 3 \times 7$$
$$45 = 3^2 \times 5$$

LCM (40, 42, 45) $= 2^3 \times 3^2 \times 5 \times 7 = 2520$ cm

So, the minimum distance that each should walk is 2520 cm.

Q13. Write the denominator of rational number $\frac{257}{5000}$ in the form of $2^m \times 5^n$, where m, n are non-negative integers. Hence, write its decimal expansion, without actual division.

Sol. Denominator of the rational number $\frac{257}{5000}$ is 5000.

$$5000 = 2^3 \times 5^4 \quad \text{which is of the form } 2^m \times 5^n$$

where $m = 3$ and $n = 4$

$$\therefore \quad \frac{257}{5000} = \frac{257}{2^3 \times 5^4} \times \frac{2}{2} = \frac{257 \times 2}{(2 \times 5)^4} = \frac{514}{10000} = 0.0514$$

Q14. Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p and q are primes.

Sol. Consider $\sqrt{p} + \sqrt{q}$ is rational and can be represented as $\sqrt{p} + \sqrt{q} = a$

$$\Rightarrow \quad (\sqrt{p}) = a - \sqrt{q}$$
$$\Rightarrow \quad (\sqrt{p})^2 = (a - \sqrt{q})^2 \quad \text{(squaring both sides)}$$
$$\Rightarrow \quad p = a^2 + q - 2a\sqrt{q}$$
$$\Rightarrow \quad 2a\sqrt{q} = a^2 + q - p$$
$$\Rightarrow \quad \sqrt{q} = \frac{a^2 + q - p}{2a}$$

As q is prime so $\sqrt{q}$ is not rational but $\frac{a^2 + q - p}{2a}$ is rational because a, p, q are non-zero integers which contradicts our consideration.

Hence, $\sqrt{p} + \sqrt{q}$ is irrational.

EXERCISE 1.4

Q1. Show that the cube of a positive integer of the form $(6q + r)$, where q is an integer and $r = 0, 1, 2, 3, 4$ and 5, which is also of the form $(6m + r)$.

Sol. By Euclid's division algorithm,

$$a = 6q + r \quad \text{...}(i)$$

where a, q and r are non-negative integers $0 \le r < 6$ i.e., $r = 0, 1, 2, 3, 4, 5$.

Cubing (*i*) both sides, we get

$$(a)^3 = (6q + r)^3$$

$\Rightarrow \quad a^3 = (6q)^3 + (r)^3 + 3(6q)^2(r) + 3(6q)(r)^2$

$= 6^3q^3 + r^3 + 3 \times 6^2q^2r + 6 \times 3qr^2$

$\Rightarrow \quad a^3 = 6[36q^3 + 18q^2r + 3qr^2] + r^3$ ...(*ii*)

When $r = 0$, then $a^3 = 6[36q^3 + 18q^2 \times 0 + 3q0^2] + 0^3$ [From (*ii*)]

$\Rightarrow \quad a^3 = 6[36q^3]$

$\Rightarrow \quad a^3 = 6m$ is perfect cube for some value of m such that $m = 36q^2$

When $r = 1$, $\quad a^3 = 6[36q^3 + 18q^2 \times 1 + 3q1^2] + 1^3$ [From (*ii*)]

$= 6[36q^3 + 18q^2 + 3q] + 1$

$\Rightarrow \quad a^3 = 6m + 1$ is perfect cube for some value of m such that $m = (36q^3 + 18q^2 + 3q)$

When $r = 2$, $\quad a^3 = 6[36q^3 + 18q^2 \times 2 + 3q \times 2^2] + 2^3$ [From (*ii*)]

$= 6[36q^3 + 36q^2 + 12q] + 6 + 2$

$= 6[36q^3 + 36q^2 + 12q + 1] + 2$

$\Rightarrow \quad a^3 = 6m + 2$ is perfect cube for some values of m such that $m = 36q^3 + 36q^2 + 12q + 1$

When $r = 3$, $\quad a^3 = 6[36q^3 + 18q^2 \times 3 + 3q \times 3^2] + 3^3$ [From (*ii*)]

$\Rightarrow \quad a^3 = 6[36q^3 + 54q^2 + 27q] + 24 + 3$

$\Rightarrow \quad a^3 = 6[36q^3 + 54q^2 + 27q + 4] + 3$

$\Rightarrow \quad a^3 = 6m + 3$

So, $(6m + 3)$ is perfect cube for specified value of m such that

$$m = 36q^3 + 54q^2 + 27q + 4$$

When $r = 4$, then eq. (*ii*) becomes

$a^3 = 6[36q^3 + 18q^2(4) + 3q\,4^2] + 4^3$

$= 6[36q^3 + 72q^2 + 48\,q] + 60 + 4$

$= 6[36q^3 + 72q^2 + 48\,q + 10] + 4$

$\Rightarrow \quad a^3 = 6m + 4$

So, $(6m + 4)$ is perfect cube for specified value of m such that

$$m = 36q^3 + 72q^2 + 48q + 10$$

When $r = 5$, eq. (*ii*) becomes as

$a^3 = 6[36q^3 + 18q^2(5) + 3q(5)^2] + (5)^3$

$= 6[36q^3 + 90q^2 + 75q] + 120 + 5$

$= 6[36q^3 + 90q^2 + 75q + 20] + 5$

$\Rightarrow \quad a^3 = 6m + 5$

$\Rightarrow$ $(6m + 5)$ is perfect cube for specified value of

$$m = 36q^3 + 90q^2 + 75q + 20$$

Hence, cubes of positive integers is of the form $(6m + r)$, where m is a specified integer and $r = 0, 1, 2, 3, 4, 5$.

Q2. Prove that one and only one out of n, $(n+2)$ and $(n+4)$ is divisible by 3, where n is any positive integer.

Sol. Consider the given numbers n, $n+2$ and $n+4$.

When $n = 1$, numbers become $1, 1+2, 1+4 = (1, 3 \text{ and } 5)$

When $n = 2$, numbers become $2, 2+2, 2+4 = (2, 4, 6)$

When $n = 3$, numbers become $= (3, 5, 7)$

When $n = 4$, numbers become $= (4, 6, 8)$

When $n = 5$, numbers become $= (5, 7, 9)$

When $n = 6$, numbers become $= (6, 8, 10)$

When $n = 7$, numbers become $= (7, 9, 11)$

From above, we observe that out of 3 numbers one is divisible by 3.

Alternate Method: Consider that if a number n is divided by 3, then we get a quotient q and remainder r then by Euclid's division algorithm,

$$n = 3q + r \quad \text{where, } 0 \le r < 3$$

At	n_1	Divisible by 3	$n_2 = n_1 + 2$	Divisible by 3	$n_3 = n_1 + 4$	Divisible by 3
$r=0$	$3q+0=3q$	Yes	$3q+2$	No	$3q+4$ $= 3q+3+1$ $= 3(q+1)+1$ $= 3m+1$	No
$r=1$	$3q+1$	No	$3q+1+2$ $= 3q+3$ $= 3(q+1)$	Yes	$3q+1+4$ $= 3q+3+2$ $= 3(q+1)+2$ $= 3m+2$	No
$r=2$	$3q+2$	No	$3q+2+2$ $= 3q+3+1$ $= 3(q+1)+1$ $= 3m+1$	No	$3q+2+4$ $= 3q+6$ $= 3(q+2)$ $= 3m$	Yes

From table, out of n_1, n_2 or n_3 one number is divisible by 3 when $r = 0, 1, 2$, are taken.

Q3. Prove that one of any three consecutive positive integers must be divisible by 3.

Sol. Consider a number n. q and r are positive integers. When n is divided by 3 the quotient is q and remainder r. So, by Euclid's division algorithm,

$$n = 3q + r \quad (0 \le r < 3) \quad \text{or } r = 0, 1, 2, 3$$

At	n_1	Divisible by 3	$n_2 = n_1 + 1$	Divisible by 3	$n_3 = n_1 + 2$	Divisible by 3
$r=0$	$3q+0$ $=3q$	Yes	$3q+1$	No	$3q+2$	No
$r=1$	$3q+1$	No	$3q+2$	No	$3q+3$ $=3(q+1)$ $=3m$	Yes
$r=2$	$3q+2$	No	$3q+3=3(q+1)$ $=3m$	Yes	$3q+4$ $= 3q+3+1$ $= 3(q+1)+1$ $= 3m+1$	No

So, one of any three consecutive positive integers is divisible by 3.

Q4. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

Sol. Let $a = n^3 - n$

$\Rightarrow$ $a = n(n^2 - 1)$

$= n(n-1)(n+1)$

$(n-1)$, n, $(n+1)$ are consecutive integers so out of three consecutive numbers at least one will be even. So, a is divisible by 2.

Sum of numbers $= (n-1) + n + (n+1)$

$= n - 1 + n + n + 1$

$= 3n$

Clearly, the sum of three consecutive numbers is divisible by 3, so any one of them must be divisible by 3.

So, out of n, $(n-1)$, $(n+1)$, one is divisible by 2 and one is divisible by 3 and

$$a = (n-1) \times n \times (n+1)$$

Hence, out of three factors of a, one is divisible by 2 and one is divisible by 3. So, a is divisible by 6 or $n^3 - n$ is divisible by 6.

Q5. Show that one and only one out of n, $(n + 4)$, $(n + 8)$, $(n + 12)$, $(n + 16)$ is divisible by 5, where n is any positive integer.

[**Hint:** Any positive integer can be written in the form $5q$, $(5q + 1)$, $(5q + 2)$, $(5q + 3)$, $(5q + 4)$]

Sol. Let a number n is divided by 5 then quotient is q and remainder is r. Then by Euclid's division algorithm,

$n = 5q + r$, where n, q, r are non-negative integers and $0 \le r < 5$

When $r = 0$, $n = 5q + 0 = 5q$

So, n is divisible by 5.

When $r = 1$, $n = 5q + 1$

$n + 2 = 5q + 1 + 2 = 5q + 3$ is not divisible by 5.

$n + 4 = (5q + 1) + 4 = 5q + 5 = 5(q + 1)$ divisible by 5.

So, $(n + 4)$ is divisible by 5.

When $r = 2$, $n = 5q + 2$

$(n + 8) = (5q + 2) + 8 = 5q + 10 = 5(q + 2) = 5m$ is divisible by 5.

So, $(n + 8)$ is divisible by 5.

When $r = 3$, $n = 5q + 3$

$n + 12 = (5q + 3) + 12 = 5q + 15 = 5(q + 3) = 5m$ is divisible by 5.

So, $(n + 12)$ is divisible by 5.

When $r = 4$, $n = 5q + 4$

$n + 16 = (5q + 4) + 16 = 5q + 20 = 5(q + 4)$

$(n + 16) = 5m$ is divisible by 5.

Hence, n, $(n + 4)$, $(n + 8)$, $(n + 12)$ and $(n + 16)$ are divisible by 5.

❑❑❑

2 Polynomials

EXERCISE 2.1

Choose the correct answer from the given four options in the following questions:

Q1. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then the value of k is

(*a*) 4/3 (*b*) –4/3 (*c*) 2/3 (*d*) –2/3

Sol. (*a*): **Main concept:** If a is root of a polynomial $f(x)$, then $f(a) = 0$.

Let $f(x) = (k-1)x^2 + kx + 1$

As -3 is a zero of $f(x)$, then

$$f(-3) = 0$$

$$\Rightarrow \quad (k-1)(-3)^2 + k(-3) + 1 = 0$$

$$\Rightarrow \quad 9k - 9 - 3k + 1 = 0$$

$$\Rightarrow \quad 9k - 3k = +9 - 1$$

$$\Rightarrow \quad 6k = 8$$

$$\Rightarrow \quad k = 4/3$$

Q2. A quadratic polynomial, whose zeroes are -3 and 4, is

(*a*) $x^2 - x - 12$ (*b*) $x^2 + x + 12$ (*c*) $\frac{x^2}{2} - \frac{x}{2} - 6$ (*d*) $2x^2 + 2x - 24$

Sol. (*c*): **Main concept:** Required quadratic polynomial

$$= x^2 - (\alpha + \beta) + \alpha\beta$$

Here, $\alpha = -3$ and $\beta = 4$

$\therefore \quad \alpha + \beta = -3 + 4 = 1$

and $\quad \alpha \cdot \beta = -3 \times 4 = -12$

$\therefore$ The quadratic polynomial is

$$= x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - 1x - 12$$

$$= \frac{x^2}{2} - \frac{x}{2} - \frac{12}{2}$$

$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

Q3. If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then

(*a*) $a = -7, b = -1$ (*b*) $a = 5, b = -1$

(*c*) $a = 2, b = -6$ (*d*) $a = 0, b = -6$

Sol. (*d*): **Main concept:** If a is zero of a polynomial $f(x)$, then $f(a) = 0$.

Let $f(x) = x^2 + (a+1)x + b$

As 2, and (−3) are zeroes of polynomial $f(x) = x^2 + (a+1)x + b$, then

$f(2) = 0$ and $f(-3) = 0$

$\Rightarrow (2)^2 + (a+1)(2) + b = 0$ | $\Rightarrow (-3)^2 + (a+1)(-3) + b = 0$

$\Rightarrow 4 + 2a + 2 + b = 0$ | $\Rightarrow 9 - 3a - 3 + b = 0$

$\Rightarrow 2a + b = -6 \quad ...(i)$ | $\Rightarrow -3a + b = -6$

$\Rightarrow 3a - b = 6 \quad ...(ii)$

$5a = 0$ [Adding (*i*) and (*ii*)]

$\Rightarrow a = 0$

But, $2a + b = -6$ [From (*i*)]

$\Rightarrow 2(0) + b = -6$

$\Rightarrow b = -6$

Hence, $a = 0$ and $b = -6$ verifies option (*d*).

Q4. The number of polynomials having zeroes as −2 and 5 is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) more than 3

Sol. (*d*): We know that if we divide or multiply a polynomial by any constant (real number), then the zeroes of polynomial remains same.

Here, $\alpha = -2$ and $\beta = +5$

$\therefore \quad \alpha + \beta = -2 + 5 = 3$ and $\alpha \cdot \beta = -2 \times 5 = -10$

So, required polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 3x - 10$

If we multiply this polynomial by any real number let 5 and 2, we get

$5x^2 - 15x - 50$

and $2x^2 - 6x - 20$

which are different polynomials having same zeroes −2 and 5.

So, we can obtain so many (infinite polynomials) from two given zeroes.

Q5. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

(*a*) $\frac{-c}{a}$ (*b*) $\frac{c}{a}$ (*c*) 0 (*d*) $\frac{-b}{a}$

Sol. (*b*): Let $f(x) = ax^3 + bx^2 + cx + d$

If α, β, γ are the zeroes of $f(x)$, then

$$\boxed{\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}}$$

One root is zero (Given) so, $\alpha = 0$.

$\Rightarrow \quad \beta\gamma = \frac{c}{a}$

Q6. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is −1, then the product of other two zeroes is

(*a*) $b - a + 1$ (*b*) $b - a - 1$ (*c*) $a - b + 1$ (*d*) $a - b - 1$

Sol. (*a*): Let $f(x) = x^3 + ax^2 + bx + c$

$\because$ Zero of $f(x)$ is -1 so

$$f(-1) = 0$$

$\Rightarrow \quad (-1)^3 + a(-1)^2 + b(-1) + c = 0$

$\Rightarrow \quad -1 + a - b + c = 0$

$\Rightarrow \quad a - b + c = 1$

$\Rightarrow \quad c = 1 + b - a$

Now, $\quad \alpha \cdot \beta \cdot \gamma = \dfrac{-d}{a} \qquad [\because c = b, \quad d = c]$

$\Rightarrow \quad -1\,\beta\gamma = \dfrac{-c}{1}$

$\Rightarrow \quad \beta\gamma = c$

$\Rightarrow \quad \beta\gamma = 1 + b - a$

Q7. The zeroes of quadratic polynomial $x^2 + 99x + 127$ are

(*a*) both positive

(*b*) both negative

(*c*) one positive and one negative

(*d*) both are equal

Sol. (*b*): Let $f(x) = x^2 + 99x + 127$

Now, $\quad b^2 - 4ac = (99)^2 - 4(1)\,127 \qquad (a = 1, b = 99, c = 127)$

$\Rightarrow \quad b^2 - 4ac = 9801 - 508$

$\Rightarrow \quad \sqrt{b^2 - 4ac} = \sqrt{9293}$

$\Rightarrow \quad \sqrt{b^2 - 4ac} = 96.4$

So, zeroes of $f(x)$, $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \dfrac{-99 \pm 96.4}{2 \times 1}$

$\Rightarrow$ Both roots will be negative as $99 > 96.4$.

Q8. The zeroes of the quadratic polynomial $x^2 + kx + k$ where $k \neq 0$

(*a*) cannot both be positive

(*b*) cannot both be negative

(*c*) are always unequal

(*d*) are always equal

Sol. (*a*): Let $f(x) = x^2 + kx + k$

For zeroes of $f(x)$, $f(x) = 0$

$\Rightarrow \quad x^2 + kx + k = 0$

But, $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \dfrac{-k \pm \sqrt{k^2 - 4 \cdot k}}{2} = \dfrac{-k \pm \sqrt{k(k-4)}}{2}$

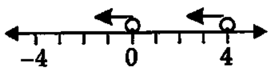
For real roots, $b^2 - 4ac > 0$

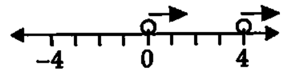
$\Rightarrow \quad k(k - 4) > 0 \qquad (k \neq 0)$

$\Rightarrow$ Either $\quad k < 0$ and $k - 4 < 0 \quad$ or $\quad k > 0$ and $(k - 4) > 0$

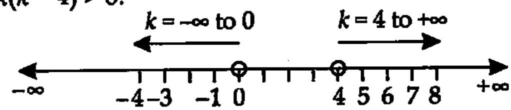
$\quad k < 0$ and $k < 4 \qquad\qquad k > 0$ and $k > 4$

$\quad k < 0, k < 4$





So, solution $k(k - 4) > 0$.



Let $k = -4$

(any point on number line)

$$x = \frac{-4 \pm \sqrt{-4(-4-4)}}{2} = \frac{-4 \pm \sqrt{32}}{2}$$

$$= \frac{-4 \pm 4\sqrt{2}}{2} = \frac{4[-1 \pm \sqrt{2}]}{2}$$

$$x = 2[-1 \pm \sqrt{2}]$$

$x_1 = 2[-1 + \sqrt{2}]$, which is positive

$x_2 = 2[-1 - \sqrt{2}]$, which is negative

Let $k = 8$ (any point on number line)

$$x = \frac{-8 \pm \sqrt{8(8-4)}}{2}$$

$$x = \frac{-8 \pm \sqrt{8 \times 4}}{2}$$

$$x = \frac{-8 \pm 4\sqrt{2}}{2}$$

$$x = \frac{+4[-2 \pm \sqrt{2}]}{2}$$

$$x = 2(-2 \pm \sqrt{2})$$

$x_1 = 2[-2 + \sqrt{2}]$, which is negative

$x_2 = 2[-2 - \sqrt{2}]$, which is negative

So, the roots cannot be both positive.

Q9. If the zeroes of the quadratic polynomial

$ax^2 + bx + c$, where, $c \neq 0$ are equal then

(*a*) *c* and *a* both have opposite signs

(*b*) *c* and *b* have opposite signs

(*c*) *c* and *a* have same sign

(*d*) *c* and *b* have the same sign

Sol. (*c*): For equal roots $b^2 - 4ac = 0$

or $\qquad b^2 = 4ac$

b^2 is always positive so $4ac$ must be positive or i.e., product of *a* and *c* must be positive i.e., *a* and *c* must have same sign either positive or negative.

Q10. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other then it

(*a*) has no linear term and the constant term is negative

(*b*) has no lineaı term and the constant term is positive

(*c*) can have a linear term but the constant term is negative.

(*d*) can have a linear term but the constant term is positive.

Sol. (*a*): Let $f(x) = x^2 + ax + b$ and α, β are the roots of it.

Then, $\beta = -\alpha$ (Given)

$$\alpha + \beta = \frac{-b}{a} \quad \text{and} \quad \alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \quad \alpha - \alpha = \frac{-a}{1} \qquad \alpha(-\alpha) = \frac{b}{1}$$

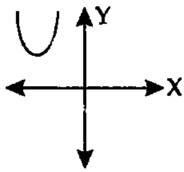
$$\Rightarrow \quad -a = 0 \qquad -\alpha^2 = b$$

$$\Rightarrow \quad a = 0 \qquad \Rightarrow \quad b < 0 \text{ or } b \text{ is negative}$$

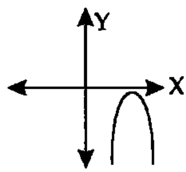
So, $f(x) = x^2 + b$ shows that it has no linear term.

Q11. Which of the following is not the graph of a quadratic polynomial?

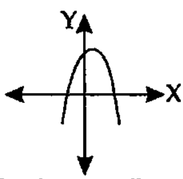
(*a*)



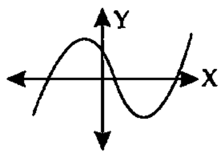
(*b*)



(*c*)



(*d*)



Sol. (*d*): Graph '*d*' intersect at three points on X-axis so the roots of polynomial of graph is three, so it is cubic polynomial. Other graphs are of quadratic polynomial. Graphs *a*, *b* have no real zeroes.

EXERCISE 2.2

Q1. Answer the following and justify.

(*i*) Can $x^2 - 1$ be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5 ?

(*ii*) What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + s, p \neq 0$?

(*iii*) If on division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero what is the relation between the degrees of $p(x)$ and $g(x)$?

(*iv*) If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?

(*v*) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?

Sol. (*i*): Let the divisor of degree 5 is $g(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + 1$

Dividend $= p(x) = x^6 + 2x^3 + x - 1,$

$q(x) = x^2 - 1$ and let remainder be $r(x)$

So, by Euclid's division algorithm

$$p(x) = g(x)\, q(x) + r(x)$$

[deg $p(x)$ is 6] = [$g(x)$ of deg 5] [$q(x)$ degree 2] + $r(x)$ of degree less than 5

$$\text{degree } p(x) = \text{degree } g(x) + \text{degree } q(x) + \text{degree } r(x)$$

$$6 = 5 + 2 + \text{any}$$

So, degree of $q(x)$ can never be 2 it may be only one.

So, $(x^2 - 1)$ can never be the quotient.

(*ii*) $p(x)$ (dividend) = $ax^2 + bx + c$

$g(x)$ (divisor) = $px^3 + qx^2 + r(x) + s$

As the degree of dividend is always greater than divisor but here degree $p(x) <$ degree $g(x)$.

When we divide $p(x)$ by $g(x)$, quotient will be zero and remainder will be $p(x)$.

(*iii*) The dividend = $p(x)$, divisor $g(x)$

quotient $q(x) = 0$

remainder = $r(x)$

Here, degree of divisor $g(x)$ is more than degree of dividend.

(*iv*) When $p(x)$ is divided by $g(x)$, the remainder is zero so the $g(x)$ is a factor of $p(x)$ and degree of $g(x)$ will be less than or equal to the degree of $p(x)$ or degree $g(x) \leq$ degree $p(x)$.

(*v*) Let $p(x) = x^2 + kx + k$

For equal zeroes, $b^2 - 4ac = 0$

$\Rightarrow \quad (k)^2 - 4(1)(k) = 0$

$\Rightarrow \quad k^2 - 4k = 0$

$\Rightarrow \quad k(k - 4) = 0$

$\Rightarrow \quad k = 0 \quad \text{or} \quad k = 4$

But $k > 1$ so $k = 4$

The given quadratic polynomial has equal zeroes at $k = 4$.

Q2. Are the following statements true or false? Justify your answers.

(*i*) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a, b and c have the same sign.

(*ii*) If the graph of polynomial intersects the X-axis at only one points it cannot be a quadratic polynomial.

(*iii*) If the graph of a polynomial intersects the X-axis at exactly two points, it need not be a quadratic polynomial.

(*iv*) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

(*v*) If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of polynomial have the same sign.

(*vi*) If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of a, b, and c is non-negative.

(*vii*) The only value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeroes is 1/2.

Sol. (*i*): **False:** Let α and β be the roots of the quadratic polynomial. If α and β are positive then $\alpha + \beta = \frac{-b}{a}$ it shows that $\frac{-b}{a}$ is negative but sum of two positive numbers (α, β) must be +ive i.e. either b or a must be negative. So a, b and c will have different signs.

(*ii*) **False:** The given statement is false, because when two zeroes of a quadratic polynomial are equal, then two intersecting points coincide to become one point.

(*iii*) **True:** If a polynomial of degree more than two has two real zeroes and other zeroes are not real or are imaginary, then graph of the polynomial will intersect at two points on x-axis.

(*iv*) **True:** Let $\beta = 0$, $\gamma = 0$

$$f(x) = (x - \alpha)(x - \beta)(x - \gamma)$$
$$= (x - \alpha)\, x \cdot x$$
$$\Rightarrow \quad f(x) = x^3 - \alpha x^2$$

which has no linear (coefficient of x) and constant terms.

(*v*) **True:** α, β, and γ are all (–)ive for cubic polynomial $ax^3 + bx^2 + cx + d$.

$$\alpha + \beta + \gamma = \frac{-b}{a} \qquad \text{...}(i)$$
$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \qquad \text{...}(ii)$$
$$\alpha\beta\gamma = \frac{-d}{a} \qquad \text{...}(iii)$$

$\because$ α, β, γ are all negative so,

$$\alpha + \beta + \gamma = -x \qquad \text{(Any negative number)}$$
$$\Rightarrow \quad \frac{-b}{a} = -x \qquad \text{[From } (i)]$$
$$\Rightarrow \quad \frac{b}{a} = x$$

So, a, b, have same sign and product of any two zeroes will be positive.

So, $$\alpha\beta + \beta\gamma + \gamma\alpha = +y \qquad \text{(Any positive number)}$$
$$\Rightarrow \quad \frac{+c}{a} = +y \qquad \text{[From } (ii)]$$

$\Rightarrow$ c and a have same sign

$$\alpha\beta\gamma = -z \qquad \text{(Any negative number)}$$
$$\Rightarrow \quad \frac{-d}{a} = -z \qquad \text{[From } (ii)]$$

$\Rightarrow \quad \frac{d}{a} = z$

So, d and a will have same sign.

Hence, signs of b, c, d are same as of a.

So, signs of a, b, c, d will be same either positive or negative.

(*vi*) **True:** As all zeroes of cubic polynomial are positive

Let $\quad f(x) = x^3 + ax^2 - bx + c$

$\therefore \quad \alpha + \beta + \gamma = +\text{ive say } +x$

$\Rightarrow \quad \frac{-b}{a} = x$

$\Rightarrow$ a and b has opposite signs ...(*i*)

$\alpha\beta + \beta\gamma + \gamma\alpha = +y$

$\Rightarrow \quad \frac{c}{a} = y$

So, signs of a and c are same. ...(*ii*)

Now, $\quad \alpha\beta\gamma = +\text{ive} = +z$

$\Rightarrow \quad \frac{-d}{a} = z$

$\Rightarrow$ a and d have opposite signs. [From (*i*)]

From (*i*), if a is positive, then b is negative.

From (*ii*) if a is positive, then c is also positive.

From (*iii*) if a is positive, then d is negative.

Hence, if zeroes α, β, γ of cubic polynomial are positive then out of a, b, c at least one is positive.

(*vii*) **False:** $\quad f(x) = kx^2 + x + k \quad (a = k, b = 1, c = k)$

For equal roots

$$b^2 - 4ac = 0$$

$$\Rightarrow \quad (1)^2 - 4(k)(k) = 0$$

$$\Rightarrow \quad 4k^2 = 1$$

$$\Rightarrow \quad k^2 = 1/4$$

$$\Rightarrow \quad k = \pm\frac{1}{2}$$

So, there are $\frac{1}{2}$ and $\frac{-1}{2}$ values of k so that the given equation has equal roots.

EXERCISE 2.3

Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and coefficients of the polynomials.

Q1. $\quad 4x^2 - 3x - 1$

Sol. Let $f(x) = 4x^2 - 3x - 1$

Splitting the middle term, we get

$$= 4x^2 - 4x + 1x - 1$$
$$= 4x(x-1) + 1(x-1)$$
$$= (x-1)(4x+1)$$

For $f(x) = 0$, we have

$$4x^2 - 3x - 1 = 0$$

or $(x-1)(4x+1) = 0$

Either $x - 1 = 0 \Rightarrow x = 1$

or $4x + 1 = 0 \Rightarrow 4x = -1 \Rightarrow x = \frac{-1}{4}$

$\therefore$ The zeroes of $f(x)$ are 1 and $\frac{-1}{4}$.

Verification: $\alpha = 1, \beta = \frac{-1}{4}$

$a = 4, b = -3$ and $c = -1$

$\therefore \quad \alpha + \beta = \frac{-b}{a}$

$\Rightarrow \quad 1 - \frac{1}{4} = \frac{-(-3)}{4}$

$\Rightarrow \quad \frac{3}{4} = \frac{3}{4}$

$\Rightarrow$ LHS = RHS

Hence, verified

$\alpha \cdot \beta = \frac{c}{a}$

$\Rightarrow \quad 1 \times \left(\frac{-1}{4}\right) = \frac{-1}{4}$

$\Rightarrow \quad \frac{-1}{4} = \frac{-1}{4}$

$\Rightarrow$ LHS = RHS

Hence, verified

Q2. $3x^2 + 4x - 4$

Sol. Let $f(x) = 3x^2 + 4x - 4$

For zeroes of $f(x)$, $f(x) = 0$

$\therefore \quad 3x^2 + 4x - 4 = 0$

Splitting the middle term, we get

$$3x^2 + 6x - 2x - 4 = 0$$

$\Rightarrow \quad 3x(x+2) - 2(x+2) = 0$

$\Rightarrow \quad (x+2)(3x-2) = 0$

$\Rightarrow \quad x + 2 = 0$ or $3x - 2 = 0$

$\Rightarrow \quad x = -2$ or $3x = +2 \Rightarrow x = \frac{2}{3}$

So, zeroes of $f(x)$ are –2, and 2/3.

Sum of roots $= \frac{-b}{a}$ $\quad (a = 3, b = 4, c = -4)$

$\Rightarrow \quad -2 + \frac{2}{3} = \frac{-4}{3}$

$\Rightarrow \quad \frac{-6+2}{3} = \frac{-4}{3}$

$$\Rightarrow \quad \frac{-4}{3} = \frac{-4}{3}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

Product of roots = $\frac{c}{a}$

$$\Rightarrow \quad -2 \times \frac{2}{3} = \frac{-4}{3}$$

$$\Rightarrow \quad \frac{-4}{3} = \frac{-4}{3}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

Q3. $5t^2 + 12t + 7$

Sol. Let $f(t) = 5t^2 + 12t + 7$

For zeroes of $f(t)$, $f(t) = 0$

$$\Rightarrow \quad 5t^2 + 12t + 7 = 0$$

$$\Rightarrow \quad 5t^2 + 7t + 5t + 7 = 0$$

$$\Rightarrow \quad t(5t + 7) + 1(5t + 7) = 0$$

$$\Rightarrow \quad (5t + 7)(t + 1) = 0$$

$$\Rightarrow \quad 5t + 7 = 0 \quad \text{or} \quad (t + 1) = 0$$

$$\Rightarrow \quad t = \frac{-7}{5} \quad \text{or} \quad t = -1$$

Verification: $\alpha = -\frac{7}{5}$ $\quad \beta = -1$

$a = 5, \quad b = 12, \quad c = +7$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \quad \frac{-7}{5} - 1 = \frac{-(+12)}{5}$$

$$\Rightarrow \quad \frac{-7-5}{5} = \frac{-12}{5}$$

$$\Rightarrow \quad \frac{-12}{5} = \frac{-12}{5}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \quad \left(\frac{-7}{5}\right)(-1) = \frac{7}{5}$$

$$\Rightarrow \quad +\frac{7}{5} = \frac{7}{5}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

Q4. $t^3 - 2t^2 - 15t$

Sol. Let $f(t) = t^3 - 2t^2 - 15t$

For zeroes of $f(t)$, $f(t) = 0$

$$\Rightarrow \quad t^3 - 2t^2 - 15t = 0$$

$$\Rightarrow \quad t[t^2 - 2 \cdot t - 15] = 0$$
$$\Rightarrow \quad t[t^2 - 5t + 3t - 15] = 0$$
$$\Rightarrow \quad t[t(t-5) + 3(t-5)] = 0$$
$$\Rightarrow \quad t(t-5)(t+3) = 0$$
$$\Rightarrow \quad t = 0 \text{ or } t - 5 = 0 \quad \text{or} \quad t + 3 = 0$$
$$\Rightarrow \quad t = 0 \text{ or } t = 5 \quad \text{or} \quad t = -3$$

So, zeroes of cubic polynomial are $\alpha = 0$, $\beta = 5$, $\gamma = -3$

Verification: $\alpha = 0$, $\beta = 5$, $\gamma = -3$

Cubic polynomial,

$f(t) = t^3 - 2t^2 - 15t$, which is of the form $at^3 + bt^2 + ct + d$

where $a = 1$, $b = -2$, $c = -15$ and $d = 0$

$$\alpha + \beta + \gamma = \frac{-b}{a}$$
$$\Rightarrow \quad 0 + 5 - 3 = \frac{-(-2)}{1}$$
$$\Rightarrow \quad 2 = 2$$
$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$
$$\Rightarrow (0)(5) + (5)(-3) + (-3)(0) = \frac{-15}{1}$$
$$\Rightarrow \quad 0 - 15 + 0 = -15$$
$$\Rightarrow \quad -15 = -15$$
$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

$$\alpha\beta\gamma = \frac{-d}{a}$$
$$\Rightarrow \quad (0)(5)(-3) = \frac{-0}{1}$$
$$\Rightarrow \quad 0 = 0$$
$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

Q5. $2x^2 + \frac{7}{2}x + \frac{3}{4}$

Sol. Let $f(x) = 2x^2 + \frac{7}{2}x + \frac{3}{4}$

For zeroes of $f(x)$, $f(x) = 0$

$$\Rightarrow \quad 2x^2 + \frac{7}{2}x + \frac{3}{4} = 0$$
$$\Rightarrow \quad 8x^2 + 14x + 3 = 0$$

[As c is positive $(+3)$ so sum of (8×3) factors should be equal to 14]

$$\Rightarrow \quad 8x^2 + 12x + 2x + 3 = 0$$
$$\Rightarrow \quad 4x(2x+3) + 1(2x+3) = 0$$
$$\Rightarrow \quad (2x+3)(4x+1) = 0$$
$$\Rightarrow \quad 2x + 3 = 0 \quad \text{or} \quad 4x + 1 = 0$$
$$\Rightarrow \quad 2x = -3 \quad \text{or} \quad 4x = -1$$

$$\Rightarrow \quad x = \frac{-3}{2} \quad \text{or} \quad x = \frac{-1}{4}$$

Verification: $\alpha = \frac{-3}{2}$ and $\beta = \frac{-1}{4}$

Quadratic polynomial $f(x) = 2x^2 + \frac{7}{2}x + \frac{3}{4}$, which is of the form $ax^2 + bx + c$.

$\therefore a = 2, \quad b = \frac{7}{2}$ and $c = \frac{3}{4}$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \quad \frac{-3}{2} - \frac{1}{4} = \frac{\frac{-7}{2}}{2}$$

$$\Rightarrow \quad \frac{-7}{4} = \frac{-7}{2} \times \frac{1}{2}$$

$$\Rightarrow \quad \frac{-7}{4} = \frac{-7}{4}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \quad \left(\frac{-3}{2}\right)\left(\frac{-1}{4}\right) = \frac{\frac{3}{4}}{2}$$

$$\Rightarrow \quad \frac{+3}{8} = \frac{3}{4} \times \frac{1}{2}$$

$$\Rightarrow \quad \frac{3}{8} = \frac{3}{8}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

Q6. $4x^2 + 5\sqrt{2}x - 3$

Sol. Let $f(x) = 4x^2 + 5\sqrt{2}x - 3$

For zeroes of $f(x)$, $f(x) = 0$

$$\Rightarrow \quad 4x^2 + 5\sqrt{2}x - 3 = 0$$

$$\Rightarrow \quad 4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3 = 0 \qquad \left[\because 4 \times 3 = 2 \times 2 \times 3 = \sqrt{2} \times \sqrt{2} \times 2 \times 3 = 6\sqrt{2} \times \sqrt{2}\right]$$

$$\Rightarrow \quad 2x[2x + 3\sqrt{2}] - 1[\sqrt{2}x + 3] = 0$$

$$\Rightarrow \quad 2\sqrt{2}x[\sqrt{2}x + 3] - 1[\sqrt{2}x + 3] = 0$$

$$\Rightarrow \quad (\sqrt{2}x + 3)(2\sqrt{2}x - 1) = 0$$

$$\Rightarrow \quad \sqrt{2}x + 3 = 0 \quad \text{or} \quad 2\sqrt{2}x - 1 = 0$$

$$\Rightarrow \quad x = \frac{-3}{\sqrt{2}} \quad \text{or} \quad x = \frac{1}{2\sqrt{2}}$$

Verification:

$\alpha = \frac{-3}{\sqrt{2}}, \beta = \frac{1}{2\sqrt{2}}, a = 4, b = 5\sqrt{2}, c = -3$

$$\alpha+\beta = \frac{-b}{a}$$

$$\Rightarrow \quad \frac{-3}{\sqrt{2}}+\frac{1}{2\sqrt{2}} = \frac{-5\sqrt{2}}{4}$$

$$\Rightarrow \quad \frac{(-6+1)}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = -\frac{5\sqrt{2}}{4}$$

$$\Rightarrow \quad \frac{-5\sqrt{2}}{4} = -\frac{5\sqrt{2}}{4}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

$$\alpha\cdot\beta = \frac{c}{a}$$

$$\Rightarrow \quad \left(-\frac{3}{\sqrt{2}}\right)\left(\frac{1}{2\sqrt{2}}\right) = \frac{-3}{4}$$

$$\Rightarrow \quad \frac{-3}{4} = \frac{-3}{4}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

Q7. $2s^2-(1+2\sqrt{2})s+\sqrt{2}$

Sol. Let $f(s)=2s^2-(1+2\sqrt{2})s+\sqrt{2}$

For zeroes of $f(s)$, $f(s)=0$

$$\Rightarrow \quad 2s^2-(1+2\sqrt{2})s+\sqrt{2} = 0 \qquad [\because\ a\times c=(2\sqrt{2})]$$

$$\Rightarrow \quad 2s^2-1s-2\sqrt{2}s+\sqrt{2} = 0 \qquad \text{[Open the brackets]}$$

$$\Rightarrow \quad s(2s-1)-\sqrt{2}\,(2s-1) = 0$$

$$\Rightarrow \quad (2s-1)(s-\sqrt{2}) = 0$$

$$\Rightarrow \quad 2s-1=0 \quad \text{or} \quad s-\sqrt{2}=0$$

$$\Rightarrow \quad s=\frac{1}{2} \quad \text{or} \quad s=\sqrt{2}$$

Verification of the relation between α, β, *a*, *b* and *c*

$\alpha=\frac{1}{2},\quad \beta=\sqrt{2},\quad a=2,\quad b=-(1+2\sqrt{2}),\quad c=\sqrt{2}$

$$\alpha+\beta = \frac{-b}{a}$$

$$\Rightarrow \quad \frac{1}{2}+\sqrt{2} = \frac{+(1+2\sqrt{2})}{2}$$

$$\Rightarrow \quad \frac{1}{2}+\sqrt{2} = \frac{1}{2}+\frac{2\sqrt{2}}{2}$$

$$\Rightarrow \quad \frac{1}{2}+\sqrt{2} = \frac{1}{2}+\sqrt{2}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

$$\alpha\cdot\beta = \frac{c}{a}$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)(\sqrt{2}) = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \quad \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

Q8. $v^2+4\sqrt{3}v-15$

Sol. Let $f(v)=v^2+4\sqrt{3}v-15$

For zeroes of $f(v)$, $f(v) = 0$

$$\Rightarrow \quad v^2 + 4\sqrt{3}v - 15 = 0$$

$$\Rightarrow \quad v^2 + 5\sqrt{3}v - 1\sqrt{3}v - 15 = 0 \qquad \begin{bmatrix} 15 = 5 \times 3 \\ = 1 \times 5 \times \sqrt{3} \times \sqrt{3} \end{bmatrix}$$

$$\Rightarrow \quad v(v + 5\sqrt{3}) - \sqrt{3}(v + 5\sqrt{3}) = 0$$

$$\Rightarrow \quad (v + 5\sqrt{3})(v - \sqrt{3}) = 0$$

$$\Rightarrow \quad (v + 5\sqrt{3}) = 0 \quad \text{or} \quad (v - \sqrt{3}) = 0$$

$$\Rightarrow \quad v = -5\sqrt{3} \quad \text{or} \quad v = \sqrt{3}$$

Verification of relations between α, β, a, b, c

$\alpha = -5\sqrt{3}$, $\beta = \sqrt{3}$, $a = 1$, $b = 4\sqrt{3}$ and $c = -15$

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \quad -5\sqrt{3} + \sqrt{3} = \frac{-4\sqrt{3}}{1}$$

$$\Rightarrow \quad -4\sqrt{3} = -4\sqrt{3}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \quad (-5\sqrt{3})(\sqrt{3}) = \frac{-15}{1}$$

$$\Rightarrow \quad -5 \times 3 = -15$$

$$\Rightarrow \quad -15 = -15$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, verified.

Q9. $y^2 + \frac{3}{2}\sqrt{5}y - 5$

Sol. Let $f(y) = y^2 + \frac{3}{2}\sqrt{5}y - 5$

For zeroes of $f(y)$, $f(y) = 0$

$$\Rightarrow \quad y^2 + \frac{3}{2}\sqrt{5}y - 5 = 0$$

$$\Rightarrow \quad 2y^2 + 3 \cdot \sqrt{5}y - 10 = 0$$

$$\Rightarrow \quad 2y^2 + 4\sqrt{5}y - 1\sqrt{5}y - 10 = 0 \qquad \begin{bmatrix} 2 \times 10 = 2 \times 2 \times 5 \\ = 2 \times 2 \times \sqrt{5} \times \sqrt{5} \\ = (4 \times 5) \end{bmatrix}$$

$$\Rightarrow \quad 2y(y + 2\sqrt{5}) - \sqrt{5}[y + 2\sqrt{5}] = 0$$

$$\Rightarrow \quad (y + 2\sqrt{5})(2y - \sqrt{5}) = 0$$

$$\Rightarrow \quad y + 2\sqrt{5} = 0 \quad \text{or} \quad 2y - \sqrt{5} = 0$$

$$\Rightarrow \quad y = -2\sqrt{5} \quad \text{or} \quad y = \frac{\sqrt{5}}{2}$$

Verification of the relations between α, β, and a, b, c

$\alpha = -2\sqrt{5}$, $\beta = \frac{\sqrt{5}}{2}$, $a = 1$, $b = \frac{3}{2}\sqrt{5}$ and $c = -5$

$$\alpha+\beta=\frac{-b}{a}$$

$$\Rightarrow \quad -2\sqrt{5}+\frac{\sqrt{5}}{2}=\frac{\frac{-3}{2}\sqrt{5}}{1}$$

$$\Rightarrow \quad \frac{-4\sqrt{5}+\sqrt{5}}{2}=\frac{-3}{2}\sqrt{5}$$

$$\Rightarrow \quad \frac{-3\sqrt{5}}{2}=\frac{-3}{2}\sqrt{5}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

$$\alpha\cdot\beta=\frac{c}{a}$$

$$\Rightarrow \quad (-2\sqrt{5})\left(\frac{\sqrt{5}}{2}\right)=\frac{-5}{1}$$

$$\Rightarrow \quad -5=-5$$

$\Rightarrow$ LHS = RHS

Hence, verified.

Q10. $7y^2-\frac{11}{3}y-\frac{2}{3}$

Sol. Let $f(y)=7y^2-\frac{11}{3}y-\frac{2}{3}$

For zeroes of $f(y)$, $f(y)=0$

$$\Rightarrow \quad 7y^2-\frac{11}{3}y-\frac{2}{3}=0$$

$$\Rightarrow \quad 21y^2-11y-2=0$$

$$\Rightarrow \quad 21y^2-14y+3y-2=0$$

$$\Rightarrow \quad 7y(3y-2)+1(3y-2)=0$$

$$\Rightarrow \quad (3y-2)(7y+1)=0$$

$$\Rightarrow \quad 3y-2=0 \quad \text{or} \quad 7y+1=0$$

$$\Rightarrow \quad y=\frac{2}{3} \quad \text{or} \quad y=\frac{-1}{7}$$

Verification of the relations between α, β, a, b and c

$$\alpha=\frac{2}{3},\quad \beta=\frac{-1}{7},\quad a=7,\quad b=-\frac{11}{3},\quad c=\frac{-2}{3}$$

$$\Rightarrow \quad \alpha+\beta=\frac{-b}{a}$$

$$\Rightarrow \quad \left(\frac{2}{3}\right)-\frac{1}{7}=\frac{+\frac{11}{3}}{7}$$

$$\Rightarrow \quad \frac{14-3}{21}=\frac{11}{3}\times\frac{1}{7}$$

$$\Rightarrow \quad \frac{11}{21}=\frac{11}{21}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

$$\alpha\cdot\beta=\frac{c}{a}$$

$$\Rightarrow \quad \left(\frac{2}{3}\right)\times\left(\frac{-1}{7}\right)=\frac{\frac{-2}{3}}{7}$$

$$\Rightarrow \quad \frac{-2}{21}=\frac{-2}{3}\times\frac{1}{7}$$

$$\Rightarrow \quad \frac{-2}{21}=\frac{-2}{21}$$

$\Rightarrow$ LHS = RHS

Hence, verified.

EXERCISE 2.4

Q1. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomial by factorisation.

(*i*) $\frac{-8}{3}, \frac{4}{3}$ (*ii*) $\frac{21}{8}, \frac{5}{16}$ (*iii*) $-2\sqrt{3}, -9$ (*iv*) $\frac{-3}{2\sqrt{5}}, \frac{-1}{2}$

Sol. Main concept: (*a*) If α, β are the zeroes of $f(x)$, then

$$f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

(*b*) The zeroes of $f(x)$ are given by $f(x) = 0$.

(*i*) $\alpha + \beta = \frac{-8}{3}$ and $\alpha \cdot \beta = \frac{4}{3}$ [Given]

$\therefore$ $f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$ [Formula]

$$= x^2 - \left(\frac{-8}{3}\right)x + \frac{4}{3}$$

Multiplying or dividing $f(x)$ by any real number does not affect the zeroes of polynomial.

So, $f(x) = 3x^2 + 8x + 4$ [Multiplying by LCM 3]

For zeroes of $f(x)$, $f(x) = 0$

$\Rightarrow$ $3x^2 + 8x + 4 = 0$

$\Rightarrow$ $3x^2 + 6x + 2x + 4 = 0$

$\Rightarrow$ $3x(x + 2) + 2(x + 2) = 0$

$\Rightarrow$ $(x + 2)(3x + 2) = 0$

$\Rightarrow$ $x + 2 = 0$ or $3x + 2 = 0$

$\Rightarrow$ $x = -2$ or $x = \frac{-2}{3}$

$\therefore$ $\alpha = -2$ and $\beta = \frac{-2}{3}$

(*ii*) $\alpha + \beta = \frac{21}{8}$ and $\alpha \cdot \beta = \frac{5}{16}$ [Given]

$f(x) = x^2 - (\alpha + \beta)x + \alpha \cdot \beta$ [Formula]

$\Rightarrow$ $f(x) = x^2 - \left(\frac{21}{8}\right)x + \left(\frac{5}{16}\right)$

Multiplying (or dividing) $f(x)$ by any real number does not affect the zeroes of $f(x)$ so, multiplying $f(x)$ by 16 (LCM), we get

$$f(x) = 16x^2 - 42x + 5$$

For zeroes of polynomial $f(x)$, $f(x) = 0$

$\Rightarrow$ $16x^2 - 42x + 5 = 0$

$\Rightarrow$ $16x^2 - 40x - 2x + 5 = 0$

$\Rightarrow$ $8x(2x - 5) - 1(2x - 5) = 0$

$\Rightarrow \quad (2x-5)(8x-1) = 0$

$\Rightarrow \quad 2x-5 = 0 \quad \text{or} \quad 8x-1 = 0$

$\Rightarrow \quad x = \frac{5}{2} \quad \text{or} \quad x = \frac{1}{8}$

$\therefore \alpha = \frac{5}{2}$ and $\beta = \frac{1}{8}$

(*iii*) $\alpha + \beta = -2\sqrt{3}$ and $\alpha\beta = -9$ [Given]

$f(x) = x^2 - (\alpha+\beta)x + \alpha\beta$ [Formula]

$= x^2 - (-2\sqrt{3})x + (-9)$

$\Rightarrow \quad f(x) = x^2 + 2\sqrt{3}x - 9$

For zeroes of polynomial $f(x)$, $f(x) = 0$

$\Rightarrow \quad x^2 + 2\sqrt{3}x - 9 = 0$

$\Rightarrow \quad x^2 + 3\sqrt{3}x - 1\sqrt{3}x - 9 = 0$

$\Rightarrow \quad x(x+3\sqrt{3}) - \sqrt{3}(x+3\sqrt{3}) = 0$

$\Rightarrow \quad (x+3\sqrt{3})(x-\sqrt{3}) = 0$

$\Rightarrow \quad x+3\sqrt{3} = 0 \quad \text{or} \quad (x-\sqrt{3}) = 0$

$\Rightarrow \quad x = -3\sqrt{3} \quad \text{or} \quad x = \sqrt{3}$

$\therefore \quad \alpha = -3\sqrt{3} \quad \text{and} \quad \beta = \sqrt{3}$

(*iv*) $\alpha + \beta = \frac{-3}{2\sqrt{5}}$ and $\alpha \cdot \beta = -\frac{1}{2}$ [Given]

$f(x) = x^2 - (\alpha+\beta)x + \alpha\beta$ [Formula]

$= x^2 - \left(\frac{-3}{2\sqrt{5}}\right)x + \left(-\frac{1}{2}\right)$

$\Rightarrow \quad f(x) = x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}$

Multiplying or dividing $f(x)$ by any real number does not affect the zeroes of $f(x)$. On multiplying $f(x)$ by $2\sqrt{5}$ (LCM), we get

$f(x) = 2\sqrt{5}x^2 + 3x - \sqrt{5}$

For zeroes of polynomial $f(x)$, $f(x) = 0$

$\Rightarrow \quad 2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$

$\Rightarrow \quad 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5} = 0$

$\Rightarrow \quad \sqrt{5}x(2x+\sqrt{5}) - 1(2x+\sqrt{5}) = 0$

$\Rightarrow \quad (2x+\sqrt{5})(\sqrt{5}x-1) = 0$

$\Rightarrow \quad (2x+\sqrt{5}) = 0 \quad \text{or} \quad \sqrt{5}x - 1 = 0$

$$\Rightarrow \quad x = \frac{-\sqrt{5}}{2} \quad \text{or} \quad x = \frac{1}{\sqrt{5}}$$

$$\therefore \quad \alpha = \frac{-\sqrt{5}}{2} \quad \text{and} \quad \beta = \frac{1}{\sqrt{5}}$$

Q2. Given that the zeroes of cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, $(a + b)$, $(a + 2b)$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial.

Sol. Main concept: $\alpha + \beta + \gamma = \frac{-b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{+c}{a}$ and $\alpha\beta\gamma = \frac{-d}{a}$

Let $f(x) = x^3 - 6x^2 + 3x + 10$ [given] ...(i)

$\alpha = a$, $\beta = a + b$ and $\gamma = a + 2b$ [Given]

But, $f(x) = ax^3 + bx^2 + cx + d$ ...(ii)

$\therefore a = 1$, $b = -6$, $c = 3$ and $d = +10$ [Comparing (i) and (ii)]

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\Rightarrow \quad a + a + b + a + 2b = \frac{+6}{1} \Rightarrow 3a + 3b = 6$$

$$\Rightarrow \quad a + b = 2$$

$$\Rightarrow \quad b = 2 - a \quad \text{...(iii)}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\Rightarrow \quad a(a + b) + (a + b)(a + 2b) + (a + 2b)(a) = \frac{3}{1}$$

$$\Rightarrow \quad a^2 + ab + a^2 + 2ab + ab + 2b^2 + a^2 + 2ab = 3$$

$$\Rightarrow \quad 3a^2 + 6ab + 2b^2 = 3$$

$$\Rightarrow \quad 3a^2 + 6a(2 - a) + 2(2 - a)^2 = 3 \quad \text{[Using (iii)]}$$

$$\Rightarrow \quad 3a^2 + 12a - 6a^2 + 2(4 + a^2 - 4a) = 3$$

$$\Rightarrow \quad -3a^2 + 12a + 8 + 2a^2 - 8a - 3 = 0$$

$$\Rightarrow \quad -a^2 + 4a + 5 = 0$$

$$\Rightarrow \quad a^2 - 4a - 5 = 0$$

$$\Rightarrow \quad a^2 - 5a + a - 5 = 0$$

$$\Rightarrow \quad a(a - 5) + 1(a - 5) = 0$$

$$\Rightarrow \quad (a + 1)(a - 5) = 0$$

$$\Rightarrow \quad (a + 1) = 0 \quad \text{or} \quad (a - 5) = 0$$

$$\Rightarrow \quad a = -1 \quad \text{or} \quad a = 5$$

Now, $b = 2 - a$ [From (iii)]

When $a = 5$, $b = 2 - 5 = -3$

When $a = -1$, $b = 2 - (-1) = 3$

If $a = -1$ and $b = 3$, then zeroes are, a, $(a + b)$, $(a + 2b)$

$$= -1, (-1 + 3), [-1 + 2(3)]$$

$$= -1, 2, 5$$

If $a = 5$, and $b = -3$, then zeroes are 5, $[5 + (-3)]$, $[5 + 2(-3)] = 5, 2, -1$

So, zeroes in both cases are $\beta = 2$, $\gamma = -1$ and $\alpha = 5$.

Q3. Given that $\sqrt{2}$ is a zero of a cubic polynomial

$6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.

Sol. Main concept: Using Euclid's division algorithm here, remainder is zero. Then quotient will be quadratic whose zeroes can be find out by factorisation.

Let $f(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

If $\sqrt{2}$ is the zero of $f(x)$, then $(x - \sqrt{2})$ will be a factor of $f(x)$. So, by remainder theorem when $f(x)$ is divided by $(x - \sqrt{2})$, the quotient comes out to be quadratic.

$$\begin{array}{r|l} & 6x^2 + 7\sqrt{2}x + 4 \\ \hline x - \sqrt{2} & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \\ & \underline{\overset{-}{}6x^3 \underset{+}{-} 6\sqrt{2}x^2} \\ & 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\ & \underline{\overset{-}{}7\sqrt{2}x^2 \underset{+}{-} 14x} \\ & 4x - 4\sqrt{2} \\ & \underline{\overset{-}{}4x \underset{+}{-} 4\sqrt{2}} \\ & 0 \end{array}$$

$\therefore \quad f(x) = (x - \sqrt{2})(6x^2 + 7\sqrt{2}x + 4)$ (By Euclid's division algorithm)

$$= (x - \sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4)$$

For zeroes of $f(x)$, $f(x) = 0$

$\therefore \quad (x - \sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4) = 0$

$\Rightarrow \quad (x - \sqrt{2})[2x(3x + 2\sqrt{2}) + \sqrt{2}(3x + 2\sqrt{2})] = 0$

$\Rightarrow \quad (x - \sqrt{2})(3x + 2\sqrt{2})(2x + \sqrt{2}) = 0$

$\Rightarrow x - \sqrt{2} = 0$ or $3x + 2\sqrt{2} = 0$ or $2x + \sqrt{2} = 0$

$\Rightarrow \quad x = \sqrt{2}$ or $x = \dfrac{-2\sqrt{2}}{3}$ or $x = \dfrac{-\sqrt{2}}{2}$

So, other two roots are $= \dfrac{-2\sqrt{2}}{3}$ and $\dfrac{-\sqrt{2}}{2}$.

Q4. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of two polynomials.

Sol. Main concept: Factor theorem and Euclid's division algorithm.

By factor theorem and Euclid's division algorithm, we get

$$f(x) = g(x) \times q(x) + r(x)$$

Let $f(x) = 2x^4 + x^3 - 14x^2 + 5x + 6$ ...(*i*)

and $g(x) = x^2 + 2x + k$

$\therefore$

$$\begin{array}{r|l} & 2x^2 - 3x - 8 - 2k \\ \hline x^2 + 2x + k & 2x^4 + x^3 - 14x^2 + 5x + 6 \\ & \underline{\underset{-}{2x^4} \underset{-}{+} 4x^3 \underset{-}{+} 2kx^2} \\ & -3x^3 - 14x^2 - 2kx^2 + 5x + 6 \\ & \underline{\underset{+}{-3x^3} \underset{+}{-} 6x^2 \quad \underset{+}{-} 3kx} \\ & -8x^2 - 2kx^2 + 5x + 3kx + 6 \\ & \underline{\underset{+}{-8x^2} \quad \underset{+}{-} 16x \quad \underset{+}{-} 8k} \\ & -2kx^2 + 21x + 3kx + 8k + 6 \\ & \underline{\underset{+}{-2kx^2} \quad \underset{+}{-} 4kx \quad \underset{+}{-} 2k^2} \\ & 21x + 7kx + 2k^2 + 8k + 6 \end{array}$$

But, $r(x) = 0$

$\therefore$ $(21 + 7k)x + 2k^2 + 8k + 6 = 0x + 0$

$\Rightarrow$ $21 + 7k = 0$ and $2k^2 + 8k + 6 = 0$

$2k^2 + 6k + 2k + 6 = 0$

$\Rightarrow$ $k = \dfrac{-21}{7}$ | $\Rightarrow$ $2k(k + 3) + 2(k + 3) = 0$

$\Rightarrow$ $(k + 3)(2k + 2) = 0$

$\Rightarrow$ $k = -3$ | $\Rightarrow$ $k + 3 = 0$ or $2k + 2 = 0$

$\Rightarrow$ $k = -3$ or $k = -1$

$\therefore$ Common solution is $k = -3$

So, $q(x) = 2x^2 - 3x - 8 - 2(-3)$

$= 2x^2 - 3x - 8 + 6$

$\Rightarrow$ $q(x) = 2x^2 - 3x - 2$

$\therefore$ $f(x) = g(x)\, q(x) + 0$

$= (x^2 + 2x - 3)(2x^2 - 3x - 2)$

$= (2x^2 - 4x + 1x - 2)(x^2 + 3x - 1x - 3)$

$$= [2x(x-2)+1\,(x-2)]\,[x(x+3)-1(x+3)]$$

$$\Rightarrow \quad f(x) = (x-2)\,(2x+1)\,(x+3)\,(x-1)$$

For zeroes of $f(x)$, $f(x) = 0$

$$\therefore \quad (x-1)\,(x-2)\,(x+3)\,(2x+1) = 0$$

$$\Rightarrow (x-1)=0, \quad (x-2)=0, \quad (x+3)=0 \text{ and } 2x+1=0$$

$$\Rightarrow x=1, x=2, x=-3 \text{ and } x = \frac{-1}{2}$$

So, zeroes of $f(x)$ are 1, 2, -3, and $\frac{-1}{2}$.

Q5. Given that $(x-\sqrt{5})$ is a factor of cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, find all the zeroes of the polynomial.

Sol. Main concept: Factor theorem, Euclid's division algorithm.

Let $f(x) = x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$

and $\quad g(x) = (x-\sqrt{5})$

$\because$ $g(x)$ is a factor of $f(x)$ so $f(x) = q(x)\,(x-\sqrt{5})$

$$
\begin{array}{r|l}
 & x^2 - 2\sqrt{5}x + 3 \\
\hline
x-\sqrt{5} & x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} \\
 & \underset{-}{x^3} \underset{+}{-} \sqrt{5}x^2 \\
\hline
 & -2\sqrt{5}x^2 + 13x - 2\sqrt{5} \\
 & \underset{+}{-} 2\sqrt{5}x^2 \underset{-}{+} 10x \\
\hline
 & +3x - 3\sqrt{5} \\
 & \underset{-}{+} 3x \underset{+}{-} 3\sqrt{5} \\
\hline
 & 0
\end{array}
$$

But, $\quad f(x) = q(x)\,g(x)$

$$\therefore \quad f(x) = (x^2 - 2\sqrt{5}x + 3)\,(x-\sqrt{5})$$

$$\Rightarrow f(x) = [x^2 - \{(\sqrt{5}+\sqrt{2}) + (\sqrt{5}-\sqrt{2})\}x + (\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})][(x)-\sqrt{5}]$$

$$= [x^2 - (\sqrt{5}+\sqrt{2})x - (\sqrt{5}-\sqrt{2})x + (\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})][x-\sqrt{5}]$$

$$= x[x-(\sqrt{5}+\sqrt{2})] - (\sqrt{5}-\sqrt{2})\,[x-(5+\sqrt{2})]\,[x-\sqrt{5}]$$

$$\Rightarrow f(x) = (x-\sqrt{5}-\sqrt{2})\,(x-\sqrt{5}+\sqrt{2})\,(x-\sqrt{5})$$

For zeroes of $f(x)$, $f(x) = 0$

$$\Rightarrow (x-\sqrt{5}-\sqrt{2})\,(x-\sqrt{5}+\sqrt{2})\,(x-\sqrt{5}) = 0$$

$$\Rightarrow (x-\sqrt{5}-\sqrt{2}) = 0 \text{ or } (x-\sqrt{5}+\sqrt{2}) = 0 \text{ or } (x-\sqrt{5}) = 0$$

$$\Rightarrow \qquad x = \sqrt{5}+\sqrt{2} \quad \text{or} \quad x = \sqrt{5}-\sqrt{2} \quad \text{or} \quad x = +\sqrt{5}$$

$\therefore$ Zeroes are $(\sqrt{5}+\sqrt{2}), (\sqrt{5}-\sqrt{2})$ and $\sqrt{5}$.

Q6. For which values of a and b are the zeroes of $q(x) = x^3 + 2x^2 + a$ also the zeroes of polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? Which zeroes of $p(x)$ are not the zeroes of $q(x)$?

Sol. Main concept: Factor theorem and Euclid's division algorithm.

By factor theorem if $q(x)$ is a factor of $p(x)$, then $r(x)$ must be zero.

$$p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$$
$$q(x) = x^3 + 2x^2 + a$$

$$\begin{array}{r|l} & x^2 - 3x + 2 \\ \hline x^3+2x^2+a & x^5 - x^4 - 4x^3 + 3x^2 + 3x + b \\ & \underset{-}{x^5} \underset{-}{+} 2x^4 \qquad \underset{-}{+} ax^2 \\ \hline & -3x^4 - 4x^3 - ax^2 + 3x^2 + 3x + b \\ & \underset{+}{-3x^4} \underset{+}{-} 6x^3 \qquad \underset{+}{-} 3ax \\ \hline & 2x^3 - ax^2 + 3x^2 + 3ax + 3x + b \\ & \underset{-}{2x^3} \quad \underset{-}{+} 4x^2 \qquad \underset{-}{+} 2a \\ \hline & -ax^2 - x^2 + 3ax + 3x - 2a + b \end{array}$$

So, by factor theorem remainder must be zero i.e.,

$$r(x) = 0$$

$$\Rightarrow \qquad -(a+1)x^2 + (3a+3)x + (b-2a) = 0x^2 + 0x + 0$$

Comparing the coefficients of x^2, x and constt. on both sides, we get

$$-(a+1) = 0 \quad \text{and} \quad 3a+3 = 0 \quad \text{and} \quad b - 2a = 0$$

$$\Rightarrow \qquad a = -1 \quad \text{and} \quad a = -1 \quad \text{and} \quad b - 2(-1) = 0$$

$$\Rightarrow \qquad b = -2$$

For $a = -1$ and $b = -2$, zeroes of $q(x)$ will be zeroes of $p(x)$.

For zeroes of $p(x)$, $p(x) = 0$

$$\Rightarrow \qquad (x^3 + 2x^2 + a)(x^2 - 3x + 2) = 0 \qquad [\because \ a = -1]$$

$$\Rightarrow \qquad [x^3 + 2x^2 - 1][x^2 - 2x - 1x + 2] = 0$$

$$\Rightarrow \qquad (x^3 + 2x^2 - 1)[x(x-2) - 1(x-2)] = 0$$

$$\Rightarrow \qquad (x^3 + 2x^2 - 1)(x-2)(x-1) = 0$$

Hence, $x = 2$ and 1 are not the zeroes of $q(x)$.

❑❑❑

3 Pair of Linear Equations in Two Variables

EXERCISE 3.1

Choose the correct answer from the given four options in the following questions:

Q1. Graphically, the pair of equations

$$6x - 3y + 10 = 0$$
$$2x - y + 9 = 0$$

represents two lines which are

(*a*) intersecting at exactly one point
(*b*) intersecting at exactly two points
(*c*) coincident
(*d*) parallel

Sol. (*d*): Here, $\frac{a_1}{a_2} = \frac{6}{2} = 3, \quad \frac{b_1}{b_2} = \frac{-3}{-1} = 3, \quad \frac{c_1}{c_2} = \frac{10}{9}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the system of linear equations is inconsistent (no solution) and graph will be a pair of parallel lines.

Q2. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have

(*a*) a unique solution
(*b*) exactly two solutions
(*c*) infinitely many solutions
(*d*) no solution

Sol. (*d*): Here, $\frac{a_1}{a_2} = \frac{1}{-3} = -\frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{2}{-6} = \frac{-1}{3}, \quad \frac{c_1}{c_2} = \frac{5}{1}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the system of linear equations has no solution.

Q3. If a pair of linear equations is consistent, then the lines will be

(*a*) parallel
(*b*) always coincident
(*c*) intersecting or coincident
(*d*) always intersecting

Sol. (*c*): Condition for consistency

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ have unique solution (consistent) *i.e.*, intersecting at one point

or $\quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad$ (consistent lines, coincident or depend)

Q4. The pair of equations $y = 0$ and $y = -7$ has

(*a*) one solution (*b*) two solutions

(*c*) infinitely many solutions (*d*) no solution

Sol. (*d*): We know that equation of the form $y = a$ is a line parallel to x-axis at a distance 'a' from it. $y = 0$ is the equation of the x-axis and $y = -7$ is is the equation of the line parallel to the x-axis. So, these two equations represent two parallel lines. Therefore, there is no solution. Hence, (*d*) is the correct answer.

Q5. The pair of equations $x = a$ and $y = b$ graphically represents lines which are

(*a*) parallel (*b*) intersecting at (b, a)

(*c*) coincident (*d*) intersecting at (a, b)

Sol. (*d*): $x = a$ is the equation of a straight line parallel to the y-axis at a distance 'a' from it. Again, $y = b$ is the equation of a straight line parallel to the x-axis at a distance 'b' from it.

So, the pair of equations $x = a$ and $y = b$ graphically represents lines which are intersecting at (a, b).

Hence, (*d*) is the correct answer.

Q6. For what value of k do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines?

(*a*) $\frac{1}{2}$ (*b*) $\frac{-1}{2}$ (*c*) 2 (*d*) -2

Sol. (*c*):

$$3x - y = -8 \quad \ldots(i)$$

$$6x - ky = -16 \quad \ldots(ii)$$

For coincident lines,

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{3}{6} = \frac{-1}{-k} = \frac{-8}{-16}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{k} = \frac{1}{2}$$

So, $k = 2$.

Q7. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is

(*a*) $\frac{-5}{4}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{15}{4}$ (*d*) $\frac{3}{2}$

Sol. (*c*): For parallel lines (or no solution)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{2k}{5} \neq \frac{2}{-1}$$

$$\Rightarrow \quad 4k = 15$$

$$\Rightarrow \quad k = \frac{15}{4}$$

Q8. The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions is

(*a*) 3 (*b*) –3 (*c*) –12 (*d*) No value

Sol. (*d*): For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{c}{6} = \frac{-1}{-2} = \frac{2}{3}$$

Ratio I II III

From ratios I and II, $2c = 6 \Rightarrow c = 3$

From ratios I and III, $3c = 12 \Rightarrow c = 4$

As from the ratios, values of c are not common. So, there is no value of c for which lines have many solutions.

Q9. One equation of a pair of dependent linear equations is $-5x + 7y = 2$. The second equation can be

(*a*) $10x + 14y + 4 = 0$ (*b*) $-10x - 14y + 4 = 0$

(*c*) $-10x + 14y + 4 = 0$ (*d*) $10x - 14y = -4$

Sol. (*d*): $-5x + 7y - 2 = 0$ ...(*i*)

$a_2x + b_2y + c_2 = 0$ ...(*ii*)

∴ For dependent system of linear equations

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{-5}{a_2} = \frac{7}{b_2} = \frac{-2}{c_2} = \frac{1}{k}$$

So, $a_2 = -5k$, $b_2 = 7k$, $c_2 = -2k$

$k = 0$, and 1, does not satisfy the required condition.

For $k = -2$, $a_2 = +10$, $b_2 = -14$ and $c_2 = +4$ satisfies the condition.

i.e., $\frac{-5}{+10} = \frac{7}{-14} = \frac{-2}{+4} = \frac{-1}{2}$ satisfies the condition.

Q10. A pair of linear equations which has a unique solution $x = 2$, $y = -3$ is

(*a*) $x + y = -1$ and $2x - 3y = -5$

(*b*) $2x + 5y = -11$ and $4x + 10y = -22$

(*c*) $2x - y = 1$ and $3x + 2y = 0$

(*d*) $x - 4y - 14 = 0$ and $5x - y - 13 = 0$

Sol. (*b* and *d*): As $x = 2, y = -3$ is unique solution of system of equations so these values must satisfy both equations.

(*a*) $x + y = -1$ and $2x - 3y = -5$

Put $x = 2$ and $y = -3$ in both the equations.

LHS $= x + y \Rightarrow 2 - 3 = -1$ (RHS)

LHS $= 2x - 3y \Rightarrow 2(2) - 3(-3) \Rightarrow 4 + 9 = 13 \neq$ RHS

(*b*) $2x + 5y = -11$ and $4x + 10y = -22$

Put $x = 2$ and $y = -3$ in both the equations.

LHS $= 2x + 5y \Rightarrow 2 \times 2 + 5(-3) \Rightarrow 4 - 15 = -11 =$ RHS

LHS $= 4x + 10y \Rightarrow 4(2) + 10(-3) \Rightarrow 8 - 30 = -22 =$ RHS

(*c*) $2x - y = 1$ and $3x + 2y = 0$

Put $x = 2$ and $y = -3$ in both the equations.

LHS $= 2x - y \Rightarrow 2(2) + 3 \Rightarrow 7 \neq$ RHS

LHS $= 3x + 2y \Rightarrow 3(2) + 2(-3) \Rightarrow 6 - 6 = 0 =$ RHS

(*d*) $x - 4y - 14 = 0$ and $5x - y - 13 = 0$

$x - 4y = 14$ and $5x - y = 13$

Put $x = 2$ and $y = -3$ in both the equations.

LHS $= x - 4y \Rightarrow 2 - 4(-3) \Rightarrow 2 + 12 = 14 =$ RHS

LHS $= 5x - y \Rightarrow 5(2) - (-3) \Rightarrow 10 + 3 = 13 =$ RHS

Hence, the pair of equations is (*b*) and (*d*).

Q11. If $x = a$, $y = b$, is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of *a* and *b* are, respectively

(*a*) 3 and 5 (*b*) 5 and 3 (*c*) 3 and 1 (*d*) -1 and -3

Sol. (*c*): If (*a*, *b*) is the solution of the given equations, then it must satisfy the given equations so,

$a - b = 2$ ...(*i*)

$a + b = 4$ ...(*ii*)

$\Rightarrow$ $2a = 6$ [Adding (*i*) and (*ii*)]

$\Rightarrow$ $a = 3$

Now, $3 + b = 4$ [From (*ii*)]

$\Rightarrow$ $b = 1$

So, $(a, b) = (3, 1)$.

Q12. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(*a*) 35 and 15 (*b*) 35 and 20 (*c*) 15 and 35 (*d*) 25 and 25

Sol. (*d*): Let the number of ₹ 1 coins $= x$

and the number of ₹ 2 coins $= y$

So, according to the question

$x + y = 50$ (*i*)

$1x + 2y = 75$ ...(*ii*)

$2x + 2y = 100$ [(*i*) $\times 2$]

$1x + 2y = 75$ [From (*ii*)]

$\underline{- \quad - \quad -}$

$x = 25$

Now, $25 + y = 50 \Rightarrow y = 25$ [From (*i*)]

So, $y = 25$ and $x = 25$.

Q13. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, (in years) of the son and the father are, respectively

(*a*) 4 and 24 (*b*) 5 and 30 (*c*) 6 and 36 (*d*) 3 and 24

Sol. (*c*): Let the present age of father be x years and the present age of son be y years.

∴ According to the question, $x = 6y$ ...(*i*)

Age of father after four years = $(x+4)$ years

and the age of son after four years = $(y+4)$ years

Now, according to the question,

$$x + 4 = 4(y+4) \quad ...(ii)$$

$$\Rightarrow x + 4 = 4y + 16$$

$$\Rightarrow 6y - 4y = 16 - 4 \quad [\because x = 6y]$$

$$\Rightarrow 2y = 12$$

$$\Rightarrow y = 6$$

$$\therefore x = 6 \times 6 = 36 \text{ years} \quad [\text{From } (i)]$$

and $y = 6$ years

So, the present ages of the son and the father are 6 years and 36 years respectively.

EXERCISE 3.2

Q1. Do the following pair of linear equations have no solution? Justify your answer.

(*i*) $2x + 4y = 3$ and $12y + 6x = 6$

(*ii*) $x = 2y$ and $y = 2x$

(*iii*) $3x + y - 3 = 0$ and $2x + \frac{2}{3}y = 2$

Sol. The system of linear equations has no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

(*i*) $2x + 4y = 3$ and $6x + 12y = 6$

Here, $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{4}{12} = \frac{1}{3}$, $\frac{c_1}{c_2} = \frac{3}{6} = \frac{1}{2}$

$\therefore \frac{2}{6} = \frac{4}{12} \neq \frac{3}{6}$

So, the given system of linear equations has no solution.

(*ii*) $x - 2y = 0$ and $2x - y = 0$

Here, $\frac{a_1}{a_2} = \frac{1}{2}$

and $\frac{b_1}{b_2} = \frac{-2}{-1} = 2$

So, the given system of linear equations does not satisfy $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

(*iii*) $3x + y - 3 = 0$ and $2x + \frac{2}{3}y = 2$

Here, $\frac{a_1}{a_2} = \frac{3}{2}$, $\frac{b_1}{b_2} = \frac{1}{2/3} = \frac{3}{2}$, $\frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$

So, the given system of linear equations does not satisfy $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Q2. Do the following equations represent a pair of coincident lines? Justifies your answer.

(*i*) $3x + \frac{1}{7}y = 3$ and $7x + 3y = 7$

(*ii*) $-2x - 3y = 1$ and $6y + 4x = -2$

(*iii*) $\frac{x}{2} + y + \frac{2}{5} = 0$ and $4x + 8y + \frac{5}{16} = 0$

Sol. Condition for coincident lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \qquad ...(i)$$

(*i*) $3x + \frac{1}{7}y = 3$ and $7x + 3y = 7$

Here, $\frac{a_1}{a_2} = \frac{3}{7}$, $\frac{b_1}{b_2} = \frac{\frac{1}{7}}{3} = \frac{1}{21}$ and $\frac{c_1}{c_2} = \frac{3}{7}$

So, the given system of linear equations does not satisfy condition (*i*).

(*ii*) $-2x - 3y = 1$ and $6y + 4x = -2$

Here, $\frac{a_1}{a_2} = \frac{-2}{4} = \frac{-1}{2}$, $\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$ and $\frac{c_1}{c_2} = \frac{1}{-2}$

So, the given system of linear equations does not satisfy given condition (*i*).

(*iii*) $\frac{x}{2} + y + \frac{2}{5} = 0$ and $4x + 8y + \frac{5}{16} = 0$

Here, $\frac{a_1}{a_2} = \frac{\frac{1}{2}}{4} = \frac{1}{8}$, $\frac{b_1}{b_2} = \frac{1}{8}$ and $\frac{c_1}{c_2} = \frac{\frac{2}{5}}{\frac{5}{16}} = \frac{32}{25}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of linear equations does not satisfy condition (*i*).

Q3. Are the following pair of linear equations consistent? Justify your answer.

(*i*) $-3x - 4y = 12$ and $4y + 3x = 12$

(*ii*) $\frac{3}{5}x - y = \frac{1}{2}$ and $\frac{1}{5}x - 3y = \frac{1}{6}$

(*iii*) $2ax + by = a$ and $4ax + 2by - 2a = 0, a, b \neq 0$

(*iv*) $x + 3y = 11$ and $2(2x + 6y) = 22$

Sol. For consistent system of linear equations $a, b \neq 0$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \text{(infinitely many solutions)}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \text{(unique solution)}$$

(*i*) $-3x - 4y = 12$ and $4y + 3x = 12$

Here, $\frac{a_1}{a_2} = \frac{-3}{3} = -1$, $\frac{b_1}{b_2} = \frac{-4}{4} = -1$ and $\frac{c_1}{c_2} = \frac{12}{12} = 1$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given pair of linear equations is inconsistent and has no solution.

(*ii*) $\frac{3}{5}x - y = \frac{1}{2}$ and $\frac{1}{5}x - 3y = \frac{1}{6}$

Here, $\frac{a_1}{a_2} = \frac{3/5}{1/5} = 3$, $\frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{1/2}{1/6} = \frac{6}{2} = 3$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, the given pair of linear equations is consistent and has unique solution.

(*iii*) $2ax + by = a$ and $4ax + 2by - 2a = 0$

Here, $\frac{a_1}{a_2} = \frac{2a}{4a} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{b}{2b} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{a}{2a} = \frac{1}{2}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the given pair of linear equations is consistent and has infinitely many solutions.

(*iv*) $x + 3y = 11$ and $2(2x + 6y) = 22$

or $x + 3y = 11$ and $4x + 12y = 22$

Here, $\frac{a_1}{a_2} = \frac{1}{4}$, $\frac{b_1}{b_2} = \frac{3}{12} = \frac{1}{4}$ and $\frac{c_1}{c_2} = \frac{11}{22} = \frac{1}{2}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given pair of linear equations is inconsistent and has no solution.

Q4. For the pair of equations, $\lambda x + 3y = -7$ and $2x + 6y = 14$, to have infinitely many solutions the value of λ should be 1. Is the statement true? Give reasons.

Sol. $\lambda x + 3y + 7 = 0$ ...(*i*)

$2x + 6y - 14 = 0$ ...(*ii*)

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{\lambda}{2}, \ \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \ \frac{c_1}{c_2} = \frac{7}{-14} = \frac{1}{-2}$$

So, $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, for any value of λ.

Hence, the given statement is not true.

Q5. For all real values of *c*, the pair of equations $x - 2y = 8$ and $5x - 10y = c$ have a unique solution. Justify whether it is true or false.

Sol. (False) System of linear equations are

$$x - 2y = 8 \quad ...(i)$$

$$5x - 10y = c \quad ...(ii)$$

$\therefore \ \frac{a_1}{a_2} = \frac{1}{5}$, $\frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5}$ and $\frac{c_1}{c_2} = \frac{-8}{-c} = \frac{8}{c}$

As $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ so system of linear equations can never have unique solution.

Hence, the given statement is false.

Q6. The line represented by $x = 7$ is parallel to x-axis. Justify whether the statement is true or not.

Sol. The line represented by $x = 7$ is of the form $x = a$. The graph of the equation is a line parallel to the *y*-axis.

Hence, the given statement is not true.

EXERCISE 3.3

Q1. For which value(s) of λ do the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have

(*i*) no solution? (*ii*) infinitely many solutions?

(*iii*) a unique solution?

Sol. $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$

(*i*) For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \quad \underset{\text{I}}{\frac{\lambda}{1}} = \underset{\text{II}}{\frac{1}{\lambda}} \neq \underset{\text{III}}{\frac{\lambda^2}{1}}$$

From ratio I and II, we get

$$\lambda^2 = 1$$

$$\Rightarrow \quad \lambda = \pm 1$$

(*ii*) For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{\lambda}{1} = \frac{1}{\lambda} = \frac{\lambda^2}{1}$$

$$\Rightarrow \quad \lambda^2 = 1 \quad \Rightarrow \quad \lambda = \pm 1$$

Also, $\frac{\lambda}{1} = \frac{\lambda^2}{1}$

$$\Rightarrow \quad \lambda^2 = \lambda$$

$$\Rightarrow \quad \lambda^2 - \lambda = 0$$

$$\Rightarrow \quad \lambda(\lambda - 1) = 0$$

$$\Rightarrow \quad \lambda = 0, \quad \lambda = 1$$

$\therefore$ Common solution for which the pair of linear equations has infinitely many solutions is $\lambda = 1$ only.

(*iii*) For unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\therefore \quad \frac{\lambda}{1} \neq \frac{1}{\lambda}$$

$$\Rightarrow \quad \lambda^2 \neq 1 \quad \text{or} \quad \lambda \neq 1, -1$$

So, for unique solution all real values except $\lambda = 1, -1$.

Q2. For which value(s) of k will the pair of equations $kx + 3y = k - 3$ and $12x + ky = k$ has no solution?

Sol.
$$kx + 3y = k - 3$$
$$12x + ky = k$$

System of eqns. will have no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow$ $$\frac{k}{12} = \frac{3}{k} \neq \frac{k-3}{k}$$

$\Rightarrow$ $$k^2 = 36$$

$\Rightarrow$ $$k = \pm 6$$

Also, $$\frac{3}{k} \neq \frac{(k-3)}{k}$$

$\Rightarrow$ $$k^2 - 3k \neq 3k$$

$\Rightarrow$ $$k^2 - 3k - 3k \neq 0$$

$\Rightarrow$ $$k^2 - 6k \neq 0$$

$\Rightarrow$ $$k(k-6) \neq 0$$

$\Rightarrow$ $$k \neq 0 \quad \text{and } k \neq 6$$

So, the value of k for which the system of linear equations has no solution is $k = -6$.

Q3. For which values of a and b, will the following pair of linear equations has infinitely many solutions?

$$x + 2y = 1$$

$$(a-b)x + (a+b)y = a+b-2$$

Sol. For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow$ $$\underset{\text{I}}{\frac{1}{(a-b)}} = \underset{\text{II}}{\frac{2}{(a+b)}} = \underset{\text{III}}{\frac{1}{(a+b-2)}}$$

From ratios I and II,

$$2a - 2b = a + b$$

$\Rightarrow$ $$a - 3b = 0 \qquad ...(i)$$

From ratios II and III,

$$2a + 2b - 4 = a + b$$

$\Rightarrow$ $$a + b = 4 \qquad ...(ii)$$

Now, solving (*i*) and (*ii*), we have

$$a - 3b = 0 \qquad ...(i)$$

$$\underset{-}{a} + \underset{-}{b} = \underset{-}{4} \qquad ...(ii) \qquad \text{[Subtracting } (ii) \text{ from } (i)]$$

$$-4b = -4$$

$\Rightarrow$ $$b = 1$$

and $$a = 4 - b \qquad \text{[From } (ii)]$$

$\Rightarrow$ $$a = 4 - 1$$

$\Rightarrow$ $$a = 3$$

Q4. Find the values of p in (*i*) to (*iv*) and p and q in (*v*) for the following pair of equations:

(*i*) $3x - y - 5 = 0$ and $6x - 2y - p = 0$, if the lines represented by these equations are parallel.

(*ii*) $-x + py = 1$ and $px - y = 1$, if the pair of equations has no solution.

(*iii*) $-3x + 5y = 7$ and $2px - 3y = 1$, if the lines represented by these equations are intersecting at a unique point.

(*iv*) $2x + 3y - 5 = 0$ and $px - 6y - 8 = 0$, if the pair of equations has a unique solution.

(*v*) $2x + 3y = 7$ and $2px + py = 28 - qy$, if the pair of equations has infinitely many solutions.

Sol. (*i*) Given equations are

$$3x - y - 5 = 0 \quad \text{...(i)}$$

$$6x - 2y - p = 0 \quad \text{...(ii)}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-5}{-p} = \frac{5}{p}$$

The lines will be parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{2} \neq \frac{5}{p}$$

$$\Rightarrow \quad p \neq 10$$

So, the given lines are parallel for all real values of p except 10.

(*ii*) Given pair of equations is

$$-x + py = 1 \quad \text{...(i)}$$

$$px - y = 1 \quad \text{...(ii)}$$

For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \quad -\frac{1}{p} = \frac{p}{-1} \neq \frac{1}{1}$$

$$\text{I} \quad \text{II} \quad \text{III}$$

From ratios I and II, $p^2 = 1$ or $p = \pm 1$

Using ratios II and III, $p \neq -1$

$\therefore$ For $p = 1$, the given equations have not any solution.

(*iii*) Pair of equations is

$$-3x + 5y = 7 \quad \text{...(i)}$$

$$2px - 3y = 1 \quad \text{...(ii)}$$

For unique solution, we have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \quad \frac{-3}{2p} \neq \frac{5}{-3}$$

$$\Rightarrow \quad 10p \neq +9 \quad \Rightarrow \quad p \neq \frac{9}{10}$$

Hence, the given equations have unique solution for all real values of p except $\frac{9}{10}$.

(*iv*) Pair of equations is

$$2x + 3y - 5 = 0 \quad \text{...(i)}$$

$$px - 6y - 8 = 0 \quad \text{...(ii)}$$

Pair of equations have unique solution if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \quad \frac{2}{p} \neq \frac{3}{-6}$$

$$\Rightarrow \quad 3p \neq -2 \times 6$$

$$\Rightarrow \quad p \neq -\frac{12}{3}$$

$$\Rightarrow \quad p \neq -4$$

Hence, the system of linear equations has unique solution for all real values of p except -4.

(*v*) Given system of linear equations is

$$2px + py = 28 - qy$$

i.e.,

$$2px + (p+q)y = 28 \quad \text{...(i)}$$

$$2x + 3y = 7 \quad \text{...(ii)}$$

The system of equations will have infinitely many solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \underset{\text{I}}{\frac{2p}{2}} = \underset{\text{II}}{\frac{p+q}{3}} = \underset{\text{III}}{\frac{28}{7}}$$

Using ratios I and II we get,

$$\frac{2p}{2} = \frac{p+q}{3}$$

$$\Rightarrow \quad 3p = p + q$$

$$\Rightarrow \quad 2p - q = 0$$

$$\Rightarrow \quad q = 2p \quad \text{...(iii)}$$

Using ratios I and III, we get

$$\frac{2p}{2} = \frac{28}{7} \quad \Rightarrow \quad p = 4$$

$\therefore \qquad q = 2p = 2 \times 4 = 8$ [From (*iii*)]

$\therefore \qquad q = 8 \quad \text{and} \quad p = 4$

Now, $$\frac{2p}{2} = \frac{p+q}{3} = \frac{28}{7}$$

$\Rightarrow$ $$\frac{p}{1} = \frac{p+q}{3} = \frac{4}{1}$$

By substituting the values of p and q, we have

$$4 = \frac{4+8}{3} = 4$$

$\Rightarrow$ $$4 = \frac{12}{3} = 4$$

$\Rightarrow$ $$4 = 4 = 4$$

Hence, the given system of equations has infinitely many solutions when $p = 4$ and $q = 8$.

Q5. Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$. Check whether the paths will cross each other or not.

Sol. Two straight paths are represented by the equations

$x - 3y = 2$ and $-2x + 6y = 5$.

For the paths to cross each other i.e., to intersect each other, we must have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Now, $\frac{a_1}{a_2} = \frac{1}{-2} = \frac{-1}{2}$ and $\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

Hence, the two straight paths do not cross each other.

Q6. Write a pair of linear equations which has the unique solution $x = -1$, and $y = 3$. How many such pairs can you write ?

Sol. For system of linear equations

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

the lines has unique solution $x = -1$ and $y = 3$ so it must satisfy the above equations.

$\therefore$ $$a_1(-1) + b_1(3) + c_1 = 0$$

and $$a_2(-1) + b_2(3) + c_2 = 0$$

$\Rightarrow$ $$-a_1 + 3b_1 + c_1 = 0 \qquad \text{...}(i)$$

and $$-a_2 + 3b_2 + c_2 = 0 \qquad \text{...}(ii)$$

The restricted values of a_1, a_2 and b_1, b_2 are only

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \qquad \text{...}(iii)$$

So, all the real values of a_1, a_2, b_1, b_2 except condition (*iii*) can form so many linear equations which will satisfy equations (*i*) and (*ii*) and have solution $x = -1$ and $y = 3$.

We can have infinite number of lines passing through $(-1, 3)$, which is the solution of intersecting lines at this $(-1, 3)$ point.

So, infinite number of pairs of system of equations are possible which has unique solution $x = -1$ and $y = 3$.

Q7. If $2x + y = 23$ and $4x - y = 19$, then find the values of $5y - 2x$ and $\frac{y}{x} - 2$.

Sol. Given equations are $\quad 2x + y = 23 \quad$ ...(*i*)

$4x - y = 19 \quad$ ...(*ii*)

Adding equations (*i*) and (*ii*), we get

$$6x = 42$$

$\Rightarrow \quad x = 7$

Now, $\quad 2(7) + y = 23 \quad$ [From (*i*)]

$\Rightarrow \quad y = 23 - 14$

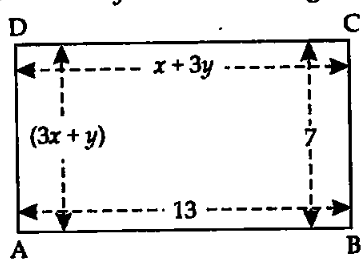
$\Rightarrow \quad y = 9$

So, $\quad 5y - 2x = 5(9) - 2(7) = 45 - 14 = 31$

and $\quad \frac{y}{x} - 2 = \frac{9}{7} - 2 = \frac{9 - 14}{7} = \frac{-5}{7}$

Hence, the values of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$ are 31 and $\frac{-5}{7}$ respectively.

Q8. Find the values of x and y in the following rectangle:



Sol. As the opposite sides of a rectangle are equal so by figure, we conclude that

$3x + y = 7 \quad$ ...(*i*)

$x + 3y = 13 \quad$ ...(*ii*)

$9x + 3y = 21 \quad$ [From (*i*)]

$x + 3y = 13$

$-\quad -\quad -$

$8x = 8 \quad$ [Subtracting (*ii*) from (*i*)]

$x = 1$

Now, $3(1)+y=7$ [From (*i*)]

$\Rightarrow$ $y=7-3$

$\Rightarrow$ $y=4$

and $x=1$

Hence, the required values of x and y are 1 and 4 respectively.

Q9. Solve the following pairs of linear equations:

(*i*) $x+y=3.3, \quad \dfrac{0.6}{3x-2y}=-1, \quad 3x-2y\neq 0$

(*ii*) $\dfrac{x}{3}+\dfrac{y}{4}=4, \quad \dfrac{5x}{6}-\dfrac{y}{8}=4$

(*iii*) $4x+\dfrac{6}{y}=15, \quad 6x-\dfrac{8}{y}=14, \quad y\neq 0$

(*iv*) $\dfrac{1}{2x}-\dfrac{1}{y}=-1, \quad \dfrac{1}{x}+\dfrac{1}{2y}=8, \quad x, y\neq 0$

(*v*) $43x+67y=-24, \quad 67x+43y=24$

(*vi*) $\dfrac{x}{a}+\dfrac{y}{b}=a+b, \quad \dfrac{x}{a^2}+\dfrac{y}{b^2}=2, \quad a, b\neq 0$

(*vii*) $\dfrac{2xy}{x+y}=\dfrac{3}{2}, \quad \dfrac{xy}{2x-y}=\dfrac{-3}{10}, \quad x+y\neq 0, \quad 2x-y\neq 0$

Sol. Some important rules for easy solution.

- Fraction in which constants are in denominators, convert the equation in the form of $ax+by=c$ by multiplying the equation both sides by LCM of denominators of equation.
- We can not multiply an equation by variable unless variable is not zero. If variables are not zero, then we can multiply by variables also.
- If in system of equations, x or y or both are in denominator and are symmetric no need to remove denominator.
- Remove decimals and again remove denominators by multiplying LCM of denominator to both sides.

(*i*) We have $x+y=3.3$ ...(*i*)

$$\frac{0.6}{3x-2y}=-1 \quad \text{...(ii)}$$

On multiplying eqn. (*i*) by 20, we get

$$20x+20y=66 \quad \text{...(iii)}$$

From eqn. (*ii*), we have

$$-3x+2y=0.6$$

On multiplying it by (–10), we get

$$30x-20y=-6 \quad \text{...(iv)}$$

Now, adding (*iii*) and (*iv*), we get

$$30x - 20y = -6 \quad ...(iv)$$
$$20x + 20y = 66 \quad ...(iii)$$
$$50x = 60$$

$\Rightarrow$ $x = \frac{60}{50} \Rightarrow x = \frac{6}{5} \Rightarrow x = 1.2$

Now, $10(1.2) + 10y = 33$ [From (*i*)]

$\Rightarrow$ $12 + 10y = 33$

$\Rightarrow$ $10y = 33 - 12$

$\Rightarrow$ $y = \frac{21}{10} \Rightarrow y = 2.1$

$\therefore$ The solution of the given system of equations is $x = 1.2$, and $y = 2.1$.

(*ii*) $\frac{x}{3} + \frac{y}{4} = 4$ ...(*i*) $\times$ LCM 12

$\frac{5x}{6} - \frac{y}{8} = 4$ ...(*ii*) $\times$ LCM 24

$$4x + 3y = 48 \quad ...(iii)$$
$$20x - 3y = 96 \quad ...(iv)$$
$$24x = 144$$ [By adding above two equations]

$\Rightarrow$ $x = \frac{144}{24} \Rightarrow x = 6$

Now, $4x + 3y = 48$ [From (*iii*)]

On putting the value of $x = 6$, we have

$4(6) + 3y = 48$

$\Rightarrow$ $3y = 48 - 24$

$\Rightarrow$ $3y = 24$

$\Rightarrow$ $y = \frac{24}{3} \Rightarrow y = 8$

So, the solution of the given equations is $x = 6$ and $y = 8$

(*iii*) $4x + \frac{6}{y} = 15$ ...(*i*) $\times$ 6 or 3

$6x - \frac{8}{y} = 14$ ...(*ii*) $\times$ 4 or 2

(As y is in denominators and symmetric so no need to remove denominator and 6 and 4 are divisible by 2 so we can multiply (*i*), (*ii*) by 3 and 2 respectively)

$$12x + \frac{18}{y} = 45 \quad ...(iii)$$

$$12x - \frac{16}{y} = 28 \quad ...(iv)$$

$$\underline{- \quad + \quad -}$$

$$\frac{18}{y} + \frac{16}{y} = 17 \text{ [By subtracting (iv) from (iii)]}$$

$$\frac{18+16}{y} = \frac{17}{1}$$

$\Rightarrow$ $$17y = 34$$

$\Rightarrow$ $$y = \frac{34}{+17} \Rightarrow y = 2$$

Now, $$12x + \frac{18}{y} = 45 \quad \text{[From (iii)]}$$

$\Rightarrow$ $$12x + \frac{18}{2} = 45 \quad [\because y = 2]$$

$\Rightarrow$ $$12x = 45 - 9$$

$\Rightarrow$ $$12x = 36 \Rightarrow x = 3 \text{ and } y = 2$$

(*iv*) Given equations are

$$\frac{1}{2x} - \frac{1}{y} = -1 \quad ...(i) \times \frac{1}{2}$$

$$\frac{1}{x} + \frac{1}{2y} = 8 \quad ...(ii) \times 1, x, y \neq 0$$

[x, y both are in denominator and symmetric so no need to convert into linear equation hence, can be eliminated directly]

Multiplying eqn. (*i*) by the coefficient of $\frac{1}{y}$ in (*ii*) and vice versa, we have

$$\frac{1}{4x} - \frac{1}{2y} = -\frac{1}{2} \quad ...(iii)$$

$$\frac{1}{x} + \frac{1}{2y} = 8 \quad ...(iv)$$

$$\frac{1}{4x} + \frac{1}{x} = 8 - \frac{1}{2} \quad \text{[Adding eqns (iii) and (iv)]}$$

$\Rightarrow$ $$\frac{1+4}{4x} = \frac{16-1}{2}$$

$\Rightarrow$ $$\frac{5}{4x} = \frac{15}{2}$$

$\Rightarrow \qquad 15 \times 4x = 5 \times 2$

$\Rightarrow \qquad x = \dfrac{5 \times 2}{15 \times 4} \Rightarrow x = \dfrac{1}{6}$

Now, $\dfrac{1}{x} + \dfrac{1}{2y} = 8$ [From (*iv*)]

$\Rightarrow \qquad \dfrac{1}{\left(\dfrac{1}{6}\right)} + \dfrac{1}{2y} = 8$ $\left[\because x = \dfrac{1}{6}\right]$

$\Rightarrow \qquad 6 + \dfrac{1}{2y} = 8 \Rightarrow \dfrac{1}{2y} = 8 - 6$

$\Rightarrow \qquad \dfrac{1}{2y} = \dfrac{2}{1} \Rightarrow 4y = 1 \Rightarrow y = \dfrac{1}{4}$

So, $x = \dfrac{1}{6}$ and $y = \dfrac{1}{4}$.

(*v*) Given pair of equations are

$43x + 67y = -24$ ...(*i*)

$67x + 43y = 24$ ...(*ii*)

$\Rightarrow \qquad 110x + 110y = 0$ [Adding (*i*) and (*ii*)]

$\Rightarrow \qquad x + y = 0$ ...(*iii*)

Subtracting (*ii*) from (*i*), we have

$-24x + 24y = -48$

$\Rightarrow \qquad -x + y = -2$ ...(*iv*)

$x + y = 0$ [From (*iii*)]

$\Rightarrow \qquad 2y = -2$ [Adding (*iii*) and (*iv*)]

$y = -1$

From (*iii*), $x + y = 0$

$\Rightarrow \qquad x + (-1) = 0$ $[\because y = -1]$

$\Rightarrow \qquad x = 1$

and $\qquad y = -1$

Hence, the solution of the given equations is $x = 1, y = -1$.

(*vi*) Pair of linear equations is,

$\dfrac{x}{a} + \dfrac{y}{b} = a + b$ ...(*i*) $\times ab$

$\dfrac{x}{a^2} + \dfrac{y}{b^2} = 2$ ...(*ii*) $\times a^2b^2$, $a, b \neq 0$

Multiplying (*i*) by LCM ab, and (*ii*) by LCM a^2b^2, we have

$bx + ay = a^2b + ab^2$ ...(*iii*) $\times a^2$ or a

$b^2x + a^2y = 2a^2b^2$ ...(*iv*) $\times a$ or 1

$$
\begin{array}{rl}
abx + a^2y = a^3b + a^2b^2 & \quad ...(v) \\
b^2x + a^2y = 2a^2b^2 & \quad ...(iv) \\
- \quad - \quad - & \\
\hline
abx - b^2x = a^3b - a^2b^2 &
\end{array}
$$

[Subtracting (*vi*) from (*v*)]

$\Rightarrow \quad bx(a-b) = a^2b(a-b)$

$\Rightarrow \quad x = \dfrac{a^2b(a-b)}{b(a-b)}$

$\Rightarrow \quad x = a^2$

Now, $\quad bx + ay = a^2b + ab^2$ [From (*iii*)]

$\Rightarrow \quad b(a^2) + ay = a^2b + ab^2$ [$\because x = a^2$]

$\Rightarrow \quad ay = a^2b + ab^2 - a^2b$

$\Rightarrow \quad ay = ab^2$

$\Rightarrow \quad y = \dfrac{ab^2}{a} \Rightarrow y = b^2$

So, the solution of the given equations is $x = a^2$ and $y = b^2$.

(*vii*) We have: $\quad \dfrac{2xy}{x+y} = \dfrac{3}{2}$ ...(*i*)

$\dfrac{xy}{2x-y} = \dfrac{-3}{10}$ ...(*ii*)

[$(x+y) \neq 0$ and $(2x-y) \neq 0$]

Inversing the eqn. (*i*), we get

$\dfrac{x+y}{2xy} = \dfrac{2}{3}$

$\Rightarrow \quad \dfrac{x}{2xy} + \dfrac{y}{2xy} = \dfrac{2}{3}$

$\Rightarrow \quad \dfrac{1}{2y} + \dfrac{1}{2x} = \dfrac{2}{3}$ ...(*iii*)

Inversing the eqn. (*ii*), we get

$\dfrac{2x-y}{xy} = \dfrac{10}{-3}$

$\Rightarrow \quad \dfrac{2x}{xy} - \dfrac{y}{xy} = \dfrac{-10}{3}$

$\Rightarrow \quad \dfrac{2}{y} - \dfrac{1}{x} = \dfrac{-10}{3}$

$\Rightarrow \quad \dfrac{2}{2y} - \dfrac{1}{2x} = \dfrac{-5}{3}$ ...(*iv*)

Now, $\quad \dfrac{2}{2y} - \dfrac{1}{2x} = \dfrac{-5}{3}$ [From (*iv*)] and $\frac{1}{2y}+\frac{1}{2x}=\frac{2}{3}$ [From (*iii*)]

$\frac{2}{2y}+\frac{1}{2y}=-\frac{5}{3}+\frac{2}{3}$ [Adding (*iii*) and (*iv*)]

$\Rightarrow \quad \frac{2+1}{2y}=\frac{-5+2}{3} \Rightarrow \frac{3}{2y}=\frac{-3}{3}$

$\Rightarrow \quad \frac{3}{2y}=-1 \Rightarrow y=-\frac{3}{2}$

Now, $\frac{2}{2y}-\frac{1}{2x}=\frac{-5}{3}$ [From (*iv*)]

$\Rightarrow \quad \frac{2}{y}-\frac{1}{x}=\frac{-10}{3}$

$\Rightarrow \quad \frac{2}{\left(\frac{-3}{2}\right)}-\frac{1}{x}=\frac{-10}{3}$ $\left[\because y=\frac{-3}{2}\right]$

$\Rightarrow \quad \frac{4}{-3}-\frac{1}{x}=\frac{-10}{3}$

$\Rightarrow \quad -\frac{1}{x}=\frac{-10}{3}+\frac{4}{3}$

$\Rightarrow \quad \frac{-1}{x}=\frac{-6}{3}$

$\Rightarrow \quad \frac{1}{x}=\frac{+2}{1} \Rightarrow 2x=1 \Rightarrow x=\frac{1}{2}$

So, $x=\frac{1}{2}$ and $y=\frac{-3}{2}$.

Q10. Find the solution of the pair of equations $\frac{x}{10}+\frac{y}{5}-1=0$ and $\frac{x}{8}+\frac{y}{6}=15$. Hence, find λ, if $y=\lambda x+5$.

Sol. Given equations are

$\frac{x}{10}+\frac{y}{5}-1=0$ ...(*i*) × 20

and $\frac{x}{8}+\frac{y}{6}=15$ ...(*ii*) × 24

i.e., $2x+4y=20$ ...(*iii*)

$3x+4y=360$ ...(*iv*)

$2x+4y=20$ ...(*iii*)

$3x+4y=360$ ...(*iv*)

$-\quad -\quad\quad -$ [Subtracting (*iv*) from (*iii*)]

$-x=-340$

$\Rightarrow \quad 4x=+340$

Now, $2x + 4y = 20$ [From (*iii*)]

$\Rightarrow$ $x + 2y = 10$

$\Rightarrow$ $340 + 2y = 10$ [$\because x = 340$]

$\Rightarrow$ $2y = 10 - 340$

$\Rightarrow$ $2y = -330$

$\Rightarrow$ $y = \dfrac{-330}{2} = -165$

and $x = 340$

Now, $y = \lambda x + 5$ [Given]

$\Rightarrow$ $-165 = \lambda(340) + 5$ [$\because y = -165$ and $x = 340$]

$\Rightarrow$ $-\lambda(340) = 5 + 165$

$\Rightarrow$ $-\lambda(340) = 170$

$\Rightarrow$ $\lambda = \dfrac{170}{-340} \Rightarrow \lambda = -\dfrac{1}{2}$

Hence, the solution of the given pair of equations is $x = 340$, $y = -165$ and $\lambda = -\dfrac{1}{2}$.

Q11. By the graphical method, find whether the following pair of equations are consistent or not. If consistent, solve them.

(*i*) $3x + y + 4 = 0$ and $6x - 2y + 4 = 0$

(*ii*) $x - 2y = 6$ and $3x - 6y = 0$

(*iii*) $x + y = 3$ and $3x + 3y = 9$

Sol. (*i*) Given equations are

$3x + y + 4 = 0$ ...(*i*)

$6x - 2y + 4 = 0$ ...(*ii*)

$\dfrac{a_1}{a_2} = \dfrac{3}{6} = \dfrac{1}{2}$, $\dfrac{b_1}{b_2} = \dfrac{1}{-2} = \dfrac{-1}{2}$, $\dfrac{c_1}{c_2} = \dfrac{4}{4} = 1$

$\therefore$ $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

So, the given pair of equations is consistent and has unique solution.

$3x + y + 4 = 0$ [From (*i*)]

$\Rightarrow$ $\boxed{y = -3x - 4}$

If $x = 0$, $y = -3(0) - 4 = 0 - 4 = -4$

$x = 1$, $y = -3(1) - 4 = -3 - 4 = -7$

$x = 2$, $y = -3(2) - 4 = -6 - 4 = -10$

x	0	1	2
y	−4	−7	−10

$6x - 2y + 4 = 0$ [From (*ii*)]

$\Rightarrow$ $3x - y + 2 = 0$

$\Rightarrow$ $-y = -3x - 2$

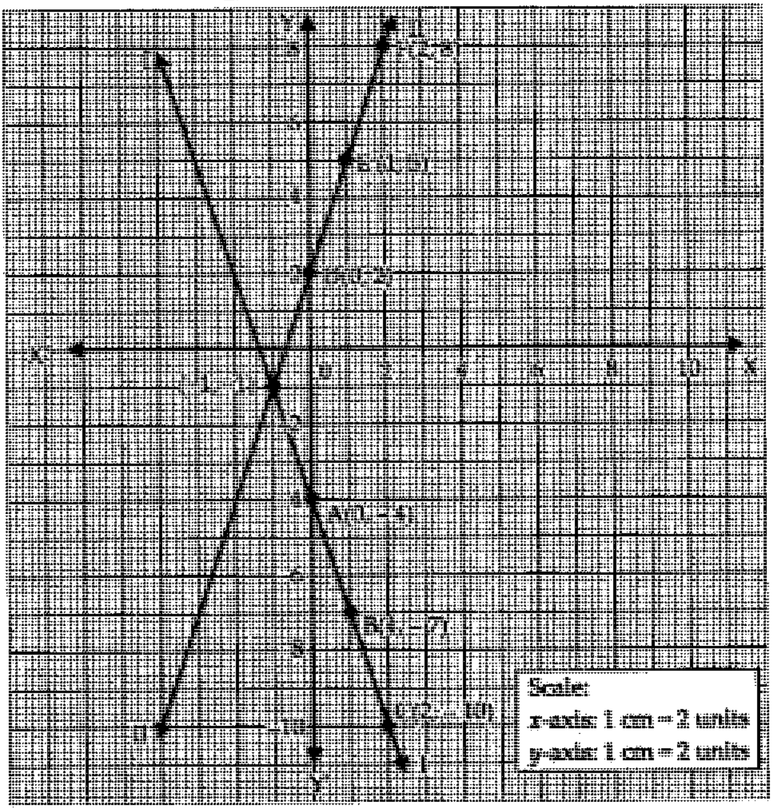
$\Rightarrow$ $\boxed{y = 3x + 2}$

If $x = 0$, $y = 3(0) + 2 = 0 + 2 = 2$

$x = 1$, $y = 3(1) + 2 = 3 + 2 = 5$

$x = 2$, $y = 3(2) + 2 = 6 + 2 = 8$

x	0	1	2
y	2	5	8



Intersecting point is $(-1, -1)$ i.e., $x = -1$ and $y = -1$

(*ii*) Given equations are,

$$x - 2y = 6 \quad \text{...}(i)$$

$$3x - 6y = 0 \quad \text{...}(ii)$$

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-2}{-6} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{6}{0}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore$ System of equations is inconsistent. Hence, the lines represented by the given equations are parallel. So, the given equations have no solution.

(*iii*) Pair of equations is

$$x + y = 3 \quad ...(i)$$
$$3x + 3y = 9 \quad ...(ii)$$

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{3}{9} = \frac{1}{3}$$

So, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the system of given equations have infinitely many solutions. Graph will be overlapping so pair of equations is consistent.
As the lines are dependent so points on graph for both equations will be same. To draw, we can take any one equation.

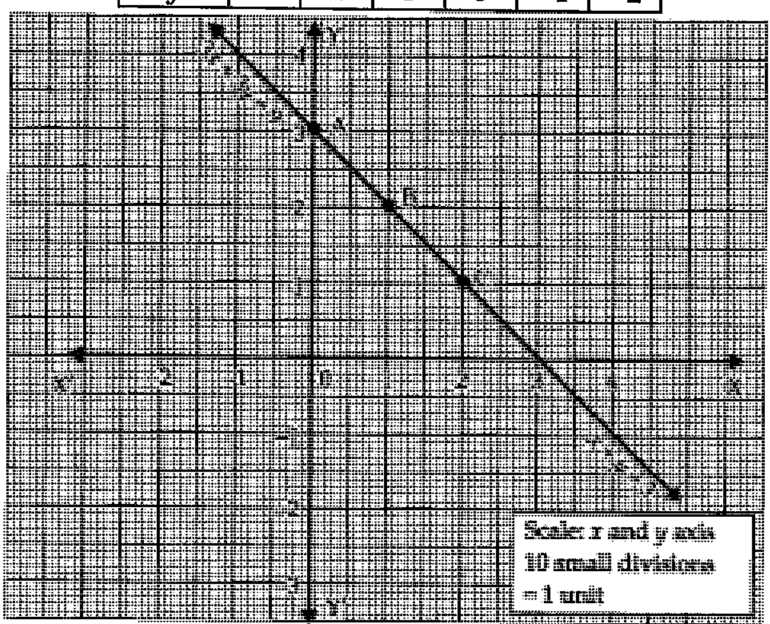
$$x + y = 3$$

$\Rightarrow$ $\boxed{y = 3 - x}$

If $x = 0$, $\quad y = 3 - 0 = 3$
$x = 1$, $\quad y = 3 - 1 = 2$
$x = 2$, $\quad y = 3 - 2 = 1$

Points for graph of equation (*i*) and (*ii*) are

x	0	1	2	3	4	5
y	3	2	1	0	−1	−2



So, the lines represented by the given equations are coinciding. Given equations are consistent.
Some solutions of system of equations are (0, 3), (1, 2), (2, 1), (3, 0), (4, –1) and (5, –2).

Q12. Draw the graph of the pair of equations $2x + y = 4$ and $2x - y = 4$. Write the vertices of the triangle formed by these lines and the y-axis. Also find the area of this triangle.

Sol. $2x + y = 4$ ...(*i*)

$\Rightarrow$ $\boxed{y = 4 - 2x}$

If $x = 0$, $y = 4 - 2(0) = 4 - 0 = 4$

$x = 1$, $y = 4 - 2(1) = 4 - 2 = 2$

$x = 2$, $y = 4 - 2(2) = 4 - 4 = 0$

x	0	1	2	3
y	4	2	0	–2
(i)	A	B	C	D

$2x - y = 4$ ...(*ii*)

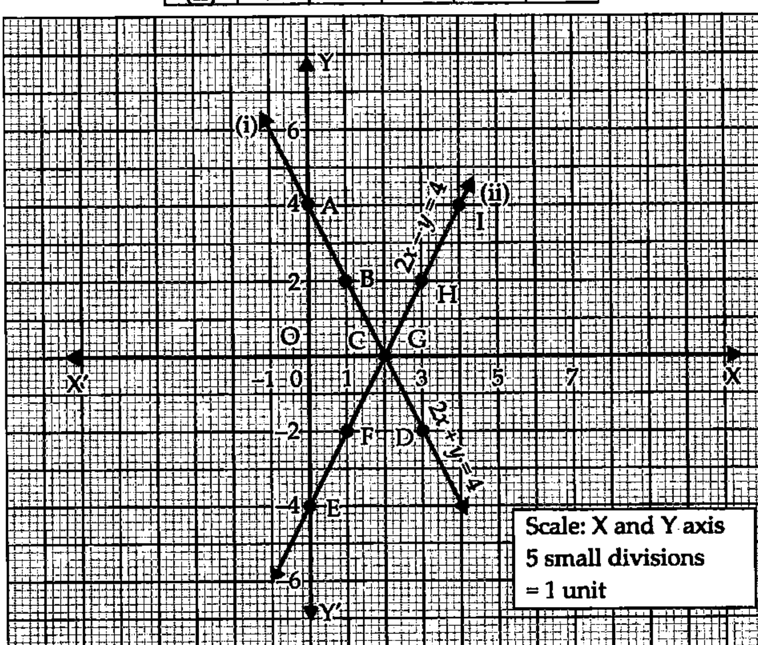
$\Rightarrow$ $\boxed{y = 2x - 4}$

If $x = 0$, $y = 2(0) - 4 = 0 - 4 = -4$

$x = 1$, $y = 2(1) - 4 = 2 - 4 = -2$

$x = 2$, $y = 2(2) - 4 = 4 - 4 = 0$

x	0	1	2	3	4
y	–4	–2	0	2	4
(ii)	E	F	G	H	I



Triangle formed by the lines with y-axis is ΔAEC. Coordinates of vertices are A(0, 4), E(0, –4) and C(2, 0).

$$\text{Area of } \Delta AEC = \frac{1}{2} \text{Base} \times \text{Altitude}$$
$$= \frac{1}{2} AE \times CO$$
$$= \frac{1}{2} \times [4-(-4)] \times (2-0)$$
$$= \frac{1}{2} \times 8 \times 2 = 8$$

$\therefore$ Area of ΔAEC = 8 square units

Q13. Write an equation of a line passing through the point representing the solution of the pair of linear equations $x+y=2$ and $2x-y=1$. How many such lines can we find?

Ans. Given pair of linear equations is

$$x+y=2 \quad ...(i)$$
$$2x-y=1 \quad ...(ii)$$

Here, $\frac{a_1}{a_2}=\frac{1}{2}$, $\frac{b_1}{b_2}=\frac{1}{-1}=-1$

$\Rightarrow$ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, the given pair of equations has unique solution.

For solution of equations, add (*i*) and (*ii*), we get

$$3x = 3$$

$\Rightarrow$ $x = \frac{3}{3} \Rightarrow x = 1$

Now, $x+y=2$ [From (*i*)]

$\Rightarrow$ $1+y=2 \Rightarrow y=1$ [$\because x=1$]

Hence, the solution of the given equations is $y=1$ and $x=1$.

Now, we have to find a line passing through (1, 1). We can make infinite linear equations passing through (1, 1). Some of the linear equations are given below:

Step I: Take any linear polynomial in x and y, let it be $8x-5y$.

Step II: Put $x=1$ and $y=1$ in the above polynomial, i.e.

$$8(1)-5(1)=8-5=3$$

Step III: So, the required equation of line passing through (1, 1) is

$$8x-5y=3$$

Some more required equations are $2x-3y=-1$, $3x-2y=1$ and $5x-2y=3$ and $x-y=0$ etc.

Q14. If $(x+1)$ is a factor of $2x^3+ax^2+2bx+1$, then find the values of a and b given that $2a-3b=4$.

Sol. Let $f(x)=2x^3+ax^2+2bx+1$

If $(x+1)$ is a factor of $f(x)$, then by factor theorem $f(-1)=0$.

$\therefore$ $f(-1)=2(-1)^3+a(-1)^2+2b(-1)+1=0$

$$\Rightarrow \quad -2 + a - 2b + 1 = 0$$

$$\Rightarrow \quad a - 2b = 1 \quad ...(i)$$

$$2a - 3b = 4 \quad ...(ii) \quad \text{[Given]}$$

$$2a - 4b = 2 \quad [(i) \times 2]$$

$$2a - 3b = 4 \quad \text{[From (ii)]}$$

$$- \quad + \quad -$$

$$-b = -2$$

$$\Rightarrow \quad b = 2$$

Now, $a - 2b = 1$ [From (*i*)]

$$\Rightarrow \quad a - 2(2) = 1 \quad [\because b = 2]$$

$$\Rightarrow \quad a = 1 + 4$$

$$\Rightarrow \quad a = 5, \quad b = 2$$

Q15. The angles of a triangle are x, y and $40°$. The difference between the two angles x and y is $30°$. Find x and y.

Sol. x, y and 40 are the measures of interior angles of a triangle.

$$\therefore \quad x + y + 40° = 180°$$

$$\Rightarrow \quad x + y = 140° \quad ...(i)$$

The difference between x and y is $30°$ so

$$x - y = 30° \quad ...(ii)$$

$$x + y = 140° \quad \text{[From (i)]}$$

$$2x = 170° \quad \text{[Adding (i) and (ii)]}$$

$$\Rightarrow \quad x = \frac{170}{2} = 85°$$

Now, $x + y = 140°$ [From (*i*)]

$$\Rightarrow \quad 85° + y = 140° \quad [\because x = 85°]$$

$$\Rightarrow \quad y = 140° - 85°$$

$$\Rightarrow \quad y = 55°$$

and $x = 85°$

Q16. Two years ago, Salim was thrice as old as his daughter and six years later he will be four years older than twice her age. How old are they now?

Sol. Let the present age of Salim be x years.

Also, let the present age of his daughter be y years.

Age of Salim 2 years ago = $(x - 2)$ years

Age of Salim's daughter 2 years ago = $(y - 2)$ years

According to the question, we have

Age of Salim was = thrice × daughter [Given]

$$\Rightarrow \quad x - 2 = 3 \times (y - 2)$$

$$\Rightarrow \quad x - 2 = 3y - 6$$

$\Rightarrow \quad x - 3y = -4 \quad ...(i)$

Age of Salim 6 years later = $(x + 6)$ years

Age of Salim's daughter 6 years later = $(y + 6)$ years

According to the question, we have

$$x + 6 = 2(y + 6) + 4$$
$$\Rightarrow \quad x + 6 = 2y + 12 + 4$$
$$\Rightarrow \quad x - 2y = 16 - 6$$
$$\Rightarrow \quad x - 2y = 10 \quad (ii)$$
$$x - 3y = -4 \quad \text{[From } (i)]$$
$$\underline{- \quad + \qquad +}$$
$$y = 14 \quad \text{[Subtracting } (i) \text{ from } (ii)]$$

Now, $\quad x - 2y = 10 \quad$ [From (ii)]

$\Rightarrow \quad x - 2(14) = 10 \quad [\because \; y = 14]$

$\Rightarrow \quad x = 10 + 28$

$\Rightarrow \quad x = 38$

$\therefore$ Age of Salim at present = 38 years

and age of Salim's daughter at present = 14 years

Q17. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Sol. Let the present age of father be x years.

Also, let the sum of present ages of two children be y years.

The age of father is (=) twice (× 2) the sum of the ages of two children

$\Rightarrow \quad x = 2 \times (y) \quad \Rightarrow \quad x = 2y$

$\Rightarrow \quad x - 2y = 0 \quad ...(i)$

Age of father 20 years later = $(x + 20)$ years

Increase in age of first children in 20 years = 20 years

Increase in age of second children in 20 years = 20 years

$\therefore$ Increase in the age of both children in 20 years = 20 + 20 = 40 years

$\therefore$ Sum of ages of both children 20 years later = $(y + 40)$

Now, according to the question, we have

Father will be (=) sum of ages of two children [Given]

$\Rightarrow \quad x + 20 = y + 40$

$\Rightarrow \quad x - y = 20 \quad ...(ii)$

$\Rightarrow \quad 2y - y = 20 \quad [(\because x = 2y)$ from $(i)]$

$\Rightarrow \quad y = 20$ years

Now, $\quad x = 2y \quad$ [From (i)]

$\Rightarrow \quad x = 2 \times 20$

$\Rightarrow \quad x = 40$

$\therefore$ Age of father is 40 years.

Q18. Two numbers are in the ratio 5:6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.

Sol. Let the numbers be $5x$ and $6x$ respectively. So, new numbers after subtracting 8 from each will be $(5x - 8)$ and $(6x - 8)$ respectively.

According to the question, ratio of new numbers is 4 : 5.

$$\therefore \quad \frac{5x-8}{6x-8} = \frac{4}{5}$$

$$\Rightarrow \quad 25x - 40 = 24x - 32$$

$$\Rightarrow \quad 25x - 24x = 40 - 32$$

$$\Rightarrow \quad x = 8$$

$\therefore$ Required numbers = $5x$ and $6x$ become $5 \times 8, 6 \times 8$

i.e., Required numbers = 40 and 48

Q19. There are some students in two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in the two halls.

Sol. Let the number of students initially in hall A be x.
and the number of students initially in hall B be y.

Case I: 10 students of hall A shifted to B

Now, number of students in hall A = $(x - 10)$

Now, number of students in hall B = $(y + 10)$

According to the question, number of students in both halls are equal.

$$\therefore \quad x - 10 = y + 10$$

$$\Rightarrow \quad x - y = 20 \quad ...(i)$$

Case II: 20 students are shifted from hall B to A, then

Number of students in hall A becomes = $x + 20$

Number of students in hall B becomes = $y - 20$

According to the question, students in hall A becomes twice of students in hall B.

$$\therefore \quad x + 20 = 2(y - 20)$$

$$\Rightarrow \quad x + 20 = 2y - 40$$

$$\Rightarrow \quad x - 2y = -60 \quad (ii)$$

$$x - y = 20 \quad \text{[From (i)]}$$

$$\underline{- \quad + \qquad -} \quad \text{[Subtracting eqn. (i) from (ii)]}$$

$$-y = -80$$

$$\Rightarrow \quad y = 80$$

Now, $\quad x - y = 20 \quad$ [From (i)]

$$\Rightarrow \quad x - 80 = 20$$

$\Rightarrow \quad x = 20 + 80$

$\Rightarrow \quad x = 100$

$\therefore$ Number of students initially in hall A = 100
and number of students initially in hall B = 80

Q20. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days and an additional charge for each day thereafter. Latika paid ₹ 22 for a book kept for six days, while Anand paid ₹ 16 for the book kept for four days. Find the fixed charges and the charge for each extra day.

Sol. Let the fixed charges for first two days = ₹x
Let the additional charges per day after 2 days = ₹ y
Latika paid ₹ 22 for six days. [Given]
2 days fixed charges + (6 – 2) days charges = 22

$\Rightarrow \quad x + 4y = 22 \quad ...(i)$

Anand paid ₹ 16 for books kept for four days.
2 day's fixed charges + (4 – 2) day's additional charges = 16

$\Rightarrow \quad x + 2y = 16 \quad ...(ii)$

$x + 2y = 16$ [From (*ii*)]
$x + 4y = 22$ [From (*i*)]
$-\quad -\quad\quad -$ [Subtracting eqn. (*i*) from (*ii*)]
$-2y = -6$

$\Rightarrow \quad y = ₹\ 3$ per day

Now, $\quad x + 2y = 16$ [From (*ii*)]

$\Rightarrow \quad x + 2(3) = 16$

$\Rightarrow \quad x = 16 - 6 \quad \Rightarrow \quad x = 10$

So, the fixed charges for first 2 days = ₹ 10
The additional charges per day after 2 days = ₹ 3 per day

Q21. In a competitive examination, 1 mark is awarded for each correct answer, while 1/2 mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?

Sol. Let the number of questions attempted correctly = x
Number of questions answered = 120
So, wrong answer attempted = $(120 - x)$
Marks awarded for right answer = $1 \times x = x$ marks
Marks deducted for $(120 - x)$ wrong answer = $\frac{1}{2}(120 - x)$
Totals marks awarded = 90

$\therefore \quad x - \frac{1}{2}(120 - x) = 90 \quad \Rightarrow \quad -60 + - = 90$

$\Rightarrow \quad x + \frac{x}{2} = 90 + 60 \quad \Rightarrow \quad \frac{3x}{2} = 150$

$$\Rightarrow \qquad x = \frac{150 \times 2}{3} \Rightarrow x = 100$$

Hence, Jayanti answered 100 questions correctly.

Q22. The angles of a cyclic quadrilateral ABCD are $\angle A = (6x + 10)°$, $\angle B = (5x)°$, $\angle C = (x + y)°$, $\angle D = (3y - 10)°$. Find x and y, and hence the values of four angles.

Sol. The sum of opposite angles of a cyclic □ABCD is 180° so

$$\angle A + \angle C = 180° \qquad \text{[Opposite } \angle\text{s of cyclic □]}$$

$$\Rightarrow \quad (6x + 10) + (x + y) = 180°$$

$$\Rightarrow \quad 6x + 10 + x + y = 180°$$

$$\Rightarrow \quad 7x + y = 170° \qquad ...(i)$$

Also, $\angle B + \angle D = 180°$

$$\Rightarrow \quad 5x + (3y - 10) = 180°$$

$$\Rightarrow \quad 5x + 3y = 180° + 10°$$

$$\Rightarrow \quad 5x + 3y = 190 \qquad (ii)$$

$$21x + 3y = 510 \qquad ...(iii) \text{ [From (i)]}$$

$$\underline{- \quad - \quad -}$$

$$-16x = -320 \qquad \text{[Subtracting eqn. (i) from (ii)]}$$

$$\Rightarrow \quad x = \frac{320}{16} \Rightarrow x = 20$$

Now, $7x + y = 170$ [From (i)]

$$\Rightarrow \quad 7(20) + y = 170$$

$$\Rightarrow \quad y = 170 - 140$$

$$\Rightarrow \quad y = 30$$

and $x = 20$

$$\therefore \quad \angle A = (6x + 10)° = (6 \times 20 + 10)° = (120 + 10)° = 130°$$

$$\angle B = (5x)° = (5 \times 20)° = 100°$$

$$\angle C = (x + y)° = (20 + 30)° = 50°$$

$$\angle D = (3y - 10)° = (3 \times 30 - 10)° = (90 - 10)° = 80°$$

Hence, the values of x and y are 20 and 30 respectively. $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are 130°, 100°, 50°, 80° respectively.

EXERCISE 3.4

Q1. Graphically, solve the following pair of equations $2x + y = 6$ and $2x - y + 2 = 0$. Find the ratio of the areas of the two triangles formed by the lines representing these equations with the x-axis and the lines with the y-axis.

Sol. Given equation is $2x + y = 6$

$$\Rightarrow \quad \boxed{y = 6 - 2x} \qquad ...(i)$$

If $x = 0, y = 6 - 2(0) = 6$

$x = 1, y = 6 - 2(1) = 6 - 2 = 4$

$x = 2, y = 6 - 2(2) = 6 - 4 = 2$

x	0	1	2
y	6	4	2
I	A	B	C

Given equation is $2x - y + 2 = 0$

$\Rightarrow$ $\boxed{y = 2x + 2}$ ...(*ii*)

If $x = 0$, $y = 2(0) + 2 = 0 + 2 = 2$

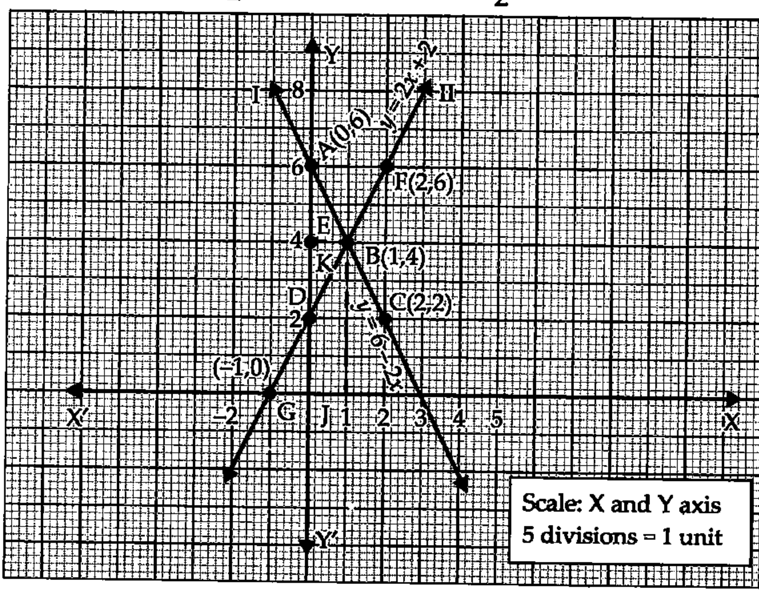
$x = 1$, $y = 2(1) + 2 = 2 + 2 = 4$

$x = 2$, $y = 2(2) + 2 = 4 + 2 = 6$

x	0	1	2
y	2	4	6
II	D	E	F

The area of Δ BGH formed by lines and X axis $= \frac{1}{2} GH \times BJ$

$$= \frac{1}{2}[3 - (-1)] \times (4 - 0) = \frac{1}{2} \times 4 \times 4 = 8 \text{ sq. units}$$



The area of Δ BAD formed by lines and Y axis $= \frac{1}{2} AD \times KB$

$$= \frac{1}{2}(6 - 2) \times (1 - 0) = \frac{1}{2} \times 4 \times 1 = 2 \text{ sq. units}$$

$\therefore$ Ratio of areas of two Δs $= \frac{\text{Area } \Delta \text{ BGH}}{\text{Area } \Delta \text{ BAD}} = \frac{8}{2} = \frac{4}{1} = 4:1$

Q2. Determine graphically the vertices of the triangle formed by the lines $y = x$, $3y = x$, and $x + y = 8$.

Sol. Given equations are

$$y = x \quad ...(i)$$
$$x = 3y \quad ...(ii)$$
$$x + y = 8 \quad ...(iii)$$
$$\Rightarrow \quad y = 8 - x \quad \text{[From (iii)]}$$

If $x = 0$, $y = 8 - 0 = 8$
$x = 1$, $y = 8 - 1 = 7$
$x = 2$, $y = 8 - 2 = 6$

$y = 8 - x$

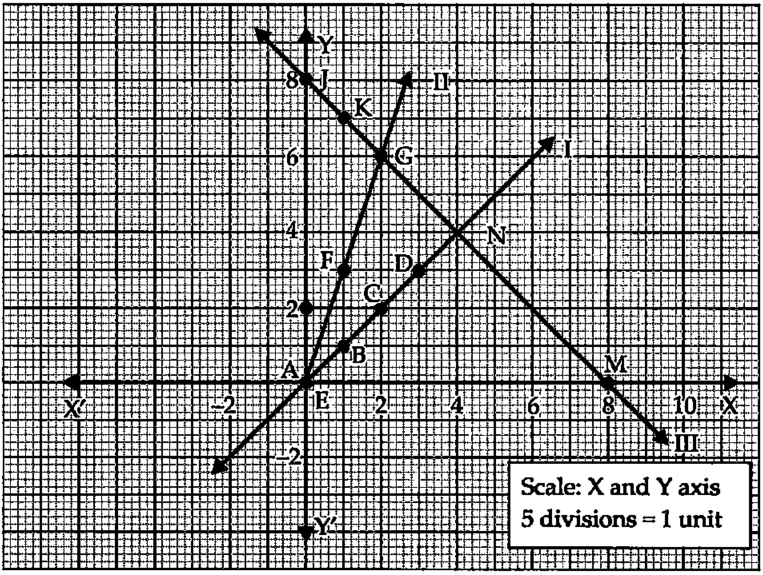
III	J	K	L	M
x	0	1	2	8
y	8	7	6	0

$y = x$

I	A	B	C	D
x	0	1	2	3
y	0	1	2	3

$x = 3y$

II	E	F	G	H
x	0	1	2	3
y	0	3	6	9



Hence, the vertices of Δ GNA formed by 3 lines are G(2, 6), N(4, 4) and A(0, 0).

Q3. Draw the graphs of the equations $x = 3$, $x = 5$, and $2x - y - 4 = 0$. Also find the area of the quadrilateral formed by the lines and the x-axis.

Sol. The given equations are

$$x = 3 \quad \text{...(i)}$$
$$x = 5 \quad \text{...(ii)}$$
$$2x - y - 4 = 0 \quad \text{...(iii)}$$
$$\Rightarrow \quad y = 2x - 4$$

If $x = 0$, then $y = 2(0) - 4 = 0 - 4 = -4$
$x = 1$, then $y = 2(1) - 4 = 2 - 4 = -2$
$x = 2$, then $y = 2(2) - 4 = 4 - 4 = 0$
$x = 3$, then $y = 2(3) - 4 = 6 - 4 = 2$
$x = 4$, then $y = 2(4) - 4 = 8 - 4 = 4$

$\Rightarrow$ $y = 2x - 4$

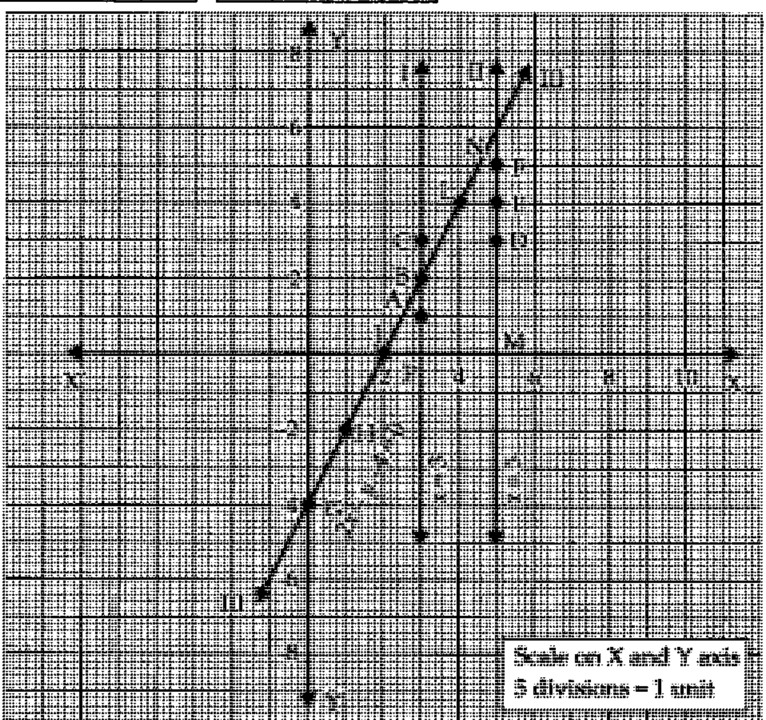
x	0	1	2	3	4
y	–4	–2	0	2	4
III	G	H	J	K	L

$x = 3$

x	3	3	3
y	1	2	3
I	A	B	C

$x = 5$

x	5	5	5
y	3	4	5
II	D	E	F



The coordinates of the vertices of the required □ PMNB are P(3,0), M(5,0), N(5,6) and B(3,2)

The quadrilateral formed by these given three lines and x-axis is $\square$ PMNB. It is trapezium. So, area of the required trapezium

$$= \frac{1}{2}(BP + MN) \times PM$$

$$= \frac{1}{2}[(2-0)+(6-0)](5-3)$$

$$= \frac{1}{2} \times 8 \times 2 = 8 \text{ square units}$$

Hence, the area of required $\square$ PMNB = 8 square units.

Q4. The cost of 4 pens and 4 pencil boxes is ₹ 100. Three times the cost of a pen is ₹ 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

Sol. Let the cost of a pen = ₹ x

Let the cost of a pencil box = ₹ y

$\therefore$ The cost of 4 pens and 4 pencil boxes = ₹ 100 [Given]

$$4x + 4y = 100 \quad \text{...}(i)$$

$$x + y = 25 \quad \text{...}(ii) \quad [\text{By dividing } (i) \text{ by } 4]$$

Accoding to the second condition, we have

$$3x = y + 15$$

$$3x - y = 15 \quad (iii)$$

$$x + y = 25 \quad (ii)$$

$$4x = 40 \quad [\text{Adding } (ii) \text{ and } (iii)]$$

$$\Rightarrow \quad x = \frac{40}{4} = 10$$

Now, $\quad x + y = 25$ [From (ii)]

$\Rightarrow \quad 10 + y = 25$ [$\because \ x = 10$)]

$\Rightarrow \quad y = 25 - 10 =$ ₹ 15

So, $x =$ ₹ 10 and $y =$ ₹ 15

Hence, the cost of a pen and a pencil box are ₹ 10 and ₹ 15 respectively.

Q5. Determine, algebraically, the vertices of the triangle formed by the lines, $3x - y = 3$, $2x - 3y = 2$ and $x + 2y = 8$.

Sol. Given linear equations are

$$3x - y = 3 \quad \text{...}(i)$$

$$2x - 3y = 2 \quad \text{...}(ii)$$

$$x + 2y = 8 \quad \text{...}(iii)$$

Let the intersecting points of lines (*i*) and (*ii*) is A, and of lines (*ii*) and (*iii*) is B and that of lines (*iii*) and (*i*) is C.

The intersecting point of (*ii*) and (*i*) can be find out by solving (*i*) and (*ii*) for (*x*, *y*).

$$3x - y = 3 \quad \text{[From (i)]}$$
$$2x - 3y = 2 \quad \text{[From (ii)]}$$

$$9x - 3y = 9 \quad \text{...(iv) [Multiplying eqn. (i) by 3]}$$
$$2x - 3y = 2 \quad \text{[From (ii)]}$$
$$\underline{- \quad + \quad\quad -}$$
$$7x = 7 \quad \text{[By subtracting (ii) from (iv)]}$$

$\Rightarrow$ $x = \frac{7}{7} \Rightarrow x = 1$

Now, $3x - y = 3$ [From (*i*)]

$\Rightarrow$ $3(1) - y = 3$ [$\because$ $x = 1$]

$\Rightarrow$ $-y = 3 - 3 \Rightarrow -y = 0 \Rightarrow y = 0$

So, intersecting point of eqns. (*i*) and (*ii*) is A(1, 0).

Similarly, intersecting point B of eqns. (*ii*) and (*iii*) can be find out as follows:

$$2x - 3y = 2 \quad \text{[From (ii)]}$$
$$x + 2y = 8 \quad \text{[From (iii)]}$$

$$2x - 3y = 2 \quad \text{[From (ii)]}$$
$$2x + 4y = 16 \quad \text{...(v) [By multiplying (iii) by 2]}$$
$$\underline{- \quad - \quad\quad -}$$
$$-7y = -14 \quad \text{[Subtracting (v) from (ii)]}$$

$\Rightarrow$ $y = \frac{14}{7} \Rightarrow y = 2$

Now, $x + 2y = 8$ [From (*iii*)]

$\Rightarrow$ $x + 2(2) = 8$

$\Rightarrow$ $x = 8 - 4$

$\Rightarrow$ $x = 4$

So, the coordinates of B are (4, 2).

Similarly, for intersecting point C of eqns. (*i*) and (*iii*), we have

$$3x - y = 3 \quad \text{[From (i)]}$$
$$x + 2y = 8 \quad \text{[From (iii)]}$$

Multiplying (*i*) by 2, we get

$$6x - 2y = 6 \quad \text{...(vi)}$$
$$\underline{x + 2y = 8} \quad \text{[From (iii)]}$$
$$7x = 14 \quad \text{[Adding (vi) and (iii)]}$$

$\Rightarrow$ $x = \frac{14}{7} \Rightarrow x = 2$

Now, $3x - y = 3$ [From (*i*)]

$\Rightarrow$ $3(2) - y = 3$

$\Rightarrow$ $-y = 3 - 6$

$\Rightarrow$ $-y = -3 \Rightarrow y = 3$

So, point C is (2, 3).

Hence, the vertices of ΔABC formed by given three linear equations are A(1, 0), B(4, 2) and C(2, 3).

Q6. Ankita travels 14 km to her home partly by rikshaw and partly by bus. She takes half an hour if she travels 2 km by rikshaw and the remaining distance by bus. On the other hand, if she travels 4 km by rikshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rikshaw and of the bus.

Sol. Let the speed of rikshaw = x km/hr

and let the speed of bus = y km/hr

Case I: Time taken by rikshaw to travel 2 km = $\frac{\text{Distance}}{\text{Speed}} = \frac{2}{x}$ hr

Time taken by bus to travel (14 – 2) km (remaining) = $\frac{12}{y}$ hr

Total time taken by rikshaw (2 km) and bus (12 km) = $\frac{1}{2}$ hr

$$\therefore \quad \frac{2}{x} + \frac{12}{y} = \frac{1}{2} \qquad \text{...}(i)$$

Case II: Time taken by rikshaw to travel 4 km = $\frac{4}{x}$ hr

Time taken by bus to travel remaining (14 – 4) km = $\frac{10}{y}$ hr

$\therefore$ Total time in case II = $\frac{1}{2}$ hr + 9 min

$$\therefore \quad \frac{4}{x} + \frac{10}{y} = \frac{1}{2}\text{hr} + \frac{9}{60}\text{hr} \Rightarrow \frac{4}{x} + \frac{10}{y} = \frac{30+9}{60}$$

$$\Rightarrow \quad \frac{4}{x} + \frac{10}{y} = \frac{39}{60} \Rightarrow \frac{4}{x} + \frac{10}{y} = \frac{13}{20} \qquad \text{...}(ii)$$

Multiplying equation (*i*) by 2, we get

$$\frac{4}{x} + \frac{24}{y} = \frac{2}{2} \qquad \text{...}(iii)$$

Now, subtracting (*iii*) from (*ii*), we get

$$\frac{4}{x} + \frac{10}{y} = \frac{13}{20}$$

$$\frac{4}{x} + \frac{24}{y} = \frac{2}{2}$$
$$\underline{- \quad - \qquad -}$$
$$\frac{10}{y} - \frac{24}{y} = \frac{13}{20} - \frac{2}{2}$$

$\Rightarrow$ $$\frac{10-24}{y} = \frac{13-20}{20} \Rightarrow -\frac{14}{y} = \frac{-7}{20}$$

$\Rightarrow$ $$7y = 14 \times 20 \Rightarrow y = \frac{14 \times 20}{7}$$

$\Rightarrow$ $$y = 40 \text{ km/hr}$$

Now, $$\frac{2}{x} + \frac{12}{y} = \frac{1}{2}$$ [From (*i*)]

$\Rightarrow$ $$\frac{2}{x} + \frac{12}{(40)} = \frac{1}{2} \Rightarrow \frac{2}{x} = \frac{1}{2} - \frac{3}{10}$$

$\Rightarrow$ $$\frac{2}{x} = \frac{5-3}{10} \Rightarrow \frac{2}{x} = \frac{2}{10}$$

$\Rightarrow$ $$x = 10 \text{ km/hr}$$

Hence, the speeds of rikshaw and bus are 10 km/hr and 40 km/hr respectively.

Q7. A person, rowing at the rate of 5 km/hr in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.

Sol. Let the speed of the stream be x km/hr.

And, the speed of the boat in still water = 5 km/hr

Speed of the boat upstream = $(5 - x)$ km/hr

Speed of the boat downstream = $(5 + x)$ km/hr

Time taken in rowing 40 km upstream = $\frac{40}{5-x}$ hrs

Time taken in rowing 40 km downstream = $\frac{40}{5+x}$ hrs

According to the question, we have

Time taken in 40 km upstream = 3 × Time taken in 40 km downstream

$\therefore$ $$\frac{40}{5-x} = \frac{3 \times 40}{5+x}$$

$\Rightarrow$ $$\frac{1}{5-x} = \frac{3}{5+x}$$

$\Rightarrow$ $$-3x + 15 = x + 5$$

$$\Rightarrow \quad -3x - x = 5 - 15$$
$$\Rightarrow \quad -4x = -10$$
$$\Rightarrow \quad x = \frac{10}{4}$$
$$\Rightarrow \quad x = 2.5 \text{ km/hr}$$

Hence, the speed of stream is 2.5 km/hr.

Q8. A motorboat can travel 30 km upstream and 28 km downstream in 7 hrs. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

Sol. Let speed of boat in still water = x km/hr
and the speed of the stream = y km/hr
Speed of motor boat upstream = $(x - y)$ km/hr
Speed of motor boat downstream = $(x + y)$ km/hr

Case I: Time taken by motor boat in 30 km upstream = $\frac{30}{x-y}$ hr

Time taken by motor boat in 28 km downstream = $\frac{28}{x+y}$ hr

$$\therefore \quad \frac{30}{(x-y)} + \frac{28}{(x+y)} = 7$$
$$\Rightarrow \quad 2\left[\frac{15}{(x-y)} + \frac{14}{(x+y)}\right] = 7$$
$$\Rightarrow \quad \frac{15}{x-y} + \frac{14}{x+y} = \frac{7}{2} \quad \ldots(i)$$

Case II: Time taken by motor boat in 21 km upstream = $\frac{21}{x-y}$ hr

Time taken by motor boat to return 21 km downstream = $\frac{21}{x+y}$ hr

$$\therefore \quad \frac{21}{x-y} + \frac{21}{x+y} = 5$$
$$\Rightarrow \quad 21\left[\frac{1}{x-y} + \frac{1}{x+y}\right] = 5$$
$$\Rightarrow \quad \frac{1}{x-y} + \frac{1}{x+y} = \frac{5}{21} \quad \ldots(ii)$$
$$\frac{15}{x-y} + \frac{14}{x+y} = \frac{7}{2} \quad \text{[From } (i)\text{]}$$

As equations (both) are symmetric to $(x - y)$ and $(x + y)$ so we can eliminate either $(x - y)$ or $(x + y)$.

Multiplying (*ii*) by 14, we get

$$\frac{14}{(x-y)} + \frac{14}{(x+y)} = \frac{70}{21} \quad \text{...(iii)}$$

$$\frac{15}{(x-y)} + \frac{14}{x+y} = \frac{7}{2} \quad \text{[From (i)]}$$

$$- \quad - \quad - \quad \text{[Subtracting (i) from (iii)]}$$

$$\frac{14}{(x-y)} - \frac{15}{(x-y)} = \frac{10}{3} - \frac{7}{2}$$

$$\Rightarrow \quad \frac{14-15}{(x-y)} = \frac{20-7\times 3}{3\times 2}$$

$$\Rightarrow \quad \frac{-1}{(x-y)} = \frac{-1}{6}$$

$$\Rightarrow \quad (x-y) = 6 \quad \text{...(iv)}$$

Now, substituting $x - y = 6$ in (*ii*), we have

$$\frac{1}{(x-y)} + \frac{1}{(x+y)} = \frac{5}{21}$$

$$\Rightarrow \quad \frac{1}{6} + \frac{1}{(x+y)} = \frac{5}{21} \quad \Rightarrow \quad \frac{1}{(x+y)} = \frac{5}{21} - \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{(x+y)} = \frac{2\times 5 - 7\times 1}{3\times 7\times 2} \quad \Rightarrow \quad \frac{1}{(x+y)} = \frac{3}{42}$$

$$\Rightarrow \quad \frac{1}{(x+y)} = \frac{1}{14}$$

$$\Rightarrow \quad x + y = 14 \quad \text{...(v)}$$

$$x - y = 6 \quad \text{[From (iv)]}$$

$$2x = 20 \quad \text{[Subtracting (iv) from (v)]}$$

$$\Rightarrow \quad x = 10 \text{ km/hr}$$

Now, $x + y = 14$ [From (*v*)]

$$\Rightarrow \quad 10 + y = 14$$

$$\Rightarrow \quad y = 4 \text{ km/hr}$$

Hence, the speed of motorboat and stream are 10 km/hr and 4 km/hr respectively.

Q9. A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.

Sol. Let the two digit number $= xy$

$$= 10x + y$$

According to the question:

$$\text{Number} = 8(x+y)-5$$

$\Rightarrow$ $$10x+y = 8x+8y-5$$

$\Rightarrow$ $$10x-8x+y-8y = -5$$

$\Rightarrow$ $$2x-7y = -5 \quad ...(i)$$

Also, $$\text{Number} = 16(x-y)+3 = 10x+y$$

$\Rightarrow$ $$10x+y = 16x-16y+3$$

$\Rightarrow$ $$-6x+17y = 3 \quad ...(ii)$$

Multiplying (*i*) by 3, we get

$$6x-21y = -15 \quad ...(iii)$$

Adding (*iii*) and (*ii*), we have

$$\begin{array}{r} -6x+17y = 3 \\ 6x-21y = -15 \\ \hline -4y = -12 \end{array}$$

$\Rightarrow$ $$y = 3$$

Now, $$2x-7y = -5 \quad \text{[From (i)]}$$

$\Rightarrow$ $$2x-7(3) = -5$$

$\Rightarrow$ $$2x = -5+21$$

$\Rightarrow$ $$2x = 16$$

$\Rightarrow$ $$x = 8$$

So, the number is $xy = 83$.

We can also find another number if possible.

$$16(y-x)+3 = 10x+y$$

$\Rightarrow$ $$16y-16x+3 = 10x+y$$

$\Rightarrow$ $$-16x-10x+16y-y = -3$$

$\Rightarrow$ $$-26x+15y = -3 \quad ...(iv)$$

$$\begin{array}{rl} -26x+15y = -3 & ...(iv) \\ 26x-91y = -65 & (i) \times 13 \\ \hline -76y = -68 & \text{[Adding above 2 eqns.]} \end{array}$$

$\Rightarrow$ $$y = \frac{68}{76} = \frac{17}{19}$$

But x, y can never be in fraction or negative.

Hence, the required number = 83

Q10. A railway half ticket costs half the full fare, but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from the station A to B costs ₹ 2530. Also, one reserved first class ticket and one reserved first class half ticket from station A to B costs ₹ 3810. Find the full first class fare from station A to B, and also the reservation charges for a ticket.

Sol. Let the cost of full fare from station A to B = ₹ x
and the reservation charges per ticket = ₹ y
Cost of one full ticket from A to B = ₹ 2530
i.e., (1 fare + 1 reservation) charges = ₹ 2530
i.e., $x + y = 2530$ ...(*i*)
Cost of 1 full and one, half ticket from station A to B = ₹ 3810
i.e., (1 full ticket) + (1/2 ticket) charges = ₹ 3810
i.e., $(x + y) + (1/2 \text{ fare} + \text{reservation}) = 3810$

i.e., $$(x+y) + \frac{1}{2}x + y = 3810$$

$$\Rightarrow \quad \frac{3}{2}x + 2y = 3810$$

$$\Rightarrow \quad 3x + 4y = 7620 \quad \text{...(ii)}$$

Multiplying (*i*) by 3, we get

$$3x + 3y = 7590 \quad \text{...(iii)}$$

Subtracting (*iii*) from (*ii*), we get

$$\begin{array}{rrl} 3x + & 4y = 7620 & \text{...(ii)} \\ 3x + & 3y = 7590 & \text{(iii)} \\ - \; - & - & \\ \hline & y = ₹\, 30 & \end{array}$$

Now, $x + y = 2530$ [From (*i*)]

$\Rightarrow \quad x + 30 = 2530$ $(\because \; y = 30)$

$\Rightarrow \quad x = 2530 - 30$

$\Rightarrow \quad x = ₹\, 2500$

Hence, full fare and reservation charges of a ticket from station A to B are ₹ 2500 and ₹ 30 respectively.

Q11. A shopkeeper sells a saree at 8% profit and a sweater at 10% discount; thereby getting a sum of ₹ 1008. If she had sold saree at 10% profit and the sweater at 8% discount. She would have got ₹ 1028, then find out the cost price of the saree and the list price (price before discount) of the sweater.

Sol. Let the cost price of a saree = ₹ x
and the list price of sweater = ₹ y

Case I:

(S.P. of saree at 8% profit) + (S.P. of a sweater at 10% discount)= ₹ 1008

$$\Rightarrow \quad \frac{(100+8)}{100}x + \frac{(100-10)}{100}y = 1008$$

$$\Rightarrow \quad 108x + 90y = 100800$$

$$\Rightarrow \quad 6x + 5y = 5600 \quad ...(i)$$

Case II:

(S.P. of saree at 10% profit) + (S.P. of a sweater at 8% discount)= ₹ 1028

$$\Rightarrow \quad \frac{(100+10)}{100}x + \frac{(100-8)}{100}y = 1028$$

$$\Rightarrow \quad 110x + 92y = 102800 \quad ...(ii)$$

Dividing (*ii*) by 2, we have

$$55x + 46y = 51400 \quad ...(iii)$$

Again, multiplying (*iii*) by 5, we get

$$275x + 230y = 257000 \quad ...(iv)$$

Multiplying (*i*) by 46, we get

$$276x + 230y = 257600 \quad ...(v)$$

Subtracting (*v*) from (*iv*), we get

$$275x + 230y = 257000 \quad ...(iv)$$

$$276x + 230y = 257600 \quad ...(v)$$

$$- \quad - \quad -$$

$$-x = 257000 - 257600$$

$$\Rightarrow \quad -x = -600$$

$$\Rightarrow \quad x = ₹\, 600$$

Now, $6x + 5y = 5600$ [From (*i*)]

$$\Rightarrow \quad 6 \times 600 + 5y = 5600 \quad [\because x = 600]$$

$$\Rightarrow \quad 5y = 5600 - 3600$$

$$\Rightarrow \quad y = \frac{2000}{5}$$

$$\Rightarrow \quad y = 400$$

Hence, the C.P. of a saree and L.P. of sweater are ₹ 600, ₹ 400 respectively.

Q12. Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum respectively. She received ₹ 1860 as annual interest. However, had she interchanged the amount of investments in two schemes, she would have received ₹ 20 more as annual interest. How much money did she invest in each scheme?

Sol. Let the money invested in scheme A = ₹ x

and the money invested in scheme B = ₹ y

Case I: Susan invested ₹ x at 8% p.a. + Susan invested ₹ y at 9% p.a. = 1860

$$\Rightarrow \quad \frac{x \times 8 \times 1}{100} + \frac{y \times 9 \times 1}{100} = 1860$$

$$\Rightarrow \quad 8x + 9y = 186000 \quad \ldots(i)$$

Case II: Interchanging the amount in schemes A and B, we have

$$\frac{9 \times x}{100} + \frac{8 \times y}{100} = (1860 + 20)$$

$$\Rightarrow \quad 9x + 8y = 188000 \quad \ldots(ii)$$

Adding (*i*) and (*ii*), we get

$$\begin{array}{r} 9x + 8y = 188000 \quad \ldots(ii) \\ 8x + 9y = 186000 \quad \ldots(i) \\ \hline 17x + 17y = 374000 \end{array}$$

$$\Rightarrow \quad x + y = 22000 \quad \ldots(iii)$$

On subtracting (*i*) and (*ii*), we get $x - y = 2000$ ...(*iv*)

Now,

$$\begin{array}{r} x - y = 2000 \quad \ldots(iv) \\ x + y = 22000 \quad \ldots(iii) \\ \hline 2x \quad = 24000 \end{array} \quad \text{[Adding (iv) and (iii)]}$$

$$\Rightarrow \quad x = ₹\,12000$$

Now, $x + y = 22000$ [From (*iii*)]

$$\Rightarrow \quad y = 22000 - 12000$$

$$\Rightarrow \quad y = ₹\,10,000$$

Hence, the amount invested in schemes A and B are ₹ 12000 and ₹ 10,000 respectively.

Q13. Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of ₹ 2 for 3 bananas and the second lot at the rate of ₹ 1 per banana and got total of ₹ 400. If he had sold the first lot at the rate of ₹ 1 per banana and the second lot at the rate of ₹ 4 for 5 bananas, his total collection would have been ₹ 460. Find the total number of bananas he had.

Sol. Let the number of bananas in lot A = x

and the number of bananas in lot B = y

Case I: S.P. of 3 bananas of lot A = ₹ 2

$$\Rightarrow \quad \text{S.P. of 1 banana of lot A} = ₹\frac{2}{3}$$

$$\Rightarrow \quad \text{S.P. of } x \text{ bananas of lot A} = \frac{2}{3}x$$

Now, S.P. of 1 banana of lot B = ₹ 1

$$\Rightarrow \quad \text{S.P. of } y \text{ bananas of lot B} = ₹\, y$$

$$\therefore \quad \frac{2x}{3} + y = 400$$

$$\Rightarrow \quad 2x + 3y = 1200 \quad ...(i)$$

Case II: $x + \frac{4}{5}y = 460$

$$\Rightarrow \quad 5x + 4y = 2300 \quad ...(ii)$$

Multiplying (*i*) by 4, we get

$$8x + 12y = 4800 \quad ...(iii)$$

Also, multiplying (*ii*) by 3, we get

$$15x + 12y = 6900 \quad ...(iv)$$

Now,

$$15x + 12y = 6900 \quad ...(iv)$$

$$8x + 12y = 4800 \quad ...(iii)$$

$$\underline{-\quad -\quad\quad -}$$

$$7x = 2100$$ [On subtracting (*iii*) from (*iv*)]

$$\Rightarrow \quad x = \frac{2100}{7}$$

$$\Rightarrow \quad x = 300$$

Now, $2x + 3y = 1200$ [From (*i*)]

$$\Rightarrow \quad 2(300) + 3y = 1200$$

$$\Rightarrow \quad 3y = 1200 - 600$$

$$\Rightarrow \quad y = \frac{600}{3}$$

$$\Rightarrow \quad y = 200$$

Hence, the total number of bananas = $(x + y) = (300 + 200) = 500$.

❑❑❑

4 Quadratic Equations

EXERCISE 4.1

Choose the correct answer from the given four options in the following questions:

Q1. Which of the following is a quadratic equation?

(a) $x^2 + 2x + 1 = (4 - x)^2 + 3$ (b) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$

(c) $(k + 1)x^2 + \frac{3}{2}x = 7$ (where $k = -1$)

(d) $x^3 - x^2 = (x - 1)^3$

Sol. (d): **Main concept used:** An equation of the form $ax^2 + bx + c = 0$ where a, b, c are real numbers and $a \neq 0$, is called a quadratic equation.

(a) $x^2 + 2x + 1 = (4 - x)^2 + 3$

$\Rightarrow x^2 + 2x + 1 = (4)^2 + (x)^2 - 2(4)(x) + 3$

$\Rightarrow 2x + 1 = 16 - 8x + 3$

$\therefore$ Coefficient of x^2 is zero or $a = 0$. So, it is not a quadratic equation.

(b) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$

$\Rightarrow -2x^2 = 10x - 2 - 2x^2 + \frac{2}{5}x$

$\Rightarrow -2x^2 + 2x^2 = 10x - 2 + \frac{2}{5}x$

$\Rightarrow 0 = 10x - 2 + \frac{2}{5}x$

As the coefficient of x^2 in the above equation is zero or $a = 0$. So, it is not a quadratic equation.

(c) $(k + 1)x^2 + \frac{3}{2}x = 7$, where $k = -1$

$\Rightarrow (-1 + 1)x^2 + \frac{3}{2}x = 7$

So, the coefficient of x^2 is zero or $a = 0$. Hence, the equation is not quadratic.

(d) $x^3 - x^2 = (x - 1)^3$

$\Rightarrow x^3 - x^2 = (x)^3 - (1)^3 - 3(x)^2(1) + 3(x)(1)^2$

$\Rightarrow x^3 - x^2 = x^3 - 1 - 3x^2 + 3x$

$\Rightarrow \quad -x^2 = -1 - 3x^2 + 3x$

$\Rightarrow \quad 2x^2 - 3x + 1 = 0$

As the coefficient of x^2 in the above equation is 3 or $a = 3$, so it is a quadratic equation.

Q2. Which of the following is not a quadratic equation?

(a) $2(x-1)^2 = 4x^2 - 2x + 1$ (b) $2x - x^2 = x^2 + 5$

(c) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ (d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

Sol. (c): **Main concept used:** An equation will not be a quadratic in which $a = 0$ in equation of the form $ax^2 + bx + c = 0$

(a) Given equation is $2(x-1)^2 = 4x^2 - 2x + 1$

$\Rightarrow \quad 2[(x)^2 + (1)^2 - 2(x)(1)] - 4x^2 + 2x - 1 = 0$

$\Rightarrow \quad 2x^2 + 2 - 4x - 4x^2 + 2x - 1 = 0$

$\Rightarrow \quad -2x^2 - 2x + 1 = 0$

$\therefore$ $a = -2$ so given equation is quadratic as, it is of the form $ax^2 + bx + c = 0$ and $a \neq 0$

(b) The given equation is $2x - x^2 = x^2 + 5$

$\Rightarrow \quad 2x - x^2 - x^2 - 5 = 0$

$\Rightarrow \quad -2x^2 + 2x - 5 = 0$

$\Rightarrow \quad 2x^2 - 2x + 5 = 0$

$\therefore$ $a = 2$ so the given so equation is quadratic, as it is of the form $ax^2 + bx + c = 0$ and $a \neq 0$.

(c) The given equation is $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$

$\Rightarrow \quad (\sqrt{2}x)^2 + (\sqrt{3})^2 + 2(\sqrt{2}x)(\sqrt{3}) + x^2 - 3x^2 + 5x = 0$

$\Rightarrow \quad 2x^2 + 3 + 2\sqrt{6}x + x^2 - 3x^2 + 5x = 0$

$\Rightarrow \quad 0 + (2\sqrt{6} + 5)x + 3 = 0$

As $a = 0$ so, the given equation is not quadratic.

(d) Given equation is $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

$\Rightarrow \quad (x^2)^2 + (2x)^2 + 2(x^2)(2x) - x^4 - 3 - 4x^3 = 0$

$\Rightarrow \quad x^4 + 4x^2 + 4x^3 - x^4 - 3 - 4x^3 = 0$

$\Rightarrow \quad 4x^2 - 3 = 0$

As $a = 2$, so the given equation is quadratic.

Q3. Which of the following equations has 2 as a root?

(a) $x^2 - 4x + 5 = 0$ (b) $x^2 + 3x - 12 = 0$

(c) $2x^2 - 7x + 6 = 0$ (d) $3x^2 - 6x - 2 = 0$

Sol. (c): **Main concept used:** Roots of equation must satisfy the given equation.

(a) Substituting $x = 2$ in the equation $x^2 - 4x + 5 = 0$, we get

$(2)^2 - 4(2) + 5 = 0$

$$\Rightarrow \quad 4-8+5=0$$
$$\Rightarrow \quad 9-8=0$$
$$\Rightarrow \quad 1=0, \text{ which is false}$$

As $x=2$ does not satisfy the given equation so 2 is not the root of the given equation.

(*b*) Substituting $x=2$ in the equation $x^2+3x-12=0$, we get

$$(2)^2+3(2)-12=0$$
$$\Rightarrow \quad 4+6-12=0$$
$$\Rightarrow \quad 10-12=0$$
$$\Rightarrow \quad -2=0, \text{ which is false}$$

As $x=2$ does not satisfy the given equation so 2 is not the root of the given equation.

(*c*) Substituting $x=2$ in the equation $2x^2-7x+6=0$, we get

$$2(2)^2-7(2)+6=0$$
$$\Rightarrow \quad 8-14+6=0$$
$$\Rightarrow \quad 14-14=0$$
$$\Rightarrow \quad 0=0, \text{ which is true}$$

As $x=2$ satisfies the given equation so 2 is the root of the given equation.

(*d*) Substituting $x=2$ in the equation $3x^2-6x-2=0$, we get

$$3(2)^2-6(2)-2=0$$
$$\Rightarrow \quad 12-12-2=0$$
$$\Rightarrow \quad -2=0, \text{ which is false}$$

As $x=2$ does not satisfy the given equation so 2 is not the root of the given equation.

Q4. If $\frac{1}{2}$ is a root of the equation $x^2+kx-\frac{5}{4}=0$, then the value of k is

(*a*) 2 (*b*) -2 (*c*) $\frac{1}{4}$ (*d*) $\frac{1}{2}$

Sol. (*a*): As $\frac{1}{2}$ is the root of the given equation so $x=\frac{1}{2}$ must satisfy the given equation.

On substituting $x=\frac{1}{2}$ in the equation $x^2+kx-\frac{5}{4}=0$, we get

$$\left(\frac{1}{2}\right)^2+k\left(\frac{1}{2}\right)-\frac{5}{4}=0$$
$$\Rightarrow \quad \frac{1}{4}+\frac{k}{2}-\frac{5}{4}=0$$
$$\Rightarrow \quad 1+2k-5=0$$

$\Rightarrow \quad +2k = +4$

$\Rightarrow \quad k = +2$

Q5. Which of the following equations has the sum of its roots as 3?

(*a*) $2x^2 - 3x + 6 = 0$ (*b*) $-x^2 + 3x - 3 = 0$

(*c*) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ (*d*) $3x^2 - 3x + 3 = 0$

Sol. (*b*): **Main concept used:** Sum of roots (α, β) of quadratic equation $ax^2 + bx + c = 0$ is $\alpha + \beta = \frac{-b}{a}$

(*a*) Given equation is $2x^2 - 3x + 6 = 0$

Here, $\alpha + \beta = \frac{3}{2} \neq 3$

So, the given equation has not the sum of roots as 3.

(*b*) Given equation is $-x^2 + 3x - 3 = 0$

Here, $\alpha + \beta = \frac{-3}{-1} = 3$

$\therefore$ The given equation has sum of its roots as 3.

(*c*) Given equation is

$$\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$$

Here, $\alpha + \beta = \frac{-\left(\frac{-3}{\sqrt{2}}\right)}{\sqrt{2}} = \frac{+3}{\sqrt{2}\sqrt{2}} = \frac{3}{2} \neq 3$

So, the given equation has not the sum of roots as 3.

(*d*) Given equation is $3x^2 - 3x + 3 = 0$

Here, $\alpha + \beta = \frac{-(-3)}{3} = 1 \neq 3$

So, the given equation has not the sum of roots as 3.

Q6. Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

(*a*) 0 only (*b*) 4 (*c*) 8 only (*d*) 0, 8

Sol.(*d*): **Main concept used:** The condition for equal roots of quadratic equation $ax^2 + bx + c = 0$ is $b^2 - 4ac = 0$.

Given equation is $2x^2 - kx + k = 0$

For equal roots, $b^2 - 4ac = 0$

$\Rightarrow \quad (-k)^2 - 4(2)(k) = 0 \qquad (a = 2, b = -k, c = +k)$

$\Rightarrow \quad k^2 - 8k = 0$

$\Rightarrow \quad k(k-8) = 0$

$\Rightarrow \quad k = 0$ or $k - 8 = 0$

$\Rightarrow \quad k = 0$ or $k = 8$

So, the values of k are 0 and 8. Hence, the answer is (*d*).

Q7. Which constant must be added and subtracted to solve the quadratic equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square?

(a) $\frac{1}{8}$ (b) $\frac{1}{64}$ (c) $\frac{1}{4}$ (d) $\frac{9}{64}$

Sol. (b): The given equation is $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$

So, to make the expression a complete square, we have to subtract $\left(\frac{1}{8}\right)^2$ or $\frac{1}{64}$.

$$\Rightarrow \quad (3x)^2 + \left(\frac{1}{8}\right)^2 + 2(3x)\left(\frac{1}{8}\right) - \sqrt{2} - \frac{1}{64} = 0$$

$$\Rightarrow \quad \left(3x + \frac{1}{8}\right)^2 = \sqrt{2} + \frac{1}{64}$$

Q8. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(a) two distinct real roots (b) two equal real roots
(c) no real roots (d) more than two real roots.

Sol. (c): **Main concept used:** After calculating $D = b^2 - 4ac$, check the following conditions:

(i) for no real roots $D < 0$ (ii) for two equal and real roots $D = 0$ (iii) for two distinct roots $D > 0$ and any quadratic equation must have only two roots.

Given equation is $2x^2 - \sqrt{5}x + 1 = 0$

$$D = b^2 - 4ac$$
$$= (-\sqrt{5})^2 - 4(2)(1) \quad (a = 2, b = -\sqrt{5}, c = 1)$$
$$= 5 - 8$$
$$\Rightarrow \quad D = -3$$

As $D < 0$ so, the given equation has no real roots.

Q9. Which of the following equations has two distinct real roots?

(a) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ (b) $x^2 + x - 5 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$ (d) $5x^2 - 3x + 1 = 0$

Sol. (b): **Main concept used:** For real distinct roots $D > 0$

(a) Given equation is $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

$$D = b^2 - 4ac$$
$$= (-3\sqrt{2})^2 - 4(2)\left(\frac{9}{4}\right) \quad \left(a = 2, b = -3\sqrt{2}, c = \frac{9}{4}\right)$$
$$= 9 \times 2 - 18$$

$\Rightarrow \quad D = 0$

As $D = 0$, so the given equation has two real equal roots.

(*b*) $x^2 + x - 5 = 0$

$D = b^2 - 4ac$

$= (1)^2 - 4(1)(-5) \qquad (a = 1, b = 1, c = -5)$

$\Rightarrow \quad D = 1 + 20$

$\Rightarrow \quad D = 21$

As $D > 0$, so the given equation has two distinct real roots.

(*c*) $x^2 + 3x + 2\sqrt{2} = 0$

$D = b^2 - 4ac$

$= (3)^2 - 4(1)(2\sqrt{2}) \qquad (a = 1, b = 3, c = 2\sqrt{2})$

$\Rightarrow \quad D = 9 - 8\sqrt{2} = 9 - 8 \times 1.414 = 9 - 11.312$

$\Rightarrow \quad D = -2.312$

As $D < 0$, so the given equation has no real roots.

(*d*) $5x^2 - 3x + 1 = 0$

$D = b^2 - 4ac$

$= (-3)^2 - 4(5)(1) \qquad (a = 5, b = -3, c = 1)$

$= 9 - 20$

$\Rightarrow \quad D = -11$

As $D < 0$, so the given equation has no real roots.

Q10. Which of the following equations has no real roots?

(*a*) $x^2 - 4x + 3\sqrt{2} = 0$ (*b*) $x^2 + 4x - 3\sqrt{2} = 0$

(*c*) $x^2 - 4x - 3\sqrt{2} = 0$ (*d*) $3x^2 + 4\sqrt{3}x + 4 = 0$

Sol. (*a*):

(*a*) Given equation is $x^2 - 4x + 3\sqrt{2} = 0$

$D = b^2 - 4ac$

$= (-4)^2 - 4(1)(3\sqrt{2}) \qquad (a = 1, b = -4, c = 3\sqrt{2})$

$= 16 - 12\sqrt{2} = 16 - 12 \times 1.414$

$= 16 - 16.968$

$\Rightarrow \quad D = -0.968$

As $D < 0$, so the given equation has no real roots.

(*b*) $x^2 + 4x - 3\sqrt{2} = 0$

$D = b^2 - 4ac$

$= (4)^2 - 4(1)(-3\sqrt{2}) \qquad (a = 1, b = 4, c = -3\sqrt{2})$

$\Rightarrow \quad D = 16 + 12\sqrt{2}$

$\therefore \quad D > 0$

Hence, the given equation has two distinct real roots.

(*c*) $x^2 - 4x - 3\sqrt{2} = 0$

$D = b^2 - 4ac$

$\Rightarrow \quad D = (-4)^2 - 4(1)(-3\sqrt{2}) \quad (a = 1, b = -4, c = -3\sqrt{2})$

$\Rightarrow \quad D = 16 + 12\sqrt{2}$

$\therefore \quad D > 0$

So, the given equation has two real distinct roots.

(*d*) $3x^2 + 4\sqrt{3}x + 4$

$D = b^2 - 4ac$

$= (4\sqrt{3})^2 - 4(3)(4) \quad (a = 3, b = 4\sqrt{3}, c = 4)$

$= 16 \times 3 - 48 = 48 - 48$

$\Rightarrow \quad D = 0$

So, the given equation has two real and equal roots.

Q11. $(x^2 + 1)^2 - x^2 = 0$ has

(*a*) four real roots (*b*) two real roots

(*c*) no real roots (*d*) one real root

Sol. (*c*): Given equation is $(x^2 + 1)^2 - x^2 = 0$

$\Rightarrow \quad (x^2)^2 + (1)^2 + 2(x^2)(1) - x^2 = 0$

$\Rightarrow \quad (x^2)^2 + 1x^2 + 1 = 0$

Let $x^2 = y$

$\Rightarrow \quad y^2 + 1y + 1 = 0$

Now, $D = b^2 - 4ac$

$= (1)^2 - 4(1)(1) = 1 - 4 \quad (a = 1, b = 1, c = 1)$

$\Rightarrow \quad D = -3$

$\Rightarrow \quad D < 0$

So, the given equation $y^2 + y + 1 = 0$ has no values of y in equation $y^2 + 1y + 1 = 0$ or if y is not real then x^2 will not be real so no values of x are real or the given equation has no real roots.

EXERCISE 4.2

Q1. State whether the following quadratic equations have two distinct real roots. Justify your answer.

(*i*) $x^2 - 3x + 4 = 0$ (*ii*) $2x^2 + x - 1 = 0$

(*iii*) $2x^2 - 6x + \frac{9}{2} = 0$ (*iv*) $3x^2 - 4x + 1 = 0$

(*v*) $(x + 4)^2 - 8x = 0$ (*vi*) $(x - \sqrt{2})^2 - 2(x + 1) = 0$

(*vii*) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$ (*viii*) $x(1 - x) - 2 = 0$

(*ix*) $(x - 1)(x + 2) + 2 = 0$ (*x*) $(x + 1)(x - 2) + x = 0$

Sol. Main concept used: Quadratic equation $ax^2 + bx + c = 0$ will have two distinct real roots if $D > 0$ or $b^2 - 4ac > 0$.

(*i*) Given quadratic equation is $x^2 - 3x + 4 = 0$

Now, $D = b^2 - 4ac$

$= (-3)^2 - 4(1)(4) \quad (a = 1, b = -3, c = 4)$

$\Rightarrow \quad D = 9 - 16$

$\Rightarrow \quad D = -7 < 0$

$\therefore \quad D < 0$

So, the given equation has no real roots.

(ii) $2x^2 + x - 1 = 0$

Now, $D = b^2 - 4ac$

$= (1)^2 - 4(2)(-1) \qquad (a = 2, b = 1, c = -1)$

$= 1 + 8$

$\Rightarrow \quad D = 9 > 0$

$\therefore \quad D > 0$

So, the given equation has two distinct real roots.

(iii) $2x^2 - 6x + \frac{9}{2} = 0$

Now, $D = b^2 - 4ac$

$\Rightarrow \quad D = (-6)^2 - 4(2)\left(\frac{9}{2}\right) \qquad \left(a = 2, b = -6, c = \frac{9}{2}\right)$

$\Rightarrow \quad D = 36 - 36$

$\Rightarrow \quad D = 0$

So, the given equation has two real and equal roots.

(iv) $3x^2 - 4x + 1 = 0$

Now, $D = b^2 - 4ac \qquad (a = 3, b = -4, c = 1)$

$= (-4)^2 - 4(3)(1) = 16 - 12$

$\Rightarrow \quad D = 4 > 0$

$\therefore \quad D > 0$

So, the given equation has two distinct real roots.

(v) $(x + 4)^2 - 8x = 0$

$\Rightarrow (x)^2 + (4)^2 + 2(x)(4) - 8x = 0$

$\Rightarrow \quad x^2 + 16 + 8x - 8x = 0$

$\Rightarrow \quad x^2 + 16 = 0$

$\Rightarrow \quad x^2 + 0x + 16 = 0$

Now, $D = b^2 - 4ac$

$= (0)^2 - 4(1)(16) \qquad (a = 1, b = 0, c = 16)$

$\Rightarrow \quad D = -64 < 0$

As $D < 0$, so the given equation has no real roots.

(vi) $(x - \sqrt{2})^2 - 2(x + 1) = 0$

$\Rightarrow \quad x^2 - 2\sqrt{2}x + 2 - 2x - 2 = 0$

$\Rightarrow \quad x^2 - (2\sqrt{2} + 2)x = 0$

Now, $D = b^2 - 4ac$

$= [-(2\sqrt{2} + 2)]^2 - 4 \times 1 \times 0$

$[\because a = 1, b = -(2\sqrt{2} + 2), c = 0]$

$$= (2\sqrt{2}+2)^2 > 0$$

As D > 0, so the given equation has real and unequal roots.

(*vii*) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$

$$D = b^2 - 4ac$$

$$\Rightarrow \quad D = \left(\frac{-3}{\sqrt{2}}\right)^2 - 4(\sqrt{2})\left(\frac{1}{\sqrt{2}}\right) \quad \left(a = \sqrt{2}, b = \frac{-3}{\sqrt{2}}, c = +\frac{1}{\sqrt{2}}\right)$$

$$= \frac{9}{2} - \frac{4}{1} = \frac{9-8}{2}$$

$$\Rightarrow \quad D = \frac{1}{2} > 0$$

$$\therefore \quad D > 0$$

Hence, the given quadratic equation has two real and distinct roots.

(*viii*) $x(1-x) - 2 = 0$

$$\Rightarrow \quad x - x^2 - 2 = 0$$

$$\Rightarrow \quad -x^2 + x - 2 = 0$$

Now, $D = b^2 - 4ac$

$$= (-1^2) - 4(-1)(-2)(a = -1, b = 1, c = -2)$$

$$= 1 - 8$$

$$\Rightarrow \quad D = -7 < 0$$

So, the given equation has no real roots.

(*ix*) $(x-1)(x+2) + 2 = 0$

$$\Rightarrow \quad x^2 + 2x - x - 2 + 2 = 0$$

$$\Rightarrow \quad x^2 + x = 0$$

$$\Rightarrow \quad x^2 + x + 0 = 0$$

Now, $D = b^2 - 4ac \quad (a = 1, b = 1, c = 0)$

$$= (1)^2 - 4(1)(0) = 1$$

$$\Rightarrow \quad D = 1 > 0$$

So, the given equation has two distinct real roots.

(*x*) $(x+1)(x-2) + x = 0$

$$\Rightarrow \quad x^2 - 2x + x - 2 + x = 0$$

$$\Rightarrow \quad x^2 - 2 = 0$$

$$\Rightarrow \quad x^2 + 0x - 2 = 0$$

Now, $D = b^2 - 4ac$

$$= (0)^2 - 4(1)(-2) \quad (a = 1, b = 0, c = -2)$$

$$\Rightarrow \quad D = 8 > 0$$

So, the given equation has two distinct real roots.

Q2. Write whether the following statements are true or false. Justify your answer.

(*i*) Every quadratic equation has exactly one root.

(*ii*) Every quadratic equation has atleast one real root.
(*iii*) Every quadratic equation has at least two roots.
(*iv*) Every quadratic equation has atmost two roots.
(*v*) If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.
(*vi*) If the coefficient of x^2 and the constant term have the same sign and if the coefficient of x term is zero, then the quadratic equation has no real roots.

Sol. (*i*) **False:** Consider a quadratic equation $x^2 - 4 = 0$ which has two distinct roots -2 and 2. So, the given statement is false.

(*ii*) **False:** Consider a quadratic equation $x^2 + 1 = 0$ which has no real root. So, the given statement is false.

(*iii*) **False:** Consider the quadratic equation $x^2 - 4x + 4 = 0$ which has only 2 as root. So, the given statement is false.

(*iv*) **True:** Consider the quadratic equation $x^2 - 5x + 6 = 0$. Put 2 and 3 in x and the quadratic expression $x^2 - 5x + 6$ becomes equal to 0. So, 2 and 3 are the roots of the quadratic equation $x^2 - 5x + 6 = 0$. So, any quadratic equation can have atmost two roots i.e., one or two roots, but not more than two.

(*v*) **True:** In quadratic equation $ax^2 + bx + c = 0$, if a and c have opposite signs, then $ac < 0$.
Therefore, $b^2 - 4ac > 0$. So, the quadratic equation has real roots.
Hence, the given statement is true.

(*vi*) **True:** In quadratic equation $ax^2 + bx + c = 0$, if a and c have same sign and $b = 0$, then $b^2 - 4ac = (0)^2 - 4ac = -4ac < 0$.
So, the quadratic equation has no real roots.
Hence, the given statement is true.

Q3. A quadratic equation with integral coefficient has integral roots. Justify your answer.

Sol. No, a quadratic equation with integral coefficients (0, ± 1, ± 2, ± 3...) can have its roots in fraction, i.e., non integral.
For example, $5x^2 + 3x - 8 = 0$ has integral coefficients (coefficients 5, 3, -8 are integers).

Now, $5x^2 + 3x - 8 = 0$

$\Rightarrow$ $5x^2 + 8x - 5x - 8 = 0$

$\Rightarrow$ $x(5x + 8) - 1(5x + 8) = 0$

$\Rightarrow$ $(5x + 8)(x - 1) = 0$

$\Rightarrow$ $5x + 8 = 0$ or $(x - 1) = 0$

Therefore, the roots are given by $x = \frac{-8}{5}$ and $x = 1$

So, the given statement is false.

Q4. Does there exist a quadratic equation, whose coefficients are rational but both of its roots are irrational? Justify your answer.

Sol. Yes, a quadratic equation having coefficients as rational number, has irrational roots.

For example, $2x^2 - 3x - 15 = 0$ has rational coefficients.

$$D = b^2 - 4ac \qquad (a = 2, b = -3, c = -15)$$
$$= (-3)^2 - 4(2)(-15) = 9 + 120$$
$$\Rightarrow \qquad D = 129$$

$\therefore$ Roots are given by
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\Rightarrow \qquad x = \frac{-(-3) \pm \sqrt{129}}{2 \times 2}$$
$$\Rightarrow \qquad x = \frac{3 \pm \sqrt{129}}{4}$$

The roots are irrational as $\sqrt{129}$ is irrational.

Q5. Does there exist a quadratic equation whose coefficients are all distinct irrationals but both the roots are rationals? Why?

Sol. Yes, there may be a quadratic equation whose coefficients are all distinct irrationals, but both the roots are rational.

For example, consider a quadratic equation having distinct irrational coefficients

$$3\sqrt{\frac{3}{2}}x^2 + \frac{5}{\sqrt{6}}x - 2\sqrt{\frac{2}{3}} = 0$$

Now,
$$D = b^2 - 4ac$$
$$= \left[\frac{5}{\sqrt{6}}\right]^2 - 4\left[\frac{3\sqrt{3}}{\sqrt{2}}\right]\left[\frac{-2\sqrt{2}}{\sqrt{3}}\right]$$
$$\left(a = \frac{3\sqrt{3}}{\sqrt{2}}, b = \frac{5}{\sqrt{6}}, c = \frac{-2\sqrt{2}}{\sqrt{3}}\right)$$
$$= \frac{25}{6} + \frac{24}{1} = \frac{25 + 144}{6}$$
$$\Rightarrow \qquad D = \frac{169}{6} \Rightarrow \sqrt{D} = \frac{13}{\sqrt{6}}$$

Roots are given by
$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-\frac{5}{\sqrt{6}} \pm \frac{13}{\sqrt{6}}}{2 \cdot \frac{3\sqrt{3}}{\sqrt{2}}}$$

$$= \frac{\frac{1}{\sqrt{6}}[-5 \pm 13]\sqrt{2}}{6\sqrt{3}} = \frac{(-5 \pm 13)\sqrt{2}}{\sqrt{2}\,\sqrt{3} \times 6\sqrt{3}}$$

$\Rightarrow \quad x = \dfrac{(-5 \pm 13)}{18}$

$\Rightarrow \quad x = \dfrac{8}{18}$ or $\dfrac{-18}{18}$

$\Rightarrow \quad x = \dfrac{4}{9}$ or -1

Hence, the roots are rational while coefficients a, b, c were irrational.

Q6. Is 0.2 a root of equation $x^2 - 0.4 = 0$? Justify your answer.

Sol. If 0.2 is a root of equation $x^2 - 0.4 = 0$, then 0.2 must satisfy the given equation.

$x^2 - 0.4 = 0$ [Given]

$\Rightarrow \quad (0.2)^2 - 0.4 = 0$

$\Rightarrow \quad 0.04 - 0.4 = 0$

$\Rightarrow \quad -0.36 \neq 0$

So, 0.2 is not a root of the given equation.

Q7. If $b = 0$, $c < 0$, is it true that the roots of $x^2 + bx + c = 0$ are numerically equal and opposite in sign? Justify your answer.

Sol. Given equation is $x^2 + bx + c = 0$

$b = 0$ [Given]

$\therefore \quad x^2 + c = 0$

$\Rightarrow \quad x^2 = -c$

$\Rightarrow \quad x = \sqrt{-c}$

As c is negative so $-c$ becomes positive or $\sqrt{-c}$ is real.

So, the roots of the given equation are

$$x = \pm\sqrt{-c}$$

or $\quad x = +\sqrt{-c}$ and $-\sqrt{-c}$ [$\because$ $(-c)$ is positive]

Hence, the roots of the given equation are real, equal and opposite in sign.

EXERCISE 4.3

Q1. Find the roots of the quadratic equations by using the quadratic formula in each of the following:

(*i*) $2x^2 - 3x - 5 = 0$ (*ii*) $5x^2 + 13x + 8 = 0$

(*iii*) $-3x^2 + 5x + 12 = 0$ (*iv*) $-x^2 + 7x - 10 = 0$

(v) $x^2 + 2\sqrt{2}x - 6 = 0$ (vi) $x^2 - 3\sqrt{5x} + 10 = 0$

(vii) $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

Sol. Main concept used: Roots of quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b \pm \sqrt{D}}{2a}$$

(i) Given equation is $2x^2 - 3x - 5 = 0$

$$D = b^2 - 4ac$$

$$\Rightarrow \quad D = (-3)^2 - 4(2)(-5) \qquad (a = 2, b = -3, c = -5)$$

$$= 9 + 40$$

$$\Rightarrow \quad \sqrt{D} = \sqrt{49}$$

$$\Rightarrow \quad \sqrt{D} = 7$$

Now, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-3) \pm 7}{2 \times 2} \quad \Rightarrow \quad x = \frac{3 \pm 7}{4}$$

$$\Rightarrow \quad x_1 = \frac{3 + 7}{4} \quad \text{and} \quad x_2 = \frac{3 - 7}{4}$$

$$\Rightarrow \quad x_1 = \frac{10}{4} = \frac{5}{2} \quad \text{and} \quad x_2 = \frac{-4}{4} = -1$$

$\therefore$ Roots of the given equation are $\frac{5}{2}$ and -1.

(ii) $5x^2 + 13x + 8 = 0$

$$D = b^2 - 4ac$$

$$= (13)^2 - 4(5)(8) \qquad (a = 5, b = 13, c = 8)$$

$$= 169 - 160$$

$$\Rightarrow \quad \sqrt{D} = \sqrt{9} = 3$$

Now, $$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(13) \pm 3}{2 \times 5}$$

$$\Rightarrow \quad x_1 = \frac{-13 + 3}{10} \quad \text{and} \quad x_2 = \frac{-13 - 3}{10}$$

$$\Rightarrow \quad x_1 = \frac{-10}{10} \quad \text{and} \quad x_2 = \frac{-16}{10}$$

$$\Rightarrow \quad x_1 = -1 \quad \text{and} \quad x_2 = \frac{-8}{5}$$

So, the roots of the given equation are -1 and $\frac{-8}{5}$.

(*iii*) $-3x^2 + 5x + 12 = 0$

$$D = b^2 - 4ac$$
$$= (5)^2 - 4(-3)(12) \qquad (a = -3, b = 5, c = 12)$$
$$= 25 + 144$$

$\Rightarrow \quad D = 169$

$\Rightarrow \quad \sqrt{D} = 13$

Now, $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-5 \pm 13}{2 \times (-3)} = \dfrac{-5 \pm 13}{-6}$

$\Rightarrow \quad x_1 = \dfrac{-5 + 13}{-6}$ and $x_2 = \dfrac{-5 - 13}{-6}$

$\Rightarrow \quad x_1 = \dfrac{8}{-6}$ and $x_2 = \dfrac{-18}{-6}$

$\Rightarrow \quad x_1 = -\dfrac{4}{3}$ and $x_2 = +3$

Hence, the roots of the given equation are $-\dfrac{4}{3}$ and 3.

(*iv*) $-x^2 + 7x - 10 = 0$

$$D = b^2 - 4ac$$
$$= (7)^2 - 4(-1)(-10) \quad (a = -1, b = 7, c = -10)$$
$$= 49 - 40 = 9$$

$\Rightarrow \quad \sqrt{D} = 3$

Now, $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-7 \pm 3}{2 \times (-1)}$

$\Rightarrow \quad x_1 = \dfrac{-7 + 3}{-2}$ and $x_2 = \dfrac{-7 - 3}{-2}$

$\Rightarrow \quad x_1 = \dfrac{-4}{-2}$ and $x_2 = \dfrac{-10}{-2}$

$\Rightarrow \quad x_1 = 2$ and $x_2 = 5$

Hence, the roots of the given quadratic equation are 2 and 5.

(*v*) $x^2 + 2\sqrt{2}x - 6 = 0$

$$D = b^2 - 4ac$$
$$= (2\sqrt{2})^2 - 4(1)(-6) \qquad (a = 1, b = 2\sqrt{2}, c = -6)$$
$$= 4 \times 2 + 24 = 8 + 24 = 32$$

$\Rightarrow \quad \sqrt{D} = \sqrt{32} \quad \Rightarrow \quad \sqrt{D} = 4\sqrt{2}$

Now, $x = \dfrac{- \pm \sqrt{D}}{2} \quad \dfrac{-2\sqrt{2} \pm 4\sqrt{2}}{2 \times 1}$

$$\Rightarrow \quad x_1 = \frac{-2\sqrt{2}+4\sqrt{2}}{2} \quad \text{and} \quad x_2 = \frac{-2\sqrt{2}-4\sqrt{2}}{2\times 1}$$

$$\Rightarrow \quad x_1 = \frac{2\sqrt{2}}{2} \quad \text{and} \quad x_2 = \frac{-6\sqrt{2}}{2}$$

$$\Rightarrow \quad x_1 = \sqrt{2} \quad \text{and} \quad x_2 = -3\sqrt{2}$$

Hence, the roots of the given equation are $\sqrt{2}$ and $-3\sqrt{2}$.

(*vi*) $x^2 - 3\sqrt{5}x + 10 = 0$

$$D = b^2 - 4ac$$

$$\Rightarrow \quad D = (-3\sqrt{5})^2 - 4(1)(10) \qquad (a = 1, b = -3\sqrt{5}, c = 10)$$

$$= 9 \times 5 - 40$$

$$\Rightarrow \quad D = 45 - 40 = 5$$

$$\Rightarrow \quad \sqrt{D} = \sqrt{5}$$

Now, $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-(-3\sqrt{5}) \pm \sqrt{5}}{2\times 1} = \dfrac{3\sqrt{5} \pm \sqrt{5}}{2}$

$$\Rightarrow \quad x_1 = \frac{3\sqrt{5}+\sqrt{5}}{2} \quad \text{and} \quad x_2 = \frac{3\sqrt{5}-\sqrt{5}}{2}$$

$$\Rightarrow \quad x_1 = \frac{4\sqrt{5}}{2} \quad \text{and} \quad x_2 = \frac{2\sqrt{5}}{2}$$

$$\Rightarrow \quad x_1 = 2\sqrt{5} \quad \text{and} \quad x_2 = \sqrt{5}$$

Hence, the roots of the given equation are $2\sqrt{5}$ and $\sqrt{5}$.

(*vii*) $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

$$D = b^2 - 4ac$$

$$= (-\sqrt{11})^2 - 4\left(\frac{1}{2}\right)(1) \qquad \left(a = \frac{1}{2}, b = -\sqrt{11}, c = 1\right)$$

$$= +11 - 2 = 9$$

$$\Rightarrow \quad \sqrt{D} = \sqrt{9} \quad \Rightarrow \quad \sqrt{D} = 3$$

Now, $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{+\sqrt{11} \pm 3}{2 \times \frac{1}{2}} = \sqrt{11} \pm 3$

$$\Rightarrow \quad x_1 = \sqrt{11} + 3 \quad \text{and} \quad x_2 = \sqrt{11} - 3$$

Hence, the roots of the given equation are $\sqrt{11}+3$ and $\sqrt{11}-3$.

Q2. Find the roots of the following quadratic equations by the factorization method.

(*i*) $2x^2 + \frac{5}{3}x - 2 = 0$ (*ii*) $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

(iii) $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$ (iv) $3x^2 + 5\sqrt{5}x - 10 = 0$

(v) $21x^2 - 2x + \frac{1}{21} = 0$

Sol. (i) $2x^2 + \frac{5}{3}x - 2 = 0$

$\Rightarrow 6x^2 + 5x - 6 = 0$

$\Rightarrow 6x^2 + 9x - 4x - 6 = 0$

$\Rightarrow 3x(2x+3) - 2(2x+3) = 0$

$\Rightarrow (2x+3)(3x-2) = 0$

$\Rightarrow 2x + 3 = 0$ or $3x - 2 = 0$

$\Rightarrow 2x = -3$ or $3x = 2$

$\Rightarrow x = \frac{-3}{2}$ or $x = \frac{2}{3}$

So, the roots of the given quadratic equation are $\frac{-3}{2}$ and $\frac{2}{3}$.

(ii) $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

$\Rightarrow 2x^2 - 5x - 3 = 0$

$\Rightarrow 2x^2 - 6x + 1x - 3 = 0$

$\Rightarrow 2x(x-3) + 1(x-3) = 0$

$\Rightarrow (x-3)(2x+1) = 0$

$\Rightarrow x - 3 = 0$ or $2x + 1 = 0$

$\Rightarrow x = 3$ or $2x = -1$ or $x = \frac{-1}{2}$

So, the roots of the quadratic equation are 3 and $\frac{-1}{2}$.

(iii) $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$

$\Rightarrow 3\sqrt{2}x^2 - 6x + 1x - \sqrt{2} = 0$

$\Rightarrow 3\sqrt{2}x(x - \sqrt{2}) + 1(x - \sqrt{2}) = 0$

$\Rightarrow (x - \sqrt{2})(3\sqrt{2}x + 1) = 0$

$\Rightarrow x - \sqrt{2} = 0$ or $3\sqrt{2}x + 1 = 0$

$\Rightarrow x = \sqrt{2}$ or $3\sqrt{2}x = -1$

$\Rightarrow x = \sqrt{2}$ or $x = \frac{-1}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\Rightarrow x = \sqrt{2}$ or $x = \frac{-\sqrt{2}}{6}$

Hence, the roots of the given equation are $\sqrt{2}$ and $\frac{-\sqrt{2}}{6}$.

(*iv*) $3x^2 + 5\sqrt{5}x - 10 = 0$

$\Rightarrow \quad 3x^2 + 6\sqrt{5}x - \sqrt{5}x - 10 = 0$

$\Rightarrow 3x(x + 2\sqrt{5}) - \sqrt{5}(x + 2\sqrt{5}) = 0$

$\Rightarrow \quad (3x - \sqrt{5})(x + 2\sqrt{5}) = 0$

$\Rightarrow \quad 3x - \sqrt{5} = 0$ or $x + 2\sqrt{5} = 0$

$\Rightarrow \quad 3x = \sqrt{5}$ or $x = -2\sqrt{5}$

$\Rightarrow \quad x = \dfrac{\sqrt{5}}{3}$ or $x = -2\sqrt{5}$

Hence, the roots of the given quadratic equation are $\dfrac{\sqrt{5}}{3}$ and $-2\sqrt{5}$.

(*v*) $21x^2 - 2x + \dfrac{1}{21} = 0$

$\Rightarrow \quad 441x^2 - 42x + 1 = 0$

$\Rightarrow \quad 441x^2 - 21x - 21x + 1 = 0$

$\Rightarrow 21x(21x - 1) - 1(21x - 1) = 0$

$\Rightarrow \quad (21x - 1)(21x - 1) = 0$

$\Rightarrow \quad (21x - 1) = 0$ or $(21x - 1) = 0$

$\Rightarrow \quad 21x = 1$ or $21x = 1$

$\Rightarrow \quad x = \dfrac{1}{21}$ or $x = \dfrac{1}{21}$

So, the roots of the given equation are $\dfrac{1}{21}$ and $\dfrac{1}{21}$.

OR

The given equation is $21x^2 - 2x + \dfrac{1}{21} = 0$

$\Rightarrow 441x^2 - 42x + 1 = 0$

[Multiplying by 21 (LCM of equation) to both sides]

As 441 and 1 are perfect squares so

$(21x)^2 + (1)^2 - 2(21x)(1) = 0$

$\Rightarrow \quad (21x - 1)^2 = 0$

$\Rightarrow \quad (21x - 1)(21x - 1) = 0$

$\Rightarrow \quad 21x - 1 = 0$ or $21x - 1 = 0$

$\Rightarrow \quad 21x = 1$ or $21x = 1$

$\Rightarrow \quad x = \dfrac{1}{21}$ or $x = \dfrac{1}{21}$

Hence, the roots of the given equation are $\dfrac{1}{21}$ and $\dfrac{1}{21}$.

EXERCISE 4.4

Q1. Find whether the following equations have real roots. If real roots exist, find them.

(i) $8x^2+2x-3=0$ (ii) $-2x^2+3x+2=0$

(iii) $5x^2-2x-10=0$ (iv) $\frac{1}{(2x-3)}+\frac{1}{(x-5)}=1,\ x\neq\frac{3}{2},5$

(v) $x^2+5\sqrt{5}x-70=0$

Sol. Main concept used: For real roots of quadratic equation

$ax^2+bx+c=0,\ b^2-4ac>0$

(i) The given equation is $8x^2+2x-3=0$

Discriminant (D) $= b^2-4ac$

$\Rightarrow$ $D=(2)^2-4(8)(-3)$ $(a=8, b=2, c=-3)$

$\Rightarrow$ $D=4+96 \Rightarrow D=100$

As $D>0$, so, roots are real.

Now, Discriminant $\sqrt{D}=10$

So, roots are $x=\frac{-b\pm\sqrt{D}}{2a}=\frac{-2\pm10}{2\times8}=\frac{-2\pm10}{16}$

$\Rightarrow$ $x_1=\frac{-2+10}{16}$ and $x_2=\frac{-2-10}{16}$

$\Rightarrow$ $x_1=\frac{8}{16}$ and $x_2=\frac{-12}{16}$

$\Rightarrow$ $x_1=\frac{1}{2}$ and $x_2=\frac{-3}{4}$

So, the roots of the given equation are $\frac{1}{2}$ and $\frac{-3}{4}$.

(ii) $-2x^2+3x+2=0$

Discriminant $D=b^2-4ac$

$\Rightarrow$ $D=(3)^2-4(-2)(2)$ $(a=-2, b=3, c=2)$

$\Rightarrow$ $D=9+16$

$\Rightarrow$ $D=25>0$

So, the given equation has real and distinct roots.

Now, $\sqrt{D}=5$

And, $x=\frac{-b\pm\sqrt{D}}{2a}=\frac{-3\pm5}{2(-2)}=\frac{-3\pm5}{-4}$

$\Rightarrow$ $x_1=\frac{-3+5}{-4}$ and $x_2=\frac{-3-5}{-4}$

$\Rightarrow$ $x_1=\frac{2}{-4}$ and $x_2=\frac{-8}{-4}$

$\Rightarrow \quad x_1 = \frac{-1}{2} \quad$ and $\quad x_2 = 2$

Hence, the roots of the given equation are 2 and $\frac{-1}{2}$.

(iii) $5x^2 - 2x - 10 = 0$

Discriminant $D = b^2 - 4ac$

$\Rightarrow \quad D = (-2)^2 - 4(5)(-10) \quad (a = 5, b = -2, c = -10)$

$= 4 + 200$

$\Rightarrow \quad D = 204 > 0$

So, the roots of the given equation are real and distinct.

Now, $\quad \sqrt{D} = \sqrt{204} \quad \Rightarrow \quad \sqrt{D} = 2\sqrt{51}$

And, $\quad x = \frac{-b \pm \sqrt{D}}{2a} = \frac{+2 \pm 2\sqrt{51}}{2 \times 5}$

$= \frac{2[1 \pm \sqrt{51}]}{10} = \frac{1 \pm \sqrt{51}}{5}$

$\Rightarrow \quad x_1 = \frac{1 + \sqrt{51}}{5} \quad$ and $\quad x_2 = \frac{1 - \sqrt{51}}{5}$

Hence, the roots of the given equation are $\frac{1+\sqrt{51}}{5}, \frac{1-\sqrt{51}}{5}$.

(iv) $\frac{1}{2x-3} + \frac{1}{x-5} = 1, \quad x \neq \frac{3}{2}, 5$

$\Rightarrow \quad \frac{(x-5) + (2x-3)}{(2x-3)(x-5)} = 1$

$\Rightarrow \quad 2x^2 - 10x - 3x + 15 = x - 5 + 2x - 3$

$\Rightarrow \quad 2x^2 - 13x + 15 = 3x - 8$

$\Rightarrow \quad 2x^2 - 13x + 15 - 3x + 8 = 0$

$\Rightarrow \quad 2x^2 - 16x + 23 = 0$

Now, $\quad D = b^2 - 4ac$

$= (-16)^2 - 4(2)(23) \; (a = 2, b = -16, c = 23)$

$\Rightarrow \quad D = 256 - 184 = 72 > 0$

$\Rightarrow \quad \sqrt{D} = \sqrt{72}$

$\Rightarrow \quad \sqrt{D} = 6\sqrt{2}$

Now, $\quad x = \frac{-b \pm \sqrt{D}}{2a} = \frac{+16 \pm 6\sqrt{2}}{2 \times 2} = \frac{16}{4} \pm \frac{6\sqrt{2}}{4}$

$\Rightarrow \quad x_1 = 4 + \frac{3}{2}\sqrt{2} \quad$ and $\quad x_2 = 4 - \frac{3}{2}\sqrt{2}$

Hence, the roots of the given quadratic equation are

$\left(4 + \frac{3}{2}\sqrt{2}\right)$ and $\left(4 - \frac{3}{2}\sqrt{2}\right)$

(v) $x^2 + 5\sqrt{5}x - 70 = 0$

$$D = b^2 - 4ac$$
$$= (5\sqrt{5})^2 - 4(1)(-70) \quad (a = 1, b = 5\sqrt{5}, c = -70)$$
$$= 25 \times 5 + 280 = 125 + 280$$
$$\Rightarrow \quad D = 405 > 0$$

So, the roots of the given equation are real and distinct.

For roots $\sqrt{D} = \sqrt{405} \Rightarrow \sqrt{D} = \sqrt{9 \times 9 \times 5}$

$$\Rightarrow \quad \sqrt{D} = 9\sqrt{5}$$

Now, $$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5\sqrt{5} \pm 9\sqrt{5}}{2 \times 1} = \frac{(-5 \pm 9)\sqrt{5}}{2}$$

$$\Rightarrow \quad x_1 = \frac{(-5+9)\sqrt{5}}{2} \quad \text{and} \quad x_2 = \frac{(-5-9)\sqrt{5}}{2}$$
$$= \frac{4\sqrt{5}}{2} \qquad\qquad = \frac{-14\sqrt{5}}{2}$$
$$= 2\sqrt{5} \qquad\qquad = -7\sqrt{5}$$

Hence, the roots of the given quadratic equation are $2\sqrt{5}$ and $-7\sqrt{5}$.

Q2. Find a natural number whose square diminished by 84, is equal to thrice of 8 more than the given number.

Sol. Let the required number be x.

According to the question,

$$x^2 - 84 = 3 \times (x + 8)$$
$$\Rightarrow \quad x^2 - 84 = 3x + 24$$
$$\Rightarrow \quad x^2 - 3x - 84 - 24 = 0$$
$$\Rightarrow \quad x^2 - 3x - 108 = 0$$
$$\Rightarrow \quad x^2 - 12x + 9x - 108 = 0$$
$$\Rightarrow \quad x(x - 12) + 9(x - 12) = 0$$
$$\Rightarrow \quad (x - 12)(x + 9) = 0$$
$$\Rightarrow \quad x - 12 = 0 \quad \text{or} \quad x + 9 = 0$$
$$\Rightarrow \quad x = 12 \quad \text{or} \quad x = -9$$

($x = -9$ is not a natural number so it is rejected.)

Hence, the required number is 12.

Q3. A natural number, when increased by 12 equals 160 times its reciprocal. Find the number.

Sol. Let the required number be x. (where $x \neq 0$)

According to the question,

$$x + 12 = \frac{1}{x} \times 160$$
$$\Rightarrow \quad x^2 + 12x = 160$$

$$\Rightarrow \quad x^2 + 12x - 160 = 0$$
$$\Rightarrow \quad x^2 + 20x - 8x - 160 = 0$$
$$\Rightarrow \quad x(x + 20) - 8(x + 20) = 0$$
$$\Rightarrow \quad (x + 20)(x - 8) = 0$$
$$\Rightarrow \quad x + 20 = 0 \quad \text{or} \quad x - 8 = 0$$
$$\Rightarrow \quad x = -20 \quad \text{or} \quad x = 8$$

But, $x = -20$ is not a natural number.
Hence, the required number is 8.

Q4. A train travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

Sol. Let the original speed of train = x km/hr
So, the new increased speed of train = $(x + 5)$ km/hr
Time taken by train in covering 360 km with original speed

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{360}{x} \text{ hr}$$

Time taken by train in covering 360 km with new speed = $\frac{360}{x+5}$ hr
According to the question,

$$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60} \text{ hr}$$
$$\Rightarrow \quad 360\left[\frac{1}{x} - \frac{1}{(x+5)}\right] = \frac{4}{5}$$
$$\Rightarrow \quad 360\left[\frac{x+5-x}{x(x+5)}\right] = \frac{4}{5}$$
$$\Rightarrow \quad \frac{90[5]}{x^2+5x} = \frac{1}{5}$$
$$\Rightarrow \quad x^2 + 5x = 90 \times 25$$
$$\Rightarrow \quad x^2 + 5x - 90 \times 25 = 0$$
$$\Rightarrow \quad x^2 + 50x - 45x - 90 \times 25 = 0$$
$$\Rightarrow \quad x(x + 50) - 45[x + 10 \times 5] = 0$$
$$\Rightarrow \quad (x + 50)(x - 45) = 0$$
$$\Rightarrow \quad x + 50 = 0 \quad \text{or} \quad x - 45 = 0$$
$$\Rightarrow \quad x = -50 \quad \text{or} \quad x = 45$$

$x = -50$ is rejected as it is negative.
Hence, the original speed of train is 45 km/hr.

Q5. If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?

Sol. Let Zeba's actual (real) age now = x years

$\therefore$ Zeba's age when she was 5 years younger than now = $(x-5)$ years

According to the question,

$$(x-5)^2 = 5x + 11$$

$\Rightarrow$ $(x)^2 + (5)^2 - 2(x)(5) - 5x - 11 = 0$

$\Rightarrow$ $x^2 + 25 - 10x - 5x - 11 = 0$

$\Rightarrow$ $x^2 - 15x + 14 = 0$

$\Rightarrow$ $x^2 - 14x - 1x + 14 = 0$

$\Rightarrow$ $x(x-14) - 1(x-14) = 0$

$\Rightarrow$ $(x-14)(x-1) = 0$

$\Rightarrow$ $x - 14 = 0$ or $x - 1 = 0$

$\Rightarrow$ $x = 14$ or $x = 1$ year

When 5 is subtracted from 1, we get negative age so $x = 1$ is rejected.

Hence, the age of Zeba is 14 years.

Q6. At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Nisha and Asha.

Sol. Let the present age of Asha $= x$ years

and the present age of her daughter Nisha = y years

At present, Asha's age, $x = (y^2) + 2$ (I)

Age of Nisha will be equal to age of her mother (x) after

= Age of Mother – Age of Daughter

$= x - y$

$= y^2 + 2 - y = y^2 - y + 2$

$\therefore$ Age of (Nisha) daughter after $(y^2 - y + 2)$ years

$= y^2 - y + 2 + y = (y^2 + 2)$ years

Age of Asha (mother) after $(y^2 - y + 2)$ years

$= x + y^2 - y + 2$

$= y^2 + 2 + y^2 - y + 2$ [From I]

$= 2y^2 - y + 4$ years

After $(y^2 - y - 2)$ years, age of Asha $= 2y^2 - y + 4 = 10y - 1$

$\Rightarrow$ $2y^2 - y - 10y + 5 = 0$

$\Rightarrow$ $2y^2 - 11y + 5 = 0$

$\Rightarrow$ $2y^2 - 10y - 1y + 5 = 0$

$\Rightarrow$ $2y(y-5) - 1(y-5) = 0$

$\Rightarrow$ $(y-5)(2y-1) = 0$

$\Rightarrow$ $y - 5 = 0$ or $2y - 1 = 0$

$\Rightarrow$ $y = 5$ or $y = \frac{1}{2}$ years

From I, we have

$$x = y^2 + 2$$

Putting $y = 5$, we have

$$x = (5)^2 + 2 = 25 + 2 = 27$$

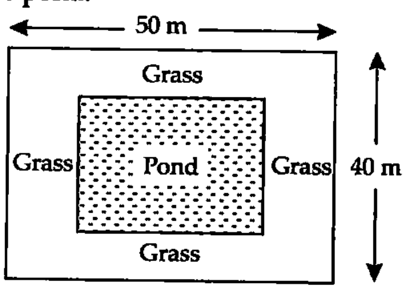
Putting $y = \frac{1}{2}$, we have

$$x = \left(\frac{1}{2}\right)^2 + 2 = 2\frac{1}{4} \text{ years}$$

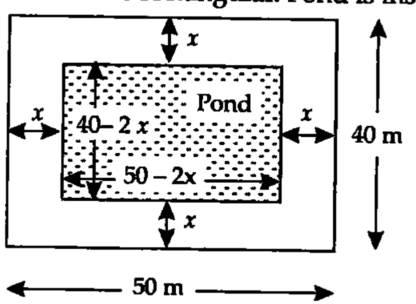
Mother's age can never be $2\frac{1}{4}$ years, so it is rejected.

Hence, the ages of Asha and Nisha are 27 years and 5 years respectively.

Q7. In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed, so that the area of grass surrounding the pond would be 1184 m^2 (see figure). Find the length and breadth of the pond.



Sol. Pond and lawn both are rectangular. Pond is inside the lawn.



Let the length of pond = $(50 - 2x)$ m

and the breadth of pond = $(40 - 2x)$ m

But, Area of grass around the pond = 1184 m^2

$\Rightarrow$ Area of Lawn – Area of Pond = 1184

$\Rightarrow$ $50 \times 40 - (50 - 2x)(40 - 2x) = 1184$

$\Rightarrow$ $2000 - (2000 - 100x - 80x + 4x^2) - 1184 = 0$

$\Rightarrow$ $2000 - (2000 - 180x + 4x^2) - 1184 = 0$

$$\Rightarrow \quad 2000 - 2000 + 180x - 4x^2 - 1184 = 0$$
$$\Rightarrow \quad 4x^2 - 180x + 1184 = 0$$
$$\Rightarrow \quad x^2 - 45x + 296 = 0$$
$$\Rightarrow \quad x^2 - 37x - 8x + 296 = 0$$
$$\Rightarrow \quad x(x - 37) - 8(x - 37) = 0$$
$$\Rightarrow \quad (x - 37)(x - 8) = 0$$
$$\Rightarrow \quad x - 37 = 0 \quad \text{or} \quad x - 8 = 0$$
$$\Rightarrow \quad x = 37 \quad \text{or} \quad x = 8$$

When $x = 37$, then

the length of pond $= 50 - 2 \times 37$
$= 50 - 74$
$= -24$ m

Length cannot be negative. So, $x = 37$ is rejected.

When $x = 8$, then

the length of pond $= 50 - 2x$
$= 50 - 2 \times 8$
$= 50 - 16$
$= 34$ m

and the breadth of the pond
$= 40 - 2x$
$= 40 - 2 \times 8$
$= 40 - 16$
$= 24$ m

Hence, the length and breadth of the pond are 34 m and 24 m respectively.

Q8. At t minutes past 2 p.m., the time needed by minute hand of a clock to show 3 p.m. was found to be 3 min. less than $\frac{t^2}{4}$ min. Find t.

Sol. Total time taken by min hand from 2 p.m. to 3 p.m. is 60 min.
After t min past 2 p.m. the time needed by min. hand of a clock to show 3 p.m. is given by 3 min less than $\frac{t^2}{4}$ min.

$$\therefore \quad t + \left(\frac{t^2}{4} - 3\right) = 60$$
$$\Rightarrow \quad 4t + t^2 - 12 = 240$$
$$\Rightarrow \quad t^2 + 4t - 252 = 0$$
$$\Rightarrow \quad t^2 + 18t - 14t - 252 = 0$$
$$\Rightarrow \quad t(t + 18) - 14(t + 18) = 0$$
$$\Rightarrow \quad (t + 18)(t - 14) = 0$$
$$\Rightarrow \quad t + 18 = 0 \quad \text{or} \quad t - 14 = 0$$
$$\Rightarrow \quad t = -18 \quad \text{or} \quad t = 14 \text{ min.}$$

Being, negative value, $t = -18$ is rejected.
Hence, $t = 14$ min.

❑❑❑

5 Arithmetic Progressions

EXERCISE 5.1

Choose the correct answer from the given four options in the following questions:

Q1. In an A.P., if $d = -4$, $n = 7$, $a_n = 4$, then a is

(*a*) 6 (*b*) 7 (*c*) 20 (*d*) 28

Sol. (*d*): **Main concept used:** $\boxed{a_n = a + (n-1)d}$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad 4 = a + (7-1)(-4)$ (By the given condition)

$\Rightarrow \quad -a = -4 - 24$

$\Rightarrow \quad a = 28$

Q2. In an A.P., if $a = 3.5$, $d = 0$, $n = 101$, then a_n will be

(*a*) 0 (*b*) 3.5 (*c*) 103.5 (*d*) 104.5

Sol. (*b*): $a_n = a + (n-1)d$

$\Rightarrow \quad a_n = 3.5 + (101 - 1) \times 0$ (By the given condition)

$\Rightarrow \quad a_n = 3.5 + 100 \times 0$

$\Rightarrow \quad a_n = 3.5$

OR

As $d = 0$ so all terms are same.

Q3. The list of numbers $-10, -6, -2, 2, \ldots$ is

(*a*) an A.P. with $d = -16$ (*b*) an A.P. with $d = 4$

(*c*) an A.P. with $d = -4$ (*d*) not an A.P.

Sol. (*b*): **Main concept used:** A series of numbers will be an A.P. if $d_1 = d_2 = d_3 = \ldots$

where $\quad d_1 = a_2 - a_1,\ d_2 = a_3 - a_2,\ d_3 = a_4 - a_3$

$d_1 = a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$d_2 = a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$

$d_3 = a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

As $\quad d_1 = d_2 = d_3 = 4$

So, the given series is an A.P. with $d = 4$.

Q4. The 11th term of an A.P. $-5, \frac{-5}{2}, 0, \frac{5}{2} \ldots$ is

(*a*) -20 (*b*) 20 (*c*) -30 (*d*) 30

Sol. (*b*): Here, $n = 11$, $a = -5$, $d = \frac{5}{2} - 0 = \frac{5}{2}$

$a_n = a + (n-1)d$

$\therefore \quad a_{11} = -5 + (11-1)\left(\frac{5}{2}\right)$

$= -5 + 10 \times \frac{5}{2} = -5 + 25 = 20$

Q5. The first four terms of an A.P. whose first term is -2 and the common difference is (-2), are

(*a*) $-2, 0, 2, 4$ (*b*) $-2, 4, -8, 16$

(*c*) $-2, -4, -6, -8$ (*d*) $-2, -4, -8, -16$

Sol. (*c*): **Main concept used:** $a_n = a + (n-1)d$

$a_1 = -2, d = -2$

$\therefore \quad a_2 = a_1 + d$

$\Rightarrow \quad a_2 = -2 - 2 = -4$

and $\quad a_3 = a_2 + d = -4 + (-2) = -6$

and $\quad a_4 = a_3 + d = -6 + (-2) = -8$

So, the first four terms are $-2, -4, -6, -8$.

Q6. The 21st term of an A.P. whose first two terms are -3 and 4 is

(*a*) 17 (*b*) 137 (*c*) 143 (*d*) -143

Sol. (*b*): **Main concept used:** $a_n = a + (n-1)d$

Here, $\quad a = a_1 = -3, a_2 = 4$

$\therefore \quad d = a_2 - a_1 = 4 - (-3) = 4 + 3 = 7$

Hence, $d = 7$

Now, $\quad a_n = a + (n-1)d$

$\Rightarrow \quad a_{21} = -3 + (21-1) \times 7 = -3 + 20 \times 7 = -3 + 140$

$\Rightarrow \quad a_{21} = 137$

Hence, (*b*) is the correct answer.

Q7. If the 2nd term of an A.P. is 13 and 5th term is 25, what is its 7th term?

(*a*) 30 (*b*) 33 (*c*) 37 (*d*) 38

Sol. (*b*): Here, $a_2 = 13$ and $a_5 = 25$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad a_2 = a + (2-1)d$

$\Rightarrow \quad 13 = a + d$

$\Rightarrow \quad a + d = 13 \quad \ldots(i)$

and $\quad a_5 = a + (5-1)d$

$\Rightarrow \quad 25 = a + 4d$

$\Rightarrow \quad a + 4d = 25 \quad \ldots(ii)$

Now, subtracting (*i*) from (*ii*), we get

$$\begin{array}{rrl} a + & 4d = 25 & \ldots(ii) \\ a + & d = 13 & \ldots(i) \\ - \quad - & - & \\ \hline 3d & = 12 & \end{array}$$

$\Rightarrow \quad d = \frac{12}{3}$

$\Rightarrow \quad d = 4$

Now, $\quad a + d = 13$ [From (*i*)]

$\Rightarrow \quad a + 4 = 13$

$\Rightarrow \quad a = 13 - 4 = 9$

Now, $\quad a_7 = a + 6d = 9 + 6(4) = 9 + 24$

$\Rightarrow \quad a_7 = 33$

Hence, (*b*) is the correct answer.

Q8. Which term of an A.P.: 21, 42, 63, 84, ... is 210?

(*a*) 9th (*b*) 10th (*c*) 11th (*d*) 12th

Sol. (*b*): Given A.P. is 21, 42, 63, 84, ...

So, $a = 21$, $d = 42 - 21 = 21$, $a_n = 210$

We know that $\quad a_n = a + (n-1)d$

$\Rightarrow \quad 210 = 21 + (n-1)21$

$\Rightarrow \quad 210 - 21 = (n-1)21$

$\Rightarrow \quad \frac{189}{21} = (n-1)$

$\Rightarrow \quad n - 1 = 9$

$\Rightarrow \quad n = 10$

Hence, (*b*) is the correct answer.

Q9. If the common difference of an A.P. is 5, then what is $a_{18} - a_{13}$?

(*a*) 5 (*b*) 20 (*c*) 25 (*d*) 30

Sol. (*c*): Here, $d = 5$.

$\because \quad a_n = a + (n-1)d.$

$\therefore \quad a_{18} - a_{13} = [a + (18-1)d] - [a + (13-1)d]$

$= a + 17d - a - 12d$

$= 5d = 5 \times 5 = 25$

Hence, (*c*) is the correct answer.

Q10. What is the common difference of an A.P. in which $a_{18} - a_{14} = 32$?

(*a*) 8 (*b*) -8 (*c*) -4 (*d*) 4

Sol. (*a*): Here, $a_{18} - a_{14} = 32$

$\Rightarrow \quad [a + (18-1)d] - [a + (14-1)d] = 32$ $[\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + 17d - a - 13d = 32$

$\Rightarrow \quad 4d = 32$

$\Rightarrow \quad d = \frac{32}{4} = 8$

Hence, (*a*) is the correct answer.

Q11. Two A.P.s have the same common difference. The Ist term of one of these is -1, and that of other is -8. The difference between their 4th terms is

(*a*) -1 (*b*) -8 (*c*) 7 (*d*) -9

Sol. (*c*): Given: $a_1 = -1$ and $a'_1 = -8$

Let d be the same common difference of two A.Ps.

So, $d_1 = d$, $d'_1 = d$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad a_4 - a'_4 = [a_1 + (4-1)\,d_1] - [\,a'_1 + (4-1)\,d'_1]$

$\Rightarrow \quad a_4 - a'_4 = (-1 + 3d) - [-8 + 3d]$

$= -1 + 3d + 8 - 3d = 7$

Hence, the required answer is (*c*).

Q12. If 7 times the 7th term of an A.P. is equal to 11 times its 11th term, then its 18th term will be

(*a*) 7 (*b*) 11 (*c*) 18 (*d*) 0

Sol. (*d*): $a_{18} = a + (18-1)d = a + 17d$

Now, $7a_7 = 11a_{11}$ [Given]

$\Rightarrow 7[a + (7-1)d] = 11[a + (11-1)d]$

$\Rightarrow \quad 7[a + 6d] = 11[a + 10d]$

$\Rightarrow \quad 7a + 42d = 11a + 110d$

$\Rightarrow \quad 0 = 11a - 7a + 110d - 42d$

$\Rightarrow \quad 0 = 4a + 68d$

$\Rightarrow \quad 0 = a + 17d$

$\Rightarrow \quad a_{18} = 0$

Hence, (*d*) is the correct answer.

Q13. The 4th term from the end of an A.P. $-11, -8, -5, \ldots, 49$ is

(*a*) 37 (*b*) 40 (*c*) 43 (*d*) 58

Sol. (*b*): Reversing the A.P., we get

$49, \ldots, -5, -8, -11$

$\therefore \quad d = -8 - (-5) = -8 + 5 = -3$

$a = 49$ and $n = 4$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad a_4 = 49 + (4-1)(-3)$

$= 49 + 3(-3) = 49 - 9$

$\Rightarrow \quad a_4 = 40$

Hence, the required value of a_4 is 40 and answer is (*b*).

Q14. The famous mathematician associated with finding the sum of the first 100 natural numbers is

(*a*) Pythagoras (*b*) Newton (*c*) Gauss (*d*) Euclid

Sol. (*c*): Gauss is the famous mathematician associated with finding the sum of first 100 natural numbers, i.e., $1 + 2 + 3 + 4 + 5 + \ldots + 100$

Here, $a = 1$, $d = 1$, $n = 100$

As $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore \quad S_{100} = \frac{100}{2}[2(1) + (100-1)1]$$
$$= \frac{100}{2}[2+99] = \frac{100 \times 101}{2} = 50 \times 101$$
$$= 5050$$

Q15. If the first term of an A.P. is -5 and the common difference is 2, then the sum of first 6 terms is

(*a*) 0 (*b*) 5 (*c*) 6 (*d*) 15

Sol. (*a*) Here, $a = -5, \quad d = 2, \quad n = 6$

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore \quad S_6 = \frac{6}{2}[2(-5) + (6-1)2]$$
$$= 3[-10 + 5 \times 2]$$
$$= 3[-10 + 10]$$
$$= 3[0]$$
$$\Rightarrow \quad S_6 = 0$$

Hence, (*a*) is the correct answer.

Q16. The sum of first 16 terms of an A.P. 10, 6, 2, ... is

(*a*) -320 (*b*) 320 (*c*) -352 (*d*) -400

Sol. (*a*): Here, $a = 10, \quad n = 16, \quad d = 6 - 10 = -4$

$$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$$
$$\therefore \quad S_{16} = \frac{16}{2}[2 \times 10 + (16-1)(-4)]$$
$$= 8[20 + 15(-4)] = 8[20 - 60] = -8 \times 40$$
$$\Rightarrow \quad S_{16} = -320$$

So, the required answer is (*a*).

Q17. In an A.P., if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is

(*a*) 19 (*b*) 21 (*c*) 38 (*d*) 42

Sol. (*c*): We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\Rightarrow \quad S_n = \frac{n}{2}[a + a + (n-1)d]$$
$$\Rightarrow \quad 399 = \frac{n}{2}[a + a_n] \qquad [a_n = \text{last term}]$$
$$\Rightarrow \quad 399 = \frac{n}{2}[1 + 20] \quad \Rightarrow \quad n = \frac{399 \times 2}{21} = 38$$

Hence, (*c*) is the correct answer.

Q18. The sum of first five multiples of 3 is

(*a*) 45 (*b*) 55 (*c*) 65 (*d*) 75

Sol. (*a*): Ist five multiples of 3 are 3, 6, 9, 12, 15, ...

Here, $a = 3$, $n = 5$, $d = 6 - 3 = 3$

$$\therefore \quad S_5 = \frac{5}{2}[2 \times 3 + (5-1)3] \qquad \left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$$

$$\Rightarrow \quad S_5 = \frac{5}{2}[6+12] = \frac{5}{2} \times 18 = 45$$

Hence, (*a*) is the correct answer.

EXERCISE 5.2

Q1. Which of the following form an A.P.? Justify your answer.

(*i*) $-1, -1, -1, -1, \ldots$ (*ii*) $0, 2, 0, 2, \ldots$

(*iii*) $1, 1, 2, 2, 3, 3, \ldots$ (*iv*) $11, 22, 33, \ldots$

(*v*) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ (*vi*) $2, 2^2, 2^3, 2^4, \ldots$

(*vii*) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

Sol. (*i*) $-1, -1, -1, -1, \ldots$

A series of numbers will be in A.P. if $d_1 = d_2 = d_3 \ldots$

So, $d_1 = -1 - (-1) = 0$

$d_2 = -1 - (-1) = 0$

$d_3 = -1 - (-1) = 0$

$\therefore$ $d_1 = d_2 = d_3 \ldots$

So, the given series form an A.P.

(*ii*) $0, 2, 0, 2, \ldots$

Given form of numbers will be in A.P. if $d_1 = d_2 = d_3 \ldots$

So, $d_1 = 2 - 0 = 2$

$d_2 = 0 - 2 = -2$

$\therefore$ $d_1 \neq d_2$

So, the given form of numbers is not an A.P.

(*iii*) $1, 1, 2, 2, 3, 3, \ldots$

Given form of numbers will form an A.P. if $d_1 = d_2 = d_3 \ldots$

So, $d_1 = 1 - 1 = 0$

$d_2 = 2 - 1 = 1$

$\therefore$ $d_1 \neq d_2$

Hence, the given form of numbers will not form an A.P.

(*iv*) $11, 22, 33, \ldots$

Given form of numbers will form an A.P. if $d_1 = d_2 = d_3 = \ldots$

So, $d_1 = 22 - 11 = 11$

$d_2 = 33 - 22 = 11$

$\therefore$ $d_1 = d_2 = 11$

Hence, the given form of numbers will form an A.P.

(v) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

Given form of numbers will form an A.P. if $d_1 = d_2 = d_3 \ldots$

So, $d_1 = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \frac{-1}{6}$

$d_2 = \frac{1}{4} - \frac{1}{3} = \frac{3-4}{12} = \frac{-1}{12}$

$\therefore$ $d_1 \neq d_2$

Hence, the given form of numbers will not form an A.P.

(vi) $2, 2^2, 2^3, 2^4, \ldots$

Given form of numbers will form an A.P. if $d_1 = d_2 = d_3 \ldots$

So, $d_1 = 2^2 - 2 = 4 - 2 = 2$

$d_2 = 2^3 - 2^2 = 8 - 4 = 4$

$d_3 = 2^4 - 2^3 = 16 - 8 = 8$

$\therefore$ $d_1 \neq d_2 \neq d_3$

Hence, the given form of numbers will not form an A.P.

(vii) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

Given form of numbers will form an A.P. if $d_1 = d_2 = d_3 \ldots$

So, $d_1 = \sqrt{12} - \sqrt{3} = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

$d_2 = \sqrt{27} - \sqrt{12} = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$

$d_3 = \sqrt{48} - \sqrt{27} = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$

$\therefore$ $d_1 = d_2 = d_3 = \sqrt{3}$

Hence, the given form of numbers will form an A.P.

Q2. Justify whether it is true to say that $-1, \frac{-3}{2}, -2, \frac{5}{2} \ldots$ forms an A.P. as $a_2 - a_1 = a_3 - a_2$.

Sol. Main concept used: A form of numbers will form an A.P. if $d_1 = d_2 = d_3 = \ldots d_n = d$.

Given form of numbers will form an A.P. if $d_1 = d_2 = d_3 = d$ otherwise not.

So, $d_1 = a_2 - a_1 = \frac{-3}{2} - (-1) = \frac{-3}{2} + 1 = \frac{-3+2}{2} = \frac{-1}{2}$

$d_2 = a_3 - a_2 = -2 - \left(\frac{-3}{2}\right) = -2 + \frac{3}{2} = \frac{-4+3}{2} = \frac{-1}{2}$

$d_3 = a_4 - a_3 = \frac{5}{2} - (-2) = \frac{5}{2} + 2 = \frac{5+4}{2} = \frac{9}{2}$

$\therefore$ $d_1 = d_2 \neq d_3$

Although $a_2 - a_1 = a_3 - a_2 = \frac{-1}{2}$ but $a_4 - a_3 \neq \frac{-1}{2}$

So, the given form of numbers will not form an A.P. Hence, the given statement is false.

Q3. For the A.P. –3, –7, –11, ..., can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20}? Give reasons for your answer.

Sol. Here, $a = -3,$

$$d_1 = -7 - (-3) = -7 + 3 = -4$$
$$d_2 = -11 - (-7) = -11 + 7 = -4$$
$$\therefore \quad d = d_1 = d_2 = -4$$

Now, $a_{30} = a + (30 - 1)d = a + 29d$

and $a_{20} = a + (20 - 1)d = a + 19d$

So, $a_{30} - a_{20} = (a + 29d) - (a + 19d) = a + 29d - a - 19d$

$\Rightarrow \quad a_{30} - a_{20} = 10d$

$$= 10 \times (-4) = -40$$

So, we can find $a_{30} - a_{20}$ without finding a_{30} and a_{20}.

Hence, $a_{30} - a_{20} = -40$.

Q4. Two A.P.s have the same common difference. The first term of one A.P. is 2, and that of the other is 7. The difference between their 10th terms is same as the difference between their 21th terms, which is the same as the difference between any two corresponding terms. Why?

Sol. Given: $a_1 = 2$ and $a'_1 = 7$

Let d be the same common difference of two A.P.s.

So, $d_1 = d$ and $d'_1 = d$

Now, $a_{10} - a'_{10} = a_1 + (10 - 1)d_1 - [a'_1 + (10 - 1)\,d'_1]$

$$= 2 + 9d - [7 + 9d] = 2 + 9d - 7 - 9d$$

$\Rightarrow \quad a_{10} - a'_{10} = -5$

Also, $a_{21} - a'_{21} = a_1 + (21 - 1)\,d_1 - [a'_1 + (21 - 1)\,d'_1]$

$$= 2 + 20d - [7 + 20d] = 2 + 20d - 7 - 20d$$

$\Rightarrow \quad a_{21} - a'_{21} = -5$

$\Rightarrow \quad a_{21} - a'_{21} = a_{10} - a'_{10} = -5$

Now, $a_n - a'_n = a_1 + (n - 1)d_1 - [\,a'_1 + (n - 1)\,d'_1]$

$$= 2 + (n - 1)d - [7 + (n - 1)d]$$
$$= 2 + nd - d - [7 + nd - d]$$
$$= 2 + nd - d - 7 - nd + d$$
$$= 2 - 7$$

$\Rightarrow \quad a_n - a'_n = -5$

Hence, the difference between any two corresponding terms of such A.P.'s is same (–5) as the difference between their 10th terms and 21st terms.

Q5. Is 0 a term of the A.P. 31, 28, 25, ...? Justify your answer.

Sol. Main concept used: $a_n = a + (n-1)d$

If we substitute the values of a_n, a, and d in the above equation and if n comes out to be a natural number then, the given a_n will be the term of the given series.

Here, $a_n = 0, \quad a = 31$

$d_1 = 28 - 31 = -3, d_2 = 25 - 28 = -3$

So, $d_1 = d_2 = -3$

$\because \quad a_n = a + (n-1)d \Rightarrow 0 = 31 + (n-1) \times (-3)$

$\Rightarrow \quad -31 = -(n-1) \times 3 \Rightarrow (n-1) = \frac{31}{3}$

$\Rightarrow \quad n = \frac{31}{3} + 1 \Rightarrow n = \frac{31+3}{3} = \frac{34}{3} = 11\frac{1}{3} \neq$ natural number.

Since n is in fraction and is not natural number so 0 (a_n) is not any term of the given A.P..

Q6. The taxi fare after each km, when the fare is ₹ 15 for the first km and ₹ 8 for each additional km, does not form an A.P., as the total fare (in ₹) after each km is 15, 8, 8, 8, ... Is the statement true? Give reasons.

Sol. 15, 8, 8, 8, ... are not the total fare for 1, 2, 3, 4, km respectively.

Total fare for Ist km = ₹ 15.

Total fare for 2 km = ₹ 15 + ₹ 8 = ₹ 23

Total fare for 3 km = ₹ 23 + ₹ 8 = ₹ 31

Total fare for 4 km = ₹ 31 + ₹ 8 = ₹ 39

$\therefore$ Total fare for 1 km, 2 km, 3 km, 4 km, ... are 15, 23, 31, 39, ... respectively.

Now, $d_1 = 23 - 15 = 8$

$d_2 = 31 - 23 = 8$

$d_3 = 39 - 31 = 8$

Hence, the total fare form an A.P. as 15, 23, 31, 39, ...

But, fare for each km does not form A.P. as 15, 8, 8, 8 ...

Q7. In which of the following situations do the lists of numbers involved form an A.P.? Give reasons for your answers.

(*i*) The fee charged from a student every month by a school for the whole session, when the monthly fee is ₹ 400.

(*ii*) The fee charged every month by a school from classes I to XII, when the monthly fee for class I is ₹ 250 and it increases by ₹ 50 for the next higher class.

(*iii*) The amount of money in the account of Varun at the end of every year when ₹ 1000 is deposited at simple interest of 10% per annum.

(*iv*) The number of bacteria in a certain food item after each second, when they double in every second.

Sol. (*i*) The fee charged from a student every month by a school is ₹ 400. So, the fee charged from a student the whole session is 400, 400, 400, 400, ... As $d_1 = d_2 = d_3 = - d_{12} = 0$ so, the series of numbers is an A.P.

(*ii*) Fee for Ist class = ₹ 250

Fee for IInd class = ₹ (250 + 50) = ₹ 300

Fee for IIIrd class = ₹ (300 + 50) = ₹ 350

Fee for IV class = ₹ (350 + 50) = ₹ 400

∴ 250, 300, 350, 400, ... is a series consisting of 12 terms.

So, $d_1 = 300 - 250 =$ ₹ 50, $d_2 = 350 - 300 =$ ₹ 50, $d_3 = 400 - 350 =$ ₹ 50

∴ $d_1 = d_2 = d_3 =$ ₹ 50

So, the list of numbers 250, 300, 350, 400, ... is in A.P.

(*iii*) $SI = \frac{PRT}{100} = \frac{1000 \times 10 \times 1}{100} =$ ₹ 100

So, ₹ 100 is credited at the end of each year in the account of Varun.

Money in the beginning of Ist year (deposited) = ₹ 1000

Money at the end of Ist year when interest credited

= 1000 + 100 = ₹ 1100

Money at the end of IInd year = 1100 + 100 = ₹ 1200

Money at the end of IIIrd your = 1200 + 100 = ₹ 1300

Money at the end of IV year = 1300 + 100 = ₹ 1400

∴ Amount of money at the end of each year starting initially from Ist year is given by 1000, 1100, 1200, 1300, 1400 ...

∴ $d_1 = d_2 = d_3 = d_4 = 100$

So, the list of numbers is an A.P.

(*iv*) Let the number of bacteria present initially = x

Then, the number of bacteria present after 1 sec = $2(x) = 2x$

Number of bacteria present after 2 sec = $2(2x) = 4x$

Number of bacteria present after 3 sec = $2(4x) = 8x$

Number of bacteria present after 4 second = $2(8x) = 16x$

So, the number of bacteria from starting to end of each second are given by $x, 2x, 4x, 8x, 16x, \ldots$

Now, $d_1 = 2x - x = x$, $d_2 = 4x - 2x = 2x$

As $d_1 \neq d_2$, so the list of numbers does not form an A.P.

Q8. Justify whether it is true to say that the following are the nth terms of an A.P.

(*i*) $2n - 3$ (*ii*) $3n^2 + 5$ (*iii*) $1 + n + n^2$

Sol. (*i*) $a_n = 2n - 3$

∴ $a_1 = 2(1) - 3 = 2 - 3 = -1$, $a_2 = 2(2) - 3 = 4 - 3 = 1$

$a_3 = 2(3) - 3 = 6 - 3 = 3$, $a_4 = 2(4) - 3 = 8 - 3 = 5$

So, $d_1 = 1-(-1) = 1+1 = 2, \qquad d_2 = 3-1 = 2, \quad d_3 = 5-3 = 2$

As $d_1 = d_2 = d_3 = 2$, hence, $a_n = 2n - 3$ form nth term of an A.P.

(*ii*) $a_n = 3n^2 + 5$

$\therefore \quad a_1 = 3(1)^2 + 5 = 3\times 1 + 5 = 3 + 5 = 8$

$a_2 = 3(2)^2 + 5 = 3\times 4 + 5 = 12 + 5 = 17$

$a_3 = 3(3)^2 + 5 = 3\times 9 + 5 = 27 + 5 = 32$

$a_4 = 3(4)^2 + 5 = 3\times 16 + 5 = 48 + 5 = 53$

$a_5 = 3(5)^2 + 5 = 3\times 25 + 5 = 75 + 5 = 80$

$\therefore \quad d_1 = a_2 - a_1 = 17 - 8 = 9, \qquad d_2 = a_3 - a_2 = 32 - 17 = 15$

$d_3 = a_4 - a_3 = 53 - 32 = 21, \quad d_4 = a_5 - a_4 = 80 - 53 = 27$

Since $d_1 \neq d_2$, so the list of numbers 8, 17, 32, 53, ... is not in A.P.

(*iii*) $a_n = 1 + n + n^2$

$\therefore \quad a_1 = 1 + (1) + (1)^2 = 1 + 1 + 1 = 3$

$a_2 = 1 + (2) + (2)^2 = 1 + 2 + 4 = 7$

$a_3 = 1 + (3) + (3)^2 = 1 + 3 + 9 = 13$

$a_4 = 1 + (4) + (4)^2 = 1 + 4 + 16 = 21$

$a_5 = 1 + (5) + (5)^2 = 1 + 5 + 25 = 31$

So, $d_1 = a_2 - a_1 = 7 - 3 = 4$

$d_2 = a_3 - a_2 = 13 - 7 = 6$

$d_3 = a_4 - a_3 = 21 - 13 = 8$

$d_4 = a_5 - a_4 = 31 - 21 = 10$

As $d_1 \neq d_2$, so the list of numbers 3, 7, 13, 21, 31, ... is not in A.P.

EXERCISE 5.3

Q1. Match the A.P.s given in column A with suitable common differences given in column B.

Column A	Column B
(A_1) 2, −2, −6, −10, ...	(B_1) 2/3
(A_2) $a = -18, n = 10, a_n = 0$	(B_2) −5
(A_3) $a = 0, a_{10} = 6$	(B_3) 4
(A_4) $a_2 = 13, a_4 = 3$	(B_4) −4
	(B_5) 2
	(B_6) 1/2
	(B_7) 5

Sol. (*i*) Here, $a_1 = 2$

$\therefore \quad d_1 = -2 - 2 = -4$

and $\quad d_2 = -6 - (-2) = -6 + 2 = -4$

Hence, A_1 matches to B_4.

(*ii*) Given: $a_n = 0, a = -18, n = 10$

Now, $a_n = a + (n-1)d$

$\Rightarrow \quad 0 = -18 + (10-1)d$

$\Rightarrow \quad -9d = -18$

$\Rightarrow \quad d = 2$

Hence, A_2 matches to B_5.

(*iii*) Given: $a = 0, a_{10} = 6$

Now, $a_n = a + (n-1)d$

$\Rightarrow \quad 6 = 0 + (10-1)d$

$\Rightarrow \quad 9d = 6$

$\Rightarrow \quad d = \frac{6}{9} \Rightarrow d = \frac{2}{3}$

Hence, A_3 matches to B_1.

(*iv*) $a_2 = 13$ [Given]

$\therefore \quad a + (2-1)d = 13$ [$\because a_n = a + (n-1)d$]

$\Rightarrow \quad a + d = 13$

$\Rightarrow \quad a = 13 - d$ ...(*i*)

Also, $a_4 = 3$ [Given]

$\therefore \quad a + (4-1)d = 3$ [$\because a_n = a + (n-1)d$]

$\Rightarrow \quad a + 3d = 3$

$\Rightarrow \quad 13 - d + 3d = 3$ [Using (*i*)]

$\Rightarrow \quad 2d = 3 - 13$

$\Rightarrow \quad 2d = -10$

$\Rightarrow \quad d = -5$

Hence, A_4 matches to B_2.

Q2. Verify that each of the following is an A.P. and then write its next three terms.

(*i*) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \cdots$ (*ii*) $5, \frac{14}{3}, \frac{13}{3}, 4, \cdots$

(*iii*) $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$ (*iv*) $a+b, (a+1)+b, (a+1)+(b+1), \ldots$

(*v*) $a, 2a+1, 3a+2, 4a+3, \ldots$

Sol. Main concept used: (*a*) List of numbers will form an A.P. if $d_1 = d_2 = d_3 \ldots, = d$ (*b*) $a_{n+1} = a_n + d$

(*i*) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \cdots$

$$d_1 = \frac{1}{4} - 0 = \frac{1}{4}, d_2 = \frac{1}{2} - \frac{1}{4} = \frac{2-1}{4} = \frac{1}{4}, d_3 = \frac{3}{4} - \frac{1}{2} = \frac{3-2}{4} = \frac{1}{4}$$

$\therefore \quad d_1 = d_2 = d_3 = \frac{1}{4}$

So, the given list of numbers form an A.P.

Now, $a_5 = a_4 + d = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

$a_6 = a_5 + d = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$

$a_7 = a_6 + d = \frac{5}{4} + \frac{1}{4} = \frac{6}{4} = \frac{3}{2}$

So, the next three terms are $1, \frac{5}{4}$ and $\frac{3}{2}$.

(*ii*) $5, \frac{14}{3}, \frac{13}{3}, 4, \cdots$

$d_1 = \frac{14}{3} - 5 = \frac{14-15}{3} = \frac{-1}{3}$

$d_2 = \frac{13}{3} - \frac{14}{3} = \frac{13-14}{3} = \frac{-1}{3}$

$d_3 = 4 - \frac{13}{3} = \frac{12-13}{3} = -\frac{1}{3}$

Since, $d_1 = d_2 = d_3 = -\frac{1}{3}$ so, the given list of numbers is in A.P.

For next 3 terms, we have

$a_5 = a_4 + d = 4 + \left(\frac{-1}{3}\right) = \frac{12-1}{3} = \frac{11}{3}$

$a_6 = a_5 + d = \frac{11}{3} + \left(\frac{-1}{3}\right) = \frac{11-1}{3} = \frac{10}{3}$

$a_7 = a_6 + d = \frac{10}{3} + \left(\frac{-1}{3}\right) = \frac{10-1}{3} = \frac{9}{3}$

Hence, the next three terms are $\frac{11}{3}, \frac{10}{3}$ and $\frac{9}{3}$.

(*iii*) $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, ...$

$d_1 = a_2 - a_1 = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

$d_2 = a_3 - a_2 = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$

$\therefore$ $d_1 = d_2 = \sqrt{3}$ verifies that the given list of numbers form an A.P.

For next three terms, we have

$a_4 = a_3 + d = 3\sqrt{3} + \sqrt{3} = 4\sqrt{3}$

$a_5 = a_4 + d = 4\sqrt{3} + \sqrt{3} = 5\sqrt{3}$

$a_6 = a_5 + d = 5\sqrt{3} + \sqrt{3} = 6\sqrt{3}$

Hence, the next three terms are $4\sqrt{3}, 5\sqrt{3}$ and $6\sqrt{3}$.

(*iv*) $a+b, (a+1)+b, (a+1)+(b+1), \dots$

$d_1 = a+1+b-(a+b) = a+1+b-a-b = 1$

$d_2 = (a+1)+(b+1)-[(a+1)+b] = a+1+b+1-a-1-b = 1$

$\therefore d_1 = d_2 = 1$ verifies that the given list of numbers form an A.P.

For next three terms, we have

$a_4 = a_3 + d = (a+1)+(b+1)+1 = (a+2)+(b+1)$

$a_5 = a_4 + d = (a+2)+(b+1)+1 = (a+2)+(b+2)$

$a_6 = a_5 + d = (a+2)+(b+2)+1 = (a+3)+(b+2)$

(*v*) $a, 2a+1, 3a+2, 4a+3, \dots$

$d_1 = a_2 - a_1 = 2a+1-a = a+1$

$d_2 = 3a+2-(2a+1) = a+2-1 = a+1$

$d_3 = 4a+3-(3a+2) = 4a+3-3a-2 = a+1$

$\Rightarrow\ d_1 = d_2 = d_3 = a+1$ verifies that the given list of numbers form an A.P.

For next three terms, we have

$a_5 = a_4 + d = 4a+3+a+1 = 5a+4$

$a_6 = a\ \ + d = 5a+4+a+1 = 6a+5$

$a_7 = a_6 + d = 6a+5+a+1 = 7a+6$

Hence, the next three terms are $(5a+4)$, $(6a+5)$ and $(7a+6)$.

Q3. Write the first three terms of the A.P.s when a and d are as given below.

(*i*) $a = \frac{1}{2}, d = \frac{-1}{6}$ (*ii*) $a = -5, d = -3$ (*iii*) $a = \sqrt{2}, d = \frac{1}{\sqrt{2}}$

Sol. Main concept used: $a_n = a + (n-1)d$

(*i*) Here, $a = \frac{1}{2}, d = \frac{-1}{6}$

We know that

$$a_n = a + (n-1)\,d$$

$$\Rightarrow a_n = \frac{1}{2} + (n-1)\left(\frac{-1}{6}\right)$$

$$\Rightarrow a_n = \frac{1}{2} - \frac{n}{6} + \frac{1}{6} = \frac{1}{2} + \frac{1}{6} - \frac{n}{6} = \frac{3+1-n}{6} \quad \Rightarrow \quad a_n = \frac{4-n}{6}$$

$$\therefore\ a_1 = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}, \quad a_2 = \frac{4-2}{6} = \frac{2}{6} = \frac{1}{3}, \quad a_3 = \frac{4-3}{6} = \frac{1}{6}$$

Hence, the required first three terms are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{6}$.

(*ii*) Here, $a = -5, d = -3$

We know that

$$a_n = a + (n-1)\,d$$

$\Rightarrow \quad a_n = -5 + (n-1)(-3) = -5 - 3n + 3 = -2 - 3n$

$\Rightarrow \quad a_n = -(2 + 3n)$

$\therefore \quad a_1 = -[2 + 3(1)] = -(2+3) = -5$

$a_2 = -[2 + 3 \times 2] = -[2+6] = -8$

$a_3 = -[2 + 3 \times 3] = -[2+9] = -11$

Hence, the first three terms are $-5, -8$ and -11.

(*iii*) Here, $a = \sqrt{2}, d = \dfrac{1}{\sqrt{2}}$

We know that $a_n = a + (n-1)d$

$$\Rightarrow \quad a_n = \sqrt{2} + (n-1)\frac{1}{\sqrt{2}} = \sqrt{2} + \frac{n}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \sqrt{2} - \frac{1}{\sqrt{2}} + \frac{n}{\sqrt{2}}$$

$$\Rightarrow \quad a_n = \frac{2 - 1 + n}{\sqrt{2}} \quad \Rightarrow \quad a_n = \frac{1+n}{\sqrt{2}}$$

$$\therefore \quad a_1 = \frac{1+1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2},$$

$$a_2 = \frac{1+2}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

and $$a_3 = \frac{1+3}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

Hence, the first three terms are $\sqrt{2}, \dfrac{3\sqrt{2}}{2}$ and $2\sqrt{2}$.

Q4. Find a, b, c such that the following numbers are in A.P.: $a, 7, b, 23, c$.

Sol. We have

$$d_1 = a_2 - a_1 = 7 - a$$
$$d_2 = a_3 - a_2 = b - 7$$
$$d_3 = a_4 - a_3 = 23 - b$$
$$d_4 = a_5 - a_4 = c - 23$$

As list of numbers is in A.P.,

so $\quad d_1 = d_2 = d_3 = d_4$

Now, $\quad d_2 = d_3$

$\Rightarrow \quad b - 7 = 23 - b$

$\Rightarrow \quad b + b = 30 \quad \Rightarrow \quad 2b = 30 \quad \Rightarrow \quad b = 15$

Now, $\quad d_2 = d_1$

$\Rightarrow \quad b - 7 = 7 - a$

$\Rightarrow \quad 15 - 7 = 7 - a \quad \Rightarrow \quad 8 = 7 - a$

$a = 7 - 8 = -1$

Now, $\quad d_4 = d_2$

$\Rightarrow \quad c - 23 = b - 7$

$\Rightarrow \quad c = 23 + 15 - 7 = 38 - 7$

$\Rightarrow \quad c = 31$

Hence, $a = -1$, $b = 15$, and $c = 31$.

Q5. Determine the A.P. whose 5th term is 19 and the difference of 8th term from 13th term is 20.

Sol. Main concept used: (*i*) $a_n = a + (n-1)d$ (*ii*) Solution of linear eqn.

Given: $a_5 = 19, \quad a_{13} - a_8 = 20$

Let us consider an A.P. whose Ist term and common difference are a and d respectively.

$$a_5 = 19 \qquad \text{[Given]}$$

$$\Rightarrow \quad a + (5-1)d = 19$$

$$\Rightarrow \quad a + 4d = 19 \qquad ...(i)$$

Also,
$$a_{13} - a_8 = 20 \qquad \text{[Given]}$$

$$\Rightarrow \quad a + (13-1)d - [a + (8-1)d] = 20$$

$$\Rightarrow \quad a + 12d - [a + 7d] = 20$$

$$\Rightarrow \quad a + 12d - a - 7d = 20$$

$$\Rightarrow \quad 5d = 20$$

$$\Rightarrow \quad d = \frac{20}{5} \quad \Rightarrow \quad d = 4$$

Now,
$$a + 4d = 19 \qquad \text{[From (i)]}$$

$$\Rightarrow \quad a + 4 \times 4 = 19$$

$$\Rightarrow \quad a = 19 - 16 = 3$$

A.P. is given by $a, a+d, a+2d, a+3d, \ldots$

Hence, the required A.P. is 3, 7, 11, 15, ...

Q6. The 26th, 11th and the last term of an A.P. are 0, 3, and $-\frac{1}{5}$ respectively. Find the common difference and the number of terms.

Sol. Consider an A.P. whose first term, common difference and last term are a, d and a_n

Given: $a_{26} = 0, \quad a_{11} = 3 \quad \text{and} \quad a_n = -\frac{1}{5}$

$$a_{26} = 0 \qquad \text{[Given]}$$

$$\Rightarrow \quad a + (26-1)d = 0$$

$$\Rightarrow \quad a + 25d = 0 \qquad ...(i)$$

$$a_{11} = 3 \qquad \text{[Given]}$$

$$\Rightarrow \quad a + (11-1)d = 3$$

$$\Rightarrow \quad a + 10d = 3 \qquad ...(ii)$$

$$a_n = -\frac{1}{5}$$

$$\Rightarrow \quad a + (n-1)d = -\frac{1}{5} \qquad ...(iii)$$

On subtracting eqn. (*ii*) from eqn. (*i*), we get

$$15d = -3$$

$$\Rightarrow \quad d = \frac{-3}{15} = \frac{-1}{5}$$

From (*ii*), $a + 10d = 3$

$$\Rightarrow \quad a + 10\left(-\frac{1}{5}\right) = 3$$

$$\Rightarrow \quad a - 2 = 3 \quad \Rightarrow \quad a = 3 + 2$$

$$\Rightarrow \quad a = 5$$

$\therefore$ From (*iii*), $a + (n-1)d = -\frac{1}{5}$

$$\Rightarrow \quad 5 + (n-1)\left(\frac{-1}{5}\right) = \frac{-1}{5}$$

$$\Rightarrow \quad 25 - (n-1) = -1$$

$$\Rightarrow \quad 25 + 1 = (n-1)$$

$$\Rightarrow \quad n - 1 = 26$$

$$\Rightarrow \quad n = 27$$

Hence, the common difference and number of terms in A.P. are $-\frac{1}{5}$ and 27 respectively.

Q7. The sum of the 5th and the 7th terms of an A.P. is 52, and the 10th term is 46. Find the A.P.

Sol. Consider an A.P. whose Ist term and common difference are a and d respectively. According to the question,

$$a_5 + a_7 = 52$$

$$\Rightarrow \quad a + (5-1)d + a + (7-1)d = 52 \qquad [\because a_n = a + (n-1)d]$$

$$\Rightarrow \quad 2a + 4d + 6d = 52$$

$$\Rightarrow \quad 2a + 10d = 52$$

$$\Rightarrow \quad a + 5d = 26 \qquad \text{...}(i)$$

Also, $a_{10} = 46$ [Given]

$$\Rightarrow \quad a + (10-1)d = 46$$

$$\Rightarrow \quad a + 9d = 46 \qquad \text{...}(ii)$$

$$\begin{array}{rcrl} a + & 5d = & 26 & \text{[From (i)]} \\ a + & 9d = & 46 & \text{[From (ii)]} \\ - \quad - & & - & \\ \hline & -4d = & -20 & \text{[Subtract (ii) from (i)]} \end{array}$$

$$\Rightarrow \quad d = \frac{20}{4}$$

$$\Rightarrow \quad d = 5$$

Now, $a + 5d = 26$ [From (*i*)]

$$\Rightarrow \quad a + 5 \times 5 = 26$$
$$\Rightarrow \quad a = 26 - 25$$
$$\Rightarrow \quad a = 1$$

A.P. is given by $a, a+d, a+2d, \ldots$

Hence, the required A.P. is given by 1, 6, 11, 16, ...

Q8. Find the 20th term of an A.P. whose 7th term is 24 less than the 11th term, first term being 12.

Sol. Consider an A.P. whose first term and common difference are 'a' and 'd' respectively.

According to the question, we have

$$a_7 = a_{11} - 24$$
$$\Rightarrow \quad a + (7-1)d + 24 = a + (11-1)d \qquad [\because a_n = a + (n-1)d]$$
$$\Rightarrow \quad a + 6d + 24 - a = 10d$$
$$\Rightarrow \quad 6d - 10d = -24$$
$$\Rightarrow \quad -4d = -24$$
$$\Rightarrow \quad d = \frac{24}{4} = 6$$

Now, $\quad a_n = a + (n-1)d$

$$\therefore \quad a_{20} = 12 + (20-1)6 \qquad [\because n = 20, a = 12, d = 6]$$
$$= 12 + 19 \times 6 = 12 + 114$$
$$\Rightarrow \quad a_{20} = 126$$

Hence, the 20th term of A.P. is 126.

Q9. If the 9th term of an A.P. is zero, prove that its 29th term is twice its 19th term.

Sol. Consider an A.P. whose first term and common difference are 'a' and 'd' respectively.

$$a_9 = 0 \qquad \text{[Given]}$$
$$\therefore \quad a + (9-1)d = 0 \qquad [\because a_n = a + (n-1)d]$$
$$\Rightarrow \quad a + 8d = 0$$
$$\Rightarrow \quad a = -8d \qquad \ldots(i)$$

We have to prove that $a_{29} = 2a_{19}$

So, $\quad a_{29} = a + (29-1)d$

$$= -8d + 28d \qquad \text{[Using equation } (i)]$$
$$\Rightarrow \quad a_{29} = 20d \qquad \ldots(ii)$$

Now, $\quad a_{19} = a + (19-1)d$

$$\Rightarrow \quad a_{19} = -8d + 18d \qquad \text{[Using } (i)]$$
$$\Rightarrow \quad a_{19} = 10d$$

But, $\quad a_{29} = 20d \qquad$ [From (ii)]

$$= 2 \times 10d$$
$$= 2 \times a_{19} \qquad [\because a_{19} = 10d]$$
$$= 2a_{19}$$
$$\therefore \qquad a_{29} = 2a_{19}$$

Hence, the 29th term is twice the 19th term in the given A.P.

Q10. Find whether 55 is a term of the A.P.: 7, 10, 13, ... or not. If yes, find which term it is.

Sol. Main concept used: 55 will be nth term of the given A.P. if value of n is only natural number.

Here, $a = 7$, $d = 10 - 7 = 3$

Let 55 is the nth term of the given A.P.

$$\therefore \qquad a_n = 55 \qquad \text{[Given]}$$
$$\Rightarrow \qquad 7 + (n-1)3 = 55 \qquad [\because a_n = a + (n-1)d]$$
$$\Rightarrow \qquad (n-1)3 = 55 - 7$$
$$\Rightarrow \qquad (n-1) = \frac{48}{3}$$
$$\Rightarrow \qquad n - 1 = 16$$
$$\Rightarrow \qquad n = 17, \text{ which is a natural number}$$

Hence, 55 is the 17th term of the given A.P.

Q11. Determine k so that $(k^2 + 4k + 8)$, $(2k^2 + 3k + 6)$ and $3k^2 + 4k + 4$ are three consecutive terms of an A.P.

Sol. Main concept used: Given numbers will be in A.P. if $d_1 = d_2 = d$

Here, $\qquad d_1 = a_2 - a_1 = 2k^2 + 3k + 6 - (k^2 + 4k + 8)$

$$= 2k^2 + 3k + 6 - k^2 - 4k - 8$$
$$\Rightarrow \qquad d_1 = k^2 - k - 2$$

Now, $\qquad d_2 = a_3 - a_2 = 3k^2 + 4k + 4 - (2k^2 + 3k + 6)$

$$= 3k^2 + 4k + 4 - 2k^2 - 3k - 6$$
$$= 3k^2 - 2k^2 + 4k - 3k - 6 + 4$$
$$\Rightarrow \qquad d_2 = k^2 + k - 2$$

As the given terms are in A.P.

$$\therefore \qquad d_2 = d_1$$
$$\Rightarrow \qquad k^2 + k - 2 = k^2 - k - 2$$
$$\Rightarrow \qquad 2k = -2 + 2$$
$$\Rightarrow \qquad 2k = 0 \quad \Rightarrow \quad k = \frac{0}{2} \quad \Rightarrow \quad k = 0$$

Hence, for $k = 0$, the given sequence of numbers will be in A.P.

Q12. Split 207 into three parts such that these are in A.P. and the product of the two smaller parts is 4623.

Sol. Main concept used: Sum of three terms is given so terms can be considered as $(a - d)$, a, $(a + d)$.

Consider an A.P. whose three consecutive terms are $(a-d), a, (a+d)$.
According to the question,

$$(a-d)+a+(a+d) = 207$$

$$\Rightarrow \quad 3a = 207$$

$$\Rightarrow \quad a = \frac{207}{3} \quad \Rightarrow \quad a = 69$$

Also, $$(a-d)(a) = 4623$$

$$\Rightarrow \quad (69-d)69 = 4623 \qquad [\because a = 69]$$

$$\Rightarrow \quad 69-d = \frac{4623}{69}$$

$$\Rightarrow \quad 69-d = 67$$

$$\Rightarrow \quad d = 69-67$$

$$\Rightarrow \quad d = 2$$

So, $$\text{A.P.} = (a-d), a, (a+d)$$
$$= (69-2), 69, (69+2)$$
$$= 67, 69, 71$$

Hence, 207 can be divided into 67, 69, 71 which form three terms of an A.P.

Q13. The angles of a triangle are in A.P. The greatest angle is twice the least. Find all the angles of the triangle.

Sol. Main concept used: (*i*) Sum of interior angles of a triangle is 180°.
(*ii*) So, 180° is divided into three parts which are in A.P. Hence, the terms of A.P. are $(a-d)$, a $(a+d)$.

$$\therefore \quad a-d+a+a+d = 180°$$

[Angle sum property of a triangle]

$$\Rightarrow \quad 3a = 180°$$

$$\Rightarrow \quad a = \frac{180°}{3} = 60°$$

Also, the greatest angle is twice of the smallest. [Given]

$$\Rightarrow \quad a+d = 2(a-d)$$

$$\Rightarrow \quad a+d = 2a-2d$$

$$\Rightarrow \quad a+d-2a+2d = 0 \quad \Rightarrow \quad -a+3d = 0$$

$$\Rightarrow \quad 3d = a \quad \Rightarrow \quad d = \frac{60°}{3} \Rightarrow d = 20° \; [\because a = 60°]$$

$\therefore$ Required parts are $a-d, a, a+d$

$$= 60° - 20°, 60°, 60° + 20°$$
$$= 40°, 60°, 80°$$

Hence, the angles of triangle are 40°, 60° and 80°.

Q14. If nth terms of two A.P.s: 9, 7, 5, ... and 24, 21, 18. ... are same, then find the value of n. Also find that term.

Sol. First A.P. series is 9, 7, 5, ...

Here, $a_1 = 9, \quad d = 7 - 9 = -2$

Now, $a_n = a + (n-1)d$

$= 9 + (n-1)(-2) = 9 - 2(n-1)$

$= 9 - 2n + 2$

$\Rightarrow \quad a_n = 11 - 2n$

Second A.P. series is 24, 21, 18, ...

Here, $a'_1 = 24, \quad d'_1 = 21 - 24 = -3$

$\therefore \quad a'_n = a'_1 + (n-1)d'$

$\Rightarrow \quad a'_n = 24 + (n-1)(-3)$

$\Rightarrow \quad a'_n = 24 - 3n + 3$

$\Rightarrow \quad a'_n = 27 - 3n$

According to the question, we have

$a_n = a'_n$

$\Rightarrow \quad 11 - 2n = 27 - 3n$

$\Rightarrow \quad 3n - 2n = 27 - 11$

$\Rightarrow \quad n = 16$

So, 16th term of Ist A.P., i.e., $a_{16} = a_1 + (n-1)d$

$\Rightarrow \quad a_{16} = 9 + (16-1)(-2)$

$= 9 - 2 \times 15 = 9 - 30$

$\Rightarrow \quad a_{16} = -21$

16th term of IInd A.P., i.e., $a'_{16} = a'_1 + (n-1)d'$

$\Rightarrow \quad a'_{16} = 24 + (16-1)(-3)$

$= 24 - 15 \times 3 = 24 - 45$

$\Rightarrow \quad a'_{16} = -21$

Hence, the 16th term of both A.P.s is equal to – 21.

Q15. If the sum of 3rd and the 8th terms of an A.P. is 7 and the sum of 7th and 14th terms is – 3, find the 10th term.

Sol. Consider an A.P. whose Ist term and common difference are *a* and *d*, respectively.

According to the question,

$a_3 + a_8 = 7$ [Given]

$\Rightarrow \quad a + (3-1)d + a + (8-1)d = 7$ $[\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + 2d + a + 7d = 7$

$\Rightarrow \quad 2a + 9d = 7$ ...(*i*)

Also, $a_7 + a_{14} = -3$ [Given]

$\Rightarrow \quad a + (7-1)d + a + (14-1)d = -3$

$\Rightarrow \quad a + 6d + a + 13d = -3$

$\Rightarrow \quad 2a + 19d = -3$ (*ii*)

Now, subtracting (*ii*) from (*i*), we get

$$2a + 19d = -3 \quad \text{...(ii)}$$
$$2a + 9d = 7 \quad \text{...(i)}$$
$$- \quad - \quad -$$
$$10d = -10$$

$\Rightarrow \quad d = -1$

Now, $\quad 2a + 9d = 7 \quad$ [Using (*i*)]

$\Rightarrow \quad 2a + 9(-1) = 7$

$\Rightarrow \quad 2a = 7 + 9 \Rightarrow a = \frac{16}{2} \Rightarrow a = 8$

$\therefore \quad a_{10} = a + (10 - 1)d = 8 + 9(-1)$

$\Rightarrow \quad a_{10} = -1$

Hence, the 10th term of A.P. is -1.

Q16. Find the 12th term from the end of the A.P.: $-2, -4, -6, \ldots -100$.

Sol. Main concept used: To find the term from end, consider the given A.P. in reverse order and find the term.

To find the term from the end consider the given A.P. in reverse order i.e., $-100, \ldots -6, -4, -2$.

Now, $\quad a = -100$

$d = a_{n+1} - a_n = -4 - (-6) = -4 + 6 = 2$

$n = 12$

$\therefore \quad a_{12} = a + (n - 1)d$

$\Rightarrow \quad a_{12} = -100 + (12 - 1)(2)$

$= -100 + 11 \times 2 = -100 + 22$

$\Rightarrow \quad a_{12} = -78$

Hence, the 12th term from the last of A.P. $-2, -4, -6, \ldots -100$ is -78.

Q17. Which term of the A.P.: 53, 48, 43, ... is the first negative term?

Sol. Given A.P. is 53, 48, 43, ...

$\therefore \quad a = 53, \quad d = 48 - 53 = -5$

Let the *n*th term of A.P. is the first negative term.

Then, $\quad a_n < 0$

$\Rightarrow \quad a + (n - 1)d < 0 \Rightarrow 53 + (n - 1)(-5) < 0$

$\Rightarrow \quad -5(n - 1) < -53 \Rightarrow 5(n - 1) > 53$

$\Rightarrow \quad 5n - 5 > 53 \Rightarrow 5n > 53 + 5$

$\Rightarrow \quad n > \frac{58}{5} \Rightarrow n > 11.6$

$\therefore \quad n = 12$

Hence, the first negative term of A.P. is 12th term, i.e.,

$a_{12} = a + (n - 1)d$

$= 53 + (12 - 1)(-5) = 53 - 5 \times 11$

$= 53 - 55 = -2$

Q18. How many numbers lie between 10 and 300, which when divided by 4 leave remainder 3?

Sol. Main concept used: Find the least and the largest required number between 10 and 300 and make an A.P.

The least number between 10 and 300 which leaves remainder 3 after dividing by 4 is 11. The largest number between 10 and 300 which leaves remainder 3 on dividing by 4 is 296 + 3 = 299.

So, Ist term or number = 11, IInd term or number = 15

IIIrd term or number = 19, last term or number = 299

$\therefore$ A.P. becomes 11, 15, 19, ..., 299

Here, $a_n = 299$, $a = 11$, $d = 15 - 11 = 4$, $n = ?$

Now, $a + (n-1)d = 299 \Rightarrow 11 + (n-1)4 = 299$

$\Rightarrow (n-1)4 = 299 - 11 \Rightarrow n - 1 = \dfrac{288}{4}$

$\Rightarrow n = 72 + 1 \Rightarrow n = 73$

Hence, the required numbers between 10 and 300 are 73.

Q19. Find the sum of the two middle most terms of an A.P. $\dfrac{-4}{3}, -1, \dfrac{-2}{3}, \ldots 4\dfrac{1}{3}$.

Sol. Main concept used: (*i*) Finding the number of terms, i.e., n (*ii*) median of n.

Given A.P. is $\dfrac{-4}{3}, -1, \dfrac{-2}{3}, \ldots, +\dfrac{13}{3}$

Here, $a = \dfrac{-4}{3}$, $d = \dfrac{-1}{1} - \left(\dfrac{-4}{3}\right) = \dfrac{-1}{1} + \dfrac{4}{3}$

$\Rightarrow d = \dfrac{-3+4}{3} = \dfrac{1}{3}$

And, $a_n = \dfrac{13}{3}$

$\Rightarrow a + (n-1)d = \dfrac{13}{3}$

$\Rightarrow \dfrac{-4}{3} + (n-1)\left(\dfrac{1}{3}\right) = \dfrac{13}{3}$

$\Rightarrow -4 + (n-1) = 13$

$\Rightarrow n - 1 = 13 + 4$

$\Rightarrow n = 17 + 1$

$\Rightarrow n = 18$

So, the middle most terms in 18 terms $= \left(\dfrac{18}{2}\right)$th and $\left(\dfrac{18}{2}\right)$th + 1

= 9th and 10th terms are middle most

So, the required sum $= a_9 + a_{10}$

$$= a + (9-1)d + a + (10-1)d$$
$$= 2a + 8d + 9d = 2a + 17d$$
$$= 2\left(\frac{-4}{3}\right) + 17\left(\frac{1}{3}\right) = \frac{-8+17}{3}$$
$$= \frac{9}{3} = 3$$

Hence, the sum of two middle most terms, i.e., $a_9 + a_{10} = 3$.

Q20. The first term of an A.P. is -5 and last term is 45. If the sum of the terms of A.P. is 120, then find the number of terms and the common difference.

Sol. Let us consider an A.P. whose first term and common difference are a and d respectively.

Here, $a = -5,\quad a_n = 45,\quad S_n = 120$

Now, $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + a + (n-1)d]$

$\Rightarrow$ $S_n = \frac{n}{2}[a + a_n]$ $\quad$ [a_n = last term]

$\Rightarrow$ $120 = \frac{n}{2}[-5 + 45] \Rightarrow 120 = \frac{n}{2} \times 40$

$\Rightarrow$ $n = \frac{120 \times 2}{40} = 6 \Rightarrow n = 6$

Hence, the number of terms = 6

Now, $a_n = a + (n-1)d \Rightarrow 45 = -5 + (6-1)d$

$\Rightarrow$ $45 + 5 = 5d \Rightarrow 5d = 50$

$\Rightarrow$ $d = \frac{50}{5} \Rightarrow d = 10$

Hence, the common difference and the number of terms in A.P. are 10 and 6 respectively.

Q21. Find the sum:

(*i*) $1 + (-2) + (-5) + (-8) + \ldots + (-236)$

(*ii*) $\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \cdots$ upto n terms

(*iii*) $\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \cdots$ upto 11 terms.

Sol. (*i*) From the given series,

$$a = 1,\quad a_n = -236$$
$$d_1 = -2 - 1 = -3,\quad d_2 = -5 - (-2) = -5 + 2 = -3$$

$$d_3 = -8-(-5) = -8+5 = -3$$

$\therefore$ $$d = d_1 = d_2 = d_3 = -3$$

Now, $$a+(n-1)d = a_n$$

$\Rightarrow$ $$1+(n-1)(-3) = -236 \Rightarrow -3(n-1) = -236-1$$

$\Rightarrow$ $$-3(n-1) = -237 \Rightarrow -(n-1) = \frac{-237}{3}$$

$\Rightarrow$ $$n-1 = +79 \Rightarrow n = 79+1 \Rightarrow n = 80$$

Now, $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$\Rightarrow$ $$S_{80} = \frac{80}{2}[2(1)+(80-1)(-3)]$$

$$= 40[2-79\times 3] = 40[2-237]$$

$$= 40[-235] = -9400$$

Hence, the sum of all terms $= -9400$

(*ii*) From the given series, we have

$$a = \left(4-\frac{1}{n}\right) \quad \text{and} \quad n = n$$

$$d_1 = \left(4-\frac{2}{n}\right)-\left(4-\frac{1}{n}\right) = 4-\frac{2}{n}-4+\frac{1}{n} = -\frac{1}{n}$$

$$d_2 = \left(4-\frac{3}{n}\right)-\left(4-\frac{2}{n}\right) = 4-\frac{3}{n}-4+\frac{2}{n} = -\frac{1}{n}$$

Now, $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{n}{2}\left[2\left(4-\frac{1}{n}\right)+(n-1)\left(-\frac{1}{n}\right)\right]$$

$$= \frac{n}{2}\left[8-\frac{2}{n}-\frac{(n-1)}{n}\right] = \frac{n}{2}\left[8-\frac{2}{n}-1+\frac{1}{n}\right]$$

$$= \frac{n}{2}\left[7-\frac{1}{n}\right] = -\left[\frac{7 \quad 1}{\quad}\right]$$

$\Rightarrow$ $$S_n = \frac{7n-1}{2}$$

(*iii*) From the given series, we have

$$a\,(\text{Ist term}) = \frac{a-b}{a+b}, \quad n = 11$$

$$d = \frac{(3a-2b)}{(a+b)}-\frac{(a-b)}{(a+b)}$$

$$= \frac{3a-2b-(a-b)}{a+b} = \frac{3a-2b-a+b}{a+b}$$

$$\Rightarrow \qquad d = \frac{2a-b}{a+b}$$

Now, $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow \qquad S_{11} = \frac{11}{2}\left[\frac{2(a-b)}{(a+b)} + (11-1)\frac{(2a-b)}{(a+b)}\right]$$

$$= \frac{11}{2(a+b)}[2a-2b+10(2a-b)]$$

$$= \frac{11}{2(a+b)}[2a-2b+20a-10b]$$

$$= \frac{11}{2(a+b)}[22a-12b]$$

$$= \frac{11}{2}\frac{(22a-12b)}{(a+b)} = \frac{11\times 2(11a-6b)}{2(a+b)}$$

$$= \frac{11(11a-6b)}{(a+b)}$$

Q22. Which term of the A.P., $-2, -7, -12, \ldots$ will be -77? Find the sum of this A.P. upto the term -77.

Sol. Given A.P. is $-2, -7, -12 \ldots -77$

Here, $a = -2, \quad a_n = -77$

$$d_1 = -7-(-2) = -7+2 = -5$$

$$d_2 = -12-(-7) = -12+7 = -5$$

Now, $$a_n = -77$$

$$\Rightarrow \qquad a+(n-1)d = -77 \quad \Rightarrow \quad -2+(n-1)(-5) = -77$$

$$\Rightarrow \qquad -[2+(n-1)5] = -77 \quad \Rightarrow \quad (2+5n-5) = 77$$

$$\Rightarrow \qquad 5n-3 = 77 \quad \Rightarrow \quad 5n = 77+3$$

$$\Rightarrow \qquad n = \frac{80}{5} \quad \Rightarrow \quad n = 16$$

So, the 16th term will be -77.

Now, $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow \qquad S_{16} = \frac{16}{2}[2(-2)+(16-1)(-5)]$$

$$= 8[-4-15\times 5] = 8[-4-75]$$

$$= 8[-79] = -632$$

Hence, the sum of the given A.P. upto -77 terms is -632.

Q23. If $a_n = 3 - 4n$, then show that $a_1, a_2, a_3, \ldots$ form an A.P. Also find S_{20}.

Sol. $a_n = 3 - 4n$ [Given]

$\therefore$ $a_1 = 3 - 4(1) = 3 - 4 = -1$

$a_2 = 3 - 4(2) = 3 - 8 = -5$

$a_3 = 3 - 4(3) = 3 - 12 = -9$

$a_4 = 3 - 4(4) = 3 - 16 = -13$

Now, $d_1 = a_2 - a_1 = -5 - (-1) = -5 + 1 = -4$

$d_2 = a_3 - a_2 = -9 - (-5) = -9 + 5 = -4$

$d_3 = a_4 - a_3 = -13 - (-9) = -13 + 9 = -4$

As $d_1 = d_2 = d_3 = -4$ so $a_1, a_2, a_3 \ldots a_n$ are in A.P.

So, $a = -1, \quad d = -4, \quad n = 20$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $S_{20} = \frac{20}{2}[2 \times (-1) + (20 - 1)(-4)]$

$= 10[-2 - 76] = 10[-78]$

$\Rightarrow$ $S_{20} = -780$

Hence, $a_1, a_2, a_3, \ldots a_n$ are in A.P. and $S_{20} = -780$.

Q24. In an A.P., if $S_n = n(4n + 1)$ then find the A.P.

Sol. Main concept used: $a_1 = S_1, \quad a_2 = S_2 - S_1, \quad a_3 = S_3 - S_2$

$S_n = n(4n + 1) = 4n^2 + n$ [Given]

$a_n = S_n - S_{n-1}$

$\Rightarrow$ $a_n = [4n^2 + n] - [4(n-1)^2 + (n-1)]$

$= 4n^2 + n - [4(n^2 + 1 - 2n) + n - 1]$

$= 4n^2 + n - [4n^2 + 4 - 8n + n - 1]$

$= 4n^2 + n - [4n^2 - 7n + 3]$

$= 4n^2 + n - 4n^2 + 7n - 3$

$\Rightarrow$ $a_n = 8n - 3$

$\therefore$ $a_1 = 8(1) - 3 = 8 - 3 = 5$

$a_2 = 8(2) - 3 = 16 - 3 = 13$

$a_3 = 8(3) - 3 = 24 - 3 = 21$

$a_4 = 8(4) - 3 = 32 - 3 = 29$

Hence, the required A.P. is 5, 13, 21, 29, ...

Q25. In an A.P. if $S_n = 3n^2 + 5n$ and $a_k = 164$, then find the value of k.

Sol. Main concept used: $a_n = S_n - S_{n-1}$

$S_n = 3n^2 + 5n$

$\therefore$ $S_{n-1} = 3(n-1)^2 + 5(n-1)$

$\Rightarrow \quad S_{n-1} = 3(n^2+1-2n)+5n-5$

$= 3n^2+3-6n+5n-5$

$\Rightarrow \quad S_{n-1} = 3n^2-n-2$

Now, $\quad a_n = S_n - S_{n-1}$

$\Rightarrow \quad a_n = 3n^2+5n-(3n^2-n-2)$

$\Rightarrow \quad a_n = 3n^2+5n-3n^2+n+2$

$\Rightarrow \quad a_n = 6n+2 \quad \Rightarrow \quad a_k = 6k+2$

$\Rightarrow \quad 164 = 6k+2 \quad \Rightarrow \quad 6k = 164-2$

$\Rightarrow \quad k = \frac{162}{6} \quad \Rightarrow \quad k = 27$

Q26. If S_n denotes the sum of first n terms of an A.P., then prove that $S_{12} = 3(S_8 - S_4)$.

Sol. Consider an A.P. whose first term and common difference are '*a*' and '*d*' respectively.

$$S_n = \frac{n}{2}[2a+(n-1)d] \quad \Rightarrow \quad S_{12} = \frac{12}{2}[2a+(12-1)d]$$

$\Rightarrow \quad S_{12} = 6[2a+11d] \quad$ ...(*i*)

and $\quad S_8 = \frac{8}{2}[2a+(8-1)d]$

$\Rightarrow \quad S_8 = 4[2a+7d] \quad$ ...(*ii*)

and $\quad S_4 = \frac{4}{2}[2a+(4-1)d]$

$\Rightarrow \quad S_4 = 2[2a+3d] \quad$ ...(*iii*)

Now, $\quad 3(S_8 - S_4) = 3[4(2a+7d)-2(2a+3d)]$ [Using eqns. (*ii*) and (*iii*)]

$= 3[8a+28d-4a-6d]$

$= 3[4a+22d]$

$= 3\times 2[2a+11d]$

$= 6[2a+11d] = S_{12}$ [Using eqn. (*i*)]

$\therefore \quad$ RHS = LHS

Hence, proved.

Q27. Find the sum of first 17 terms of an A.P. whose 4th and 9th terms are -15, and -30 respectively.

Sol. $a_4 = -15, \quad a_9 = -30, \quad S_{17} = ?$

Consider an A.P. whose Ist term and common difference are a and d respectively.

$a_4 = -15$ [Given]

$\Rightarrow \quad a+(4-1)d = -15$ $[\because a_n = a+(n-1)d]$

$\Rightarrow \quad a + 3d = -15$ ...(i)

Also, $a_9 = -30$ [Given]

$\Rightarrow \quad a + (9-1)d = -30$ $[\because a_n = a + (n-1)d]$

$\Rightarrow \quad a + 8d = -30$ ...(ii)

Subtracting (i) from (ii), we get

$$\begin{array}{r} a + 8d = -30 \quad \text{...(ii)} \\ a + 3d = -15 \quad \text{[From (i)]} \\ - \quad - \quad + \\ \hline 5d = -15 \end{array}$$

$\Rightarrow \quad d = \frac{-15}{5} = -3$

Now, $a + 3d = -15$ [From (i)]

$\Rightarrow \quad a + 3(-3) = -15$

$\Rightarrow \quad a = -15 + 9$

$\Rightarrow \quad a = -6$

$S_{17} = ?$

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{17} = \frac{17}{2}[2(-6) + (17-1)(-3)]$

$= \frac{17}{2}[-12 + 16(-3)] = \frac{17}{2}[-12 - 48]$

$= \frac{17}{2}(-60) = -17 \times 30$

$\Rightarrow \quad S_{17} = -510$

Q28. If sum of first 6 terms of an A.P. is 36 and that of the first 16 terms is 256, find the sum of the first 10 terms.

Sol. Consider the A.P. whose first term and common difference are 'a' and 'd' respectively.

$S_6 = 36$ [Given]

$\therefore \quad \frac{6}{2}[2a + (6-1)d] = 36$ $\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow \quad 2a + 5d = \frac{36}{3}$

$\Rightarrow \quad 2a + 5d = 12$ ...(i)

Also, $S_{16} = 256$ [Given]

$\Rightarrow \quad \frac{16}{2}[2a + (16-1)d] = 256$

$\Rightarrow \quad 2a + 15d = \frac{256}{8}$

$\Rightarrow \quad 2a + 15d = 32 \quad ...(ii)$

Subtracting (*i*) from (*ii*), we get

$$\begin{aligned} 2a + 15d &= 32 \quad ...(ii) \\ 2a + 5d &= 12 \quad [\text{From } (i)] \\ - \quad - \quad & \quad - \\ \hline 10d &= 20 \end{aligned}$$

$\Rightarrow \quad d = 2$

Now, $\quad 2a + 5d = 12 \quad$ [From (*i*)]

$\Rightarrow \quad 2a + 5(2) = 12$

$\Rightarrow \quad 2a = 12 - 10 \quad \Rightarrow \quad a = \frac{2}{2} \quad \Rightarrow \quad a = 1$

So, $\quad S_{10} = \frac{10}{2}[2a + (10-1)d]$

$= 5[2(1) + 9(2)] = 5[2 + 18] = 5[20] = 100$

$\Rightarrow \quad S_{10} = 100$

Hence, the sum of first 10 terms is 100.

Q29. Find the sum of all the 11 terms of an A.P. whose middle most term is 30.

Sol. Number of terms are 11, so $n = 11$

$$\text{Middle term} = \frac{11+1}{2} = \frac{12}{2} = 6\text{th term}$$

Also, middle term = 30 $\quad$ [Given]

$\therefore \quad a_6 = 30 \quad$ [Given]

$\Rightarrow \quad a + (6-1)d = 30 \quad [\because \; a_n = a + (n-1)d]$

$\Rightarrow \quad a + 5d = 30 \quad ...(i)$

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{11} = \frac{11}{2}[2a + (11-1)d] = \frac{11}{2}[2a + 10d]$

$= \frac{11 \times 2}{2}[a + 5d] \quad$ [Using (*i*)]

$= 11 \times 30$

$\Rightarrow \quad S_{11} = 330$

Hence, the sum of all 11 terms is 330.

Q30. Find the sum of last 10 terms of the A.P. 8, 10, 12, ..., 126.

Sol. To find out the sum of last 10 terms, we will reverse the order of the given A.P. and get 126, ..., 12, 10, 8

So, $\quad a = 126, \quad d = 10 - 12 = -2, \quad n = 10$

$\therefore \quad S_{10} = \frac{10}{2}[2(126) + (10-1)(-2)] \quad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$$= 5[252 + 9(-2)] = 5[252 - 18]$$
$$= 5 \times 234$$
$$\Rightarrow \quad S_{10} = 1170$$

Hence, the sum of 10 terms from the end of A.P. 8, 10, 12, ..., 126 is 1170.

Q31. Find the sum of first seven numbers which are multiples of 2 as well as of 9. **[Hint:** Take the L.C.M. of 2 and 9]

Sol. The numbers which are multiples of 2 as well as of 9 are 18, 36, 54, ... 7 terms

So, $n = 7, \quad a = 18, \quad d = 36 - 18 = 18$

$$\therefore \quad S_7 = \frac{7}{2}[2(18) + (7-1)(18)] \qquad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$
$$= \frac{7 \times 18}{2}[2 + 6]$$
$$= 7 \times 9 \times 8 = 7 \times 72$$
$$\Rightarrow \quad S_7 = 504$$

Hence, the sum of first 7 numbers which are multiple of 2 as well as 9, i.e., multiples of 18 is 504.

Q32. How many terms of the A.P.: –15, –13, –11, ... are needed to make the sum –55? Explain the reason for double answer.

Sol. Given A.P. is – 15, – 13, – 11, ...

$\therefore \quad S_n = -55, \quad a = -15, \quad n = ?$

$$d = -13 - (-15) = -13 + 15 = 2 \quad \Rightarrow \quad d = 2$$

But, $$S_n = -55$$
$$\Rightarrow \quad \frac{n}{2}[2a + (n-1)d] = -55$$
$$\Rightarrow \quad n[2(-15) + (n-1)(2)] = -55 \times 2$$
$$\Rightarrow \quad n[-30 + 2(n-1)] = -110$$
$$\Rightarrow \quad n[-30 + 2n - 2] + 110 = 0$$
$$\Rightarrow \quad -30n + 2n^2 - 2n + 110 = 0$$
$$\Rightarrow \quad 2n^2 - 32n + 110 = 0$$
$$\Rightarrow \quad n^2 - 16n + 55 = 0$$
$$\Rightarrow \quad n^2 - 11n - 5n + 55 = 0$$
$$\Rightarrow \quad n(n-11) - 5(n-11) = 0$$
$$\Rightarrow \quad (n-11)(n-5) = 0$$
$$\Rightarrow \quad n - 11 = 0 \quad \text{or} \quad n - 5 = 0$$
$$\Rightarrow \quad n = 11 \quad \text{or} \quad n = 5$$

So, 5 or 11 terms of A.P. are needed to make the sum – 55.

Q33. The sum of first n terms of an A.P. whose first term is 8 and the common difference is 20 is equal to sum of first $2n$ terms of another A.P. whose first term is -30, and common difference is 8. Find n.

Sol. For AP I: $a = 8, d = +20$ | For AP II: $a' = -30, d' = 8$

According to the question, $S_n = S'_{2n}$

$$\Rightarrow \quad \frac{n}{2}[2a+(n-1)d] = \frac{2n}{2}[2a'+(2n-1)d']$$

$$\Rightarrow \quad [2(8)+(n-1)20] = 2[2(-30)+(2n-1)8]$$

$$\Rightarrow \quad 2\times 8+n\times 20-20 = 2[-60+16n-8]$$

$$\Rightarrow \quad 16+20n-20 = 2[-68+16n]$$

$$\Rightarrow \quad 20n-4 = -136+32n$$

$$\Rightarrow \quad -32n+20n = -136+4$$

$$\Rightarrow \quad -12n = -132$$

$$\Rightarrow \quad n = \frac{132}{12} = 11$$

Hence, the required value of n is 11.

Q34. Kanika was given her pocket money on Jan. 1, 2008. She puts ₹ 1 on day 1, ₹ 2 on day 2, ₹ 3 on day 3, and continued doing so till the end of the month, from this money into her piggy bank. She also spent ₹ 204 of her pocket money, and found that at the end of the month she still had ₹ 100 with her. How much was her pocket money for the month?

Sol. Let the pocket money of Kanika for the month be ₹ x.

Out of x, the money which she deposited in piggy bank and spent = ₹ 204

Money put in piggy bank from Jan. 1 to Jan. 31 = $1+2+3+4+\cdots+31$

So, $a = 1$, $d = 1$, $n = 31$

$$\text{Now,} \quad S_{31} = \frac{31}{2}[2(1)+(31-1)(1)] \qquad \left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$$

$$= \frac{31}{2}[2+30]$$

$$\Rightarrow \quad S_{31} = \frac{31\times 32}{2} = 31\times 16 \quad \Rightarrow \quad S_{31} = 496$$

$\therefore$ Money deposited in piggy bank = ₹ 496

Money spent = ₹ 204

Money which she still have = ₹ 100

$\therefore \quad x - 496 - 204 = 100$

$$\Rightarrow \quad x = 100 + 496 + 204 = 800$$

Hence, her monthly pocket money is ₹ 800.

Q35. Yasmeen saves ₹ 32 during the first month, ₹ 36 in the second month and ₹ 40 in 3rd month. If she continues to save in this manner, in how many months will she save ₹ 2000?

Sol. During Ist month, savings of Yasmeen = ₹ 32
During IInd month, savings of Yasmeen = ₹ 36
During IIIrd month, savings of Yasmeen = ₹ 40
During IVth month, savings of Yasmeen = ₹ 44

$$\therefore \quad 32 + 36 + 40 + 44 + \dots = 2000$$

Also, $a = 32$, $\quad d = 36 - 32 = 4$

Now, $S_n = 2000$

$$\Rightarrow \quad \frac{n}{2}[2a + (n-1)d] = 2000 \quad \Rightarrow \quad n[2(32) + (n-1)4] = 2000 \times 2$$

$$\Rightarrow \quad n[64 + 4n - 4] = 4000 \quad \Rightarrow \quad n[4n + 60] = 4000$$

$$\Rightarrow \quad 4n[n + 15] = 4000 \quad \Rightarrow \quad n[n + 15] = \frac{4000}{4}$$

$$\Rightarrow \quad n^2 + 15n - 1000 = 0 \quad \Rightarrow \quad n^2 + 40n - 25n - 1000 = 0$$

$$\Rightarrow \quad n[n + 40] - 25[n + 40] = 0 \quad \Rightarrow \quad (n + 40)(n - 25) = 0$$

$$\Rightarrow \quad n + 40 = 0 \quad \text{or} \quad n - 25 = 0$$

$$\Rightarrow \quad n = -40 \quad \text{or} \quad n = 25$$

Rejecting $n = -40$, we have $n = 25$.

Hence, in 25 months she saves ₹ 2000.

EXERCISE 5.4

Q1. The sum of the first five terms of an A.P. and the sum of the first seven terms of the same A.P. is 167. If the sum of the first ten terms of this A.P. is 235, find the sum of its first twenty terms.

Sol. Consider an A.P. whose first term and the common difference are a and d respectively.

According to the question:

$$S_5 + S_7 = 167 \quad \text{[Given]}$$

$$\Rightarrow \quad \frac{5}{2}[2a + (5-1)d] + \frac{7}{2}[2a + (7-1)d] = 167$$

$$\Rightarrow \quad 5\{2a + 4d\} + 7\{2a + 6d\} = 167 \times 2$$

On multiplying both sides by $\frac{1}{2}$, we get

$$\frac{1}{2}[10a + 20d + 14a + 42d] = 167$$

$$\Rightarrow \quad \frac{1}{2}[24a + 62d] = 167$$

$$\Rightarrow \quad \frac{1}{2} \times 2[12a + 31d] = 167$$

$$\Rightarrow \quad 12a + 31d = 167 \qquad \text{...(i)}$$

Also, $$S_{10} = 235 \qquad \text{[Given]}$$

$$\Rightarrow \quad \frac{10}{2}[2a + (10-1)d] = 235$$

$$\Rightarrow \quad 5[2a + 9d] = 235$$

$$\Rightarrow \quad 2a + 9d = \frac{235}{5}$$

$$\Rightarrow \quad 2a + 9d = 47 \qquad \text{...(ii)}$$

Multiplying (*ii*) by 6, we have

$$12a + 54d = 282 \qquad \text{...(iii)}$$

Now, subtracting (*i*) from (*iii*), we get

$$\begin{array}{rl} 12a + 54d = 282 & \text{(iii)} \\ 12a + 31d = 167 & \text{[From (i)]} \\ - \quad - \qquad - & \\ \hline 23d = 115 & \end{array}$$

$$\Rightarrow \quad d = \frac{115}{23} \Rightarrow d = 5$$

Now, $$2a + 9d = 47 \qquad \text{[From (ii)]}$$

$$\Rightarrow \quad 2a + 9 \times 5 = 47$$

$$\Rightarrow \quad 2a = 47 - 45 \Rightarrow 2a = 2 \Rightarrow a = 1$$

$$\therefore \quad S_{20} = \frac{20}{2}[2a + (20-1)d] \quad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$= 10[2 \times (1) + 19(5)] = 10[2 + 95] = 10 \times 97$$

$$\Rightarrow \quad S_{20} = 970$$

Hence, the sum of first twenty terms is 970.

Q2. Find the

(*i*) sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

(*ii*) sum of those integers from 1 to 500 which are multiples of 2 as well as of 5.

(*iii*) sum of those integers from 1 to 500 which are multiples of 2 or 5.

[**Hint:** These numbers will be: multiples of 2 + multiples of 5 – multiples of 2 as well as of 5.]

Sol. (*i*) Integers which are multiples of 2 as well as 5 are multiples of 10, i.e., 10, 20, 30, ..., 490. [∵ Between 1 and 500]

$\therefore\ a = 10,\quad d = 10,\quad a_n = 490$

Now, $a_n = 490$ $\quad [\because a_n = a + (n-1)d]$

$\Rightarrow\ a + (n-1)d = 490$

$\Rightarrow\ 10 + (n-1)10 = 490$

$\Rightarrow\ 1 + (n-1) = \frac{490}{10}$

$\Rightarrow\ n = 49$

$\because\ S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore\ S_{49} = \frac{49}{2}[2 \times 10 + (49-1)10]$

$= \frac{49}{2} \times 10[2 + 48] = 49 \times 5 \times 50$

$\Rightarrow\ S_{49} = 12250$

(*ii*) Multiple of 2 as well as of 5 are multiples of $2 \times 5 = 10$. Multiples of 10 from (not between) 1 to 500 are 10, 20, 30, 40, ..., 500.

$\therefore\ a = 10,\quad d = 10,\quad a_n = 500$

Now, $a_n = a + (n-1)d = 500$

$\Rightarrow\ 10 + (n-1)10 = 500$

$\Rightarrow\ 1 + n - 1 = 50$

$\Rightarrow\ n = 50$

So, $S_{50} = \frac{50}{2}[2 \times 10 + (50-1)10]$

$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= \frac{50 \times 10}{2}[2 + 49] = 50 \times 5 \times 51$

$\Rightarrow\ S_{50} = 12750$

(*iii*) Sum of integers which are multiples of 2 or 5 only (not of 10)

= Sum of integers which are multiples of 2 + Sum of integers which are multiples of 5 − Sum of integers which are multiples of 10

$= (2 + 4 + 6 + \ldots + 500) + (5 + 10 + 15 + 20 + \ldots + 500) - (10 + 20 + 30 + \ldots + 500)$

$= S_1 + S_2 - S_3$

For $S_1 = 2 + 4 + 6 + \ldots + 500$, we have

$a = 2,\quad d = 2,\quad a_n = 500$

$\therefore\ a + (n-1)d = 500 \Rightarrow 2 + (n-1)2 = 500$

$\Rightarrow\ 2[1 + (n-1)] = 500 \Rightarrow 2n = 500$

$\Rightarrow \quad n = 250$

$\therefore \quad S_1 = S_{250}$

$\Rightarrow \quad S_1 = S_{250} = \frac{250}{2}[2 \times 2 + (250-1)(2)]$

$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= 125[4 + 249 \times 2]$

$\Rightarrow \quad S_1 = 125[4 + 498]$

$\Rightarrow \quad S_1 = 125 \times 502 = 62750$

For $S_2 = 5 + 10 + 15 + 20 + \cdots + 500$, we have

$a = 5, \quad d = 5, \quad a_n = 500$

$\therefore \quad a + (n-1)d = 500 \quad \Rightarrow \quad 5 + (n-1)5 = 500$

$\Rightarrow \quad 5[1 + n - 1] = 500 \quad \Rightarrow \quad n = 100$

$\therefore \quad S_2 = S_{100}$

$\Rightarrow \quad S_2 = S_{100} = \frac{100}{2}[2a + (n-1)d]$

$= 50[2(5) + (100-1)5] = 50[10 + 99 \times 5]$

$= 50[10 + 495] = 50 \times 505$

$\Rightarrow \quad S_2 = 25250$

For $S_3 = 10 + 20 + 30 + \ldots + 500$, we have

$a = 10, \quad d = 10, \quad a_n = 500$

$\therefore \quad a + (n-1)d = 500 \qquad [\because \quad a_n = a + (n-1)d]$

$\Rightarrow \quad 10 + (n-1)10 = 500$

$\Rightarrow \quad 10[1 + n - 1] = 500$

$\Rightarrow \quad n = \frac{500}{10} = 50$

Now, $\quad S_3 = S_{50} = \frac{50}{2}[2a + (n-1)d]$

$= 25[2(10) + (50-1)10]$

$= 25[20 + 490]$

$= 25 \times 510$

$\Rightarrow \quad S_3 = 12750$

Hence, the sum of the required integers $= S_1 + S_2 - S_3$

$= 62750 + 25250 - 12750$

$= 88000 - 12750 = 75250$

Q3. The 8th term of an A.P. is half its second term and 11th term exceeds one third of its fourth term by '1'. Find the 15th term.

Sol. Consider the first term and common difference as a and d respectively.

$$a_8 = \frac{1}{2}a_2 \qquad \text{[Given]}$$

$$\Rightarrow \quad a + (8-1)d = \frac{1}{2}[a + (2-1)d] \qquad [\because a_n = a + (n-1)d]$$

$$\Rightarrow \quad 2(a + 7d) = a + d$$

$$\Rightarrow \quad 2a + 14d - a - d = 0$$

$$\Rightarrow \quad a + 13d = 0 \qquad ...(i)$$

Now, $$a_{11} = \frac{1}{3}a_4 + 1 \qquad \text{[Given]}$$

$$\Rightarrow \quad a + (11-1)d = \frac{1}{3}[a + (4-1)d] + 1$$

$$\Rightarrow \quad (a + 10d) = \frac{1}{3}(a + 3d) + 1$$

$$\Rightarrow \quad 3(a + 10d) = a + 3d + 3$$

$$\Rightarrow \quad 3a + 30d - a - 3d = 3$$

$$\Rightarrow \quad 2a + 27d = 3 \qquad ...(ii)$$

Multiplying (*i*) by 2, we have

$$2a + 26d = 0 \qquad ...(iii)$$

Now, subtraction (*iii*) from (*ii*), we get

$$2a + 27d = 3 \qquad ...(ii)$$

$$2a + 26d = 0 \qquad (iii)$$

$$- \quad - \quad -$$

$$d = 3$$

Now, $$a + 13d = 0 \qquad \text{[From (i)]}$$

$$\Rightarrow \quad a + 13 \times 3 = 0$$

$$\Rightarrow \quad a = -39$$

Now, we know that $a_n = a + (n-1)d \Rightarrow a_{15} = -39 + (15-1)3$

$$= -39 + 14 \times 3 = -39 + 42$$

$$\Rightarrow \quad a_{15} = 3$$

Q4. An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the A.P.

Sol. Consider an A.P. whose first term and common difference are 'a' and 'd' respectively.

Total terms = 37

The middle most term $= \dfrac{37+1}{2} = \dfrac{38}{2} = 19\text{th term}$

So, the sum of the three middle most terms $= a_{18} + a_{19} + a_{20}$

$$= a + (18-1)d + a + (19-1)d + a + (20-1)d$$
$$= 3a + 17d + 18d + 19d$$
$\Rightarrow \quad 225 = 3a + 54d \quad ...(i)$

$\Rightarrow \quad a + 18d = 75$

The sum of the last three terms $= a_{37} + a_{36} + a_{35} = 429$ [Given]

$$= a + (37-1)d + a + (36-1)d + a + (35-1)d = 429$$
$\Rightarrow \quad 3a + 36d + 35d + 34d = 429$

$\Rightarrow \quad 3a + 105d = 429$

$\Rightarrow \quad a + 35d = 143 \quad ...(ii)$

Now, subtracting (*i*) from (*ii*), we get

$$\begin{aligned} a + 35d &= 143 \quad ...(ii) \\ a + 18d &= \ \ 75 \quad (i) \\ - \quad - \quad & \quad - \\ \hline 17d &= \ \ 68 \end{aligned}$$

$\Rightarrow \quad d = 4$

Now, $\quad a + 18d = 75$ [Using (*i*)]

$\Rightarrow \quad a + 18 \times 4 = 75$

$\Rightarrow \quad a = 75 - 72 = 3$

$\therefore \ a = 3$ and $d = 4$

Hence, the required A.P. is $a, a+d, a+2d, a+3d... = 3, 7, 11, 15 ...$

Q5. Find the sum of the integers between 100 and 200 that are (*i*) divisible by 9 (*ii*) not divisible by 9.

[**Hint (*ii*):** These numbers will be: Total numbers – Total numbers divisible by 9.]

Sol. (*i*) Numbers between 100 – 200 divisible by 9 are 108, 117, 125, ...198

Here, $a = 108$, $d = 117 - 108 = 9$ and $a_n = 198$

$\Rightarrow \quad a + (n-1)d = 198 \quad [a_n = a + (n-1)d]$

$\Rightarrow \ 108 + (n-1)9 = 198 \quad \Rightarrow \quad 9[12 + n - 1] = 198$

$\Rightarrow \quad 11 + n = \frac{198}{9} \quad \Rightarrow \quad n = 22 - 11 \quad \Rightarrow \quad n = 11$

Now, $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{11} = \frac{11}{2}[2(108) + (11-1)(9)]$

$$= \frac{11}{2}[216 + 99 - 9] = \frac{11}{2}[216 + 90]$$
$$= \frac{11}{2} \times 306$$

$\Rightarrow \quad S_{11} = 1683$

(ii) Numbers between 100 and 200 = 101, 102, 103, ...199
Here, $a = 101$, $d = 1$, $a_n = 199$

$\Rightarrow \quad a + (n-1)d = 199 \quad \Rightarrow \quad 101 + (n-1)(1) = 199$

$\Rightarrow \quad (n-1) = 199 - 101 = 98$

$\Rightarrow \quad n = 99$

Now, $S_{99} = \frac{99}{2}[2 \times 101 + (99-1)(1)]$

$$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$= \frac{99}{2}[202 + 98] = \frac{99}{2} \times 300 = 99 \times 150 = 14850$$

So, the sum of integers between 100 and 200 which are not divisible by 9 = 14850 − 1683 = 13167.

Q6. The ratio of the 11th term to the 18th term of an A.P. is 2 : 3. Find the ratio of the 5th term to the 21st term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms.

Sol. Consider an A.P. whose first term and common difference are a and d respectively.

$$a_{11} : a_{18} = 2 : 3 \qquad \text{[Given]}$$

$$\Rightarrow \quad \frac{a + 10d}{a + 17d} = \frac{2}{3} \qquad [\because a_n = a + (n-1)d]$$

$\Rightarrow \quad 3a + 30d = 2a + 34d$

$\Rightarrow \quad 3a - 2a = 34d - 30d$

$\Rightarrow \quad a = 4d$

To find:

$$\frac{a_5}{a_{21}} = \frac{a + 4d}{a + 20d} = \frac{4d + 4d}{4d + 20d} = \frac{8d}{24d} = \frac{1}{3}$$

$\therefore \quad a_5 : a_{21} = 1 : 3$

Now, $$\frac{S_5}{S_{21}} = \frac{\frac{5}{2}[2a + (5-1)d]}{\frac{21}{2}[2a + (21-1)d]} = \frac{5[2(4d) + 4d]}{21[2(4d) + 20d]} = \frac{5[8d + 4d]}{21[8d + 20d]}$$

$$= \frac{5 \times 12d}{21 \times 28d} = \frac{5}{7 \times 7} = \frac{5}{49} = 5 : 49$$

$\therefore \quad S_5 : S_{21} = 5 : 49$

Q7. Show that the sum of an A.P. whose first term is a, the second term b and the last term c, is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$

Sol. Here, a (Ist term) $= a$, $d = (b - a)$, $a_n = c$

As $a_n = c$

$\Rightarrow \quad a + (n-1)d = c \qquad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad (n-1)(b-a) = c - a$

$$\Rightarrow \quad (n-1) = \frac{(c-a)}{b-a}$$

$$\Rightarrow \quad n = \frac{c-a}{b-a} + 1 = \frac{c-a+b-a}{b-a}$$

$$\Rightarrow \quad n = \frac{(b+c-2a)}{b-a} \qquad ...(i)$$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{(b+c-2a)}{2(b-a)}\left[2a + \left\{\frac{b+c-2a}{b-a} - 1\right\}(b-a)\right] \qquad \text{[Using } (i)]$$

$$= \frac{(b+c-2a)}{2(b-a)}\left[2a + \left\{\frac{b+c-2a-b+a}{(b-a)}\right\} \times (b-a)\right]$$

$$= \frac{(b+c-2a)}{2(b-a)}[2a + c - a]$$

$$\Rightarrow \quad S_n = \frac{(b+c-2a)}{2(b-a)}(a+c)$$

Hence proved.

Q8. Solve the equation $-4 + (-1) + 2 + \ldots + x = 437$.

Sol. Given series is $-4 + (-1) + 2 + \ldots + x$

So, $d_1 = -1 - (-4) = -1 + 4 = 3$, $d_2 = 2 - (-1) = 2 + 1 = 3$

$\therefore$ Given list of numbers are in A.P. $\qquad [\because d = d_1 = d_2 = 3]$

Here, $\quad a = -4$ and $a_n = x$

As $a_n = x$

$\Rightarrow \quad a + (n-1)d = x \qquad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad -4 + (n-1)(3) = x$

$\Rightarrow \quad (n-1)3 = x + 4$

$$\Rightarrow \quad (n-1) = \frac{x+4}{3}$$

$$\Rightarrow \quad n = \frac{x+4}{3} + 1 = \frac{x+4+3}{3}$$

$\Rightarrow \quad n = \dfrac{x+7}{3}$ ...(i)

$\therefore \quad S_n = \dfrac{n}{2}[2a+(n-1)d]$

$\Rightarrow \quad S_n = \dfrac{(x+7)}{(2\times 3)}\left[2(-4)+\dfrac{(x+4)3}{3}\right]$ [Using (i)]

$= \dfrac{(x+7)}{6}[-8+x+4] \Rightarrow S_n = \dfrac{(x+7)(x-4)}{6}$

But, $S_n = 437 \Rightarrow \dfrac{(x+7)(x-4)}{6} = 437$

$\Rightarrow \quad x^2+3x-28 = 437\times 6$

$\Rightarrow \quad x^2+3x-28-2622 = 0$

$\Rightarrow \quad x^2+3x-2650 = 0$

$\Rightarrow \quad x^2+53x-50x-2650 = 0$

$\Rightarrow \quad x(x+53)-50(x+53) = 0$

$\Rightarrow \quad (x+53)(x-50) = 0$

$\Rightarrow \quad x = -53$ or $x = 50$

Rejecting the negative value $x = -53$, we have $x = 50$.

So, $x = 50$ is the required value as forward terms are positive.

Q9. Jaspal Singh repays his total loan of ₹ 118000 by paying every month starting with the first instalment of ₹ 1000. If he increases the instalment by ₹ 100 every month, what amount will be paid by him in the 30th instalment? What amount of loan does he still have to pay after the 30th instalment?

Sol. Monthly instalment paid by Jaspal Singh are 1000, 1100, 1200, ... 30 terms

$\therefore \; a = 1000, \quad d = 100, \quad a_n = ?, \quad n = 30$

$\Rightarrow \quad a_n = a+(n-1)d = 1000+(30-1)\,100$

$= 100\,[10+29] = 3900$

So, the amount paid by him in 30th instalment = ₹ 3900.

Total amount of all 30 instalments paid

$= 1000+1100+1200+\ldots+3900$

Here, $a = 1000, \quad d = 100, \quad n = 30$

$\therefore \quad S_n = \dfrac{n}{2}[2a+(n-1)d] \Rightarrow S_{30} = \dfrac{30}{2}[2\times 1000+(30-1)100]$

$= 15\,[2000+2900]$

$\Rightarrow \quad S_{30} = 15 \times 4900 = ₹\,73500$

So, the loan amount left after 30th instalment

$= ₹\,118000 - ₹73500 = ₹\,44500$

Hence, he has to pay ₹ 44500 after 30th instalment.

Q10. The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags.

Ruchi kept her books where the flags were stored. She could carry only one flag at a time. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag?

Sol. 27 flags are to be fixed at intervals of 2 m.

Position of the middle most flag $= \frac{27+1}{2}$ th flag $= \frac{28}{2}$ th flag = 14th flag

This means that 13 flags are to be fixed before the middle most 14th flag and 13 flags are to be fixed after the 14th flag.

Distance between flags = 2 m

Distance covered by placing a first flag = 2 + 2 = 4 m

Distance covered to place IInd flag = 4 + 4 = 8 m

Distance covered to place IIIrd flag = 6 + 6 = 12 m

So, the total distance covered to place 13 flags on either side is given by

$$S_{13} = 4 + 8 + 12 + \ldots \text{ 13 terms}$$

Here, $a = 4, \quad d = 4, \quad n = 13$

$$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d] \quad \Rightarrow \quad S_{13} = \frac{13}{2}[2(4) + (13-1)(4)]$$

$$= \frac{13}{2}[8 + 48] = \frac{13}{2} \times 56 = 13 \times 28$$

$$\Rightarrow \quad S_{13} = 364$$

Distance covered by Ruchi for other side 13 flags = 364 m

Hence, the total distance to place 27 flags and pickup her books

$= 364 \times 2 = 728$ m

Maximum distance which she travelled carrying a flag = Distance covered in fixing Ist or 27th flag

$= (13 \times 2)$ m = 26 m.

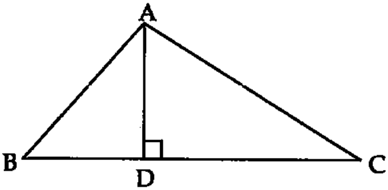
❑❑❑

6 Triangles

EXERCISE 6.1

Choose the correct answer from the given four options:

Q1. In the given figure, if $\angle BAC = 90°$ and $AD \perp BC$. Then,



(*a*) $BD \cdot DC = BC^2$
(*b*) $AB \cdot AC = BC^2$
(*c*) $BD \cdot CD = AD^2$
(*d*) $AB \cdot AC = AD^2$

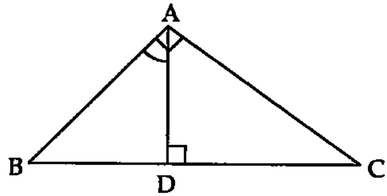
Sol. (*c*): In ΔADC and ΔADB,

$\angle BDA = \angle ADC = 90°$ [Given]

$\angle B = \angle DAC = (90° - C)$

$\therefore \quad \Delta ADB \sim \Delta CDA$

[By AA similarity critierion]



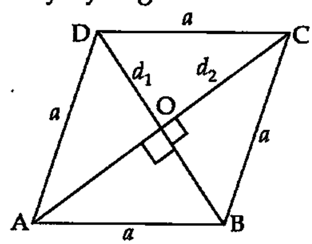
$$\Rightarrow \quad \frac{AD}{CD} = \frac{AB}{CA} = \frac{DB}{DA}$$

$\therefore \quad AD^2 = BD \cdot DC$

Q2. The lengths of the diagonals of a rhombus are 16 cm and 12 cm. Then, the length of the side of the rhombus is

(*a*) 9 cm (*b*) 10 cm (*c*) 8 cm (*d*) 20 cm

Sol. (*b*): Let the length of the side of the rhombus is *a* cm.
As the diagonals of rhombus bisect at 90° so by Pythagoras theorem in right angled ΔOAB,



$$a^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$$

$$= \left(\frac{12}{2}\right)^2 + \left(\frac{16}{2}\right)^2$$

$$= (6)^2 + (8)^2 = 36 + 64$$

$\Rightarrow \quad a^2 = 100$

$\Rightarrow \quad a = 10$ cm

Q3. If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true?

(*a*) $BC \cdot EF = AC \cdot FD$ (*b*) $AB \cdot EF = AC \cdot DE$
(*c*) $BC \cdot DE = AB \cdot EF$ (*d*) $BC \cdot DE = AB \cdot FD$

Sol. (*c*): $\Delta ABC \sim \Delta EDF$ [Given]

$$\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF} \quad ...(i)$$

So, every statement will be true if it satisfies the above relation, *i.e.*, LHS from option and RHS from (*i*).

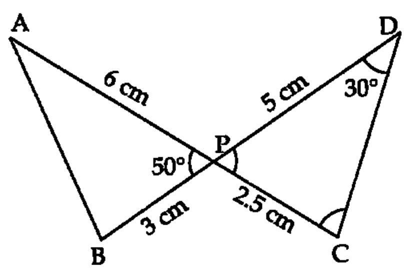
(*a*) $BC \cdot EF = AC \cdot DF$ True
(*b*) $AB \cdot EF = AC \cdot DE$ True
(*c*) $BC \cdot DE = AB \cdot EF$ False
(*d*) $BC \cdot DE = AB \cdot DF$ True

Q4. If in two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

(*a*) $\Delta PQR \sim \Delta CAB$ (*b*) $\Delta PQR \sim \Delta ABC$
(*c*) $\Delta CBA \sim \Delta PQR$ (*d*) $\Delta BCA \sim \Delta PQR$

Sol. (*a*): Here, vertex P corresponds to vertex C, vertex Q corresponds to vertex A and vertex R corresponds to vertex B. Symbolically, we write the similarity of these two triangles as $\Delta PQR \sim \Delta CAB$.
Hence, (*a*) is the correct answer.

Q5. In the given figure, two line segments AC and BD intersect each other at P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, $\angle APB = 50°$ and $\angle CDP = 30°$, then $\angle PBA$ is equal to



(*a*) 50° (*b*) 30°
(*c*) 60° (*d*) 100°

Sol. (*d*): Considering ΔAPB and ΔDPC

$$\frac{PA}{PC} = \frac{6.0}{2.5} = \frac{12}{5}$$

$$\frac{PB}{PD} = \frac{3}{5} \neq \frac{PA}{PC}$$

So, the above solution is rejected.

Now, $$\frac{PA}{PD} = \frac{6}{5}$$

$$\frac{PB}{PC} = \frac{3.0}{2.5} = \frac{6}{5}$$

$\Rightarrow$ $$\frac{PA}{PD} = \frac{PB}{PC}$$

$\angle APB = \angle CPD = 50°$ [Vertically opp $\angle$s]

$\therefore$ $\Delta APB \sim \Delta DPC$ [By SAS similarity criterion]

$\angle PBA = \angle PCD$ [$\because$ Corresponding $\angle$s of similar Δs are equal]

In ΔDPC, $\angle DPC = \angle APB = 50°$ [Vertically opp. $\angle$s]

$\angle D = 30°$

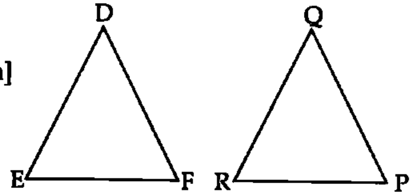
$\therefore \quad \angle PCD = \angle C = 180° - 50° - 30° = 180 - 80° = 100°$

$\Rightarrow \quad \angle PBA = 100°$ verifies the option (*d*).

Q6. If in two triangles DEF and PQR, $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not true?

(*a*) $\frac{EF}{PR} = \frac{DF}{PQ}$ (*b*) $\frac{DE}{PQ} = \frac{EF}{RP}$ (*c*) $\frac{DE}{QR} = \frac{DF}{PQ}$ (*d*) $\frac{EF}{RP} = \frac{DE}{QR}$

Sol. (*b*): In ΔDEF and ΔPQR,



$\left.\begin{aligned} \angle D &= \angle Q \\ \angle E &= \angle R \end{aligned}\right\}$ [Given]

$\therefore \quad \angle F = \angle P$

$\therefore \quad \Delta DEF \sim \Delta QRP$

$\therefore \quad \frac{DE}{QR} = \frac{DF}{QP} = \frac{EF}{RP}$

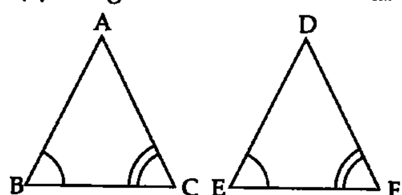
Hence, (*b*) is not true.

Q7. In ΔABC and ΔDEF, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$. Then, the two triangles are

(*a*) congruent but not similar (*b*) similar but not congruent

(*c*) neither congruent nor similar (*d*) congruent as well as similar

Sol. (*b*): In ΔABC and ΔDEF,



$\left.\begin{aligned} \angle B &= \angle E \\ \angle C &= \angle F \end{aligned}\right\}$ [Given]

$\therefore \quad \Delta ABC \sim \Delta DEF$

[By AA similarity criterion]

So, AB and DE sides are corresponding sides.

But, $AB = 3DE$ [Given]

So, ΔABC cannot be congruent to ΔDEF.

So, Δs are similar but not congruent.

Q8. It is given that $\Delta ABC \sim \Delta PQR$, with $\frac{BC}{QR} = \frac{1}{3}$. Then $\frac{ar(\Delta PRQ)}{ar(\Delta BCA)}$ is equal to

(*a*) 9 (*b*) 3 (*c*) $\frac{1}{3}$ (*d*) $\frac{1}{9}$

Sol. (*a*): $\Delta ABC \sim \Delta PQR$ [Given]

$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$ [By area theorem]

or $= \frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \frac{9}{1}$

Hence, verifies option (*a*).

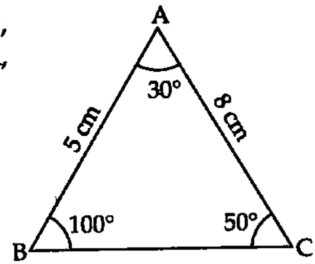
Q9. It is given that $\Delta ABC \sim \Delta DFE$, $\angle A = 30°$, $\angle C = 50°$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm, then which of the following is true?

(*a*) $DE = 12$ cm, $\angle F = 50°$

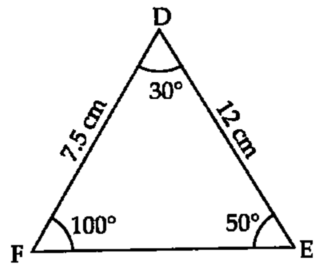
(*b*) $DE = 12$ cm, $\angle F = 100°$

(*c*) $EF = 12$ cm, $\angle D = 100°$

(*d*) $EF = 12$ cm, $\angle D = 30°$



Sol. (*b*): $\Delta ABC \sim \Delta DEF$ [Given]



$$\therefore \quad \frac{AB}{DF} = \frac{AC}{DE} = \frac{BC}{FE}$$

$$\Rightarrow \quad \frac{5}{7.5} = \frac{8}{DE} = \frac{BC}{EF}$$

$$\Rightarrow \quad DE = \frac{8 \times 7.5}{5} = 12 \text{ cm}$$

Now, $\angle A = \angle D = 30°$

$\angle B = \angle F = 180° - 30° - 50° = 100°$

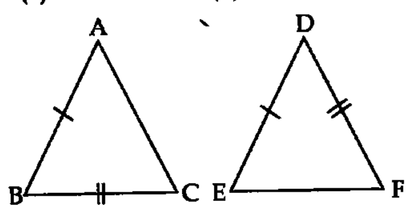
$\angle C = \angle E = 50°$

$\therefore$ Verifies the option (*b*) i.e., $DE = 12$ cm, $\angle F = 100°$.

Q10. If in ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when

(*a*) $\angle B = \angle E$ (*b*) $\angle A = \angle D$ (*c*) $\angle B = \angle D$ (*d*) $\angle A = \angle F$

Sol. (*c*): In ΔABC and ΔDEF,



$$\frac{AB}{DE} = \frac{BC}{FD}$$

Angle formed by AB and BC is $\angle B$.

Angle formed by DE and FD is $\angle D$.

So, $\angle B = \angle D$

$\therefore$ $\Delta ABC \sim \Delta EDF$ [By SAS similarity criterion]

Hence, (*c*) is the correct answer.

Q11. If $\Delta ABC \sim \Delta QRP$, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{9}{4}$, $AB = 18$ cm and $BC = 15$ cm, then PR is equal to

(*a*) 10 cm (*b*) 12 cm (*c*) $\frac{20}{3}$ cm (*d*) 8 cm

Sol. (*a*): $\because$ $\Delta ABC \sim \Delta QRP$ [Given]

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \frac{BC^2}{RP^2} = \frac{AB^2}{QR^2} \quad \text{[By area theorem]}$$

$$\Rightarrow \quad \frac{9}{4} = \frac{15^2}{RP^2} = \frac{18^2}{QR^2}$$

$\Rightarrow \qquad RP^2 = \dfrac{15 \times 15 \times 4}{9}$

$\Rightarrow \qquad RP^2 = 100$

$\Rightarrow \qquad RP = 10$ cm

Hence, verifies the option (*a*).

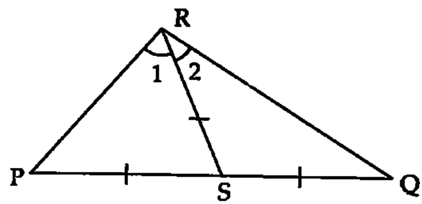
Q12. If S is a point on side PQ, of a ΔPQR such that PS = SQ = RS, then

(*a*) $PR \cdot QR = RS^2$

(*b*) $QS^2 + RS^2 = QR^2$

(*c*) $PR^2 + QR^2 = PQ^2$

(*d*) $PS^2 + RS^2 = PR^2$



Sol. (*c*): In ΔPQR,

$$PS = SQ = RS$$

Now, in ΔPSR,

$$PS = SR$$

$\therefore \qquad \angle P = \angle 1$

[Angles opposite to equal sides in a triangle are equal]

Similarly, in $\angle SRQ$,

$$\angle Q = \angle 2$$

Now, in ΔPQR,

$\angle P + \angle Q + \angle R = 180°$ [Angle sum property of a triangle]

$\Rightarrow \qquad \angle 1 + \angle 2 + (\angle 1 + \angle 2) = 180°$

$\Rightarrow \qquad 2(\angle 1 + \angle 2) = 180°$

$\Rightarrow \qquad \angle 1 + \angle 2 = 90°$

$\Rightarrow \qquad \angle PRQ = 90°$

By Pythagoras theorem, we have

$$PQ^2 = PR^2 + RQ^2$$

Hence, verifies the option (*c*).

EXERCISE 6.2

Q1. Is the triangle with sides 25 cm, 5 cm, and 24 cm a right triangle? Give reasons for your answer.

Sol. False: By converse of Pythagoras theorem, this Δ will be right angle triangle if

$$(25)^2 = (5)^2 + (24)^2$$

$\Rightarrow \qquad 625 = 25 + 576$

$\Rightarrow \qquad 625 \neq 601$

So, the given triangle is not right angled triangle.

Q2. It is given that $\Delta DEF \sim \Delta RPQ$. Is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$? Why?

Sol. False: When $\Delta DEF \sim \Delta RPQ$, each angle of a triangle will be equal to the corresponding angle of similar triangle so

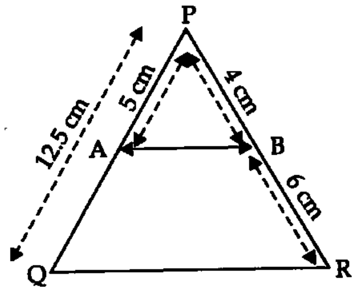
$$\angle D = \angle R$$

$$\angle E = \angle P$$
$$\angle F = \angle Q$$

So, $\angle D = \angle R$ is true but $\angle F \neq \angle P$.

Hence, it is not true that $\angle D = \angle R$ and $\angle F = \angle P$.

Q3. A and B are respectively the points on the sides PQ and PR of a ΔPQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm and PB = 4 cm. Is AB || QR? Give reasons for your answer.



Sol. True: By converse of BPT, AB will be parallel to QR if AB, divides PQ and PR in the same ratio i.e.,

$$\frac{AP}{AQ} = \frac{PB}{BR}$$

$$\Rightarrow \quad \frac{5}{12.5 - 5} = \frac{4}{6}$$

$$\Rightarrow \quad \frac{5.0}{7.5} = \frac{2}{3} \quad \text{or} \quad \frac{2}{3} = \frac{2}{3}$$

So, AB is parallel to QR. Hence, the given statement AB || QR is true.

Q4. In the given figure, BD and CE intersect each other at P. Is $\Delta PBC \sim \Delta PDE$? Why?

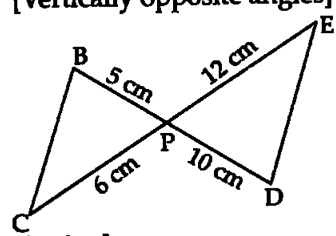
Sol. True: In ΔPBC and ΔPDE, we have

$$\angle BPC = \angle DPE \quad \text{[Vertically opposite angles]}$$

$$\frac{BP}{PD} = \frac{5}{10} = \frac{1}{2}$$

$$\frac{PC}{PE} = \frac{6}{12} = \frac{1}{2}$$

$$\therefore \quad \frac{BP}{PD} = \frac{PC}{PE}$$



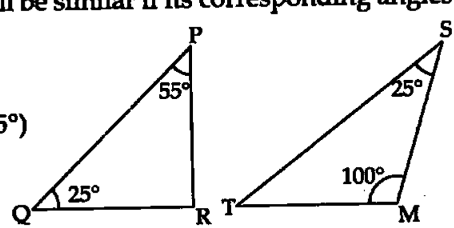
Hence, $\Delta BPC \sim \Delta DPE$ [By SAS similarity criterion]

Hence, the given statement is true.

Q5. In ΔPQR and ΔMST, $\angle P = 55°$, $\angle Q = 25°$, $\angle M = 100°$, $\angle S = 25°$. Is $\Delta QPR \sim \Delta TSM$? Why?

Sol. False: ΔQPR and ΔTSM will be similar if its corresponding angles are equal

$$\angle Q = 25°$$
$$\angle P = 55°$$
$$\Rightarrow \quad \angle R = 180° - (25° + 55°)$$
$$= 180° - 80°$$
$$\Rightarrow \quad \angle R = 100°$$
$$\angle S = 25°$$



$$\angle M = 100^\circ$$

$\Rightarrow$ $$\angle T = 180^\circ - (100^\circ + 25^\circ) = 55^\circ$$

$\therefore$ $$\angle Q \neq \angle T$$
$$\angle P \neq \angle S$$
$$\angle R \neq \angle M$$

So, ΔQPR is not similar to ΔTSM. So, the given statement $\Delta QPR \sim \Delta TSM$ is false.

Q6. Is the following statement true? Why?

"Two quadrilaterals are similar if their corresponding angles are equal".

Sol. False: Two quadrilaterals will be similar if their corresponding angles as well as ratio of sides are also equal. So, the given statement is false.

Q7. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

Sol. True: Let the two sides of ΔABC are $AB = 3$ cm, $AC = 4$ cm and perimeter $AB + BC + AC = 13$ cm, then $BC = 13 - 7 = 6$ cm.

According to the question, the sides of another ΔDEF are

$$DE = 3 \times 3 = 9,$$
$$DF = 3 \times 4 = 12,$$

and $$DE + DF + EF = 3 \times 13 = 39$$

So, $$EF = 39 - 12 - 9 = 18$$

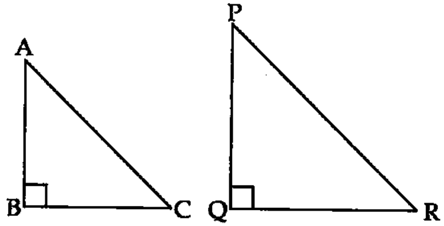
$\therefore$ $$\frac{DE}{AB} = \frac{9}{3} = \frac{3}{1}$$
$$\frac{DF}{AC} = \frac{12}{4} = \frac{3}{1}$$
$$\frac{EF}{BC} = \frac{18}{6} = \frac{3}{1}$$

$\therefore$ $$\frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC} = \frac{3}{1}$$

As the ratio of corresponding sides in two Δs are same then $\Delta DEF \sim \Delta ABC$ by SSS similarity criterion.

Hence, the triangles are similar or the given statement is true.

Q8. If in two right triangles, one of the acute angles of one triangle is equal to an acute angle of the other triangle, can you say that two triangles will be similar? Why?



Sol. True: In ΔABC and ΔPQR,

$\angle B = \angle Q = 90°$ [Given]

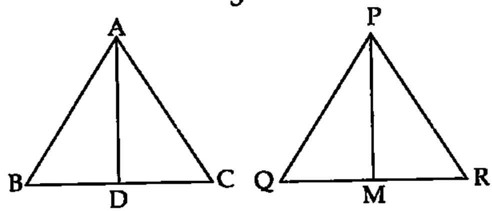
$\angle C = \angle R$ [Given]

$\therefore$ $\Delta ABC \sim \Delta PQR$ [By AA similarity criterion]

Hence, the statement that two triangles are similar is true.

Q9. The ratio of the corresponding altitudes of two similar triangles is $\frac{3}{5}$. Is it correct to say that ratio of their areas is $\frac{6}{5}$? Why?

Sol. False: If two triangles are similar, then the ratio of areas of two triangles will be equal to the square of the ratios of their corresponding sides or altitudes or angle bisectors,



If $\Delta ABC \sim \Delta PQR$, then

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AD}{PM}\right)^2$$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{3}{5}\right)^2$$

$$= \frac{9}{25} \neq \frac{6}{5}$$

So, the given statement is false.

Q10. D is the point on side QR of ΔPQR such that $PD \perp QR$. Will it be correct to say that $\Delta PQD \sim \Delta RPD$? Why?

Sol. False: In ΔPDQ and ΔPDR,

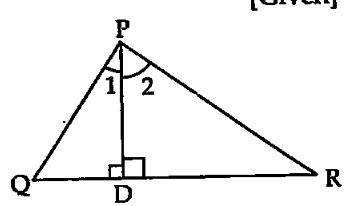
$PD \perp QR$ [Given]

$\therefore$ $\angle PDQ = \angle PDR = 90°$

PD does not bisect $\angle P$.

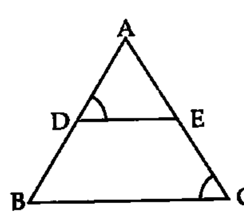
$\therefore$ $\angle 1 \neq \angle 2$

$\angle Q \neq \angle R$ [$\because PQ \neq QR$]



Any ratio of sides are also not equal. So, ΔPDQ is not similar to ΔPDR. Hence, the given statement is false.

Q11. In the given figure, $\angle D = \angle C$, then is it true that $\Delta ADE \sim \Delta ACB$? Why?



Sol. True: In ΔADE and ΔABC,

$\angle D = \angle C$ [Given]

$\angle A = \angle A$ [Common]

$\therefore \Delta ADE \sim \Delta ACB$ [By AA similarity criterion]

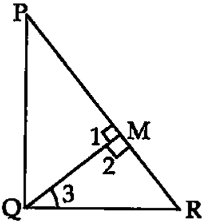
Q12. Is it true to say that if in two triangls, an angle of one triangle is equal to an angle of another triangle and, two sides of one triangle are proportional to the two sides of the other triangle, then triangles are similar? Give reasons for your answer.

Sol. False: Here, the ratio of two sides of a triangle is equal to the ratio of corresponding two sides of other triangle, although the one angle of one triangle is equal to one angle of other triangle but, not included angles of proportional sides are equal.

So, triangles are not similar. Hence, the given statement is false.

EXERCISE 6.3

Q1. In a ΔPQR, $PR^2 - PQ^2 = QR^2$ and M is a point on side PR such that $QM \perp PR$. Prove that $QM^2 = PM \times MR$.



Sol. Given: In ΔPQR,

$$PR^2 - PQ^2 = QR^2$$

$\Rightarrow$ $PR^2 = PQ^2 + QR^2$

$\Rightarrow$ PR is hypotenuse.

Also, $QM \perp PR$

To Prove: $MQ^2 = MP \times MR$

Proof: In ΔPQR,

$PR^2 - PQ^2 = QR^2$ [Given]

$\Rightarrow$ $PR^2 = PQ^2 + QR^2$

$\therefore$ $\angle PQR = 90°$ [By conv. of Pythagoras theorem]

In ΔQMP and ΔQMR, [$\because$ Sides QM, MP and MR form these]

$QM \perp PR$

$\therefore$ $\angle 1 = \angle 2 = 90°$

$\angle 3 = 90° - \angle R$

$\angle P = 90° - \angle R$

$\Rightarrow$ $\angle 3 = \angle P$

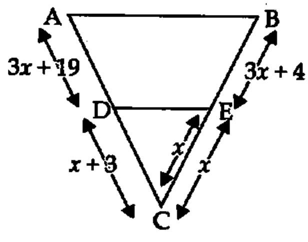
$\therefore$ $\Delta QMP \sim \Delta QMR$ [By AA similarity criterion]

$\Rightarrow$ $\dfrac{PQ}{QR} = \dfrac{PM}{QM} = \dfrac{QM}{RM}$

$\Rightarrow$ $QM^2 = PM \times RM$

Hence, proved.

Q2. Find the value of x for which DE || AB in the given figure.



Sol. In ΔABC, DE || AB.

$\Rightarrow$ $\dfrac{AD}{DC} = \dfrac{BE}{EC}$

$\Rightarrow$ $\dfrac{3x+19}{x+3} = \dfrac{3x+4}{x}$

$\Rightarrow$ $x(3x+19) = (x+3)(3x+4)$

$\Rightarrow$ $3x^2 + 19x = 3x^2 + 4x + 9x + 12$

$\Rightarrow$ $3x^2 - 3x^2 + 19x - 13x = 12$

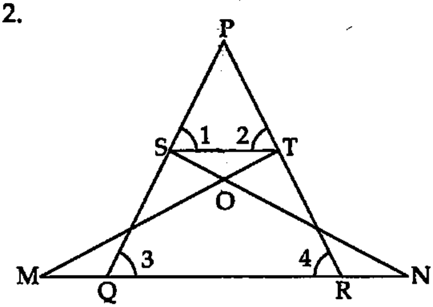
$\Rightarrow \quad 6x = 12$

$\Rightarrow \quad x = \frac{12}{6}$

$\Rightarrow \quad x = 2$

Hence, the required value of x is 2.

Q3. In the given figure, $\angle 1 = \angle 2$ and $\Delta NQS \cong \Delta MTR$. Prove that $\Delta PTS \sim \Delta PRQ$.



Sol. Given: In ΔPQR, point S is on PQ and T is on PR such that $\angle 1 = \angle 2$

and $\Delta NSQ \cong \Delta MTR$

To prove: $\Delta PTS \sim \Delta PRQ$

Proof: $\Delta NSQ \cong \Delta MTR$ [Given]

$\therefore \quad SQ = TR$ [CPCT] (I)

$\angle 1 = \angle 2$ [Given]

$\therefore \quad PT = PS$ [Sides opposite to equal angles in ΔPTS] (II)

$\Rightarrow \quad \frac{PT}{TR} = \frac{PS}{SQ}$ [From (I), (II)]

$\therefore \quad ST \parallel QR$ [By converse of BPT]

Now, in ΔPTS and ΔPRQ, we have

$ST \parallel QR$ [Proved above]

$\angle 1 = \angle 3$ [Corresponding $\angle$s]

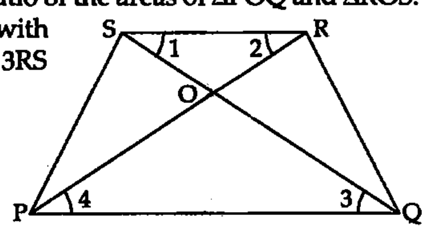
$\angle 2 = \angle 4$ [Corresponding $\angle$s]

$\therefore \quad \Delta PTS \sim \Delta PRQ$ [By AA similarity criterion]

Hence, proved.

Q4. Diagonals of a trapezium PQRS intersect each other at the point O, $PQ \parallel RS$ and $PQ = 3RS$. Find the ratio of the areas of ΔPOQ and ΔROS.

Sol. Given: PQRS is a trapezium with $PQ \parallel RS$ and $PQ = 3RS$



To find: $\frac{ar(\Delta POQ)}{ar(\Delta ROS)}$

Proof: In ΔPOQ and ΔROS,

$PQ \parallel RS$ [Given]

$\therefore \quad \angle 1 = \angle 3$ [Alt. int. $\angle$s]

$\angle 2 = \angle 4$ [Alt. int. $\angle$s]

$\therefore \quad \Delta POQ \sim \Delta ROS$ [By AA similarity criterion]

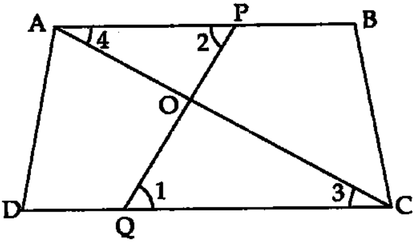
So, $\frac{ar(\Delta POQ)}{ar(\Delta ROS)} = \left(\frac{PQ}{RS}\right)^2$ [By area theorem]

But, $PQ = 3RS$ [Given]

$$\Rightarrow \quad \frac{ar(\Delta POQ)}{ar(\Delta ROS)} = \left(\frac{3RS}{RS}\right)^2 = \frac{9}{1}$$

Hence, the required ratio is 9:1.

Q5. In the given figure, if AB || DC, and AC and PQ intersect each other at O, prove that $OA \cdot CQ = OC \cdot AP$



Sol. Given: □ABCD,

AB || DC

and PQ intersect AC at O (in figure)

To Prove: $OA \cdot CQ = OC \cdot AP$

Proof: In ΔOPA and ΔOQC,

$$\left.\begin{array}{l}\angle 1 = \angle 2 \\ \angle 3 = \angle 4\end{array}\right\}$$ [Alt. int. ∠s]

$\therefore \quad \Delta OPA \sim \Delta OQC$ [By AA similarity criterion]

$$\Rightarrow \quad \frac{OQ}{OP} = \frac{OC}{OA} = \frac{QC}{PA}$$

$$\Rightarrow \quad OA \cdot CQ = OC \cdot PA$$

Hence, proved.

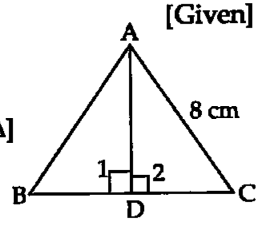
Q6. Find the altitude of an equilateral triangle of side 8 cm.

Sol. ΔABC is an equilateral triangle. [Given]

$AB = BC = AC = 8$ cm [Given]

$AD \perp BC$ [Given]

$\therefore \quad \angle 1 = \angle 2 = 90°$



In ΔADB and ΔADC,

$AB = AC$ [Sides of an equilateral Δ]

$\angle 1 = \angle 2 = 90°$

$AD = AD$ [Common]

$\therefore \quad \Delta ADB \cong \Delta ADC$ [By RHS congruence criterion]

$\Rightarrow \quad BD = DC$ [CPCT]

$$\therefore \quad BD = DC = \frac{BC}{2} = \frac{AB}{2} = \frac{8}{2} = 4 \text{ cm}$$

$\therefore$ By Pythagoras theorem, we have

$$AD^2 + BD^2 = AB^2$$

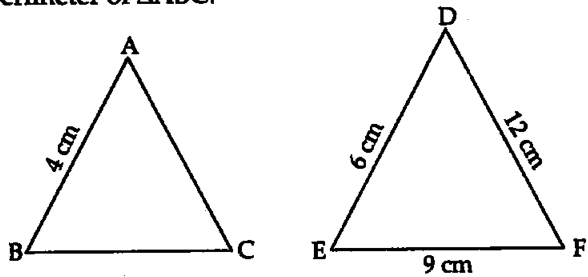
$$\Rightarrow \quad AD^2 + (4)^2 = (8)^2$$

$$\Rightarrow \quad AD^2 = 64 - 16$$

$$\Rightarrow \quad AD^2 = 48$$

$$\Rightarrow \quad AD = 4\sqrt{3} \text{ cm}$$

Q7. If $\Delta ABC \sim \Delta DEF$, AB = 4 cm, DE = 6 cm, EF = 9 cm, FD = 12 cm, then find the perimeter of ΔABC.



Sol. Given: In ΔABC and ΔDEF,

$$AB = 4 \text{ cm}, \quad DE = 6 \text{ cm}$$
$$EF = 9 \text{ cm}, \quad FD = 12 \text{ cm}$$

To find: Perimeter of ΔABC

Proof: $\Delta ABC \sim \Delta DEF$ [Given]

$\therefore$ $$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

$\Rightarrow$ $$\frac{4}{6} = \frac{AC}{12} = \frac{BC}{9}$$

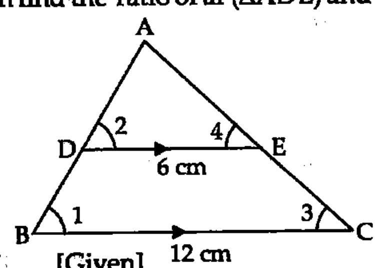
$\Rightarrow$ $$AC = \frac{4}{6} \times 12 = 8 \text{ cm}$$

and $$BC = \frac{4}{6} \times 9 = 6 \text{ cm}$$

$\therefore$ The perimeter of $\Delta ABC = AB + BC + AC$

$$= 4 \text{ cm} + 6 \text{ cm} + 8 \text{ cm} = 18 \text{ cm}$$

Q8. In the given figure, if DE || BC, then find the ratio of ar (ΔADE) and ar (□DECB).



Sol. Given: In ΔABC, in which

DE || BC

and DE = 6 cm and BC = 12 cm

To find: $$\frac{ar(\Delta ADE)}{ar(\square DECB)}$$

In ΔADE and ΔABC,

DE || BC [Given]

$\therefore$ $\angle 1 = \angle 2$, $\angle 3 = \angle 4$ [Corresponding angles]

$\therefore$ $\Delta ADE \sim \Delta ABC$ [By AA similarity critrion]

Now, $$\frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \left(\frac{BC}{DE}\right)^2$$

[$\because$ Ratio of the areas of two similar triangles is equal to the squares of the ratio of their corresponding sides]

$$\Rightarrow \quad \frac{ar(\square DECB) + ar(\Delta ADE)}{ar(\Delta ADE)} = \left(\frac{12}{6}\right)^2$$

$$\Rightarrow \quad \frac{ar(\square DECB)}{ar(\Delta ADE)} + \frac{ar(\Delta ADE)}{ar(\Delta ADE)} = (2)^2$$

$$\Rightarrow \quad \frac{ar(\square DECB)}{ar(\Delta ADE)} + 1 = 4$$

$$\Rightarrow \quad \frac{ar(\square DECB)}{ar(\Delta ADE)} = 4 - 1 = 3$$

$$\Rightarrow \quad \frac{ar(\Delta ADE)}{ar(\square DECB)} = \frac{1}{3}$$

Hence, the required ratio is 1 : 3.

Q9. ABCD is a trapezium in which AB || DC and P, Q are points on AD and BC respectively such that PQ || DC. If PD = 18 cm, BQ = 35 cm and QC = 15 cm, find AD.

Sol. Given: ABCD is a trapezium in which

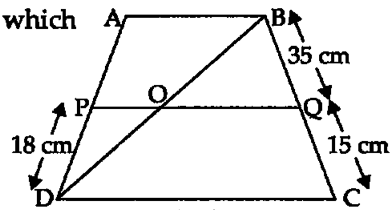
AB || CD and

PQ || DC (See figure)

Also, PD = 18 cm,

BQ = 35 cm and QC = 15 cm

To find: AD



Proof: In trapezium ABCD,

AB || CD

PQ || DC

∴ AB || CD || PQ (I)

In ΔBCD,

OQ || CD [From (I)]

$$\therefore \quad \frac{BO}{OD} = \frac{BQ}{QC} \quad \text{(II) [By BPT]}$$

Similarly, in ΔDAB,

PO || AB [From (I)]

$$\therefore \quad \frac{BO}{OD} = \frac{AP}{PD} \quad \text{(III) [By BPT]}$$

From (II) and (III)

$$\frac{AP}{PD} = \frac{BQ}{QC}$$

$$\Rightarrow \quad \frac{AP}{18} = \frac{35}{15}$$

$$\Rightarrow \quad AP = \frac{35}{15} \times 18 = 7 \times 6$$

$\Rightarrow \qquad AP = 42 \text{ cm}$

$\therefore \; AD = AP + PD = 42 \text{ cm} + 18 \text{ cm} = 60 \text{ cm}$

Q10. Corresponding sides of two similar triangles are in the ratio 2 : 3. If the area of the smaller triangle is 48 cm^2, then find the area of the larger triangle.

Sol. If $\Delta ABC \sim \Delta DEF$, then by area theorem,

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2$$

But, $\qquad AB : DE = 2 : 3$

and $\qquad ar(\Delta ABC)$ (smaller) $= 48 \text{ cm}^2$

$$\therefore \qquad \frac{48}{ar(\Delta DEF)} = \left(\frac{2}{3}\right)^2$$

$$\Rightarrow \qquad ar(\Delta DEF) = \frac{48 \times 9}{4} = 108 \text{ cm}^2$$

Q11. In a ΔPQR, N is the point on PR such that $QN \perp PR$. If $PN \times NR = QN^2$, then prove that $\angle PQR = 90°$.

Sol. Given: ΔPQR in which $QN \perp PR$ and $PN \times NR = QN^2$.

To Prove: $\angle PQR = 90°$

Proof: In ΔQNP and ΔQNR,

$QN \perp PR$ [Given]

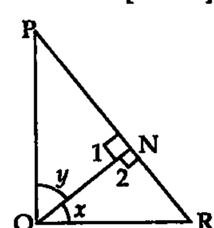
$\therefore \qquad \angle 1 = \angle 2 = 90°$

$QN^2 = NR \times NP$ [Given]

$\Rightarrow \qquad \frac{QN}{NR} = \frac{NP}{QN}$ or $\frac{QN}{NP} = \frac{NR}{QN}$

$\therefore \qquad \Delta PNQ \sim \Delta QNR$

[By SAS similarity criterion]



$\angle P = \angle RQN = x$ (I)

$\angle 1 = \angle 2 = 90°$

$\angle PQN = \angle R = y$ (II)

In ΔPQR, we have

$\angle P + \angle PQR + \angle R = 180°$ [Angle sum property of a triangle]

$\Rightarrow \qquad x + x + y + y = 180°$ [Using (I) and (II)]

$\Rightarrow \qquad 2x + 2y = 180°$

$\Rightarrow \qquad x + y = 90°$

$\Rightarrow \qquad \angle PQR = 90°$

Hence, proved.

Q12. Areas of two similar triangles are 36 cm^2 and 100 cm^2. If the length of a side of the larger triangle is 20 cm, find the length of the corresponding side of the similar triangle.

Sol. Here, $ar(\Delta ABC) = 36 \text{ cm}^2$, $ar(\Delta DEF) = 100 \text{ cm}^2$, $DE = 20$ cm, $AB = ?$

If $\Delta ABC \sim \Delta DEF$, then by area theorem $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2$

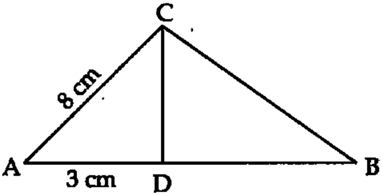
$$\Rightarrow \quad \frac{36}{100} = \left(\frac{AB}{DE}\right)^2$$

$$\Rightarrow \quad \frac{6}{10} = \left(\frac{AB}{DE}\right) \qquad \text{[Taking square root]}$$

$$\text{or} \quad \frac{6}{10} = \frac{AB}{20} \Rightarrow AB = \frac{6 \times 20}{10} = 12 \text{ cm}$$

$\therefore$ AB = 12 cm. Hence, side of smaller Δ is 12 cm.

Q13. In the given figure, if $\angle ACB = \angle CDA$, AC = 8 cm, AD = 3 cm, then find BD.



Sol. In ΔACD and ΔACB, we have

$$\angle CDA = \angle ACB \quad \text{[Given]}$$

$$\angle A = \angle A \quad \text{[Common]}$$

$$\therefore \quad \Delta ACD \sim \Delta ACB \qquad \text{[By AA similarity criterion]}$$

$$\text{So,} \quad \frac{AC}{AB} = \frac{DC}{BC} = \frac{AD}{AC} \Rightarrow \frac{8}{AB} = \frac{DC}{BC} = \frac{3}{8}$$

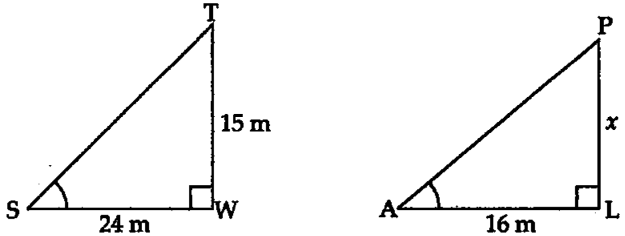
$$\text{Now,} \quad \frac{8}{AB} = \frac{3}{8} \Rightarrow AB = \frac{8 \times 8}{3} = \frac{64}{3}$$

$$BD = AB - AD = \frac{64}{3} - 3 = \frac{64 - 9}{3}$$

$$= \frac{55}{3} \text{ cm} = 18.33 \text{ cm}$$

Hence, BD = 18.33 cm.

Q14. A 15 m high tower casts a shadow 24 m long at a certain time and at the same time a telephone pole casts a shadow 16 m long. Find the height of the telephone pole.



Sol. Let TW = 15 m be the tower and SW = 24 m be its shadow. Also, let PL be the telephone pole and AL = 16 m be its shadow.

Let PL = x metres.

In ΔTWS and ΔPLA,

$$\angle W = \angle L = 90^\circ$$

$$\angle S = \angle A \quad \text{[Each = Angular elevation of sun]}$$

$$\therefore \quad \Delta TWS \sim \Delta PLA$$

$$\Rightarrow \quad \frac{TW}{PL} = \frac{TS}{PA} = \frac{WS}{LA}$$

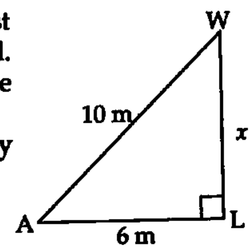
$$\Rightarrow \quad \frac{15}{x} = \frac{24}{16}$$

$$\Rightarrow \quad x = \frac{15 \times 16}{24} = 5 \times 2$$

$$\Rightarrow \quad x = 10 \text{ m}$$

Hence, the height of the pole is 10 m.

Q15. Foot of a 10 m long ladder leaning against a vertical wall is 6 m away from the base of wall. Find the height of the point on the wall where the top of the ladder reaches.



Sol. As wall WL = x m is vertically up so by Pythagoras theorem,

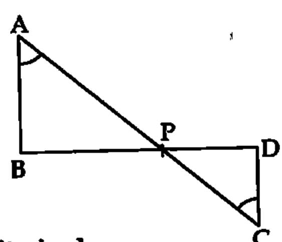
$$x^2 = 10^2 - 6^2 = 100 - 36$$

$$\Rightarrow \quad x^2 = 64$$

$$\Rightarrow \quad x = 8 \text{ m}$$

EXERCISE 6.4

Q1. In the given figure, if $\angle A = \angle C$, AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm, then find the lengths of PD and CD.



Sol. In ΔABP and ΔCDP,

$$\angle A = \angle C \quad \text{[Given]}$$

$$\angle 1 = \angle 2$$

[Vertically opposite angles]

$$\therefore \quad \Delta ABP \sim \Delta CDP \quad \text{[By AA similarity criterion]}$$

$$\Rightarrow \quad \frac{AB}{CD} = \frac{AP}{CP} = \frac{BP}{DP}$$

$$\Rightarrow \quad \frac{6}{y} = \frac{12}{4} = \frac{15}{x}$$

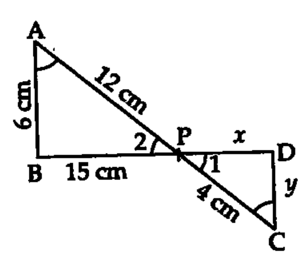
$$\Rightarrow \quad \frac{6}{y} = \frac{12}{4}$$

$$\Rightarrow \quad y = \frac{6}{3} = 2 \text{ cm}$$

$$\Rightarrow \quad \frac{15}{x} = \frac{12}{4}$$

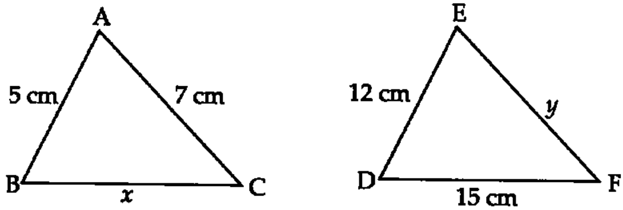
$$\Rightarrow \quad \frac{15}{3} = x$$

$$\Rightarrow \quad x = 5 \text{ cm}$$



$\therefore$ PD = 5 cm and DC = 2 cm

Q2. It is given that $\Delta ABC \sim \Delta EDF$ such that AB = 5 cm, AC = 7 cm, DF = 15 cm and DE = 12 cm. Find the lengths of the remaining sides of the triangles.



Sol. $\Delta ABC \sim \Delta EDF$ [Given]

$\therefore \quad \frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$

$\Rightarrow \quad \frac{5}{12} = \frac{7}{y} = \frac{x}{15}$

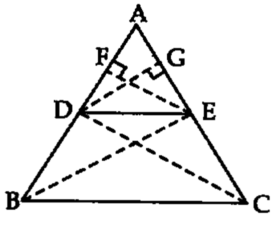
$\Rightarrow \quad \frac{5}{12} = \frac{7}{y}$

$\Rightarrow \quad y = \frac{7 \times 12}{5} = \frac{84}{5} = 16.8 \text{ cm}$

and $\quad x = \frac{5 \times 15}{12} = \frac{25}{4} = 6.25 \text{ cm}$

Hence, the length of BC = 6.25 cm and EF = 16.8 cm.

Q3. Prove that, if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio.



Sol. Given: In ΔABC,

$DE \parallel BC$

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw $EF \perp AB$ and $DG \perp AC$.

Join DC and BE.

Proof: $\frac{ar(\Delta ADE)}{ar(\Delta DBE)} = \frac{\frac{1}{2} AD \times EF}{\frac{1}{2} DB \times EF} = \frac{AD}{DB}$ (I)

and $\frac{ar(\Delta AED)}{ar(\Delta ECD)} = \frac{\frac{1}{2} AE \times DG}{\frac{1}{2} EC \times DG} = \frac{AE}{EC}$ (II)

Note that ΔDBE and ΔECD are on same base DE and between same parallel lines DE and BC.

$\therefore$ $\quad$ ar (ΔDBE) = ar (ΔECD) $\quad$ (III)

From equations (II) and (III), we have

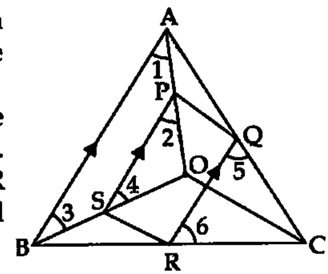
$$\frac{ar(\Delta AED)}{ar(\Delta DBE)} = \frac{AE}{EC} \quad \text{(IV)}$$

From equations (I) and (IV), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence, proved.

Q4. In the given figure, if PQRS is a parallelogram and AB || PS, then prove that OC || SR.



Sol. Given: In ΔABC, O is any point in the interior of ΔABC. OA, OB, OC are joined. PQRS is a parallelogram such that P, Q, R and S lies on segments OA, AC, BC and OB and PS || AB.

To Prove: $\quad$ OC || SR

Proof: In ΔOAB and ΔOPS

PS || AB $\quad$ [Given]

$\therefore$ $\quad$ $\angle 1 = \angle 2$, $\angle 3 = \angle 4$ $\quad$ [Corresponding angles]

$\therefore$ $\quad$ $\Delta OPS \sim \Delta OAB$ $\quad$ [By AA similarity criterion]

$$\Rightarrow \quad \frac{OP}{OA} = \frac{OS}{OB} = \frac{PS}{AB} \quad \text{(I)}$$

PQRS is a parallelogram so PS || QR. $\quad$ (II)

$\Rightarrow$ $\quad$ QR || AB $\quad$ (III) $\quad$ [From (I), (II]

In ΔCQR and ΔCAB,

QR || AB $\quad$ (III)

$\therefore$ $\quad$ $\angle CAB = \angle 5$, $\angle CBA = \angle 6$ $\quad$ [Corresponding angles]

$\therefore$ $\quad$ $\Delta CQR \sim \Delta CAB$ $\quad$ [By AA similarity criterion]

$$\Rightarrow \quad \frac{CQ}{CA} = \frac{CR}{CB} = \frac{QR}{AB}$$

PQRS is a parallelogram.

$\therefore$ $\quad$ PS || QR

$$\therefore \quad \frac{PS}{AB} = \frac{CR}{CB} = \frac{CQ}{CA} \quad \text{(IV)}$$

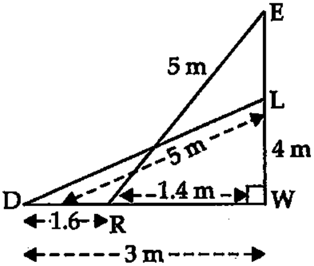
$$\Rightarrow \quad \frac{CR}{CB} = \frac{OS}{OB} \quad \text{[From (I) and (IV)]}$$

These are the ratios of two sides of ΔBOC and are equal so by converse of BPT, SR ||OC.

Hence, proved.

Q5. A 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

Sol. In figure ELW is a wall. DL and RE are two positions of ladder of length 5 cm.



Case I: In right angled ΔLWD,

$$DW^2 = DL^2 - LW^2$$

$$\Rightarrow \quad DW^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$$\Rightarrow \quad DW = 3 \text{ m}$$

Case II: $RW = DW - DR = 3 - 1.6 = 1.4$ m

In right angled triangle RWE,

$$EW^2 = RE^2 - RW^2 = 5^2 - 1.4^2 = 25 - 1.96 = 23.04$$

$$EW = \sqrt{23.04} = 4.8 \text{ m.}$$

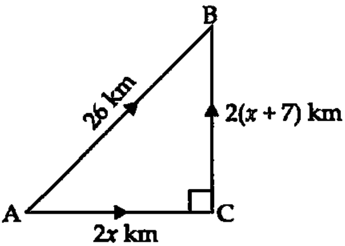
$\therefore$ The distance by which the ladder shifted upward = EL = 4.8 m − 4 m = 0.8 m

Hence, the ladder would slide upward on wall by 0.8 m.

Q6. For going to a city B from city A, there is route via city C, such that $AC \perp CB$, $AC = 2x$ km, and $CB = 2(x + 7)$ km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A, after the construction of the highway.

Sol. Distance saved by direct highway = (AC + BC) − AB

$\because$ $AC \perp BC$ so by Pythagoras theorem



$$AC^2 + BC^2 = AB^2$$

$$\Rightarrow \quad (2x)^2 + [2(x+7)]^2 = 26^2$$

$$\Rightarrow \quad 2^2x^2 + 2^2(x+7)^2 = 676$$

$$\Rightarrow \quad 4x^2 + 4(x^2 + 49 + 14x) = 676$$

$$\Rightarrow \quad 4[x^2 + x^2 + 49 + 14x] = 676$$

$$\Rightarrow \quad 2x^2 + 14x + 49 = \frac{676}{4}$$

$$\Rightarrow \quad 2x^2 + 14x + 49 = 169$$

$$\Rightarrow \quad 2x^2 + 14x + 49 - 169 = 0$$

$$\Rightarrow \quad 2x^2 + 14x - 120 = 0$$

$$\Rightarrow \quad x^2 + 7x - 60 = 0$$

$$\Rightarrow \quad x^2 + 12x - 5x - 60 = 0$$

$$\Rightarrow \quad x(x + 12) - 5(x + 12) = 0$$

$\Rightarrow \quad (x+12)(x-5) = 0$

$\Rightarrow \quad x+12 = 0 \quad$ or $\quad x-5=0$

$\Rightarrow \quad x = -12 \quad$ or $\quad x = 5$

(rejected)

$\therefore$ The required distance $= AC + BC - AB$

$= 2x + 2x + 14 - 26$

$= 4x - 12$

$= 4 \times 5 - 12 = 20 - 12 \qquad [\because x = 5]$

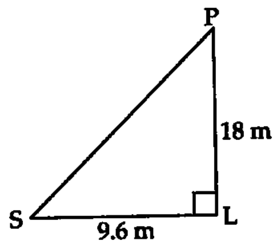
$= 8$ km

Hence, the distance saved by highway is 8 km.

Q7. A flag pole 18 m high casts a shadow 9.6 m long. Find the distance of the top of the pole from the far end of the shadow.

Sol. Pole PL = 18 m casts shadow LS = 9.6 m

The required distance between top of pole and far end of shadow is equal to PS as pole is vertical so $\angle L = 90°$.



$\therefore$ By Pythagoras theorem,

$$PS^2 = 18^2 + 9.6^2$$

$$\Rightarrow \quad PS^2 = 324 + 92.16 = 416.16$$

$$\Rightarrow \quad PS = \sqrt{416.16}$$

$$\Rightarrow \quad PS = 20.4 \text{ m}$$

Hence, the required distance = 20.4 m

Q8. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3 m, then find how far she is away from the base of the pole.

Sol. In ΔLPS and ΔNWS,

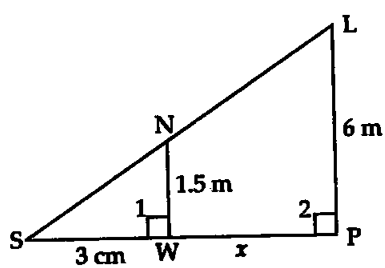
Bulb L is fixed at a height of 6 m above the road SP.

Woman and pole are vertical.

$\therefore \quad \angle 1 = \angle 2 = 90°$

$\angle S = \angle S \qquad$ [Common]

$\therefore \quad \Delta LPS \sim \Delta NWS \qquad$ [By AA similarity criterion]



$$\Rightarrow \quad \frac{LP}{NW} = \frac{LS}{NS} = \frac{PS}{WS}$$

$$\Rightarrow \quad \frac{6 \text{ m}}{1.5 \text{ m}} = \frac{LS}{NS} \quad \frac{}{3}$$

$$\Rightarrow \quad \frac{6}{1.5} = \frac{3+x}{3}$$

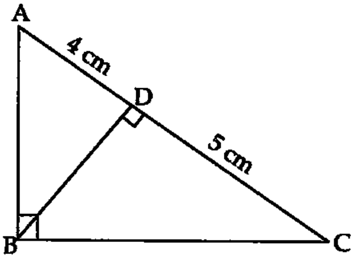
$$\Rightarrow \quad 4.5 + 1.5x = 18$$

$$\Rightarrow \quad 1.5x = 18 - 4.5$$

$$\Rightarrow \quad x = \frac{13.5}{1.5} = 9 \text{ m}$$

Hence, the woman is 9 m away from the pole.

Q9. In the given figure, ABC, is a triangle right angled at B and $BD \perp AC$. If AD = 4 cm, and CD = 5 cm then find BD and AB.



Sol. In ΔABC,

$\angle ABC = 90°$ [Given]

$BD \perp AC$ [Hypotenuse]

$\therefore \quad BD^2 = DA \times DC$

$\Rightarrow \quad BD^2 = 4 \times 5$

$\Rightarrow \quad BD = 2\sqrt{5}$ cm

In right angled ΔBDA,

$BD \perp AC$ [Given]

$\therefore \quad \angle BDA = 90°$

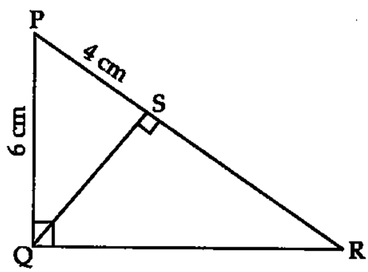
$\Rightarrow \quad AB^2 = AD^2 + BD^2$ [By Pythagoras theorem]

$= 4^2 + (2\sqrt{5})^2$

$= 16 + 20 = 36$

$\Rightarrow \quad AB = 6$ cm

Q10. In the given figure, PQR is a right triangle right angled at Q and $QS \perp PR$. If PQ = 6 cm and PS = 4 cm, then find QS, RS and QR.



Sol. In ΔPQR,

$\angle PQR = 90°$ [Given]

$QS \perp PR$

[From vertex Q to hypotenuse PR]

$\therefore \quad QS^2 = PS \times SR \quad$ (I) [By theorem]

Now, in ΔPSQ, we have

$QS^2 = PQ^2 - PS^2$ [By Pythagoraus theorem]

$= 6^2 - 4^2$

$= 36 - 16$

$\Rightarrow \quad QS^2 = 20$

$\Rightarrow \quad QS = 2\sqrt{5}$

$QS^2 = PS \times SR \quad$ (I)

$\Rightarrow \quad (2\sqrt{5})^2 = 4 \times SR$

$\Rightarrow \quad \frac{20}{4} = SR$

$\Rightarrow \quad SR = 5$ cm

Now, $\quad QS \perp PR$

$\therefore \quad \angle QSR = 90°$

$\Rightarrow \quad QR^2 = QS^2 + SR^2$ [By Pythagoras theorem]

$= (2\sqrt{5})^2 + 5^2$

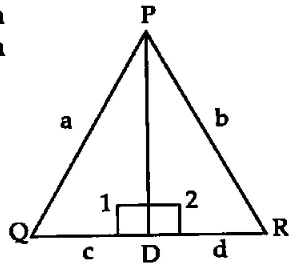
$= 20 + 25$

$\Rightarrow \quad QR^2 = 45$

$\Rightarrow \quad QR = 3\sqrt{5}$ cm

Hence, $QS = 2\sqrt{5}$, $RS = 5$ cm and $QR = 3\sqrt{5}$ cm.

Q11. In ΔPQR, $PD \perp QR$ such that D lies on QR, if $PQ = a$, $PR = b$, $QD = c$ and $DR = d$, then prove that $(a + b)(a - b) = (c + d)(c - d)$



Sol. Given: In ΔPQR, $PD \perp QR$ so $\angle 1 = \angle 2$.

$PQ = a$, $PR = b$, $QD = c$ and $DR = d$.

To Prove: $(a + b)(a - b) = (c + d)(c - d)$

Proof: In right angle ΔPDQ,

$PD^2 = PQ^2 - QD^2$

[By Pythagoras theorem]

$\Rightarrow \quad PD^2 = a^2 - c^2$ (I)

Similarly, in right angled ΔPDR,

$PD^2 = PR^2 - DR^2$ [By Pythagoras theorem]

$\Rightarrow \quad PD^2 = b^2 - d^2$ (II)

From (I) and (II), we have

$a^2 - c^2 = b^2 - d^2$

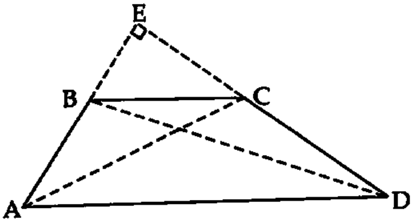
$\Rightarrow \quad a^2 - b^2 = c^2 - d^2$

$\Rightarrow \quad (a - b)(a + b) = (c - d)(c + d)$

Hence, proved.

Q12. In a quadrilateral ABCD, $\angle A + \angle D = 90°$. Prove that $AC^2 + BD^2 = AD^2 + BC^2$

[Hint: Produce AB and DC to meet at E.]



Sol. Given: A quadrilateral ABCD in which $\angle A + \angle D = 90°$.

To Prove: $AC^2 + BD^2 = AD^2 + BC^2$

Construction: Join AC and BD. Produce AB and DC to meet at E.

Proof: In ΔADE,

$\angle BAD + \angle CDA = 90°$ [Given]

$\therefore \quad \angle E = 90°$ [Int. angles of a Δ]

By Pythagoras theorem in ΔADE and ΔBCE,

$AD^2 = AE^2 + DE^2$ (I)

$BC^2 = BE^2 + EC^2$ (II)

Adding (I) and (II), we get

$$AD^2 + BC^2 = AE^2 + EC^2 + DE^2 + BE^2 \quad \text{(III)}$$

By Pythagoras theorem in ΔECA and ΔEBD,

$$AC^2 = AE^2 + CE^2 \quad \text{(IV)}$$

$$BD^2 = BE^2 + DE^2 \quad \text{(V)}$$

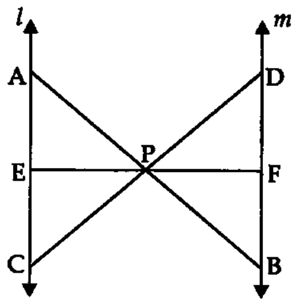
$\Rightarrow \quad AC^2 + BD^2 = AE^2 + BE^2 + CE^2 + DE^2$ (VI)[Adding (IV) and (V)]

$\Rightarrow \quad AC^2 + BD^2 = AD^2 + BC^2$ [Using (III)]

Hence, proved.

Q13. In the given figure, $l \parallel m$ and line segments AB, CD, and EF are concurrent at point P.

Prove that: $\dfrac{AE}{BF} = \dfrac{AC}{BD} = \dfrac{CE}{FD}$

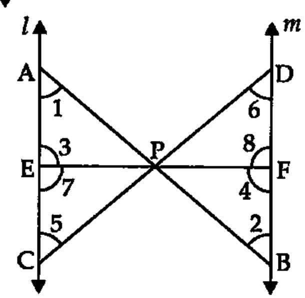


Sol. Given: $l \parallel m$

Line segments AB, CD and EF intersect at P.

Points A, E and C are on line l.

Points D, F and B are on line m.



To Prove: $\dfrac{AE}{BF} = \dfrac{AC}{BD} = \dfrac{CE}{FD}$

Proof: In ΔAEP and ΔBFP,

$l \parallel m$ [Given]

$\left.\begin{array}{l}\angle 1 = \angle 2 \\ \angle 3 = \angle 4\end{array}\right\}$ [Alternate interior angles] [Same reason]

$\therefore \quad \Delta AEP \sim \Delta BFP$ [By AA similarity criterion]

$$\Rightarrow \quad \frac{AE}{BF} = \frac{AP}{BP} = \frac{EP}{FP} \quad \text{(I)}$$

In ΔCEP and ΔDFP,

$l \parallel m$ [Given]

$\left.\begin{array}{l}\angle 7 = \angle 8 \\ \angle 5 = \angle 6\end{array}\right\}$ [Alternate interior angles]

$\therefore \quad \Delta CEP \sim \Delta DFP$ [By AA similarity criterion]

$$\Rightarrow \quad \frac{CE}{DF} = \frac{CP}{DP} = \frac{EP}{FP} \quad \text{(II)}$$

In ΔACP and ΔBDP,

$l \parallel m$ [Given]

$\left.\begin{array}{l}\angle 1 = \angle 2 \\ \angle 5 = \angle 6\end{array}\right\}$ [Alternate interior angles]

$\therefore \quad \Delta ACP \sim \Delta BDP$ [By AA similarity criterion]

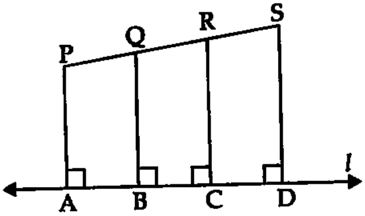
$$\Rightarrow \quad \frac{AC}{BD} = \frac{AP}{BP} = \frac{CP}{DP} \quad \text{(III)}$$

$$\Rightarrow \quad \frac{AP}{PB} = \frac{AC}{BD} = \frac{CP}{DP} = \frac{CE}{DF} = \frac{EP}{FP} = \frac{AE}{BF}$$

$$\Rightarrow \quad \frac{AC}{BD} = \frac{AE}{BF} = \frac{CE}{DF}$$

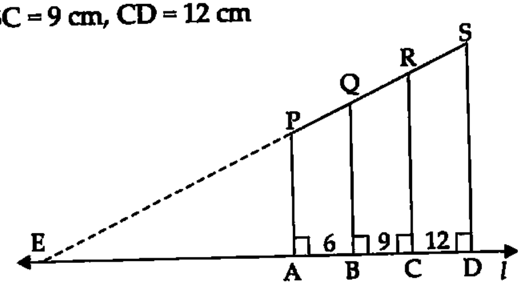
Hence, proved.

Q14. In the given figure, PA, QB, RC, and SD are all perpendiculars to line 'l', AB = 6 cm, BC = 9 cm, CD = 12 cm and SP = 36 cm. Find PQ, QR and RS.



Sol. Given: PA, QB, RC and SD are perpendiculars on line l.

AB = 6 cm, BC = 9 cm, CD = 12 cm



To find: PQ, QR and RS

Construction: Produce SP and l to meet each other at E.

Proof: In ΔEDS,

$AP \parallel BQ \parallel DS \parallel CR$ [Given]

$\therefore \quad PQ : QR : RS = AB : BC : CD$

$PQ : QR : RS = 6 : 9 : 12$

Let $PQ = 6x$

then $QR = 9x$

and $RS = 12x$

$\therefore \quad PQ + QR + RS = 36$ cm

$\Rightarrow \quad 6x + 9x + 12x = 36$

$\Rightarrow \quad 27x = 36$

$\Rightarrow \quad x = \frac{36}{27} = \frac{4}{3}$

$\therefore \quad PQ = 6 \times \frac{4}{3} = 8$ cm

$QR = 9 \times \frac{4}{3} = 12$ cm

$RS = 12 \times \frac{4}{3} = 16$ cm

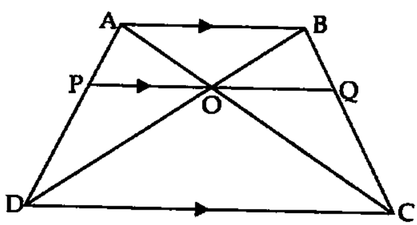
Q15. 'O' is the point of intersection of the diagonals AC and BD of a trapezium ABCD with AB || CD. Through 'O', a line PQ is drawn parallel to AB meeting AD in P and BC in Q. Prove that PO = QO.

Sol. Given: In trapezium ABCD, AB || DC.

Diagonals BD and AC intersect at O and POQ || DC || AB

To Prove: PO = QO

Proof: In ΔABD,



PO || AB [Given]

$\therefore \quad \frac{AP}{PD} = \frac{BO}{OD}$ (I)

Similarly, in ΔBDC,

OQ || DC

$\therefore \quad \frac{BO}{OD} = \frac{BQ}{QC}$ (II)

From (I) and (II), we have

$$\frac{AP}{PD} = \frac{BQ}{QC}$$

$\Rightarrow \quad \frac{AP}{PD} + 1 = \frac{BQ}{QC} + 1$ [Adding 1 on both sides]

$\Rightarrow \quad \frac{AP + PD}{PD} = \frac{BQ + QC}{QC}$

$\Rightarrow \quad \frac{AD}{PD} = \frac{BC}{QC}$ or $\frac{PD}{AD} = \frac{QC}{BC}$ (III)

In ΔDOP and ΔDBA,

AB || PO [Given]

$\therefore \quad \angle DPO = \angle DAB$
$\angle DOP = \angle DBA$ } [Corresponding angles]

$\therefore \quad \Delta DOP \sim \Delta DBA$ [By AA similarity criterion]

$\Rightarrow \quad \frac{PO}{AB} = \frac{DP}{DA}$ (IV)

Similarly, ΔCOQ ~ ΔCAB [By AA similarity criterion]

$\therefore \qquad \dfrac{OQ}{AB} = \dfrac{QC}{BC}$ (V)

From (III), (IV) and (V), we have

$$\frac{PO}{AB} = \frac{OQ}{AB}$$

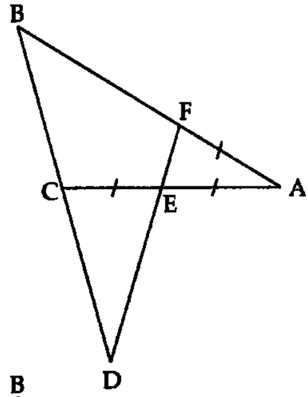
$\Rightarrow \qquad PO = OQ$

Hence, proved.

Q16. In the given figure, the line segment DF intersect the side AC of ΔABC at the point E such that E is mid point of AC and $\angle AFE = \angle AEF$.

Prove that: $\dfrac{BD}{CD} = \dfrac{BF}{CE}$.

[Hint: Take point G on AB such that CG || DF.]



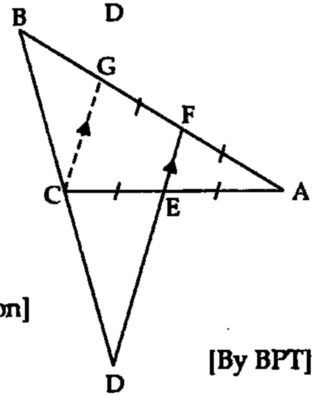
Sol. In the given figure of ΔABC,

$$EA = AF = EC$$

EF and BC meets at D.

To Prove: $\dfrac{BD}{CD} = \dfrac{BF}{CE}$

Construction: Draw CG || EF.



Proof: In ΔACG, CG || EF.

$\because$ E is mid-point of AC

$\therefore$ F will be the mid point of AG.

$\Rightarrow \qquad FG = FA$

But, $\qquad EC = EA = AF \qquad$ [Given]

$\therefore \qquad FG = FA = EA = EC \qquad$ (I)

In ΔBCG and BDF,

CG || EF [By construction]

$\therefore \qquad \dfrac{BC}{CD} = \dfrac{BG}{GF}$ [By BPT]

$\Rightarrow \qquad \dfrac{BC}{CD} + 1 = \dfrac{BG}{GF} + 1 \Rightarrow \dfrac{BC + CD}{CD} = \dfrac{BG + GF}{GF}$

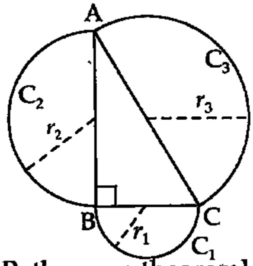
$\Rightarrow \qquad \dfrac{BD}{CD} = \dfrac{BF}{GF}$

But, $\qquad FG = CE$ [From (I)]

$\Rightarrow \qquad \dfrac{BD}{CD} = \dfrac{BF}{CE}$

Hence, proved.

Q17. Prove that the area of the semi-circle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of semi-circles drawn on the other two sides of the triangle.



Sol. Given: In figure, ΔABC is right angled at B. Three semi-circles taking as the sides BC, AB and AC of triangle ABC as diameter C_1, C_2 and C_3 are drawn.

To Prove: Area of semicircles $(C_1 + C_2)$ = Area of semi-circle C_3

Proof: In ΔABC,

$$\angle B = 90°$$

$\therefore$ $BC^2 + AB^2 = AC^2$ [By Pythagoras theorem]

$\Rightarrow$ $(2r_1)^2 + (2r_2)^2 = (2r_3)^2$

[From figure as BC, AB and AC are diameters]

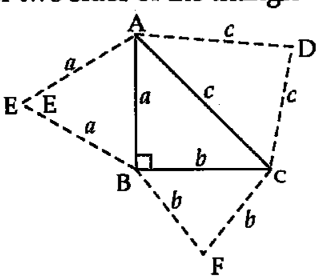
$\Rightarrow$ $4(r_1^2 + r_2^2) = 4r_3^2 \Rightarrow r_1^2 + r_2^2 = r_3^2$

$\Rightarrow$ $\frac{1}{2}\pi r_1^2 + \frac{1}{2}\pi r_2^2 = \frac{1}{2}\pi r_3^2$

ar (semi-circle C_1) + ar (semi-circle C_2) = ar (semi-circle C_3)

Hence, proved.

Q18. Prove that the area of the equilateral triangle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the equilateral triangles drawn on the other two sides of the triangle.



Sol. Given: A right triangle ABC.

Let AB = a, BC = b, AC = c and B = ∠90°.

Equilateral triangles with sides AB = a, BC = b and AC = c are drawn respectively.

To Prove: Area of equilateral triangle with side hypotenuse (c) is equal to the area of equilateral triangles with side a and b.

or $\frac{\sqrt{3}}{4}c^2 = \frac{\sqrt{3}}{4}a^2 + \frac{\sqrt{3}}{4}b^2$

Proof: In ΔABC,

$\angle ABC = 90°$ [Given]

$\therefore$ $AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow$ $c^2 = a^2 + b^2$

$\Rightarrow$ $\frac{\sqrt{3}}{4}c^2 = \frac{\sqrt{3}}{4}a^2 + \frac{\sqrt{3}}{4}b^2$ [Multiplying by $\frac{\sqrt{3}}{4}$ to both sides]

$\Rightarrow$ (Area of equilateral Δ with side c) = (Area of equilateral Δ with side a) + (Area of equilateral Δ with side b)

Hence, the area of equilateral Δ with hypotenuse is equal to the sum of areas of equilateral triangles on other two sides.

Hence, proved.

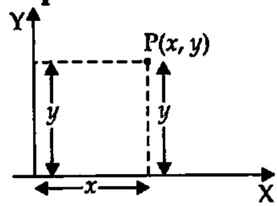
❑❑❑

7 Coordinate Geometry

EXERCISE 7.1

Choose the correct answer from the given four options:

Q1. The distance of the point P(2, 3) from x-axis is



(*a*) 2 (*b*) 3

(*c*) 1 (*d*) 5

Sol. (*b*): The perpendicular distance of P(2, 3) from x-axis is equal to the y coordinate so, it is 3 units. verifies ans. (*b*).

Q2. The distance between the points A(0, 6) and B(0, – 2) is

(*a*) 6 (*b*) 8 (*c*) 4 (*d*) 2

Sol. (*b*): $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-0)^2 + (-2-6)^2} = \sqrt{0 + (-8)^2} = \sqrt{64}$

$\Rightarrow$ AB = 8 units

Hence, verifies Ans (*b*).

Q3. The distance of the point P(– 6, 8) from the origin is

(*a*) 8 (*b*) $2\sqrt{7}$ (*c*) 10 (*d*) 6

Sol. (*c*): Coordinates of origin are O(0, 0) and P(– 6, 8)

$\therefore \quad (OP)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$= (-6-0)^2 + (8-0)^2 = 36 + 64$

$OP = \sqrt{100}$

$\Rightarrow$ OP = 10 units. verifies ans. (*c*).

Q4. The distance between the points (0, 5) and (–5, 0) is

(*a*) 5 (*b*) $5\sqrt{2}$ (*c*) $2\sqrt{5}$ (*d*) 10

Sol. (*b*): Let A(0, 5) and B(–5, 0) are the two points.

Then, $AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

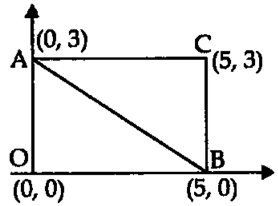
$= (-5-0)^2 + (0-5)^2 = 25 + 25$

$\Rightarrow AB^2 = 50$

$\Rightarrow AB = 5\sqrt{2}$ units. verifies ans. (*b*).

Q5. AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0), and B(5, 0). The length of its diagonal is

(*a*) 5 (*b*) 3 (*c*) $\sqrt{34}$ (*d*) 4



Sol. (*c*): A (0, 3) and B(5, 0)

The length of diagonal = AB

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (5-0)^2 + (0-3)^2$$
$$= 25 + 9$$
$\Rightarrow$ $AB = \sqrt{34}$ verifies Ans. (*c*).

Q6. The perimeter of a triangle with vertices (0, 4), (0, 0), and (3, 0) is

(*a*) 5 (*b*) 12 (*c*) 11 (*d*) $7+\sqrt{5}$

Sol. (*b*): Perimeter of $\Delta ABC = AB + BC + AC$

Let A(0, 4), B(0, 0), C(3, 0) be the three vertices of ΔABC.

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (0-0)^2 + (0-4)^2 = 0 + 16$$
$\Rightarrow$ $AB = \sqrt{16} = 4$ cm

$$AC^2 = (3-0)^2 + (0-4)^2 = 9 + 16$$
$\Rightarrow$ $AC^2 = 25$

$\Rightarrow$ $AC = 5$ cm

$$BC^2 = (3-0)^2 + (0-0)^2 = 9 + 0$$
$\Rightarrow$ $BC^2 = 9$

$\Rightarrow$ $BC = 3$ cm

$\therefore$ Perimeter = 4 cm + 5 cm + 3 cm = 12 cm

Hence, verifies Ans. (*b*).

Q7. The area of triangle with vertices A(3, 0), B(7, 0), and C(8, 4) is

(*a*) 14 (*b*) 28 (*c*) 8 (*d*) 6

Sol. (*c*): Area (A) of ΔABC whose vertices are A(3, 0), B(7, 0) and C(8, 4) is given by

$$\text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$= \frac{1}{2}[3(0-4) + 7(4-0) + 8(0-0)]$$
$$= \frac{1}{2}[-12 + 28 + 0] = \frac{1}{2}[16] = 8 \text{ sq.units}$$

Hence, verifies the Ans. (*c*).

Q8. The points (–4, 0), (4, 0) and (0, 3) are the vertices of a

(*a*) right triangle (*b*) isosceles triangle

(*c*) equilateral triangle (*d*) scalene triangle

Sol. (*b*): Let the vertices of ΔABC are A(–4, 0), B(4, 0) and C(0, 3).

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$\Rightarrow$ $AB^2 = [4-(-4)]^2 + (0-0)^2 = 64 + 0 = 64$

$\Rightarrow$ $AB = 8$ cm

$$AC^2 = [0-(-4)]^2 + (3-0)^2 = 16 + 9 = 25$$

$\Rightarrow \quad AC^2 = 25$

$\Rightarrow \quad AC = 5$ cm

$BC^2 = (0-4)^2 + (3-0)^2 = 16 + 9 = 25$

$\Rightarrow \quad BC^2 = 25$

$\Rightarrow \quad BC = 5$ cm

$\therefore \quad AC = BC = 5$ cm and $AB = 8$ cm

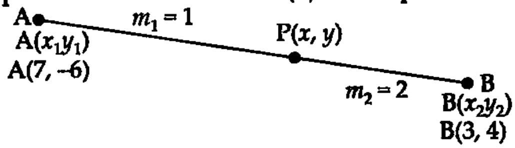
Hence, the triangle is an isosceles triangle. So, verifies ans. (*b*).

Q9. The point which divides the line segment joining the points (7, – 6) and (3, 4) in ratio 1 : 2 internally lies in the

(*a*) Ist quadrant (*b*) IInd quadrant

(*c*) IIIrd quadrant (*d*) IVth quadrant

Sol. (*d*):



$$\boxed{x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}} \qquad \boxed{y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}}$$

$$\Rightarrow \quad x = \frac{1(3) + 2(7)}{1+2} = \frac{3+14}{3} \qquad y = \frac{1(4) + 2(-6)}{1+2} = \frac{4-12}{3}$$

$$\Rightarrow \quad x = \frac{17}{3} \qquad y = \frac{-8}{3}$$

$P\left(\frac{17}{3}, \frac{-8}{3}\right)$ verifies the Ans. (*d*).

Q10. The point which lies on the perpendicular bisector of the line segment joining the points A(– 2, – 5) and B(2, 5) is

(*a*) (0, 0) (*b*) (0, 2) (*c*) (2, 0) (*d*) (–2, 0)

Sol. (*a*): The perpendicular bisector of AB will pass through the mid- point of AB. Mid-point of $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

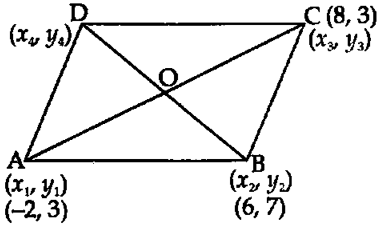
$$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0, 0)$$

So, the perpendicular bisector passes through (0, 0).

Q11. The fourth vertex D of a parallelogram ABCD whose three vertices are A(– 2, 3), B(6, 7), and C(8, 3) is

(*a*) (0, 1) (*b*) (0, –1) (*c*) (– 1, 0) (*d*) (1, 0)

Sol. (*b*): We know that the diagonals AC and BD of parallelogram ABCD bisect each other.



OR

$$\begin{bmatrix}\text{The mid point}\\ \text{of diagonal AC}\end{bmatrix} = \begin{bmatrix}\text{Mid point of}\\ \text{diagonal BD}\end{bmatrix}$$

$$\Rightarrow \quad \left(\frac{-2+8}{2}, \frac{3+3}{2}\right) = \left(\frac{x_4+6}{2}, \frac{y_4+7}{2}\right)$$

$$\Rightarrow \quad \left(\frac{6}{2}, \frac{6}{2}\right) = \left(\frac{x_4+6}{2}, \frac{y_4+7}{2}\right)$$

$$\Rightarrow \quad (3, 3) = \left(\frac{x_4+6}{2}, \frac{y_4+7}{2}\right)$$

Comparing both sides, we have

$$\frac{x_4+6}{2} = 3 \quad \text{and} \quad \frac{y_4+7}{2} = 3$$

$$\Rightarrow \quad x_4 + 6 = 6 \qquad \Rightarrow \quad y_4 + 7 = 6$$

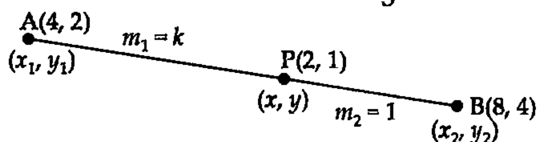
$$\Rightarrow \quad x_4 = 0 \qquad \Rightarrow \quad y_4 = 6 - 7 = -1$$

$\therefore$ The fourth vertex of parallelogram is (0, – 1) verifies ans. (*b*).

Q12. If the point P(2, 1) lies on the line segment joining points A(4, 2) and B(8, 4), then

(*a*) $AP = \frac{1}{3}AB$ (*b*) $AP = PB$ (*c*) $PB = \frac{1}{3}AB$ (*d*) $AP = \frac{1}{2}AB$

Sol. (*d*):



$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \qquad\qquad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

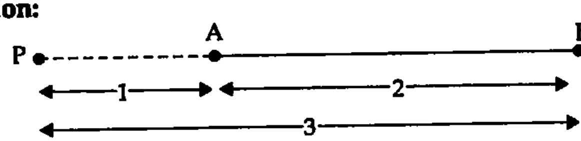
$$\Rightarrow \quad 2 = \frac{k(8) + 1(4)}{k+1} \qquad\qquad 1 = \frac{k(4) + 1(2)}{k+1}$$

$$\Rightarrow \quad 8k + 4 = 2k + 2 \qquad\qquad 4k + 2 = k + 1$$

$$\Rightarrow \quad 6k = -2 \qquad\qquad 3k = -1$$

$$\Rightarrow \quad k = \frac{-1}{3} \qquad\qquad k = \frac{-1}{3}$$

Verification:



$\therefore \quad \frac{AP}{PB} = \frac{-1}{3}$

$\Rightarrow \quad AP = -1$ i.e., 1 part outside AB

and $\quad PB = 3$

$\therefore \quad AP = 1x$ unit

and $\quad AB = 3x - 1x = 2x$ units

So, $\quad AP = \frac{1}{2} AB$

$\Rightarrow \quad 1 = \frac{1}{2} \times 2 \Rightarrow 1 = 1$, which is true

Hence, verifies the ans. (*d*).

Q13. If $P\left(-, 4\right)$ is the mid point of the line segment joining the points Q(– 6, 5) and R (– 2, 3), then the value of '*a*' is

(*a*) –4 (*b*) –12 (*c*) 12 (*d*) –6

Sol. (*b*): P(*x*, *y*) is mid-point of QR then

$$\left(\frac{a}{3}, 4\right) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\Rightarrow \quad \left(\frac{a}{3}, 4\right) = \left(\frac{-6-2}{2}, \frac{5+3}{2}\right)$$

$$\Rightarrow \quad \frac{a}{3} = \frac{-8}{2}$$

$$\Rightarrow \quad a = -4 \times 3 = -12$$

Verifies the ans. (*b*).

Q14. The perpendicular bisector of the line segment joining the points A(1, 5) and B(4, 6) cuts *y*-axis at

(*a*) (0, 13) (*b*) (0, – 13) (*c*) (0, 12) (*d*) (13, 0)

Sol. (*a*): The given points are A(1, 5) and B(4, 6).

The perpendicular bisector of the line segment joining the points A(1, 5) and B(4, 6) cuts the *y*-axis at P(0, *y*).

Now, $\quad AP = BP \Rightarrow AP^2 = BP^2$

$$\therefore \quad 1 + (y-5)^2 = 16 + (y-6)^2$$

$$\Rightarrow \quad 1 + y^2 - 10y + 25 = 16 + y^2 - 12y + 36$$

$$\Rightarrow \quad -10y + 26 = -12y + 52$$

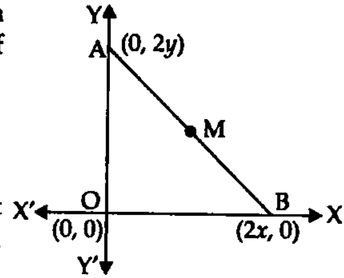
$$\Rightarrow \quad 12y - 10y = 52 - 26$$

$$\Rightarrow \quad 2y = 26$$

$$\Rightarrow \quad y = 26 \div 2 = 13$$

So, the required point is (0, 13).
Hence, (*a*) is the correct answer.

Q15. The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the figure is



(*a*) (x, y) (*b*) (y, x)

(*c*) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (*d*) $\left(\frac{y}{2}, \frac{x}{2}\right)$

Sol. (*a*): In a right triangle, the mid-point of the hypotenuse is equidistant from the three vertices of triangle.

Mid-point of A(2*x*, 0) and B(0, 2*y*) is

$$= \left(\frac{2x+10}{2}, \frac{0+2y}{2}\right) = (x, y)$$

Hence, (*a*) is the correct answer.

Q16. A circle drawn with origin as the centre passes through $\left(\frac{13}{2}, 0\right)$. The point which does not lie in the interior of the circle is

(*a*) $\left(\frac{-3}{4}, 1\right)$ (*b*) $\left(2, \frac{7}{3}\right)$ (*c*) $\left(5, \frac{-1}{2}\right)$ (*d*) $\left(-6, \frac{5}{2}\right)$

Sol. (*d*): Radius of circle $= \sqrt{\left(\frac{13}{2}-0\right)^2 + (0-0)^2} = \frac{13}{2} = 6.5$ units

(*a*) Distance of point $\left(\frac{-3}{4}, 1\right)$ from (0, 0) is

$$= \sqrt{\left(\frac{-3}{4}-0\right)^2 + (1-0)^2} = \sqrt{\frac{9}{16}+1} = \sqrt{\frac{25}{16}} = \frac{5}{4} = 1.25 \text{ units}$$

The distance $1.25 < 6.5$. So, the point $\left(\frac{-3}{4}, 1\right)$ lies in the interior of the circle.

(*b*) Distance of point $\left(2, \frac{7}{3}\right)$ from (0, 0) is

$$= \sqrt{(2-0)^2 + \left(\frac{7}{3}-0\right)^2} = \sqrt{4+\frac{49}{9}} = \sqrt{\frac{85}{9}} = \frac{9.2195}{3} = 3.0731 < 6.25$$

So, the point $\left(2, \frac{7}{3}\right)$ lies in the interior of the circle.

(*c*) Distance of point $\left(5, -\frac{1}{2}\right)$ from (0, 0) is

$$= \sqrt{(5-0)^2 + \left(-\frac{1}{2}-0\right)} = \sqrt{25+\frac{1}{4}} = \sqrt{\frac{101}{4}} = \frac{10.0498}{2} = 5.0249 < 6.5$$

So, the point $\left(5, \frac{-1}{2}\right)$ lies in the interior of the circle.

(*d*) Distance of point $\left(-6, \frac{5}{2}\right)$ from (0, 0) is

$$= \sqrt{(-6-0)^2 + \left(\frac{5}{2}-0\right)^2} = \sqrt{36 + \frac{25}{4}} = \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5 \text{ units}$$

So, $\left(-6, \frac{5}{2}\right)$ lies on the circle. It does not lie in the interior of the circle.

Hence, (*d*) is the correct answer.

Q17. A line intersects the *y*-axis and *x*-axis at points P and Q respectively. If (2, –5) is the mid-point of PQ, then co-ordinates of P and Q are respectively.

(*a*) (0, –5) and (2, 0)
(*b*) (0, 10) and (–4, 0)
(*c*) (0, 4) and (–10, 0)
(*d*) (0, –10) and (4, 0)

Sol. (*d*): P lies on *y*-axis so co-ordinates of P are (0, *y*).

Similarly, co-ordinates of Q lies on *x*-axis = Q(*x*, 0)

Mid-point of PQ is

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M(2, -5), \text{ which is given}$$

$$\Rightarrow \quad M\left(\frac{0+x}{2}, \frac{y+0}{2}\right) = M(2, -5)$$

$$\Rightarrow \quad \left(\frac{x}{2}, \frac{y}{2}\right) = (2, -5)$$

Comparing both sides, we get

$$\frac{x}{2} = 2 \quad \text{and} \quad \frac{y}{2} = -5$$

$$\Rightarrow \quad x = 4 \quad \text{and} \quad y = -10$$

Hence, the co-ordinates of P(0, –10) and Q(4, 0) verifies ans. (*d*).

Q18. The area of the triangle with vertices $(a, b+c)$ $(b, c+a)$ and $(c, a+b)$ is

(*a*) $(a+b+c)^2$ (*b*) 0 (*c*) $a+b+c$ (*d*) abc

Sol. (*b*): If the vertices of ΔABC are

$$A(x_1, y_1) = A(a, b+c)$$
$$B(x_2, y_2) = B(b, c+a)$$
$$C(x_3, y_3) = C(c, a+b)$$

Then, Area of $\Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}[a\{c+a-(a+b)\} + b\{a+b-(b+c)\} + c\{b+c-(c+a)\}]$$

$$= \frac{1}{2}[a(c-b) + b(a-c) + c(b-a)]$$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}[ac - ab + ab - bc + bc - ac]$

$\Rightarrow$ Area of $\Delta ABC = 0$ So, verifies the option (*b*).

Q19. If the distance between the points (4, *p*) and (1, 0) is 5, then the value of *p* is

(*a*) 4 only   (*b*) ± 4 (*c*) -4 only   (*d*) 0

Sol. (*b*): According to the question, the distance between A(4, *p*) and B(1, 0) is 5 units.

$$\therefore \quad AB = 5 \text{ units}$$
$$\Rightarrow \quad (AB)^2 = (5)^2$$
$$\Rightarrow \quad (4-1)^2 + (p-0)^2 = 25$$
$$\Rightarrow \quad (3)^2 + (p)^2 = 25$$
$$\Rightarrow \quad p^2 = 25 - 9$$
$$\Rightarrow \quad p^2 = 16$$
$$\Rightarrow \quad p = \pm 4$$
Hence, verifies the ans. (*b*).

Q20. If the points A(1, 2), O(0, 0) and C(*a*, *b*) are collinear, then

(*a*) $a = b$ (*b*) $a = 2b$ (*c*) $2a = b$ (*d*) $a = -b$

Sol. (*c*): Points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3\ y_3)$ will be collinear if the area of ΔABC is zero so, A(1, 2), B(0, 0), C(*a*, *b*) will collinear if area $\Delta ABC = 0$

$$\text{or} \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$
$$\Rightarrow \quad \frac{1}{2}[1(0-b) + 0(b-2) + a(2-0)] = 0$$
$$\Rightarrow \quad \frac{1}{2}(-b + 2a) = 0$$
$$\Rightarrow \quad \frac{-b}{2} + a = 0$$
$$\Rightarrow \quad -b + 2a = 0$$
$$\Rightarrow \quad 2a = b$$

Hence, verifies the ans. (*c*).

EXERCISE 7.2

State whether the following statements are true or false. Justify your answer.

Q1. ΔABC with vertices A(−2, 0), B(2, 0) and C(0, 2) is similar to ΔDEF with vertices D(−4, 0), E(4, 0) and F(0, 4).

Sol. True: $\Delta ABC \sim \Delta DEF$ if $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = k$

In ΔABC,

$$AB^2 = [2 - (-2)]^2 + [0 - (0)]^2 = (4)^2 + 0 = (4)^2$$
$$\Rightarrow \quad AB = 4 \text{ units}$$
$$BC^2 = (0-2)^2 + (2-0)^2 = 4 + 4 = 8$$

$\Rightarrow \quad BC = 2\sqrt{2}$ units

$AC^2 = [0-(-2)]^2 + (2-0)^2 = 2^2 + 2^2 = 4 + 4 = 8$

$\Rightarrow \quad AC = 2\sqrt{2}$ units

In ΔDEF,

$DE^2 = [4-(-4)]^2 + (0-0)^2 = (8)^2$

$\Rightarrow \quad DE = 8$ units

$EF^2 = (0-4)^2 + (4-0)^2 = 4^4 + 4^2 = 16 + 16 = 32$

$\Rightarrow \quad EF = 4\sqrt{2}$ units

$DF^2 = [0-(-4)]^2 + (4-0)^2 = 16 + 16 = 32$

$\Rightarrow \quad DF = 4\sqrt{2}$ units

Now, $\dfrac{AB}{DE} = \dfrac{4}{8} = \dfrac{1}{2}$

$\dfrac{BC}{EF} = \dfrac{2\sqrt{2}}{4\sqrt{2}} = \dfrac{1}{2}$

$\dfrac{AC}{DF} = \dfrac{2\sqrt{2}}{4\sqrt{2}} = \dfrac{1}{2}$

$\therefore \quad \dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF} = \dfrac{1}{2}$

Hence, $\Delta ABC \sim \Delta DEF$.

Q2. Point P(– 4, 2) lies on the line segment joining the points A(– 4, 6) and B(– 4, – 6).

Sol. True: We observe that x-coordiante is same i.e., equal to (– 4) so line is parallel to y-axis. y-coordinate of P i.e., 2 lies between 6 and – 6 of A and B respectively. Hence, P lies between and on AB.

OR

Point P(– 4, 2) will lie on the line AB if area of ΔABP is zero.

$\therefore$ i.e., $ar(\Delta ABP) = 0$

$\Rightarrow \quad \dfrac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \quad \dfrac{1}{2}[-4(-6-2) - 4(2-6) - 4(6+6)] = 0$

$\Rightarrow \quad [-4(-8) - 4(-4) - 4(12)] = 0$

$\Rightarrow \quad 32 + 16 - 48 = 0$

$\Rightarrow \quad 48 - 48 = 0$, which is true.

Hence, point P lies on the line joining A and B.

Q3. The points (0, 5), (0, – 9) and (3, 6) are collinear.

Sol. False: Three points A, B, and C will be collinear if the area of $\Delta ABC = 0$

$\Rightarrow \quad \dfrac{1}{2}[0(-9-6) + 0(6-5) + 3(5-(-9))] = 0$

$\Rightarrow \qquad 0+0+3(14) = 0$

$\Rightarrow \qquad 42 \neq 0$, which is false.

Hence, the given points are not collinear.

Q4. Point P(0, 2) is the point of intersection of y-axis and perpendicular bisector of line segment joining the points A(– 1, 1) and B(3, 3).

Sol. False: As the point P(0, 2) is the point of intersection of y-axis and perpendicular bisector of the line joining the points A(– 1, 1) and B(3, 3), then point P must be equidistant from A and B. So, we must write PA = PB.

$$PA = \sqrt{(-1-0)^2 + (1-2)^2} = \sqrt{1+1} = \sqrt{2} \text{ units}$$

$$PA = \sqrt{(3-0)^2 + (3-2)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$

$\therefore \qquad PA \neq PB$

Hence, the given statement is false.

Q5. Points A(3, 1), B(12, – 2) and C(0, 2) cannot be the vertices of a triangle.

Sol. True: Points A, B, C can form a triangle if the sum of any two sides is greater than the third side.

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow \quad AB^2 = (12-3)^2 + (-2-1)^2 = 81 + 9 = 90$$

$$\Rightarrow \quad AB = 3\sqrt{10} \text{ units}$$

$$BC^2 = (0-12)^2 + [2-(-2)]^2 = 144 + 16 = 160$$

$$\Rightarrow \quad BC = 4\sqrt{10} \text{ units}$$

$$AC^2 = (0-3)^2 + (2-1)^2 = 9 + 1 = 10 \Rightarrow AC = \sqrt{10} \text{ units}$$

$$\therefore \quad AC = \sqrt{10} \text{ units}, \quad AB = 3\sqrt{10} \text{ units} \quad \text{and} \quad BC = 4\sqrt{10} \text{ units}$$

Now, $\quad AB + AC = \sqrt{10} + 3\sqrt{10} = 4\sqrt{10}$ units $= BC$

So, A, B, C points cannot form a Δ.

Q6. Points A(4, 3), B(6, 4), C(5, – 6) and D(–3, 5) are the vertices of a parallelogram.

Sol. False: The diagonals of parallelogram bisect each other so, ABCD will be a parallelogram if

mid-point of diagonal AC = mid-point of diagonal BD

$$\Rightarrow \quad \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{x_1' + x_2'}{2}, \frac{y_1' + y_2'}{2}\right)$$

$$\Rightarrow \quad \left(\frac{4+5}{2}, \frac{-6+3}{2}\right) = \left(\frac{6-3}{2}, \frac{4+5}{2}\right)$$

$$\Rightarrow \quad \left(\frac{9}{2}, \frac{-3}{2}\right) \neq \left(\frac{3}{2}, \frac{9}{2}\right)$$

Hence, ABCD is not a parallelogram.

Q7. A circle has its centre at the origin and a point P(5, 0) lies on it. The point Q(6, 8) lies outside the circle.

Sol. True: If the distance of Q from the cente O(0, 0) is greater than the radius then point Q lies in the exterior of the circle. Point P(5, 0) lies on the circle and centre is at O(0, 0) so radius = OP

$$OP^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (5 - 0)^2 + (0 - 0)^2$$
$$\Rightarrow \quad OP^2 = 5^2$$
$$\Rightarrow \quad OP = 5 \text{ units}$$

Now, $OQ^2 = (6 - 0)^2 + (8 - 0)^2 = 36 + 64 = 100$

$$\Rightarrow \quad OQ = 10 \text{ units}$$
$$\therefore \quad OQ > OP \text{ (radius)}$$

So, point Q lies exterior to circle.

Q8. The point A(2, 7) lies on the perpendicular bisector of line segment joining the points P(6, 5) and Q(0, – 4).

Sol. False: Any point (A) on perpendicular bisector will be equidistant from P and Q so

$$PA = QA$$

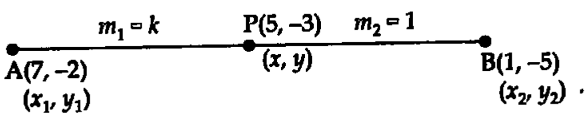
or $$PA^2 = QA^2$$

$$\Rightarrow (2 - 6)^2 + [7 - (5)]^2 = (2 - 0)^2 + [7 - (-4)]^2$$
$$\Rightarrow \quad (-4)^2 + (2)^2 = 2^2 + (11)^2$$
$$\Rightarrow \quad 16 + 4 = 4 + 121$$
$$\Rightarrow \quad 20 \neq 125$$

So, A does not lie on the perpendicular bisector of PQ.

Q9. Point P(5, –3) is one of the two points of trisection of the line segment joining the points A(7, –2) and B(1, –5).

Sol. True



Let point P divides the line AB in ratio $k : 1$ then

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \quad \text{and} \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\Rightarrow \quad x = \frac{k(1) + 1(7)}{(k+1)}, \qquad y = \frac{k(-5) + 1(-2)}{k+1}$$

$$\Rightarrow \quad 5 = \frac{k+7}{k+1}, \qquad -3 = \frac{-5k-2}{k+1}$$

$$\Rightarrow \quad 5k + 5 = k + 7, \qquad -5k - 2 = -3k - 3$$

$$\Rightarrow \quad 4k = 7 - 5, \qquad -2k = -3 + 2$$

$$\Rightarrow \quad k = \frac{2}{4} = \frac{1}{2}, \qquad k = \frac{-1}{-2} = \frac{1}{2}$$

So, P divides AB in 1 : 2 ratio.
Hence, P is one point of trisection of AB.

Q10. Points A(−6, 10), B(−4, 6) and C(3, −8) are collinear such that $AB = \frac{2}{9}AC$.

Sol. True: Points A, B and C will be collinear if ar (ΔABC) = 0

$$\text{ar } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[-6\{6-(-8)\} - 4(-8-10) + 3(10-6)] = 0$$

$$\Rightarrow \quad -6(14) - 4(-18) + 3(4) = 0$$

$$\Rightarrow \quad -84 + 72 + 12 = 0$$

$$\Rightarrow \quad -84 + 84 = 0, \text{ which is true}$$

So, points A, B and C are collinear.

$$AC^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$= (3+6)^2 + (-8-10)^2 = 81 + 324$$

$$\Rightarrow \quad AC = \sqrt{405} = 9\sqrt{5} \text{ units}$$

$$AB^2 = [-4-(-6)]^2 + (6-10)^2$$

$$= (-4+6)^2 + (-4)^2$$

$$= (2)^2 + (-4)^2 = 4 + 16$$

$$\Rightarrow \quad AB^2 = 20$$

$$\Rightarrow \quad AB = 2\sqrt{5} \text{ units}$$

Now, $$AB = \frac{2}{9}AC$$

$$\text{R.H.S.} = \frac{2}{9} \times 9\sqrt{5}$$

$$= 2\sqrt{5}$$

$$= AB$$

Hence, $AB = \frac{2}{9}AC$ is true.

Q11. The point P(−2, 4) lies on a circle of radius 6 and centre (3, 5).

Sol. False: The point P(−2, 4) lies on a circle if distance between P and centre is equal to the radius so distance of P from centre O(3, 5) will be

$$OP^2 = (-2-3)^2 + (4-5)^2$$

$$\Rightarrow \quad OP^2 = 25 + (-1)^2$$

$$\Rightarrow \quad OP = \sqrt{26} \neq \text{radius } 6$$

So, P does not lie on the circle. It will lie inside the circle.

Q12. The points A(−1, −2), B(4, 3), C(2, 5) and D(−3, 0) in that order form a rectangle.

Sol. True: ABCD will form a rectangle if

(*i*) it is a parallelogram. (*ii*) diagonals are equal.

For parallelogram: Diagonals bisect each other.

i.e., Mid point of AC = Mid point of BD is

i.e., $\left(\frac{-1+2}{2}, \frac{-2+5}{2}\right) = \left(\frac{4-3}{2}, \frac{3+0}{2}\right)$

$\Rightarrow \left(\frac{1}{2}, \frac{3}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$

Hence, ABCD is a parallelogram.

Now, Diagonal AC $= \sqrt{(2+1)^2 + (5+1)^2} = \sqrt{9+49}$

$\Rightarrow$ AC $= \sqrt{58}$ units

and Diagonal BD $= \sqrt{(-3-4)^2 + (0-3)^2}$

$\Rightarrow$ BD $= \sqrt{49+9}$ units

$\Rightarrow$ BD $= \sqrt{58}$ units

$\therefore$ Diagonal AC = Diagonal BD

Hence, ABCD is a rectangle.

EXERCISE 7.3

Q1. Name the type of triangle formed by the points A(–5, 6), B(– 4, – 2) and C(7, 5).

Sol. A(– 5, 6), B(– 4, – 2), C(7, 5)

$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\Rightarrow AB^2 = (-4+5)^2 + (-2-6)^2$

$= (1)^2 + (-8)^2 = 1 + 64 = 65$

$\Rightarrow AB = \sqrt{65}$ units

$AC^2 = (7+5)^2 + (5-6)^2$

$\Rightarrow AC^2 = (12)^2 + (-1)^2 \Rightarrow AC^2 = 144 + 1$

$\Rightarrow AC = \sqrt{145}$ units

$BC^2 = (7+4)^2 + (5+2)^2 = 11^2 + 7^2 = 121 + 49$

$\Rightarrow BC = \sqrt{170}$ units

As $AB \neq BC \neq AC$ so scalene triangle.

$\because AC^2 + AB^2 = 145 + 65 = 210 \neq BC^2$, so it is not a right angled Δ

So, a scalene Δ will be formed.

Q2. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ from point (7, – 4). How many such points are there?

Sol. Let point P(x, 0) be a point on x-axis, and A be the point (7, – 4).

So, $AP = 2\sqrt{5}$ [Given]

$\Rightarrow AP^2 = 4 \times 5 = 20$

$\Rightarrow (x-7)^2 + [0-(-4)]^2 = 20$

$\Rightarrow x^2 + 49 - 14x + 16 = 20$

$$\Rightarrow \quad x^2 - 14x - 20 + 65 = 0$$
$$\Rightarrow \quad x^2 - 14x + 45 = 0$$
$$\Rightarrow \quad x^2 - 9x - 5x + 45 = 0$$
$$\Rightarrow \quad x(x-9) - 5(x-9) = 0$$
$$\Rightarrow \quad (x-9)(x-5) = 0$$
$$\Rightarrow \quad x - 9 = 0 \quad \text{or} \quad x - 5 = 0$$
$$\Rightarrow \quad x = 9 \quad \text{or} \quad x = 5$$

Hence, there are two such points on x-axis whose distance from $(7, -4)$ is $2\sqrt{5}$. Hence, required points are $(9, 0)$, $(5, 0)$.

Q3. What type of quadrilateral do the points A(2, −2), B(7, 3), C(11, −1) and D(6, −6) taken in that order, form?

Sol. (*i*) A qudrilateral is a parallelogram, if mid points of diagonals AC and BD are same.

(*ii*) A parallelogram is not a rectangle, if diagonals $AC \neq BD$.

(*iii*) A parallelogram may be a rhombus if AB = BC.

(*iv*) If in a parallelogram diagonals are equal, then it is rectangle.

In a rectangle if the sides AB = BC, then the rectangle is a square.

For parallelogram with vertices A(2, −2), B(7, 3), C(11, −1), D(6, −6).

mid point of AC = mid point of BD

$$\Rightarrow \quad \left(\frac{2+11}{2}, \frac{-2-1}{2}\right) = \left(\frac{7+6}{2}, \frac{3-6}{2}\right)$$
$$\Rightarrow \quad \left(\frac{13}{2}, \frac{-3}{2}\right) = \left(\frac{13}{2}, \frac{-3}{2}\right), \text{ which is true.}$$

Hence, ABCD is a parallelogram.

Now, we will check whether AC = BD

$$\text{or} \quad AC^2 = BD^2$$
$$\Rightarrow \quad (11-2)^2 + (-1+2)^2 = (6-7)^2 + (-6-3)^2$$
$$\Rightarrow \quad (9)^2 + (1)^2 = (-1)^2 + (-9)^2$$
$$\Rightarrow \quad 81 + 1 = 1 + 81$$
$$\Rightarrow \quad 82 = 82, \text{ which is true.}$$

As the diagonals are equal so it is a rectangle or square.

Now, we will check whether adjacent sides AB = BC

$$\text{or} \quad AB^2 = BC^2$$
$$\Rightarrow \quad (7-2)^2 + (3+2)^2 = (11-7)^2 + (-1-3)^2$$
$$\Rightarrow \quad 5^2 + 5^2 = (4)^2 + (-4)^2$$
$$\Rightarrow \quad 25 + 25 = 16 + 16$$
$$\Rightarrow \quad 50 \neq 32, \text{ which is false.}$$

So, ABCD is not a square. Hence, ABCD is a rectangle.

Q4. Find the value of a, if the distance between the points A(−3, −14) and B(a, −5) is 9 units.

Sol. Consider A(−3, −14) and B(a, −5).

According to the question, $AB = 9$

$\Rightarrow$ $AB^2 = 81$

$\Rightarrow$ $(a+3)^2 + (-5+14)^2 = 81$

$\Rightarrow$ $a^2 + 9 + 6a + (9)^2 = 81$

$\Rightarrow$ $a^2 + 6a + 9 = 81 - 81$

$\Rightarrow$ $(a+3)^2 = 0$

$\Rightarrow$ $a + 3 = 0$

$\Rightarrow$ $a = -3$

Q5. Find a point which is equidistant from the points A(–5, 4) and B(– 1, 6). How many such points are there?

Sol. Let P(x, y) is equidistant from A(–5, 4) and B(– 1, 6), then

$$PA = PB$$

$\Rightarrow$ $PA^2 = PB^2$

$\Rightarrow$ $(x+5)^2 + (y-4)^2 = (x+1)^2 + (y-6)^2$

$\Rightarrow$ $x^2 + 25 + 10x + y^2 + 16 - 8y = x^2 + 1 + 2x + y^2 + 36 - 12y$

$\Rightarrow$ $41 + 10x - 8y = 37 + 2x - 12y$

$\Rightarrow$ $8x + 4y + 4 = 0$

$\Rightarrow$ $2x + 1y + 1 = 0$ (I)

The above equation shows that infinite points are equidistant from AB, because all the points on perpendicular bisector of AB will be equidistant from AB.

$\Rightarrow$ One such point which is equidistant from A and B is the mid-point M of AB *i.e.,*

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M\left(\frac{-5-1}{2}, \frac{4+6}{2}\right)$$

$$M\left(\frac{-6}{2}, \frac{10}{2}\right)$$

$$M(-3, 5)$$

So, (– 3, 5) is equidistant from points A and B.

Q6. Find the coordinates of the point Q on the x-axis which lies on the perpendicular bisector of the line segment joining the points A(– 5, – 2) and B(4, – 2). Name the type of triangle formed by the points Q, A and B.

Sol. Let Q(x, 0) be a point on x-axis which lies on the perpendicular bisector of AB.

$\therefore$ $QA = QB$

$\Rightarrow$ $QA^2 = QB^2$

$\Rightarrow$ $(-5-x)^2 + (-2-0)^2 = (4-x)^2 + (-2-0)^2$

$\Rightarrow$ $(x+5)^2 + (-2)^2 = (4-x)^2 + (-2)^2$

$\Rightarrow$ $x^2 + 25 + 10x + 4 = 16 + x^2 - 8x + 4$

$$\Rightarrow \quad 10x + 8x = 16 - 25$$
$$\Rightarrow \quad 18x = -9$$
$$\Rightarrow \quad x = \frac{-9}{18} = \frac{-1}{2}$$

Hence, the point Q is $\left(\frac{-1}{2}, 0\right)$.

Now, $$QA^2 = \left[-5 + \frac{1}{2}\right]^2 + [-2 - 0]^2$$
$$= \left(\frac{-9}{2}\right)^2 + \frac{4}{1}$$
$$\Rightarrow \quad QA^2 = \frac{81}{4} + \frac{4}{1} = \frac{81 + 16}{4} = \frac{97}{4}$$
$$\Rightarrow \quad QA = \sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2} \text{ units}$$

Now, $$QB^2 = \left(4 + \frac{1}{2}\right)^2 + (-2 - 0)^2 = \left(\frac{9}{2}\right)^2 + (-2)^2$$
$$\Rightarrow \quad QB^2 = \frac{81}{4} + \frac{4}{1} = \frac{81 + 16}{4} = \frac{97}{4}$$
$$\Rightarrow \quad QB = \sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2} \text{ units}$$

and $$AB = \sqrt{(4 + 5)^2 + [-2 - (-2)]^2} = \sqrt{(9)^2} = 9 \text{ units}$$
$$\Rightarrow \quad AB = 9 \text{ units}$$

As $$QA = QB$$

So, ΔQAB is an isosceles Δ.

Q7. Find the value of m if the points $(5, 1)$, $(-2, -3)$ and $(8, 2m)$ are collinear.

Sol. Points A, B, C will be collinear if the area of $\Delta ABC = 0$.

i.e., $$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$
$$\Rightarrow \quad \frac{1}{2}[5(-3 - 2m) - 2(2m - 1) + 8(1 + 3)] = 0$$
$$\Rightarrow \quad -15 - 10m - 4m + 2 + 32 = 0$$
$$\Rightarrow \quad -14m - 15 + 34 = 0$$
$$\Rightarrow \quad -14m + 19 = 0$$
$$\Rightarrow \quad -14m = -19$$
$$\Rightarrow \quad m = \frac{19}{14}$$

Hence, the required value of $m = \frac{19}{14}$.

Q8. If the point A(2, – 4) is equidistant from P(3, 8) and Q(–10, y), then find the values of y. Also find distance PQ.

Sol. According to the question,

$$PA = QA$$

$$\Rightarrow \quad PA^2 = QA^2$$

$$\Rightarrow \quad (3-2)^2 + (8+4)^2 = (-10-2)^2 + (y+4)^2$$

$$\Rightarrow \quad 1^2 + 12^2 = (-12)^2 + y^2 + 16 + 8y$$

$$\Rightarrow \quad y^2 + 8y + 16 - 1 = 0$$

$$\Rightarrow \quad y^2 + 8y + 15 = 0$$

$$\Rightarrow \quad y^2 + 5y + 3y + 15 = 0$$

$$\Rightarrow \quad y(y+5) + 3(y+5) = 0$$

$$\Rightarrow \quad (y+5)(y+3) = 0$$

$$\Rightarrow \quad y + 5 = 0 \quad \text{or } y + 3 = 0$$

$$\Rightarrow \quad y = -5 \quad \text{or} \quad y = -3$$

So, the co-ordinates are P(3, 8), $Q_1(-10, -3)$, $Q_2(-10, -5)$.

Now, $$PQ_1^2 = (3+10)^2 + (8+3)^2 = 13^2 + 11^2$$

$$\Rightarrow \quad PQ_1^2 = 169 + 121$$

$$\Rightarrow \quad PQ_1 = \sqrt{290} \text{ units}$$

and $$PQ_2^2 = (3+10)^2 + (8+5)^2 = 13^2 + 13^2$$

$$= 13^2[1+1]$$

$$\Rightarrow \quad PQ_2^2 = 13^2 \times 2$$

$$\Rightarrow \quad PQ_2 = 13\sqrt{2} \text{ units}$$

Hence, $y = -3, -5$, and PQ = $\sqrt{290}$ units and $13\sqrt{2}$ units.

Q9. Find the area of the triangle whose vertices are (– 8, 4), (– 6, 6) and (– 3, 9).

Sol. Vertices of ΔABC are A(– 8, 4), B(– 6, 6) and C(– 3, 9).

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{1}{2}[-8(6-9) - 6(9-4) - 3(4-6)]$$

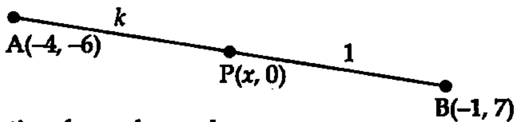
$$= \frac{1}{2}[-8(-3) - 6(5) - 3(-2)]$$

$$= \frac{1}{2}[24 - 30 + 6] = 0$$

Hence, the area of given triangle is zero.

Q10. In what ratio does the x-axis divides the line segment joining the points (– 4, – 6) and (– 1, 7)? Find the coordinates of the point of division.

Sol. Point P(x, 0) on x-axis intersects the line joining the points A(– 4, – 6) and B(– 1, 7). Let P divides the line in the ratio $k : 1$.



Using the section formula, we have

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \qquad \text{(I)}$$

$\Rightarrow$ $$\frac{0}{1} = \frac{k(7) + 1(-6)}{k+1}$$

$\Rightarrow$ $$7k - 6 = 0$$

$\Rightarrow$ $$k = \frac{6}{7}$$

$\Rightarrow$ $$m_1 = 6 \text{ and } m_2 = 7$$

Again, using the section formula, we have

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$\Rightarrow$ $$x = \frac{6(-1) + 7(-4)}{6+7} = \frac{-6-28}{13}$$

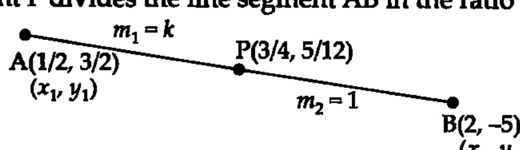
$\Rightarrow$ $$x = \frac{-34}{13}$$

Now, $$y = \frac{6(7) + 7(-6)}{6+7} = \frac{42-42}{13} = 0 \qquad \text{[From (I)]}$$

$\therefore$ Hence, the required point of intersection is $\left(\frac{-34}{13}, 0\right)$.

Q11. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{3}, \frac{3}{2}\right)$ and $B(2, -5)$.

Sol. Let point P divides the line segment AB in the ratio $k : 1$, then



The coordinates of P, by section formula are

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2},\ y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$\therefore$ $$x = \frac{k(2) + 1\left(\frac{1}{2}\right)}{k+1}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{2k+\frac{1}{2}}{k+1}$$

$$\Rightarrow \quad 8k+2 = 3k+3$$

$$\Rightarrow \quad 8k-3k = 3-3$$

$$\Rightarrow \quad 5k = 1$$

$$\Rightarrow \quad k = \frac{1}{5}$$

$$\Rightarrow \quad m_1 = 1 \text{ and } m_2 = 5$$

Now,

$$y = \frac{m_1(y_2)+m_2(y_1)}{m_1+m_2}$$

$\Rightarrow$

$\Rightarrow$

$\therefore$ y-coordinate of P is $\left(\frac{5}{12}\right)$

$$y = \frac{1(-5)+5\left(\frac{3}{2}\right)}{1+5} = \frac{-5+\frac{15}{2}}{6}$$

$$= \frac{\frac{-10+15}{2}}{6}$$

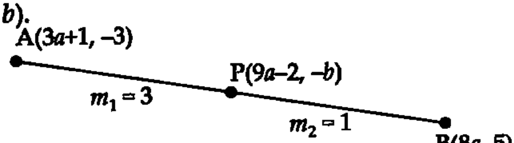
$$\Rightarrow \quad y = \frac{5}{2}\times\frac{1}{6} = \frac{5}{12}$$

y-coordinate of P is $\left(\frac{5}{12}\right)$.

Hence, P divides AB in ratio 1 : 5.

Q12. If point $P(9a-2, -b)$ divides the line segment joining the points $A(3a+1, -3)$ and $B(8a, 5)$ in the ratio 3 : 1, then find the values of a and b.

Sol. Point $P(9a-2, -b)$ divides the line segment joining the points $A(3a+1, -3)$ and $B(8a, 5)$ in the ratio 3 : 1. But, the coordinates of P are $(9a-2, -b)$.



Using section formula, we have

$$9a-2 = \frac{3(8a)+1(3a+1)}{3+1}$$

$$= \frac{24a+3a+1}{4}$$

$$\Rightarrow \quad 36a-8 = 27a+1$$

$$\Rightarrow \quad 36a-27a = 8+1$$

$$\Rightarrow \quad 9a = 9$$

$$\Rightarrow \quad a = \frac{9}{9} = 1$$

$$-b = \frac{3(+5)+1(-3)}{3+1}$$

$$\Rightarrow \quad -b = \frac{+15-3}{4} = \frac{12}{4}$$

$$\Rightarrow \quad b = -3$$

Hence, $a = +1$ and $b = -3$

Q13. If (a, b) is mid-point of the line segment joining points A(10, – 6) and B$(k, 4)$ and $a-2b=18$, then find the value of k and the distance AB.

Sol. Let P(a, b) is the mid-point of the line-segment joining the points A(10, –6) and B$(k, 4)$. Therefore, P(a, b) divides the line segment joining the points A(10, – 6) and B$(k, 4)$ in the ratio 1 : 1.

$$\Rightarrow \quad a = \frac{10+k}{2} \quad \text{(I)} \quad \text{and} \quad b = \frac{-6+4}{2}$$

$$\Rightarrow \quad b = \frac{-2}{2}$$

$$\Rightarrow \quad b = -1 \qquad \text{(II)}$$

But, $\quad a - 2b = 18 \qquad$ (III) [Given]

$\Rightarrow \quad a - 2(-1) = 18 \qquad$ [Using (II)]

$\Rightarrow \quad a = 18 - 2 \Rightarrow a = 16$

But, $\quad a = \dfrac{10+k}{2} \qquad$ [From (I)]

$$\Rightarrow \quad 16 = \frac{10+k}{2}$$

$$\Rightarrow \quad 10 + k = 32$$

$$\Rightarrow \quad k = 32 - 10$$

$$\Rightarrow \quad k = 22$$

Now, the co-ordinates of A and B are given by A (10, – 6) and B(22, 4).

$$\therefore \quad AB^2 = (22-10)^2 + (4+6)^2$$

$$= 12^2 + 10^2 = 144 + 100$$

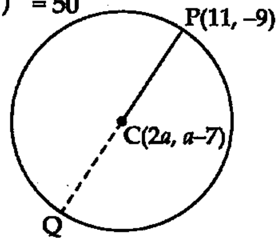
$$\Rightarrow \quad AB^2 = 244$$

$$\Rightarrow \quad AB = 2\sqrt{61} \text{ units}$$

Hence, the required value of $k = 22$, $a = 16$, $b = -1$ and AB $= 2\sqrt{61}$ units.

Q14. If the centre of circle is $(2a, a-7)$ then find the values of a if the circle passes through the point (11, – 9) and has diameter $10\sqrt{2}$ units.

Sol. Let C$(2a, a-7)$ be the centre of the circle and it passes through the point P(11, – 9).



$$\therefore \quad PQ = 10\sqrt{2}$$

$$\Rightarrow \quad CP = 5\sqrt{2}$$

$$\Rightarrow \quad CP^2 = (5\sqrt{2})^2 = 50$$

$$\Rightarrow \quad (2a-11)^2 + (a-7+9)^2 = 50$$

$$\Rightarrow \quad (2a)^2 + (11)^2 - 2(2a)(11) + (a+2)^2 = 50$$

$$\Rightarrow \quad 4a^2 + 121 - 44a + (a)^2 + (2)^2 + 2(a)(2) = 50$$

$$\Rightarrow \quad 5a^2 - 40a + 125 = 50$$

$$\Rightarrow \quad a^2 - 8a + 25 = 10$$

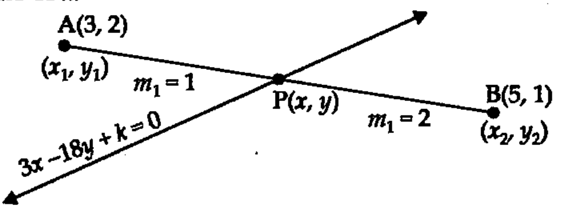
$$\Rightarrow \quad a^2 - 8a + 25 - 10 = 0$$

$$\Rightarrow \quad a^2 - 8a + 15 = 0$$
$$\Rightarrow \quad a^2 - 5a - 3a + 15 = 0$$
$$\Rightarrow \quad a(a-5) - 3(a-5) = 0$$
$$\Rightarrow \quad (a-5)(a-3) = 0$$
$$\Rightarrow \quad a - 5 = 0 \quad \text{or} \quad a - 3 = 0$$
$$\Rightarrow \quad a = 5 \quad \text{or} \quad a = 3$$

Hence, the required values of a are 5 and 3.

Q15. The line segment joining the points A(3, 2) and B(5, 1) is divided at the point P in the ratio of 1 : 2 and it lies on the line $3x - 18y + k = 0$. Find the value of k.

Sol.



P divides AB in the ratio 1 : 2. Then, the coordinates of P(x, y) are given by

$$x = \frac{m_1(x_2) + m_2(x_1)}{m_1 + m_2} \quad \text{and} \quad y = \frac{m_1(y_2) + m_2(y_1)}{m_1 + m_2}$$

$$\Rightarrow \quad x = \frac{1(5) + 2(3)}{1+2} = \frac{5+6}{3} \qquad \Rightarrow \quad y = \frac{1(1) + 2(2)}{1+2} = \frac{1+4}{3}$$

$$\Rightarrow \quad x = \frac{11}{3} \qquad \Rightarrow \quad y = \frac{5}{3}$$

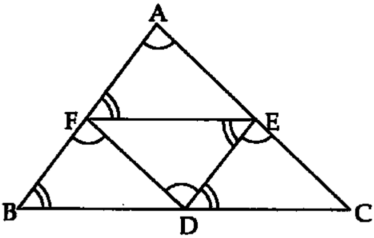
But, $P\left(\frac{11}{3}, \frac{5}{3}\right)$ P(x, y) lies on the line $3x - 18y + k = 0$

$$\therefore \quad 3\left(\frac{11}{3}\right) - 18\left(\frac{5}{3}\right) + k = 0$$
$$\Rightarrow \quad \frac{33}{3} - \frac{90}{3} + k = 0$$
$$\Rightarrow \quad 33 - 90 + 3k = 0$$
$$\Rightarrow \quad 3k = 90 - 33$$
$$\Rightarrow \quad 3k = 57$$
$$\Rightarrow \quad k = \frac{57}{3}$$
$$\Rightarrow \quad k = 19$$

Hence, the required value of $k = 19$.

Q16. If $D\left(\frac{-1}{2}, \frac{5}{2}\right)$, E(7, 3) and $F\left(\frac{7}{2}, \frac{7}{2}\right)$ are the mid-points of sides of ΔABC, find the area of ΔABC.

Sol. In ΔABC, D is mid point of BC, E is mid point of AC, and F is mid point of AB.



$\therefore \quad \Delta DEF \cong \Delta AFE \cong \Delta FBD \cong \Delta EDC$

So, area of $\Delta ABC = 4$ (area of ΔDEF)

The mid-points of sides of ΔABC are given by $D\left(\frac{-1}{2}, \frac{5}{2}\right)$, $E(7, 3)$, and $F\left(\frac{7}{2}, \frac{7}{2}\right)$.

$$\therefore \quad \text{Area } \Delta DEF = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}\left[-\frac{1}{2}\left(3 - \frac{7}{2}\right) + 7\left(\frac{7}{2} - \frac{5}{2}\right) + \frac{7}{2}\left(\frac{5}{2} - 3\right)\right]$$

$$= \frac{1}{2}\left[-\frac{1}{2}\left(\frac{-1}{2}\right) + 7(1) + \frac{7}{2}\left(\frac{-1}{2}\right)\right]$$

or

$$= \frac{1}{2}\left[\frac{1}{4} + 7 - \frac{7}{4}\right]$$

$$= \frac{1}{2}\left[\frac{1 + 28 - 7}{4}\right]$$

$$= \frac{1}{2}\left(\frac{29 - 7}{4}\right)$$

$$= \frac{22}{8} = \frac{11}{4}$$

$$\therefore \quad \text{Area of } \Delta ABC = 4 \times \text{Area } \Delta DEF$$

$$= 4 \times \frac{11}{4}$$

$$= 11 \text{ square units}$$

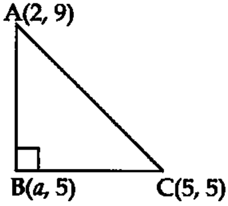
Hence, the required area of ΔABC is 11 square units.

Q17. The points A(2, 9), B(*a*, 5) and C(5, 5) are the vertices of a ΔABC right angled at B. Find the values of *a* and hence the area of ΔABC.

Sol. ΔABC is right angled at B.

$\therefore$ By Pythagoras theorem,

$$AB^2 + BC^2 = AC^2 \quad \text{...(I)}$$



$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow \quad AB^2 = (a - 2)^2 + (5 - 9)^2$$

$$\Rightarrow \quad AB^2 = (a)^2 + (2)^2 - 2(a)(2) + (-4)^2$$

$$= a^2 + 4 - 4a + 16$$

$$\Rightarrow \quad AB^2 = a^2 - 4a + 20$$

$$BC^2 = (a-5)^2 + (5-5)^2$$
$$= (a)^2 + (5)^2 - 2(a)(5) + 0^2$$
$\Rightarrow$ $$BC^2 = a^2 + 25 - 10a$$
$$AC^2 = (5-2)^2 + (5-9)^2$$
$$= 3^2 + (-4)^2$$
$$= 9 + 16 = 25$$
$\Rightarrow$ $$AC = \sqrt{25} = 5 \text{ units}$$
$\therefore$ $$a^2 - 4a + 20 + a^2 + 25 - 10a = (5)^2 \quad \text{[From (I)]}$$
$\Rightarrow$ $$2a^2 - 14a + 45 - 25 = 0$$
$\Rightarrow$ $$2a^2 - 14a + 20 = 0$$
$\Rightarrow$ $$a^2 - 7a + 10 = 0$$
$\Rightarrow$ $$a^2 - 5a - 2a + 10 = 0$$
$\Rightarrow$ $$a(a-5) - 2(a-5) = 0$$
$\Rightarrow$ $$(a-5)(a-2) = 0$$
$\Rightarrow$ $$a - 5 = 0 \quad \text{or} \quad a - 2 = 0$$
$\Rightarrow$ $$a = 5 \quad \text{or} \quad a = 2$$

If $a = 5$ then B(5, 5) and C(5, 5) and BC = 0, which is not possible.
Hence, $a = 2$.

Now, $$AB^2 = a^2 - 4a + 20$$
$$= (2)^2 - 4(2) + 20$$
$$= 4 - 8 + 20$$
$\Rightarrow$ $$AB^2 = 24 - 8$$
$\Rightarrow$ $$AB^2 = 16$$
$\Rightarrow$ $$AB = 4 \text{ units}$$
And, $$BC^2 = a^2 + 25 - 10a$$
$$= (2)^2 + 25 - 10(2) \quad [\because a = 2]$$
$$= 4 + 25 - 20 = 29 - 20 = 9$$
$\Rightarrow$ $$BC^2 = 9$$
$\Rightarrow$ $$BC = 3 \text{ units}$$

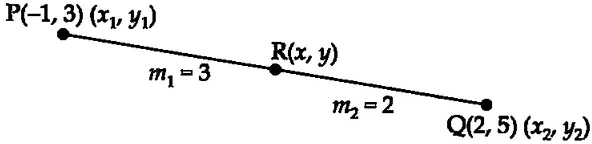
$\therefore$ Area of right angled triangle ABC $= \frac{1}{2}$ base $\times$ altitude
$$= \frac{1}{2} BC \times AB$$
$$= \frac{1}{2} \times 3 \times 4$$
$$= 6 \text{ square units}$$

Hence, the value of $a = 2$ and area of ΔABC is 6 sq. units.

Q18. Find the coordinates of the point R on the line segment joining the points P(– 1, 3) and Q(2, 5) such that $PR = \frac{3}{5} PQ$.

Sol. $PR = \frac{3}{5}PQ$ [Given]

$\Rightarrow \quad \frac{5}{3} = \frac{PQ}{PR}$



$\Rightarrow \quad \frac{5}{3} = \frac{PR + RQ}{PR}$

$\Rightarrow \quad \frac{5}{3} = \frac{PR}{PR} + \frac{RQ}{PR}$

$\Rightarrow \quad \frac{QR}{PR} = \frac{5}{3} - 1 = \frac{5-3}{3}$

$\Rightarrow \quad \frac{QR}{PR} = \frac{2}{3}$

or $\quad \frac{PR}{QR} = \frac{3}{2}$ or $PR : QR = 3 : 2$

$\therefore \quad m_1 = 3$ and $m_2 = 2$

Now, the coordinates of point R are given by

$x = \frac{m_1(x_2) + m_2(x_1)}{m_1 + m_2}$ and $y = \frac{m_1(y_2) + m_2(y_1)}{m_1 + m_2}$

$\Rightarrow \quad x = \frac{3(2) + 2(-1)}{3+2} = \frac{6-2}{5}$ $\quad \Rightarrow \quad y = \frac{3(5) + 2(3)}{3+2} = \frac{15+6}{5}$

$\Rightarrow \quad x = \frac{4}{5}$ $\quad \Rightarrow \quad y = \frac{21}{5}$

Hence, the required coordinates of R are $\left(\frac{4}{5}, \frac{21}{5}\right)$.

Q19. Find the value of k if the points $A(k + 1, 2k)$, $B(3k, 2k + 3)$ and $C(5k - 1, 5k)$ are collinear.

Sol. Points A, B, and C will be collinear if area of $\Delta ABC = 0$

$\Rightarrow \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \quad \frac{1}{2}[(k+1)\{2k + 3 - 5k\} + 3k\{5k - 2k\} + (5k-1)\{2k - (2k+3)\}] = 0$

$\Rightarrow \quad (k+1)(-3k+3) + 3k(3k) + (5k-1)(2k-2k-3) = 0$

$\Rightarrow -3(k+1)(k-1)+3(3k^2)-3(5k-1)=0$

Divide by 3 on both sides, we have

$$[(k+1)(-k+1)+3k^2+(5k-1)(-1)]=0$$

$\Rightarrow \quad 1-k^2+3k^2-5k+1=0$

$\Rightarrow \quad 2k^2-5k+2=0$

$\Rightarrow \quad 2k^2-4k-1k+2=0$

$\Rightarrow \quad 2k(k-2)-1(k-2)=0$

$\Rightarrow \quad (k-2)(2k-1)=0$

$\Rightarrow \quad k-2=0 \quad$ or $\quad 2k-1=0$

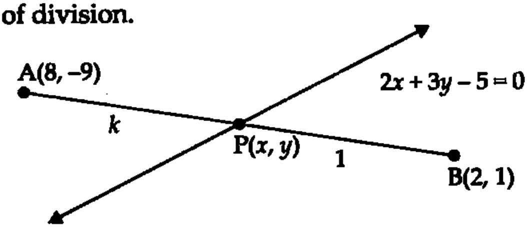
$\Rightarrow \quad k=2 \quad$ or $\quad 2k=1$

$\Rightarrow \quad k=2 \quad$ or $\quad k=\frac{1}{2}$

Hence, the required value of k are 2 and $\frac{1}{2}$.

Q20. Find the ratio in which the line $2x+3y-5=0$ divides the line segment joining the points $(8,-9)$ and $(2,1)$. Also find the coordinates of the point of division.

Sol.



$2x+3y-5=0 \quad$ ...(I)

Let the line given by equation I divides AB at $P(x, y)$ in the ratio $k:1$. Then, using the section formula, the coordinates of P are given by

$$x=\frac{m_1(x_2)+m_2(x_1)}{m_1+m_2} \quad \text{and} \quad y=\frac{m_1(y_2)+m_2(y_1)}{m_1+m_2}$$

$$\Rightarrow \quad x=\frac{k(2)+1(8)}{(k+1)} \quad \text{and} \quad y=\frac{k(1)+1(-9)}{k+1}$$

$$\Rightarrow \quad x=\frac{2k+8}{k+1} \quad \text{and} \quad y=\frac{k-9}{k+1}$$

$\Rightarrow P(x, y)=\left(\frac{2k+8}{k+1}, \frac{k-9}{k+1}\right)$ lies on line I so P must satisfy equation (I)

So substitute $x=\frac{2k+8}{k+1}$ and $y=\frac{k-9}{k+1}$ in equation I

$$\Rightarrow \quad 2\left(\frac{2k+8}{k+1}\right)+3\left(\frac{k-9}{k+1}\right)-5=0$$

On multiplying by $(k+1)$ in above equation both sides, we get

$$2(2k+8)+3(k-9)-5(k+1)=0$$

$$\Rightarrow \quad 4k+16+3k-27-5k-5=0$$

$$\Rightarrow \quad 2k-16=0$$

$$\Rightarrow \quad k=\frac{16}{2}=8$$

$\therefore$ Point of intersection is given by $P\left(\frac{2k+8}{k+1}, \frac{k-9}{k+1}\right)$

$$=P\left(\frac{2\times 8+8}{8+1}, \frac{8-9}{8+1}\right)$$

$$=P\left(\frac{16+8}{9}, \frac{-1}{9}\right)$$

$$=P\left(\frac{24}{9}, \frac{-1}{9}\right)$$

$$=P\left(\frac{8}{3}, \frac{-1}{9}\right)$$

Hence, line of eqn. (I) divides AB in ratio 8 : 1 at $P\left(\frac{8}{3}, \frac{-1}{9}\right)$.

EXERCISE 7.4

Q1. If (– 4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle.

Sol. Let A(– 4, 3), B(4, 3) and C(x, y) are the three vertices of ΔABC.

As the triangle is equilateral,

so $\quad AC = BC = AB$

or $\quad AC^2 = BC^2 = AB^2 \quad$ (I)

Now, $\quad AB^2 = (4+4)^2+(3-3)^2$

$\Rightarrow \quad AB^2 = (8)^2 = 64$

$\Rightarrow \quad AB = 8$ units $\quad$ (II)

$AC^2 = (x+4)^2+(y-3)^2$

$= (x)^2+(4)^2+2(x)(4)+(y)^2+(3)^2-2(y)(3)$

$= x^2+y^2+8x-6y+16+9$

$\Rightarrow \quad AC^2 = x^2+y^2+8x-6y+25 \quad$ (III)

$BC^2 = (x-4)^2+(y-3)^2$

$= (x)^2+(4)^2-2(x)(4)+(y)^2+(3)^2-2(y)(3)$

$= x^2+y^2-8x-6y+16+9$

$\Rightarrow \quad BC^2 = x^2+y^2-8x-6y+25 \quad$ (IV)

Now, $AC^2 = AB^2$ [From (I)]

$\Rightarrow \quad x^2 + y^2 + 8x - 6y + 25 = 64$ [From (III), (II)]

$\Rightarrow \quad x^2 + y^2 + 8x - 6y = 64 - 25$

$\Rightarrow \quad x^2 + y^2 + 8x - 6y = 39$ (V)

Again, $BC^2 = AB^2$ [From (I)]

$\Rightarrow \quad x^2 + y^2 - 8x - 6y + 25 = 64$ [From (II), (IV)]

$\Rightarrow \quad x^2 + y^2 - 8x - 6y = 64 - 25$

$\Rightarrow \quad x^2 + y^2 - 8x - 6y = 39$ (VI)

Subtracting (V) from (VI), we have

$$x^2 + y^2 - 8x - 6y = 39 \quad \text{(VI)}$$
$$x^2 + y^2 + 8x - 6y = 39 \quad \text{(V)}$$
$$- \quad - \quad - \quad + \quad -$$
$$-16x = 0$$

$\Rightarrow \quad x = 0$

Putting $x = 0$ in (V), we have

$(0)^2 + y^2 + 8(0) - 6y = 39$

$\Rightarrow \quad y^2 - 6y - 39 = 0$

$D = b^2 - 4ac \qquad (a = 1, b = -6, c = -39)$

$= (-6)^2 - 4(1)(-39) = 36 + 156$

$\Rightarrow \quad D = 192$

$\Rightarrow \quad \sqrt{D} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3}$

$\Rightarrow \quad \sqrt{D} = 8\sqrt{3}$

$\therefore \quad y = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{6 \pm 8\sqrt{3}}{2 \times 1} = \dfrac{2(3 \pm 4\sqrt{3})}{2}$

$\Rightarrow \quad y_1 = 3 + 4\sqrt{3}$ and $y_2 = 3 - 4\sqrt{3}$

Hence, the third vertex of ΔABC may be $C(0, 3 + 4\sqrt{3})$ and $C'(0, 3 - 4\sqrt{3})$.

Now, $C(0, 3 + 4\sqrt{3})$

$= C(0, 3 + 4 \times 1.732)$

$= C(0, 3 + 6.9)$

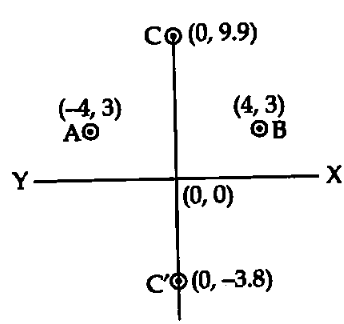
$= C(0, 9.9)$

and $C'(0, 3 - 4\sqrt{3})$

$= C'(0, 3 - 4 \times 1.732)$

$= C'(0, 3 - 6.9)$

$= C'(0, -3.9)$

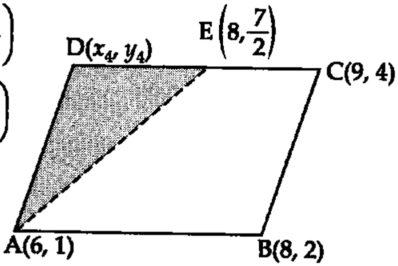


So, the required point so that origin lies inside it is $(0, 3 - 4\sqrt{3})$.

Q2. A(6, 1), B(8, 2) and C(9, 4) are three vertices of a parallelogram ABCD. If E is the mid point of DC, then find the area of ΔADE.

Sol. ABCD is a parallelogram so

[Mid point of diagonal BD] = [Mid point of diagonal AC]



$\therefore$ Mid point of BD$= \left(\dfrac{x_4+8}{2}, \dfrac{y_4+2}{2}\right)$

and Mid point of AC $= \left(\dfrac{6+9}{2}, \dfrac{1+4}{2}\right)$

$\Rightarrow \dfrac{x_4+8}{2} = \dfrac{15}{2}$ and $\dfrac{y_4+2}{2} = \dfrac{5}{2}$

$\Rightarrow \quad x_4 = 15-8$ and $y_4 = 5-2$

$\Rightarrow \quad x_4 = 7$ and $y_4 = 3$

$\therefore \quad D = (7, 3)$

$$\text{Mid point of DC is } E\left(\frac{x_4+9}{2}, \frac{y_4+4}{2}\right)$$

$$= E\left(\frac{7+9}{2}, \frac{3+4}{2}\right)$$

$$= E\left(\frac{16}{2}, \frac{7}{2}\right) = E\left(8, \frac{7}{2}\right)$$

Now, Area of $\Delta ADE = \dfrac{1}{2}\left[6\left(3-\dfrac{7}{2}\right)+7\left(\dfrac{7}{2}-1\right)+8(1-3)\right]$

$$= \frac{1}{2}\left[6\left(\frac{-1}{2}\right)+7\left(\frac{5}{2}\right)+8(-2)\right]$$

$$= \frac{1}{2}\left(3+\frac{35}{2}-16\right) = \frac{1}{2}\left(\frac{-6+35-32}{2}\right)$$

$$= \frac{1}{2}\times\frac{(-3)}{2} = \frac{-3}{4} \text{ sq units} = \frac{3}{4} \text{ sq. units}$$

[In magnitude]

Hence, the area of ΔADE is $\dfrac{3}{4}$ sq. units.

Q3. The points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of ΔABC.

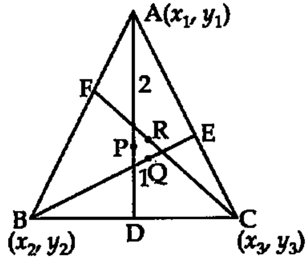
(*i*) The median from A meets BC at D. Find the coordinates of the point D.

(*ii*) Find the coordinates of the point P on AD such that AP : PD = 2 : 1.

(*iii*) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.

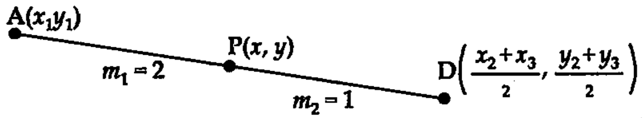
(*iv*) What are the coordinates of the centroid of the ΔABC?

Sol. (*i*) Median from A meets BC at D *i.e.*, D is the mid-point of BC.



So, the coordinates of D are given by $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

(*ii*)



The coordinates of the point P on AD such that AP : PD = 2 : 1 are given by

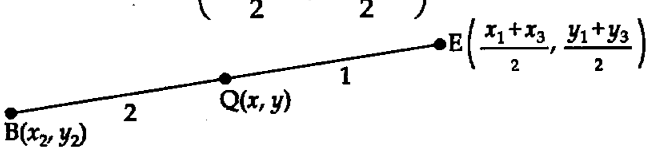
$$x = \frac{2\left(\frac{x_2 + x_3}{2}\right) + 1(x_1)}{2+1}, \qquad y = \frac{2\left(\frac{y_2 + y_3}{2}\right) + 1(y_1)}{2+1}$$

$$\Rightarrow \quad x = \frac{x_2 + x_3 + x_1}{3}, \qquad y = \frac{y_2 + y_3 + y_1}{3}$$

$\therefore$ P$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$ is the required point.

(*iii*) (*a*) Median BE meets the side AC at its mid-point E.

$\therefore$ Coordinates of E are $\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$.



Now, the coordinates of Q such that BE is median and BQ : QE = 2 : 1 are given by

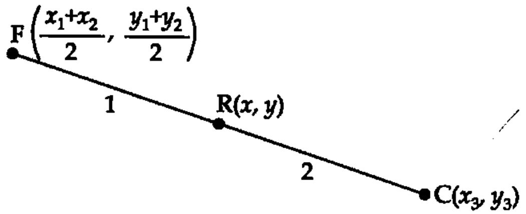
$$x = \frac{2\left(\frac{x_1 + x_3}{2}\right) + 1(x_2)}{2+1}, \qquad y = \frac{2\left(\frac{y_1 + y_3}{2}\right) + 1(y_2)}{2+1}$$

$$\Rightarrow \quad x = \frac{x_1 + x_3 + x_2}{3}, \qquad y = \frac{y_1 + y_3 + y_2}{3}$$

$\therefore$ The coordinates of point Q on median BE such at QB : QE = 2 : 1 are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

(*b*) Median CF meets the side AB at its mid-point F.

$\therefore$ Coordinate of F are $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$.



Now, the coordinates of R such that CF is median and CR : RF = 2 : 1 are given by

$$x = \frac{1(x_3) + 2\left(\dfrac{x_1+x_2}{2}\right)}{1+2}, \qquad y = \frac{1(y_3) + 2\left(\dfrac{y_1+y_2}{2}\right)}{1+2}$$

$$\Rightarrow \qquad x = \frac{x_3+x_1+x_2}{3}, \qquad y = \frac{y_3+y_1+y_2}{3}$$

So, the coordinates of point R on the median CF such that CR : RF = 2 : 1 are $\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$

(*iv*) Coordinates of centroid G of ΔABC are

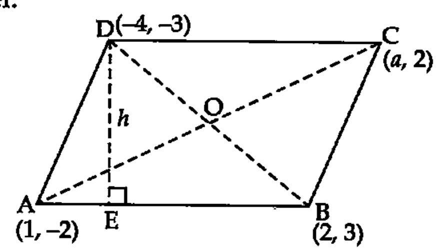
$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

It is observed that coordinates of P, Q, R and G are same.

Hence, the medians intersect at the same point *i.e.*, centroid which divides the medians in the ratio 2 : 1.

Q4. If the points A(1, –2), B(2, 3), C(*a*, 2) and D(–4, –3) form a parallelogram, find the value of *a* and height of the parallelogram taking AB as base.

Sol. As ABCD is a parallelogram and diagonals of parallelogram bisect each other.



OR

The mid points of diagonals of parallelogram will coincide i.e.,

Mid-point of diagonal AC = Mid-point of diagonal BD

$$\Rightarrow \left(\frac{1+a}{2}, \frac{-2+2}{2}\right) = \left(\frac{-4+2}{2}, \frac{-3+3}{2}\right)$$

$$\Rightarrow \left(\frac{1+a}{2}, 0\right) = \left(\frac{-2}{2}, 0\right)$$

$$\Rightarrow \frac{1+a}{2} = \frac{-2}{2}$$

$$\Rightarrow a = -2-1 = -3$$

Hence, the value of a is -3.

Now, Area of ΔABD = $\frac{1}{2}$ base $\times$ altitude

$$\Rightarrow \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = \frac{1}{2}AB \times h$$

$$\Rightarrow \frac{1}{2}[1\{3-(-3)\}+2\{-3-(-2)\}-4(-2-3)] = -\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\Rightarrow \frac{1}{2}[(3+3)+2(-3+2)-4(-5)] = \frac{h}{2}\sqrt{(2-1)^2+(3+2)^2}$$

$$\Rightarrow \frac{1}{2}[6+2(-1)+20] = \frac{h}{2}\sqrt{(1)^2+(5)^2}$$

$$\Rightarrow \frac{1}{2}[6-2+20] = \frac{h}{2}\sqrt{1+25}$$

$$\Rightarrow \frac{1}{2}[26-2] = \frac{h}{2}\sqrt{26}$$

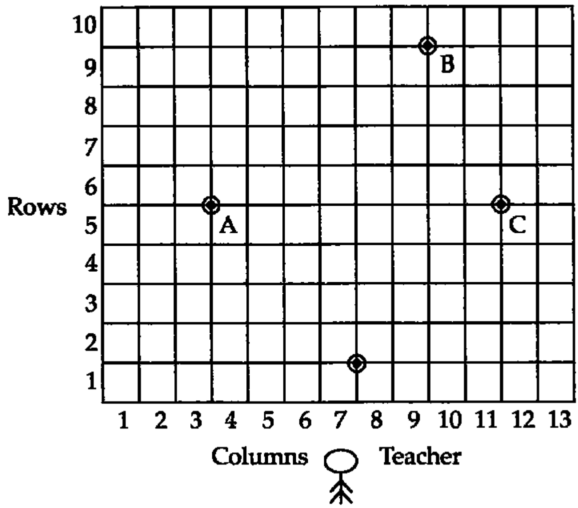
$$\Rightarrow h\sqrt{26} = 24$$

$$\Rightarrow h = \frac{24}{\sqrt{26}} \times \frac{\sqrt{26}}{\sqrt{26}} = \frac{24\sqrt{26}}{26}$$

$$\Rightarrow h = \frac{12}{13}\sqrt{26} \text{ units}$$

Hence, the perpendicular distance between parallel sides AB and CD is $\frac{12\sqrt{26}}{13}$ units.

Q5. Student of a school are standing in rows and columns in their playground for a drill practice. A, B, C, D are the positions of four students as shown in the figure. Is it possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B C and D? If so, what should be his position?



Sol. Coordinates of A, B, C and D from graph are A(3, 5), B(7, 9), C(11, 5), and D(7, 1).

To find the shape of □ABCD:

$$AB^2 = (7-3)^2 + (9-5)^2 = 4^2 + 4^2 = 4^2(1+1)$$

$\Rightarrow \quad AB = 4\sqrt{2}$ units

$$BC^2 = (11-7)^2 + (5-9)^2 = (4)^2 + (-4)^2 = 4^2(1+1)$$

$\Rightarrow \quad BC = 4\sqrt{2}$ units

$$CD^2 = (7-11)^2 + (1-5)^2 = (-4)^2 + (-4)^2 = 4^2 + 4^2$$

$\Rightarrow \quad CD = 4\sqrt{2}$ units

$$DA^2 = (7-3)^2 + (1-5)^2 = 4^2 + (-4)^2 = 4^2 + 4^2$$

$\Rightarrow \quad DA = \sqrt{4^2(1+1)} = 4\sqrt{2}$ units

$\therefore \quad AB = BC = CD = DA = 4\sqrt{2}$ units.

So, ABCD will be either square or rhombus.

Now, Diagonal $AC = \sqrt{(11-3)^2 + (5-5)^2}$

$\Rightarrow \quad AC = \sqrt{(8)^2 + (0)^2}$

$\Rightarrow \quad AC = 8$ units

and diagonal $BD = \sqrt{(7-7)^2 + (1-9)^2} = \sqrt{(0)^2 + (8)^2} = \sqrt{0 + (8)^2} = \sqrt{8^2}$

$\Rightarrow \quad BD = 8$ units

$\therefore \quad$ Diagonal AC = Diagonal BD

So, the given quadrilateral ABCD is a square. The point which is equidistant from point A, B, C, D of a square ABCD will be at the intersecting point of diagonals and diagonals bisect each other.

Hence, the required point O equidistant from A, B, C, D is mid point of any diagonal $= \left(\frac{7+7}{2}, \frac{9+1}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$.

Hence, the required point is (7, 5).

Q6. Ayush starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. What is the extra distance travelled by Ayush in reaching his office? (Assume that all distances covered are in straight lines). If the house is situated at (2, 4), bank at (5, 8) school at (13, 14) and office at (13, 26) and coordinates are in km.

Sol. Consider the coordinates of house H(2, 4), bank B(5, 8), school S(13, 14) and office O(13, 26).

$$\text{Distance } HB^2 = (5-2)^2 + (8-4)^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$\Rightarrow \quad HB = 5 \text{ km}$$

$$\text{Distance } BS^2 = (13-5)^2 + (14-8)^2 = 8^2 + 6^2 = 64 + 36$$

$$\Rightarrow \quad BS^2 = 100$$

$$\Rightarrow \quad BS = 10 \text{ km}$$

$$\text{Distance } SO^2 = (13-13)^2 + (26-14)^2 = 0^2 + 12^2 = 12^2$$

$$\Rightarrow \quad SO = 12 \text{ km}$$

Total distance travelled by Ayush from house to bank to school and then to office

$$= HB + BS + SO$$

$$= 5 + 10 + 12 = 27 \text{ km}$$

Direct distance from house to office = HO

$$\Rightarrow \quad HO^2 = (13-2)^2 + (26-4)^2 = (11)^2 + (22)^2$$

$$\Rightarrow \quad HO^2 = 121 + 484$$

$$\Rightarrow \quad HO = \sqrt{605} = 24.6 \text{ km}$$

So, extra distance travelled by Ayush = 27 km – 24.6 km = 2.4 km.

Hence, extra distance travelled by Ayush = 2.4 km

❑❑❑

8 Introduction to Trigonometry and its Applications

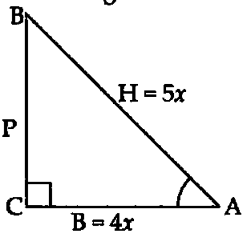
EXERCISE 8.1

Choose the correct answer from the given four options:

Q1. If $\cos A = \frac{4}{5}$, then the value of tan A is

(*a*) $\frac{3}{5}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{4}{3}$ (*d*) $\frac{5}{3}$

Sol. (*b*): $\cos A = \frac{4}{5} = \frac{B}{H} = \frac{4x}{5x}$



$P^2 + B^2 = H^2$ (By Pythagoras theorem)

$\Rightarrow \quad P^2 + (4x)^2 = (5x)^2$

$\Rightarrow \quad P^2 = 25x^2 - 16x^2$

$\Rightarrow \quad P^2 = 9x^2$

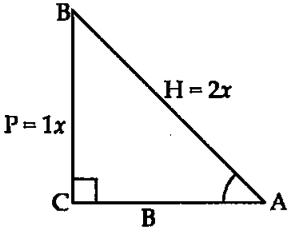
$\Rightarrow \quad P = 3x$

$\therefore \quad \tan A = \frac{P}{B} = \frac{3x}{4x} = \frac{3}{4}$, which verifies option (*b*).

Q2. If $\sin A = \frac{1}{2}$, then the value of cot A is

(*a*) $\sqrt{3}$ (*b*) $\frac{1}{\sqrt{3}}$ (*c*) $\frac{\sqrt{3}}{2}$ (*d*) 1

Sol. (*a*): $\sin A = \frac{1}{2} = \frac{P}{H} = \frac{1x}{2x}$



$B^2 + P^2 = H^2$

$\Rightarrow \quad B^2 + (1x)^2 = (2x)^2$

$\Rightarrow \quad B^2 = 4x^2 - 1x^2$

$\Rightarrow \quad B^2 = 3x^2 \Rightarrow B = \sqrt{3}x$

$\therefore \quad \cot A = \frac{B}{P} = \frac{\sqrt{3}x}{1x} = \sqrt{3}$

Hence, right option is (*a*).

Q3. The value of the expression

$\text{cosec}\,(75° + \theta) - \sec\,(15° - \theta) - \tan\,(55° + \theta) + \cot\,(35° - \theta)$ is

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) $\frac{3}{2}$

Sol. (*b*): $(75° + \theta)$ and $(15° - \theta)$, are complements of each other. Similarly, $(55° + \theta)$ and $(35° - \theta)$ are also complements.

So, $\text{cosec}\,(75° + \theta) - \sec\,(15° - \theta) - \tan\,(55° + \theta) + \cot\,(35° - \theta)$

$= \text{cosec}\,[90° - (15° - \theta)] - \sec\,(15° - \theta) - \tan\,(55° + \theta) + \cot\,[90° - (55° + \theta)]$

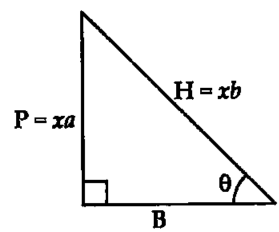
$= \sec(15° - \theta) - \sec(15° - \theta) - \tan(55° + \theta) + \tan(55° + \theta)$
$= 0$

Hence, right option is (*b*).

Q4. Given that $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to

(*a*) $\frac{b}{\sqrt{b^2 - a^2}}$ (*b*) $\frac{b}{a}$ (*c*) $\frac{\sqrt{b^2 - a^2}}{b}$ (*d*) $\frac{a}{\sqrt{b^2 - a^2}}$

Sol. (*c*): $\sin\theta = \frac{a}{b} = \frac{P}{H} = \frac{ax}{bx}$ [Given]



By Pythagoras theorem,

$B^2 + P^2 = H^2$

$\Rightarrow B^2 + (ax)^2 = (bx)^2$

$\Rightarrow B^2 = b^2x^2 - a^2x^2$

$\Rightarrow B^2 = x^2(b^2 - a^2)$

$\Rightarrow B = x\sqrt{b^2 - a^2}$

$\therefore \cos\theta = \frac{B}{H} = \frac{x\sqrt{(b^2 - a^2)}}{bx} = \frac{\sqrt{b^2 - a^2}}{b}$,

which verifies the option (*c*).

Q5. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to

(*a*) $\cos\beta$ (*b*) $\cos 2\beta$ (*c*) $\sin\alpha$ (*d*) $\sin 2\alpha$

Sol. (*b*): $\cos(\alpha + \beta) = 0$ [Given]

$\Rightarrow \cos(\alpha + \beta) = \cos 90°$

$\Rightarrow \alpha + \beta = 90°$

$\Rightarrow \alpha = 90° - \beta$

Now, $\sin(\alpha - \beta) = \sin(90° - \beta - \beta)$
$= \sin(90° - 2\beta)$

$\Rightarrow \sin(\alpha - \beta) = \cos 2\beta$

Hence, verifies the option (*b*).

Q6. The value of $(\tan 1° \tan 2° \tan 3° \ldots \tan 89°)$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) $\frac{1}{2}$

Sol. (*b*): $(\tan 1° \tan 2° \tan 3° \ldots \tan 89°)$

$= (\tan 1° \tan 89°)(\tan 2° \tan 88°)(\tan 3° \tan 87°) \ldots (\tan 45° \tan 45°)$

$= [\tan 1° \tan(90° - 1)][\tan 2° \tan(90° - 2)][\tan 3° \tan(90° - 3)] \ldots \tan 45° \tan(90° - 45°)$

$= \tan 1° \cot 1° \tan 2° \cot 2° \tan 3° \cot 3° \ldots \tan 45° \cot 45°$

$= \tan 1° \times \frac{1}{\tan 1°} \tan 2° . \frac{1}{\tan 2°} \tan 3° \cdot \frac{1}{\tan 3°} \cdots \frac{\tan 45°}{\tan 45°}$

$= 1 \cdot 1 \cdot 1 \cdot 1 . \ldots 1 \cdot 1$

$= 1$

Hence, verifies the option (*b*).

Q7. If $\cos 9\alpha = \sin \alpha$ and $9\alpha < 90°$, then the value of $\tan 5\alpha$ is

(*a*) $\frac{1}{\sqrt{3}}$ (*b*) $\sqrt{3}$ (*c*) 1 (*d*) 0

Sol. (*c*): $\cos 9\alpha = \sin \alpha$

$\Rightarrow \cos 9\alpha = \cos (90° - \alpha)$

$\Rightarrow 9\alpha = 90° - \alpha$

$\Rightarrow 10\alpha = 90°$

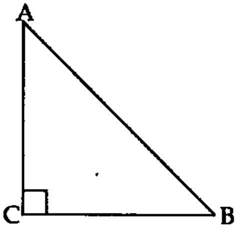
$\Rightarrow \alpha = 9°$

$\therefore \tan 5\alpha = \tan 5 \times 9° = \tan 45° = 1$

Hence, verifies the option (*c*).

Q8. If ΔABC is right angled at C, then the value of $\cos (A + B)$ is

(*a*) 0 (*b*) 1 (*c*) $\frac{1}{2}$ (*d*) $\frac{\sqrt{3}}{2}$



Sol. (*a*): $\angle A + \angle B + \angle C = 180°$

(Angle sum property of a triangle)

$\Rightarrow (A + B) + 90° = 180°$

$\Rightarrow \angle A + \angle B = 90°$

$\Rightarrow \cos (A + B) = \cos 90° = 0$

Hence, verifies the option (*a*).

Q9. If $\sin A + \sin^2 A = 1$, then the value of the expression $(\cos^2 A + \cos^4 A)$ is

(*a*) 1 (*b*) $\frac{1}{2}$ (*c*) 2 (*d*) 3

Sol. (*a*): $\sin A + \sin^2 A = 1$ [Given]

$\Rightarrow \sin A = (1 - \sin^2 A)$

$\Rightarrow \sin A = \cos^2 A$

$\Rightarrow \sin^2 A = \cos^4 A$ [Squaring both sides]

$\Rightarrow 1 - \cos^2 A = \cos^4 A$ [$\because \sin^2 A = 1 - \cos^2 A$]

$\Rightarrow 1 = \cos^2 A + \cos^4 A$

Hence, verifies the option (*a*).

Q10. Given that $\sin \alpha = \frac{1}{2}, \cos \beta = \frac{1}{2}$, then value of $(\alpha + \beta)$ is

(*a*) 0° (*b*) 30° (*c*) 60° (*d*) 90°

Sol. (*d*): $\sin \alpha = \frac{1}{2}$ [Given]

$\Rightarrow \sin \alpha = \sin 30°$

$\Rightarrow \alpha = 30°$

Also, $\cos \beta = \frac{1}{2}$

$\Rightarrow \cos \beta = \cos 60°$

$\Rightarrow \beta = 60°$

$\therefore \alpha + \beta = 30° + 60° = 90°$

Hence, verifies the option (*d*).

Q11. The value of the expression

$$\left[\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°\right] \text{ is}$$

(*a*) 3 (*b*) 2 (*c*) 1 (*d*) 0

Sol. (*b*): Here, the complement angle of each angle is available. So, by using formula for complementary angles, we get

$$\frac{\sin^2 22° + \sin^2(90° - 22°)}{\cos^2 22° + \cos^2(90° - 22°)} + \sin^2 63° + \cos 63° \sin(90° - 63°)$$

$$\Rightarrow \frac{\sin^2 22° + \cos^2 22°}{\cos^2 22° + \sin^2 22°} + \sin^2 63° + \cos 63° \cos 63°$$

$$\Rightarrow \frac{1}{1} + \sin^2 63° + \cos^2 63° \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow 1 + 1 = 2$$

Hence, verifies the option (*b*).

Q12. If $4 \tan\theta = 3$, then $\left[\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right]$ is equal to

(*a*) $\frac{2}{3}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{3}{4}$

Sol. (*c*): $\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}$

$$= \frac{4\left(\frac{\sin\theta}{\cos\theta}\right) - 1}{4\left(\frac{\sin\theta}{\cos\theta}\right) + 1}$$ (Dividing the numerator and denominator throughout by $\cos\theta$)

$$= \frac{4\tan\theta - 1}{4\tan\theta + 1} = \frac{3-1}{3+1} \qquad [\because 4\tan\theta = 3]$$

$= \frac{2}{4} = \frac{1}{2}$. Hence, verifies the option (*c*).

Q13. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(*a*) 1 (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{4}$

Sol. (*c*): $\sin\theta - \cos\theta = 0$ [Given]

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \sin\theta = \sin(90° - \theta)$

$\Rightarrow \theta = 90° - \theta$

$\Rightarrow 2\theta = 90°$

$\Rightarrow \theta = 45°$

Now, $\sin^4\theta + \cos^4\theta = (\sin 45°)^4 + (\cos 45°)^4$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

Hence, verifies the option (*c*).

Q14. sin (45° + θ) – cos (45° – θ) is equal to

(*a*) 2 cos θ (*b*) 0 (*c*) 2 sin θ (*d*) 1

Sol. (*b*): (45° + θ) and (45° – θ) are complementary angles.

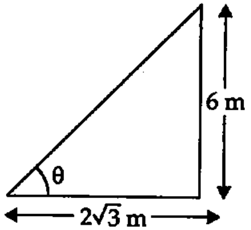
∴ By using formulae of complementary angles,

$$\sin(45° + \theta) - \cos(45° - \theta) = \sin(45° + \theta) - \cos[90° - (45° + \theta)]$$
$$= \sin(45° + \theta) - \sin(45° + \theta)$$
$$= 0$$

Hence, verifies the option (*b*).

Q15. If a pole 6 m high casts a shadow of $2\sqrt{3}$ m long on the ground, then the Sun's elevation is

(*a*) 60° (*b*) 45°
(*c*) 30° (*d*) 90°



Sol. (*a*): $\tan\theta = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \frac{\sqrt{3}}{1}$

⇒ $\tan\theta = \tan 60°$

⇒ $\theta = 60°$

Hence, the right option is (*a*).

EXERCISE 8.2

Write True or False and justify your answer in each of the following:

Q1. $\frac{\tan 47°}{\cot 43°} = 1$

Sol. True: 47° and 43° are complementary angles.

$$\therefore \frac{\tan 47°}{\cot 43°} = \frac{\tan 47°}{\cot(90° - 47°)} = \frac{\tan 47°}{\tan 47°} = 1$$

Hence, the given expression is true.

Q2. The value of the expression ($\cos^2 23° - \sin^2 67°$) is positive.

Sol. False: 23° and 67° are complementary angles so

$$\cos^2 23° - \sin^2 67° = \cos^2 23° - \sin^2(90° - 23°)$$
$$= \cos^2 23° - \cos^2 23°$$
$$= 0$$

So, the value of the given expression is not positive. Hence, the given statement is false.

Q3. The value of the expression (sin 80° – cos 80°) is negative.

Sol. False: 80° is near to 90°, sin 90° = 1 and cos 90° = 0

So, the given expression sin 80° – cos 80° > 0

So, the value of the given expression is positive. So, the given statement is false.

Q4. $\sqrt{(1-\cos^2\theta)\sec^2\theta} = \tan\theta$

Sol. True: $\quad \text{LHS} = \sqrt{(1-\cos^2\theta)\sec^2\theta} = \sqrt{\sin^2\theta \cdot \dfrac{1}{\cos^2\theta}}$

$$= \sqrt{\frac{\sin^2\theta}{\cos^2\theta}} = \tan\theta = \text{RHS}$$

Hence, the given expression is true.

Q5. If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A = 1$

Sol. True:

$$\cos A + \cos^2 A = 1 \qquad \text{[Given]}$$

$\Rightarrow \quad \cos A = 1 - \cos^2 A$

$\Rightarrow \quad \cos A = \sin^2 A$

$\Rightarrow \quad \cos^2 A = \sin^4 A$

Now, $\quad \text{LHS} = \sin^2 A + \sin^4 A$

$= \cos A + \cos^2 A$

$= 1 = \text{RHS}$

Hence, the given statement is true.

Q6. $(\tan\theta + 2)(2\tan\theta + 1) = 5\tan\theta + \sec^2\theta$

Sol. False:

$$\begin{aligned} \text{LHS} &= (\tan\theta + 2)(2\tan\theta + 1) \\ &= \tan\theta(2\tan\theta + 1) + 2(2\tan\theta + 1) \\ &= 2\tan^2\theta + \tan\theta + 4\tan\theta + 2 \\ &= 2\tan^2\theta + 5\tan\theta + 2 \\ &= 2(\tan^2\theta + 1) + 5\tan\theta \\ &= 2\sec^2\theta + 5\tan\theta \neq \text{RHS} \end{aligned}$$

Hence, the given statement is false.

Q7. If the length of the shadow of a tower is increasing, then the angle of elevation of the sun is also increasing.

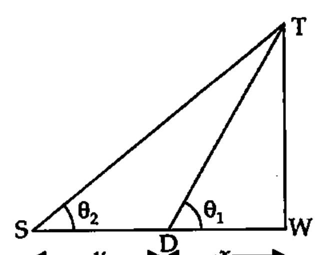
Sol. False: The shadow of a tower on the ground increases from x to $(x + y)$ when angle of elevation of the sun changes from θ_1 to θ_2.

$\therefore$ θ_1 is the exterior angle of ΔTSD

so $\qquad \theta_1 > \theta_2$

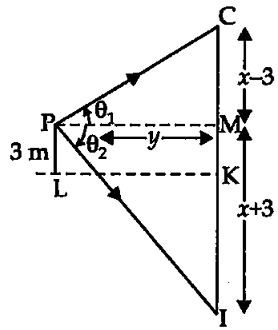
So, on increasing the length of shadow the angle of elevation decreases.

Hence, the given statement is false.



Q8. If a man standing on a platform 3 m above the surface of a lake observes a cloud and its reflection in the lake, then the angle of elevation of the cloud is equal to the angle of depression of its reflection.

Sol. False: The observer is at the platform (P) 3 m above the surface LK of the lake.
He observes the angle of elevation of cloud C from P and its reflection image in the lake is formed at I. The observer measures the angle of depression of image (I) θ_2. Draw PM ⊥ on the vertical line passing through the cloud and its image.



CK = KI = x by the prop. of reflection

$$CM = CK - MK = x - 3$$
$$MI = KI + MK = x + 3$$

Now, $\tan\theta_1 = \dfrac{x-3}{y}$ and $\tan\theta_2 = \dfrac{x+3}{y}$

$\Rightarrow$ $y = \dfrac{x-3}{\tan\theta_1}$ and $y = \dfrac{x+3}{\tan\theta_2}$

$\Rightarrow$ $\dfrac{x+3}{\tan\theta_2} = \dfrac{x-3}{\tan\theta_1}$

$\Rightarrow$ $\tan\theta_2 = \left(\dfrac{x+3}{x-3}\right)\tan\theta_1$

$\Rightarrow$ $\tan\theta_1 \neq \tan\theta_2$

or $\theta_1 \neq \theta_2$

Alternate Method: By the property of image formation, the distance of image and the object are equal from the reflecting surface.

So, $KC = KI$

$\Rightarrow$ $MI \neq MC$

$\Rightarrow$ $\Delta MPC \not\equiv \Delta MPI$

so $\theta_1 \neq \theta_2$

Q9. The value of 2 sin θ can be $\left(a + \dfrac{1}{a}\right)$, where a is a positive number, and $a \neq 1$.

Sol. False: Consider 'a' and '$\dfrac{1}{a}$' as positive numbers and $a \neq 0$

Arithmetic mean (AM) of a and $\dfrac{1}{a} = \dfrac{\left(a + \dfrac{1}{a}\right)}{2}$

Geometric mean (GM) of a and $\dfrac{1}{a} = \sqrt{a \times \dfrac{1}{a}} = 1$

$\because$ AM > GM

$\therefore$ $\dfrac{\left(a + \dfrac{1}{a}\right)}{2} > 1$

$$\Rightarrow \quad \left(a+\frac{1}{a}\right) > 2$$

Let $\quad a+\frac{1}{a} = 2\sin\theta$

$\Rightarrow \quad 2\sin\theta > 2$

$\Rightarrow \quad \sin\theta > 1$

which can never be possible.

Hence, our consideration that $a+\frac{1}{a}=2\sin\theta$ is false.

Alternate Method: a is positive and $a \neq 1$ *i.e.*, a can be above 0 to all real and values except 1.

Let $0 < a < 1$ then $\frac{1}{a}$ will be more than 1 so $a+\frac{1}{a} > 2$ for any value of a.

Let $\quad a = 0.2 \Rightarrow a+\frac{1}{a} = 0.2+\frac{1}{0.2} = 5.2$

$\quad a = 0.9 \Rightarrow a+\frac{1}{a} = 0.9+\frac{1}{0.9} = 0.9+1.111 = 2.011$

Put $a+\frac{1}{a} = 2\sin\theta$

$\therefore \quad 2\sin\theta > 2$

$\Rightarrow \quad \sin\theta > 1$

which is impossible so $2\sin\theta \neq a+\frac{1}{a}$

Hence, the given statement is false. If we take any value of a more than one, then the value of $a+\frac{1}{a}$ is always greater than 2 which repeats the result.

Q10. $\cos\theta = \frac{a^2+b^2}{2ab}$, where a and b are two distinct numbers such that $ab > 0$.

Sol. False: Consider two numbers a^2 and b^2 then

Arithmetic mean (AM) of a^2 and $b^2 = \frac{a^2+b^2}{2}$

Geometric mean (GM) of a^2 and $b^2 = \sqrt{a^2 \times b^2}$

$\Rightarrow \quad GM = ab$

$\because \quad AM > GM$

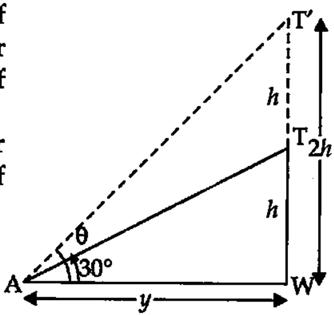
$\therefore \quad \frac{a^2+b^2}{2} > ab$

$\Rightarrow \quad \frac{a^2+b^2}{2ab} > 1$

$$\Rightarrow \quad \cos\theta > 1 \qquad \left(\text{Given } \cos\theta = \frac{a^2+b^2}{2ab}\right)$$

But, the value of cos θ can never be greater than 1.
So, the given expression is false.

Q11. The angle of elevation of the top of a tower is 30°. If the height of the tower is doubled, then the angle of elevation of its top will also be doubled.



Sol. False: Let the height of the tower is h. For the observer at A the angle of elevation is equal to 30°.

$$\tan 30° = \frac{h}{y}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{y}$$

$$\Rightarrow \quad y = h\sqrt{3}$$

Now, the height of the tower increases to $2h$.
Now, let the new angle of elevation at A becomes θ then

$$\tan\theta = \frac{2h}{y}$$

$$\Rightarrow \quad \tan\theta = \frac{2h}{h\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \frac{2}{\sqrt{3}}$$

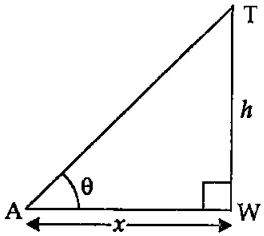
But, $\tan 60° = \sqrt{3}$

$$\Rightarrow \quad \tan 60° = \sqrt{3} \neq \frac{2}{\sqrt{3}}$$

So, $\theta \neq 60°$

Hence, angle of elevation will not be doubled or the given statement is false.

Q12. If the height of a tower and the distance of the point of observation from its foot, both are increased by 10%, then the angle of elevation of its top remains: unchanged.



Sol. True: Let height h of tower TW makes an angle of elevation θ to observer at A and the distance from foot of tower to the observer is x.

$$\therefore \quad \tan\theta = \frac{h}{x} \qquad \text{(I)}$$

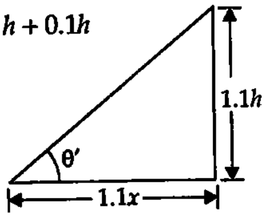
Now, h and x increases by 10%

$\therefore \quad h' = h + 10\% \text{ of } h = h + \frac{10}{100} \times h = h + 0.1h$

$\Rightarrow \quad h' = 1.1\,h$

Similarly, $\quad x' = 1.1x$

$\therefore \quad \tan\theta' = \frac{1.1h}{1.1x} = \frac{h}{x} \quad \text{(II)}$



From (I) and (II), we get

$\tan\theta = \tan\theta'$

$\Rightarrow \quad \theta = \theta'$

Hence, the given statement is true.

EXERCISE 8.3

Prove the following questions (from Q1 to Q7):

Q1. $\frac{\sin\theta}{(1+\cos\theta)} + \frac{(1+\cos\theta)}{\sin\theta} = 2\,\text{cosec}\,\theta$

Sol. (*i*) If there is (+)ve or (–)ve sign in D^r and N^r of the expression, then keep the expression in brackets.

(*ii*) LHS is more difficult than RHS so we will start from LHS.

(*iii*) Use identities, if applicable.

(*iv*) Convert the expression into $\sin\theta$ and $\cos\theta$.

$$\text{LHS} = \frac{\sin\theta}{(1+\cos\theta)} + \frac{(1+\cos\theta)}{\sin\theta}$$

$$= \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta + (1)^2 + (\cos\theta)^2 + 2(1)(\cos\theta)}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1+1+2\cos\theta}{\sin\theta\,(1+\cos\theta)} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2+2\cos\theta}{\sin\theta\,(1+\cos\theta)} = \frac{2\,(1+\cos\theta)}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\,\text{cosec}\,\theta$$

$\therefore \quad \text{LHS} = \text{RHS}$

Hence, proved.

Q2. $\dfrac{\tan A}{1+\sec A} - \dfrac{\tan A}{1-\sec A} = 2\,\text{cosec}\,A$

Sol. We will start from LHS.

[We can take tan A as common and (1 – sec A) (1 + sec A) makes $(1-\sec^2 A = -\tan^2 A)$ so formulae will be used.]

$$\text{LHS} = \frac{\tan A}{(1+\sec A)} - \frac{\tan A}{(1-\sec A)}$$

$$= \tan A\left[\frac{1}{(1+\sec A)} - \frac{1}{(1-\sec A)}\right] \quad \text{[Taking tan A common]}$$

$$= \tan A\left[\frac{1-\sec A-(1+\sec A)}{(1+\sec A)(1-\sec A)}\right] \quad \text{[Taking LCM]}$$

$$= \tan A\left[\frac{-2\sec A}{(1-\sec^2 A)}\right] = \frac{\tan A(-2\sec A)}{-(\sec^2 A-1)}$$

$$= \frac{-\tan A\, 2\sec A}{-\tan^2 A} = \frac{2\sec A}{\tan A}$$

$$= \frac{2\times\dfrac{1}{\cos A}}{\dfrac{\sin A}{\cos A}}$$

$$= \frac{2}{\sin A} = 2\,\text{cosec}\,A = \text{RHS}$$

Hence, proved.

Q3. If $\tan A = \dfrac{3}{4}$, then $\sin A \cos A = \dfrac{12}{25}$.

Sol. $\tan A = \dfrac{3}{4}$ [Given]

$\Rightarrow$ $\tan A = \dfrac{P}{B} = \dfrac{3x}{4x}$

$H^2 = P^2 + B^2$ [By Pythagoras theorem]

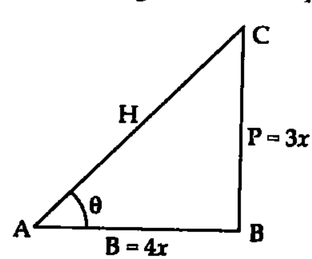
$= (3x)^2 + (4x)^2 = 9x^2 + 16x^2$

$\Rightarrow$ $H^2 = 25x^2$

$\Rightarrow$ $H = 5x$

$\therefore$ $\sin A = \dfrac{P}{H} = \dfrac{3x}{5x} = \dfrac{3}{5}$

and $\cos A = \dfrac{B}{H} = \dfrac{4x}{5x} = \dfrac{4}{5}$



$\therefore \quad \text{LHS} = \sin A \cos A$

$= \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$

$= \text{RHS}$

Hence, verified.

Q4. $(\sin\alpha + \cos\alpha)(\tan\alpha + \cot\alpha) = \sec\alpha + \operatorname{cosec}\alpha$

Sol. $\text{LHS} = (\sin\alpha + \cos\alpha)(\tan\alpha + \cot\alpha)$

$$= (\sin\alpha + \cos\alpha)\left(\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}\right)$$

$$= (\sin\alpha + \cos\alpha)\left[\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha\cos\alpha}\right]$$

$$= (\sin\alpha + \cos\alpha)\frac{1}{\sin\alpha\cos\alpha} \qquad [\because \sin^2\alpha + \cos^2\alpha = 1]$$

$$= \frac{\sin\alpha}{\sin\alpha\cos\alpha} + \frac{\cos\alpha}{\sin\alpha\cos\alpha}$$

$$= \frac{1}{\cos\alpha} + \frac{1}{\sin\alpha}$$

$= \sec\alpha + \operatorname{cosec}\alpha$

$= \text{RHS}$

Hence, proved.

Q5. $(\sqrt{3} + 1)(3 - \cot 30^\circ) = \tan^3 60^\circ - 2\tan 60^\circ$

Sol. $\text{LHS} = (\sqrt{3} + 1)(3 - \cot 30^\circ)$

$= (\sqrt{3} + 1)(3 - \sqrt{3})$

$= \sqrt{3}(3 - \sqrt{3}) + 1(3 - \sqrt{3})$

$= 3\sqrt{3} - 3 + 3 - \sqrt{3}$

$= 2\sqrt{3}$

$\text{RHS} = \tan^3 60^\circ - 2\sin 60^\circ$

$$= (\sqrt{3})^3 - 2 \times \frac{\sqrt{3}}{2}$$

$= 3\sqrt{3} - \sqrt{3}$

$= 2\sqrt{3}$

$\Rightarrow \quad \text{LHS} = \text{RHS}$

Hence, proved.

Q6. $1 + \frac{\cot^2\alpha}{(1 + \operatorname{cosec}\alpha)} = \operatorname{cosec}\alpha$

Sol. $\text{LHS} = 1 + \frac{\cot^2\alpha}{(1 + \operatorname{cosec}\alpha)}$

$$\therefore \quad = 1+\frac{(\text{cosec}^2\alpha-1)}{(1+\text{cosec}\,\alpha)} \qquad [\because \cot^2\alpha = \text{cosec}^2\alpha - 1]$$

$$= 1+\frac{(\text{cosec}\,\alpha-1)(\text{cosec}\,\alpha+1)}{(\text{cosec}\,\alpha+1)} \quad [\because a^2-b^2=(a-b)(a+b)]$$

$$= 1+\text{cosec}\,\alpha-1$$

$$= \text{cosec}\,\alpha$$

$$= \text{RHS}$$

Hence, proved.

Q7. $\tan\theta + \tan(90^\circ-\theta) = \sec\theta\sec(90^\circ-\theta)$

Sol. $\text{LHS} = \tan\theta + \tan(90^\circ-\theta)$

$$= \tan\theta + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} \qquad [\because \sin^2\theta+\cos^2\theta=1]$$

$$= \text{cosec}\,\theta\,.\,\sec\theta$$

$\Rightarrow \quad \text{LHS} = \text{cosec}\,\theta\cdot\sec\theta$

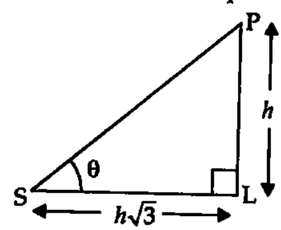
$\text{RHS} = \sec\theta\sec(90^\circ-\theta)$

$$= \sec\theta\,\text{cosec}\,\theta$$

$\Rightarrow \quad \text{LHS} = \text{RHS}$

Hence, verified.

Q8. Find the angle of elevation of the sun when the shadow of a pole h m high is $\sqrt{3}h$ m long.



Sol. Height of pole = PL = h

Length of shadow = SL = $h\sqrt{3}$

$\therefore \quad \tan\theta = \dfrac{h}{h\sqrt{3}}$

$$= \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad \tan\theta = \tan 30^\circ$

$\Rightarrow \quad \theta = 30^\circ$

Hence, the angle of elevation of sun is 30°.

Q9. If $\sqrt{3}\tan\theta = 1$, then find the value of $\sin^2\theta - \cos^2\theta$.

Sol. $\sqrt{3}\tan\theta = 1$ [Given]

$\Rightarrow \quad \tan\theta = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = \tan 30^\circ$

$\Rightarrow \quad \theta = 30°$

So, $\sin^2\theta - \cos^2\theta = \sin^2 30° - \cos^2 30°$ $[\because \theta = 30°]$

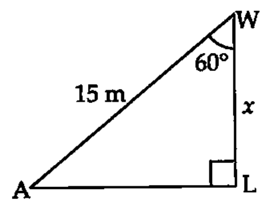
$$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = \frac{-2}{4} = \frac{-1}{2}$$

Q10. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, find the height of the wall.

Sol. Consider the vertical wall WL = x m (let)

Length of inclined ladder AW = 15 m [Given]

Ladder makes an angle of 60° with the wall.



$\therefore \quad \frac{x}{15} = \cos 60°$

$\Rightarrow \quad \frac{x}{15} = \frac{1}{2}$

$\Rightarrow \quad x = \frac{15}{2} = 7.5$

Hence, the height of the wall = 7.5 m.

Q11. Simplify: $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

Sol. $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$$= (\sec^2\theta)(1 - \sin^2\theta)$$
$$= \sec^2\theta \cos^2\theta$$
$$= \frac{1}{\cos^2\theta} \times \cos^2\theta$$
$$= 1$$

Q12. If $2\sin^2\theta - \cos^2\theta = 2$, then find the value of θ.

Sol. Given: $2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow \quad 2\sin^2\theta - (1 - \sin^2\theta) = 2$

$\Rightarrow \quad 2\sin^2\theta - 1 + \sin^2\theta = 2$

$\Rightarrow \quad 3\sin^2\theta = 2 + 1$

$\Rightarrow \quad \sin^2\theta = \frac{3}{3} = 1$

$\Rightarrow \quad \sin\theta = +1, -1$

$\Rightarrow \quad \sin\theta = 1$	$\Rightarrow \quad \sin\theta = -1$
$\Rightarrow \quad \sin\theta = \sin 90°$	$\Rightarrow \quad \sin\theta = -\sin 90°$
$\Rightarrow \quad \theta = 90°$	$\Rightarrow \quad -\sin(-\theta) = -\sin 90°$
	$\Rightarrow \quad \sin(-\theta) = \sin 90°$
	$\Rightarrow \quad -\theta = 90°$
	$\Rightarrow \quad \theta = -90°$
	i.e., $\quad \theta = 360° - 90° = 270°$

Hence, $\theta = 90°$ and $270°$

Q13. Show that $\dfrac{\cos^2(45°+\theta)+\cos^2(45°-\theta)}{\tan(60°+\theta)\tan(30°-\theta)}=1.$

Sol. $(45°+\theta)$, $(45°-\theta)$ and $(60°+\theta)$, $(30°-\theta)$ are complementary angles so by using complementary angle formulae, we get

$$\frac{\cos^2(45°+\theta)+\cos^2(45°-\theta)}{\tan(60°+\theta)\tan(30°-\theta)}$$

$$=\frac{\cos^2(45°+\theta)+\cos^2[90°-(45°+\theta)]}{\tan(60°+\theta)\tan[90°-(60°+\theta)]}$$

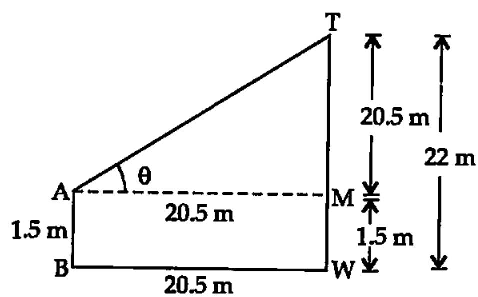
$$[\because\quad 90°-(45°+\theta)=90°-45°-\theta=45°-\theta]$$

$$=\frac{\cos^2(45°+\theta)+\sin^2(45°+\theta)}{\tan(60°+\theta)\cdot\cot(60°+\theta)}$$

$$=\frac{1}{\tan(60°+\theta)\cdot\dfrac{1}{\tan(60°+\theta)}}=\frac{1}{1}=1=\text{RHS}$$

Hence, proved.

Q14. An observer 1.5 m tall is 20.5 m away from a tower 22 m high. Determine the angle of elevation of the top of the tower from the eye of the observer.



Sol. Height of tower (TW) = 22 m [Given]

Height of observer (AB) = 1.5 m [Given]

Distance between foot of tower and observer (BW) = 20.5 m [Given]

Let $\theta = \angle$ of elevation of the observer at the top of the tower

Now, $\quad TM = 22\text{ m} - 1.5\text{ m} = 20.5\text{ m}$

$\quad AM = 20.5\text{ m}$

$\therefore \quad \tan\theta = \dfrac{20.5}{20.5}=1$

$\Rightarrow \quad \tan\theta = 1$

$\Rightarrow \quad \tan\theta = \tan 45°$

$\Rightarrow \quad \theta = 45°$

Hence, the angle of elevation of the top of the tower from observer's eye is 45°.

Q15. Show that $\tan^4\theta+\tan^2\theta=\sec^4\theta-\sec^2\theta$.

Sol. $\tan^2\theta$ or $\tan^4\theta$ can be converted into $\sec^2\theta$

So, $\quad \text{LHS} = \tan^4\theta+\tan^2\theta$

$= \tan^2\theta(\tan^2\theta + 1)$
$= (\sec^2\theta - 1)\cdot\sec^2\theta$
[$\because$ $\tan^2\theta = \sec^2\theta - 1$ and $\tan^2\theta + 1 = \sec^2\theta$]
$= \sec^4\theta - \sec^2\theta$
$=$ RHS

Hence, proved.

EXERCISE 8.4

Q1. If $\text{cosec}\,\theta + \cot\theta = p$, then prove that $\cos\theta = \dfrac{p^2-1}{p^2+1}$.

Sol. $\text{cosec}\,\theta + \cot\theta = p$ [Given]

$\Rightarrow \quad \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta} = p$

$\Rightarrow \quad \dfrac{1+\cos\theta}{\sin\theta} = p$

$\Rightarrow \quad \dfrac{(1+\cos\theta)^2}{\sin^2\theta} = p^2$

$\Rightarrow \quad \dfrac{(1+\cos\theta)^2}{(1-\cos^2\theta)} = p^2$

$\Rightarrow \quad \dfrac{(1+\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = p^2$

$\Rightarrow \quad \dfrac{(1+\cos\theta)}{(1-\cos\theta)} = \dfrac{p^2}{1}$

[By using the rule of componendo and dividendo $\dfrac{a}{b} = \dfrac{c}{d}$ can be written as $\dfrac{a+b}{a-b} = \dfrac{c+d}{c-d}$]

So, by using componendo and dividendo, we have

$$\frac{(1+\cos\theta)+(1-\cos\theta)}{(1+\cos\theta)-(1-\cos\theta)} = \frac{p^2+1}{p^2-1}$$

$\Rightarrow \quad \dfrac{2}{2\cos\theta} = \dfrac{p^2+1}{p^2-1}$ $\left(\text{By invertendo } \dfrac{a}{b} = \dfrac{c}{d};\ \dfrac{b}{a} = \dfrac{d}{c}\right)$

$\Rightarrow \quad \cos\theta = \dfrac{p^2-1}{p^2+1}$

Hence, proved.

Q2. Prove that $\sqrt{\sec^2\theta + \text{cosec}^2\theta} = \tan\theta + \cot\theta$.

Sol. LHS $= \sqrt{\sec^2\theta + \text{cosec}^2\theta}$

$$= \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}} = \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta}}$$

$$= \sqrt{\frac{1}{\sin^2\theta \cdot \cos^2\theta}} = \frac{1}{\sin\theta\cos\theta}$$

$$= \operatorname{cosec}\theta \sec\theta$$

$$\text{RHS} = \tan\theta + \cot\theta$$

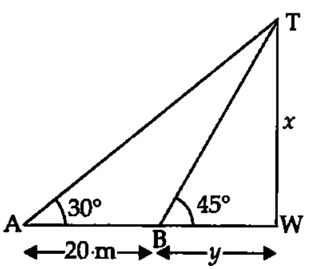
$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta \cdot \cos\theta}$$

$$= \operatorname{cosec}\theta \cdot \sec\theta = \text{LHS}$$

Hence, proved.

Q3. The angle of elevation of the top of a tower from a certain point is 30°. If the observer moves 20 m towards the tower, the angle of elevation of the top increases by 15°. Find the height of the tower.



Sol. Consider the height of the vertical tower (TW) = x m (let)

Ist position of observer at A makes angle of elevation at the top of tower is 30°.

Now, observer moves towards the tower at new position B such that AB = 20 m. Let BW = y.

Now, angle of elevation of the top of tower is increased by 15° *i.e.*, it becomes 30° + 15° = 45°.

In ΔTWB, we have

$$\tan 45° = \frac{x}{y}$$

$$\Rightarrow \quad 1 = \frac{x}{y} \Rightarrow x = y \qquad \text{(I)}$$

Now, ΔTWA, we have

$$\tan 30° = \frac{x}{20+y}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{20+x} \qquad \text{[From (I)]}$$

$$\Rightarrow \quad \sqrt{3}x = 20 + x$$

$$\Rightarrow \quad \sqrt{3}x - x = 20$$

$$\Rightarrow \quad x(\sqrt{3} - 1) = 20$$

$$\Rightarrow \quad x = \frac{20}{(\sqrt{3}-1)} \times \frac{\sqrt{3} \quad 1}{(\sqrt{3}+1)}$$

$$\Rightarrow \quad x = \frac{20(\sqrt{3}+1)}{3-1} = \frac{20(\sqrt{3}+1)}{2}$$

$$\Rightarrow \quad x = 10(1.732+1)$$

$$\Rightarrow \quad x = 10 \times 2.732 = 27.32 \text{ m}$$

Hence, the height of the tower is 27.32 m.

Q4. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then prove that $\tan\theta = 1$, or $\frac{1}{2}$.

Sol. To solve an equation in θ, we have to change it into one trigonometric ratio.

Given trigonometric equation is

$$1 + \sin^2\theta = 3\sin\theta\cos\theta$$

$$\Rightarrow \quad \frac{1}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$$

[Dividing by $\sin^2\theta$ both sides]

$$\Rightarrow \quad \operatorname{cosec}^2\theta + 1 = 3\cot\theta$$

$$\Rightarrow \quad 1 + \cot^2\theta + 1 - 3\cot\theta = 0$$

$$\Rightarrow \quad \cot^2\theta - 3\cot\theta + 2 = 0$$

$$\Rightarrow \quad \cot^2\theta - 2\cot\theta - 1\cot\theta + 2 = 0$$

$$\Rightarrow \quad \cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$$

$$\Rightarrow \quad (\cot\theta - 2)(\cot\theta - 1) = 0$$

$$\Rightarrow \quad \cot\theta - 2 = 0 \quad \text{or} \quad (\cot\theta - 1) = 0$$

$$\Rightarrow \quad \cot\theta = 2 \quad \text{or} \quad \cot\theta = 1$$

$$\Rightarrow \quad \tan\theta = \frac{1}{2} \quad \text{or} \quad \tan\theta = 1$$

Hence, $\tan\theta = \frac{1}{2}$, or 1.

Q5. Given that $\sin\theta + 2\cos\theta = 1$, then prove that $2\sin\theta - \cos\theta = 2$.

Sol. $\sin\theta + 2\cos\theta = 1$ [Given]

On squaring both sides, we get

$$(\sin\theta)^2 + (2\cos\theta)^2 + 2(\sin\theta)(2\cos\theta) = 1$$

$$\Rightarrow \quad \sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = 1$$

$$\Rightarrow \quad 1 - \cos^2\theta + 4(1 - \sin^2\theta) + 4\sin\theta\cos\theta = 1$$

$$\Rightarrow \quad 1 - \cos^2\theta + 4 - 4\sin^2\theta + 4\sin\theta\cos\theta = 1$$

$$\Rightarrow \quad -\cos^2\theta - 4\sin^2\theta + 4\sin\theta\cos\theta = -4$$

$$\Rightarrow \quad \cos^2\theta + 4\sin^2\theta - 4\sin\theta\cos\theta = 4$$

$$\Rightarrow \quad (\cos\theta)^2 + (2\sin\theta)^2 - 2(\cos\theta)(2\sin\theta) = 4$$

$$\Rightarrow \quad (2\sin\theta - \cos\theta)^2 = 2^2$$

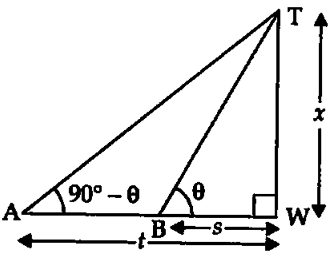
Taking square root both sides, we have

$$2\sin\theta - \cos\theta = 2$$

Hence, proved.

Q6. The angle of elevation of the top of a tower from two points distant s and t from its foot are complementary. Prove that the height of tower is $\sqrt{st}$.

Sol. Let the height of the vertical tower (TW) = x m



Points of observation A and B are at distances 't' and 's' from the foot of tower.

The angles of elevation of top of the tower from observation points A and B are $(90° - \theta)$ and θ which are complementary.

In ΔTWB, we have

$$\tan\theta = \frac{x}{s} \qquad \text{(I)}$$

Now, in ΔTWA, we have

$$\tan(90° - \theta) = \frac{x}{t}$$

$$\Rightarrow \quad \cot\theta = \frac{x}{t}$$

$$\Rightarrow \quad \cot\theta \cdot \tan\theta = \frac{x}{t} \cdot \frac{x}{s} \qquad \text{[Multiply by (I)]}$$

$$\Rightarrow \quad \frac{1}{\tan\theta} \cdot \tan\theta = \frac{x^2}{st}$$

$$\Rightarrow \quad \frac{x^2}{st} = 1$$

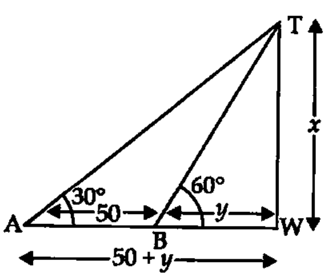
$$\Rightarrow \quad x^2 = st$$

$$\Rightarrow \quad x = \sqrt{st}$$

Hence, proved.

Q7. The shadow of a tower standing on a level plane is found to be 50 m. longer when Sun's elevation is 30° than when it is 60°. Find the height of the tower.

Sol. Let a tower TW of light x (let) is standing vertically upright on a level plane ABW. A and B are two positions of observation when angle of elevation changes from 30° to 60° respectively.



Let $\qquad BW = y$

$$AB = 50 \text{ m} \qquad \text{[Given]}$$

In ΔTWB, we have

$$\tan 60° = \frac{x}{y}$$

$$\Rightarrow \quad \sqrt{3} = \frac{x}{y}$$

$$\Rightarrow \quad x = \sqrt{3}y \qquad \text{(I)}$$

Now, in ΔTWA, we have

$$\tan 30^\circ = \frac{x}{y+50} \qquad \text{(II)}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{\sqrt{3}y}{y+50} \qquad \text{[From (I)]}$$

$$\Rightarrow \quad 3y = y + 50$$

$$\Rightarrow \quad 3y - y = 50$$

$$\Rightarrow \quad 2y = 50$$

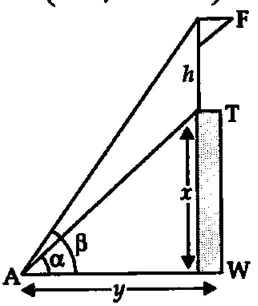
$$\Rightarrow \quad y = 25 \qquad \text{(I)}$$

Now, $x = \sqrt{3}y$

$$\Rightarrow \quad x = \sqrt{3} \times 25$$

$$\Rightarrow \quad x = 25\sqrt{3} \text{ m}$$

Q8. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height 'h'. At a point on the plane, the angles of elevation of the bottom and top of the flag staff are α and β, respectively. Prove that the height of the tower is $\left(\frac{h \tan \alpha}{\tan \beta - \tan \alpha}\right)$.



Sol. Let the height of vertical tower (TW) = x.
And, the height of flag staff (TF) = h (Given)
The angle of elevation at A on ground from the base and top of flag staff are α, β respectively.

Let AW = y

In ΔTWA, we have

$$\tan \alpha = \frac{x}{y}$$

$$\Rightarrow \quad y = \frac{x}{\tan \alpha} \qquad \text{(I)}$$

Now, in ΔFWA, we have

$$\tan \beta = \frac{x+h}{y}$$

$$\Rightarrow \quad y \tan \beta = x + h$$

$$\Rightarrow \quad \frac{x \tan \beta}{\tan \alpha} = x + h \qquad \text{[From (I)]}$$

$$\Rightarrow \quad x \tan \beta = x \tan \alpha + h \tan \alpha$$

$$\Rightarrow \quad x(\tan \beta - \tan \alpha) = h \tan \alpha$$

$$\Rightarrow \quad x = \frac{h \tan \alpha}{\tan \beta - \tan \alpha}$$

Hence, proved.

Q9. If $\tan\theta + \sec\theta = l$, then prove that $\sec\theta = \dfrac{l^2+1}{2l}$.

Sol. [Recall identity $\sec^2\theta - \tan^2\theta = 1$
and now change $\sec\theta + \tan\theta$ to $\sec^2\theta - \tan^2\theta$ by multiplying and dividing the given expression to $(\sec\theta - \tan\theta)$.

$$\sec\theta + \tan\theta = l \qquad \text{[Given]} \quad \text{(I)}$$

$$\Rightarrow \quad (\sec\theta + \tan\theta)\frac{(\sec\theta - \tan\theta)}{\sec\theta - \tan\theta} = l$$

$$\Rightarrow \quad \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} = l \qquad [\because 1 + \tan^2\theta = \sec^2\theta]$$

$$\Rightarrow \quad \frac{1}{\sec\theta - \tan\theta} = l$$

$$\text{or} \quad \sec\theta - \tan\theta = \frac{1}{l} \qquad \text{(II)}$$

Now, get $\sec\theta$ by eliminating $\tan\theta$ from (I) and (II).
It can be obtained by adding (I) and (II).

$$\Rightarrow \quad 2\sec\theta = l + \frac{1}{l}$$

$$\Rightarrow \quad 2\sec\theta = \frac{l^2+1}{l}$$

$$\Rightarrow \quad \sec\theta = \frac{l^2+1}{2l}$$

Hence, proved.

Q10. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, then prove that $q(p^2 - 1) = 2p$.

Sol.

$$\sin\theta + \cos\theta = p \qquad \text{(I)}$$

$$\sec\theta + \operatorname{cosec}\theta = q \qquad \text{(II)}$$

[IInd expression can be changed into $\sin\theta$, $\cos\theta$ and eliminate trigonometric ratio from (I) and (II)]

$$\sec\theta + \operatorname{cosec}\theta = q$$

$$\Rightarrow \quad \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q$$

$$\Rightarrow \quad \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = \frac{q}{1}$$

$$\Rightarrow \quad \frac{p}{\sin\theta\cos\theta} = q \qquad \text{[Using (I)]}$$

$$\Rightarrow \quad \sin\theta\cos\theta = \frac{p}{q} \qquad \text{(III)}$$

$$\sin\theta + \cos\theta = p \qquad \text{[From (I)]}$$

$$\Rightarrow \quad (\sin\theta + \cos\theta)^2 = p^2 \qquad \text{[Squaring both sides]}$$

$$\Rightarrow \quad \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = p^2$$

$$\Rightarrow \quad 1 + 2\cdot\frac{p}{q} = p^2$$

$[\because \sin^2\theta + \cos^2\theta = 1$ and using (III)]

$$\Rightarrow \quad q + 2p = p^2 q$$

$$\Rightarrow \quad 2p = p^2 q - q$$

$$\Rightarrow \quad 2p = q(p^2 - 1)$$

Hence, proved.

Q11. If $a\sin\theta + b\cos\theta = c$, then prove that

$$a\cos\theta - b\sin\theta = \sqrt{a^2 + b^2 - c^2}.$$

Sol. $a\sin\theta + b\cos\theta = c$ [Given]

On squaring both sides, we get

$$(a\sin\theta)^2 + (b\cos\theta)^2 + 2(a\sin\theta)(b\cos\theta) = c^2$$

$$\Rightarrow \quad a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = c^2$$

$$\Rightarrow \quad a^2(1-\cos^2\theta) + b^2(1-\sin^2\theta) + 2ab\sin\theta\cos\theta = c^2$$

$$\Rightarrow \quad a^2 - a^2\cos^2\theta + b^2 - b^2\sin^2\theta + 2ab\sin\theta\cos\theta = c^2$$

$$\Rightarrow \quad -a^2\cos^2\theta - b^2\sin^2\theta + 2ab\sin\theta\cos\theta = c^2 - a^2 - b^2$$

$$\Rightarrow \quad a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta = a^2 + b^2 - c^2$$

$$\Rightarrow \quad (a\cos\theta)^2 + (b\sin\theta)^2 - 2(a\cos\theta)(b\sin\theta) = a^2 + b^2 - c^2$$

$$\Rightarrow \quad (a\cos\theta - b\sin\theta)^2 = a^2 + b^2 - c^2$$

$$\Rightarrow \quad a\cos\theta - b\sin\theta = \pm\sqrt{a^2 + b^2 + c^2}$$

(Taking square root both sides)

Hence, $a\cos\theta - b\sin\theta = \sqrt{a^2 + b^2 - c^2}$

Hence, proved.

Q12. Prove that $\dfrac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \dfrac{1-\sin\theta}{\cos\theta}$

Sol. Recall identity $\sec^2\theta - \tan^2\theta = 1$

$$\text{LHS} = \frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$$

$$= \frac{\sec^2\theta - \tan^2\theta + \sec\theta - \tan\theta}{1+\sec\theta+\tan\theta}$$

$$= \frac{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta) + (\sec\theta-\tan\theta)}{1+\sec\theta+\tan\theta}$$

$[\because a^2 - b^2 = (a-b)(a+b)]$

$$= \frac{(\sec\theta-\tan\theta)[\sec\theta+\tan\theta+1]}{(\sec\theta+\tan\theta+1)}$$

$$= \sec\theta - \tan\theta$$

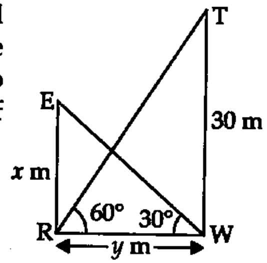
$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta} = \text{RHS}$$

Hence, proved.

Q13. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60°, and the angle of elevation of the top of second tower from the foot of first tower is 30°. Find the distance between the two towers and also the height of the other tower.

Sol. Two vertical towers TW = 30 m and ER = x m (let) are standing on a horizontal plane RW = y(let). The angle of elevation from R to top of 30 m high tower is 60° and the angle of elevation of second tower from W is 30°.



In ΔERW,

$$\tan 30° = \frac{x}{y}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{y}$$

$$\Rightarrow \quad y = \sqrt{3}x \qquad \text{(I)}$$

Now, In ΔTWR,

$$\tan 60° = \frac{30°}{y} \qquad \text{(I)}$$

$$\Rightarrow \quad \sqrt{3} = \frac{30}{\sqrt{3}x} \qquad \text{[From (I)]}$$

$$\Rightarrow \quad 3x = 30$$

$$\Rightarrow \quad x = 10 \text{ m}$$

Now, $y = \sqrt{3}x$

$$\Rightarrow \quad y = \sqrt{3}(10)$$

$$\Rightarrow \quad y = 1.732 \times 10$$

$$\Rightarrow \quad y = 17.32 \text{ m}$$

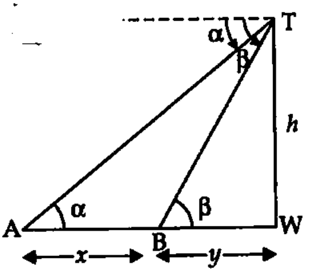
Hence, the distance between the two towers is 17.32 m and the height of the second tower is 10 m.

Q14. From the top of a tower h m high, the angles of depression of two objects, which are in line with the foot of the tower are α and β, $(\beta > \alpha)$. Find the distance between the two objects.

Sol. Consider a vertical tower TW = h m. Two objects A and B are x m apart in the line joining A, B, and W and BW = y (let).

The angle of depression from the top a tower to objects A and B are α, and β respectively.

In ΔTWB, we have



$$\tan\beta = \frac{h}{y}$$

$$\Rightarrow \quad y = \frac{h}{\tan\beta} \qquad \text{(I)}$$

Now, in ΔTWA,

$$\tan\alpha = \frac{h}{x+y}$$

$$\Rightarrow \quad \tan\alpha\,(x+y) = h$$

$$\Rightarrow \quad \left(x + \frac{h}{\tan\beta}\right)\tan\alpha = h \qquad \text{[From(I)]}$$

$$\Rightarrow \quad x\tan\alpha + \frac{h\tan\alpha}{\tan\beta} = h$$

$$\Rightarrow \quad x\tan\alpha = h - \frac{h\tan\alpha}{\tan\beta}$$

$$\Rightarrow \quad x\tan\alpha = \frac{h\tan\beta - h\tan\alpha}{\tan\beta}$$

$$\Rightarrow \quad x = \frac{h[\tan\beta - \tan\alpha]}{\tan\alpha\cdot\tan\beta}$$

$$\Rightarrow \quad x = h\left[\frac{\tan\beta}{\tan\alpha\cdot\tan\beta} - \frac{\tan\alpha}{\tan\alpha\cdot\tan\beta}\right]$$

$$\Rightarrow \quad x = h\left[\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right]$$

$$\Rightarrow \quad x = h[\cot\alpha - \cot\beta]$$

Hence, the distance between the two objects is $h(\cot\alpha - \cot\beta)$ m.

Q15. A ladder rests against a vertical wall at an inclination α to the horizontal. Its foot is pulled away from the wall through a distance p, so that its upper end slides a distance q down the wall and then the ladder makes an angle β with horizontal. Show that $\frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$.

Sol. Consider a vertical wall WB. Two positions AW and LD of a ladder as shown in figure such that $LA = p$, $WD = q$ and $LD = AW = z$. Angle of inclination of ladder at two positions A and L are α and β respectively.
Let $AB = y$ and $DB = x$.

In ΔABW, we have

$$\sin\alpha = \frac{x+q}{z}$$

and $$\cos\alpha = \frac{y}{z}$$

In ΔLBD, we have

$$\sin\beta = \frac{x}{z}$$

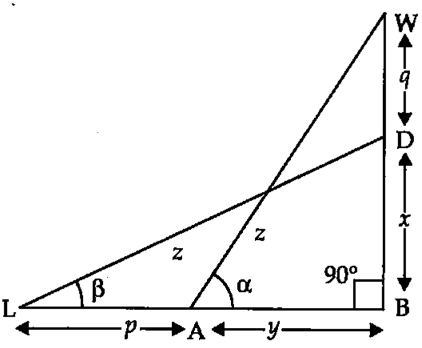
and $$\cos\beta = \frac{y+p}{z}$$

Taking RHS = $\dfrac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$

$$= \dfrac{\dfrac{y+p}{z} - \dfrac{y}{z}}{\dfrac{x+q}{z} - \dfrac{x}{z}} = \dfrac{\dfrac{y+p-y}{z}}{\dfrac{x+q-x}{z}}$$

$$= \frac{p}{z} \div \frac{q}{z} = \frac{p}{z} \times \frac{z}{q}$$

$$= \frac{p}{q} = \text{LHS}$$



Hence, proved.

Q16. The angle of elevation of the top of a vertical tower from a point on the ground is 60°. From another point 10 m vertically above the first, its angle of elevation is 45°. Find the height of the tower.

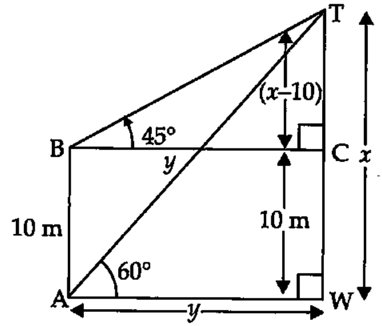
Sol. Let the height of the vertical tower TW = x m.

It stands on a horizontal plane AW = y.

Also BC = y.

Observation point B is 10 m above the first observation point A.

The angles of elevation from point of observations A and B are 60° and 45° respectively.



$$TC = x - 10$$

In right angled triangle TBC, we have

$$\tan 45° = \frac{x-10}{y}$$

$$\Rightarrow \quad 1 = \frac{x-10}{y}$$

$$\Rightarrow \quad y = x - 10 \qquad \text{(I)}$$

Now, in ΔTAW,

$$\tan 60° = \frac{x}{y}$$

$$\Rightarrow \quad \sqrt{3} = \frac{x}{x-10} \qquad \text{[From (I)]}$$

$$\Rightarrow \quad \sqrt{3}x - 10\sqrt{3} = x$$

$$\Rightarrow \quad \sqrt{3}x - x = 10\sqrt{3}$$

$$\Rightarrow \quad x(\sqrt{3} - 1) = 10\sqrt{3}$$

$$\Rightarrow \quad x = \frac{10\sqrt{3}}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{\sqrt{3}+1}$$

$$\Rightarrow \qquad x = \frac{10(3+\sqrt{3})}{(\sqrt{3})^2 - 1}$$

$$= \frac{10 \times (3 + 1.732)}{3-1} = \frac{10 \times 4.732}{2} = 10 \times 2.366$$

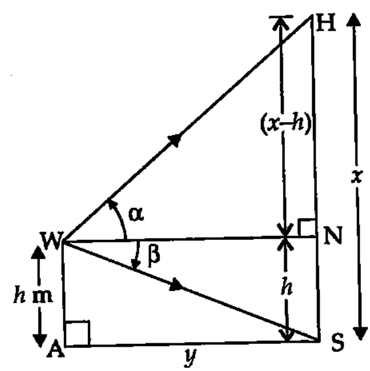
$$\Rightarrow \qquad x = 23.66 \text{ m}$$

Hence, the height of the tower = 23.66 m.

Q17. A window of a house is h m above the ground. From the window, the angles of elevation and depression of the top and the bottom of another house situated on the opposite side of the lane are found to be α and β respectively. Prove that the height of the other house is $h\,(1 + \tan\alpha \cot\beta)$ m.

Sol. Window W, h m above the ground point A, another house HS = x(m), AS = y m away from observation window, AS = WN = y (let), NS = h, HN = $(x - h)$.

Angle of elevation and depression of top and bottom of house HS from window W are α, β respectively.



In right angled ΔWNS,

$$\tan\beta = \frac{h}{y}$$

$$\Rightarrow \qquad y = \frac{h}{\tan\beta} \qquad \text{(I)}$$

Now, in right angled ΔHNW,

$$\Rightarrow \qquad \tan\alpha = \frac{x-h}{y}$$

$$\Rightarrow \qquad y\tan\alpha = x - \text{m}\, h$$

$$\Rightarrow \qquad \frac{h\tan\alpha}{\tan\beta} = x - h \quad \text{[From (I)]}$$

$$\Rightarrow \qquad h\tan\alpha = x\tan\beta - h\tan\beta$$

$$x\tan\beta = h\tan\alpha + h\tan\beta$$

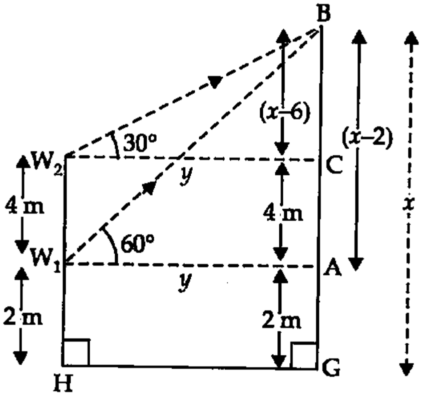
$$\Rightarrow \qquad x = \frac{h(\tan\alpha + \tan\beta)}{\tan\beta}$$

$$\Rightarrow \qquad x = h\left[\frac{\tan\alpha}{\tan\beta} + \frac{\tan\beta}{\tan\beta}\right]$$

$$\Rightarrow \qquad x = h\,[\tan\alpha \cdot \cot\beta + 1]$$

Hence, the height of the house on the other side of the observer is $h\,[1 + \tan\alpha \,.\, \cot\beta]$ m.

Q18. The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be 60° and 30°, respectively. Find the height of the balloon above the ground.



Sol. Let B be a balloon at a height GB = x m.

Let W_1 be the window, which is 2 m above the ground H.

$\therefore \quad W_1H = 2$ m

$\Rightarrow \quad AG = 2$ m

Let W_2 be the second window, which is the 4 m above the window W_1.

$\therefore \quad W_2W_1 = AC = 4$ m

The angles of elevation of balloon B from W_1, W_2 are 60° and 30° respectively.

$$BA = (x-2) \text{ m}$$
$$BC = x-2-4 = (x-6) \text{ m}$$

In right angled ΔW_2CB, we have

$$\tan 30° = \frac{x-6}{y}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x-6}{y}$$

$$\Rightarrow \quad y = \sqrt{3}(x-6) \qquad \text{(I)}$$

Now, in right angled ΔW_1AB,

$$\tan 60° = \frac{x-2}{y}$$

$$\Rightarrow \quad \sqrt{3} = \frac{x-2}{y}$$

$$\Rightarrow \quad \sqrt{3}y = (x-2) \qquad \text{(II)}$$

$$\Rightarrow \quad \sqrt{3}\cdot\sqrt{3}(x-6) = x-2 \qquad \text{[From (I)]}$$

$$\Rightarrow \quad 3x-18 = x-2$$

$$\Rightarrow \quad 3x - x = 18-2$$

$$\Rightarrow \quad 2x = 16$$

$$\Rightarrow \quad x = 8 \text{ m}$$

Hence, the height of the balloon above the ground is 8 m.

❑❑❑

9 Circles

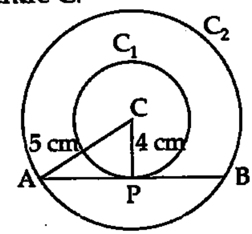
EXERCISE 9.1

Choose the correct answer from the given four options:

Q1. If the radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is the tangent to the other circle is

(*a*) 3 cm (*b*) 6 cm (*c*) 9 cm (*d*) 1 cm

Sol. (*b*): C_1, C_2 are concentric circles with their centre C.



Chord AB of circle C_2 touches C_1 at P

AB is tangent at P and PC is radius at P.

So, $CP \perp AB$.

$\Rightarrow \angle P = 90°$, $CP = 4$ cm and $CA = 5$ cm (Given)

$\therefore$ In right angle ΔPAC,

$$AP^2 = AC^2 - PC^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$\Rightarrow$ $AP = 3$ cm

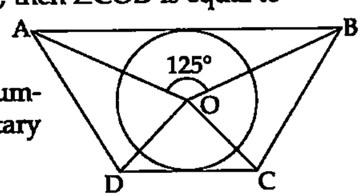
Perpendicular from centre to chord bisects the chord.

So, $AB = 2AP = 2 \times 3 = 6$ cm. Hence, verifies option (*b*).

Q2. In the given figure, if $\angle AOB = 125°$, then $\angle COD$ is equal to

(*a*) 62.5° (*b*) 45°

(*c*) 35° (*d*) 55°



Sol. (*d*): We know that a quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

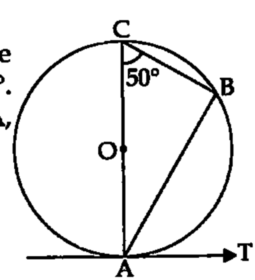
$$\therefore \quad \angle AOB + \angle COD = 180°$$

$$125° + \angle COD = 180°$$

$$\angle COD = 180° - 125° = 55°.$$

Hence, verifies option (*d*).

Q3. In the given figure, AB is a chord of the circle and AOC is its diameter, such that $\angle ACB = 50°$. If AT is the tangent to the circle at the point A, then $\angle BAT$ is equal to



(*a*) 65° (*b*) 60°

(*c*) 50° (*d*) 40°

Sol. (*c*): AC is diameter.

$\Rightarrow$ $\angle B = 90°$ ($\angle$ in a semi-circle)

$\therefore$ $\angle BAC = 180° - \angle C - \angle B$ [Angle sum property of a triangle]

$\Rightarrow$ $\angle BAC = 180° - 50° - 90° = 180° - 140° = 40°$

Tangent AT at A and radius OA at A arc at 90°.

So, $\angle OAT = 90°$

$\therefore \quad \angle OAB + \angle BAT = 90°$

$\Rightarrow \quad 40° + \angle BAT = 90°$

$\Rightarrow \quad \angle BAT = 90° - 40°$

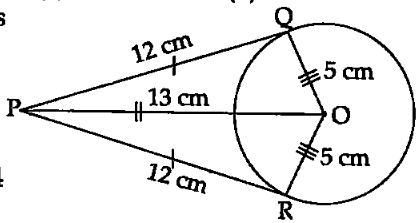
$\Rightarrow \quad \angle BAT = 50°.$

Hence, verifies option (*c*).

Q4. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is

(*a*) 60 cm² (*b*) 65 cm² (*c*) 30 cm² (*d*) 32.5 cm²

Sol. (*a*): PQ is tangent and QO is radius at contact point Q.



$\therefore \quad \angle PQO = 90°$

$\therefore$ By Pythagoras theorem,

$PQ^2 = OP^2 - OQ^2$

$= 13^2 - 5^2 = 169 - 25 = 144$

$\Rightarrow \quad PQ = 12$ cm

$\therefore \quad \Delta OPQ \cong \Delta OPR$ [By SSS criterion of congruence]

$\therefore \quad$ Area of ΔOPQ = ar ΔOPR

Area of quadrilateral QORP = 2 ar (ΔOPR)

$= 2 \times \frac{1}{2}$ base $\times$ altitude

$= RP \times OR = 12 \times 5 = 60$ cm²

Hence, verifies the option (*a*).

Q5. At one end A of diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is

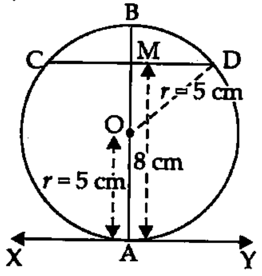
(*a*) 4 cm (*b*) 5 cm (*c*) 6 cm (*d*) 8 cm

Sol. (*d*): XAY is tangent and AO is radius at contact point A of circle.

AO = 5 cm

$\therefore \quad \angle OAY = 90°$

CD is another chord at distance (perpendicular) of 8 cm from A and CMD || XAY meets AB at M.



Join OD.

OD = 5 cm

OM = 8 − 5 = 3 cm

$\angle OMD = \angle OAY = 90°$

Now, in right angled ΔOMD,

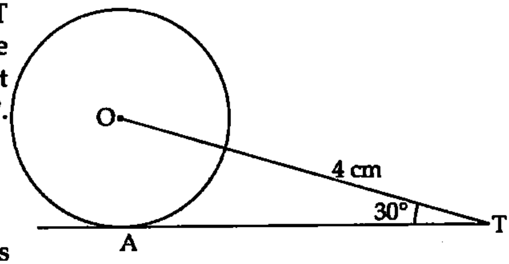
$MD^2 = OD^2 - MO^2 = 5^2 - 3^2 = 25 - 9 = 16$

$\Rightarrow \quad MD = 4$ cm

Perpendicular from centre O of circle bisect the chord. So CD = 2MD = 2 × 4 = 8 cm.

Hence, length of chord CD = 8 cm, which verifies option (*d*).

Q6. In the given figure, AT is a tangent to the circle with centre 'O' such that OT = 4 cm and ∠OTA = 30°. Then AT is equal to

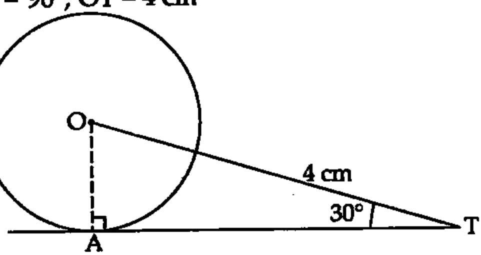


(*a*) 4 cm (*b*) 2 cm

(*c*) $2\sqrt{3}$ cm (*d*) $4\sqrt{3}$ cm

Sol. (*c*): Join OA. OA is radius and AT is tangent at contact point A.

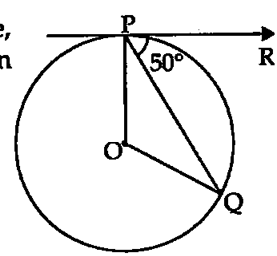
So, ∠OAT = 90°, OT = 4 cm [Given]



Now, $\frac{AT}{4} = \frac{\text{Base}}{\text{Hypotenuse}} = \cos 30° \Rightarrow AT = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$ cm.

Hence, verifies the option (*c*).

Q7. In the given figure, 'O' is the centre of circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then ∠POQ is equal to



(*a*) 100° (*b*) 80°

(*c*) 90° (*d*) 75°

Sol. (*a*): OP is radius and PR is tangent at P.

So, ∠OPR = 90°

⇒ ∠OPQ + 50° = 90°

⇒ ∠OPQ = 90° − 50°

⇒ ∠OPQ = 40°

In ΔOPQ,

OP = OQ [Radii of same circle]

∴ ∠Q = ∠OPQ = 40°

[Angles opposite to equal sides are equal]

But, ∠POQ = 180° − ∠P − ∠Q

= 180° − 40° − 40° = 180° − 80° = 100°

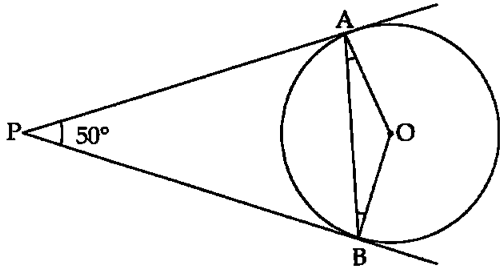
⇒ ∠POQ = 100°.

Hence, verifies the option (*a*).

Q8. In the given figure, if PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$, then $\angle OAB$ is equal to

(*a*) 25° (*b*) 30°

(*c*) 40° (*d*) 50°



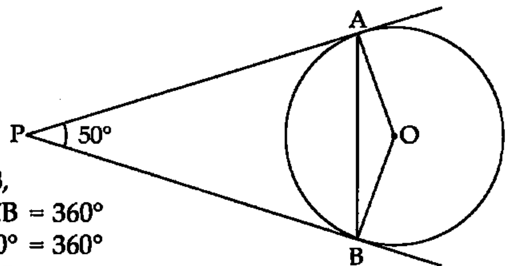
Sol. (*a*): In ΔOAB, we have

$OA = OB$ [Radii of same circle]

$\therefore \quad \angle OAB = \angle OBA$ [Angles opposite to equal sides are equal]

As OA and PA are radius and tangent respectively at contact point A.

So, $\angle OAP = 90°$. Similarly, $\angle OBP = 90°$



Now, in quadrilateral PAOB,

$$\angle P + \angle A + \angle O + \angle B = 360°$$

$$\Rightarrow \quad 50° + 90° + \angle O + 90° = 360°$$

$$\Rightarrow \quad \angle O = 360° - 90° - 90° - 50°$$

$$\Rightarrow \quad \angle O = 130°$$

Again, in ΔOAB,

$$\angle O + \angle OAB + \angle OBA = 180°$$

$$\Rightarrow \quad 130° + \angle OAB + \angle OAB = 180° \qquad [\because \angle OBA = \angle OAB]$$

$$\Rightarrow \quad 2\angle OAB = 180° - 130° = 50°$$

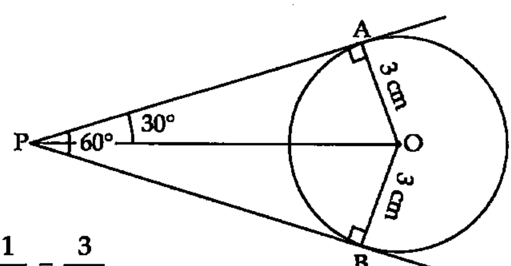
$$\Rightarrow \quad \angle OAB = 25°$$

Hence, $\angle OAB = 25°$ which verifies option (*a*).

Q9. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to

(*a*) $\frac{3}{2}\sqrt{3}$ cm (*b*) 6 cm (*c*) 3 cm (*d*) $3\sqrt{3}$ cm

Sol. (*d*): $\because$ OA and PA are the radius and the tangent respectively at contact point A of a circle of radius OA = 3 cm. So, $\angle PAO = 90°$.

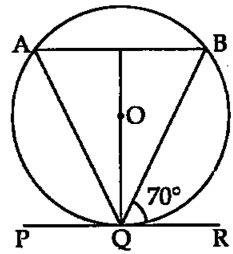


In right angled ΔPOA,

$$\tan 30° = \frac{OA}{PA} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3}{PA}$$

$\Rightarrow \quad PA = 3\sqrt{3}$ which verifies the option (*d*).

Q10. In the given figure, if PQR is the tangent to a circle at Q, whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70°$, then $\angle AQB$ is equal to

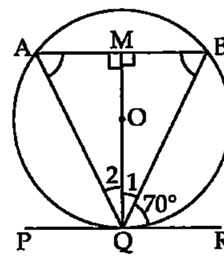


(*a*) 20° (*b*) 40°
(*c*) 35° (*d*) 45°

Sol. (*b*): AB || PQR [Given]

$\angle B = \angle BQR = 70°$ [Alternate interior angles]

and $\angle OQR = \angle AMQ$ [Alternate interior angles]



As PQR and OQ are tangent and radius at contact point Q

$\therefore \quad \angle OQR = 90°$

$\Rightarrow \quad \angle 1 + \angle 70° = 90°$

$\Rightarrow \quad \angle 1 = 90° - 70° = 20°$

$\therefore$ $\angle AMO = 90°$ and perpendicular from centre to chord bisect the chord

So, $\quad MA = MB$

$\angle QMA = \angle QMB$ [Each 90°]

$MQ = MQ$ [Common]

$\therefore \quad \Delta QMA \cong \Delta QMB$ [By SAS criterion of congruence]

$\Rightarrow \quad \angle A = \angle B$

$\Rightarrow \quad \angle A = 70°$ [$\because \angle B = 70°$]

$\therefore \quad \angle A + \angle AMQ + \angle 2 = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad 70° + 90° + \angle 2 = 180°$

$\Rightarrow \quad \angle 2 = 180° - 160°$

$\Rightarrow \quad \angle 2 = 20°$

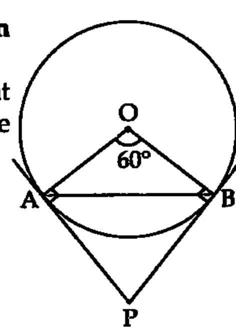
$\therefore \quad \angle AQB = \angle 1 + \angle 2 = 20° + 20° = 40°$

Hence, verifies option (*b*).

EXERCISE 9.2

Write True or False and justify your answer in each of the following:

Q1. If a chord AB subtends an angle of 60° at the centre of a circle, then the angle between the tangents at A and B is also 60°.



Sol. False: Chord AB subtends $\angle 60°$ at O.

$\therefore$ AP and OA are tangent and radius at A.

$\therefore \quad \angle OAP = 90°$

Similarly, $\quad \angle OBP = 90°$

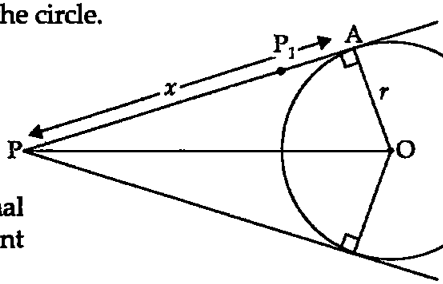
In quadrilateral OAPB,

$$\angle O + \angle P + \angle OAP + \angle OBP = 360^\circ$$
$$\Rightarrow \quad 60^\circ + \angle P + 90^\circ + 90^\circ = 360^\circ$$
$$\Rightarrow \quad \angle P = 360^\circ - 240^\circ$$
$$\Rightarrow \quad \angle P = 120^\circ$$

Hence, the given statement is false.

Q2. The length of tangent from an external point on a circle is always greater than the radius of the circle.

Sol. False: Consider any point P external to a circle away from O.



Now, draw tangent PA on the circle. Clearly,

PA > r [∵ P is external to circle and P is at sufficient distance]

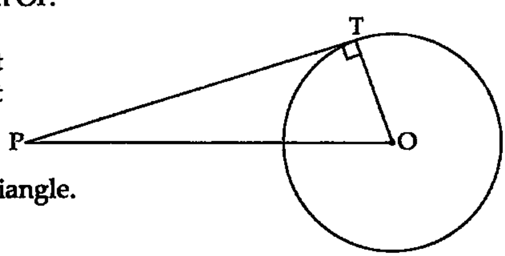
Now, again consider any point P_1 on the tangent AP very near to contact point A of tangent PA, $P_1A < AO$

So, it is clear that the length of the tangent PA and P_1A are greater and smaller respectively than radius OA.

Hence, the length of the tangent from an external point of a circle may or may not be greater than the radius of the circle. Hence, the given statement is false.

Q3. The length of the tangent from an external point P on a circle with centre O is always less than OP.

Sol. True:



PT and OT are the tangent and radius respectively at contact point T.

So, $\angle OTP = 90^\circ$

$\Rightarrow$ ΔOPT is right angled triangle.

Again, in ΔOPT

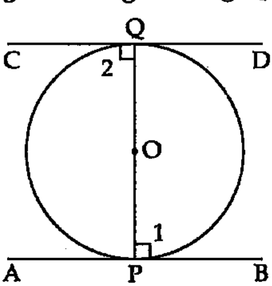
∵ $\angle T > \angle O$

∴ $OP > PT$ [Side opposite to greater angle is larger]

Hence, the given statement is true.

Q4. The angle between two tangents to a circle may be 0°.

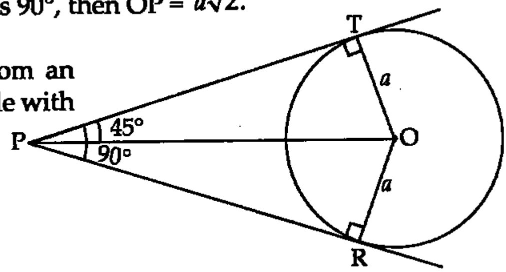
Sol. True:



Consider the diameter POQ of a circle with centre O. The tangent at P and Q are drawn, as we know the radius and tangent at contact point are perpendicular so $\angle 1 = \angle 2 = 90^\circ$. These

are alternate angles so the tangent APB || CQD *i.e.*, angle between two tangents to a circle may be zero.
Hence, the given statement is true.

Q5. If the angle between two tangents drawn from a point P to a circle of radius 'a' and centre O is 90°, then OP = $a\sqrt{2}$.



Sol. True.
Consider a tangent PT from an external point P on a circle with radius 'a'.
OT and PT are radius and tangent respectively at contact point T.

$\therefore \quad \angle T = 90°$

As $\quad \Delta OPT \cong \Delta OPR \quad$ [By SSS criterion of congruence]

$\therefore \quad \angle OPT = \angle OPR = \dfrac{90°}{2} = 45°$

$\therefore$ In right angle ΔOPT,

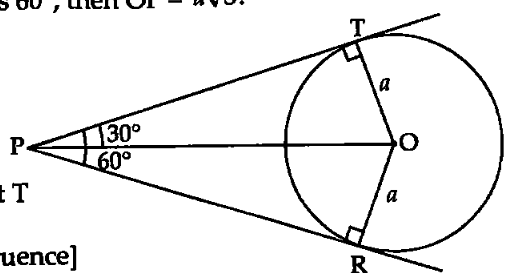
$$\sin 45° = \frac{OT}{OP}$$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}} = \dfrac{a}{OP}$

$\Rightarrow \quad OP = \sqrt{2}a.$

Hence, the given statement is true.

Q6. If the angle between two tangents drawn from a point P to a circle of radius 'a' and centre O is 60°, then OP = $a\sqrt{3}$.



Sol. False: PT and OT are tangent and radius respectively at contact point T.

$\therefore \quad \angle OTP = 90°$

$\Rightarrow \Delta OTP$ is right angle Δ at T

As $\quad \Delta OPT \cong \Delta OPR$

[By SSS criterion of congruence]

$\Rightarrow \quad \angle OPT = \angle OPR = \dfrac{1}{2} \times 60° = 30°$

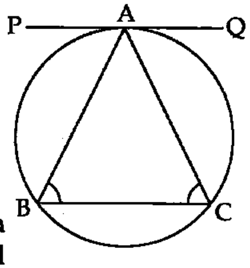
$\therefore$ In right angle ΔOPT,

$$\sin 30° = \frac{OT}{OP} \Rightarrow \frac{1}{2} = \frac{a}{OP} \Rightarrow OP = 2a$$

Hence, the given statement is false.

Q7. The tangent to the circumcircle of an isosceles ΔABC at A, in which AB = AC, is parallel to BC.

Sol. True.



A ΔABC, inscribed in a circle in which $AB = AC$.

PAQ is tangent at A.

AB is chord.

$\therefore \quad \angle PAB = \angle C \quad ...(i)$

$\because$ Angle $\angle PAB$ formed by chord (AB) with tangent is equal to the angle $\angle C$ formed by chord AC in alternate segment.

In ΔABC,

$AB = AC$ [Given]

$\therefore \quad \angle B = \angle C$ [$\because$ Angles opposite to equal sides are equal] ...(*ii*)

From (*i*) and (*ii*), $\angle B = \angle PAB$

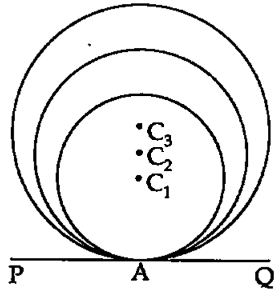
These are alternate interior angles.

So, $PAQ \parallel BC$

Hence, the given statement is true.

Q8. If a number of circles touch a given line segment PQ at a point A, then their centres lies on the perpendicular bisector of PQ.

Sol. False:



C_1A and PAQ are radius and tangent at contact point A.

$\therefore \quad \angle C_1AP = 90° \Rightarrow C_1A \perp PQ$

Similarly, $\angle C_2AP = 90° \Rightarrow C_2A \perp PQ$

$\angle C_3AP = 90° \Rightarrow C_3A \perp PQ$

We know that perpendicular on any point of a segment PQ may be only one.

So, point segments C_1A, C_2A, C_3A, C_4A, ... will be on a line.

$\Rightarrow C_1A$, C_2A, C_3A, C_4A will lie on a line, which is perpendicular on PQ at A.

As A is not mid point of PQ. So, the perpendicular AB will not be perpendicular bisector of PQ.

Hence, the given statement is false.

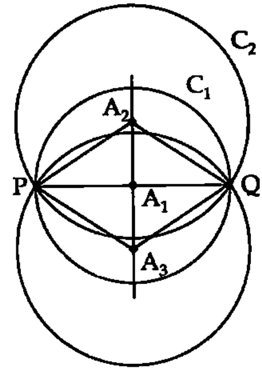
Q9. If a number of circles pass through the end points P and Q of a line segment PQ, then their centres lie on the perpendicular bisector of PQ.

Sol. True: Centre of any circle passing through the end points P and Q of a line segment are equidistant from P and Q.

$\therefore$ $A_1P = A_1Q$

$A_2P = A_2Q$

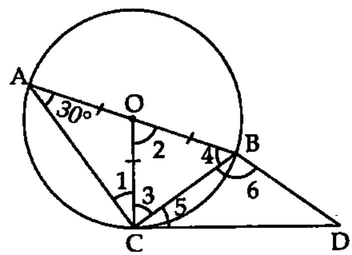
$A_3P = A_3Q$



as we know that any point on perpendicular bisector of a segment is equidistant from the end points of the segment. Hence, A_1, A_2, A_3 points are the centres of circles passing through the end points P and Q of a segment PQ or the centres of circles lie on the perpendicular bisector of PQ.

Q10. AB is a diameter of a circle and AC is its chord such that ∠BAC = 30°. If the tangent at C intersects AB extended at D, then BC = BD.

Sol. True:



CD is a tangent at contact point C. AOB is diameter which meets tangent produced at D.

Chord AC makes ∠A = 30° with diameter AB.

To prove: BD = BC

Proof: In ΔOAC,

OA = OC = r [Radii of same circle]

∠1 = ∠A [∠s opp. to equal sides are equal]

⇒ ∠1 = 30° [∵ ∠A = 30°]

Exterior ∠BOC = ∠2 = ∠1 + ∠A = (30° + 30°) = 60°

Now, in ΔOCB,

OC = OB [Radii of same circle]

∴ ∠3 = ∠4 [Angles opposite to equal sides are equal]

∠3 + ∠4 + ∠COB = 180°

⇒ ∠3 + ∠3 + 60° = 180° [Angle sum property of triangle]

⇒ 2 ∠3 = 180° – 60° = 120°

⇒ ∠3 = 60° = ∠4

∠6 + ∠4 = 180° [Linear pair axiom]

⇒ ∠6 = 180° – ∠4

= 180° – 60°

⇒ ∠6 = 120°

∵ Tangent CD and radius CO are at contact point C.

∴ ∠OCD = 90°

⇒ ∠3 + ∠5 = 90°

⇒ 60° + ∠5 = 90°

⇒ ∠5 = 30°

Now, in ΔBCD, we have

$\angle D + \angle 5 + \angle 6 = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle D = 180° - \angle 5 - \angle 6$

$= 180° - 30° - 120° = 180° - 150°$

$\Rightarrow \angle D = 30°$

$\therefore \angle D = \angle 5 = 30°$

$\Rightarrow BC = BD$

[Sides opposite to equal ∠s of a triangle are equal]

Hence, verifies the given statement true.

EXERCISE 9.3

Q1. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

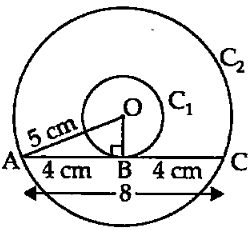
Sol. Given: Two concentric circles C_1 and C_2 with centre O.

Chord AC of circle C_2 is tangent of circle C_1 at B.

We know that tangent AC and radius BO at point B are perpendicular.

∴ Perpendicular from centre to chord bisects the chord.

$\therefore AB = CB = \frac{AC}{2} = \frac{8}{2} = 4 \text{ cm}$



In right angle ΔABO,

$OB^2 = OA^2 - AB^2$

[By Pythagoras theorem]

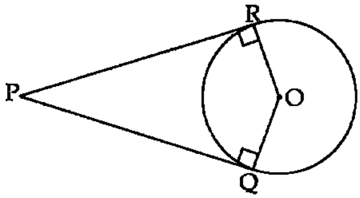
$= 5^2 - 4^2 = 25 - 16 = 9$

$\Rightarrow OB = 3 \text{ cm}$

Hence, radius of circle C_1 is 3 cm.

Q2. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

Sol. Given: Tangents PR and PQ from an external point P to a circle with centre O.



To prove: Quadrilateral QORP is cyclic.

Proof: RO and RP are the radius and tangent respectively at contact point R.

$\therefore \angle PRO = 90°$

Similarly, $\angle PQO = 90°$

In quadrilateral QORP, we have

$\angle P + \angle R + \angle O + \angle Q = 360°$

$\Rightarrow \angle P + \angle 90° + \angle O + \angle 90° = 360°$

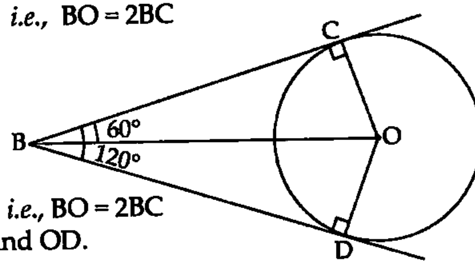
$\Rightarrow \angle P + \angle O = 360° - 180° = 180°$

These are opposite angles of quadrilateral QORP and are supplementary.

∴ Quadrilateral QORP is cyclic. Hence, proved.

Q3. If from an external point B of a circle with centre 'O', two tangents BC, BD are drawn such that $\angle DBC = 120°$, prove that

$$BC + BD = BO, \text{ i.e., } BO = 2BC$$



Sol. Given: A circle with centre O.
Tangents BC and BD are drawn from an external point B such that $\angle DBC = 120°$.

To prove: $BC + BD = BO$, *i.e.*, $BO = 2BC$

Construction: Join OB, OC and OD.

Proof: In ΔOBC and ΔOBD, we have

$OB = OB$ [Common]

$OC = OD$ [Radii of same circle]

$BC = BD$ [Tangents from an external point are equal in length] ...(*i*)

$\therefore \quad \Delta OBC \cong \Delta OBD$ [By SSS criterion of congruence]

$\Rightarrow \quad \angle OBC = \angle OBD$ (CPCT)

$\therefore \quad \angle OBC = \frac{1}{2}\angle DBC = \frac{1}{2} \times 120°$ [$\because \angle CBD = 120°$ given]

$\Rightarrow \quad \angle OBC = 60°$

OC and BC are radius and tangent respectively at contact point C.

So, $\angle OCB = 90°$

Now, in right angle ΔOCB, $\angle OBC = 60°$

$\therefore \quad \cos 60° = \frac{BC}{BO}$

$\Rightarrow \quad \frac{1}{2} = \frac{BC}{BO}$

$\Rightarrow \quad OB = 2BC$

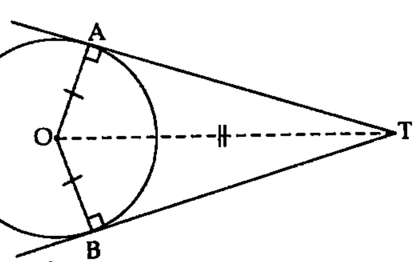
Hence, proved (*ii*) part.

$\Rightarrow \quad OB = BC + BC$

$\Rightarrow \quad OB = BC + BD$ [$\because BC = BD$ from (*i*)]

Hence, proved.

Q4. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.



Sol. Given: Two intersecting lines AT and BT intersect at T. A circle with centre O touches the above lines at A and B.

To prove: OT bisects the $\angle ATB$.

Construction: Join OA and OB.

Proof: OA is radius and AT is tangent at A.

$\therefore$ $\angle OAT = 90°$

Similarly, $\angle OBT = 90°$

In ΔOTA and ΔOTB, we have

$\angle OAT = \angle OBT = 90°$

$OT = OT$ [Common]

$OA = OB$ [Radii of same circle]

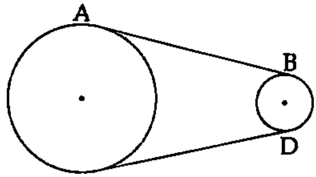
$\therefore$ $\Delta OTA \cong \Delta OTB$ [By RHS criterion of congruence]

$\Rightarrow$ $\angle OTA = \angle OTB$ [CPCT]

$\Rightarrow$ Centre of circle 'O' lies on the angle bisector of $\angle ATB$.

Hence, proved.

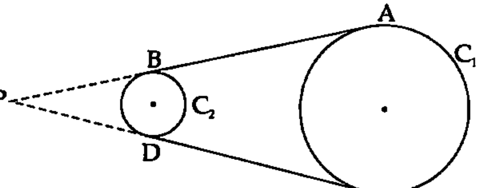
Q5. In the given figure, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD.



Sol. Given: Circles C_1 and C_2 of radii r_1 and r_2 respectively and $r_1 < r_2$. AB and CD are two common tangents.

To prove: AB = CD

Construction: Produce AB and CD upto point P where both tangents meet.



Proof: Tangents from an external point to a circle are equal.

For circle C_1, $PB = PD$ ...(*i*)

and for circle C_2, $PA = PC$ ...(*ii*)

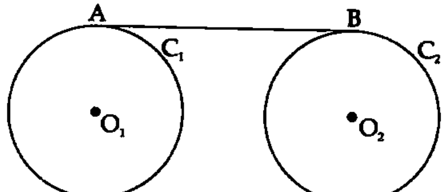
Subtracting (*i*) from (*ii*), we have

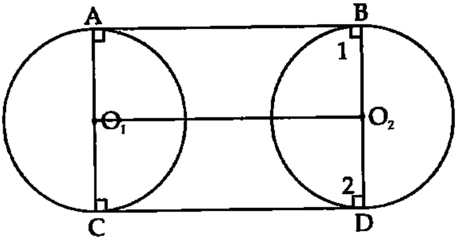
$$PA - PB = PC - PD$$

$\Rightarrow$ $AB = CD$.

Hence, proved.

Q6. In Question 5 above, if radii of the two circles are equal, prove that AB = CD.





Sol. Given: Two circles of equal radii, two common tangents, AB and CD on circles C_1 and C_2.

To prove: AB = CD

Construction: Join O_1A, O_1C and O_2B and O_2D. Also, join O_1O_2.

Proof: Since tangent at any point of a circle is perpendicular to the radius to the point of contact.

$\therefore \qquad \angle O_1AB = \angle O_2BA = 90°$

As $O_1A = O_2B$, so O_1ABO_2 is a rectangle.

Since opposite sides of a rectangle are equal,

$\therefore \qquad AB = O_1O_2 \qquad$ ...(*i*)

Similarly, we can prove that O_1CDO_2 is a rectangle.

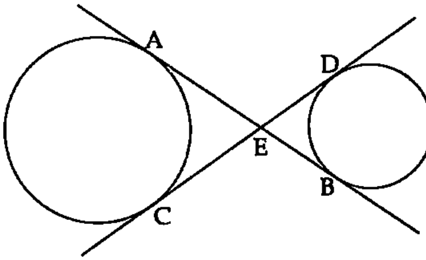
$\therefore \qquad O_1O_2 = CD \qquad$ ...(*ii*)

From (*i*) and (*ii*), we get

$$AB = CD.$$

Hence, proved.

Q7. In the given figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD.



Sol. Given: Two non-intersecting circles are shown in the figure. Two intersecting tangents AB and CD intersect at E. E point is between the circles and outside also.

To prove: AB = CD

Proof: We know that tangents drawn from an external point (E) to a circle are equal. Point E is outside of both the circles.

So, $\qquad EA = EC \qquad$ ...(*i*)

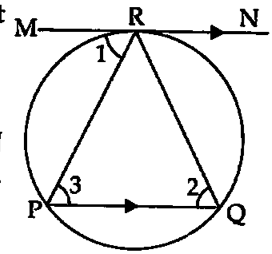
$\qquad EB = ED \qquad$ ...(*ii*)

$\Rightarrow \qquad EA + EB = EC + ED \qquad$ [Adding (*i*) and (*ii*)]

$\Rightarrow \qquad AB = CD$

Hence, proved.

Q8. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.



Sol. Given: In a circle a chord PQ and a tangent MRN at R such that QP || MRN

To prove: R bisects the arc PRQ.

Construction: Join RP and RQ.

Proof: Chord RP subtends $\angle 1$ with tangent MN and $\angle 2$ in alternate segment of circle so $\angle 1 = \angle 2$.

MRN || PQ

$\therefore \quad \angle 1 = \angle 3$ [Alternate interior angles]

$\Rightarrow \quad \angle 2 = \angle 3$

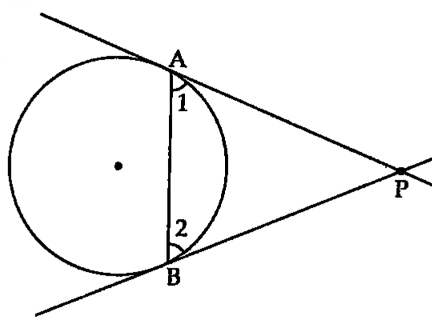
$\Rightarrow \quad PR = RQ$ [Sides opp. to equal $\angle$s in ΔRPQ]

$\because$ Equal chords subtend equal arcs in a circle so

arc PR = arc RQ

or R bisect the arc PRQ. Hence, proved.

Q9. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.



Sol. Given: A chord AB of a circle, tangents AP and BP at A and B respectively are drawn.

To prove: $\angle PAB = \angle PBA$

Proof: We know that tangents drawn from an external point P to a circle are equal so PA = PB.

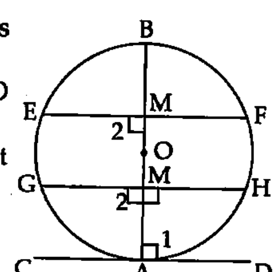
$\Rightarrow \angle 2 = \angle 1$

[Angles opposite to equal sides of a triangle are equal]

Hence, tangents PA and PB make equal angles with chord AB.

Hence, proved.

Q10. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.



Sol. Given: A circle with centre O and AOB is diameter.

CAD is a tangent at A. Chord EF || tangent CAD

To prove: AB bisects any chord EF || CAD.

Proof: OA radius is perpendicular to tangent CAD.

$\therefore \quad \angle 1 = 90°$

CAD || EF [Given]

$\therefore \quad \angle 1 = \angle 2 = 90°$ [Alternate interior angles]

Point M is on diameter which passes through centre O.

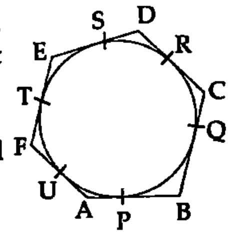
$\because$ Perpendicular drawn from centre to chord bisect the chord.

Hence, AB bisects any chord EF || CAD.

EXERCISE 9.4

Q1. If a hexagon ABCDEF circumscribe a circle, then prove that

$$AB + CD + EF = BC + DE + FA$$



Sol. Given: A circle inscribed in a hexagon ABCDEF. Sides, AB, BC, CD, DE and DF touches the circle at P, Q, R, S, T and U respectively.

To prove: $AB + CD + EF = BC + DE + FA$

Proof: We know that tangents from an external point to a circle are equal.

Here, vertices of hexagon are outside the circle so

$$AP = AU$$
$$BP = BQ$$
$$CQ = CR$$
$$DR = DS$$
$$ES = ET$$
$$FT = FU$$

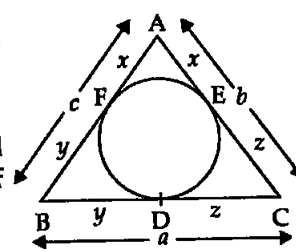
$$LHS = AB + CD + EF = (AP + PB) + (DR + CR) + (ET + TF)$$

By using above results, we have

$$LHS = AB + CD + EF = AU + BQ + DS + CQ + ES + FU$$
$$= AU + FU + BQ + CQ + DS + ES$$
$$= AF + BC + DE.$$

Hence, proved.

Q2. Let s denotes the semi-perimeter of a ΔABC in which $BC = a$, $CA = b$, $AB = c$. If a circle touches the sides BC, CA, AB at D, E, F respectively, prove that $BD = s - b$.



Sol. Given: A circle inscribed in ΔABC touches the sides BC, CA and AB at D, E, F respectively.

To prove: $BD = s - b$

Proof: Tangents drawn from an external point to the circle are equal. Vertices of ΔABC are in the exterior of circle. So,

$$AF = AE = x$$
$$BF = BD = y$$
$$CD = CE = z$$

Now, $AB + BC + CA = c + a + b$

$\Rightarrow$ $AF + BF + BD + DC + AE + CE = a + b + c$

$\Rightarrow$ $x + y + y + z + x + z = a + b + c$

$\Rightarrow$ $2x + 2y + 2z = a + b + c$

$\Rightarrow$ $2(x + y + z) = a + b + c$

$\Rightarrow$ $x + y + z = \dfrac{a + b + c}{2}$

$\Rightarrow \quad x + y + z = s$ [Given]

$\Rightarrow \quad y = s - (x + z) \Rightarrow y = s - x - z$

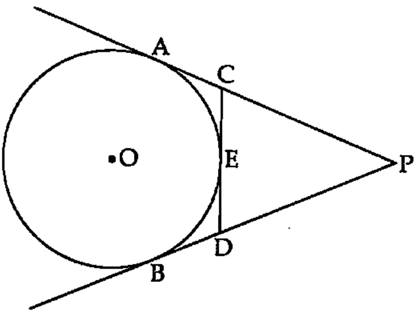
$\Rightarrow \quad y = s - (AE + EC)$

$\Rightarrow \quad = s - AC$

$\Rightarrow \quad BD = s - b$

Hence, proved.

Q3. From an external point P, two tangents PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D respectively. If PA = 10 cm, find the perimeter of ΔPCD.



Sol. Given: A circle with centre O. PA, PB are tangents from an external point P. A tangent CD at E intersect AP and PB at C and D respectively.

To find: Perimeter of ΔPCD.

Method: Tangents drawn from an external point to a circle are equal.

$\therefore$ PA = PB = 10 cm [Given]

CA = CE

DE = DB

Perimeter of ΔPCD = PC + PD + CD

= PC + PD + CE + DE

= PC + CE + PD + DE

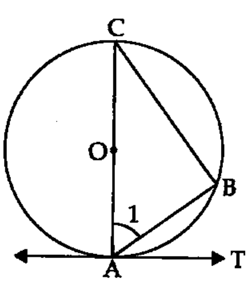
= PC + CA + PD + DB

= PA + PB

= 10 + 10 = 20 cm

$\therefore$ Perimeter of ΔPCD = 20 cm.

Q4. If AB is a chord of a circle with centre O. AOC is a diameter and AT is the tangent at A as shown in figure. Prove that ∠BAT = ∠ACB.



Sol. Given: Chord AB, diameter AOC and tangent at A of a circle with centre O.

To prove: ∠BAT = ∠ACB

Proof: Radius OA and tangent AT at A are perpendicular.

$\therefore \quad \angle OAT = 90°$

$\Rightarrow \quad \angle BAT = 90° - \angle 1$ ...(*i*)

AOC is diameter.

$\therefore \quad \angle B = 90°$

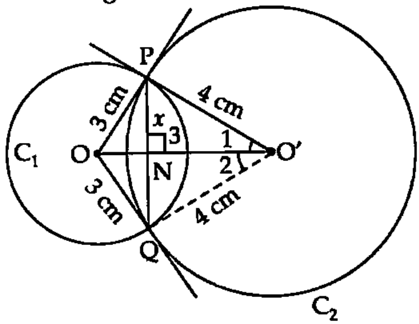
$\Rightarrow \quad \angle C + \angle 1 = 90°$

$\Rightarrow \quad \angle C = 90° - \angle 1$ ...(*ii*)

From (*i*) and (*ii*), we get

$$\angle BAT = \angle ACB.$$ Hence, proved.

Q5. Two circles with centres O and O′ of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and O′P are tangents to the two circles. Find the length of common chord PQ.



Sol. PO′ is tangent on circle C_1 at P. OP is tangent on circle C_2 at P. As radius OP and tangent PO′ are at a point of contact P

$\therefore \quad \angle P = 90°$

So, by Pythagoras theorem in right angled $\Delta OPO'$,

$OO'^2 = OP^2 + PO'^2 = 3^2 + 4^2 = 9 + 16$
$= 25$ cm

$\Rightarrow \quad OO' = 5$ cm

$\Delta OO'P \cong \Delta OO'Q$ [By SSS criterion of congruence]

$\Rightarrow \quad \angle 1 = \angle 2$

$\Delta O'NP \cong \Delta O'NQ$ [By SAS criterion of congruence]

$\Rightarrow \quad \angle 3 = \angle O'NQ$ [CPCT]

$\Rightarrow \quad \angle 3 = \angle O'NQ = 90°$ [Linear Pair axiom]

Let $ON = y$, then $NO' = (5 - y)$

Let $PN = x$

By Pythagoras theorem in ΔPNO and $\Delta PNO'$, we have

$x^2 + y^2 = 3^2$ ...(*i*)

$x^2 + (5 - y)^2 = 4^2$

$x^2 + 25 + y^2 - 10y = 16$ ...(*ii*)

$x^2 + y^2 = 9$ [From (*i*)]

$- \quad - \quad -$

$25 - 10y = 7$ [Subtract (*i*) from (*ii*)]

$\Rightarrow \quad -10y = 7 - 25$

$\Rightarrow \quad -10y = -18$

$\Rightarrow \quad y = 1.8$

But, $\quad x^2 + y^2 = 3^2$ [From (*i*)]

$\Rightarrow \quad x^2 + (1.8)^2 = 3^2$

$\Rightarrow \quad x^2 = 9 - 3.24$

$\Rightarrow \quad x^2 = 5.76$

$\Rightarrow \quad x = 2.4$

$\therefore$ The perpendicular drawn from the centre bisects the chord.

$\therefore \quad PQ = 2PN = 2x$

$= 2 \times 2.4$

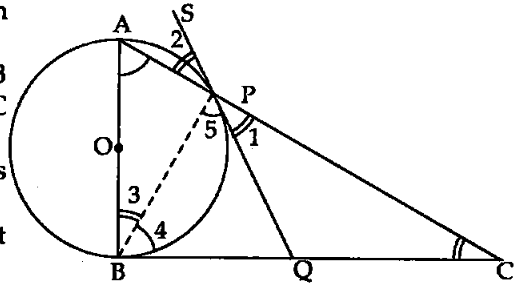
$\Rightarrow \quad PQ = 4.8$ cm

Q6. In a right triangle ABC in which $\angle B = 90°$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at P bisects BC.

Sol. Given: ΔABC in which $\angle B = 90°$

Circle with diameter AB intersect the hypotenuse AC at P.

A tangent SPQ at P is drawn to meet BC at Q.



To prove: Q is mid point of BC.

Construction: Join PB.

Proof: SPQ is tangent and AP is chord at contact point P.

$\therefore$ $\angle 2 = \angle 3$ [Angles in alternate segment of circle]

$\angle 2 = \angle 1$ [Vertically opposite angles]

$\Rightarrow$ $\angle 3 = \angle 1$ ...(*i*) [From above two relations]

$\angle ABC = 90°$ [Given]

OB is radius so, BC will be tangent at B.

$\therefore$ $\angle 3 = 90° - \angle 4$ ...(*ii*)

$\angle APB = 90°$ [$\angle$ in a semi circle]

$\Rightarrow$ $\angle C = 90° - \angle 4$ ...(*iii*)

From (*ii*) and (*iii*), $\angle C = \angle 3$

Using (*i*), $\angle C = \angle 1$

$\Rightarrow$ $CQ = QP$ ...(*iv*) [Sides opp. to = $\angle$s in ΔQPC]

$\angle 4 = 90° - \angle 3$

$\angle 5 = 90° - \angle 1$ [From fig.]

$\angle 3 = \angle 1$

$\therefore$ $\angle 4 = \angle 5$

$\Rightarrow$ $PQ = BQ$ ...(*v*) [Sides opp. to equal angles in ΔQPB]

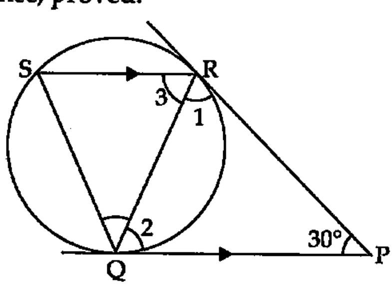
From (*iv*) and (*v*),

$BQ = CQ$

Therefore, Q is mid-point of BC. Hence, proved.

Q7. In the given figure, tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30°$. A chord RS is drawn parallel to tangent PQ. Find the $\angle RQS$.

[**Hint:** Draw a line through Q and perpendicular to QP.]



Sol. In ΔPRQ, PQ and PR are tangents from an external point P to circle.

$\therefore$ $PR = PQ$

$\Rightarrow$ $\angle 2 = \angle 1$ [$\angle$s opp. to equal sides in ΔPRQ are equal]

$\angle 1 + \angle 2 + \angle RPQ = 180°$ [Int. $\angle$s of Δ]

$\Rightarrow$ $\angle 1 + \angle 1 + 30° = 180°$

$\Rightarrow$ $2\angle 1 = 180° - 30°$

$\Rightarrow$ $\angle 1 = \dfrac{150°}{2}$

$\therefore$ $\angle 1 = \angle 2 = 75°$

Tangent $PQ \parallel SR$ [Given]

$\therefore$ $\angle 2 = \angle 3 = 75°$ [Alternate interior angles]

PQ is tangent at Q and QR is chord at Q.

$\therefore$ $\angle S = \angle 2 = 75°$ [$\angle$s in alternate segment of circle]

In ΔSRQ,

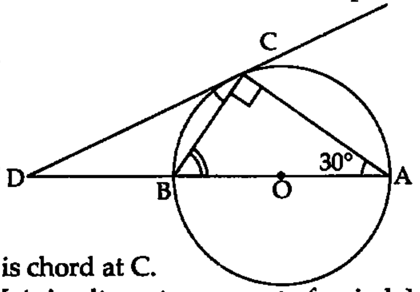
$\angle S + \angle 3 + \angle SQR = 180°$ [Angle sum property of a triangle]

$\Rightarrow$ $75° + 75° + \angle SQR = 180°$

$\Rightarrow$ $\angle SQR = 180° - 150°$

$\Rightarrow$ $\angle SQR = 30°$

Q8. AB is a diameter and AC is chord of a circle with centre O such that $\angle BAC = 30°$. The tangent at C intersects extended AB at a point D. Prove that BC = BD.



Sol. Given: A circle with centre O.

A tangent CD at C.

Diameter AB is produced to D.

BC and AC chords are joined, $\angle BAC = 30°$.

To prove: $BC = BD$

Proof: DC is tangent at C and, CB is chord at C.

$\therefore$ $\angle DCB = \angle BAC$ [$\angle$s in alternate segment of a circle]

$\Rightarrow$ $\angle DCB = 30°$ ...(*i*) [$\because \angle BAC = 30°$ (Given)]

AOB is diameter. [Given]

$\therefore$ $\angle BCA = 90°$ [Angle in a semi circle]

$\therefore$ $\angle ABC = 180° - 90° - 30° = 60°$

In ΔBDC,

Exterior $\angle ABC = \angle D + \angle BCD$

$\Rightarrow$ $60° = \angle D + 30°$

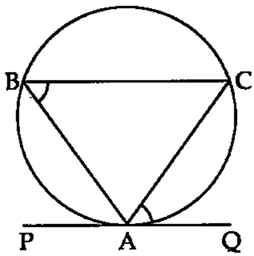
$\Rightarrow$ $\angle D = 30°$ ...(*ii*)

$\therefore$ $\angle DCB = \angle D = 30°$ [From (*i*), (*ii*)]

$\Rightarrow$ $BD = BC$ [$\because$ Sides opposite to equal angles are equal in a triangle]

Hence, proved.

Q9. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.



Sol. Given: arc BAC in which A is mid point of arc BAC.

PAQ is tangent at A.

To prove: BC || PAQ

Proof: PAQ is tangent and CAB is an arc at contact point A.

∴ $\angle CAQ = \angle B$ ...(*i*)

[Angles in alternate segment of a circle]

A is mid point of arc BAC.

∴ min. arc AB = min. arc AC

⇒ Chord AB = Chord AC [Equal arcs subtend equal chords]

⇒ $\angle C = \angle B$ ...(*ii*) [Angles opp. to equal sides in ΔABC are equal]

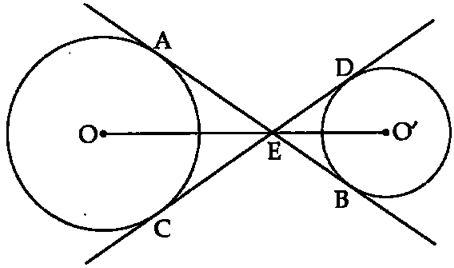
⇒ $\angle C = \angle CAQ$ [From (*i*) and (*ii*)]

These are alternate interior angles and are equal.

∴ BC || PAQ.

Hence, proved.

Q10. In the given figure, the common tangents, AB and CD to two circles with centres O and O′ intersect at E. Prove that the points O, E and O′ are collinear.

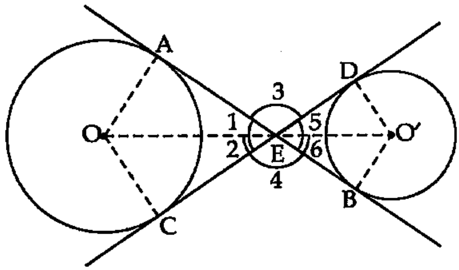


Sol. Given: Two circles (non intersecting) with their centres O and O′.

Two common tangents AB and CD intersect at E between the circles.

To prove: O, E, O′ points are collinear.

Construction: Join OA, OC, O′D, O′B and EO and EO′



Proof: In ΔAEO and ΔCEO,

$OE = OE$ [Common]

$OA = OC$ [Radii of same circle]

$EA = EC$ [Tangents from an external point to a circle are equal in length]

$\therefore \quad \angle OEA \cong \angle OEC$ [By SSS criterion of congruence]

$\Rightarrow \quad \angle OEA = \angle OEC$ [CPCT]

$\therefore \quad \angle 1 = \angle 2$ [CPCT]

Similarly, $\angle 5 = \angle 6$

and $\angle 3 = \angle 4$ [Vertically opposite angles]

Since sum of angles at a point = 360°

$\therefore \quad \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360^\circ$

$\Rightarrow \quad 2(\angle 1 + \angle 3 + \angle 5) = 360^\circ$

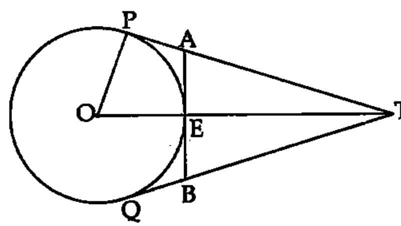
$\Rightarrow \quad \angle 1 + \angle 3 + \angle 5 = 180^\circ$

$\Rightarrow \quad \angle OEO' = 180^\circ$

$\therefore$ OEO′ is a straight line.

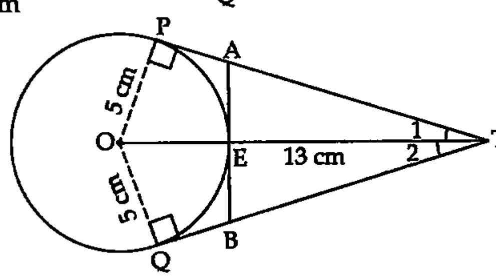
Hence, O, E and O′ are collinear.

Q11. In the given figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E, find the length of AB.



Sol. $OP = OQ = 5$ cm

$OT = 13$ cm



OP and PT are radius and tangent respectively at contact point P.

$\therefore \quad \angle OPT = 90^\circ$

So, by Pythagoras theorem, in right angled ΔOPT,

$PT^2 = OT^2 - OP^2 = 13^2 - 5^2$

$= 169 - 25 = 144$

$\Rightarrow \quad PT = 12$ cm.

AP and AE are two tangents from an external point A to a circle.

$\therefore \quad AP = AE$

AEB is tangent and OE is radius at contact point E.

So, $\quad AB \perp OT$ ...(i)

So, by Pythagoras theorem, in right angled ΔAET,

$AE^2 = AT^2 - ET^2$

$\Rightarrow \quad AE^2 = (PT - PA)^2 - [TO - OE]^2$

$$= (12 - AE)^2 - (13 - 5)^2$$

$$\Rightarrow \quad AE^2 = (12)^2 + (AE)^2 - 2(12)(AE) - (8)^2$$

$$\Rightarrow AE^2 - AE^2 + 24AE = 144 - 64$$

$$\Rightarrow \quad 24AE = 80$$

$$\Rightarrow \quad AE = \frac{80}{24} \text{ cm}$$

$$\Rightarrow \quad AE = \frac{10}{3} \text{ cm}$$

In ΔTPO and ΔTQO,

$OT = OT$ [Common]

$PT = QT$ [Tangents from T]

$OP = OQ$ [Radii of same circle]

$\therefore \quad \Delta TPO \cong \Delta TQO$ [By SSS criterion of congruence]

$\Rightarrow \quad \angle 1 = \angle 2$ ...(*ii*) [CPCT]

In ΔETA and ΔETB,

$ET = ET$ [Common]

$\angle TEA = \angle TEB = 90°$ [From (*i*)]

$\angle 1 = \angle 2$ [CPCT] [From (*ii*)]

$\therefore \quad \Delta ETA \cong \Delta ETB$ [By ASA criterion of congruence]

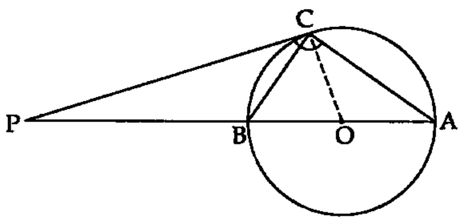
$\Rightarrow \quad AE = BE$ [CPCT]

$$\Rightarrow \quad AB = 2AE = 2 \times \frac{10}{3}$$

$$\Rightarrow \quad AB = \frac{20}{3} \text{ cm}.$$

Hence, the required length is $\frac{20}{3}$ cm.

Q12. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle PCA = 110°$, find $\angle CBA$. [**Hint:** Join C with centre O].



Sol. OC and CP are radius and tangent respectively at contact point C.

So, $\angle OCP = 90°$

$\angle OCA = \angle ACP - \angle OCP$

$\Rightarrow \quad \angle OCA = 110° - 90°$

$\Rightarrow \quad \angle OCA = 20°$

In ΔOAC,

$OA = OC$ [Radii of same circle]

$\therefore \quad \angle OCA = \angle A = 20°$ [$\because$ Angles opposite to equal sides are equal]

CP and CB are tangent and chord of a circle.

$\therefore$ $\angle CBP = \angle A$ [Angles in alternate segments are equal]

In ΔCAP,

$\angle P + \angle A + \angle ACP = 180°$ [Angle sum property of a triangle]

$\Rightarrow$ $\angle P + 20° + 110° = 180°$

$\Rightarrow$ $\angle P = 180° - 130°$

$\Rightarrow$ $\angle P = 50°$

In ΔBPC,

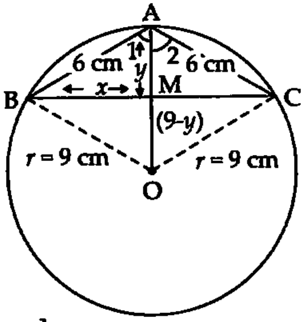
Exterior angle $\angle CBA = \angle P + \angle BCP$

$\Rightarrow$ $\angle CBA = 50° + 20°$

$\Rightarrow$ $\angle CBA = 70°$

Q13. If an isosceles ΔABC in which $AB = AC = 6$ cm is inscribed in a circle of radius 9 cm, find the area of the triangle.

Sol. In figure, ΔABC has $AB = AC = 6$ cm.



In ΔOAB and ΔOAC,

$AB = AC$ [Given]

$OA = OA$ [Common]

$OB = OC$ [Radii of same circle]

$\therefore$ $\Delta OAB \cong \Delta OAC$

[By SSS criterion of congruence]

$\Rightarrow$ $\angle 1 = \angle 2$ [CPCT]

In ΔAMC and ΔAMB,

$\angle 1 = \angle 2$ [Proved above]

$AM = AM$ [Common]

$AB = AC$ [Given]

$\therefore$ $\Delta AMB \cong \Delta AMC$ [By SAS criterion of congruence]

$\Rightarrow$ $\angle AMB = \angle AMC = 90°$ [CPCT and Linear pair axiom]

Now, Area of $\Delta ABC = \frac{1}{2} BC \times AM$

Let $BM = x$ and $AM = y$,

then $MO = OA - AM$

$\Rightarrow$ $MO = OA - AM$

$\Rightarrow$ $MO = 9 - y$

In right angled ΔBMA and ΔBMO,

$x^2 + y^2 = 6^2$ ...(*i*) [By Pythagoras theorem]

$x^2 + (9 - y)^2 = 9^2$

$x^2 + (9)^2 + (y)^2 - 2(9)(y) = 81$

$\Rightarrow$ $x^2 + 81 + y^2 - 18y = 81$

$\Rightarrow$ $x^2 + y^2 - 18y = 0$ ...(*ii*)

Now, subtract (*i*) from (*ii*)

$$\begin{array}{rcl} x^2 + y^2 - 18y &=& 0 \\ x^2 + y^2 &=& 36 \\ - \quad - & & - \\ \hline -18y &=& -36 \end{array}$$

$\Rightarrow \quad y = \dfrac{-36}{-18}$

$\Rightarrow \quad y = 2 \text{ cm} \Rightarrow AM = 2 \text{ cm}$

But, $\quad x^2 + y^2 = 36$ [From (*i*)]

$\Rightarrow \quad x^2 + (-2) = 36$

$\Rightarrow \quad x^2 = 36 - 4 = 32$

$\Rightarrow \quad x = \sqrt{32} = 4\sqrt{2} \text{ cm}$

$\therefore \quad BC = 2x = 2 \times 4\sqrt{2} = 8\sqrt{2} \text{ cm}$

($\because$ Perpendicular from centre to chord bisects the chord)

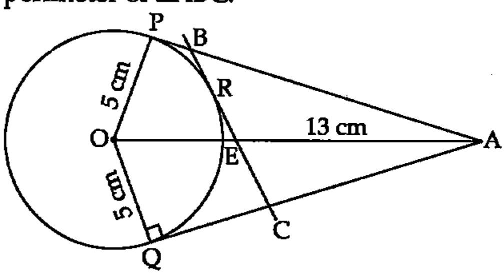
$\therefore \quad \text{Area of } \Delta ABC = \dfrac{1}{2} \times 2 \times 8\sqrt{2}$

$\Rightarrow \quad \text{Area of } \Delta ABC = 8\sqrt{2} \text{ cm}^2$

Q14. A is a point at a distance 13 cm from the centre 'O' of a circle of radius 5 cm. AP and AQ are the tangents to circle at P and Q. If a tangent BC is drawn at point R lying on minor arc PQ to intersect AP at B and AQ at C. Find the perimeter of ΔABC.

Sol. $OA = 13$ cm

$OP = OQ = 5$ cm



OP and PA are radius and tangent respectively at contact point P.

$\therefore \quad \angle OPA = 90^\circ$

In right angled ΔOPA by Pythagoras theorem

$$PA^2 = OA^2 - OP^2 = 13^2 - 5^2 = 169 - 25 = 144$$

$\Rightarrow \quad PA = 12$ cm

Points A, B and C are exterior to the circle and tangents drawn from an external point to a circle are equal so

$$PA = QA$$
$$BP = BR$$
$$CR = CQ$$

$$\begin{aligned} \text{Perimeter of } \Delta ABC &= AB + BC + AC \\ &= AB + BR + RC + AC \quad \text{[From figure]} \\ &= AB + BP + CQ + AC = AP + AQ \\ &= AP + AP = 2AP = 2 \times 12 = 24 \text{ cm} \end{aligned}$$

So, the perimeter of ΔABC = 24 cm.

❑❑❑

10 Constructions

EXERCISE 10.1

Choose the correct answer from the given four options:

Q1. To divide a line segment AB in the ratio 5 : 7, first a ray AX is drawn so that ∠BAX is an acute angle and then at equal distances points are marked on the ray AX such that the minimum number of these points is

(*a*) 8 (*b*) 10 (*c*) 11 (*d*) 12

Sol. (*d*): Minimum number of the points marked = 5 + 7 = 12 verifies option (*d*).

Q2. To divide a line segment AB in ratio 4 : 7, a ray AX is drawn first such that ∠BAX is an acute angle and then points A_1, A_2, A_3, ... are located at equal distances on the ray AX and the point B is joined to

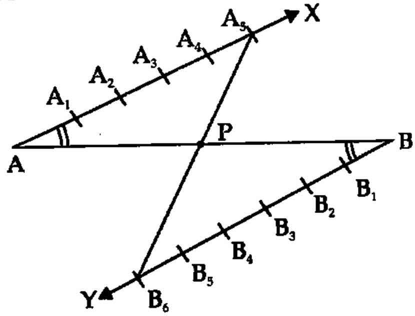
(*a*) A_{12} (*b*) A_{11} (*c*) A_{10} (*d*) A_9

Sol. (*b*): We have to divide the constructed line into 7 + 4 = 11 equal parts and 11th part will be joined to B. Verifies the option (*b*).

Q3. To divide a line segment AB in the ratio 5 : 6, draw a ray AX such that ∠BAX is an acute angle, then draw a ray BY parallel to AX, and the points, A_1, A_2, A_3 ... and B_1, B_2, B_3 ... are located at. Equal distances on ray AX and BY, respectively. Then the points joined are

(*a*) A_5 and B_6 (*b*) A_6 and B_5 (*c*) A_4 and B_5 (*d*) A_5 and B_4

Sol. (*a*): In the figure, segment AB of given length is divided into 2 parts of ratio 5 : 6 in following steps:



(*i*) Draw a line-segment AB of given length.

(*ii*) Draw an acute angle BAX as shown in figure either up side or down side.

(*iii*) Draw angle ∠ABY = ∠BAX on other side of AX, *i.e.*, down side.

(*iv*) Divide AX into 5 equal parts by using compass.

(*v*) Divide BX into same distance in 6 equal parts as AX was divided.

(*vi*) Now, join A_5 and B_6 which meet AB at P. P divides AB in ratio AP : PB = 5 : 6.

Q4. To construct a triangle similar to a given ΔABC with its sides $\frac{3}{7}$ of the corresponding sides of ΔABC, first draw a ray BX such that $\angle CBX$ is an acute angle and X lies on the opposite side of A with respect to BC. Then locate points $B_1, B_2, B_3, \ldots$ on BX at equal distances and next step is to join

(*a*) B_{10} to C (*b*) B_3 to C (*c*) B_7 to C (*d*) B_4 to C

Sol. (*c*): Here, ratio is $\frac{3}{7} < 1$ so resultant figure will be smaller than original so, last 7th part is to be joined to C, so that parallel line from third part of BX meet on BC without producing. So, verifies the option (*c*).

Q5. To construct a triangle similar to a given ΔABC with its sides $\frac{8}{5}$ of the corresponding sides of ΔABC draw a ray BX such that $\angle CBX$ is an acute angle and X is on the opposite side of A with respect to BC. The minimum number of points to be located at equal distances on the ray BX

(*a*) 5 (*b*) 8 (*c*) 13 (*d*) 3

Sol. (*b*): To construct a triangle similar to a given triangle ABC with its sides $\frac{8}{5}$ of the corresponding sides of ΔABC, the minimum number of parts in which BX is divided in 8 equal parts. Verifies the option (*b*).

Q6. To draw a pair of tangents to a circle which are inclined to each other at an angle of 60°, it is required to draw tangents at end points of those two radii of the circle, the angle between them should be

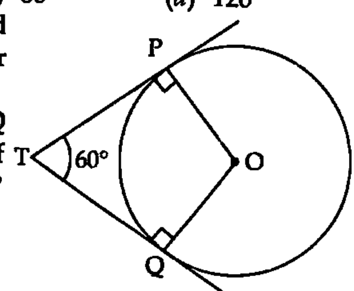
(*a*) 135° (*b*) 90° (*c*) 60° (*d*) 120°

Sol. (*d*): We know that tangent and radius at contact point are perpendicular to each other.

So, $\angle P$ and $\angle Q$ in quadrilateral TPOQ formed by tangents and radii will be of 90° each. So, the sum of $\angle T + \angle O = 180°$ as $T = 60°$ (Given)

$\therefore \angle O = 180° - 60° = 120°$

Verifies the option (*d*).



EXERCISE 10.2

Write True or False and give reason for your answer in each of the following:

Q1. By geometrical construction, it is possible to divide a line segment in ratio $\sqrt{3} : \frac{1}{\sqrt{3}}$.

Sol. True: On multiplying or dividing a given ratio by a real number, the ratio remains same.

On multiplying the given ratio by $\sqrt{3}$ we get $\sqrt{3}.\sqrt{3} : \frac{1}{\sqrt{3}}.\sqrt{3}$ or $3:1$

Hence, the given ratio $\sqrt{3} : \frac{1}{\sqrt{3}}$ is possible to divide a line in ratio $3:1$ in place of $\sqrt{3} : \frac{1}{\sqrt{3}}$.

Q2. To construct a triangle similar to a given ΔABC with its sides $\frac{7}{3}$ of the corresponding sides of ΔABC, draw a ray BX making acute angle with BC and X lies on the opposite side of A with respect to BC. The points $B_1, B_2, \ldots B_7$ are located at equal distances on BX, B_3 is joined to C and then a line segment B_6C' is drawn parallel to B_3C where C′ lies on BC produced. Finally, the line segment A′C′ is drawn parallel to AC.

Sol. False: Given ratio is $\frac{7}{3} > 1$ so, the resulting triangle will be larger than given as $B_7C' \parallel B_3C$ and BX is equally divided into 7 parts as $(7 > 3)$.

Construction: (*i*) Draw given triangle with given specifications.

(*ii*) Draw an acute angle CBX.

(*iii*) Divide BX into 7 equal parts and mark them $B_1, B_2, B_3, \ldots B_7$.

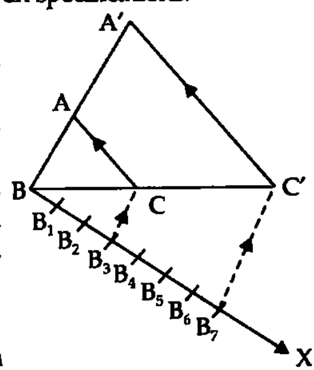
(*iv*) Produce BC and BA as shown in figure.

(*v*) Join B_3C.

(*vi*) Draw $B_7C' \parallel B_3C$, C′ is on BC produced.

(*vii*) Draw $C'A' \parallel AC$. A′ on BA produced $\Delta A'BC'$ is required triangle i.e., $\frac{\Delta A'BC'}{\Delta ABC} = \frac{3}{7}$.

Here, $B_7C' \parallel B_3C$. But in Question $B_6C' \parallel B_3C$, which is false.



Q3. A pair of tangents can be constructed from a point P to a circle of radius 3.5 cm situated at a distance of 3 cm from the centre.

Sol. False: Any tangent on a circle can be drawn only if the distance of point to draw tangent is equal to or more than radius of circle. Here, radius of circle is 3.5 cm and point is at 3 cm from centre which is inside the circle. So, no tangent can be drawn if point is inside the circle.

Q4. A pair of tangents can be constructed to a circle inclined at an angle of 170°.

Sol. True: A pair of tangents can be constructed if the angle between the tangents is between zero and less than 180°. Because the sum of angles between tangents and radii on tangent are supplementary.

So, a pair of tangents can be constructed to circle inclined at an angle of 170°.

EXERCISE 10.3

Q1. Draw a line segment of length 7 cm. Find a point P on it which divides it in the ratio 3 : 5.

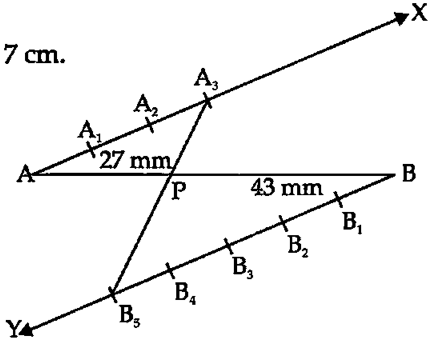
Sol. Steps of construction:

(*i*) Draw a line-segment AB = 7 cm.

(*ii*) Draw AX || BY such that ∠A and ∠B are acute angles.

(*iii*) Divide AX and BY in 3 and 5 parts equally by compass and mark A_1, A_2, A_3, B_1, B_2, B_3, B_4 and B_5 respectively.

(*iv*) Join A_3B_5 which intersect AB at P and divides AP : PB = 3 : 5.



Hence, P is the required point on AB which divide it in 3 : 5.

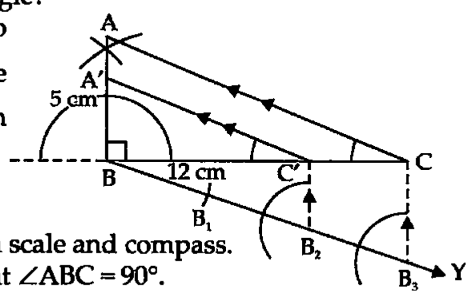
Verification (Justification): In ΔAA_3P and ΔBB_5P

	AX || BY	[By construction]
	$\angle A = \angle B$	[Alt. angles]
	$\angle A_3PA = \angle B_5PB$	[Vertically opp. angles]
∴	$\Delta AA_3P \sim \Delta BB_5P$	[By AA criterion of similarly]
⇒	$\frac{AA_3}{BB_5} = \frac{AP}{BP}$	[Let each equal part = x cm ∴ $AA_1 = A_1A_2 = B_1B_2 \ldots = x$]
⇒	$\frac{3x}{5x} = \frac{AP}{BP}$	
⇒	AP : BP = 3 : 5.	

Hence, verified.

Q2. Draw a right angled ΔABC in which BC = 12 cm, AB = 5 cm, and ∠B = 90°. Construct a triangle similar to it and of scale factor $\frac{2}{3}$. Is the new triangle also a right triangle?

Sol. Here, scale factor or ratio factor is $\frac{2}{3} < 1$, so triangle to be constructed will be smaller than given ΔABC.



Steps of construction:

(*i*) Draw BC = 12 cm.

(*ii*) Draw ∠CBA = 90° with scale and compass.

(*iii*) Cut BA = 5 cm such that ∠ABC = 90°.

(*iv*) Join AC. ΔABC is the given triangle.

(*v*) Draw an acute ∠CBY such that A and Y are in opposite direction with respect to BC.

(*vi*) Divide BY in 3 equal segments by marking arc at same distance at B_1, B_2 and B_3.
(*vii*) Join B_3C.
(*viii*) Draw $B_2C' \parallel B_3C$ by making equal alternate angles at B_2 and B_3.
(*ix*) From point C′, draw $C'A' \parallel CA$ by making equal alternate angles at C and C′.

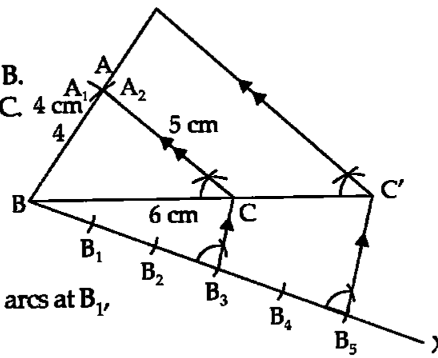
$\Delta A'BC'$ is the required triangle of scale factor $\frac{2}{3}$. This triangle is also a right triangle.

Q3. Draw a ΔABC in which BC = 6 cm, CA = 5 cm and AB = 4 cm. Construct a triangle similar to it and of scale factor $\frac{5}{3}$.

Sol. Here, scale factor is $\frac{5}{3} > 1$, so the resulting figure will be larger.

Steps of construction:

(*i*) Draw BC = 6 cm.
(*ii*) Draw arc $BA_1 = 4$ cm from B.
(*iii*) Draw arc $CA_2 = 5$ cm from C.
(*iv*) Arc CA_2 and BA_1 intersect at A.
(*v*) Join AB and AC.
(*vi*) Draw acute angle CBX below BC.
(*vii*) Cut BX into equal parts by arcs at B_1, B_2, B_3, B_4 and B_5.



(*viii*) Join B_3C.
(*ix*) Draw $B_5C' \parallel B_3C$ by making alternate angles. C′ is on BC produced.
(*x*) Draw $C'A' \parallel CA$ which meet BA produced at A′. Now, $\Delta A'BC'$ is the required triangle.

Justification:

$\Delta BCB_3 \sim \Delta BC'B_5$ [By AA criterion of similarity]

$\therefore \quad \frac{BB_3}{BB_5} = \frac{BC}{BC'}$ $[BB_1 = B_1B_2 = ... x$
$\therefore BB_3 = 3x$ and $BB_5 = 5x]$

$\Rightarrow \quad \frac{3x}{5x} = \frac{BC}{BC'}$

$\Delta ABC \sim \Delta A'BC'$ [By AA criterion of similarity]

$\Rightarrow \quad \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC}{BC'}$

Q4. Construct a pair of tangents to a circle of radius 4 cm from a point which is at a distance of 6 cm from the centre of circle.

Sol. The distance of point from which tangents to be drawn should be more than radius so that tangents can be drawn.

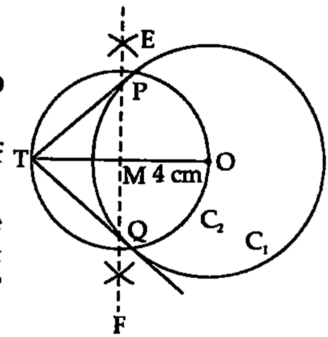
Steps of construction:

(i) Draw a line-segment OT = 6 cm.

(ii) Draw a circle of radius 4 cm taking O as centre.

(iii) Draw perpendicular bisector EF of OT which meets OT at M.

(iv) Taking MT as radius and M as centre draw a circle C_2 which intersect C_1 at P and Q. Join TP and TQ. Then, TP and TQ are the required tangents.



EXERCISE 10.4

Q1. Two line-segments AB and AC include an angle of 60°, where AB = 5 cm and AC = 7 cm. Locate points P and Q on AB and AC respectively such that $AP = \frac{3}{4}AB$ and $AQ = \frac{1}{4}AC$. Join P and Q and measure the length PQ.

Sol. (i) Draw ∠BAC = 60° such that AB = 5 cm and AC = 7 cm.

(ii) Draw acute angle CAX and mark X_1, X_2, X_3 and X_4 equally spaced.

(iii) Join X_4C.

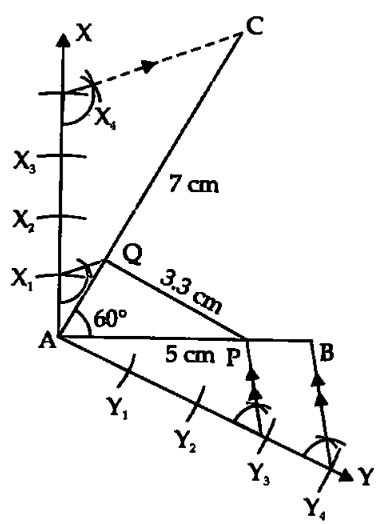
(iv) Draw $X_1Q \parallel X_4C$.

(v) Similarly, draw ∠BAY and divide AY in 4 equal parts *i.e.*, Y_1, Y_2, Y_3 and Y_4.

(vi) Join Y_4B and draw $Y_3P \parallel Y_4B$.

(vii) Join PQ and measure it.

(viii) PQ is equal to 3.3 cm.



Q2. Draw a parallelogram ABCD in which BC = 5 cm, AB = 3 cm and ∠ABC = 60°. Divide it into triangles BCD and ΔABD, by diagonal BD. Construct the triangle BD′C′ similar to ΔBDC with scale factor $\frac{4}{3}$. Draw the line segment D′A′ parallel to DA, where A′ lies on extended side BA. Is A′BC′D′ a parallelogram?

Sol. Steps of construction:

(i) Draw a line segment AB = 3 cm.

(ii) Make ∠ABC = 60° such that BC = 5 cm.

(iii) Draw CD ∥ AB and AD ∥ BC, ▭ABCD is the required parallelogram.

(iv) Join diagonal BD and produce it.

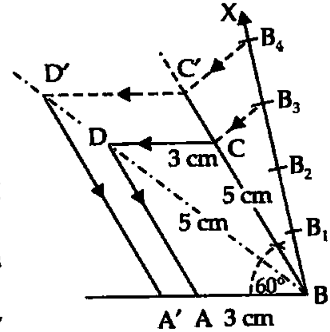
(v) Make acute angle CBX on opposite of D with respect to BC.

(vi) Mark (equi spaced) B_1, B_2, B_3, B_4 by compass.

(vii) Join B_3C and draw $B_3C \parallel B_4C'$ on BC produced.

(viii) Again, draw $C'D' \parallel CD$, where D′ is on BD produced.

(ix) Now, draw $D'A' \parallel DA$ where A′ is on BA produced. Parallelogram A′BC′D′ is similar to parallelogram ABCD with scale factor $\frac{4}{3}$.



Q3. Draw two concentric circles of radii 3 cm and 5 cm. Taking a point on outer circle construct the pair of tangents to the other. Measure the length of a tangent and verify it by actual calculation.

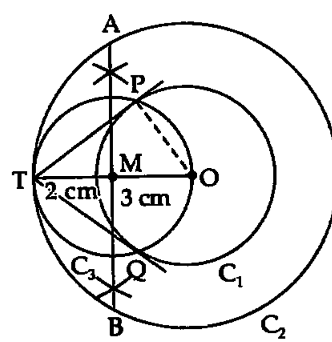
Sol. Steps of construction:

(i) Draw two concentric circles C_1, C_2 of radii 3 cm and 5 cm respectively taking 'O' as centre.

(ii) Draw perpendicular bisector AB of OT. T is any point on C_2.

(iii) Draw circle C_3 taking radius TM = OM and M as centre.

(iv) Circle C_3 intersect the circle C_1 at P and Q. Join TP and TQ. These are the required tangents. TP = TQ = 4.1 cm by measuring.



Mathematically length of tangent: Join OP. OP and TP are radius and tangent respectively at contact point P. So, $\angle TPO = 90°$.

By Pythagoras theorem in ΔTPO,

$$PT^2 = OT^2 - OP^2 = 5^2 - 3^2 = 25 - 9 = 16$$

$$\Rightarrow \quad PT = 4 \text{ cm}$$

Difference in measurement and by mathematical calculation

$$PT = 4.1 \text{ cm} - 4 \text{ cm} = 0.1 \text{ cm.}$$

Q4. Draw an isosceles ΔABC in which AB = AC = 6 cm and BC = 5 cm. Construct a triangle PQR similar to ΔABC in which PQ = 8 cm. Also justify the construction.

Sol. We have to draw

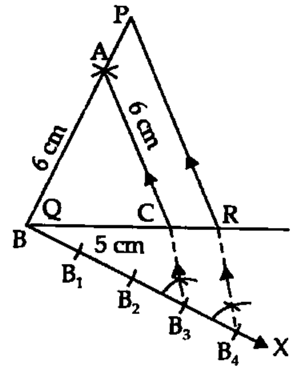
$\Delta PQR \sim \Delta ABC$

$PQ = 8$ cm

$\therefore \quad \frac{PQ}{AB} = \frac{8}{6} = \frac{4}{3} \quad (\because AB = 6 \text{ cm})$

So, $PQ = QR = 8$ cm

So, we have to draw $\Delta PQR \sim \Delta ABC$ with scale factor $\frac{4}{3} > 1$ resulting ΔPQR will be larger than ΔABC.



Steps of Construction:

(i) Draw $BC = 5$ cm

(ii) Draw two arcs of 6 cm each from B and C in same direction let it be upside.

(iii) Join AB and AC.

(iv) Draw acute $\angle CBX$ and mark B, B_1, B_2, B_3, B_4 with compass.

(v) Join B_3C and draw $B_4R \parallel B_3C$, R is on BC produced.

(vi) Again, draw $RP \parallel CA$. P is on BA produced.

Therefore, $\Delta PQR \sim \Delta ABC$ with $PQ = PR = 8$ cm. It's scale factor is $\frac{4}{3}$.

Q5. Draw a ΔABC in which $AB = 5$ cm, $BC = 6$ cm and $\angle ABC = 60°$. Construct a triangle similar to ΔABC with scale factor $\frac{5}{7}$. Justify the construction.

Sol. Scale factor $\frac{5}{7} < 1$, so the resulting Δ will be smaller than ΔABC.

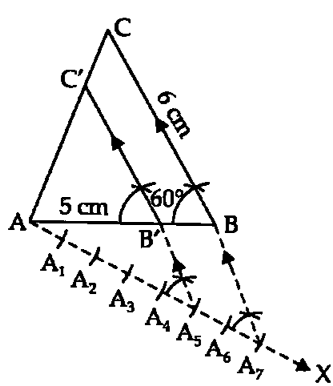
Steps of construction:

(i) Draw $AB = 5$ cm.

(ii) Draw $\angle ABC = 60°$, cut $BC = 6$ cm and join AC.

(iii) Draw acute $\angle BAX$ and mark it equispaced marks $A_1, A_2, ..., A_7$ as shown in figure.

(iv) Join A_7B and draw $A_5B' \parallel A_7B$. B' is on segment AB.



Draw $B'C' \parallel BC$, point C' is on AC.

$\Delta AB'C' \sim \Delta ABC$ with scale factor $\frac{5}{7}$.

Justification: In $\Delta AA_5B'$ and AA_7B,

$A_7B \parallel A_5B'$

$\therefore \quad \angle A_5 = \angle A_7$ [Corresponding $\angle$s]

$\angle BAA_5 = \angle BAA_7$ [Common]

$\therefore \quad \Delta AA_5B' \sim \Delta AA_7B$ [By AA criterion of similarity]

$\Rightarrow \quad \frac{AB'}{AB} = \frac{AA_5}{AA_7} = \frac{5x}{7x} = \frac{5}{7}$ ...(*i*)

where $x = AA_1 = A_1A_2 = ...A_6A_7$

Similarly, $\Delta AB'C' \sim \Delta ABC$ [By AA criterion of similarity]

$\Rightarrow \quad \frac{AB'}{AB} = \frac{AC'}{AC} = \frac{B'C'}{BC}$

$\Rightarrow \quad \frac{5}{7} = \frac{AC'}{AC} = \frac{B'C'}{BC}$

Hence, $\Delta AB'C' \sim \Delta ABC$ with scale factor $\frac{5}{7}$.

Q6. Draw a circle of radius 4 cm. Construct a pair of tangents to it, the angle between which is 60°. Also, justify the construction. Measure the distance between the centre of the circle and the point of intersection of tangents.

Sol. Angle between tangents is 60°. So angles between their radii is $180° - 60° = 120°$.

As the angles between tangents and their corresponding radii are supplementary.

Steps of construction:

(*i*) Draw a circle of radius 4 cm.

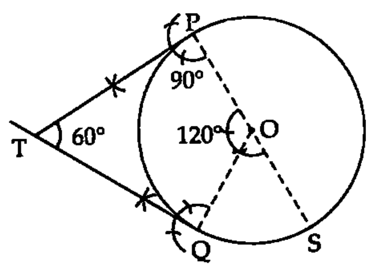
(*ii*) Draw any diameter POS.

(*iii*) Draw OQ making $\angle AOC = 120°$.

(*iv*) Draw tangent at P by drawing $\angle OPT = 90°$.

(*v*) Similarly, draw $\angle OQT$ equal to 90° to draw tangent.

(*vi*) Both PT, QT tangents intersect at T and make angle of 60°.



Hence, the two tangents on circle are TP and TQ inclined at 60°.

Justification: Because the radius OP and tangent PT at contact point makes angle $\angle TPO = 90°$.

Similarly, $\angle TQO = 90°$

In quadrilateral TPOQ,

$\angle T + \angle P + \angle O + \angle Q = 360°$

$\Rightarrow \angle T + 90° + 120° + 90° = 360°$ [$\because \angle O = 120°$ by construction]

$\Rightarrow \quad \angle T = 360° - 300°$

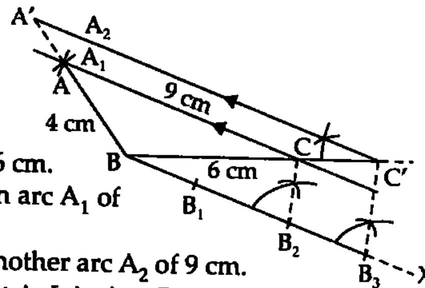
$\Rightarrow \quad \angle T = 60°$.

Hence, verified.

Q7. Draw a ΔABC in which AB = 4 cm, BC = 6 cm, and AC = 9 cm. Construct a triangle similar to ΔABC with scale factor $\frac{3}{2}$. Justify the construction. Are the two triangles congruent? Note that, all the three angles and two sides of the two triangles are equal.

Sol. Scale factor $\frac{3}{2} > 1$

So, the resulting figure will be greater than ΔABC.



Steps of construction:

(*i*) Draw line segment BC = 6 cm.

(*ii*) From B as centre, draw an arc A_1 of 6 cm.

(*iii*) From C as centre, draw another arc A_2 of 9 cm.

(*iv*) Arcs A_1 and A_2 intersect at A. Join A to B and C.

(*v*) Make an acute angle of $\angle CBX$ on other side of A.

(*vi*) Make the equispaced marks B_1, B_2, B_3 with compass.

(*vii*) Join B_2C and draw $B_3C' \parallel B_2C$, where C′ is on BC produced.

(*viii*) Draw $CA \parallel C'A'$, where A′ is on BA produced.

$\therefore \Delta A'BC' \sim \Delta ABC$ with scale factor $\frac{3}{2}$.

Justification: In $\Delta BB_3C'$ and ΔBB_2C

$\angle B = \angle B$ [Common]

$B_3C' \parallel B_2C$ [By construction]

$\therefore \angle BB_2C = \angle BB_3C'$ [Corresponding angles)

$\therefore \Delta BB_3C' \sim \Delta BB_2C$ [By AA criterion of similarity]

$\Rightarrow \frac{BC'}{CB} = \frac{BB_3}{BB_2} = \frac{3x}{2x} = \frac{3}{2}$ [$\because BB_1 = B_1B_2 = B_2B_3 = x$]

$\Rightarrow \frac{BC'}{BC} = \frac{3}{2}$

In ΔABC and $\Delta A'BC'$,

$\angle B = \angle B$ [Common]

$\because A'C' \parallel AC$

$\therefore \angle A'C'B = \angle ACB$ [Corresponding angles]

$\therefore \Delta ABC \sim \Delta A'BC'$ [By AA criterion of similarity]

$\Rightarrow \frac{A'C'}{AC} = \frac{A'B}{AB} = \frac{C'B}{BC}$

$\Rightarrow \frac{A'C'}{AC} = \frac{A'B}{AB} = \frac{3}{2}$

Hence, proved.

❑❑❑

11 Areas Related to Circles

EXERCISE 11.1

Choose the correct answer from the given four options:

Q1. If the sum of the areas of two circles with radii R_1 and R_2 in equal to the area of a circle of radius R, then

(*a*) $R_1 + R_2 = R$ (*b*) $R_1^2 + R_2^2 = R^2$

(*c*) $R_1 + R_2 < R$ (*d*) $R_1^2 + R_2^2 < R^2$

Sol. (*b*): Area of circle with radius R

= Area of circle with radius R_1 + Area of circle with radius R_2

$\Rightarrow \quad \pi R^2 = \pi R_1^2 + \pi R_2^2$

$\Rightarrow \quad \pi R^2 = \pi(R_1^2 + R_2^2)$

$\Rightarrow \quad R^2 = R_1^2 + R_2^2$

Hence, verifies the option (*b*).

Q2. If the sum of circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then

(*a*) $R_1 + R_2 = R$ (*b*) $R_1 + R_2 > R$

(*c*) $R_1 + R_2 < R$

(*d*) Nothing definite can be said about the relation among R_1, R_2 and R.

Sol. (*a*): According to the given condition,

$2\pi R = 2\pi R_1 + 2\pi R_2$

$\Rightarrow \quad 2\pi R = 2\pi(R_1 + R_2)$

$\Rightarrow \quad R = R_1 + R_2$

Hence, verifies the option (*a*).

Q3. If the circumference of a circle and the perimeter of a square are equal, then

(*a*) Area of circle = Area of the square

(*b*) Area of circle > Area of the square

(*c*) Area of circle < Area of the square

(*d*) Nothing definite can be said about the relation between the areas of the circle and square.

Sol. (*b*): According to the given condition,

Circumference of circle = Perimeter of square

$\Rightarrow \quad 2\pi r = 4a$ (where a = side of the square)

$\Rightarrow \quad \pi r = 2a$

$\Rightarrow \quad \frac{22}{7} r = 2a \Rightarrow r = \frac{7}{22} \times 2a$

$$\Rightarrow \quad r = \frac{7}{11}a$$

Now, area of circle = πr^2

$$= \frac{22}{7} \times \frac{7}{11}a \times \frac{7}{11}a = \frac{14}{11}a^2 = \frac{14}{11} \text{ area of square}$$

$\Rightarrow$ Area of circle = 1.2 area of square

$\therefore$ Area of circle > Area of the square

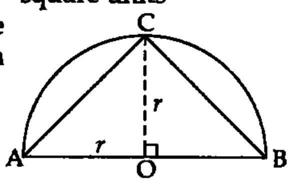
Hence, verifies the option (*b*).

Q4. Area of the largest triangle that can be inscribed in a semicircle of radius r units is

(*a*) r^2 square units (*b*) $\frac{1}{2}r^2$ square units

(*c*) $2r^2$ square units (*d*) $\sqrt{2}r^2$ square units

Sol. (*a*): Base AB of triangle ABC in semicircle is constant, i.e., equal to $2r$, and maximum altitude may be equal to r.



$$\therefore \quad \text{Area of triangle} = \frac{1}{2} \text{base} \times \text{altitude}$$

$$= \frac{1}{2} AB \times OC = \frac{1}{2}(2r) \times r = r^2$$

$\therefore$ Area of triangle in semicircle = r^2 square units.

Hence, verifies the option (*a*).

Q5. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

(*a*) 22 : 7 (*b*) 14 : 11 (*c*) 7 : 22 (*d*) 11 : 14

Sol. (*b*): Let r be the radius of a circle and side of a square is 'a', then according to the given condition,

$$2\pi r = 4a$$

$$\Rightarrow \quad 2 \times \frac{22}{7} r = 4a$$

$$\Rightarrow \quad r = \frac{4a \times 7}{2 \times 22} = \frac{7}{11}a$$

$$\therefore \quad \frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{a^2} = \frac{22}{7} \times \frac{7}{11} \times \frac{7}{11} \times \frac{a^2}{a^2}$$

$$\Rightarrow \quad \frac{\text{Area of circle}}{\text{Area of square}} = \frac{14}{11}$$

Hence, the required ratio is 14 : 11.

Verifies the option (*b*).

Q6. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be

(*a*) 10 m (*b*) 15 m (*c*) 20 m (*d*) 24 m

Sol. (*a*): Let the radius of the new park be R.

$\therefore$ Area of new park = Area of old park I + Area of park II

$$\Rightarrow \quad \pi R^2 = \pi r_1^2 + \pi r_2^2 \qquad \left[r_1 = \frac{16}{2} = 8 \text{ m}, \; r_2 = \frac{12}{2} = 6 \text{ m} \right]$$

$$\Rightarrow \quad \pi R^2 = \pi[r_1^2 + r_2^2]$$

$$\Rightarrow \quad \pi R^2 = \pi[8^2 + 6^2]$$

$$\Rightarrow \quad R^2 = 64 + 36$$

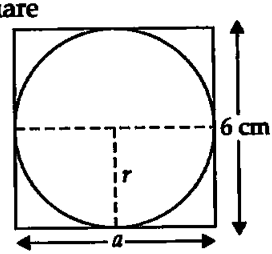
$$\Rightarrow \quad R = \sqrt{100} = 10 \text{ m}$$

Hence, verifies the option (*a*).

Q7. Area of the circle that can be inscribed in a square of side 6 cm is

(*a*) 36π cm^2 (*b*) 18π cm^2 (*c*) 12π cm^2 (*d*) 9π cm^2

Sol. (*d*): Diameter of the circle inscribed in a square = Side of square



$$\therefore \quad 2r = a$$

$$r = \frac{a}{2} = \frac{6}{2} = 3 \text{ cm}$$

$$\therefore \quad \text{Area of circle} = \pi r^2 = \pi(3)^2 = 9\pi$$

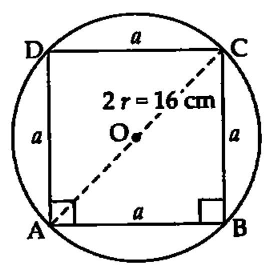
$$= 9\pi \text{ cm}^2$$

Hence, verifies the option (*d*).

Q8. The area of the square that can be inscribed in a circle of radius 8 cm is

(*a*) 256 cm^2 (*b*) 128 cm^2 (*c*) $64\sqrt{2}$ cm^2 (*d*) 64 cm^2

Sol. (*b*): Let the side of square be *a* cm.



Radius of the circle = 8 cm

$$\therefore \quad \text{Diameter} = 2r = 2 \times 8 = 16 \text{ cm}$$

By Pythagoras theorem in right ΔABC,

$$a^2 + a^2 = (16)^2$$

$$\Rightarrow \quad 2a^2 = 256$$

$$\Rightarrow \quad a^2 = \frac{256}{2} = 128$$

$$\text{Area of square} = a^2 = 128 \text{ cm}^2$$

Hence, verifies the option (*b*).

Q9. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is

(*a*) 56 cm (*b*) 42 cm (*c*) 28 cm (*d*) 16 cm

Sol. (*c*): According to the given condition,

Circumference of circle = Sum of circumferences of two circles $\left[r_1 = \frac{36}{2} = 18 \text{ cm}, \; r_2 = \frac{20}{2} = 10 \text{ cm} \right]$

$$\Rightarrow \quad 2\pi R = 2\pi r_1 + 2\pi r_2$$

$$\Rightarrow \quad 2\pi R = 2\pi(r_1 + r_2)$$

$$\Rightarrow \quad R = r_1 + r_2 = 18 + 10$$

$$\Rightarrow \quad R = 28 \text{ cm, which verifies the option } (c).$$

Q10. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is

(*a*) 31 cm (*b*) 25 cm (*c*) 62 cm (*d*) 50 cm

Sol. (*d*): According to the question,

$$\pi R^2 = \pi r_1^2 + \pi r_2^2 \qquad \begin{bmatrix} r_1 = 24 \text{ cm} \\ r_2 = 7 \text{ cm} \end{bmatrix}$$

$\Rightarrow$ $\pi(R^2) = \pi(r_1^2 + r_2^2)$

$\Rightarrow$ $R^2 = r_1^2 + r_2^2 = (24)^2 + (7)^2 = 576 + 49 = 625$

$\Rightarrow$ $R = \sqrt{625}$

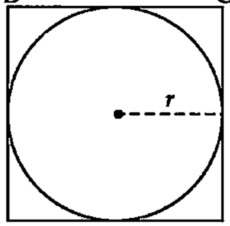
$\Rightarrow$ $R = 25$

$\therefore$ Diameter $= 2R = 2 \times 25 = 50$ cm.

Hence, verifies the option (*d*).

EXERCISE 11.2

Q1. Is the area of circle inscribed in a square of side *a* cm, πa^2 cm^2? Give reasons for your answer.



Sol. False: The radius of the circle inscribed in a square of side *a* cm is $r = \dfrac{a}{2}$

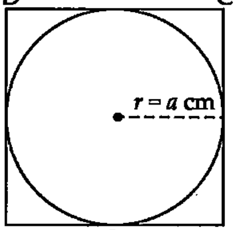
$\therefore$ Area of circle $= \pi r^2$

$$= \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4} \text{ cm}^2$$

$$\neq \pi a^2 \text{ cm}^2$$

Hence, the given statement is false.

Q2. Will it be true to say that the perimeter of the square circumscribing a circle of radius *a* cm is 8*a* cm? Give reasons for your answer.



Sol. True: Side of square = Diameter of circle

$\therefore$ $AB = 2a$

So, the perimeter of square

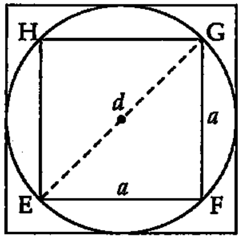
$= 4 \times AB$

$= 4 \times 2a$

$= 8a$ cm

Hence, the given statement is true.

Q3. In the given figure, a square is inscribed in a circle of diameter *d* and another square is circumscribing the circle. Is the area the outer square four times the area of inner square? Give reasons for your answer.



Sol. False: Let the side of the smaller square is *a* units and that of bigger square is *b* units.

Diameter of circle $= d$

So, diagonal of square EFGH = d

Then, by Pythagoras theorem,

$$a^2 + a^2 = d^2$$

$$\Rightarrow \quad 2a^2 = d^2$$

$$\Rightarrow \quad a^2 = \frac{d^2}{2}$$

$\therefore$ Area of small square = $a^2 = \dfrac{d^2}{2}$

Side of outer square = b = Diameter of circle

$\therefore$ Area of outer square = $b^2 = d^2$

$$= \frac{2}{2}d^2 = 2 \times \frac{1}{2}d^2$$

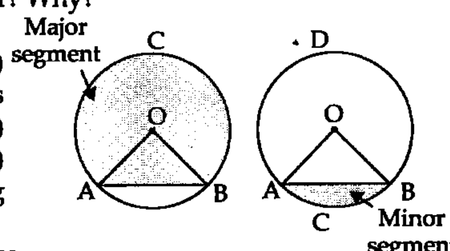
$\Rightarrow$ Area of larger square = 2 Area of smaller square

So, the given statement is false.

Q4. Is it true to say that area of a segment of a circle is less than the area of its corresponding sector? Why?

Sol. False:

Area of major segment (ACB) is always greater than its corresponding sector (OACB) and area of minor segment (ACB) is smaller than its corresponding minor sector (OACB).



Hence, the given statement is false.

Q5. Is it true that the distance travelled by circular wheel of diameter d cm in one revolution is $2\pi d$ cm? Why?

Sol. False: Distance travelled by wheel in one revolution is equal to the circumference of wheel = $2\pi r = \pi(2r) = \pi d$.

Hence, the given statement is false.

Q6. In covering a distance s metres, a circular wheel of radius r m makes $\dfrac{s}{2\pi r}$ revolutions. Is the statement true? Why?

Sol. True: Distance covered by a circular wheel in n revolutions = $2\pi r n$ where n = number of revolutions

$$\therefore \quad s = 2\pi r n \quad \text{or} \quad n = \frac{s}{2\pi r}.$$

Hence, verifies the given statement true.

Q7. The numerical values of the area of a circle is greater than the numerical value of its circumference. Is this statement true? Why?

Sol. False. Let the radius of circle is r $(0 < r < 2)$. Then, the area of circle $A = \pi r^2$ for $r = 1.5$, $A = \pi \times (1.5)^2$

$$\therefore \quad A = 2.25\pi$$

Circumference (C) = $2\pi r = 2 \times \pi \times 1.5$

$\Rightarrow \quad C = 3.0\pi$

$\therefore \quad C > A$

So, the area of a circle is not always greater than its circumference.
Hence, the given statement is false.

Q8. If the length of an arc of a circle of radius r is equal to that of an arc of a circle of radius $2r$, then the angle of the corresponding sector of the first circle is double the angle of the corresponding sector of other circle. Is this statement false? Why?

Sol. False: Consider two circles C_1 and C_2 of radii r and $2r$ respectively.
Let the lengths of two arcs be l_1 and l_2.

$$l_1 = \widehat{AB} \text{ of } C_1 = \frac{2\pi r\theta_1}{360°}$$

$$l_2 = \widehat{CD} \text{ of } C_2 = \frac{2\pi r'\theta_2}{360°} = \frac{2\pi . 2r\theta_2}{360°}$$

According to question,

$$l_1 = l_2$$

$$\Rightarrow \quad \frac{2\pi r\theta_1}{360°} = \frac{2\pi . 2r\theta_2}{360°}$$

$$\Rightarrow \quad \theta_1 = 2\theta_2$$

i.e., Angle of sector of the Ist circle is twice the angle of the sector of the other circle.
Hence, the given statement is true.

Q9. The areas of two sectors of two different circles with equal corresponding arc lengths are equal. Is this statement true? Why?

Sol. False. Consider two circles of radii, r_1, r_2 of arc length, l_1 and l_2, and their corresponding angles of sectors θ_1, θ_2 respectively.

$$l_1 = \frac{2\pi r_1\theta_1}{360°}, \quad l_2 = \frac{2\pi r_2\theta_2}{360°}$$

Now, $\quad l_1 = l_2 \quad$ [Given]

$$\Rightarrow \quad \frac{2\pi r_1\theta_1}{360°} = \frac{2\pi r_2\theta_2}{360°}$$

$$\Rightarrow \quad r_1\theta_1 = r_2\theta_2 = x \quad \text{[say]}$$

Area of sectors A_1 and A_2 are given by

$$A_1 = \frac{\pi r_1^2\theta_1}{360°}, \quad A_2 = \frac{\pi r_2^2\theta_2}{360°}$$

$$\therefore \quad \frac{A_1}{A_2} = \frac{\frac{\pi r_1\theta_1 r_1}{360°}}{\frac{\pi r_2\theta_2 r_2}{360°}} = \frac{xr_1}{xr_2} = \frac{r_1}{r_2}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{r_1}{r_2}$$

Area of sector can be equal when $\frac{r_1}{r_2} = 1$ i.e., equal radii. So, the areas of sectors of two circles of same arcs length are not equal.

Hence, the given statement is false.

Q10. The areas of two sectors of two different circles are equal. Is it necessary that their corresponding arc lengths are equal? Why?

Sol. False: Using usual abbreviations for sectors

$$A_1 = A_2 \quad \text{[Given]}$$

$$\Rightarrow \quad \frac{\pi r_1^2 \theta_1}{360°} = \frac{\pi r_2^2 \theta_2}{360°}$$

$$\Rightarrow \quad r_1^2 \theta_1 = r_2^2 \theta_2$$

$$\Rightarrow \quad \frac{\theta_1}{\theta_2} = \frac{r_2^2}{r_1^2}$$

Now,
$$\frac{l_1}{l_2} = \frac{2\pi r_1 \theta_1}{2\pi r_2 \theta_2} = \frac{r_1}{r_2} \times \frac{r_2^2}{r_1^2} = \frac{r_2}{r_1} \Rightarrow \frac{l_1}{l_2} = \frac{r_2}{r_1}$$

Hence, arcs length can be equal if $\frac{r_2}{r_1} = 1$ i.e., $r_1 = r_2 = r$.

Hence, the given statement is false.

Q11. Is the area of the largest circle that can be drawn inside a rectangle of length a cm and breadth b cm ($a > b$) is πb^2 cm^2? Why?

Sol. False: The diameter of circle that can be drawn inside the rectangle is equal to the breadth of rectangle.

The length of the rectangle = a cm

The breadth of the rectangle = b cm

$\therefore$ Diameter of circle = b cm

$$\Rightarrow \quad r = \frac{b}{2} \text{ cm}$$

$$\therefore \quad \text{Area of circle A} = \pi r^2 = \pi\left(\frac{b}{2}\right)^2 = \frac{1}{4}\pi b^2 \text{ cm}^2$$

Hence, the given statement is false.

Q12. Circumferences of two circles are equal. Is it necessary that their areas be equal? Why?

Sol. True. $\because$
$$2\pi r_1 = 2\pi r_2 \quad \text{[Given]}$$

$$\Rightarrow \quad r_1 = r_2 = r \quad \text{[say]}$$

Now,
$$\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{r^2}{r^2} = 1$$

$\therefore$ $A_1 = A_2$ *i.e.,* their areas are equal.

Hence, the given statement is true.

Q13. Areas of the two circles are equal. Is it necessary that their circumferences are equal? Why?

Sol. True: $\because$ $A_1 = A_2$ [Given]

$\Rightarrow$ $\pi r_1^2 = \pi r_2^2$

$\Rightarrow$ $r_1^2 = r_2^2$

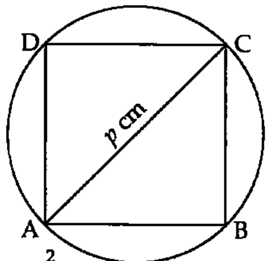
$\Rightarrow$ $r_1 = r_2 = r$ (say) [taking square root]

Now, $\frac{C_1}{C_2} = \frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{r}{r} = \frac{1}{1}$

Hence, $C_1 = C_2$

Hence, the given statement is true.

Q14. Is it true to say that area of a square inscribed in a circle of diameter p cm is p^2 cm^2? Why?



Sol. False: The diameter of the circle is p cm.

So, the diagonal of the square will be p cm.

Now, $AB^2 + BC^2 = AC^2$

$\Rightarrow$ $AB^2 + AB^2 = p^2$

$\Rightarrow$ $2AB^2 = p^2 \Rightarrow AB = \frac{p}{\sqrt{2}}$

Now, Area of square $= \frac{p}{\sqrt{2}} \times \frac{p}{\sqrt{2}} = \frac{p^2}{2}$ cm^2

Hence, the given statement is false.

EXERCISE 11.3

Q1. Find the radius of a circle whose circumference is equal to the sum of the circumferences of two circles of radii 15 cm and 18 cm.

Sol.

Circle I (C_1)	Circle II (C_2)	Circle III (C)
$r_1 = 15$ cm	$r_2 = 18$ cm	$r = ?$

According to the question,

$$C_1 + C_2 = C$$

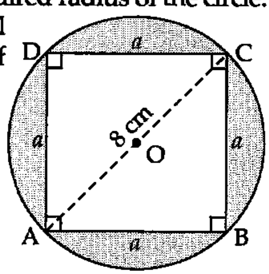
$\Rightarrow$ $2\pi r_1 + 2\pi r_2 = 2\pi r$

$\Rightarrow$ $2\pi [r_1 + r_2] = 2\pi r$

$\Rightarrow$ $r_1 + r_2 = r \Rightarrow 15 + 18 = r$

$\Rightarrow$ $r = 33$ cm is the required radius of the circle.

Q2. In the given figure, a square of diagonal 8 cm is inscribed in a circle. Find the area of shaded region.



Sol. Let the side of the square be a cm.

So, radius of the circle, $r = OA = \frac{AC}{2}$

$\Rightarrow$ $r = \frac{8}{2} = 4$ cm

So, in right angled ΔABC,

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow \quad a^2 + a^2 = 8^2$$

$$\Rightarrow \quad 2a^2 = 64 \Rightarrow a^2 = \frac{64}{2} \Rightarrow a^2 = 32$$

Area of shaded part = Area of circle – Area of square

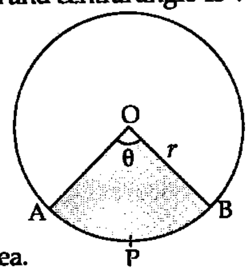
$$= \pi r^2 - a^2 = \frac{22}{7} \times 4 \times 4 - 32$$

$$= 16\left[\frac{22}{7} - \frac{2}{1}\right] = 16\left[\frac{22-14}{7}\right] = \frac{16 \times 8}{7} = \frac{128}{7}$$

Area of shaded region = 18 – cm

Q3. Find the area of a sector of a circle of radius 28 cm and central angle 45°.

Sol. Sector (OAPBO) of a circle is given whose



radius (r) = 28 cm

and central angle(θ) = 45°

$$\therefore \quad \text{Area of sector (A)} = \frac{\pi r^2 \theta}{360°}$$

$$= \frac{22}{7} \times \frac{28 \times 28 \times 45°}{360°}$$

$\Rightarrow$ Area of sector = 308 cm^2 is the required area.

Q4. The wheel of a motor cycle is of radius 35 cm. How many revolution per minute must the wheel make so as to keep a speed of 66 km/hr?

Sol. Speed of the wheel (v) = 66 km/hr

$$v = 66 \times \frac{5}{18} = \frac{55}{3} \text{ m/s}$$

$$r = 35 \text{ cm} = 0.35 \text{ m}$$

$$n = ?$$

$$t = 1 \text{ min} = 60 \text{ sec}$$

Now,
$$v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r n}{t} \Rightarrow v = \frac{2\pi r n}{t}$$

$$\Rightarrow \quad n = \frac{v.t}{2\pi r} = \frac{\frac{55}{3} \times 60}{2 \times \frac{22}{7} \times 0.35} = \frac{55 \times 60 \times 7 \times 100}{3 \times 2 \times 22 \times 35}$$

$\Rightarrow$ n = 500 revolutions per min.

Q5. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20 m × 16 m. Find the area of the field in which the cow can graze.

Sol. Field is rectangular. So if cow is tied at its vertex, it will graze the field in the shape of sector. The length of rope is less than length and breadth of rectangle. So, the required area is of sector.

Area of field grazed by cow

$= \text{Area of sector}$

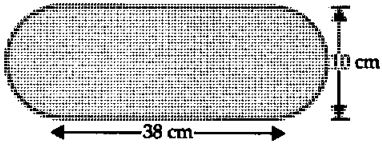
$\Rightarrow \qquad A = \dfrac{\pi r^2\theta}{360°}$

where $\qquad r = 14\text{ m}, \quad \theta = 90°$

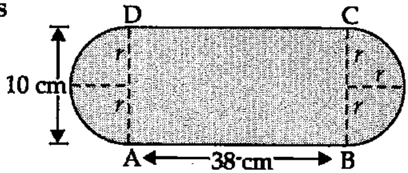
$\therefore \qquad A = \dfrac{22}{7} \times \dfrac{14 \times 14 \times 90°}{360°} = 11 \times 14 \Rightarrow A = 154\text{ m}^2.$

So, the required area grazed by cow is 154 m^2.

Q6. Find the area of flower bed (with semi-circular ends) shown in figure.



Sol. The figure has two semi-circles and one rectangle.



The radius of the semi-circles $= \dfrac{10}{2}$ cm

$\Rightarrow \qquad r = 5\text{ cm}$

The length of the rectangle

$= \qquad l = 38\text{ cm}$

$\text{Breadth} = b = 10\text{ cm}$

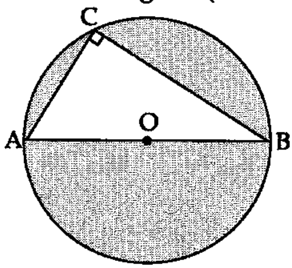
Area of the flower bed = Area of rectangle + Area of two semi-circles

$= l \times b + 2 \times \dfrac{\pi r^2}{2} = l \times b + \pi r^2$

$= 38 \times 10 + \dfrac{22}{7} \times 5 \times 5 = (380 + 25\pi)\text{ cm}^2$

Hence, the area of flower bed is $(380 + 25\pi)\text{ cm}^2$.

Q7. In the given figure, AB is diameter of circle, AC = 6 cm and BC = 8 cm. Find the area of the shaded region. ($\pi = 3.14$).



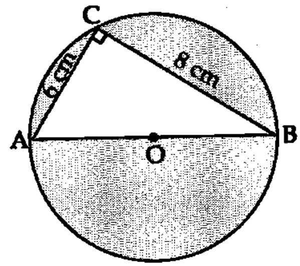
Sol. Identify the figure as a circle, and a right angled triangle (and semicircle, segments also) because AOB is diameter and angle in semicircle is 90°.

So, $\qquad \angle C = 90°.$

In right angled ΔABC,

$b = \text{base} = BC = 8\text{ cm}$

a = altitude = AC = 6 cm



By Pythagoras theorem in right ΔABC,

$$AB^2 = BC^2 + AC^2$$
$$= 8^2 + 6^2 = 64 + 36$$
$$\Rightarrow \quad AB^2 = 100 \text{ cm}^2$$
$$\Rightarrow \quad AB = 10 \text{ cm}$$

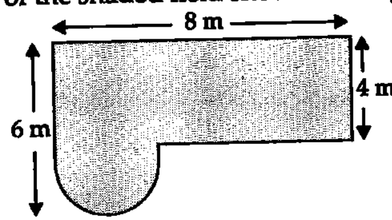
Hence, $r = \dfrac{10}{2} = 5$ cm

$\therefore$ Area of shaded region = Area of circle − Area of right ΔABC

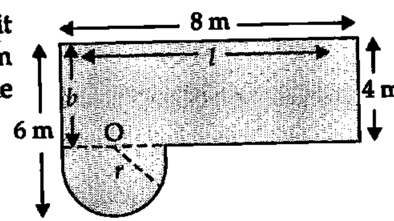
$$= \pi r^2 - \frac{1}{2}\text{Base} \times \text{Alt.}$$
$$= 3.14 \times 5 \times 5 - \frac{1}{2} \times 8 \times 6$$
$$= 3.14 \times 25 - 8 \times 3 = (78.50 - 24) \text{ cm}^2 = 54.50 \text{ cm}^2$$

$\therefore$ Area of shaded region = 54.50 cm^2

Q8. Find the area of the shaded field shown in the given figure.



Sol. Redraw the figure and divide it into well known shapes. It is clear from the figure that there is one semicircle and one rectangle.



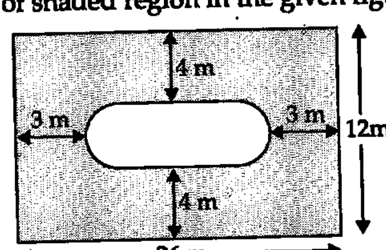
Rectangle	Circle
$l = 8$ m	$r = 6 - 4 = 2$ m
$b = 4$ m	

Area of shaded region = Area of rectangle + Area of semicircle

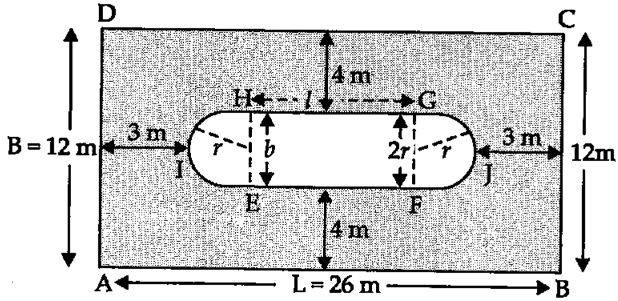
$$= l \times b + \frac{\pi r^2}{2} = 8 \times 4 + \pi \times \frac{2 \times 2}{2} = (32 + 2\pi) \text{ m}^2$$

Hence, the required area of shaded region = $(32 + 2\pi)$ m^2.

Q9. Find the area of shaded region in the given figure.



Sol. Redraw the given figure and identify the shapes (well known) from figure.



There are two rectangles ABCD and EFGH and two semicircles EHI and GFJ. For dimensions of shapes,

Rectangle ABCD	Rectangle EFGH	Semicircles HEI and GFJ
$L = 26$ m	$l = 26 - 3 - 3 - 2r$	$r = \frac{b}{2}$
$B = 12$ m	$\Rightarrow \quad l = 26 - 6 - 2r$	$\Rightarrow r = \frac{4}{2}$ m
	$\Rightarrow \quad l = 20 - 2r$	$\Rightarrow r = 2$ m
	$\Rightarrow \quad l = 20 - 2r = 20 - 2 \times 2 = 20 - 4$	
	$\Rightarrow \quad l = 16$ m	
	$b = 12 - 4 - 4$	
	$\Rightarrow \quad b = 12 - 8 = 4$ m	

Area of required shaded region

= Area of rectangle ABCD – [Area of 2 semicircles + Area of rectangle EFGH ...(*i*)

$$\Rightarrow \text{ Area of shaded region } = L \times B - \left[2.\frac{\pi r^2}{2} + l \times b\right]$$

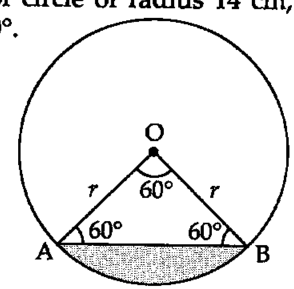
$$= 26 \times 12 - [\pi r^2 + l \times b]$$

$$= 26 \times 12 - [\pi \times 2 \times 2 + 16 \times 4] = 312 - 4\pi - 64 = (248 - 4\pi)\ \text{m}^2$$

Hence, the area of shaded region = $(248 - 4\pi)$ m^2.

Q10. Find the area of the minor segment of circle of radius 14 cm, when the angle of corresponding sector is 60°.

Sol. Shaded region is a minor segment.



In ΔOAB,

$\theta = 60°, \quad OA = OB = r = 14$ cm

$\therefore$ Area of minor segment

= Area of sector – Area of ΔOAB

In ΔOAB,

$OA = OB$ [Radii of same circle]

Let $\angle A = \angle B = x°$ [(∵ Angles opposite to equal sides are equal]

$$\angle O + \angle A + \angle B = 180° \quad \text{[Angle sum property of a triangle]}$$

$$\Rightarrow \quad 60° + x + x = 180°$$

$$\Rightarrow \quad 2x = 180° - 60°$$

$$\Rightarrow \quad x = \frac{120°}{2} \Rightarrow x = 60°$$

So, ΔOAB is an equilateral Δ with side 14 cm.

$$\text{Area of minor segment} = \frac{\pi r^2 \theta}{360°} - \text{Area of } \Delta OAB$$

$$\text{So, Area of minor segment} = \frac{\pi r^2 \theta}{360} - \frac{\sqrt{3}}{4} r^2$$

$$= \frac{22 \times 14 \times 14 \times 60°}{7 \times 360°} - \frac{\sqrt{3}}{4} \times 14 \times 14$$

$$= \frac{22 \times 14}{3} - 49\sqrt{3} = \left(\frac{308}{3} - 49\sqrt{3}\right) \text{cm}^2$$

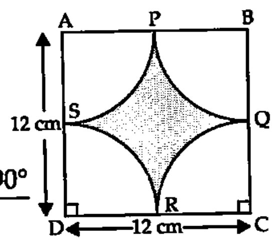
$$= (102.666 - 84.870)\ \text{cm}^2 = 17.796\ \text{cm}^2$$

Q11. Find the area of the shaded region in figure, where arcs drawn with centres A, B, C and D intersect in pairs at mid points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square ABCD. (Use $\pi = 3.14$)

Sol. From figure,

Area of shaded part = Area of square ABCD − Area of 4 sectors

Side of the square ABCD = 12 cm, $\theta = 90°$



Radii of the sectors $r = \frac{12}{2} = 6$ cm.

∴ Area of shaded region

= Area of square ABCD − Area of 4 sectors

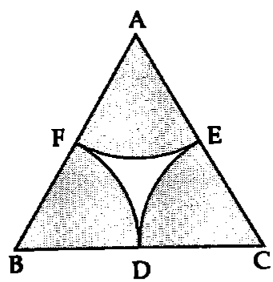
$$= (12)^2 - 4.\frac{\pi r^2 \theta}{360} = 12 \times 12 - \frac{4 \times 3.14 \times 6 \times 6 \times 90°}{360°}$$

$$= 6 \times 6\ [2 \times 2 - 3.14 \times 1 \times 1] = 36\ [4 - 3.14]$$

⇒ Area of shaded region = $36 \times 0.86\ \text{cm}^2 = 30.96\ \text{cm}^2$

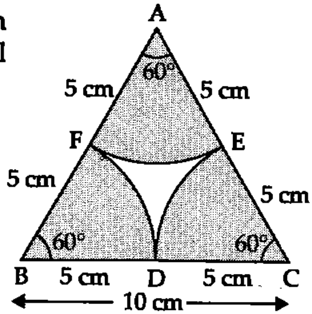
Hence, the required area is $30.96\ \text{cm}^2$.

Q12. In the given figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm to intersect the sides BC, CA and AB at their respective mid points D, E and F respectively. Find the are of the shaded region. (Use $\pi = 3.14$)



Sol. From the given figure, area of the shaded part is equal to the sum of areas of three sectors at points A, B and C.

As ΔABC is equilateral triangle of side 10 cm and radius of the sector is half of the side. All the three sectors are identical.



$$\theta = 60°$$

Radius of each sector $(r) = \frac{10}{2} = 5$ cm

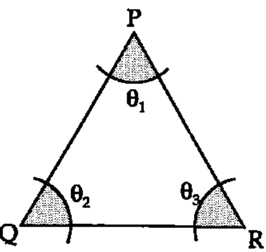
$\therefore$ Area of shaded part = 3.(Area of sector)

$$= \frac{3 \times \pi r^2 \theta}{360°} = \frac{3 \times 3.14 \times 5 \times 5 \times 60°}{360°}$$

$= 1.57 \times 25 = 39.25 \text{ cm}^2$

Hence, the required area is 39.25 cm^2.

Q13. In the given figure, arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.



Sol. The area of the shaded region is equal to the sum of areas of three sectors of same radius but of different angles θ_1, θ_2 and θ_3.

$$\angle\theta_1 + \angle\theta_2 + \angle\theta_3 = 180° \quad [\text{Int. } \angle\text{s of } \Delta]$$

$\therefore$ Area of shaded region $= \frac{\pi r_1^2 \theta_1}{360°} + \frac{\pi r_2^2 \theta_2}{360°} + \frac{\pi r_3^2 \theta_3}{360°}$

where $r_1 = r_2 = r_3 = r = 14$ cm

$$= \frac{\pi r^2}{360°}(\theta_1 + \theta_2 + \theta_3)$$

$$= \frac{22}{7} \times \frac{14 \times 14}{360°} \times 180° = 22 \times 14 = 308 \text{ cm}^2$$

$\therefore$ Area of shaded region $= 308 \text{ cm}^2$

Hence, required area $= 308 \text{ cm}^2$.

Q14. A circular park is surrounded by a road 21 m wide. If the radius of the park is 105 m, then find the area of the road.

Sol. Circular road and park are concentric circles.

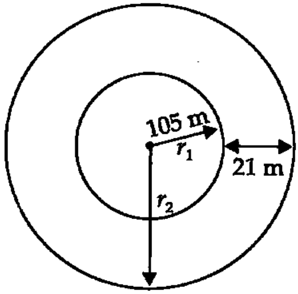
Radius of the park $= r_1 = 105$ m

Width of road = 21 m

Radius of circular road and park $= r_2$

$= 105 \text{ m} + 21 \text{ m} = 126$ m

So, Area of road = Area of park and road − Area of park



$$= \pi r_2^2 - \pi r_1^2$$

$$= \pi[r_2^2 - r_1^2]$$

$$= \frac{22}{7}[(126)^2 - (105)^2]$$

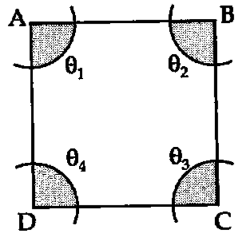
$$= \frac{22}{7}[126 - 105][126 + 105]$$

$$= \frac{22}{7} \times 21 \times 231$$

$$= 22 \times 3 \times 231 = 66 \times 231 = 15246 \text{ m}^2$$

$\therefore$ Area of road = 15246 m^2

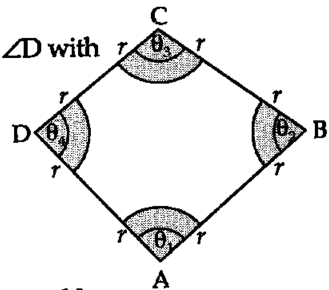
Q15. In the given figure, arcs have been drawn of radius 21 cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of shaded region.



Sol. Specification of quadrilateral are not given, so quadrilateral may be of any shape.

As the radius of all 4 arcs are same equal to $r = 21$ cm but of different angles $\angle A$, $\angle B$, $\angle C$ and $\angle D$.

So, there are four sectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ with $r = 21$ cm.



$\therefore$ Area of shaded region

$$= \frac{\pi r_1^2(\theta_1)}{360°} + \frac{\pi r_2^2(\theta_2)}{360°} + \frac{\pi r_3^2(\theta_3)}{360°} + \frac{\pi r_4^2(\theta_4)}{360°}$$

$\because r_1 = r_2 = r_3 = r_4 = r$ and

$$\angle\theta_1 + \angle\theta_2 + \angle\theta_3 + \angle\theta_4 = 360°$$

[Interior $\angle$s of a quad.]

So, Area of shaded region $= \frac{\pi r^2(\theta_1)}{360°} + \frac{\pi r^2(\theta_2)}{360°} + \frac{\pi r^2(\theta_3)}{360°} + \frac{\pi r^2(\theta_4)}{360°}$

$$= \frac{\pi r^2}{360°}(\theta_1 + \theta_2 + \theta_3 + \theta_4)$$

$$= \frac{\pi r^2}{360°}(360°)$$

$$= \pi r^2 = \frac{22}{7} \times 21 \times 21 \quad = 22 \times 63 = 1386 \text{ cm}^2$$

Hence, the area of shaded region = 1386 cm^2.

Q16. A piece of wire 20 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle.

Sol. Arc is a part of circle that makes 60° between radii at end points A and B of wire.

So, it forms the shape of a sector.

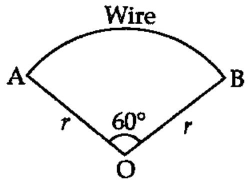
$r = ?$ $\quad l = 20$ cm

$\therefore$ Length of arc $l = \frac{2\pi r\theta}{360°}$

$$\Rightarrow \quad 20 \text{ cm} = \frac{2 \times \pi \times r \times 60^\circ}{360^\circ}$$

$$\Rightarrow \quad 2\pi r = 20 \times 6$$

$$\Rightarrow \quad r = \frac{120}{2\pi} = \frac{60}{\pi} \text{ cm}$$



Hence, radius $(r) = \frac{60}{\pi}$ cm.

EXERCISE 11.4

Q1. The area of a circular playground is 22176 m^2. Find the cost of fencing this ground at the rate of ₹ 50 per m.

Sol. Fencing is made on circumference ($2\pi r$) of circular field. So, we require radius for it.

Area of the circular playground = 22176 m^2

$$\Rightarrow \quad \pi r^2 = 22176$$

$$\Rightarrow \quad \frac{22}{7} r^2 = 22176$$

$$\Rightarrow \quad r^2 = \frac{7 \times 22176}{22} \Rightarrow r^2 = \sqrt{7 \times 1008}$$

$$\Rightarrow \quad r = \sqrt{7 \times 7 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2}$$

$$\Rightarrow \quad r = 7 \times 3 \times 2 \times 2$$

$$\Rightarrow \quad r = 84 \text{ m}$$

$\therefore$ Length of fencing = Circumference of circle

$$= 2\pi r = 2 \times \frac{22}{7} \times 84 = 24 \times 22 \text{ m}$$

So, Cost of fencing = ₹ $50 \times 24 \times 22$ = ₹ 26400

Hence, cost of fencing = ₹ 26400.

Q2. The diameters of front and rear wheels of a tractor are 80 cm and 2 m, respectively. Find the number of revolutions that rear wheel will make in covering a distance in which the front wheel makes 1400 revolutions.

Sol. $\begin{bmatrix}\text{Distance travelled by} \\ \text{rear wheel}\end{bmatrix} = \begin{bmatrix}\text{Distance travelled by} \\ \text{front wheel}\end{bmatrix}$

$$\Rightarrow \quad 2\pi r_1 n_1 = 2\pi r_2 n_2$$

$$\Rightarrow \quad r_1 n_1 = r_2 n_2 \qquad \text{(I)}$$

Front wheel	**Rear wheel**
$r_2 = 80 \text{ cm} = 0.8 \text{ m}$	$r_1 = 2 \text{ m}$
$n_2 = 1400$ revolutions	$n_1 = ?$

$\therefore$ From (I), we get

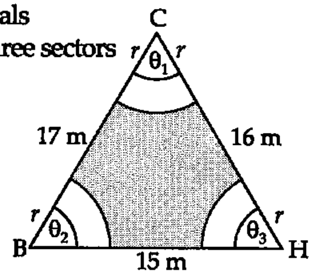
$$2 \times n_1 = 0.8 \times 1400$$

$$\Rightarrow \quad n_1 = \frac{0.8 \times 1400}{2} = 0.4 \times 1400 \Rightarrow n_1 = 560$$

Hence, the number of revolutions made by rear wheel = 560.

Q3. Sides of a triangular field are 15 m, 16 m and 17 m. With the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7 m each to graze in the field.
Find the area of the field which cannot be grazed by three animals.

Sol. Since with the three corners of the field a cow, a buffalo and a horse and tied separately with ropes of length 7 m each to graze in the field.



Area of field which cannot be grazed by animals

$$= \text{Area of } \Delta BCH - \text{Area of three sectors}$$

Here, $a = 15$ m, $b = 16$ m, $c = 17$ m

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{15+16+17}{2}$$

$$\Rightarrow \quad s = \frac{48}{2} = 24 \text{ m}$$

$$\text{Area of } \Delta BCH = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{24(24-15)(24-16)(24-17)}$$

$$= \sqrt{24 \times 9 \times 8 \times 7}$$

$$= \sqrt{\underline{2\times2}\times\underline{2}\times3\times\underline{3\times3}\times\underline{2}\times\underline{2\times2}\times7}$$

$$\Rightarrow \quad ar(\Delta BCH) = 24\sqrt{21} \text{ m}^2$$

$$\text{Area of 3 sectors} = \frac{\pi r^2\theta_1}{360°} + \frac{\pi r^2\theta_2}{360°} + \frac{\pi r^2\theta_3}{360°}$$

$$= \frac{\pi r^2}{360°}[\theta_1 + \theta_2 + \theta_3]$$

$$= \frac{22}{7} \times \frac{7\times7}{360°} \times 180° \qquad (\because \theta_1 + \theta_2 + \theta_3 = 180°)$$

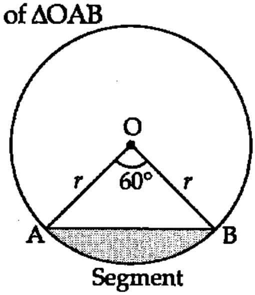
$$= 77 \text{ m}^2$$

$\therefore$ Area of 3 sectors grazed by animals $= 77$ m^2.

Hence, the area which cannot be grazed by 3 animals is equal to $\left(24\sqrt{21} - 77\right)$ m^2.

Q4. Find the area of the segment of a circle of radius 12 cm whose corresponding sector has central angle 60°. (Use $\pi = 3.14$).

Sol. Area of minor segment = Area of sector – Area of ΔOAB



In ΔOAB,

$$\theta = 60°$$

$$OA = OB = r = 12 \text{ cm}$$

$$\angle B = \angle A = x$$

[$\angle$s opp. to equal sides are equal]

$$\Rightarrow \quad \angle A + \angle B + \angle O = 180°$$

$$\Rightarrow \quad x + x + 60° = 180°$$

$$\Rightarrow \quad 2x = 180° - 60°$$

$$\Rightarrow \qquad x = \frac{120°}{2} = 60°$$

$\therefore$ ΔOAB is equilateral Δ with each side $(a) = 12$ cm

$$\text{Area of the equilateral } \Delta = \frac{\sqrt{3}}{4}a^2$$

$$\text{Area of minor segment} = \text{Area of the sector} - \text{Area of } \Delta OAB$$

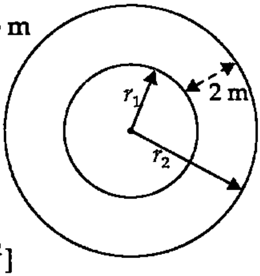
$$= \frac{\pi r^2 \theta}{360°} - \frac{\sqrt{3}}{4}a^2$$

$$= \frac{3.14 \times 12 \times 12 \times 60°}{360°} - \frac{\sqrt{3}}{4} \times 12 \times 12$$

$$= 6.28 \times 12 - 36\sqrt{3}$$

$\therefore$ Area of minor segment $= \left(75.36 - 36\sqrt{3}\right)$ cm^2.

Q5. A circular pond is 17.5 m in diameter. It is surrounded by a 2 m wide path. Find the cost of constructing the path at the rate of ₹ 25 per m^2.



Sol. Radius of the circular pond $r_1 = \frac{17.5}{2}$ m $= 8.75$ m

Width of path = 2 m

$\therefore$ Radius of the path including pond

$$r_2 = 8.75 + 2 = 10.75 \text{ m}$$

$$\text{Area of path} = \pi r_2^2 - \pi r_1^2 = \pi[r_2^2 - r_1^2]$$

Cost of constructing the path = ₹ 25 $\pi(r_2^2 - r_1^2)$

$$\therefore \quad \text{Required cost} = ₹\,25 \times \frac{22}{7}[(10.75)^2 - (8.75)^2]$$

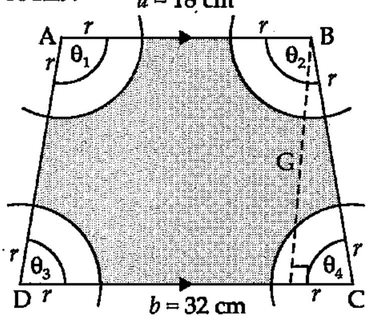
$$= 25 \times \frac{22}{7}[10.75 - 8.75][10.75 + 8.75]$$

$$= 25 \times \frac{22}{7} \times 2 \times 19.5 = \frac{50 \times 22 \times 19.5}{7}$$

$$= \frac{1100 \times 19.5}{7} = \frac{21450}{7} = ₹\,3064.29$$

Hence, the cost of constructing path is ₹ 3064.29.

Q6. In the given figure, ABCD is a trapezium with AB || CD. AB = 18 cm, DC = 32 cm and distance between AB and DC is 14 cm. If arcs of equal radii 7 cm with centres A, B, C and D have been drawn, then find the area of the shaded region of the figure.



Sol. In the given figure, there are 4 sectors and one trapezium.

4 sectors — $r = 7$ cm

Trapezium — $a = 18$ cm, $b = 32$ cm, $h = 14$ cm

Area of shaded part = Area of trapezium – Area of 4 sectors

$$= \frac{(a+b)h}{2} - \left[\frac{\pi r^2\theta_1}{360°} + \frac{\pi r^2\theta_2}{360°} + \frac{\pi r^2\theta_3}{360°} + \frac{\pi r^2\theta_4}{360°}\right]$$

$$= \frac{(18+32)\times 14}{2} - \frac{\pi r^2}{360°}(\theta_1 + \theta_2 + \theta_3 + \theta_4)$$

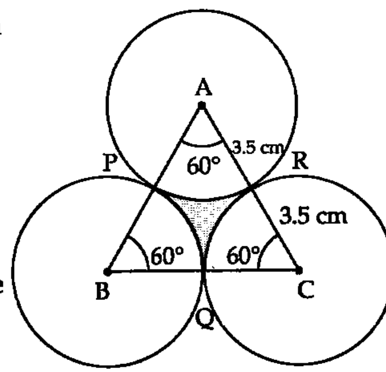
$$= \frac{50\times 14}{2} - \frac{22}{7}\times\frac{7\times 7}{360°}\times 360°$$

$\Rightarrow$ Area of shaded region $= 350 - 22 \times 7 = 350 - 154 = 196$ cm^2

Hence, the area of shaded region is 196 cm^2.

Q7. Three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between these circles.

Sol. Area of equilateral triangle with side 7 cm



$$= \frac{\sqrt{3}}{4}\times (7)^2 \text{ cm}^2$$

$$= \left(\frac{\sqrt{3}\times 49}{4}\right)\text{cm}^2$$

$$= 21.2176 \text{ cm}^2$$

Area of one sector with central angle 60° and radius 3.5 cm

$$= \frac{60°}{360°}\times \pi \times (3.5)^2$$

$$= \frac{\pi}{6}(12.25) \text{ cm}^2$$

$\therefore$ Area of three such sectors $= 3 \times \frac{\pi}{6}(12.25)$ cm^2

$$= \frac{\pi}{2}(12.25) \text{ cm}^2$$

$$= \frac{22}{14}(12.25) \text{ cm}^2 = 19.25 \text{ cm}^2$$

Now, area enclosed between these circles = Area of Δ – Area of three sectors

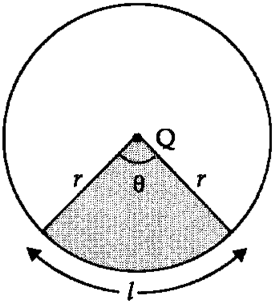
$$= 21.2176 \text{ cm}^2 - 19.25 \text{ cm}^2$$

$$= 1.9676 \text{ cm}^2$$

Q8. Find the area of sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm.

Sol. Here, $l = 3.5$ cm

$r = 5$ cm



Length of arc $l = \frac{2\pi r\theta}{360°}$

$\Rightarrow \quad 3.5 = \frac{2 \times \pi \times 5 \times \theta}{360°}$

$\Rightarrow \quad \frac{\pi\theta}{36} = 3.5$

$\Rightarrow \quad \theta = \frac{3.5 \times 36}{\pi}$

Now, Area of sector $= \frac{\pi r^2\theta}{360°} = \frac{\pi \times 5 \times 5 \times 35 \times 36}{360° \times \pi \times 10}$

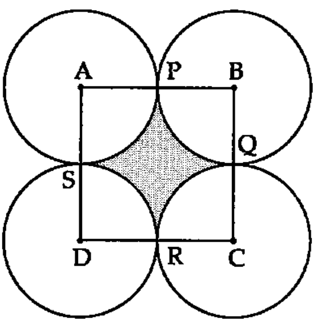
$= \frac{25 \times 35}{100} = \frac{875}{100} = 8.75 \text{ cm}^2$

$\therefore$ Area of sector $= 8.75 \text{ cm}^2$

Q9. Four circular cardboard pieces of radii 7 cm are placed on a paper in such a way that each piece touches other two pieces. Find the area of the portion enclosed between these pieces.

Sol. As we know that point of contact of two circles lies on the line joining their centres.

So, the line segments AB, BC, CD and AD will pass through the corresponding point of contact P, Q, R, S respectively.



As SD and AS are radius at contact point S. So, AD will be perpendicular to the tangent through S. It implies that interior angles of quadrilateral are 90° each.

As radius of each circle is equal.

So quadrilateral ABCD will be square with side $2r = 2 \times 7 = 14$ cm.

In the given figure, there is a square and 4 sectors.

4 sectors	**square**
$r = 7$ cm	$a = 2r = 2 \times 7 = 14$
$\theta = 90°$	$\Rightarrow a = 14$ cm

Area enclosed between circles = Area of square – Area of 4 sectors

$= a^2 - 4 \cdot \frac{\pi r^2\theta}{360°}$

$= 14 \times 14 - \frac{4 \times \pi \times 7 \times 7 \times 90°}{360°}$

$\therefore$ Required Area $= (196 - 49\pi)\ cm^2$

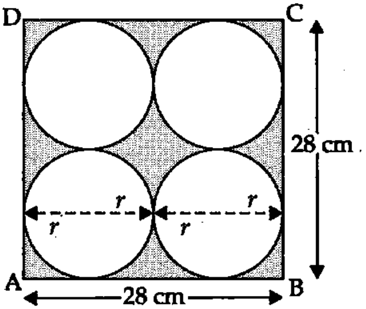
$$= \left(196 - 49 \times \frac{22}{7}\right) cm^2$$

$$= (196 - 7 \times 22)\ cm^2 = (196 - 154)\ cm^2$$

$$= 42\ cm^2$$

Hence, the area enclosed between circles $= 42\ cm^2$.

Q10. On a square cardboard sheet of area 784 cm^2, four congruent circular plates of maximum size are placed such that each circular plate touches the other two plates and each side of square sheet is tangent to two circular plates. Find the area of the square sheet not covered by the circular plates.



Sol. Let a be the side of square ABCD.

Area of square ABCD $= 784\ cm^2$

$\Rightarrow \quad a^2 = 784 \Rightarrow a = \sqrt{784}$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 7 \times 7}$

$= 2 \times 2 \times 7 \Rightarrow a = 28$ cm

Now, in four circles,

$4r = AB$

$\Rightarrow \quad 4r = 28$ cm

$\Rightarrow \quad r = 7$ cm

Area enclosed between circles and square

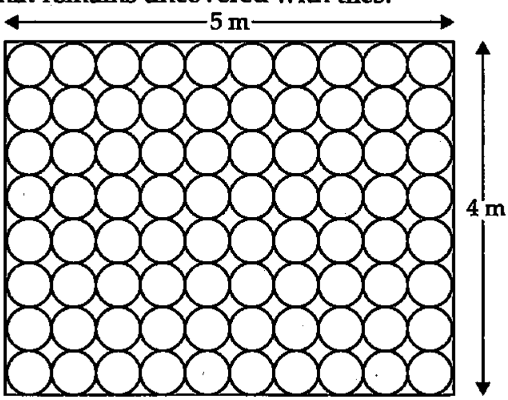
$= $ Area of square $-$ Area of 4 circles

$= 784 - 4\pi r^2$

$$= 784 - 4 \times \frac{22}{7} \times 7 \times 7 = 784 - 616 = 168\ cm^2$$

Hence, the area of square sheet not covered by circular plates is 168 cm^2.

Q11. Floor of a room is of dimensions 5 m × 4 m and it is covered with circular tiles of diameters 50 cm each as shown in figure. Find the area of the floor that remains uncovered with tiles. (Use $\pi = 3.14$)



Sol. As the diameter of circular tile is 50 cm each, then $r = \frac{0.5}{2} = 0.25$ m

Number of tiles length wise $= \frac{5\text{ m}}{0.5\text{ m}} = 10$ tiles

Number of tiles width wise $= \frac{4\text{ m}}{0.5\text{ m}} = 8$ tiles

So, 10 tiles are length wise and 8 tiles are width wise.

So, total number of tiles $= 10 \times 8 = 80$.

$\therefore$ Area of floor not covered by tiles

$= $ Area of rectangular floor – Area of 80 tiles

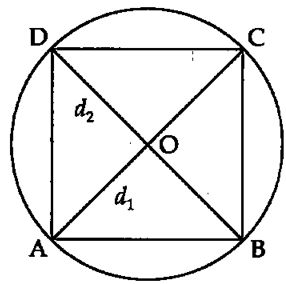
$= 5 \times 4 - 80\pi r^2 = 20 - 80 \times \pi \times 0.25 \times 0.25$

$= 20 - \frac{80 \times 314 \times 25 \times 25}{100 \times 100 \times 100} = 20 - \frac{157}{10} = 20 - 15.7 = 4.3\text{ m}^2$

Hence, the area of floor not covered by tiles $= 4.3\text{ m}^2$.

Q12. All the vertices of a rhombus lie on a circle. Find the area of the rhombus if area of the circle is 1256 cm^2. (Use $\pi = 3.14$)

Sol. All the vertices of a rhombus lie on a circle so rhombus is a square and its diagonals are of length $2r$ cm.



Area of circle $= 1256\text{ cm}^2$

$\Rightarrow \quad \pi r^2 = 1256\text{ cm}^2$

$\Rightarrow \quad r^2 = \frac{1256}{\pi}$

$\Rightarrow \quad r^2 = \frac{1256 \times 100}{314} = 400$

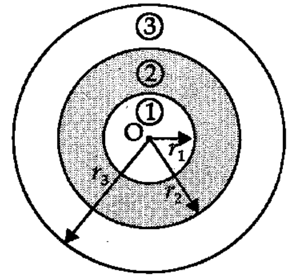
$\Rightarrow \quad r = \sqrt{400} = 20$ cm

$\therefore \quad$ Area of rhombus $= \frac{1}{2} d_1 d_2 = \frac{1}{2} \times 2r \times 2r$

$= 2r^2 = 2 \times 20 \times 20$

$\Rightarrow \quad$ Area of rhombus $= 800\text{ cm}^2$.

Q13. An archery target has three regions formed by three concentric circles as shown in figure. If the diameters of the concentric circles are in the ratio 1 : 2 : 3, then find the ratio of the areas of three regions.



Sol. $\quad d_1 : d_2 : d_3 = 1 : 2 : 3 \quad$ [given]

$= 1d : 2d : 3d \quad$ [$\times$ by d]

$\Rightarrow \quad d_1 : d_2 : d_3 = \frac{d}{2} : \frac{2d}{2} : \frac{3d}{2} \quad \left[\times \text{ by } \frac{1}{2}\right]$

$\Rightarrow \quad r_1 : r_2 : r_3 = r, 2r, 3r$

$\therefore \quad r_1 = r, \quad r_2 = 2r, \quad r_3 = 3r$

Now, $\quad A_1 = \pi r^2$

$$A_2 = \pi(2r)^2 = 4\pi r^2$$
$$A_3 = \pi(3r)^2 = 9\pi r^2$$

Area of innermost circle = $\pi r_1^2 = \pi r^2$

Area of region between first and second circles

$$= A_2 - A_1 = 4\pi r^2 - \pi r^2 = 3\pi r^2$$

Area of region between second and third circles

$$= A_3 - A_2 = 9\pi r^2 - 4\pi r^2 = 5\pi r^2$$

$\therefore$ Required ratio = $\pi r^2 : 3\pi r^2 : 5\pi r^2$

On dividing all the three ratios by πr^2, we get the required ratio of areas of three regions as 1 : 3 : 5.

Q14. The length of minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time 6 : 05 am and 6 : 40 am.

Sol. Time difference = (6 : 40 am – 6 : 05 a.m) = 35 min.

Time swept by min hand is 35 min.

Length of min. hand will be radius of circle swept.

$\therefore$ $r = 5$ cm

In 60 minutes time, area swept by min. hand = πr^2

In 1 minute time, area swept by min. hand = $\frac{\pi r^2}{60}$

In 35 minutes time, area swept by min. hand = $\frac{\pi r^2}{60} \times 35$

$\therefore$ Required area swept by min. hand = $\frac{22}{7} \times \frac{5 \times 5 \times 35}{60}$

$$= \frac{11 \times 25}{6} = \frac{275}{6} = 45\frac{5}{6} \text{ cm}^2$$

Hence, the required area swept by the min. hand is $45\frac{5}{6}$ cm^2.

Q15. Area of sector of central angle 200° of a circle is 770 cm^2. Find the length of the corresponding arc of this sector.

Sol. In the given sector,

$$\theta = 200°, \quad A = 770 \text{ cm}^2 \quad \text{and} \quad l = ?$$

$$A = \frac{\pi r^2 \theta}{360°}$$

$$\Rightarrow \quad r^2 = \frac{A \times 360°}{\pi\theta} = \frac{770 \times 360° \times 7}{22 \times 200°} = 21 \text{ cm} \Rightarrow r = 21 \text{ cm}$$

Now, length of arc $l = \frac{2\pi r\theta}{360°}$

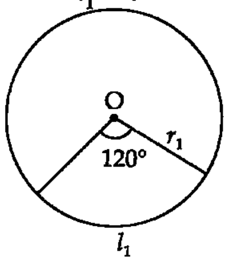
$$= 2 \times \frac{22}{7} \times \frac{21 \times 200°}{360°} = \frac{220}{3} \text{ cm} = 73\frac{1}{3} \text{ cm}$$

Hence, the required length of arc = $73\frac{1}{3}$ cm.

Q16. The central angles of two sectors of circles of radii 7 cm and 21 cm are respectively 120° and 40°. Find the areas of the two sectors as well as the lengths of the corresponding arcs. What do you observe?

Sol. For the first circle, we have

$$r_1 = 7 \text{ cm}$$
$$\theta_1 = 120°$$
$$A_1 = ?$$
$$l_1 = ?$$

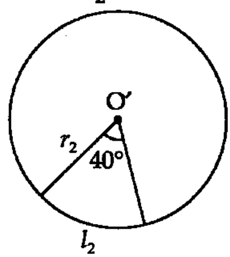


$$A_1 = \frac{\pi r_1^2 \theta_1}{360°} = \frac{22}{7} \times \frac{7 \times 7 \times 120°}{360°}$$
$$= \frac{154}{3} \text{ cm}^2$$

$$\text{Now, } l_1 = \frac{2\pi r_1 \theta_1}{360°} = 2 \times \frac{22}{7} \times \frac{7 \times 120°}{360°}$$
$$= \frac{2 \times 22}{3} = \frac{44}{3} \text{ cm}$$

For the second circle, we have

$$r_2 = 21 \text{ cm}$$
$$\theta_2 = 40°$$
$$A_2 = ?$$
$$l_2 = ?$$



$$A_2 = \frac{\pi r_2^2 \theta_2}{360°} = \frac{22}{7} \times \frac{21 \times 21 \times 40°}{360°}$$
$$= 22 \times 7 = 154 \text{ cm}^2$$

$$l_2 = \frac{2\pi r_2 \theta_2}{360°} = 2 \times \frac{22}{7} \times \frac{21 \times 40°}{360°}$$
$$= \frac{44}{3} \text{ cm}$$

Hence, the length of arcs of two given circles are equal but area of IInd circle is three times that of Ist i.e., unequal.

Q17. Find the area of the shaded region given in figure here.

Sol. Identification of shapes of figures:

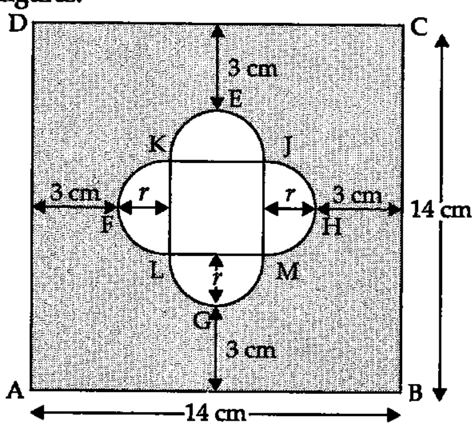
(*i*) 4 semi circles of radius r

(*ii*) square ABCD of side 14 cm

(*iii*) square JKLM of side $2r$

From figure,

$$AB = 3 + 3 + r + 2r + r$$
$$\Rightarrow \quad 14 = 6 + 4r$$
$$\Rightarrow \quad 4r = 14 - 6$$
$$\Rightarrow \quad 4r = 8$$
$$\Rightarrow \quad r = \frac{8}{4} = 2 \text{ cm}$$



So, the area of shaded region

= Area of square – Area of 4 semi circles – Area of square (JKLM)

One square ABCD	One square JKLM	Four semi-circles
$a_1 = 14$ cm	$a_2 = 2r$ $\Rightarrow a_2 = 2 \times 2$ $\Rightarrow a_2 = 4$ cm	$r = 2$ cm

$$\therefore \quad \text{Required area} = a_1^2 - 4 \times \frac{\pi r^2}{2} - a_2^2$$

$$= 14 \times 14 - \frac{4 \times \pi \times 2 \times 2}{2} - 4 \times 4$$

$$= 196 - 16 - 8\pi = (180 - 8\pi)\ \text{cm}^2$$

Hence, the shaded area = $(180 - 8\pi)$ cm^2.

Q18. Find the number of revolutions made by circular wheel of area 1.54 m^2 in rolling a distance of 176 m.

Sol. Distance covered by wheel in n revolutions with radius $r = 2\pi r n$.

$$\therefore \quad 2\pi r n = 176\ \text{m} \qquad ...(i)$$

Area of wheel (circular) = 1.54 m^2

$$\Rightarrow \quad \pi r^2 = 1.54$$

$$\Rightarrow \quad r^2 = \frac{1.54}{\pi} = \frac{154 \times 7}{22 \times 100} = \frac{7 \times 7}{10 \times 10}$$

$$\Rightarrow \quad r = 0.7\ \text{m}$$

Now, $\quad 2\pi r n = 176$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times 0.7 \times n = 176 \qquad \text{[from (i)]}$$

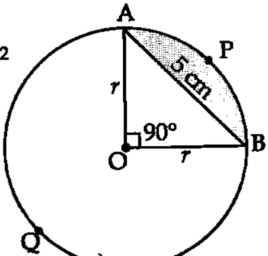
$$\Rightarrow \quad n = \frac{176 \times 7 \times 10}{2 \times 22 \times 7} = 40$$

Hence, $n = 40$ revolutions.

Q19. Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending angle of 90° at the centre.

Sol. Chord AB = 5 cm divides the circle in two segments minor segment APB, and major segment AQB. We have to find out the difference in area of major and minor segment.

Here, $\theta = 90°$.



Area of $\Delta OAB = \frac{1}{2}$ Base × Altitude $= \frac{1}{2} r \times r = \frac{1}{2} r^2$

Area of minor segment APB

$$= \frac{\pi r^2 \theta}{360°} - \text{Area of } \Delta AOB$$

$$= \frac{\pi r^2 90°}{360°} - \frac{1}{2} r^2$$

$$\Rightarrow \quad \text{Area of minor segment} = \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right) \quad \text{...(i)}$$

$$\text{Area of major segment AQB} = \text{Area of circle} - \text{Area of minor segment}$$

$$= \pi r^2 - \left[\frac{\pi r^2}{4} - \frac{r^2}{2}\right]$$

$$\Rightarrow \text{Area of major segment AQB} = \left[\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right] \quad \text{...(ii)}$$

Difference between areas of major and minor segment

$$= \left(\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right) - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$$

$$= \frac{3}{4}\pi r^2 + \frac{r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2}$$

$$\Rightarrow \quad \text{Required area} = \frac{2}{4}\pi r^2 + r^2 = \frac{1}{2}\pi r^2 + r^2$$

In right angle ΔOAB,

$$r^2 + r^2 = AB^2 \Rightarrow 2r^2 = 5^2 \Rightarrow r^2 = \frac{25}{2}$$

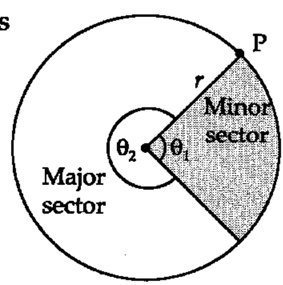
$$\text{So, required area} = \left[\frac{1}{2}\pi\frac{25}{2} + \frac{25}{2}\right] = \left[\frac{25\pi}{4} + \frac{25}{2}\right] \text{cm}^2$$

Q20. Find the difference of the areas of a sector of angle 120° and its corresponding major sector of a circle of radius 21 cm.

Sol.

For minor sector	For major sector
$r_1 = 21$ cm	$r_2 = 21$ cm
$\theta_1 = 120°$	$\theta_2 = 360° - 120° = 240°$

Difference in areas of major and minor sectors



$$= \frac{\pi r^2}{360°}(\theta_2 - \theta_1) \quad [\because r_1 = r_2 = r]$$

$$= \frac{22 \times 21 \times 21}{7 \times 360°}[240° - 120°]$$

$$= \frac{22 \times 21 \times 21 \times 120°}{7 \times 360°}$$

$$= 462 \text{ cm}^2$$

Hence, the difference in areas of major and minor sectors of given circle is 462 cm^2.

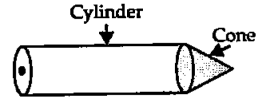
❑❑❑

12 Surface Areas and Volumes

EXERCISE 12.1

Choose the correct answer from the given four options:

Q1. A cylindrical pencil sharpened at one edge is the combination of



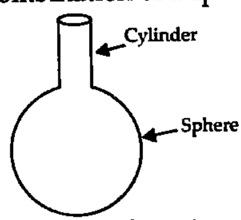
(*a*) a cone and a cylinder
(*b*) frustrum of a cone and a cylinder
(*c*) a hemisphere and a cylinder
(*d*) two cylinders

Sol. (*a*): The sharpened part of the pencil is cone and unsharpened part is cylinder.

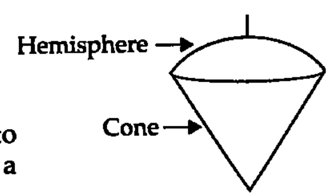
Q2. A surahi is the combination of

(*a*) a sphere and a cylinder
(*b*) a hemisphere and a cylinder
(*c*) two hemispheres
(*d*) a cylinder and a cone

Sol. (*a*): A surahi is the combination of a sphere and a cylinder.



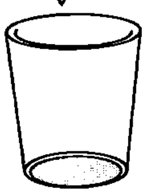
Q3. A plumbline (Sahul) is the combination of



(*a*) a cone and a cylinder
(*b*) a hemisphere and a cone
(*c*) frustrum of a cone and a cylinder
(*d*) sphere and cylinder

Sol. (*b*): Plumbline is an instrument used to check the verticality of an object. It is a combination of a hemisphere and a cone.

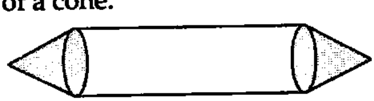
Q4. The shape of a glass (Tumbler) (see figure) is usually in the form of a



(*a*) cone
(*b*) frustrum of a cone
(*c*) cylinder
(*d*) sphere

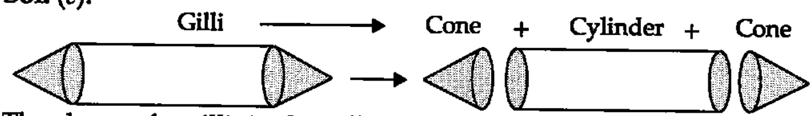
Sol. (*b*): The radius of the lower circular part is smaller than the upper part. So, it is frustrum of a cone.

Q5. The shape of a gilli, in the gilli-danda game (see in figure) is the combination of



(*a*) two cylinders (*b*) a cone and a cylinder
(*c*) two cones and a cylinder (*d*) two cylinders and a cone

Sol. (*c*):

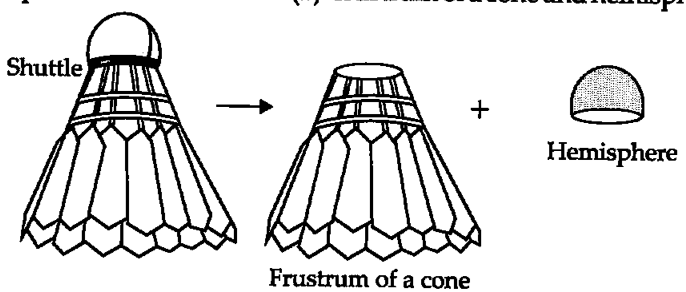


The shape of a gilli, in the gilli-danda game is a combination of two cones and a cylinder.

Q6. A shuttle cock used for playing badminton has the shape of the combination of

(*a*) a cylinder and a sphere (*b*) a cylinder and a hemisphere
(*c*) a sphere and a cone (*d*) frustrum of a cone and hemisphere

Sol. (*d*):

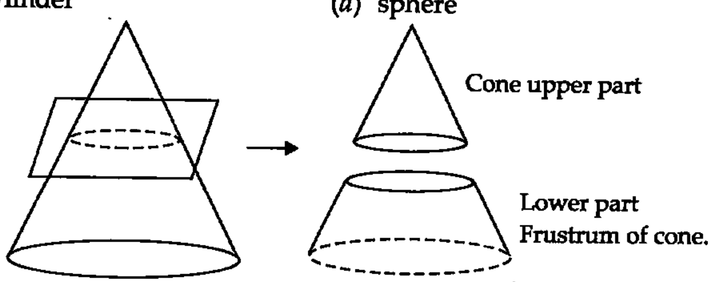


A shuttle cock used for playing badminton has the shape of the combination of frustrum of a cone and a hemisphere.

Q7. A cone is cut through a plane parallel to its base and then the cone that is formed on one side of that plane is removed. The new part that is left over on the other side of the plane is called

(*a*) a frustrum of a cone (*b*) cone
(*c*) cylinder (*d*) sphere

Sol. (*a*):

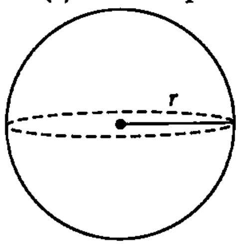


The new part that is left over on the other side of the plane is called a frustrum of a cone.

Q8. A hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then the number of marbles that the cube can accommodate is

(*a*) 142296 (*b*) 142396 (*c*) 142496 (*d*) 142596

Sol. (*a*): Let the spherical marble has radius *r*.

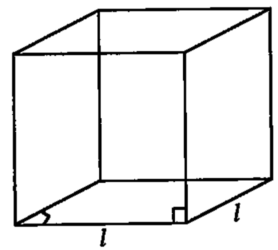


Diameter of the marble = 0.5 cm

$\Rightarrow r = \frac{0.5}{2}$ cm

$\Rightarrow r = 0.25$ cm

Cube $l = 22$ cm



Let *n* marbles can fill the cube.

$$\therefore \quad \text{Volume of } n \text{ marbles} = \left(1 - \frac{1}{8}\right) \text{ part of volume of cube}$$

$$\Rightarrow \quad n.\frac{4}{3}\pi r^3 = \frac{7}{8} \times l^3$$

$$\Rightarrow \quad n = \frac{7l^3}{8} \times \frac{3}{4\pi r^3} = \frac{7 \times 3 \times 22 \times 22 \times 22 \times 7}{8 \times 4 \times 22 \times 0.25 \times 0.25 \times 0.25}$$

$$\Rightarrow \quad n = \frac{7 \times 3 \times 22 \times 22 \times 22 \times 100 \times 100 \times 100 \times 7}{8 \times 4 \times 22 \times 25 \times 25 \times 25}$$

$$= 7 \times 3 \times 22 \times 22 \times 2 = 42 \times 487 \times 7$$

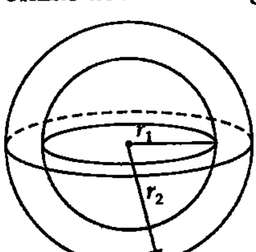
$$n = 142296$$

So, cube can accommodate upto 142296 marbles so right option is 142296, i.e., (*a*) other options are more than 142296. So, cannot accommodate.

Q9. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recasted into the form of a cone of base diameter 8 cm. The height of cone is

(*a*) 12 cm (*b*) 14 cm (*c*) 15 cm (*d*) 18 cm

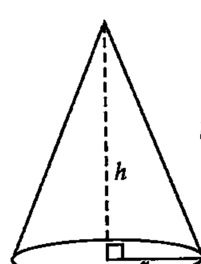
Sol. (*b*): **Main concept:** During recasting a shape into another it's volume does not change.



Spherical shell

$r_1 = \frac{4}{2} = 2$ cm

$r_2 = \frac{8}{2} = 4$ cm



Cone

$r = \frac{8}{2} = 4$ cm

$h = ?$

During recasting volume remains same so

Volume of cone = Volume of hollow spherical shell

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3$

$\Rightarrow \quad \frac{\pi}{3} r^2 h = \frac{\pi}{3} \times 4\left[r_2^3 - r_1^3\right]$

$\Rightarrow \quad r^2 h = 4\,[r_2^3 - r_1^3]$

$\Rightarrow \quad 4 \times 4\,h = 4[(4)^3 - (2)^3]$

$\Rightarrow \quad 4h = 64 - 8$

$\Rightarrow \quad h = \frac{56}{4}$

$\Rightarrow \quad h = 14$ cm

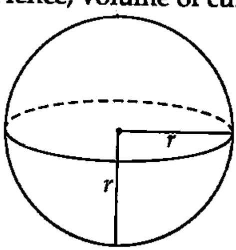
Hence, right option is (*b*).

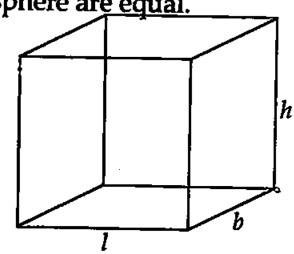
Q10. A solid piece of iron in the form of a cuboid of dimensions 49 cm × 33 cm × 24 cm, is moulded to form a solid sphere. The radius of the sphere is

(*a*) 21 cm (*b*) 23 cm (*c*) 25 cm (*d*) 19 cm

Sol. (*a*): Solid cuboid of iron is moulded into solid sphere.

Hence, volume of cuboid and sphere are equal.





∴ Volume of sphere (solid) = Volume of cuboid

$\Rightarrow \quad \frac{4}{3}\pi r^3 = l \times b \times h$

$\Rightarrow \quad r^3 = \frac{l \times b \times h \times 3}{4 \times \pi} = \frac{49 \times 33 \times 24 \times 3 \times 7}{4 \times 22}$

$\Rightarrow \quad r^3 = 7 \times 7 \times 7 \times 3 \times 3 \times 3$

$\Rightarrow \quad r = 21$ cm.

Hence, right option is 21 cm *i.e.*, option (*a*).

Q11. A mason constructs a wall of dimensions 270 cm × 300 cm × 350 cm with the bricks each of size 22.5 cm × 11.25 cm × 8.75 cm and it is assumed that $\frac{1}{8}$ space is covered by the mortar. Then the number of bricks used to construct the wall is

(*a*) 11100 (*b*) 11200 (*c*) 11000 (*d*) 11300

Sol. (*b*): The volume of the wall covered by mortar = $\frac{1}{8}$ part

So, the volume covered by bricks of wall = $\left(1-\frac{1}{8}\right)$ volume of wall

$= \frac{7}{8}$ volume of wall

Bricks (Cuboid)	Wall (Cuboid)
$l_1 = 22.5$ cm	$l = 270$ cm
$b_1 = 11.25$ cm	$b = 300$ cm
$h_1 = 8.75$ cm	$h = 350$ cm

Let n be the number of bricks.

According to the question, we have

$$\text{Volume of bricks} = \frac{7}{8}\ \text{Volume of wall (cuboid)}$$

$$\Rightarrow \quad n \times l_1 \times b_1 \times h_1 = \frac{7}{8} l \times b \times h$$

$$\Rightarrow \quad n = \frac{7 \times l \times b \times h}{8 \times l_1 \times b_1 \times h_1} = \frac{7 \times 270 \times 300 \times 350}{8 \times 22.5 \times 11.25 \times 8.75}$$

$$\Rightarrow \quad n = \frac{7 \times 270 \times 300 \times 350 \times 10 \times 100 \times 100}{8 \times 225 \times 1125 \times 875}$$

$$\Rightarrow \quad n = 2 \times 4 \times 350 \times 4 = 32 \times 350 = 11200 \text{ bricks}$$

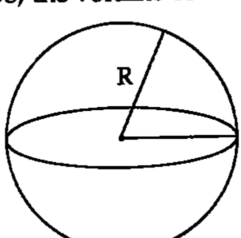
Hence, right option is (*b*).

Q12. Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is

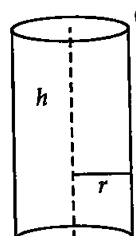
(*a*) 4 cm (*b*) 3 cm (*c*) 2 cm (*d*) 6 cm

Sol. (*c*): Solid cylinder is recasted into 12 spheres.

So, the volume of 12 spheres will be equal to cylinder.



12 Spheres
R = ?



Cylinder
$r = \frac{2}{2} = 1$ cm
$h = 16$ cm

$\therefore$ Volume of 12 spheres = Volume of cylinder

$$\Rightarrow \quad \frac{4}{3}\pi R^3 \times 12 = \pi r^2 h$$

$$\Rightarrow \quad 12 \times \frac{4}{3} R^3 = r^2 h$$

$$\Rightarrow \quad R^3 = \frac{3r^2 h}{4 \times 12} = \frac{3 \times 1 \times 1 \times 16}{4 \times 12} = 1$$

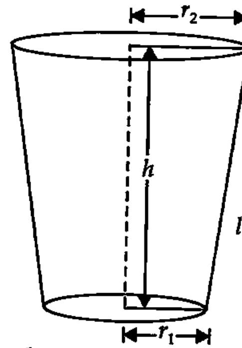
$\Rightarrow \quad R = 1$ cm

Hence, diameter (2R) is 2 cm. So, right option is (*c*).

Q13. The radii of the top and bottom of a bucket of slant height 45 cm are 28 cm and 7 cm respectively. The curved surface area of the bucket is

(*a*) 4950 cm^2 (*b*) 4951 cm^2 (*c*) 4952 cm^2 (*d*) 4953 cm^2

Sol. (*a*): Here, $r_1 = 7$ cm, $r_2 = 28$ cm, $l = 45$ cm



Curved surface area of bucket $= \pi l(r_1 + r_2)$

$$= \frac{22}{7} \times 45[7 + 28]$$

$$= \frac{22}{7} \times 45 \times 35$$

$\Rightarrow$ Curved surface area of bucket

$$= 22 \times 45 \times 5 \text{ cm}^2$$
$$= 4950 \text{ cm}^2$$

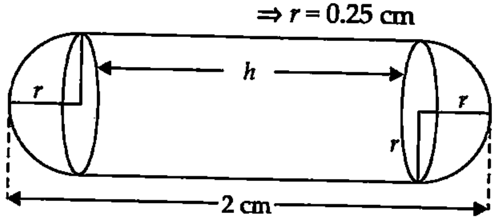
Hence, right option is (*a*).

Q14. A medicine capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of entire capsule is 2 cm. The capacity of the capsule is

(*a*) 0.36 cm^3 (*b*) 0.35 cm^3 (*c*) 0.34 cm^3 (*d*) 0.33 cm^3

Sol. (*a*): Capsule consists of

2 Hemispheres	Cylinder
$r = 0.25$ cm	$r = \frac{0.5}{2}$ cm
	$\Rightarrow r = 0.25$ cm



Total length of capsule $= r + h + r$

$\Rightarrow \quad 2 \text{ cm} = 2r + h$

$\Rightarrow \quad 2 = 2 \times 0.25 + h \Rightarrow h = 2 - 0.5 = 1.5$ cm

Volume of capsule = Vol. of two hemispheres + Vol. of cylinder

$$= 2 \times \left(\frac{4}{3}\pi r^3 \times \frac{1}{2}\right) + \pi r^2 h = \frac{4}{3}\pi r^3 + \pi r^2 h$$

$$= \pi r^2 \left[\frac{4}{3}r + h\right] = \frac{22}{7} \times 0.25 \times 0.25 \left[\frac{4}{3} \times 0.25 + \frac{15}{10}\right]$$

$$= \frac{22}{7} \times 0.25 \times 0.25 \left[\frac{4}{3} \times \frac{25}{100} + \frac{3}{2}\right]$$

$$= \frac{22}{7}\times 0.25\times 0.25\left[\frac{1}{3}+\frac{3}{2}\right]$$

$$= \frac{22}{7}\times 0.25\times 0.25\left[\frac{2+9}{6}\right]$$

$$= \frac{22\times 25\times 25\times 11}{7\times 6\times 100\times 100} = \frac{121}{42\times 8} = \frac{121}{336}$$

$\therefore$ Volume of capsule $= 0.36 \text{ cm}^3$

Hence, verifies the option (*a*).

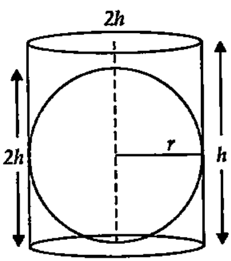
Q15. If two solid hemispheres of same base radius *r* are joined together along their bases, then curved surface area of this new solid is

(*a*) $4\pi r^2$ (*b*) $6\pi r^2$ (*c*) $3\pi r^2$ (*d*) $8\pi r^2$

Sol. (*a*): When two hemispheres of equal radii are joined base to base new solid becomes sphere and curved surface area of sphere is $4\pi r^2$. So, the right option is (*a*).

Q16. A right circular cylinder of radius *r* cm and height *h* cm (where $h > 2r$) just encloses a sphere of diameter

(*a*) *r* cm (*b*) 2*r* cm
(*c*) *h* cm (*d*) 2*h* cm



Sol. (*b*): As the cylinder just encloses the sphere so the radius or diameter of cylinder and sphere are equal *i.e.*, 2*r* and height $h > 2r$.

Hence, verifies the option (*b*).

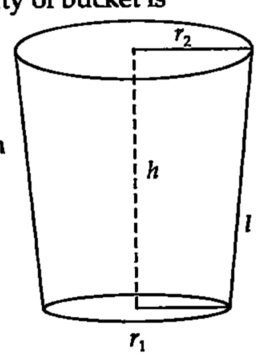
Q17. During conversion of a solid from one shape to another, the volume of new sphere will

(*a*) increase (*b*) decrease
(*c*) remains unaltered (*d*) be doubled

Sol. (*c*): During reshaping a solid, the volume of new solid will be equal to old one or remains unaltered.

Q18. The diameters of two circular ends of the bucket are 44 cm and 24 cm. The height of bucket is 35 cm. The capacity of bucket is

(*a*) 32.7 L (*b*) 33.7 L
(*c*) 34.7 L (*d*) 31.7 L



Sol. (*a*): Bucket is frustrum of a cone.

Here, $r_1 = \frac{24}{2} = 12 \text{ cm}$, $r_2 = \frac{44}{2} = 22 \text{ cm}$, $h = 35 \text{ cm}$

The volume of the bucket is given by

$$V = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{1}{3}\times\frac{22}{7}\times 35[12^2 + 22^2 + 12\times 22]$$

$$= \frac{1 \times 22 \times 35}{3 \times 7}[144 + 484 + 264]$$

$$= \frac{22 \times 35 \times 892}{3 \times 7} = \frac{22 \times 5 \times 892}{3} = \frac{110 \times 892}{3}\ \text{cm}^3$$

$$= \frac{110 \times 892}{3 \times 1000} = \frac{9812}{300} = 32.706 \text{ litre.}$$

It is close to option (*a*).

Q19. In a right circular cone, the cross-section made by a plane parallel to the base is a

(*a*) circle (*b*) frustrum of a cone
(*c*) sphere (*d*) hemisphere

Sol. (*a*): In a right circular cone, if any cut is made parallel to its base, we get a circle. Hence, verifies option (*a*).

Q20. Volumes of two spheres are in the ratio 64 : 27. The ratio of their surface areas is

(*a*) 3 : 4 (*b*) 4 : 3 (*c*) 9 : 16 (*d*) 16 : 9

Sol. (*d*): $\frac{V_1}{V_2} = \frac{64}{27}$ [V_1, V_2 are the volumes of two spheres]

$\Rightarrow$ $\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$ [r_1, r_2 are the radii of spheres]

$\Rightarrow$ $\left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{3}\right)^3 \Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$

Now, the ratio of their surface areas is given by

$$\frac{\text{T.S.A}_1}{\text{T.S.A}_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Hence, the required ratio 16 : 9 verifies option (*d*).

EXERCISE 12.2

Write True or False and justify your answer in the following:

Q1. Two identical solid hemispheres of equal base radius r cm are stuck together along their bases. The total surface area of the combination is $6\pi r^2$.

Sol. False: When two hemispheres of equal bases are stuck base to base it forms a sphere and total surface area of resulting sphere is $4\pi r^2$. Hence, the given statement is false.

Q2. A solid cylinder of radius r and height h is placed over other cylinder of same height and radius. The total surface area of the shape so formed is $4\pi rh + 4\pi r^2$.

Sol. False: When two identical cylinders of same radius 'r' and height 'h' are stuck base (circular) to base, then the resulting cylinder will have $h' = 2h$, $r' = r$

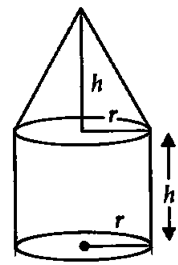
$\therefore$ $$\text{T.S.A} = 2\pi r'(r' + h') = 2\pi r(r + 2h) = 2\pi r^2 + 2\pi r.2h$$
$$= 4\pi rh + 2\pi r^2$$

Hence, the given statement is false.

Q3. A solid cone of radius r and height h is placed over a solid cylinder having same base radius and height as that of a cone. The total surface area of the combined solid is $\pi r\left[\sqrt{r^2 + h^2} + 3r + 2h\right]$.

Sol. False:

Cone	Cylinder
Radius = r	Radius = r
Height = h	Height = h



Total surface area of the combined solid

= Curved surface area of cone + Curved surface area of cylinder + Area of the base of cylinder

$= \pi rl + 2\pi rh + \pi r^2 = \pi r[l + 2h + r]$

$\because$ $l = \sqrt{r^2 + h^2}$

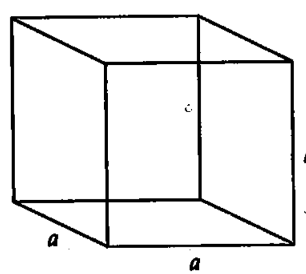
$\therefore$ Total surface area of the combined solid

$= \pi r\left[\sqrt{r^2 + h^2} + 2h + r\right]$ which is not according to the given statement.

Hence, the given statement is false.

Q4. A solid ball is exactly fitted inside the cubical box of side a. The volume of the ball is $\frac{4}{3}\pi a^3$.

Sol. False: Clearly from figure when a ball (spherical) is exactly fitted inside the cubical box then diameter of the ball becomes equal to side of cube so



Diameter = $d = a$

$\Rightarrow$ Radius = $r = \frac{a}{2}$

$\therefore$ Volume of spherical ball $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{4}{3}\pi\frac{a^3}{8} = \frac{1}{6}\pi a^3 \neq \frac{4}{3}\pi a^3$$

Hence, the given statement is false.

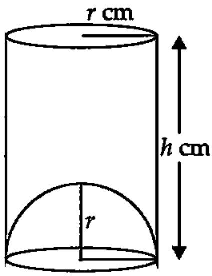
Q5. The volume of the frustrum of a cone is $\frac{1}{3}\pi h[r_1^2 + r_2^2 - r_1 r_2]$, where h is the vertical height of the frustrum and r_1, r_2 are the radii of the ends.

Sol. False: As we know that the volume of the frustrum

$$V = \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1r_2] \neq \frac{1}{3}\pi h[r_1^2 + r_2^2 - r_1r_2]$$

Hence, the given statement is false.

Q6. The capacity of a cylindrical vessel with a hemispherical portion raised upward, at the bottom as shown in figure is $\frac{\pi r^2}{3}(3h - 2r)$.



Sol. True:

	Cylinder	Hemisphere
	Radius = r	Radius = r
	Height = h	

Capacity of vessel = Volume of cylinder – Volume of hemisphere

$$= \pi r^2 h - \frac{2}{3}\pi r^3$$

$\Rightarrow$ Volume of vessel $= \frac{\pi r^2}{3}[3h - 2r]$

which is equal to the volume given in the statement.

Hence, the given statement is true.

Q7. The curved surface area of a frustrum of a cone is $\pi l(r_1 + r_2)$, where $l = \sqrt{h^2 + (r_1 + r_2)^2}$, r_1, r_2 are the radii of the two ends of frustrum and h is vertical height.

Sol. False: We know that the curved surface area of frustrum $= \pi l[r_1 + r_2]$

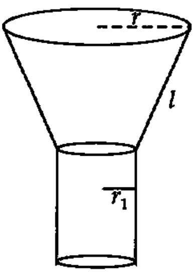
where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

But, $l = \sqrt{h^2 + (r_1 + r_2)^2}$ in the given statement.

So, the given statement is false.

Q8. An open metallic bucket is in the shape of a frustrum of a cone, mounted on a hollow cylindrical base made of same metallic sheet. The surface area of the metallic sheet used is equal to the curved surface area of frustrum of a cone + area of circular base + curved surface area of cylinder.

Sol. True: The surface area of the sheet used for vessel will be equal to the total surface area of cylinder excluding the top and only curved surface area of frustrum of a cone.



So, total surface area of vessel

= Curved surface area of frustrum + Curved surface area of cylinder + Area of base of cylinder

It is equal to the surface area of the metallic sheet given in the statement.

Hence, the given statement is true.

EXERCISE 12.3

Q1. Three metallic solid cubes whose edges are 3 cm, 4 cm and 5 cm are melted and formed into a single cube. Find the edges of the cube so formed.

Sol. Volume of new cube will be equal to the sum of volumes of three cubes in recasting process.

Cube (New)	Cube I	Cube II	Cube III
$a = ?$	$a_1 = 3$ cm	$a_2 = 4$ cm	$a_3 = 5$ cm

$$V = V_1 + V_2 + V_3$$

$\Rightarrow \quad a^3 = a_1^3 + a_2^3 + a_3^3$

$\Rightarrow \quad a^3 = (3)^3 + (4)^3 + (5)^3 = 27 + 64 + 125$

$\Rightarrow \quad a^3 = 216 = (6)^3$

$\Rightarrow \quad a = 6$ cm.

Hence, the edge of new recasted cube is 6 cm.

Q2. How many shots each having diameter 3 cm, can be made from a cuboidal lead solid of dimensions 9 cm × 11 cm × 12 cm?

Sol.

Cuboid	n spherical shots
$l = 12$ cm	$r = \frac{3}{2} = 1.5$ cm
$b = 11$ cm	
$h = 9$ cm	

Lead cuboid is recasted in lead shots (spherical) so

Volume of n spherical shots = Vol. of cuboid.

$\Rightarrow \quad n.\frac{4}{3}\pi r^3 = l \times b \times h$

$\Rightarrow \quad n \times \frac{4}{3} \times \frac{22}{7} \times 1.5 \times 1.5 \times 1.5 = 9 \times 11 \times 12$

$\Rightarrow \quad n = \frac{9 \times 11 \times 12 \times 3 \times 7 \times 1000}{4 \times 22 \times 15 \times 15 \times 15}$

$\Rightarrow \quad n = 3 \times 7 \times 4 = 84$

Hence, 84 lead shots can be made.

Q3. A bucket is in the shape of a frustrum of a cone and holds 28.490 liters of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.

Sol. Here, $r_1 = 21$ cm, $r_2 = 28$ cm, $h = ?$

$V = 28.490 \text{ L} = 28.490 \times 1000 \text{ cm}^3$

$\Rightarrow \quad V = 28490 \text{ cm}^3$ [given]

Now, $\quad V = \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2] = 28490$

$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times h[(21)^2 + (28)^2 + (21)(28)] = 28490$

$$\Rightarrow \quad \frac{22}{3\times 7}\times 7^2 h[3^2+4^2+3\times 4] = 28490$$

$$\Rightarrow \quad \frac{22\times 7\times 7}{3\times 7}h[9+16+12] = 28490$$

$$\Rightarrow \quad \frac{22\times 7\times 7\times 37h}{3\times 7} = 28490$$

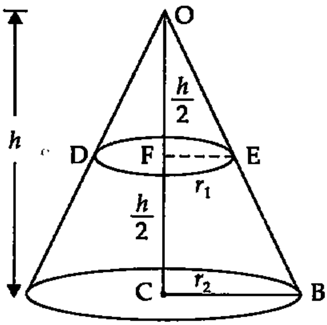
$$\Rightarrow \quad h = \frac{28490\times 3\times 7}{22\times 7\times 7\times 37} = 15$$

Hence, the height of frustrum = 15 cm.

Q4. A cone of radius 8 cm and height 12 cm is divided into two parts by a plane through the mid-point of its axis parallel to its base. Find the ratio of the volumes of two parts.

Sol.

Cone I (ABO)	Cone II (ODE)	Frustrum (DEBA)
$r = 8$ cm	$r_1 = 4$ cm	$r_1 = 4$ cm
$h = 12$ cm	$h_1 = \frac{h}{2} = \frac{12}{2}$	$r_2 = 8$ cm
	$\Rightarrow h_1 = 6$ cm	$h_2 = 6$ cm



$\Delta OBC \sim \Delta OEF$ [By AA criterion of similarity]

$$\therefore \quad \frac{r_1}{r_2} = \frac{h_1}{h_2} \Rightarrow \frac{r_1}{8} = \frac{h/2}{h} = \frac{1}{2}$$

$$\Rightarrow \quad r_1 = -\ \ 4 \text{ cm}$$

$$\frac{\text{Vol. of frustrum (DEBA)}}{\text{Vol. of cone (ODE)}} = \frac{\frac{1}{3}\pi h_2[r_1^2 + r_2^2 + r_1 r_2]}{\frac{1}{3}\pi r_1^2 h_1}$$

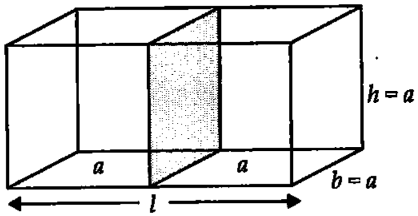
$$= \frac{6[4^2+8^2+4\times 8]}{4\times 4\times 6} = \frac{(16+64+4\times 8)}{4\times 4} = \frac{112}{4\times 4} = \frac{7}{1}$$

$\therefore$ Volume of frustrum : Volume of smaller cone = 7 : 1.

Q5. Two identical cubes each of volume 64 cm^3 are joined together end to end. What is the surface area of resulting cuboid?

Sol. Two identical cubes of side a are joined end to end to form a cuboid then

2 Cubes	**Cuboid**
Let length = a units	$l = 2a$ units
and breadth = a units	$b = a$ units
	$h = a$ units



So, the surface area of the resulting cuboid

$= 2[lb + lh + bh]$

$= 2[2a.a + 2a.a + a.a] = 2[2a^2 + 2a^2 + a^2]$

$= 10a^2$ ...(*i*)

Volume of the cube = 64 cm^3

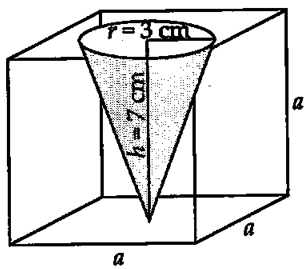
$\Rightarrow \quad a^3 = (4)^3$

$\Rightarrow \quad a = 4$ cm

$\therefore$ Total surface area of cuboid $= 10 \times 4 \times 4$ [From (*i*)]

Hence, the required surface area = 160 cm^2.

Q6. **From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume of remaining solid.**



Sol. Cone (cavity) $r = 3$ cm, $h = 7$ cm

Cube side $(a) = 7$ cm

Vol. of remaining solid

= Vol. of cube – Vol. of cone

$= a^3 - \frac{1}{3}\pi r^2 h$

$= 7 \times 7 \times 7 - \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7$

$= 343 - 66 = 277$ cm^3

Hence, the volume of remaining solid = 277 cm^3.

Q7. **Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.**

Sol. When two identical cones are joined base to base, the total surface area of new solid becomes equal to the sum of curved surface areas of both the cones.

So, total surface area of solid $= \pi rl + \pi rl = 2\pi rl$

In two cones, $r = 8$ cm, $h = 15$ cm

Now, $l^2 = r^2 + h^2 = 8^2 + 15^2 = 64 + 225 = 289$

$\Rightarrow \quad l^2 = (17)^2 \Rightarrow l = 17$ cm

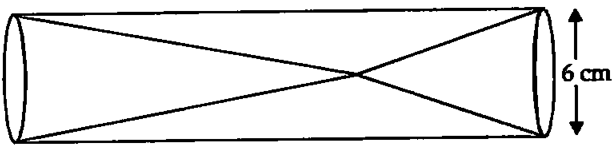
$\therefore$ Total surface area of solid $= 2\pi rl$

$= 2 \times \pi \times 8 \times 17 = 272\pi$ cm^2 = 854.857 cm^3

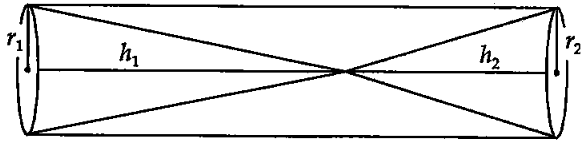
Hence, the surface area of new solid = 854.857 cm^3.

Q8. **Two solid cones A and B are placed in a cylindrical tube as shown in the figure. The ratio of their capacities is 2 : 1. Find the heights and capacities of cones. Also, find the volume of the remaining portion of the cylinder.**

21 cm



Sol. As the ratio of volumes of cone c_1 and c_2 is 2 : 1, their radii are same equal to $r = \frac{6}{2} = 3$ cm.



$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$\Rightarrow \quad \frac{2}{1} = \frac{(3)^2 h_1}{(3)^2 h_2}$$

$$\Rightarrow \quad h_1 = 2h_2 \qquad \text{...(i)}$$

Also, $h_1 + h_2 = 21$ cm

$$\Rightarrow \quad 2h_2 + h_2 = 21$$

$$\Rightarrow \quad 3h_2 = 21 \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad h_2 = 7 \text{ cm}$$

Now, $h_1 = 21 \text{ cm} - 7 \text{ cm} = 14$ cm ...(ii)

Hence, height of cone I = 14 cm and height of cone II = 7 cm.

Cone I	**Cone II**	**Cylinder**
$r_1 = \frac{6}{3} = 3$ cm	$r_2 = 3$ cm	$r = 3$ cm
$h_1 = 14$ cm	$h_2 = 7$ cm	$h = 21$ cm

$$\text{Volume of cone I} = \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 14$$

$$= 132 \text{ cm}^3$$

$$\text{Volume of cone II} = \frac{1}{3}\pi r_2^2 h_2 = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7$$

$$= 22 \times 3 = 66 \text{ cm}^3$$

Volume of remaining portion of tube

$$= \text{Vol. of cylinder} - \text{Vol. of cone I} - \text{Vol. of cone II}$$

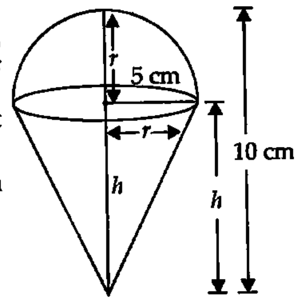
$$= \pi r^2 h - 66 - 132$$

$$= \frac{22}{7} \times 3 \times 3 \times 21 - 198$$

$$= 22 \times 27 - 198 = 594 - 198 = 396 \text{ cm}^3$$

Hence, the required volume is 396 cm^3.

Q9. An ice-cream cone full of ice-cream having radius 5 cm, and height 10 cm, as shown in figure. Calculate the volume of ice-cream, provided that its $\frac{1}{6}$ part is left unfilled with ice cream.



Sol. Ice-cream cone can be considered as a hemisphere on a cone.

Hemisphere	**Cone**
$r = 5$ cm	$r = 5$ cm
	$h = 10 - 5 = 5$ cm

$\frac{1}{6}$ part of ice-cream is left unfilled.

So, Vol. of ice-cream $= \left(1 - \frac{1}{6}\right)$ [Volume of cone and hemisphere]

$$= \frac{5}{6}\left[\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right] = \frac{5}{6} \times \frac{1}{3}\pi r^2 [h + 2r]$$

$$= \frac{5 \times 22 \times 5 \times 5}{6 \times 3 \times 7}[5 + 2 \times 5] = \frac{5 \times 22 \times 5 \times 5 \times 15}{6 \times 3 \times 7}$$

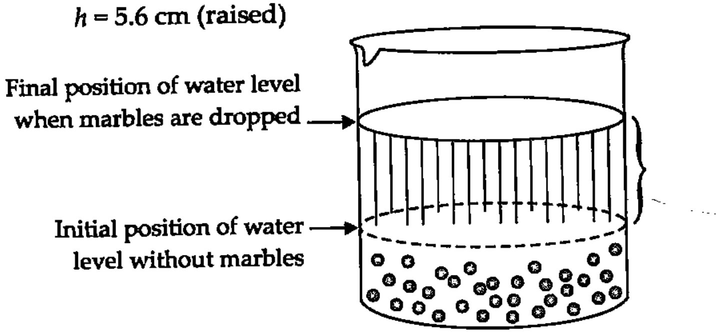
$$= \frac{55 \times 125}{21} = \frac{6875}{21} \cong 327.4 \text{ cm}^3$$

Hence, the volume of ice-cream in cone is 327.4 cm^3.

Q10. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm, containing some water. Find the number of marbles that should be dropped into the beaker so that water level rises by 5.6 cm.

Sol. When marbles are dropped in beaker filled partially with water, the volume of water raised in beaker will be equal to the volume of n marbles. The shape of water raised in beaker is cylindrical.

Cylindrical beaker	**Spherical marbles**
$r = \frac{7}{2} = 3.5$ cm	$R = \frac{1.4}{2} = 0.7$ cm
$h = 5.6$ cm (raised)	



Beaker

∴ Vol. of n spherical balls = Vol. of water raised in cylinders

$$\Rightarrow \quad n \times \frac{4}{3}\pi R^3 = \pi r^2 h$$

$$\Rightarrow \quad n \times \frac{4}{3} \times 0.7 \times 0.7 \times 0.7 = 3.5 \times 3.5 \times 5.6$$

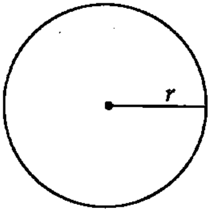
$$\Rightarrow \quad n = \frac{35 \times 35 \times 56 \times 3}{4 \times 7 \times 7 \times 7}$$

$$\Rightarrow \quad n = 50 \times 3 = 150$$

Hence, required number of marbles = 150.

Q11. How many spherical lead shots each of diameter 4.2 cm can be obtained from a solid rectangular lead piece with dimensions 66 cm, 42 cm and 21 cm?

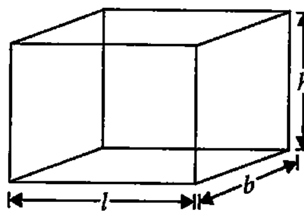
Sol.



n spherical shots

$r = \frac{4.2}{2}$

$\Rightarrow r = 2.1$ cm



Cuboid
$l = 66$ cm
$b = 42$ cm
$h = 21$ cm

Spherical lead shots are recasted from cuboid of lead. So, volume of n spherical lead shots is equal to the volume of cuboid.

∴ Volume of n spherical lead shots = Vol. of lead cuboid

$$\Rightarrow \quad n \times \frac{4}{3}\pi r^3 = l \times b \times h$$

$$\Rightarrow \quad n \times \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1 = 66 \times 42 \times 21$$

$$\Rightarrow \quad n = \frac{66 \times 42 \times 21 \times 3 \times 7 \times 1000}{4 \times 22 \times 21 \times 21 \times 21}$$

$$\Rightarrow \quad n = 3 \times 500 = 1500$$

Hence, the number of shots are 1500.

Q12. How many spherical lead shots of diameter 4 cm can be made out of a solid cube of lead whose edge measures 44 cm?

Sol. Solid cube is recasted into spherical lead shots.

Cube	**Spherical lead shots**
$a = 44$ cm	$r = \frac{4}{2} = 2$ cm

∴ Vol. of n spherical lead shots = Vol. of cube

$$\Rightarrow \quad n . \frac{4}{3}\pi r^3 = a^3$$

$$\Rightarrow \quad n \times \frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2 = 44 \times 44 \times 44$$

$$\Rightarrow \qquad n = \frac{44 \times 44 \times 44 \times 3 \times 7}{4 \times 22 \times 2 \times 2 \times 2} = 121 \times 21 \Rightarrow n = 2541$$

Hence, the number of lead shots are 2541.

Q13. A wall 24 m long, 0.4 m thick and 6 m high is constructed with the bricks each of dimensions 25 cm × 16 cm × 10 cm. If the mortar occupies $\frac{1}{10}$th of the volume of the wall, then find the number of bricks used in constructing the wall.

Sol. Wall is 24 m long, 0.4 m thick and 6 m high.

So, volume of wall $= 24 \text{ m} \times 0.4 \text{ m} \times 6 \text{ m} = 57.6 \text{ m}^3$

Since $\frac{\ }{10}$th of the volume of the wall is occupied by mortar, so the volume of bricks in the wall

$$= \left(1 - \frac{1}{10}\right) \text{ part of the wall.}$$

$$= \frac{9}{10} \text{th part of the wall}$$

$$= \frac{9}{10} \times 57.6 \text{ m}^3 = 51.84 \text{ m}^3$$

$$\text{Volume of one brick} = 25 \text{ cm} \times 16 \text{ cm} \times 10 \text{ cm}$$

$$= \frac{25}{100} \times \frac{16}{100} \times \frac{10}{100} \text{ m}^3 = 0.004 \text{ m}^3$$

$$\therefore \text{ Required number of bricks} = \frac{\text{Volume of bricks in the wall}}{\text{Volume of one brick}}$$

$$= \frac{51.84}{0.004} = 12960$$

So, 12960 bricks are used in constructing the wall.

Q14. Find the number of metallic circular discs with 1.5 cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Sol. Required number of metallic discs

$$= \frac{\text{Volume of right circular cylinder}}{\text{Volume of one metallic circular disc}}$$

$$= \frac{\pi \left(\frac{4.5}{2}\right)^2 \times 10}{\pi \left(\frac{15}{20}\right)^2 \times 0.2}$$

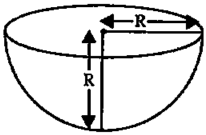
$$= (2.25)^2 \times \frac{400}{225} \times 50$$

$$= \frac{225}{100} \times \frac{225}{100} \times \frac{400}{225} \times 50 = 450$$

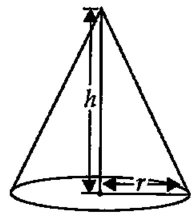
EXERCISE 12.4

Q1. A solid metallic hemisphere of radius 8 cm is melted and recasted into a right circular cone of base radius 6 cm. Determine the height of the cone.

Sol.



Hemisphere
R = 8 cm



Cone
$r = 6$ cm
$h = ?$

As the hemisphere is recasted into a cone. So,

Volume of cone = Volume of hemisphere

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi R^3$$

$$\Rightarrow \quad r^2 h = 2R^3$$

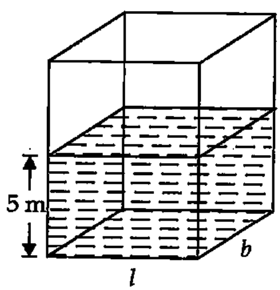
$$\Rightarrow \quad h = \frac{2R^3}{r^2} = \frac{2\times8\times8\times8}{6\times6} = \frac{32\times8}{9}$$

$$= \frac{256}{9} = 28.44 \text{ cm} \Rightarrow h = 28.44 \text{ cm}.$$

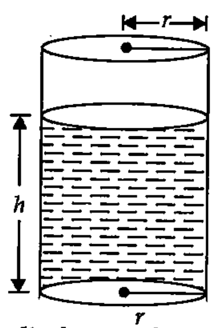
Hence, the height of the cone is 28.44 cm.

Q2. A rectangular water tank of base 11 m × 6 m contains water upto a height of 5 m. If the water in tank is transferred to a cylindrical tank of radius 3.5 m, find the height of the water level in the tank.

Sol.



Rectangular water tank
$l = 11$ m
$b = 6$ m
H = 5 m



Cylindrical tank
$r = 3.5$ m
$h = ?$

Water is transferred from cuboid to cylinder, so, the volume of water in both the vessels will be same.

$$\therefore \quad \pi r^2 h = l \times b \times H$$

$$\Rightarrow \quad \frac{22}{7}\times3.5\times3.5\times h = 11\times6\times5$$

$$\Rightarrow \quad h = \frac{11 \times 6 \times 5 \times 7 \times 100}{22 \times 35 \times 35} = \frac{60}{7}$$

$$\Rightarrow \quad h \cong 8.6 \text{ m (approx.)}$$

Hence, the height of water level in cylindrical tank is 8.6 m.

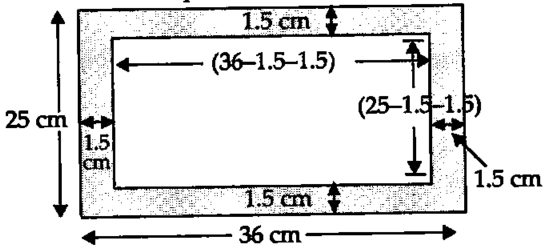
Q3. How many cubic centimetres of iron is required to construct an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm provided the thickness of the iron is 1.5 cm. If one cubic centimetre of iron weighs 7.5 g, then find the weight of the box.

Sol. External dimensions — **Internal dimensions**

$l_2 = 36$ cm — $l_1 = 36 - 1.5 - 1.5 = 36 - 3 = 33$ cm

$b_2 = 25$ cm — $b_1 = 25 - 3 = 22$ cm

$h_2 = 16.5$ cm — $h_1 = 16.5 - 1.5 = 15$ cm



Volume of iron in the open box

$$= l_2 b_2 h_2 - l_1 b_1 h_1$$

$$= (36 \times 25 \times 16.5) - (33 \times 22 \times 15)$$

$$= 9 \times 5 \times 11 \left[\frac{36 \times 25 \times 165}{10 \times 9 \times 5 \times 11} - \frac{33 \times 22 \times 15}{9 \times 5 \times 11}\right]$$

$\Rightarrow$ Volume of iron in the open box

$$= 9 \times 5 \times 11 \left[\frac{4 \times 5 \times 15}{10} - 22\right]$$

$$= 45 \times 11\,[30 - 22] = 495 \times 8 = 3960 \text{ cm}^3$$

Volume of iron is 3960 cm^3.

1 cm^3 of iron weighs = 7.5 gm

So, 3960 cm^3 of iron will weigh $= \frac{3960 \times 75}{10} = 396 \times 75$ gm

$$= \frac{396 \times 75}{1000} \text{ kg} = \frac{297}{10} \text{ kg}$$

Hence, the weight of the box = 29.7 kg

Q4. The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen is used up on writing 3300 words on an average. How many words can be written in a bottle of ink containing one-fifth of a litre?

Sol. Let n times the barrel of pen is filled.

Radius of barrel, $r = \frac{5 \text{ mm}}{2} = \frac{5}{20} \text{ cm} = \frac{1}{4} \text{ cm}$

Height of barrel, $h = 7$ cm

$\therefore \quad n \times \text{Volume of barrel} = \text{Volume of Ink}$

$$\Rightarrow \quad n \times \pi r^2 h = \frac{1}{5} \times \text{one litre}$$

$$\Rightarrow \quad n\, \pi r^2 h = \frac{1}{5} \times 1000 \text{ cm}^3$$

$$\Rightarrow \quad n \times \frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} \times 7 = 200 \text{ cm}^3$$

$$\Rightarrow \quad n = \frac{200 \times 7 \times 4 \times 4}{22 \times 7} = \frac{1600}{11}$$

Ink in one full barrel can write words = 3300

So, n barrels can write words = $3300n$

$$= 3300 \times \frac{1600}{11} = 4{,}80{,}000$$

Hence, the required number of words = 4,80,000 words.

Q5. Water flows at the rate of 10 m per min. through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the base is 40 cm and depth 24 cm?

Sol. When a fluid (water) flows through a pipe of area of cross-section A with velocity v, then volume of water coming from pipe in time t

= Area of cross-section × Length

$= A \times v.t$ $\quad$ [$\because$ V = Area of base × Height]

Cylinder

$A = \pi r^2$

$r = \frac{5 \text{ mm}}{2} = \frac{5}{2000} \text{ m}$

$r = \frac{1}{400} \text{ m}$

$v = 10 \text{ m/min} \Rightarrow v = \frac{10}{60} \text{ m/s} = \frac{1}{6} \text{ m/s}$

Cone

$R = \frac{40}{2} \text{ cm} = 0.2 \text{ m}$

$H = 24 \text{ cm} = 0.24 \text{ m}$

$\therefore \quad$ Volume of flowing water = Volume of cone

$$\Rightarrow \text{Area of base} \times \text{height (dist.)} = \frac{1}{3}\pi R^2 H$$

$$\Rightarrow \quad A \times v.t = \frac{1}{3}\pi R^2 H$$

$$\Rightarrow \quad \pi r^2.v.t = \frac{1}{3}\pi R^2 H$$

$$\Rightarrow \quad r^2.v.t = \frac{1}{3} R^2 H$$

$$\Rightarrow \quad \frac{1}{400}\times\frac{1}{400}\times\frac{1}{6}t = \frac{1}{3}\times 0.2\times 0.2\times 0.24$$

$$\Rightarrow \quad t = \frac{2\times 2\times 24\times 400\times 400\times 6}{3\times 10000}$$

$$\Rightarrow \quad t = 4\times 24\times 4\times 4\times 2 \text{ sec}$$

$$= \frac{4\times 24\times 4\times 4\times 2}{60}\text{min} = \frac{512}{10} = 51.2\,\text{min}$$

$$= 51 \text{ min} + 0.2 \text{ min} = 51 \text{ min} + 0.2\times 60 \text{ sec}$$

$$\Rightarrow \quad t = 51 \text{ min and } 12 \text{ sec.}$$

Hence, conical tank will fill in 51 min and 12 sec.

Q6. A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of rice. How much canvas cloth is required to just cover the heap?

Sol. Heap of rice is in shape of cone, so

$$r = \frac{9}{2}\text{ m} = 4.5 \text{ m}$$

$$h = 3.5 \text{ m}$$

$$\therefore \quad V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times\frac{9}{2}\times\frac{9}{2}\times 3.5$$

$$\Rightarrow \quad V = \frac{22\times 9\times 9\times 35}{3\times 7\times 2\times 2\times 10} = \frac{33\times 9}{4} = \frac{297}{4}$$

$$\Rightarrow \quad V = 74.25 \text{ m}^3$$

Hence, volume of rice = 74.25 m^3.

For Canvas:

Area of canvas = Curved surface area of cone

$= \pi r l$

But, $l^2 = r^2 + h^2 = (4.5)^2 + (3.5)^2 = 20.25 + 12.25$

$\Rightarrow \quad l^2 = 32.50$

$\Rightarrow \quad l = \sqrt{32.5} = 5.7 \text{ m}$

$\therefore$ Area of canvas $= \frac{22}{7}\times 4.5\times 5.7 = 80.614$

$\Rightarrow$ Area of canvas $= 80.61 \text{ m}^2$

Q7. A factory manufactures 1,20,000 pencils daily. The pencils are cylindrical in shape each of length 25 cm and circumference of base as 1.5 cm. Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at ₹ 0.05 per dm^2.

Sol. Shape of the pencil is cylindrical.

Here, $h = 25$ cm, $\quad 2\pi r = 1.5$ cm

Curved surface area of one pencil $= 2\pi rh$

$\therefore$ Curved surface area of 1,20,000 pencils

$$= 1,20,000 \times 2\pi rh = 1,20,000 \times 1.5 \times 25 \text{ cm}^2$$

$$= \frac{1,20,000 \times 15 \times 25}{10 \times 100} \text{dm}^2 = 600 \times 75 \text{ dm}^2$$

$\therefore$ Cost of colouring = ₹ $600 \times 75 \times 0.05$ = ₹ 2250

Hence, cost of colouring is ₹ 2250.

Q8. Water is flowing at the rate of 15 km/hr through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm?

Sol. Main concept: Volume of flowing water = $A.v.t$

Here, A = Area of cross-section of pipe or flowing water

v = Speed of water

t = Time

Here, A is equivalent to area of base and height equal to distance ($v.t$) and we know that V = area of base × height.

Flowing water	**Pond**
$A = \pi r^2$ (circular pipe)	$l = 50$ m
$r = \frac{14}{2}$ cm $= 7$ cm $= 0.07$ m	$b = 44$ m
$v = 15$ km/hr $= 15000$ m/hr	$h = 21$ cm $= 0.21$ m

$\therefore$ Volume of flowing water = Volume of same water in pond

$\Rightarrow$ $A.v.t = l \times b \times h$

$\Rightarrow$ $\pi r^2.v.t = l \times b \times h$

$\Rightarrow$ $\frac{22}{7} \times 0.07 \times 0.07 \times 15000t \text{ (hrs.)} = 50 \times 44 \times 0.21$

$\Rightarrow$ $$t = \frac{50 \times 44 \times 0.21 \times 7}{22 \times 0.07 \times 0.07 \times 15000}$$

$$= \frac{50 \times 44 \times 21 \times 7 \times 100}{22 \times 7 \times 7 \times 15000} \Rightarrow t = 2 \text{ hours}$$

Hence, time required is 2 hours.

Q9. A solid iron cuboidal block of dimensions 4.4 m × 2.6 m × 1 m is recasted into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

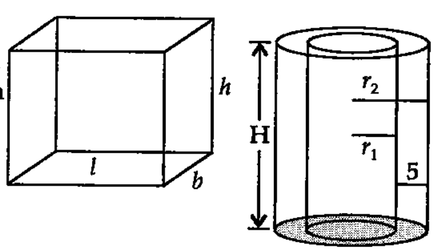
Sol.

Cuboid

$l = 4.4$ m $= 440$ cm

$b = 2.6$ m $= 260$ cm

$h = 1$ m $= 100$ cm



Cylindrical pipe

Thickness = 5 cm

$r_1 = 30$ cm

$r_2 = 30 + 5 = 35$ cm

H = ?

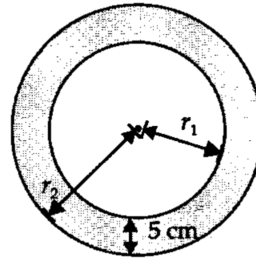
Cuboid is recasted into hollow cylindrical pipe.

$\therefore$ Volume of cuboid = Volume of cylindrical pipe (hollow)

$\Rightarrow \quad l\,bh = \pi r_2^2 H - \pi r_1^2 H$

$\Rightarrow \quad l\,bh = \pi H[r_2^2 - r_1^2]$

$\Rightarrow \quad 440 \times 260 \times 100 = \frac{22}{7} \times H\,[35^2 - 30^2]$



$\Rightarrow \quad 440 \times 260 \times 100 = \frac{22}{7} \times H\,[1225 - 900]$

$\Rightarrow \quad 440 \times 260 \times 100 = \frac{22}{7} \times H\,[325]$

$\Rightarrow \quad H = \frac{100 \times 440 \times 260 \times 7}{22 \times 325}$ m

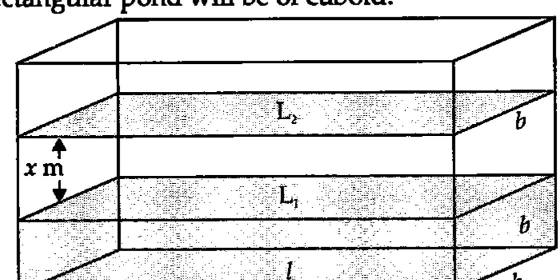
$= \frac{1600 \times 7}{100}$ m

$\Rightarrow \quad H = 112$ m

Hence, the length of pipe is 112 m.

Q10. 500 persons are taking a dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04 m^3?

Sol. Let the rise of water level in the pond be x m. The shape of water rise in rectangular pond will be of cuboid.



Here, $l = 80$ m, $b = 50$ m, $h = x$ m

Number of persons = 500

Let the water level before the persons took a dip was at L_1. Now, when 500 persons dipped into the pond, water level rises from L_1 to L_2 of height x m. The volume of water between two levels will be equal to the water displaced by 500 persons.

$\therefore$ Volume of water raised = Volume of cuboid

$\Rightarrow \quad l \times b \times h = 500 \times 0.04$

$\Rightarrow \quad 80 \times 50 \times x = 500 \times 0.04$

$\Rightarrow \quad x = \frac{500 \times 0.04}{80 \times 50} = \frac{1}{200}\text{ m} = \frac{100}{200}\text{ cm} = 0.5\text{ cm}$

$\Rightarrow \quad x = 0.5 \text{ cm}$

Hence, the rise of water level in the pond is 0.5 cm.

Q11. 16 glass spheres each of radius 2 cm are packed into cuboidal box of internal dimensions 16 cm × 8 cm × 8 cm and then the box is filled with water. Find the volume of water filled in the box.

Sol. **Cuboidal box** — $l = 16$ cm, $b = 8$ cm, $h = 8$ cm

16 spheres — $r = 2$ cm

Volume of water filled in the box = Volume of box – Volume of 16 glass spheres

$\therefore$ Vol. of water filled in the box $= lbh - 16.\frac{4}{3}\pi r^3$

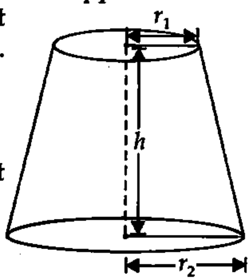
$$= 16 \times 8 \times 8 - \frac{16 \times 4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2$$

$$= 16 \times 8\left[\frac{16 \times 8 \times 8}{16 \times 8} - \frac{4 \times 22 \times 2 \times 2 \times 2}{3 \times 7 \times 1 \times 8}\right]$$

$$= 16 \times 8\left[8 - \frac{88}{21}\right] = 16 \times 8\left[\frac{168 - 88}{21}\right]$$

$$= \frac{16 \times 8 \times 80}{21} = \frac{10240}{21} = 487.6 \text{ cm}^3$$

Q12. A milk container of height 16 cm is made of metal sheet in the form of a frustrum of a cone with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk at the rate of ₹ 22 per L, which the container can hold.



Sol. Here, $r_1 = 8$ cm

$r_2 = 20$ cm

$h = 16$ cm

$\therefore$ Volume of milk = Volume of frustrum as it is filled completely

$$= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 \times r_2]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 16\,[8^2 + 20^2 + 8 \times 20]$$

$$= \frac{22 \times 16}{21}[64 + 400 + 160]$$

$$= \frac{352}{21} \times 624 = \frac{352 \times 208}{7} = \frac{73216}{7}$$

$$= 10459.428 \text{ cm}^3 = 10.459 \text{ litre}$$

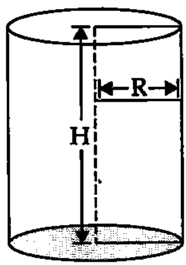
Volume of milk = 10.459 litre

$\therefore$ Cost of milk = ₹ 22 × 10.459 = ₹ 230.107

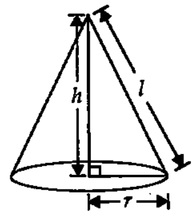
Hence, the cost of milk = ₹ 230.107.

Q13. A cylindrical bucket of height 32 cm and base radius 18 cm is filled with sand. This bucket is emptied on the ground, and a conical heap of sand is formed. If the height of conical heap is 24 cm, find the radius and slant height of the heap.

Sol. By identifying the shapes, we have cone and cylinder. On reshaping from cylindrical to conical, the volume of sand emptied out remains same.



Cylinder
R = 18 cm
H = 32cm



Cone (heap)
$r = ?$
$h = 24$ cm
$l = ?$

$\therefore$ Volume of conical heap = Volume of cylinder

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \pi R^2 H \quad \Rightarrow \quad \frac{1}{3} r^2 h = R^2 H$$

$$\Rightarrow \quad r^2 = \frac{3R^2H}{h} = \frac{3 \times 18 \times 18 \times 32}{24}$$

$$\Rightarrow \quad r^2 = 18 \times 18 \times 2 \times 2$$

$$\Rightarrow \quad r = 18 \times 2 \text{ cm} = 36 \text{ cm}$$

Radius of conical heap is 36 cm.

Now, $\quad l^2 = r^2 + h^2 = 36 \times 36 + 24 \times 24$

$$\Rightarrow \quad l^2 = 4 \times 4\,[9 \times 9 + 6 \times 6] = 4 \times 4\,[81 + 36]$$

$$= 4 \times 4 \times 117$$

$$\Rightarrow \quad l = \sqrt{4 \times 4 \times 3 \times 3 \times 13} = 4 \times 3\sqrt{13}$$

$$\Rightarrow \quad l = 12\sqrt{13} \cong 12 \times 3.60555$$

$$\Rightarrow \quad l = 43.2666 \text{ cm}$$

Hence, radius and slant height are 36 cm and 43.2666 cm respectively.

Q14. A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of cylinder. The diameter and height of cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, then find the total surface area and volume of rocket. (Use $\pi = 3.14$)

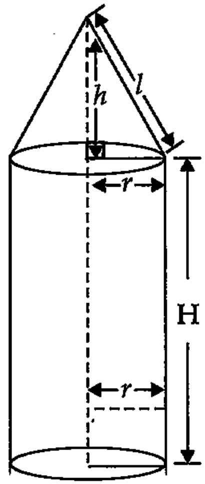
Sol.

Cylinder	Cone
$r = \frac{6}{2} = 3$ cm	$r = 3$ cm
H = 12 cm	$l = 5$ cm

$\therefore \quad l^2 = r^2 + h^2$ or $h^2 = l^2 - r^2$

$$= 5^2 - 3^2 = 25 - 9$$

$\Rightarrow$ $$h = \sqrt{16} = 4 \text{ cm}$$



Now, Volume of rocket

$$= \text{Volume of cylinder} + \text{Volume of cone}$$

$$= \pi r^2 H + \frac{1}{3}\pi r^2 h = \pi r^2 \left[H + \frac{1}{3}h\right]$$

$$= 3.14 \times 3 \times 3 \left[12 + \frac{1}{3} \times 4\right]$$

$$= 3.14 \times 9 \left[\frac{40}{3}\right] = 3.14 \times 3 \times 40 = 376.8 \text{ cm}^3$$

$\therefore$ Volume of Rocket = 376.8 cm^3

Total surface area of rocket

= Curved surface area of cylinder + Curved surface area of cone + Area of base of cylinder

[As it is closed (Given)]

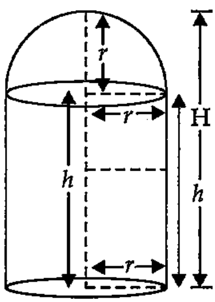
$$= 2\pi r H + \pi r l + \pi r^2 = \pi r [2H + l + r]$$

$$= 3.14 \times 3[2 \times 12 + 5 + 3] = 3.14 \times 3 \times 32 = 301.44 \text{ cm}^2$$

Hence, the surface area of rocket is 301.44 cm^2.

Q15. A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ m^3 of air. If the internal diameter of dome is equal to its total height above the floor, find the height of the building.

Sol. **Dome (hemisphere)** Radius = r — **Cylinder** Radius = r



$$H = 2r \quad \text{[given]}$$

$\Rightarrow$ $$h + r = 2r$$

$\Rightarrow$ $$h = 2r - r = r$$

$$\text{Volume of building} = 41\frac{19}{21} = \frac{880}{21} \text{ m}^3$$

$\Rightarrow$ Vol. of cylinder + Vol. of hemisphere = $\frac{880}{21}$ m^3

$\Rightarrow$ $$\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{880}{21}$$

$\Rightarrow$ $$\pi r^2 r + \frac{2}{3}\pi r^3 = \frac{880}{21} \quad [\because h = r]$$

$\Rightarrow$ $$\frac{5}{3}\pi r^3 = \frac{880}{21} \Rightarrow \frac{5}{3} \times \frac{22}{7} \times r^3 = \frac{880}{21}$$

$\Rightarrow$ $$r^3 = \frac{880 \times 3 \times 7}{21 \times 5 \times 22} \Rightarrow r^3 = 8$$

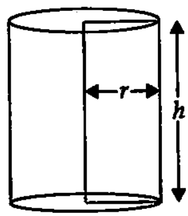
$\Rightarrow$ $$r = 2 \text{ m}$$

Hence, height of the building is $2 \times 2 = 4$ m. [$\because$ H = 2r (Given)]

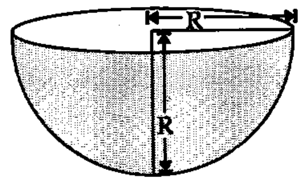
Q16. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped bottles each of radius 1.5 cm and height 4 cm. How many bottles are needed to empty the bowl?

Sol.

***n* cylindrical bottles**	**Hemisphere**
$r = 1.5$ cm	$R = 9$ cm
$h = 4$ cm	



Cylindrical bottle



Hemispherical bowl

As the volume of liquid does not change

So, Volume of n bottles = Volume of hemisphere

$$\Rightarrow \quad n\pi r^2 h = \frac{2}{3}\pi R^3 \Rightarrow nr^2h = \frac{2}{3}R^3$$

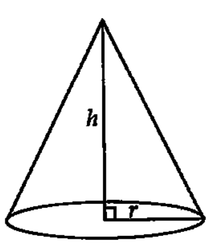
$$\Rightarrow \quad n \times 1.5 \times 1.5 \times 4 = \frac{2}{3} \times 9 \times 9 \times 9$$

$$\Rightarrow \quad n = \frac{2 \times 9 \times 9 \times 9 \times 100}{3 \times 15 \times 15 \times 4} = 54$$

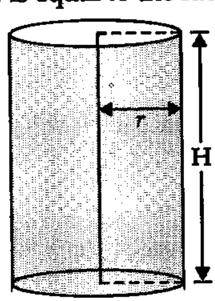
Hence, 54 bottles are needed.

Q17. A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm such that it touches the bottom. Find the volume of water left in cylinder, if the radius of the cylinder is equal to the radius to the cone.

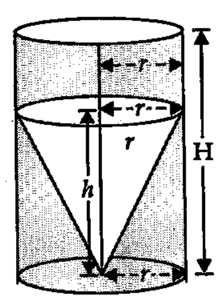
Sol.



Cone



Cylinder



Cone & Cylinder

Cone	**Cylinder**
$r = 60$ cm	$R = r = 60$ cm
$h = 120$ cm	$H = 180$ cm

Cone is placed inside the cylindrical vessel full of water. So, the volume of water from cylinder will over flow equal to the volume of cone.

Hence, the water left in cylinder = Vol. of cylinder – Vol. of cone ...(i)

Volume of water left after immersing the cone into cylinder full of water

$$= \text{Volume of cylinder} - \text{Volume of cone}$$

$$= \pi R^2 H - \frac{1}{3}\pi r^2 h$$

∴ Required volume of water in cylinder

$$= \pi r^2 H - \frac{1}{3}\pi r^2 h \qquad [\because R = r]$$

$$= \pi r^2\left[H - \frac{1}{3}h\right] = \frac{22}{7} \times 60 \times 60\left[180 - \frac{120}{3}\right]$$

$$= \frac{22}{7} \times 60 \times 60 \times 140 \text{ cm}^3$$

$$= \frac{22 \times 60 \times 60 \times 140}{7 \times 100 \times 100 \times 100} = \frac{22 \times 72}{1000} = \frac{1584}{1000}$$

∴ Vol. of water in cylinder = 1.584 m^3

Hence, required volume of water left = 1.584 m^3.

Q18. Water flows through a cylindrical pipe, whose inner radius is 1 cm at the rate of 80 cm per second in an empty cylindrical tank, the radius of whose base is 40 cm. What is the rise of water level in tank in half an hour?

Sol. Main concept: Volume of flowing water = $A.v.t$

Area of base = A = Area of cross-section of flowing water

height = distance = speed × time = $v.t$

Flowing water is filled in cylindrical tank. Hence, the volume of flowing water is equal to volume of water in cylindrical tank.

Flowing water	**Cylinder**
$A = \pi r^2$ (cylinder)	$R = 40$ cm
$v = 80$ cm/s	$H = x$

$r = 1$ cm and $t = \frac{1}{2}\text{hr} = \frac{1}{2} \times 3600 \text{ sec} = 1800 \text{ sec}$

∴ Volume of water in cylindrical tank = Volume of flowing water

$$\Rightarrow \quad \pi R^2 H = A.v.t$$

$$\Rightarrow \quad \pi R^2 H = \pi r^2 v.t$$

$$\Rightarrow \quad 40 \times 40 \times x = 1 \times 1 \times 80 \times 1800$$

$$\Rightarrow \quad x = \frac{80 \times 1800}{40 \times 40} = 5 \times 18 = 90 \text{ cm}$$

Hence, the rise of water level in cylindrical tank is 90 cm.

Q19. The rain water from a roof of dimensions 22 m × 20 cm drains into a cylindrical vessel having diameter of the base 2 m and height 3.5 m. If the rain water collected from the roof just fill the cylindrical vessel, then find the rainfall in cm.

Sol.	**Cuboid**	**Cylinder**
	$l = 22\text{ m} = 2200\text{ cm}$	$r = \frac{2}{2} = 1\text{ m} = 100\text{ cm}$
	$b = 20\text{ m} = 2000\text{ cm}$	$h = 3.5\text{ m} = 350\text{ cm}$
	$H = x\text{ cm}$	

If water from roof is not allowed to flow, then water level on roof rises upto x cm (let) then volume of cuboidal shape of water will be equal to the volume of cylinder.

$\therefore$ Volume of rain water = Volume of cylinder

$$\Rightarrow \quad l \times b \times h = \pi r^2 h$$

$$\Rightarrow \quad 2200 \times 2000 \times x = \frac{22}{7} \times 100 \times 100 \times 350$$

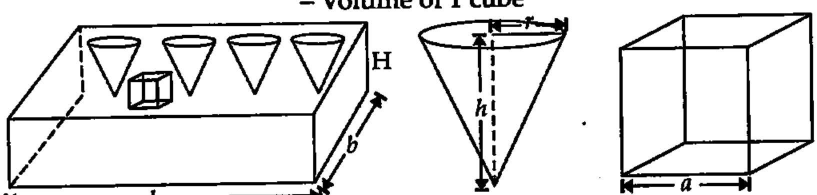
$$\Rightarrow \quad x = \frac{22 \times 100 \times 100 \times 350}{7 \times 2200 \times 2000} = \frac{5}{2} = 2.5\text{ cm}$$

Hence, the rainfall is 2.5 cm.

Q20. A pen stand made of wood is in the shape of a cuboid with four conical depressions and a cubical depression to hold the pens and pins respectively. The dimensions of cuboid are 10 cm, 5 cm, 4 cm. The radius of each of the conical depressions is 0.5 cm and depth is 2.1 cm. The edge of the cubical depression is 3 cm. Find the volume of the wood in the entire stand.

Sol. From a cuboidal piece of wood, depressions (4 cones and 1 cube) are made.

So, the volume of wood= Volume of cuboid – Volume of 4 cones – Volume of 1 cube



$l = 10$ cm
$b = 5$ cm
$H = 4$ cm

$r = 0.5$ cm
$h = 2.1$ cm

$a = 3$ cm

Hence, the volume of the wood in the entire pen stand

$$= l \times b \times h - \left(\frac{1}{3}\pi r^2 h\right) \times 4 - a^3$$

$$= 10 \times 5 \times 4 - \frac{4}{3} \times \frac{22}{7} \times \frac{5 \times 5 \times 21}{1000} - (3)^3$$

$$= 200 - \frac{11 \times 5 \times 4}{100} - 27 = 200 - 2.20 - 27 = 200 - 29.2 = 170.8\text{ cm}^3$$

So, the volume of the wood in the pen stand after making depressions is 170.8 cm^3.

❑❑❑

13 Statistics and Probability

EXERCISE 13.1

Choose the correct answer from the given four options in the following questions:

Q1. In the formula $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$ for finding the mean of grouped data d_i's are the deviations from a of

(*a*) lower limits of the classes
(*b*) upper limits of the classes
(*c*) mid points of the classes
(*d*) frequencies of the class marks

Sol. (*c*): In the given formula, a is assumed mean from class marks (x_i) and $d_i = x_i - a$

Hence, d_i is the deviation of class mark (mid-value) from the assumed mean 'a'. Hence, verifies the option (*c*).

Q2. While computing mean of grouped data, we assume that the frequencies are

(*a*) evenly distributed over all the classes
(*b*) centred at the class marks of the classes
(*c*) centred at the upper limits of the classes
(*d*) centred at the lower limits of the classes.

Sol. (*b*): In grouping the data from ungrouped data all the observations between lower and upper limits of class marks are taken in one group then mid value or class mark is taken for further calculation.

Hence frequencies or observations must be centred at the class marks of the classes.

Hence, verifies the option (*b*).

Q3. If x_i's are the mid points of the class intervals of grouped data, f_i's are the corresponding frequencies and $\bar{x}$ is the mean, then $\sum (f_i x_i - \bar{x})$ is equal to

(*a*) 0 (*b*) −1 (*c*) 1 (*d*) 2

Sol. (*a*): $\because \quad \bar{x} = \sum_{i=1}^{n} \frac{f_i x_i}{n}$

$$\therefore \quad \sum_{i=1}^{n} f_i x_i = n\bar{x} \qquad \text{(I)}$$

$$\sum_{i=1}^{n} \bar{x} = \bar{x} + \bar{x} + \bar{x} + \cdots n \text{ times}$$

$$\Rightarrow \quad \sum_{i=1}^{n} \bar{x} = n\bar{x} \qquad \text{(II)}$$

From (I) and (II), we have

$$\sum_{i=1}^{n} f_i x_i = \sum_{i=1}^{n} \bar{x}$$

$$\Rightarrow \quad \sum_{i=1}^{n} f_i x_i - \sum_{i=1}^{n} \bar{x} = 0$$

$$\Rightarrow \quad \sum_{i=1}^{n} (f_i x_i - \bar{x}) = 0 \quad \text{or} \quad \sum f_i x_i - \bar{x} = 0$$

Hence, verifies the option (*a*).

Q4. In the formula $\bar{x} = a + h\left(\dfrac{\sum f_i u_i}{\sum f_i}\right)$, for finding the mean of grouped frequency distribution, $\mu_i =$

(*a*) $\dfrac{x_i + a}{h}$ (*b*) $h(x_i - a)$ (*c*) $\dfrac{x_i - a}{h}$ (*d*) $\dfrac{a - x_i}{h}$

Sol. (*c*): $\bar{x} = a + h\left[\dfrac{\sum f_i u_i}{\sum f_i}\right]$

$u_i = \dfrac{(x_i - a)}{h}$ verifies the option (*c*).

Q5. The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its

(*a*) mean (*b*) median (*c*) mode (*d*) all of these

Sol. (*b*): The point of intersection of the less than type and of the more than type cumulative frequency curves give the median on abscissa as on X-axis we take the upper or lower limits respectively and on Y-axis we take cumulative frequency.

Hence verifies the option (*b*).

Q6. For the following distribution:

Class	0–5	5–10	10–15	15–20	20–25
Frequency	10	15	12	20	9

the sum of lower limits of median class and modal class is

(*a*) 15 (*b*) 25 (*c*) 30 (*d*) 35

Sol. (*b*):

Class	Frequency	Cumulative frequency
0–5	10	10
5–10	15	25
10–15	12	37
15–20	20	57
20–25	9	66

The modal class is the class having the maximum frequency.
The maximum frequency 20 belongs to class (15–20).
Here, $n = 66$

So, $\frac{n}{2} = \frac{66}{2} = 33$

33 lies in the class 10 – 50.
Therefore, 10 – 15 is the median class.
So, sum of lower limits of (15–20) and (10–15) is (15 + 10) = 25 verifies the option (*b*).

Q7. Consider the following frequency distribution:

Class	0–5	6–11	12–17	18–23	24–29
Frequency	13	10	15	8	11

the upper limit of the median class is

(*a*) 7 (*b*) 17.5 (*c*) 18 (*d*) 18.5

Sol. (*b*):

Class	Frequency	Cumulative frequency
0.5–5.5	13	13
5.5–11.5	10	23
11.5–17.5	15	38
17.5–23.5	8	46
23.5–29.5	11	57

The median of 57 (odd) observations $= \frac{57+1}{2} = \frac{58}{2} = 29$th term

29th term lies in class 11.5–17.5.
So, upper limit is 17.5 verifies option (*b*).

Q8. For the following distribution the modal class is

(*a*) 10–20 (*b*) 20–30 (*c*) 30–40 (*d*) 50–60

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

Sol. (*c*):

Marks	Number of students or Frequency	f_i
0–10	3 − 0 = 3	3
10–20	12–3 = 9	9
20–30	27 − 12 = 15	15
30–40	57 − 27 = 30	30
40 − 50	75 − 57 = 18	18
50 − 60	80 − 75 = 5	5

Modal class has maximum frequency (30) in class 30–40.
Hence, verifies the option (*c*).

Q9. Consider the data

Class	65–85	85–105	105–125	125–145	145–165	165–185	185–205
Frequency	4	5	13	20	14	7	4

The difference of the upper limit of the median class and the lower limit of the modal class is

(*a*) 0 (*b*) 19 (*c*) 20 (*d*) 38

Sol. (*c*):

Class	Frequency	Cumulative frequency
65–85	4	4
85–105	5	9
105–125	13	22
125–145	20	42
145–165	14	56
165–165	7	63
185–205	4	67

Hence, $n = 67$ (odd)

So, Median $= \dfrac{67+1}{2} = 34$

34 lies in class 125 – 145.
So, median class is 125 – 145 and upper limit is 145.
Now, the maximum frequency is 20 and it lies in class 125 – 145 (Modal class).
Lower limit of modal class = 125.

Hence, the required difference 145 – 125 = 20, verifies the option (*c*).

Q10. The times, in seconds, taken by 150 athletes to run a 110 m hurdle race are tabulated below–

Class	13.8–14	14–14.2	14.2–14.4	14.4–14.6	14.6–14.8	14.8–15
Frequency	2	4	5	71	48	20

The number of athletes who completed the race in less than 14.6 seconds is:

(*a*) 11 (*b*) 71 (*c*) 82 (*d*) 130

Sol. (*c*): The number of athletes who completed the race in less than 14.6 sec = 2 + 4 + 5 + 71 = 82.

Hence verifies the option (*c*).

Q11. Consider the following distribution:

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

The frequency of the class 30–40 is

(*a*) 3 (*b*) 4 (*c*) 48 (*d*) 51

Sol. (*a*):

Class	Number of Students	*f*
0–10	63 – 58 = 5	5
10–20	58 – 55 = 3	3
20–30	55 – 51 = 4	4
30–40	51 – 48 = 3	3
40–50	48 – 42 = 6	6
50–60	42 – 0 = 42	42

Hence the frequency of 30–40 class interval is 3 which verifies the option (*a*).

Q12. If an event cannot occur, then its probability is

(*a*) 1 (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{2}$ (*d*) 0

Sol. (*d*): An event that cannot occur has 0 probability, such an event is called impossible event. Hence, (*d*) is the correct answer.

Q13. Which of the following cannot be the probability of an event?

(*a*) $\frac{1}{3}$ (*b*) 0.1 (*c*) 3% (*d*) $\frac{17}{16}$

Sol. (*d*): Probability of any event cannot be more than one or negative as $\frac{17}{16} > 1$.

Hence, verifies the option (*d*).

Q14. An event is very unlikely to happen. Its probability is closest to

(*a*) 0.0001 (*b*) 0.001 (*c*) 0.01 (*d*) 0.1

Sol. (*a*): The probability of the event which is very unlikely to happen will be very close to zero. So it's probability is 0.0001 which is minimum among the given values.

Hence, verifies the option (*a*).

Q15. If the probability of an event is p, then the probability of its complementary event will be

(*a*) $p-1$ (*b*) p (*c*) $1-p$ (*d*) $1-\frac{1}{p}$

Sol. (*c*): Probability of an event + Probability of its complementary event = 1

$\therefore$ p + Probability of complement = 1

or Probability of complement = $1-p$

Hence, verifies the option (*c*).

Q16. The probability expressed as a percentage of a particular occurrence can never be

(*a*) less than 100 (*b*) less than 0

(*c*) greater than 1 (*d*) anything but a whole number

Sol. (*b*): $\because$ Probability lies between 0 and 1 and when it is converted into percentage it will be between 0 and 100. So, cannot be negative.

So, verifies the option (*b*).

Q17. If P(A) denotes the probability of an event A, then

(*a*) $P(A) < 0$ (*b*) $P(A) > 1$

(*c*) $0 \le P(A) \le 1$ (*d*) $-1 \le P(A) \le 1$

Sol. (*c*): As the probability of an event can be between 0 and 1. Hence, verifies the option (*c*).

Q18. If a card is selected from a deck of 52 cards, then the probability of its being a red face card is

(*a*) $\frac{3}{26}$ (*b*) $\frac{3}{13}$ (*c*) $\frac{2}{13}$ (*d*) $\frac{1}{2}$

Sol. (*a*): In a deck of 52 cards, there are 26 red cards.

Number of red face cards = 3 of hearts + 3 of diamonds
= 6

So, probability of having a red face card = $\frac{6}{52} = \frac{3}{26}$

Hence, verifies the option (*a*).

Q19. The probability that a non leap year selected at random will contains 53 sundays is

(*a*) $\frac{1}{7}$ (*b*) $\frac{2}{7}$ (*c*) $\frac{3}{7}$ (*d*) $\frac{5}{7}$

Sol. (*a*): Number of days in non leap year = 365

$$\text{Number of weeks} = \frac{365}{7} = 52\frac{1}{7} = 52 \text{ weeks}$$

Number of days left = 1

i.e., may be any of 7 days which from Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday so T(E) = 7

$$F(E) = 1 \text{ (Sunday)}$$

$$P(F) = \frac{F(E)}{T(E)} = \frac{1}{7}$$

Hence, verifies option (*a*).

Q20. When a die is thrown, the probability of getting an odd number less than 3 is

(*a*) $\frac{1}{6}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{2}$ (*d*) 0

Sol. (*a*): Total number of outcomes favourable for event E are (1, 2, 3, 4, 5, 6) *i.e.*, T(E) = 6

A number which is odd and less than 3 is 1 so, F(E) = 1

So, probability $P(E) = \frac{F(E)}{T(E)} = \frac{1}{6}$

Hence, verifies option (*a*).

Q21. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is

(*a*) 4 (*b*) 13 (*c*) 48 (*d*) 51

Sol. (*d*): Favourable event E is all cards except the ace of heart and ace of heart is only one. Hence, the number of outcomes favourable for event E are 52 – 1 = 51, verifies the option (*d*).

Q22. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is

(*a*) 7 (*b*) 14 (*c*) 21 (*d*) 28

Sol. (*b*): T(E) = 400

Number of outcomes favourable for event E, *i.e.*, F(E) = ?

$$P(F) = 0.035$$

$$\therefore \quad P(F) = \frac{F(E)}{T(E)} \Rightarrow 0.035 = \frac{F(E)}{400}$$

So, F(E) = 0.035 × 400 = 14 eggs. So, the number of bad eggs are 14.

Hence, verifies the option (*b*).

Q23. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?

(*a*) 40 (*b*) 240 (*c*) 480 (*d*) 750

Sol. (*c*): T(E) = 6000

$$F(E) = ?$$
$$P(F) = 0.08$$
$$\therefore \quad P(F) = \frac{F(E)}{T(E)} \Rightarrow 0.08 = \frac{F(E)}{6000}$$
$$\therefore \quad F(E) = 6000 \times 0.08 = 480$$

Hence, verifies the option (*c*).

Q24. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is

(*a*) $\frac{1}{5}$ (*b*) $\frac{3}{5}$ (*c*) $\frac{4}{5}$ (*d*) $\frac{1}{3}$

Sol. (*a*): T(E) = 40

Number of outcomes favourable for event E are 5, 10, 15, 20, 25, 30, 35, 40 *i.e.*, F(E) = 8

$$P(F) = \frac{F(E)}{T(E)} = \frac{8}{40} = \frac{1}{5}$$

Hence, verifies option (*a*).

Q25. Someone is asked to take a number from 1 to 100. The probability that it is a prime is

(*a*) $\frac{1}{5}$ (*b*) $\frac{6}{25}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{13}{50}$

Sol. (*c*): T(E) = 100

F(E) prime numbers (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97)

$$F(E) = 25$$
$$P(F) = \frac{F(E)}{T(E)} = \frac{25}{100} = \frac{1}{4}$$ Hence, verifies option (*c*).

Q26. A school has five houses, A B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from A, B and C is

(*a*) $\frac{4}{23}$ (*b*) $\frac{6}{23}$ (*c*) $\frac{8}{23}$ (*d*) $\frac{17}{23}$

Sol. (*b*): T(E) = 23

$$F(E) = \text{not from A, B, C i.e.} = 23 - (4 + 8 + 5)$$
$$F(E) = 23 - 17 = 6$$
$$\therefore \quad P(F) = \frac{6}{23}$$ verifies the option (*b*).

EXERCISE 13.2

Q1. The median of an ungrouped data and the median calculated, when the same data is grouped are always the same. Do you think that this is a correct statement? Give reason.

Sol. The median of an ungrouped data and the median calculated when the same data is grouped are not always the same because the median for ungrouped data is calculated by arranging the data in increasing or decreasing order. But for calculating the median of a grouped data, the formula used is based on the assumption that the observations are uniformly distributed in the classes.

Q2. In calculating the mean of grouped data, grouped in classes of equal width, we may use the formula

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

where a = assumed mean.

a must be one of the mid-points of the classes. Is the last statement correct? Justify your answer.

Sol. Not always. Assumed mean can be considered any convenient number which makes calculation easy.

Q3. Is it true to say that the mean, mode and median of grouped data will always be different? Justify your answer.

Sol. Not always. The median, mean and mode can be the same.

They may be equal if number of observations are odd and are equispaced.

Q4. Will the median class and modal class of grouped data always be different? Justify your answer.

Sol. The median and modal class may be same if modal class is median class which is not always possible as the number of frequencies may be maximum in any class.

So given statement is not true.

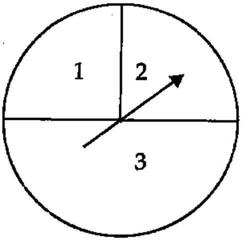
Q5. In a family having three children, there may be no girl, one girl, two girls or three girls. So, the probability of each is 1/4. Is this correct? Justify your answer.

Sol. False: In a family of three children events are (b, b, b,) (g, b, b), (g, g, b), (g, g, g)

T(E) = 4

The probability of each is not 1/4, because the outcomes are not equally likely.

Q6. A game consists of spinning an arrow which comes to rest pointing at one of the regions (1, 2, or 3) see figure. Are the outcomes 1, 2 and 3 equally likely to occur? Give reason.



Sol. The area of region 3 is double either of 1 or 2 and area of 1 and 2 are equal so no. of outcomes (or probability) of region 3 is double of either 1 or 2.

So the outcomes of 1, 2, 3 are not equally likely to occur.

Q7. Apoorv throws two dice once and compules the product of the numbers appearing on the dice. Peehu throws one die and squares the number that appears on it. Who has better chance of getting the number 36? Why?

Sol. For Apoorv T(E) = 36

Favourable is only (6, 6) *i.e.*, F(E) = 1

then P(F) by Apoorv $= \frac{F(E)}{T(E)} = \frac{1}{36}$

Now for Peehu T′(E) = 6

$$F'(E) = 1$$

$$P'(A) = \frac{F'(E)}{T'(E)} = \frac{1}{6}$$

$$\frac{1}{6} > \frac{1}{36}$$

$$\therefore \quad P'(A) > P(A)$$

Hence, Peehu has the better chance.

Q8. When we toss a coin, there are two possible outcomes—head or tail. Therefore, the probability of each outcome is 1/2. Justify your answer.

Sol. There are two outcomes of equally in all manner. So probability of both head and tail are equal to 1/2 each.

Hence, the given statement is true.

Q9. A student says that if you throw a die, it will show up 1 or not 1. Therefore, the probability of getting 1 and the probability of getting not 1 each is equal to 1/2. Is this correct? Give reason.

Sol. A dice can be thrown in 6 different equally likely ways. Possible outcomes are given by S = {1, 2, 3, 4, 5, 6}.

P(getting 1) $= \frac{1}{6}$ and P(not getting 1) $= \frac{5}{6}$.

Hence, the given statement is not correct.

Q10. I toss three coins together. The possible outcomes are no heads, 1 head, 2 heads and 3 heads. So, I say that probability of no heads is 1/4. What is wrong with this conclusion?

Sol. Three coins are tossed together.

Total outcomes T(E) $= 2^3 = 8$

(T T H), (T H H), (H T H), (H H T), (H T T), (T H T) and (H H H), (T T T), so, the number of favourable outcomes for event (getting no head) = 1

$\therefore$ Probability (getting no head) $= \frac{1}{8}$

Hence the given statement is wrong $\left(\because \frac{1}{8} \neq \frac{1}{4}\right)$.

Q11. If you toss a coin 6 times and it comes down head on each occasion. Can you say that the probability of getting a head is 1? Given reasons.

Sol. A coin is tossed 6 times so

$$T(E) = 6$$

In total six events, number of outcomes for getting head are 3 so F(E) = 3 again

$$P(F) \text{ getting head} = \frac{3}{6} = \frac{1}{2}$$

Hence, the given statement is false.

Q12. Sushma tosses a coin 3 times and gets tail each time. Do you think that the outcome of next toss will be tail? Give reasons?

Sol. As the coin is tossed 3 times and gets tail each time but it is not necessary that 4th time will be tail it may be either tail or head in any further toss.

Hence, the given statement is false.

Q13. If I toss a coin 3 times and get head each time, should I except a tail to have a higher chance in the 4th toss? Give reason in support of your answer.

Sol. As we know that a coin has two equal chances always either head or tail. So next time on tossing he can get either tail or head.

So, the given statement is false.

Q14. A bag contains slips numbered from 1 to 100. If Fatima chooses a slip at random from the bag, it will either be an odd number or an even number. Since this situation has only two possible outcomes, so, the probability of each is 1/2. Justify.

Sol. From 1 to 100 numbers, there are 50 even and 50 odd numbers.

Total number of outcomes T(E) = 100

Number of outcomes favourable for event E = F(E) = 50

$$\text{So,} \quad P(F) = \frac{50}{100} = \frac{1}{2}$$

Similarly, the probability of getting odd numbers is $\frac{1}{2}$. Hence the probability of getting odd and even each is —. Hence, the given statement is true.

EXERCISE 13.3

Q1. Find the mean of the distribution:

Class	1–3	3–5	5–7	7–10
Frequency	9	22	27	17

Sol.

Class	Class mark (x_i)	Frequency (f_i)	$f_i x_i$
1–3	2	9	18
3–5	4	22	88

5–7	6	27	162
7–10	8.5	17	144.5
		$\Sigma f_i = 75$	$\Sigma f_i x_i = 412.5$

$$\text{Mean } \bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{412.5}{75} = 5.5$$

Hence, the mean of the given distribution is 5.5.

Q2. Calculate the mean of the scores of 20 students in a mathematics test:

Marks	10–20	20–30	30–40	40–50	50–60
No. of students	2	4	7	6	1

Sol.

Marks	**Class Mark (x_i)**	**No. of students (f_i)**	$f_i x_i$
10–20	15	2	30
20–30	25	4	100
30–40	35	7	245
40–50	45	6	270
50–60	55	1	55
		$\Sigma f_i = 20$	$\Sigma f_i x_i = 700$

$$\therefore \quad \text{Means } \bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{700}{20} = 35$$

Q3. Calculate the mean of the following data:

Class	4–7	8–11	12–15	16–19
Frequency	5	4	9	10

Sol. Class marks of these classes are same, so no need to convert given data to continuous.

Class	**Class marks (x_i)**	$d_i = x_i - a$	**Frequency (f_i)**	$f_i d_i$
4–7	5.5	−4	5	−20
8–11	9.5 = a	0	4	0
12–15	13.5	+4	9	36
16–19	17.5	+8	10	80
			$\Sigma f_i = 28$	$\Sigma f_i d_i = 96$

a = assumed mean, d_i = deviation from mean

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 9.5 + \frac{96}{28} = 9.5 + 3.43$$

$$\therefore \quad \bar{x} = 12.93$$

Hence, the mean = 12.93.

Q4. The following table gives the number of pages written by Sarika for completing her own book for 30 days:

No. of pages written per day	16–18	19–21	22–24	25–27	28–30
No. of days	1	3	4	9	13

Find the mean number of pages written per day.

Sol. No need to change the class-intervals into continuous intervals as Class marks of continuous and discontinuous classes are same. d_i is deviation from assumed mean.

Class interval	Mid Value (x_i)	$d_i = (x_i - a)$	No. of days (f_i)	$f_i d_i$
16–18	17	−6	1	−6
19–21	20	−3	3	−9
22–24	$a = 23$	0	4	0
25–27	26	3	9	27
28–30	29	6	13	78
			$\Sigma f_i = 30$	$\Sigma f_i d_i = 90$

a = assumed mean, $a = 23$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$= 23 + \frac{90}{30} = 23 + 3 = 26$$

$\therefore \quad \bar{x} = 26$

Hence, the mean of pages written per day is 26.

Q5. The daily income of a sample of 50 employees are tabulated as follows:

Income (in ₹)	1–200	201–400	401–600	601–800
No. of employees	14	15	14	7

Find the mean daily income of employees.

Sol. No need to convert discontinuous classes into continuous for class mark because class mark of both C.I. are same and gives same result of $\bar{x}$.

C.I.	x_i	$d_i = (x_i - a)$	f_i	$f_i d_i$
1–200	100.5	−200	14	−2800
201–400	300.5 = a	0	15	0
401–600	500.5	+200	14	2800
601–800	700.5	+400	7	2800
			$\Sigma f_i = 50$	$\Sigma f_i d_i = 2800$

a = assumed mean $\qquad d_i = x_i - a$

Let $\qquad a = 300.5$

$$\therefore \quad \bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 300.5 + \frac{2800}{50} = ₹\ 356.5$$

$$\bar{x} = ₹\ 356.5$$

Hence, the average daily income of employees is ₹ 356.5.

Q6. An aircraft has 120 passenger seats. The number of seats occupied during 100 flights is given in the following table:

No. of seats	100–104	104–108	108–112	112–116	116–120
Frequency	15	20	32	18	15

Determine the mean number of seats occupied over the flights.

Sol. Let a = assumed mean

d_i = deviation of x_i from assumed mean = $x_i - \bar{a}$

f_i = frequencies (No. of passengers)

C.I. = Number of seats occupied in that flight

x_i = Class mark of ith C.I.

C.I.	x_i	$d_i = (x_i - a)$	f_i	$f_i d_i$
100–104	102	−8	15	−120
104–108	106	−4	20	−80
108–112	110 = a	0	32	0
112–116	114	4	18	72
116–120	118	8	15	120
			$\Sigma f_i = 100$	$\Sigma f_i d_i = -8$

Here, $a = 110$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 110 + \frac{-8}{100} = 110 - 0.08$$

$\bar{x}$ = 109.92, but, seat cannot be in decimal, so,

$$\Rightarrow \quad \bar{x} = 109$$

Hence, the mean number of seats occupied over the flights is 109.

Q7. The weights (in kg) of 50 wrestlers are recorded in the following table:

Weight in Kg	100–110	110–120	120–130	130–140	140–150
No. of Wrestlers	4	14	21	8	3

Find the mean weight of wrestlers.

Sol. a = assumed mean from x_i (weight in kg) = 125

x_i = class mark of classes (in kg)

d_i = deviation of x_i from $a = (x_i - a)$ (kg)

f_i = frequency (no. of wrestlers)

(C.I.) class interval = Number of wrestlers

C.I.	x_i	$d_i = (x_i - a)$	f_i	$f_i d_i$
100–110	105	–20	4	–80
110–120	115	–10	14	–140
120–130	125 = a	0	21	0
130–140	135	10	8	80
140–150	145	20	3	60
			$\Sigma f_i = 50$	$\Sigma f_i d_i = -80$

$a = 125$ kg

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$\bar{x} = 125 + \frac{(-80)}{50} = 125 - \frac{8}{5} = 125 - 1.6$$

$\Rightarrow$ $\bar{x} = 123.4$ kg

Hence, the mean weight of wrestlers = 123.4 kg

Q8. The mileage (km/litre) of 50 cars of the same model was tested by a manufacturer and details are tabulated as given below:

Mileage (km/l)	10–12	12–14	14–16	16–18
Number of cars (f_i)	7	12	18	13

Find the mean mileage.

The manufacturer claimed that the mileage of the model was 16 km L^{-1}. Do you agree with this claim?

Sol. $d_i = x_i - a$

x_i = class mark and a = assumed mean.

C.I.	x_i	$d_i = (x_i - a)$	f_i	$f_i d_i$
10–12	11	–2	7	–14
12–14	13 = a	0	12	0
14–16	15	2	18	36
16–18	17	4	13	52
			$\Sigma f_i = 50$	$\Sigma f_i d_i = 74$

$a = 13$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 13 + \frac{74}{50} = 13 + 1.48 = 14.48 \text{ km L}^{-1}$$

Hence, mean mileage of car is 14.48 km/litre.

So, the manufacturer's is statement is wrong that mileage is 16 km L^{-1}.

Q9. The following is the distribution of weights (in kg) of 40 persons.

Weight (in kg)	40–45	45–50	50–55	55–60	60–65	65–70	70–75	75–80
No. of person	4	4	13	5	6	5	2	1

Construct a cumulative frequency distribution (of the less than type) table for the above data.

Sol.

C.I.	f_i	Weight (in kg)	Cumulative frequency
40–45	4	less than 45	4 + 0 = 4
45–50	4	less than 50	4 + 4 = 8
50–55	13	less than 55	13 + 8 = 21
55–60	5	less than 60	5 + 21 = 26
60–65	6	less than 65	6 + 26 = 32
65–70	5	less than 70	5 + 32 = 37
70–75	2	less than 75	2 + 37 = 39
75–80	1	less than 80	1 + 39 = 40

Q10. The following table show the cumulative frequency distribution of marks of 800 students in an examination:

Marks	Number of Students	Marks	Number of Students
Below 10	10	Below 60	570
Below 20	50	Below 70	670
Below 30	130	Below 80	740
Below 40	270	Below 90	780
Below 50	440	Below 100	800

Construct a frequency distribution table for the data above.

Sol.

Marks	Number of students	C.I. (Marks)	Frequency
Below 10	10	0–10	10 – 0 = 10
Below 20	50	10–20	50–10 = 40
Below 30	130	20–30	130–50 = 80
Below 40	270	30–40	270–130 = 140
Below 50	440	40–50	440 – 270 = 170
Below 60	570	50–60	570 – 440 = 130
Below 70	670	60–70	670–570 = 100
Below 80	740	70–80	740 – 670 = 70
Below 90	780	80–90	780 – 740 = 40
Below 100	800	90–100	800–780 = 20

Q11. Form the frequency distribution table from the following data:

Marks (out of 90)	Number of students (c.f)
More than or equal to 80	4
More than or equal to 70	6
More than or equal to 60	11

More than or equal to 50	17
More than or equal to 40	23
More than or equal to 30	27
More than or equal to 20	30
More than or equal to 10	32
More than or equal to 0	34

Sol.

Marks (out of 90)	No. of Students	C.I.	No. of Students (f_i)
More than or equal to 0	34	0–10	34 – 32 = 2
More than or equal to 10	32	10–20	32 – 30 = 2
More than or equal to 20	30	20–30	30 – 27 = 3
More than or equal to 30	27	30–40	27 – 23 = 4
More than or equal to 40	23	40–50	23 – 17 = 6
More than or equal to 50	17	50–60	17 – 11 = 6
More than or equal to 60	11	60–70	11 – 6 = 5
More than or equal to 70	6	70–80	6 – 4 = 2
More than or equal to 80	4	80–90	4 – 0 = 4

Q12. Find the unknown entries *a, b, c, d e* and *f* in the following distribution of heights of students in a class.

Height (in cm)	Frequency	Cumulative frequency
150–155	12	*a*
155–160	*b*	25
160–165	10	*c*
165–170	*d*	43
170–175	*e*	48
175–180	2	*f*
Total	50	

Sol.

Height (in cm)	Frequency (f_i)	*c.f.* (given)	*c.f.* calculated from (f_i)
150–155	12	*a*	12
155–160	*b*	25	$12 + b$
160–165	10	*c*	$12 + b + 10$
165–170	*d*	43	$22 + b + d$
170–175	*e*	48	$22 + b + d + e$
175–180	2	*f*	$22 + 2 + b + d + e$

Comparing the two *c.f.* (cumulative frequency) calculated one and given one $a = 12$

$$12 + b = 25 \Rightarrow b = 25 - 12 = 13 \qquad \Rightarrow b = 13$$
$$c = 12 + b + 10 = 12 + 13 + 10 = 35 \qquad \Rightarrow c = 35$$
$$22 + b + d = 43 \Rightarrow d = 43 - 22 - b = 21 - 13 = 8 \qquad \Rightarrow d = 8$$
$$22 + b + d + e = 48 \Rightarrow e = 48 - 22 - 13 - 8 = 48 - 43 = 5 \qquad \Rightarrow e = 5$$
$$f = 24 + b + d + e = 24 + 13 + 8 + 5 = 24 + 26 = 50 \Rightarrow f = 50$$

Q13. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day:

Age (in years)	10–20	20–30	30–40	40–50	50–60	60–70
No. of patients	60	42	55	70	53	20

Form

(*i*) less than type cumulative frequency distribution.

(*ii*) more than type cumulative frequency distribution.

Sol. (*i*) For less than type cumulative frequency (*c.f.*), it is clear from the table that patients less than 10 years of age are zero and less than 20 years are 60, and for less than 30 it will include from 10 – 30 (i.e., 60 + 42) i.e. less than 30 are 102 and so on.

(*ii*) For more than type *c.f.* the Ist C.I. is 10–20 so more than 10 will include all 300 patients or from the last C.I. (60–70) we observe that patients more than or equal to 70 are zero, more than 60 or equal to 60 patient are 20, and more than or equal to 50 are 20 + 53 = 73 and so on.

Less than type		**More than type**	
Age of Patients (in years)	**Number of Patients**	**Age of Patients (in years)**	
Less than 10	0	More than or equal to 10	60 + 240 = 300
Less than 20	60 + 0 = 60	More than or equal to 20	42 + 198 = 240
Less than 30	42 + 60 = 102	More than or equal to 30	55 + 143 = 198
Less than 40	55 + 102 = 157	More than or equal to 40	70 + 73 = 143
Less than 50	70 + 157 = 227	More than or equal to 50	53 + 20 = 73
Less than 60	53 + 227 = 280	More than or equal to 60	20 + 0 = 20
Less than 70	20 + 280 = 300	More than or equal to 70	0 = 0

Q14. Given below is a cumulative frequency distribution showing the marks secured by 50 students of a class:

Marks	Below 20	Below 40	Below 60	Below 80	Below 100
No. of students	17	22	29	37	50

Form the frequency distribution table for the data.

Sol. Class size is 40 – 20 = 20

So below 20 means C.I. is 0 – 20 and frequency is 17.

Frequency 22 includes 0 – 20 and 20 – 40 both class intervals.

Hence, the frequency between 20–40 is (22 – 17) = 5

Frequency 29 includes all 0–10, 10–20 and 20–60 class intervals.

So, 40–60 = 29 – 22 = 7

Marks (C.I.)	Number of students (f_i)
0–20	$17 - 0 = 17$
20–40	$22 - 17 = 5$
40–60	$29 - 22 = 7$
60–80	$37 - 29 = 8$
80–100	$50 - 37 = 13$

Q15. Weekly income of 600 families is tabulated below:

Weekly income (in ₹)	Number of families
0–1000	250
1000–2000	190
2000–3000	100
3000–4000	40
4000–5000	15
5000–6000	5
Total	600

Compute the median income.

Sol. For calculating the median of grouped data, we first form *c.f.* table.

Weekly income (₹)	No. of families f_i	*c.f.*
0–1000	250	250
1000–2000	190	440
2000–3000	100	540
3000–4000	40	580
4000–5000	15	595
5000–6000	5	600

The median of 600 (even) obser. = mean of 300 and 301 obs.

= Median lies in range (1000–2000)

So Median class = 1000 – 2000

$$\text{Median} = \frac{l + \left(\frac{n}{2} - c.f.\right) \times h}{f},$$

where,

l = lower limit of median class = 1000

n = Total no. of observations = 600

$c.f.$ = $c.f.$ preceding the median class = 250

h = the class size = 2000 – 1000 = 1000

f = frequency of median class = 190

$$\therefore \text{ Median} = 1000 + \frac{\left(\frac{600}{2} - 250\right) \times 1000}{190}$$

$$= 1000\left[1 + \frac{50}{190}\right] = 1000\,[1 + 0.26315]$$

$$= 1000\,[1.26315] = 1263.15$$

Hence, the median income of family is ₹ 1263.15 per week.

Q16. The maximum bowling speeds, in km per hour, of 33 players at a cricket coaching centre are given as follows.

Speed (in km/h)	85–100	100–115	115–130	130–145
Number of players	11	9	8	5

Calculate the median bowling speed.

Sol. To calculate median we form *c.f.* table.

Speed (in km/h) (C.I.)	No. of players (f_i)	*c.f.*
85–100	11	11
100–115	9	20
115–130	8	28
130–145	5	33

N = No. of observations = 33.

Median obs. of 33 odd observations $= \frac{33+1}{2} = \frac{34}{2} = 17\text{th obs}$.

17th obs. lies in class 100–115

$\therefore \quad l = 100, \quad f = 9, \quad c.f. = 11, \quad h = 100 - 85 = 15$

$$\therefore \qquad \text{Median} = l + \frac{\left(\frac{N}{2} - c.f.\right)h}{f}$$

$$= 100 + \frac{\left(\frac{33}{2} - 11\right)15}{9} = 100 + \frac{(16.5 - 11)\,15}{9}$$

$$= 100 + \frac{5.5 \times 15}{9} = 100 + \frac{82.5}{9}$$

$$= 100 + 9.166 = 109.17 \text{ km/h}$$

Hence, the median bowling speed is 109.17 km/h.

Q17. The monthly income of 100 families are given below:

Income (in ₹)	**Number of families**	**Income (in ₹)**	**Number of families**
0–5000	8	20000–25000	3
5000–10000	26	25000–30000	3
10000–15000	41	30000–350000	2
15000–20000	16	35000–40000	1

Calculate the modal income.

Sol. For modal income, we have to calculate mode.

The mode of grouped data $= l + \left[\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] h$

Modal class = class having maximum frequency *i.e.*, 41 is class (10000–15000)

f_0 = frequency preceding the modal class = 26
f_1 = frequency of modal class = 41
f_2 = frequency of class succeeding the modal class = 16
l = lower limit of modal class = 10000
h = 5000

$$\therefore \quad \text{Mode} = 10000 + \frac{(41-26)\,5000}{2\times 41 - 26 - 16}$$

$$= 5000\left[2 + \frac{15}{82-42}\right] = 5000\left[2 + \frac{15}{40}\right]$$

$$= 5000\,[2 + 0.375] = 5000 \times 2.375 = 11875$$

Hence, the modal income is ₹ 11,875 per month.

Q18. The weight of coffee in 70 packets are shown in the following table.

Weight (in g)	200–201	201–202	202–203	203–204	204–205	205–206
No. of packets	12	26	20	9	2	1

Determine the modal weight.

Sol.

C.I.	(f_i)	C.I.	(f_i)
200–201	12	203–204	9
201–202	26	204–205	2
202–203	20	205–206	1

Modal class = (201–202) [$\because$ maximum frequency is 26]

$f_0 = 12$
$f_1 = 26$
$f_2 = 20$
$h = 201 - 200 = 1$
$l = 201$

$$\therefore \quad \text{Mode} = l + \frac{(f_1 - f_0)\,h}{(2f_1 - f_0 - f_2)}$$

$$\text{Mode} = 201 + \frac{(26-12)\times 1}{(2\times 26 - 12 - 20)} = 201 + \frac{14}{(52-32)}$$

$$= 201 + \frac{14}{20} = 201 + 0.7$$

$$\text{Mode} = 201.7$$

Hence, the modal weight is 201.7 g.

Q19. Two dice are thrown at the same time. Find the probability of getting

(*i*) same number on both dice.

(*ii*) different numbers on both dice.

Sol. (*i*) Let E be the event of getting same number on both dice.

Total number of all possible outcomes T(E) = 36

No. of outcomes favourable to E, F(E) = 6

F(E) are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

(*ii*) Probability of getting different number on both the dice

= 1 – Probability of getting same number on both the dice

$$\therefore \quad = 1 - \frac{1}{6} = \frac{5}{6}$$

Q20. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is

(*i*) 7? (*ii*) a prime number? (*iii*) 1?

Sol. Total number of all positive outcomes when two dice are thrown simultaneously T(E) = 36

(*i*) The sum of the numbers appearing on both dice is 7. So, combinations are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1)

$$\Rightarrow \quad F(E) = 6$$

$$\therefore \quad \text{Required probability} = P(E) = \frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

(*ii*) Sum of numbers on both dice is a prime number, *i.e.*, (1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)

Hence, number of outcomes favourable to E = F(E) = 15

$$\Rightarrow \quad P(F) = \frac{F(E)}{T(E)} = \frac{15}{36} = \frac{5}{12}$$

(*iii*) Sum of two numbers on both dice 1, F(E) = 0

$$\therefore \quad P(F) = 0$$

Q21. Two dice are thrown together. Find the probability that the product of the numbers on the top of dice is

(*i*) 6 (*ii*) 12 (*iii*) 7

Sol. Main concept: The two dice are not identical, so, (4, 3) and (3, 4) will be different outcomes.

To get favourable outcomes: Choose I entry from 1 to 6, then place 1 to 6 at IInd place if given condition satisfies.

Total number of all possible outcomes if two dice are thrown together = T(E) = 36

(i) Let E be the event of getting the product on 6.
Number of outcomes favourable to event E are (1, 6), (2, 3), (3, 2), (6, 1), F(E) = 4

$$\Rightarrow P(E) = \frac{F(E)}{T(E)} = \frac{4}{36} = \frac{1}{9}$$

(ii) Number of outcomes favourable when product of numbers on both dice is 12, (2, 6), (3, 4), (4, 3), (6, 2)

$\therefore$ $F(E) = 4$

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{4}{36} = \frac{1}{9}$$

(iii) Let E be the event of getting the product of numbers on both dice is 7

$\therefore$ $F(E) = 0$

$\Rightarrow$ $P(E) = 0$

Q22. Two dice are thrown at the same time and the product of the numbers appearing on them is noted. Find the probability that the product is less than 9.

Sol. Total number of all possible outcomes when two dice thrown together T(E) = 36

Product of the numbers on both dice is less than 9 so favourable outcomes are (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1), (4, 2), (5, 1), (6, 1)

$\therefore$ $F(E) = 16$

$$\Rightarrow \quad P(E) = \frac{F(E)}{T(E)} = \frac{16}{36} = \frac{4}{9}$$

Q23. Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3 respectively. They are thrown and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.

Sol. Total number of all possible outcomes, *i.e.*, T(E) = 36.

(i) Number of favourable outcomes when
Sum of numbers on two dice is 2 are (1, 1), (1, 1) *i.e.*, F(E) = 2

$$P_1(E) = \frac{F(E)}{T(E)} = \frac{2}{36} = \frac{1}{18}$$

(ii) Number of favourable outcomes when
Sum of numbers on two dice is 3 are (1, 2), (1, 2), (2, 1), (2, 1)

$\therefore$ $F(E) = 4$

$$\Rightarrow \quad P_2(E) = \frac{F(E)}{T(E)} = \frac{4}{36} = \frac{1}{9}$$

(iii) Number of favourable outcomes when
Sum of numbers on two dice is 4 are (1, 3), (1, 3), (2, 2), (2, 2), (3, 1), (3, 1)

$$F(E) = 6.$$

$$\therefore \quad P_3(E) = \frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

(*iv*) Number of favourable outcomes when
Sum of numbers on two dice is 5 are (2, 3), (2, 3), (3, 2), (3, 2), (4, 1), (4, 1)

$$F(E) = 6$$

$$\therefore \quad P_4(E) = \frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

(*v*) Number of favourable outcomes when
Sum of numbers on two dice is 6 *i.e.*, (3, 3), (3, 3), (4, 2), (4, 2), (5, 1), (5, 1)

$$F(E) = 6$$

$$\therefore \quad P_5(E) = \frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

(*vi*) Number of favourable outcomes when
Sum of numbers on both dice is 7 are (4, 3), (4, 3) (6, 1), (6, 1), (5, 2), (5, 2)

$$F(E) = 6$$

$$P_6(E) = \frac{F(E)}{T(E)} = \frac{6}{36} = \frac{1}{6}$$

(*vii*) Number of favourable outcomes when
Sum of numbers on both dice is 8 are (5, 3), (5, 3), (6, 2), (6, 2)

$$F(E) = 4$$

$$\therefore \quad P_7(E) = \frac{F(E)}{T(E)} = \frac{4}{36} = \frac{1}{9}$$

(*viii*) Number of favourable outcomes when
Sum of numbers on both dice is 9 are (6, 3), (6, 3) *i.e.*, F(E) = 2

$$\therefore \quad P_8(E) = \frac{F(E)}{T(E)} = \frac{2}{36} = \frac{1}{18}$$

Q24. A coin is tossed two times. Find the probability of getting almost one head.

Sol. Total number of possible outcomes if a coin is tossed 2 times (HH), (HT), (TH), (TT) *i.e.*, $T(E) = 2^2 = 4$

No. of favourable outcomes of getting almost one head *i.e.*,

$$F(E) = 3$$

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{3}{4}$$

Q25. A coin is tossed 3 times. List the possible outcomes, find the probability of getting

(*i*) all heads (*ii*) at least two heads

Sol. Total number of possible outcomes when if a coin is tossed Number of favourable outcomes of getting (HHH), (HHT), (HTH), (THH) (TTT) (TTH) (THT) (HTT) so T(E) = 8

(*i*) All head *i.e.*, F(E) = 1

$$\therefore \qquad P(E) = \frac{F(E)}{T(E)} = \frac{1}{8}$$

(*ii*) Number of favourable outcomes of getting at least 2 heads

$$F(E) = 4$$

$$\therefore \qquad P(E) = \frac{F(E)}{T(E)} = \frac{4}{8} = \frac{1}{2}$$

Q26. Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2.

Sol. Total number of possible outcomes when 2 dice (option 6 each) are tossed together = $6^2 = 36$

$$\therefore \qquad T(E) = 36$$

Number of favourable outcomes of getting the difference of the numbers as (1, 3), (2, 4), (3, 5), (4, 6), (3, 1), (4, 2), (5, 3), (6, 4)

$$\therefore \qquad F(E) = 8$$

$$\Rightarrow \qquad P(E) = \frac{F(E)}{T(E)} = \frac{8}{36} = \frac{2}{9}$$

Q27. A bag contains 10 red, 5 blue and 7 green balls. A ball is drawn at random, Find the probability of this ball being a

(*i*) red ball (*ii*) green ball (*iii*) not a blue ball

Sol. No. of red balls = 10

Number of blue balls = 5

Number of green balls = 7

Total number of balls T(E) = (10 + 5 + 7) = 22

(*i*) Number of favourable outcomes of getting a red ball = F(E) = 10

$$\therefore \qquad P(E) = \frac{F(E)}{T(E)} = \frac{10}{22} = \frac{5}{11}$$

(*ii*) Number of favourable outcomes of getting a green ball = F(E) = 7

$$\therefore \qquad P(E) = \frac{F(E)}{T(E)} = \frac{7}{22}$$

(*iii*) Number of favourable outcomes of not getting a blue ball = F(E) = 22 – 5 = 17

$$\therefore \qquad P(E) = \frac{F(E)}{T(E)} = \frac{17}{22}$$

Q28. The King, Queen, and Jack of clubs are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is

(*i*) a heart (*ii*) a king

Sol. Total number of cards after removing King, Queen and Jack of club
T(E) = 52 – 3 = 49

(*i*) Number of favourable outcomes of getting a card of heart (any) = F(E) = 13

$$\therefore \qquad P_1(E) = \frac{F(E)}{T(E)} = \frac{13}{49}$$

(*ii*) Number of favourable outcomes of getting a card of King *i.e.*, F(E) = (4 –1) = 3

$$\therefore \qquad P_2(E) = \frac{F(E)}{T(E)} = \frac{3}{49}$$

Q29. Refer to Q. 28. What is the probability that the card is

(*i*) a club? (*ii*) 10 of heart?

Sol. Total number of cards = T(E) = 52 – 3 = 49

(*i*) Number of favourable outcomes of getting a club
F(E) = 13 – 3 = 10

$$\therefore \qquad P(E) = \frac{F(E)}{T(E)} = \frac{10}{49}$$

(*ii*) Number of favourable outcomes of getting 10 of heart F(E) = 1

$$\therefore \qquad P(E) = \frac{F(E)}{T(E)} = \frac{1}{49}$$

Q30. All the jacks, queens and kings are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1, similar value for other cards, find the probability that the card has a value

(*i*) 7 (*ii*) greater than 7 (*iii*) less than 7

Sol. Out of 52 playing cards, 4 Jacks, 4 queens and 4 kings are removed.
∴ Total number of cards removed = 3 × 4 =12
Total number of cards remained = 52 – 12 = 40

$$\therefore \qquad T(E) = 40$$

As ace has been given value 1, and similar value for other cards.
So, all the four aces are numbered by 1 and so on.

(*i*) Number of favourable outcomes of getting a card that has a value 7 = 4

$$\therefore \qquad F(E) = 4$$

$$\therefore \qquad P_1(E) = \frac{F(E)}{T(E)} = \frac{4}{40} = \frac{1}{10}$$

(*ii*) The numbers greater than 7 are 8, 9, and 10
So, number of favourable outcomes of getting a card that has a value of greater than 7 = 3 × 4 = 12

$$\therefore \qquad F_2(E) = 12$$

$$\therefore \qquad P_2(E) = \frac{F_2(E)}{T(E)} = \frac{12}{40} = \frac{3}{10}$$

(*iii*) The numbers less than 7 are = 1, 2, 3, 4, 5, 6

So, number of favourable outcomes of getting a card that was a value less than 7 = $F_3(E) = 6 \times 4 = 24$

$$\therefore \quad P_3(E) = \frac{F_3(E)}{T(E)} = \frac{24}{40} = \frac{3}{5}$$

Q31. An integer is chosen between 0 and 100. What is the probability that it is

(*i*) divisible by 7? (*ii*) not divisible by 7?

Sol. (*i*) Numbers between 0 and 100 divisible by 7 are 7, 14, 21, ..., 98 = (AP)

Here, $a_n = 98$, $a = 7$, $d = 7$

$$a_n = a + (n-1)\,d$$

$$\Rightarrow \quad 98 = 7 + (n-1)\,7$$

$$\Rightarrow \quad 98 - 7 = (n-1)\,7$$

$$\Rightarrow \quad \frac{91}{7} = (n-1)$$

$$\Rightarrow \quad (n-1) = 13$$

$$\Rightarrow \quad n = 13 + 1 = 14$$

$$\therefore \quad F(E) = 14 \quad \text{and} \quad T(E) = 99$$

$$\therefore \quad P(E) = \frac{14}{99}$$

(*ii*) Number of favourable outcomes of getting a number which is not divisible by 7 = 99 − 14 = 85

$$\therefore \quad F(E) = 85$$

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{85}{99}$$

Q32. Cards with numbers 2 to 101 are placed in a box. A card is selected at random. Find the probability that the card has

(*i*) an even number (*ii*) a square number

Sol. (*i*) Total number of the cards (101 − 1) = 100

$$\therefore \quad T(E) = 100$$

Out of 100 cards, even number cards are 50

$$\therefore \quad F(E) = 50$$

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{50}{100} = \frac{1}{2}$$

(*ii*) Square numbers from 2 to 101 are 4, 9, 16, 25, 36, 49, 64, 81, 100

$$\therefore \quad F(E) = 9$$

$$\therefore \quad P(E) = \frac{F(E)}{T(E)} = \frac{9}{100}$$

Q33. A letter of English alphabets is chosen at random. Determine the probability that the letter is a consonant.

Sol. In 26 English alphabets there are 5 vowels and 21 consonants.

So, number of favourable outcomes of getting a consonant *i.e.*, F(E) = 21

Total alphabets = T(E) = 26

$$\text{Probability (getting a consonant)} = \frac{F(E)}{T(E)} = \frac{21}{26}$$

Q34. There are 1000 sealed envelopes in a box, 10 of them contain a cash prize of ₹ 100 each, 100 of them contain a cash prize of ₹ 50 each and 200 of them contain a cash prize of ₹ 10 each and rest do not contain any cash prize. If they are well shuffled and an envelope is picked up out, what is the probability that it contains no cash prize?

Sol. Total number of envelopes, T(E) = 1000

Number of envelopes containing cash prizes = 200 + 100 + 10 = 310

So, number of envelopes containing no cash prize = 1000 – 310 = 690

i.e., F(E) = 690

∴ Probability of getting an envelope of no cash prize = P(E)

$$= \frac{F(E)}{T(E)} = \frac{690}{1000} = \frac{69}{100}$$

Q35. Box 'A' contains 25 slips of which 19 are marked ₹ 1 and other are marked ₹ 5 each. Box B contains 50 slips of which 45 are marked ₹ 1 each and others are marked ₹ 13 each. Slip of both boxes are poured into a third box and reshuffled. A slip is drawn at random. What is the probability that it is marked other than ₹ 1.

Sol. Total number of slips poured in third box = 25 + 50 = 75

∴ T(E) = 75

Number of slips in third box marked ₹ 1 = 19 + 45 = 64

Hence, the number of favourable outcomes of drawing a slip from IIIrd box other than ₹ 1

= 75 – 64 = 11

∴ F(E) = 11

$$\therefore \quad \text{Required probability } P(E) = \frac{F(E)}{T(E)} = \frac{11}{75}$$

Q36. A carton of 24 bulbs contain 6 defective bulbs one bulb is drawn at random. What is the probability that the bulb is not defective? If the bulb selected is defective and it is not replaced and a second bulb is selected at random from the rest, what is the probability that the second bulb is defective?

Sol. Total bulbs in carton = 24 ⇒ T(E) = 24

Defective bulbs = 6

Number of favourable outcomes of drawing a bulb which is not defective = 24 – 6 = 18 ⇒ F(E) = 18

$\therefore$ Probability that bulb is not defective = $P(E) = \frac{F(E)}{T(E)} = \frac{18}{24} = \frac{3}{4}$

According to the question selected bulb is defective and not replaced.

So, the total remaining bulb = 23 $\Rightarrow$ $T'(E) = 23$

Number of favourable outcomes of drawing a second defective bulb *i.e.,* $F'(E) = 6 - 1 = 5$

$$\therefore \quad P'(E) = \frac{F'(E)}{T'(E)} = \frac{5}{23}$$

Q37. A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is lost at random. Find the probability that it is a

(*i*) triangle (*ii*) square

(*iii*) square of blue colour (*iv*) triangle of red colour

Sol. Total no. of triangles = 8

Triangles with blue colour = 3

Triangles with red colour = 8 − 3 = 5

Total no. of squares = 10

Squares with blue colour = 6

Squares with red colour = 10 − 6 = 4

(*i*) Number of favourable outcomes for the event that lost figure is triangle

i.e., $F(E) = 8$

Total figures (square and triangle) = 8 + 10 = 18

i.e., $T(E) = 18$

$\therefore$ Probability (getting a triangle) $P(E) = \frac{F(E)}{T(E)} = \frac{8}{18} = \frac{4}{9}$

(*ii*) Number of favourable outcomes for the events that squares is lost

i.e., $F(E) = 10$

$T(E) = 8 + 10 = 18$

$\therefore$ P (getting a square) = $P(E) = \frac{10}{18} = \frac{5}{9}$

(*iii*) Number of favourable outcomes for the events that lost figure is square of blue colour

i.e., $F(E) = 6$

$T(E) = 18$

P(E) (getting a blue square) = $\frac{F(E)}{T(E)}$

$$\therefore \quad P(E) = \frac{6}{18} = \frac{1}{3}$$

(*iv*) Number of favourable outcomes for the event that lost figure is triangle of red colour = 5 i.e., $F(E) = 5$

$T(E) = 18$

$$P(E) = \frac{F(E)}{T(E)} = \frac{5}{18}$$

Q38. In a game, the entry fee is ₹ 5. The game consists of a tossing a coin 3 times. If one or two heads show, Sweta gets her entry fee back. If she throws 3 heads, she receives double entry fees. Otherwise she will lose. For tossing a coin three times, find the probability that she

(*i*) loses the entry fee (*ii*) gets double entry fee

(*iii*) just gets her entry fee

Sol. One coin is tossed 3 times so total number of favourable outcomes $= 2^3 = 8$, which are (HHH), (HHT), (HTH), (THH) and (replacing $H \rightarrow T$ and $T \rightarrow H$) (TTT), (TTH), (THT), (HTT)

(*i*) Losing the game means getting no head

Number of favourable outcomes of getting no head = F(E) = 1

So, P (Losing the entry fee) *i.e.*, $P(E) = \frac{F(E)}{T(E)} = \frac{1}{8}$

(*ii*) Gets double entry fee back by getting 3 heads

Number of favourable outcomes of getting 3 heads i.e., F(E) = 1

$\therefore$ (getting double entry fee) $P(E) = \frac{F(E)}{T(E)} = \frac{1}{8}$

(*iii*) Just gets her entry fees back by getting either one or two heads. Number of favourable outcomes of getting either one or two heads *i.e.*, F(E) = 6

(just getting entry fee) *i.e.*, $P(E) = \frac{F(E)}{T(E)} = \frac{6}{8}$

$$\therefore \quad P(E) = \frac{3}{4}$$

Q39. A die has six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and total scores are recorded.

(*i*) How many different scores are possible?

(*ii*) What is the probability of getting a total of 7?

Sol. Total number of possible outcomes $= 6^2 = 36$

(*i*) Number of favourable outcomes are (0, 0), (0, 1), (0, 6), (1, 0), (1, 1), (1, 6), (6, 0), (6, 1), (6, 6) *i.e.*, 9.

Total of both numbers are 0, 1, 6, 2, 7, 12

So, 6 differentiate scores are possible.

(*ii*) Number of favourable outcomes for getting a total of 7 are 2

$\Rightarrow \quad F(E) = 2$

Probability of getting sum 7 = $\frac{F(E)}{T(E)}$

Total no. of all possible outcomes of getting sum either (0, 1, 2, 6, 7 and 12) = 6

$\therefore$ (sum of numbers on both dice is 7) = $\frac{2}{6} = \frac{1}{3}$

Hence, the probability of getting sum on both dice 7 is = $\frac{1}{3}$

Q40. A lot consists of 48 mobile phones of which 42 are good, 3 have only minor defects and 3 have major defects. Varnika will buy a phone, if it is good, but the trader will only buy a mobile, if it has no major defect. One phone is selected at random from the lot. What is probability that it is

(*i*) Acceptable to Varnika?

(*ii*) Acceptable to trader?

Sol. Total number of mobile phones = T(E) = 48

(*i*) Let E be the event that Varnika's selected mobile should be good.

$\therefore$ Number of favourable outcomes for event E = F(E) = 42

$$\therefore \quad P\text{ (for good mobile)} = \frac{F(E)}{T(E)} = \frac{42}{48} = \frac{7}{8}$$

(*ii*) Trader buys a phone which has no major defect.

No. of phones with major defect = 3

$\therefore$ Phones which do not have major defects = 48 – 3 = 45

$\therefore$ F(E) = 45

$$\Rightarrow \quad P(E) = \frac{F(E)}{T(E)} = \frac{45}{48} = \frac{15}{16}$$

Q41. A bag contains 24 balls of which x are red, $2x$ are white and $3x$ are blue. A ball is selected at random. What is the probability that it is

(*i*) not red (*ii*) white

Sol. Total number of balls = 24

Number of red balls = x

Number of white balls = $2x$

Number of blue balls = $3x$

Total balls = 24

$\therefore \quad 1x + 2x + 3x = 24$

$\Rightarrow \quad 6x = 24$

$\Rightarrow \quad x = 4$

So, Number of red balls = $x = 1 \times 4 = 4$

Number of white balls = $2x = 2 \times 4 = 8$

Number of blue balls = $3x = 3 \times 4 = 12$

(*i*) Randomly selected ball is not red

$\therefore$ Number of favourable outcomes for the event that ball is not red = 24 – 4 = 20

$$\therefore \qquad F(E) = 20$$

$$\text{and} \qquad T(E) = 24$$

$$\therefore \qquad P(\text{not red}) = \frac{F(E)}{T(E)} = \frac{20}{24} = \frac{5}{6}$$

$$\therefore \qquad P(E) = \frac{5}{6} \text{ is the required probability}$$

(*ii*) Number of favourable outcomes for the event that the selected ball is white, F(E) = 8

$$\therefore \qquad P(\text{ball is white}) = \frac{8}{24} = \frac{1}{3}$$

Q42. At a fete, cards bearing numbers 1 to 1000, one number on one card, are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square greater than 500, the player wins a prize. What is the probability that

(*i*) the first player wins the prize?

(*ii*) the second player wins a prize, if the first has won?

Sol. (*i*) First player can select a card from a box in 1000 ways.

Perfect square greater than 500 are 529, 576, 625, 676, 729, 784, 841, 900, 961

$$= (23)^2, (24)^2, (25)^2, (26)^2, (27)^2, (28)^2, (29)^2, (30)^2, (31)^2$$

$\therefore$ F(E) = 9

So, the probability P(E) that the first player wins the prize will be

$$P(E) = \frac{F(E)}{T(E)} = \frac{9}{1000} = 0.009$$

(*ii*) For IInd player, the card selected by earlier player is not replaced.

$\therefore$ Total number of cards for IInd player = 1000 – 1 = 999

$$\therefore \qquad T'(E) = 999$$

As the first player wins the prize. So, cards having perfect square greater than 500 become one less.

So, number of favourable outcomes for IInd player to win a prize = 9 – 1

$$\text{i.e.,} \qquad F'(E) = 8$$

$$\therefore \qquad \text{Probability} = P'(E) = \frac{F'(E)}{T'(E)} = \frac{8}{999}$$

(winning second player a prize)

EXERCISE 13.4

Q1. Find the mean marks of the students for the following distribution:

Marks	Number of Students	Marks	Number of Students
0 and above	80	60 and above	28
10 and above	77	70 and above	16
20 and above	72	80 and above	10
30 and above	65	90 and above	8
40 and above	55	100 and above	0
50 and above	43		

Sol.

Marks	c.f.	Marks C.I.	x_i	$d_i = x_i - a$	f_i	$f_i d_i$
0 and above	80	0–10	5	−50	80 − 77 = 3	−150
10 and above	77	10–20	15	−40	77 − 72 = 5	−200
20 and above	72	20–30	25	−30	72 − 65 = 7	−210
30 and above	65	30–40	35	−20	65 − 55 = 10	−200
40 and above	55	40–50	45	−10	55 − 43 = 12	−120
50 and above	43	50–60	55	0	43 − 28 = 15	0
60 and above	28	60–70	65	10	28 − 16 = 12	120
70 and above	16	70–80	75	20	16 − 10 = 6	120
80 and above	10	80–90	85	30	10 − 8 = 2	60
90 and above	8	90–100	95	40	8 − 0 = 8	320
100 and above	0	100–110	105	50	0	0
					$\Sigma f_i = 80$	$\Sigma f_i d_i = -260$

a = assumed mean = 55

$\Sigma f_i d_i = -260$

$\Sigma f_i = 80$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 55 - \frac{260}{80} = 55 - \frac{13}{4} = 55 - 3.25$$

$\Rightarrow \quad \bar{x} = 51.75$

This method is called deviation method.

Hence, the mean marks of students = 51.75.

Q2. Determine the mean of the following distribution:

Marks	Number of Students	Marks	Number of Students
Below 10	5	Below 60	60
Below 20	9	Below 70	70
Below 30	17	Below 80	78
Below 40	29	Below 90	83
Below 50	45	Below 100	85

Sol. From the given table, we observe that the students getting marks below 10 are 5, and marks cannot be negative. So, 5 students lie in (0–10) class interval.

Similarly, no. of students getting marks below 20 are 9 and below 10 are 5 so number of students getting marks 10–20 are (9 – 5) = 4. So,

Marks	*c.f.*	C.I.	x_i	$u_i = \dfrac{(x_i - a)}{h}$	f_i	f_iu_i
Below 10	5	0–10	5	–4	5 – 0 = 5	–20
Below 20	9	10–20	15	–3	9 – 5 = 4	–12
Below 30	17	20–30	25	–2	17 – 9 = 8	–16
Below 40	29	30–40	35	–1	29 – 17 = 12	–12
Below 50	45	40–50	45	0	45 – 29 = 16	0
Below 60	60	50–60	55	1	60 – 45 = 15	15
Below 70	70	60–70	65	2	70 – 60 = 10	20
Below 80	78	70–80	75	3	78 – 70 = 8	24
Below 90	83	80–90	85	4	83 – 78 = 5	20
Below 100	85	90–100	95	5	85 – 83 = 2	10
					$\Sigma f_i = 85$	$\Sigma f_iu_i = 29$

a = assumed mean = 45

$\Sigma f_i = 85$

$\Sigma f_iu_i = 29$

$$\bar{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right)h \quad \text{[Step deviation method]}$$

$$\Rightarrow \quad \bar{x} = 45 + \frac{29}{85} \times 10 = 45 + \frac{58}{17} = 45 + 3.41 = 48.41$$

$$\Rightarrow \quad \bar{x} = 48.41 \text{ marks}$$

Hence, the average marks of students are 48.41.

Q3. Find the mean age of 100 residents of a town from the following data:

Age	Number of persons	Age	Number of persons
equal and above 0	100	equal and above 40	25
equal and above 10	90	equal and above 50	15
equal and above 20	75	equal and above 60	5
equal and above 30	50	equal and above 70	0

Sol. Age above 70 years is zero. So, last C.I. is 60–70. Age above zero is 100 and above 10 is 90 so age of persons in class interval (0–10) is (100 – 90) and so on.

Age	c.f.	C.I.	x_i	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
equal and above 0	100	0–10	5	–3	100 – 90 = 10	–30
equal and above 10	90	10–20	15	–2	90 – 75 = 15	–30
equal and above 20	75	20–30	25	–1	75 – 50 = 25	–25
equal and above 30	50	30–40	35	0	50 – 25 = 25	0
equal and above 40	25	40–50	45	1	25 – 15 = 10	10
equal and above 50	15	50–60	55	2	15 – 5 = 10	20
equal and above 60	5	60–70	65	3	5 – 0 = 5	15
equal and above 70	0					
					$\Sigma f_i = 100$	$\Sigma f_i u_i = -40$

a = assumed mean = 35

$\Sigma f_i = 100$

$\Sigma f_i u_i = -40$

$h = 10$

$$\therefore \quad \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h \quad \text{[Step deviation method]}$$

$$\Rightarrow \quad \bar{x} = 35 + \frac{(-40)(10)}{100}$$

$$= 35 - \frac{400}{100} = 31$$

$$\Rightarrow \quad \bar{x} = 31 \text{ years}$$

Hence, the mean age of 100 persons = 31 years

Q4. The weight of tea in 70 packets are shown in the following table:

Weight (in g)	200–201	201–202	202–203	203–204	204–205	205–206
Number of packets	13	27	18	10	1	1

Find the mean weight of packets.

Sol.

C.I.	x_i	d_i	f_i	$f_i d_i$
200–201	200.5	–2	13	–26
201–202	201.5	–1	27	–27
202–203	202.5	0	18	0
203–204	203.5	1	10	10
204–205	204.5	2	1	2
205–206	205.5	3	1	3
			$\Sigma f_i = 70$	$\Sigma f_i d_i = -38$

$a = 202.5$

$\therefore \quad \bar{x} = a + \dfrac{\sum f_i d_i}{\sum f_i}$ (By deviation method)

$$= 202.5 - \frac{38}{70} = 202.5 - 0.5428 = 201.9572$$

$\Rightarrow \quad \bar{x} = 201.96$ (approx.)

Hence, the mean weight of packets is 201.96 g.

Q5. Refer to Q4 above. Draw the less than type ogive for the data, and use it to find the mean weight and median weight.

Sol.

C.I.	f_i	Weight	c.f.
		less than 200	0 = 0
200–201	13	less than 201	13 + 0 = 13
201–202	27 = f	less than 202	27 + 13 = 40
202–203	18	less than 203	18 + 40 = 58
203–204	10	less than 204	10 + 58 = 68
204–205	1	less than 205	1 + 68 = 69
205–206	1	less than 206	1 + 69 = 70

$$\text{Median} = l + \frac{\left(\frac{N}{2} - c.f.\right)h}{f}$$

The median class of 70 even obs. $= \dfrac{70}{2} = 35$th obs.
35th obs. lies in 201–202 class

$\therefore \quad l = 201, \quad h = 1$

$N = 70, \quad f = 27$

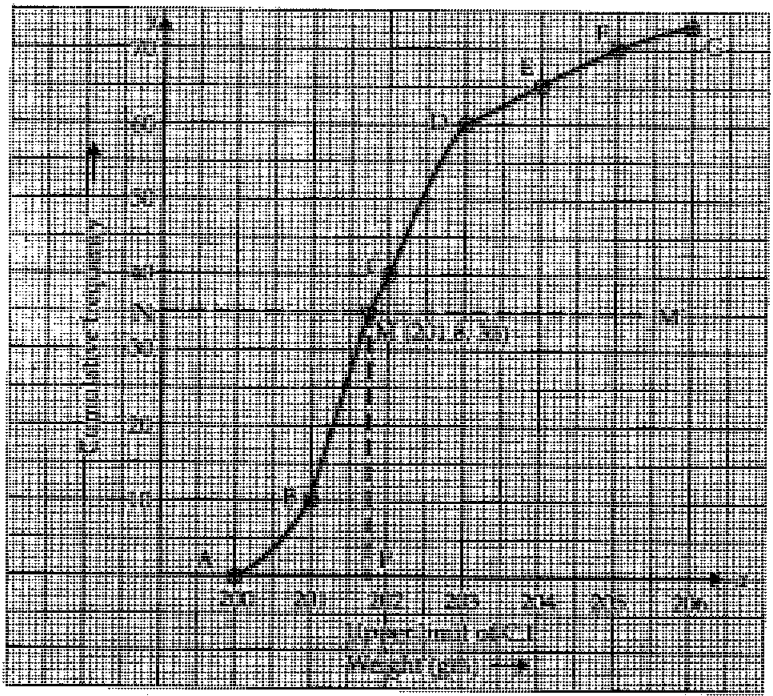
$c.f. = 13$

$$\therefore \quad \text{Median} = 201 + \frac{\left(\frac{70}{2} - 13\right)1}{27} = 201 + \frac{(35-13)}{27}$$

$$= 201 + \frac{22}{27} = 201 + 0.8148 = 201.8148$$

Hence, the median weight is 201.8148 g.

Points for less than type ogive are A(200, 0), B(201, 13), C(202, 40), D(203, 58), E(204, 68), F(205, 69), G(206, 70).



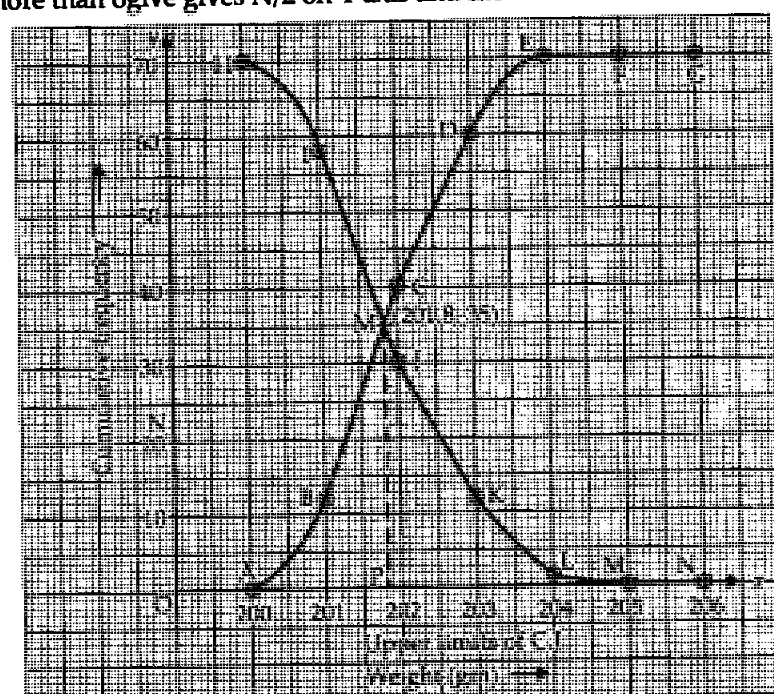
To find out median from graph take $\dfrac{N}{2} = \dfrac{70}{2} = 35$ at Y-axis and draw a line NM′ parallel to X-axis. Which meet the plotted graph at M; Draw MP perpendicular to X-axis. It meets on X-axis at 201.8 which is the median of data.

Q6. Refer to Q.5 above. Draw less than type and more than type ogives for the data and use them to find the median weight.

Sol.

C.I.	f_i wt (in gm)	c.f. less than	weight (in g)	c.f. more than	Points for more than ogive
200	0 (less than 200)	0	more than or equal 200	70	H(200, 70)
200–201	13 (less than 201)	13	more than or equal 201	57	I(201, 57)
201–202	27 (less than 202)	40	more than or equal 202	30	J(202, 30)
202–203	18 (less than 203)	58	more than or equal 203	12	K(203, 12)
203–204	10 (less than 204)	68	more than or equal 204	2	L(204, 2)
204–205	1 (less than 205)	69	more than or equal 205	1	M(205, 1)
205–206	1 (less than 206)	70	more than or equal 206	0	N(206, 0)

Graph (ogive) must be smooth having no edge. In both the graphs, upper limits of class intervals are taken on X-axis and cumulative frequency is taken on Y-axis. The intersection point of less than and more than ogive gives N/2 on Y-axis and median on X-axis.



Hence, the median weight of packets is 201.8 g.

Q7. The table below shows the salaries of 280 persons. Calculate the median and mode of the data.

Salary (in thous. ₹)	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45–50
No. of Persons	49	133	63	15	6	7	4	2	1

Sol.

Salary (₹ 1000) (C.I.)	No. of persons (f_i)	c.f.
5–10	49	$49+0=49$
10–15	133	$133+49=182$
15–20	63	$63+182=245$
20–25	15	$15+245=260$
25–30	6	$6+260=266$
30–35	7	$7+266=273$
35–40	4	$4+273=277$
40–45	2	$2+277=279$
45–50	1	$1+279=280$

(*i*) **Median:** Median class $= \frac{280}{2} = 140$ (*c.f.*) obs.

Median class is 10–15.

$\therefore \quad l=10, \quad N=280, \quad h=5, \quad f=133, \quad c.f.=49$

$$\text{Median} = l + \frac{\left(\frac{N}{2} - c.f.\right)h}{f}$$

$$\therefore \quad \text{Median} = 10 + \frac{\left(\frac{280}{2} - 49\right)5}{133} = 10 + \frac{(140-49)\times 5}{133}$$

$$= 10 + \frac{91\times 5}{133} = 10 + \frac{455}{133} = 10 + 3.4210 = 13.421$$

$= 13.421$ (₹ in 1000)

$\therefore$ Median $= 13.421 \times 1000 =$ ₹ 13, 421

(*ii*) **Mode:** Modal class (of maximum frequency) is (10–15)

$$\text{Mode} = l + \frac{(f_1 - f_0)h}{(2f_1 - f_0 - f_2)}$$

$$= 10 + \frac{(133-49)\times 5}{(2\times 133 - 49 - 63)}$$

$(f_0 = 49, \quad f_1 = 133, \quad f_2 = 63, \quad h = 5)$

$$= 10 + \frac{84\times 5}{266-112} = 10 + \frac{84\times 5}{154} = 10 + \frac{30}{11}$$

$= 10 + 2.727 = 12.727$ (₹ in 1000)

$\Rightarrow$ Mode $= 12.727 \times 1000 =$ ₹ 12,727

Hence, the median and mode of the salaries are ₹ 13,421 and ₹ 12,727 respectively.

Q8. The mean of following frequency distribution is 50, but the frequencies f_1 and f_2 in classes (20–40) and (60–80) respectively are not known. Find these frequencies, if the sum of all the frequencies is 120.

Class	0–20	20–40	40–60	60–80	80–100
Frequency	17	f_1	32	f_2	19

Sol. Mean of observations is 50.

C.I.	x_i	f_i	$d_i = (x_i - a)$	$f_i d_i$
0–20	10	17	−20	−340
20–40	30	f_1	0	0
40–60	50	32	20	+640
60–80	70	f_2	40	$+40f_2$
80–100	90	19	60	+1140
		$\Sigma f_i = 120$		$\Sigma f_i d_i = 1440 + 40f_2$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$a =$ Assumed mean $= 30$, $\bar{x} = 50$ (Given)

$$\Rightarrow \quad 50 = 30 + \frac{1440 + 40f_2}{120}$$

$$\Rightarrow \quad 50 - 30 = \frac{1440}{120} + \frac{40f_2}{120}$$

$$\Rightarrow \quad 20 = 12 + \frac{f_2}{3}$$

$$\Rightarrow \quad 20 - 12 = \frac{f_2}{3}$$

$$\Rightarrow \quad 8 \times 3 = f_2$$

$$\Rightarrow \quad f_2 = 24$$

From frequencies, we have $= 17 + f_1 + 32 + f_2 + 19 = 120$ (Given)

$$\Rightarrow \quad 68 + f_1 + f_2 = 120 \qquad (f_2 = 24)$$

$$\Rightarrow \quad 68 + f_1 + 24 = 120$$

$$\Rightarrow \quad 92 + f_1 = 120$$

$$\Rightarrow \quad f_1 = 120 - 92$$

$\Rightarrow$ $f_1 = 28$

and $f_2 = 24$

Q9. The median of the following data is 50. Find the values of p and q, if sum of all the frequencies is 90.

Marks	f
20–30	p
30–40	15
40–50	25
50–60	20
60–70	q
70–80	8
80–90	10

Sol. Here, median of observations is 50 so, we have to calculate the values of p and q.

Marks (C.I.)	f_i	c.f.
20–30	p	p
30–40	15	$p+15$
40–50	25	$p+15+25=p+40$
50–60	20	$p+40+20=p+60$
60–70	q	$p+q+60$
70–80	8	$p+q+60+8=p+q+68$
80–90	10	$p+q+68+10=p+q+78$
	$\Sigma f_i = 90$	

Now, $p+15+25+20+q+8+10=90$ (Given)

$\Rightarrow$ $78+p+q=90$

$\Rightarrow$ $p+q=90-78=12$ (I)

The median is 50. (Given)

$\therefore$ Median class is (50–60)

$\therefore$ $l=50$, $c.f.=(p+40)$, $f=20$, $h=10$

$$\text{Median} = l + \frac{\left(\frac{N}{2}-c.f\right)h}{f}$$

$$\therefore \quad \text{Median} = 50 + \frac{[45-(40+p)]\times 10}{20} \quad \left(\because \frac{N}{2}=\frac{90}{2}=45\right)$$

$$\Rightarrow \quad 50 = 50 + \frac{(45-40-p)}{2}$$

$\Rightarrow \quad 50 - 50 = \dfrac{5-p}{2}$

$\Rightarrow \quad 5 - p = 0$

$\Rightarrow \quad p = 5$

But, $\quad p + q = 12$ [From (I)]

$\Rightarrow \quad q = 12 - 5 = 7$

$\therefore \quad p = 5$ and $q = 7$.

Q10. The distribution of heights (in cm) of 96 children is given below:

Height (in cm)	Number of children	Height (in cm)	Number of children
124–128	5	144–148	12
128–132	8	148–152	6
132–136	17	152–156	4
136–140	24	156–160	3
140–144	16	160–164	1

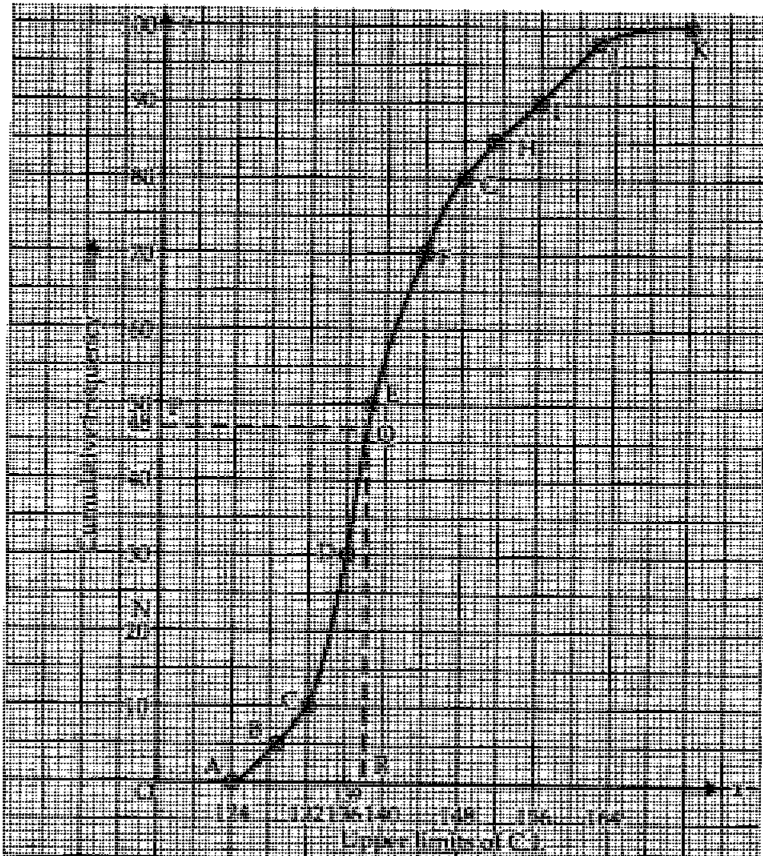
Draw a less than type cumulative frequency curve for this data and use it to compute median height of the children.

Sol. From given table, we have,

Height (in cm)	No. of children	Points for less than type ogive
less than 124	$0+0=0$	A(124, 0)
less than 128	$5+0=5$	B(128, 5)
less than 132	$8+5=13$	C(132, 13)
less than 136	$17+13=30$	D(136, 30)
less than 140	$24+30=54$	E(140, 54)
less than 144	$16+54=70$	F(144, 70)
less than 148	$12+70=82$	G(148, 82)
less than 152	$6+82=88$	H(152, 88)
less than 156	$4+88=92$	I(156, 92)
less than 160	$3+92=95$	J(160, 95)
less than 164	$1+95=96$	K(164, 96)

By plotting the graph with the above points, we get less than type ogive. Taking $y = \dfrac{N}{2} = \dfrac{96}{2} = 48$ at point P draw a line PQ parallel to x-axis and draw QR $\perp$ on x-axis. Point R on x-axis gives the value of median of the given observations.

Hence, the median height of observations is 139.2 cm.



Q11. The size of agricultural holdings in a survey of 200 families is given in the following table:

Size of agricultural holdings (in Hectare)	Number of families
0–5	10
5–10	15
10–15	30
15–20	80
20–25	40
25–30	20
30–35	5

Compute median and mode size of the holdings.

Sol.

C.I. (in hectare)	f_i (No. of families)	$c.f.$
0–5	10	10
5–10	15	25

C.I. (in hectare)	f_i (No. of families)	c.f.
10–15	30	55
15–20	80	135 → Median class
20–25	40	175
25–30	20	195
30–35	5	200

Sol. (*i*) Median class = $\frac{200}{2}$th observation = 100th observation *i.e.,* (15–20)

$$\text{Median} = l + \frac{\left(\frac{N}{2} - c.f.\right)h}{f}, \text{ where}$$

l = lower limit of median class = 15
N = Total number of observations = 200
c.f. = *c.f.* preceding the median class = 55
f = frequency of median class = 80
h = 5

$$\therefore \quad \text{Median} = 15 + \frac{\left(\frac{200}{2} - 55\right)5}{80} = 15 + \frac{(100-55)5}{80}$$

$$= 15 + \frac{45 \times 5}{80} = 15 + \frac{45}{16} = 15 + 2.8125$$

$\therefore$ Median = 17.8125 hectare

(*ii*) **Mode:** Maximum frequency in the given table is 80. So, modal class is (15–20)

$$\text{Mode} = l + \frac{(f_1 - f_0)h}{(2f_1 - f_0 - f_2)}$$

Here, $l = 15$, N/2 = 100, $f_0 = 30$, $f_1 = 80$, $f_2 = 40$

$$\therefore \quad \text{Mode} = 15 + \frac{(80-30) \times 5}{2 \times 80 - 30 - 40} = 15 + \frac{50 \times 5}{160 - 70}$$

$$= 15 + \frac{50 \times 5}{90} = 15 + \frac{25}{9} = 15 + 2.77 = 17.77$$

$\therefore$ Mode = 17.77 hectare.

Q12. The annual rainfall record of a city for 66 days is given in the following table:

Rainfall (cm)	0–10	10–20	20–30	30–40	40–50	50–60
No. of days	22	10	8	15	5	6

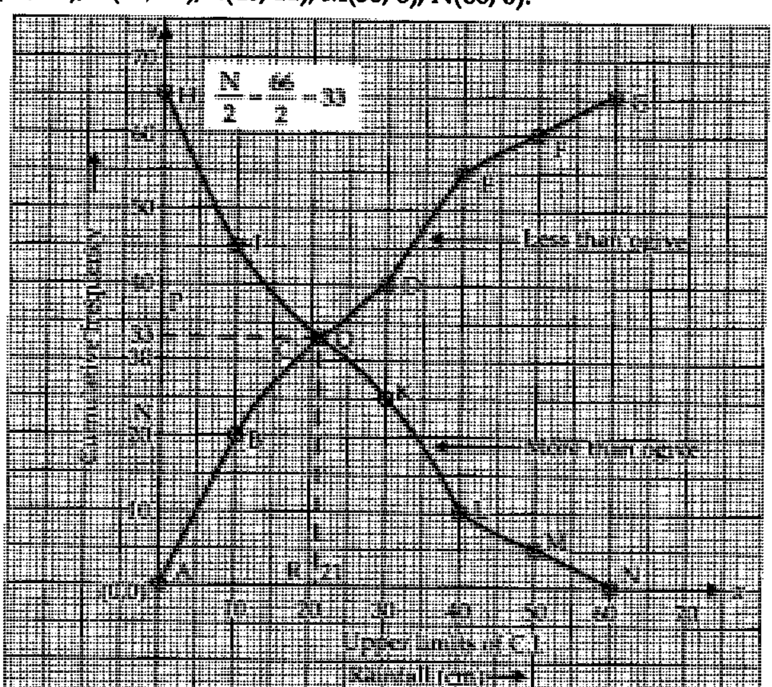
Calculate the median rainfall using ogives (more than type and less than type)

Sol. From the given table, we observe that the lowest limit is 0 so less than 0 rainfall is zero. The highest limit is 60 so more than 60 or 60 rainfall is zero.

Rainfall (cm)	No. of days (*c.f.*)	Rainfall (cm)	No. of days (*c.f.*)
less than 0	0	more than or equal 0	66
less than 10	$22 + 0 = 22$	more than or equal 10	$66 - 22 = 44$
less than 20	$10 + 22 = 32$	more than or equal 20	$44 - 10 = 34$
less than 30	$8 + 32 = 40$	more than or equal 30	$34 - 8 = 26$
less than 40	$15 + 40 = 55$	more than or equal 40	$26 - 15 = 11$
less than 50	$5 + 55 = 60$	more than or equal 50	$11 - 5 = 6$
less than 60	$6 + 60 = 66$	more than or equal 60	$6 - 6 = 0$

Co-ordinates on graph for less than type ogive are A(0, 0), B(10, 22), C(20, 32), D(30, 40), E(40, 55), F(50, 60) and G(60, 66).

Co-ordinates for more than type ogive are H(0, 66), I(10, 44), J(20, 34), K(30, 26), L(40, 11), M(50, 6), N(60, 0).



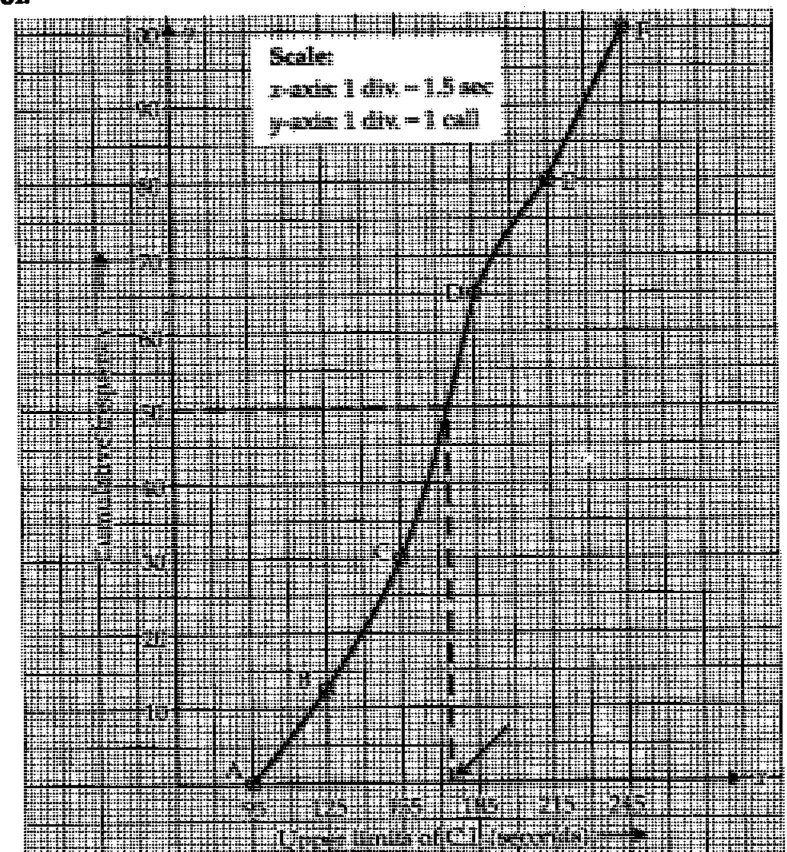
Both more than and less that type ogives intersect at point Q(21, 33). Hence, the median is 21 cm rainfall.

Q13. The following is the frequency distribution of duration for 100 calls made on a mobile phone.

Duration (in seconds)	Number of calls
95–125	14
125–155	22
155–185	28
185–215	21
215–245	15

Calculate the average duration (in sec) of a call and also find the median from the cumulative frequency curve.

Sol.



Duration (in sec) C.I.	No. of calls (f_i)	c.f.	x_i	$d_i = (x_i - a)$	$u_i = \frac{d_i}{h}$	$f_i u_i$	Points for ogive
Less than 95	0	0	0				A(95, 0)
95–125	14	14	110	−60	−2	−28	B(125, 14)
125–155	22	36	140	−30	−1	−22	C(155, 36)
155–185	28	64	170	0	0	0	D(185, 64)
185–215	21	85	200	30	1	21	E(215, 85)
215–245	15	100	230	60	2	30	F(245, 100)
	$\Sigma f_i = 100$					$\Sigma f_i u_i = 1$	

Here, $a = 170$, $h = 30$, $\Sigma f_i u_i = 1$, $\Sigma f_i = N = 100$

(i) Mean $\bar{x} = a + \frac{(\sum f_i u_i)h}{\sum f_i}$

$$= 170 + \frac{1 \times 30}{100} = 170 + 0.3 = 170.3$$

$\therefore$ $\bar{x}$ = 170.3 seconds

Hence, the average duration for a call is 170.3 seconds.

(ii) **Median:** Median class $= \left(\frac{N}{2}\right)$th observation $= \frac{100}{2}$th observation = 50th observation

After plotting the ogive, median can be find out by taking y axis at $\frac{N}{2} = \frac{100}{2} = 50$ calls. Note the call time on x-axis corresponding to 50 calls which is shown by arrows *i.e.*, 170.

Hence, the median time is 170 seconds.

Q14. 50 students enter for a school Javelin throw competition. The distance (in metre) thrown are recorded below.

Distance (m)	0–20	20–40	40–60	60–80	80–100
No. of students	6	11	17	12	4

(i) Construct a cumulative frequency table.

(ii) Draw cumulative frequency curve (less than type) and calculate the median distance thrown by using this curve.

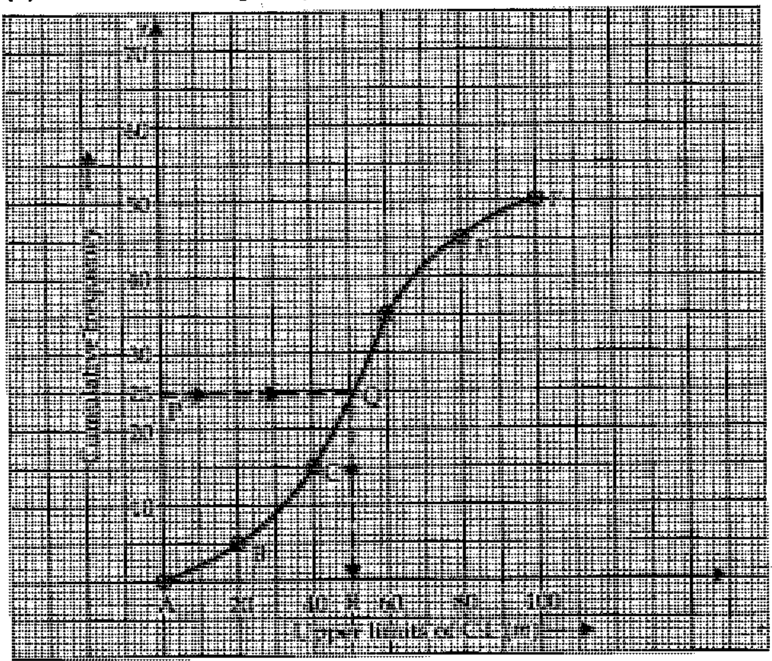
(iii) Calculate the median distance by using the formula for median.

(iv) Are the median distance calculated in (ii) and (iii) same ?

Sol. (i) Cumulative frequency table

Distance (m) C.I.	No. of students (f_i)	*c.f.*	Points of less than ogive
less than 0	0	0	A(0, 0)
0–20	6	6	B(20, 6)
20–40	11	17	C(40, 17)
40–60	17	34	D(60, 34)
60–80	12	46	E(80, 46)
80–100	4	50	F(100, 50)

(*ii*) Cumulative frequency (less than type) curve



To obtain median distance from less than or cumulative frequency ogive, we have to find out the distance of $\frac{50}{2} = 25$ observations from Y-axis and its corresponding distance on X-axis. On *x*-axis R (50 m) is the median distance.

(*iii*) Median by formula

The median class is 25th obs. that lies in 40–60 class

$\therefore \quad l = 40$

$N = 50$
$c.f. = 17$ (Preceding the median class)
$h = 20$
$f = 17$ (median class)

$$\therefore \text{ Median} = l + \frac{\left(\frac{N}{2} - c.f.\right)h}{f}$$

$$= 40 + \frac{\left(\frac{50}{2} - 17\right)20}{17} = 40 + \frac{(25-17)20}{17}$$

$$= 40 + \frac{8 \times 20}{17} = 40 + \frac{160}{17} = 40 + 9.41176 = 49.41176$$

$\therefore$ Median distance = 49.41176 metre

(*iv*) Median distance calculated by formula and graph are almost equal *i.e.*, differ only by 0.588 m. So, we can say that the median distance calculated in part (*ii*) and (*iii*) are same.

❑❑❑